ECON 3123: Macroeconomic Theory I

# Tutorial Note 7: Labour Market and Phillips Curve

Teaching Assistant: Harlly Zhou

## A Simple Model of the Labour Market

**Price Determination** Consider a production function

$$Y = AN.$$

The **marginal product of labour** (MPL) is $\frac{\partial Y}{\partial N} = A$. Suppose that the cost of hiring an extra worker is $W$. Then the marginal cost of production is $\frac{W}{A}$. Let $m$ be the markup (due to monopolistic power). Then the price level will be

$$P = (1+m)\frac{W}{A}.$$

**Wage Determination** Assume that the nominal wage is

$$W = AP^e F(u, z),$$

where $A$ is the MPL, $P^e$ is the expected price level, and $F$ is a function decreasing in unemployment rate $u$, and increasing in $z$, a variable capturing all other factors.

**Natural Rate of Unemployment** From the price determination equation, we have

$$\frac{W}{P} = \frac{A}{1+m}.$$

From the wage determination equation, **since $P^e = P$ in the medium run**, we have

$$\frac{W}{P} = AF(u, z).$$

In a $\left(u, \frac{W}{P}\right)$ diagram, the pricing curve is a horizontal line, and the wage curve is a downward-sloping curve, as is shown in Figure 1. The equilibrium unemployment rate that is reached by the two curves is called the **natural rate of unemployment**.

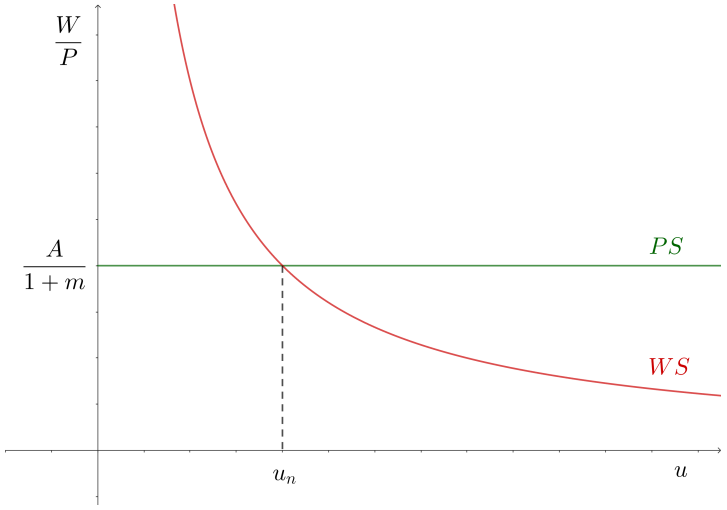


Figure 1: Natural Rate of Unemployment

**Example 1.** *Consider a linear production function*

$$Y = AN.$$

*Let the wage setting equation be*

$$\frac{W}{P^e} = AF(u, z),$$

*where $z$ is the union power and*

$$F(u, z) = 1 - \alpha u_t + \phi z_t.$$

*Suppose that the union can bargain for a real minimum wage at*

$$\frac{W}{P} \geq \underline{w}_t = \omega z_t$$

*(1) When the minimum wage is not hit, find out the equilibrium real wage and write the natural rate of unemployment (not by log approximation) as a function of $z_t$ and parameters.*

(2) *Suppose that the constraint binds. Find the natural rate of unemployment as a function of $z_t$ and parameters.*

(3) *Find the threshold for $z_t$ where the minimum wage constraint binds. Denote it as $\bar{z}$.*

(4) *When the constraint binds, how does the natural rate of unemployment change with union power? Explain the economic intuition.*

**Example 2.** *Consider the following production function*

$$Y = A \log N.$$

*The wage determination equation, in the medium run equilibrium, is*

$$\frac{W}{P} = AF(u, z) \equiv A(1 - \alpha u_t + z).$$

(1) *Let $m$ be the markup. Write the price setting equation, i.e., write $P$ as a function of $N$.*

(2) *Let the population $L$ be normalized to 1 and assume that it is constant. Find a quadratic equation characterizing the natural rate of unemployment. Which solution to the equation you should keep? Why?*

*(3) Suppose that the labour productivity $A$ increases. Using a diagram, explain what happens to the new equilibrium real wage and the new natural rate of unemployment.*

**Exercise 1.** *Chapter 7, Question 5 in Blanchard, Olivier (2021), Macroeconomics, 8th ed., Pearson.*

## Deriving the Phillips Curve

Recall that the labour market equilibrium is the intersection of

$$\begin{aligned} \text{Wage setting:} \quad & \frac{W}{P^e} = AF(u, z) \\ \text{Price setting:} \quad & P = (1+m)\frac{W}{A}. \end{aligned}$$

Combining the two curves, we obtain

$$P = P^e(1+m)F(u, z).$$

Assume that

$$F(u, z) = 1 - \alpha u + z.$$

Then

$$P_t = P_t^e(1+m)(1-\alpha u_t + z).$$

Dividing both sides by $P_{t-1}$, we obtain

$$\log \frac{P_t}{P_{t-1}} = \log \frac{P_t^e}{P_{t-1}} + \log(1+m) + \log(1-\alpha u_t + z).$$

By approximation, we have the **Phillips curve**:

$$\pi_t = \pi_t^e + (m+z) - \alpha u_t.$$

**Original Phillips Curve** Assume that inflation does not persist: $\pi^e = \bar{\pi}$,meaning that past inflation rates are not informative to predict new inflation. Then the Phillips curve becomes

$$\pi_t = (\bar{\pi} + m + z) - \alpha u_t.$$

Denote $\beta := \bar{\pi} + m + z$. Then we have

$$\pi_t = \beta - \alpha u_t.$$

However, there are periods when this original Phillips curve is not supported by the data.

**Accelerationist Phillips Curve** Assume that inflation persists: $\pi^e = \pi_{t-1}$, meaning that people use the inflation from the past period as the indicator of expected inflation. Then the Phillips curve becomes

$$\pi_t = \pi_{t-1} + (m + z) - \alpha u_t.$$

Denote $\gamma := m + z$. Then we have

$$\Delta\pi_t = \gamma - \alpha u_t.$$

The period now fits.

**Inflation Expectations** As in the previous two versions of PC, we have different methods to form expectations for inflation. Combining the two methods, we can assume that

$$\pi_t = (1 - \theta)\bar{\pi} + \theta\pi_{t-1}.$$

**Example 3.** *Continue with Example 1.*

*(1) Derive the Phillips curve.*

*(2) Write the accelerationist version of the Phillips curve. Compare it with*

$$\Delta\pi_t = \gamma - \alpha u_t.$$

*What is the difference? Where does it come from? How do they differ economically?*

**Exercise 2.** *Chapter 8, Question 5 in Blanchard, Olivier (2021), Macroeconomics, 8th ed., Pearson.*

## Natural Rate of Unemployment Revisited

Recall the the natural rate of unemployment is the rate at which $P = P^e$, *i.e.*, the labour market is at medium-run equilibrium. Since $P = P^e$, we also have $\pi = \pi^e$ in medium-run equilibrium. Then the PC becomes

$$0 = (m + z) - \alpha u_n,$$

which yields

$$u_n = \frac{m + z}{\alpha}.$$

Then the PC (not at equilibrium) can be rewritten as

$$\pi_t - \pi_t^e = -\alpha(u_t - u_n).$$

**Wage Indexation** **Wage indexation** is a provision that automatically increases wages in line with inflation. Suppose that there is a $\lambda$ proportion of indexed contracts. Then we have two types of wage settings:

$$\text{Indexed Wage: } W_1 = AP_tF(u_t, z)$$
$$\text{Normal Wage: } W_2 = AP_t^eF(u_t, z).$$

Then the price setting becomes

$$P_t = (1+m)\frac{\lambda W_1 + (1-\lambda)W_2}{A} = [\lambda P_t + (1-\lambda)P_t^e]F(u,z)(1+m),$$

which yields the following PC curve:

$$\pi_t = [\lambda \pi_t + (1-\lambda)\pi_t^e] - \alpha(u_t - u_n).$$

Rearranging the terms, we get

$$\pi_t - \pi_t^e = -\frac{\alpha}{1-\lambda}(u_t - u_n).$$

**Example 4.** *Continue with example 1 and 2.*

*(1) Find the natural rate of unemployment using the Phillips curve. Check your answer with Example 1 part (2). Do they match? Rewrite the Phillips curve using the natural rate of unemployment.*

*(2) Suppose that now half of the contracts are indexed. Write the Phillips curve equation.*

*(3) What is the effect of wage indexation on the relation between inflation and unemployment?*

**Exercise 3.** *Chapter 8, Question 6 in Blanchard, Olivier (2021), Macroeconomics, 8th ed., Pearson.*