ECON 3123: Macroeconomic Theory I

Tutorial Note 5: Asset Pricing

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Discounted Cash Flow

The key concept is the **time value of money**. In words, it says that one dollar today does not equal one dollar tomorrow. Based on the interest rate in each time period, we can **discount** the future cash flow into **present value**. Why we need to do this? Suppose we have a 2-year bond with 2% coupon and a 3 year bond with 2.1% coupon. If we would like to compare the price of the two bonds, it is not clear which one has a higher price. Therefore, discounting allows us to compare values of assets that have different rate, time to maturity, and other factors influencing cash flow.

Suppose that in year $t+j$, the cash flow is $\$z_{t+j}$. Then the value of the asset with maturity $t+\tau$ at t will be

$$\$V_t = \$z_t + \frac{1}{1+i_t}\$z_{t+1} + \frac{1}{(1+i_t)(1+i_{t+1})}\$z_{t+2} + \cdots .$$

This complicated formula simply says that the present value of an asset is the sum of total **discount cash flow** in the future.

Now we restrict our discussion on constant interest rate i and same payment each period $\$z$.

Annuity and Perpetuity **Annuity** is a contract or financial product that provides a stream of periodic payments over time. **Perpetuity** is a special type of annuity that is *without maturity date*, that is, makes payments forever. The formula of pricing an annuity with maturity $t+\tau$ is

$$\$V_t = \$z \sum_{j=0}^{\tau-1} \frac{1}{(1+i)^j} = \$z \frac{1 - \frac{1}{(1+i)^\tau}}{1 - \frac{1}{1+i}}.$$

To price a perpetuity, we take $\tau \to +\infty$, and we have

$$\$V_t = \frac{\$z}{i}$$

When there is uncertainty, we make everything in expectation. When the payments are real, we discount using real interest rates.

Example 1. *Suppose there is an asset that pays you \$1,000 at the end of each year from 2026 to 2030. The one year nominal interest rate is 3% from 2025 to 2026, while it is expected that the nominal interest rate will be reduced to 2% from the end of 2026 (2026 to 2027 and onwards). What will be a no-arbitrage price of the asset at the end of 2025?*

Bond Pricing and Bond Yield

Given a one-year zero coupon risk-free bond and a two-year zero coupon risk-free bond, no-arbitrage condition implies that

$$\$P_{2,t}(1+i_{1,t}) = \$P^e_{1,t+1},$$

meaning that the value of the 2-year bond after year should equal the value of a 1-year bond in the next year. Then we have the following relation:

$$(1+i_{2,t})^2 = (1+i_{1,t})(1+i^e_{1,t+1}),$$

which can be approximated by

$$i_{2,t} \approx \frac{1}{2}(i_{1,t} + i^e_{1,t+1}).$$

If we consider the default risk, then we need to add risk premium to the two-year rate:

$$i_{2,t} \approx \frac{1}{2}(i_{1,t} + i^e_{1,t+1} + x).$$

Exercise 1. *Derive the formula for the bond yield of a three-year zero-coupon risk-free bond using only one-year zero-coupon risk-free bond.*

Example 2. *True or false: A prediction of recession is consistent with a downward-sloping yield curve.*

What if you are pricing a bond with coupon given the annual yield? You add up two streams of future cash flows: the periodic coupon and the lump-sum face value in the end.

Stock Pricing