ECON 3123: Macroeconomic Theory I

Tutorial Note 5: Asset Pricing

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Discounted Cash Flow

The key concept is the **time value of money**. In words, it says that one dollar today does not equal one dollar tomorrow. Based on the interest rate in each time period, we can **discount** the future cash flow into **present value**. Why we need to do this? Suppose we have a 2-year bond with 2% coupon and a 3 year bond with 2.1% coupon. If we would like to compare the price of the two bonds, it is not clear which one has a higher price. Therefore, discounting allows us to compare values of assets that have different rate, time to maturity, and other factors influencing cash flow.

Suppose that in year $t+j$, the cash flow is $\$z_{t+j}$. Then the value of the asset with maturity $t+\tau$ at t will be

$$\$V_t = \$z_t + \frac{1}{1+i_t}\$z_{t+1} + \frac{1}{(1+i_t)(1+i_{t+1})}\$z_{t+2} + \cdots .$$

This complicated formula simply says that the present value of an asset is the sum of total **discount cash flow** in the future.

Now we restrict our discussion on constant interest rate i and same payment each period $\$z$.

Annuity and Perpetuity **Annuity** is a contract or financial product that provides a stream of periodic payments over time. **Perpetuity** is a special type of annuity that is *without maturity date*, that is, makes payments forever. The formula of pricing an annuity with maturity $t+\tau$ is

$$\$V_t = \$z \sum_{j=0}^{\tau-1} \frac{1}{(1+i)^j} = \$z \frac{1-\frac{1}{(1+i)^\tau}}{1-\frac{1}{1+i}}.$$

To price a perpetuity, we take $\tau \to +\infty$, and we have

$$\$V_t = \frac{\$z}{i}$$

When there is uncertainty, we make everything in expectation. When the payments are real, we discount using real interest rates.

Example 1. *Suppose there is an asset that pays you \$1,000 at the end of each year from 2026 to 2030. The one year nominal interest rate is 3% from 2025 to 2026, while it is expected that the nominal interest rate will be reduced to 2% from the end of 2026 (2026 to 2027 and onwards). What will be a no-arbitrage price of the asset at the end of 2025?*

Bond Pricing and Bond Yield

Given a one-year zero coupon risk-free bond and a two-year zero coupon risk-free bond, no-arbitrage condition implies that

$$\$P_{2,t}(1+i_{1,t}) = \$P^e_{1,t+1},$$

meaning that the value of the 2-year bond after year should equal the value of a 1-year bond in the next year. Then we have the following relation:

$$(1+i_{2,t})^2 = (1+i_{1,t})(1+i^e_{1,t+1}),$$

which can be approximated by

$$i_{2,t} \approx \frac{1}{2}(i_{1,t} + i^e_{1,t+1}).$$

If we consider the default risk, then we need to add risk premium to the two-year rate:

$$i_{2,t} \approx \frac{1}{2}(i_{1,t} + i^e_{1,t+1} + x).$$

Exercise 1. *Derive the formula for the bond yield of a three-year zero-coupon risk-free bond using only one-year zero-coupon risk-free bond.*

Example 2. *True or false: A prediction of recession is consistent with a downward-sloping yield curve.*

What if you are pricing a bond with coupon given the annual yield? You add up two streams of future cash flows: the periodic coupon and the lump-sum face value in the end.

Stock Pricing

Instead of the no-arbitrage argument we had in class, a simpler way is to look at stock pricing from the perspective of discounted cash flow. Since firms pay dividends to the investors infinitely, we simply add up the discounted values of dividends each period:

$$\$Q_t = \$D_t + \sum_{j=1}^{+\infty} \$D_{t+j} \frac{1}{\prod_{k=1}^{j}(1 + r_{t+k} + x)}.$$

Theoretically, as long as there is no shock to the market, the stock price should be constant. However, stock prices always fluctuate in reality. This is because of arbitrage.

Exercise 2. *Chapter 14, Question 6 in Blanchard, Olivier (2021), Macroeconomics, 8th ed., Pearson.*

Example 3. *Chapter 14, Question 7 in Blanchard, Olivier (2021), Macroeconomics, 8th ed., Pearson.*