THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

ECON 3123 Final Exam

Date: Dec 10, 2025

Time allowed: 120 minutes

Not to be taken away.

Instructions:

- Answer ALL the questions. Write your answers on the answer book. Anything written on the question book will NOT be graded.
- Use one separate page for each question, and separate subquestions within one question by 1 to 2 blank lines.
- The exam has 5 questions and 1 bonus question. The total score of the exam will not exceed 100, meaning that if you get 95 points in the previous parts and 10 points in the bonus question, you will not get 105 points but only 100 points.
- Make sure that all your handweritngs are legible. Anything that cannot be understood by the grader will not be graded.
- Make sure that your answer is clear and concise. Please only write within the area provided for the question. No answer outside the area will be graded.
- Please submit BOTH the question book and the answer book after the exam.

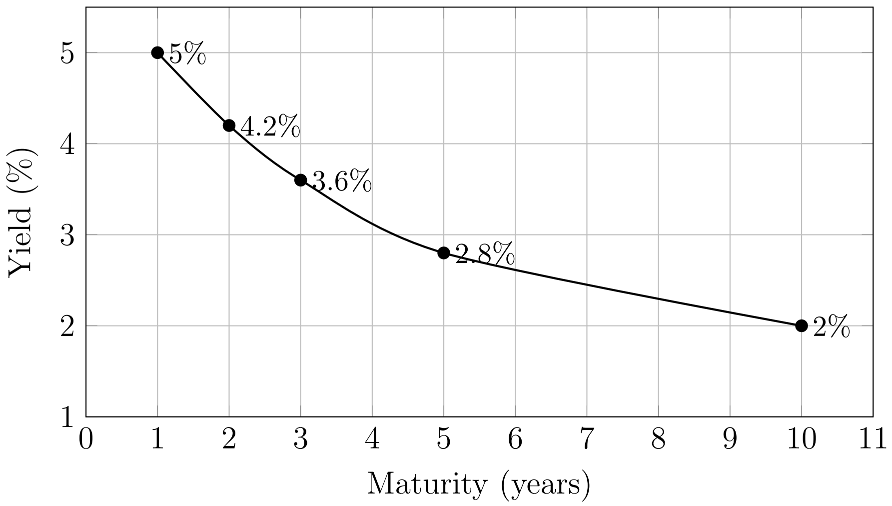
DO NOT OPEN UNTIL INSTRUCTED!

Name:

Student ID:

Question 1: Yield curve and bond pricing (12 points)

Here is the yield curve of a zero-coupon bond with face value 100.



Denote the expected 1-year yield of a bond issued in year t as i_t^e. For example, the expected 1-year yield of a bond issued in year 2 and matured in year 3 is denoted as i_2^e. Suppose that the risk premium is constantly 1%.

(1) (3 points) Using the no-arbitrage argument, derive i_2^e by considering arbitrage opportunities between 2-year bond and 3-year bond. (Hint: Consider log approximation.)

(2) (6 points) Suppose that $\bar{i} = i_3^e = i_4^e$. Using the no-arbitrage argument, derive $\bar{i}$ by considering arbitrage opportunities between 3-year bond and 5-year bond. (Hint: Consider log approximation.)

(3) (3 points) Do you predict that this economy will experience a boom or a recession from the yield curve? Explain your answer.

Question 2: Productivity shock in the medium run (30 points)

Suppose that the production function is

$$Y = \mathcal{A}\sqrt{N},$$

where Y is output, $\mathcal{A}$ is productivity, and N is labour input.

(1) (4 points) Derive the price setting equation. (Hint: The marginal product of labour is $\frac{\partial Y}{\partial N} = \frac{\mathcal{A}}{2\sqrt{N}}$.)

Normalize the total labour force to be $L = 1$. Suppose that the wage setting equation is

$$\frac{W}{P^e} = \mathcal{A}(1 - \alpha u + z),$$

where W is the nominal wage, P^e is the expected price level, $\alpha \in (0, 1)$ is a constant, u is the unemployment rate, and z is the variable capturing all other factors.

(2) (6 points) Suppose there is a positive productivity shock, that is, $\mathcal{A}$ increases. Use a **labour market diagram** (the unemployment-real wage diagram) to illstrate how the equilibrium real wage changes.

(3) (4 points) Derive the Phillips curve.

(4) (4 points) Derive the PC relation.

(5) (6 points) Assume that after the positive productivity shock, the new short run equilibrium output is higher than the new medium run equilibrium. Use an $IS - LM - PC$ diagram to illustrate the short-run effect on inflation π, investment I, and real interest rate r.

(6) (6 points) To stabilize the output, what should the central bank do? Use the same diagram in the previous question to compare the resulting inflation π, investment I, and real interest rate r of your monetary policy with the initial equilibrium (before the shock).

Question 3: Fixed exchange rate regime (25 points)

Suppose that the economy is summarized by the following equations:

- IS curve: $Y = 20 - 2i$.
- Real interest rate: $\epsilon = E\frac{P}{P^*}$.
- Uncovered interest parity: $1 + i_t = (1 + i_t^*)\dfrac{E}{\bar{E}^e}$.

Assume that in the short run prices are given so that $\epsilon = E$. The economy is operating under a fixed exchange rate regime at $E = 0.5$. The initial domestic interest rate is 1%.

(1) (4 points) In the $IS - LM - UIP$ diagram provided in the answer book, complete the UIP diagram and label the curves. Denote the initial equilibrium point by A, and label all the curves with subscript A.

Suppose that the foreign economy has a negative shock so that its central bank decreases its nominal interest rate to $i_B^* = 0.5\%$ to keep the output Y^* unchanged.

(2) (6 points) In the same diagram, show how the domestic central bank should respond in order to keep the exchange rate fixed. Denote the new equilibrium by point B. If you shift any curve, label the new curve with subscript B.

Due to the decrease in i^*, the international investors expect the domestic economy is going to revalue its currency. On average, the investors have an expectation of $\bar{E}^e = 0.8$.

(3) (4 points) Calculate the domestic nominal interest rate that the central bank should target to maintain the fixed exchange rate regime. Can this nominal interest rate be targeted?

(4) (7 points) Can the central bank still maintain fixed exchange rate regime? Draw in the same diagram to explain your answer. If you shift any curve, label the new curve with subscript C.

(5) (4 points) Suppose now the domestic central bank gives up the absolute fixed exchange rate regime but an interval of exchange rates. Find the largest lower bound for this interval.

Question 4: Harrod-Domar Model (21 points)

Suppose that the production function is

$$Y = AK^{\beta},$$

where $\beta > 0$.

Let s denote the saving rate and δ denote the depreciation rate. The law of motion of capital is

$$K_{t+1} = sY_t + (1-\delta)K_t.$$

(1) (4 points) Derive the closed-form solution for the steady-state aggregate capital K_{ss}.

(2) (5 points) Find the golden-rule level of saving rate s_G when $\beta = 0.5$.

(3) (4 points) Return to the general case when $\beta > 0$ is arbitrary. Write the growth rate of the economy g_t as a function of K_t.

(4) (8 points) Discuss the effects of different values of β on g_t in the diagram. Denote the curves, axes and steady-state points clearly. (Hint: You need to discuss three cases.)

Question 5: Speculation in foreign exchange market (12 points)

In class, we learnt the uncovered interest parity. However, it may not hold in reality because there are always fluctuations in the foreign exchagne rate that you cannot predict.

In order to hedge the risks, the market invented a contract called **forward**. The idea of the forward contract is as follows. Suppose you hold US dollar (USD) and would like to buy some British pound (GBP) in 1 year. By signing a forward contract today, you will be able to buy the GBP at a specified exchange rate at the maturity date, in this case, 1 year later. Suppose that the forward contract specifies an exchange rate of 1.5 USD/GBP. Then, 1 year later, even the actual exchange rate is ridiculously high at 15 USD/GBP, you can still make the exchange at 1.5 USD/GBP.

From the example, you may already realize that if the forward rate is not deliberately set, there will be opportunities for arbitrage. This question asks you to implement the no-arbitrage condition to calculate the forward rate, and to investigate the opportunities for speculation.

Suppose that the 1-year zero-coupon risk-free US bond has 2% return, and the 1-year zero-coupon risk-free British bond has 1% return. The current exchange rate is 1.3 USD/GBP.

(1) (4 points) Using the no-arbitrage condition, derive the forward exchange rate.

(2) (8 points) Suppose that the forward exchange rate is what you have calculated in part (1), but you are really confident that the exchange rate will turn out to be 1.25 USD/GBP. If you are allowed to borrow 1,300,000 USD or, equivalently 1,000,000 GBP now, which one would you choose to make a profit via speculation? What is your strategy? How much will be your profit in terms of the currency you initially borrowed?

Bonus Question: Micro-foundation for Harrod-Domar Model (10 points)

In question 4, we give a form of production function $Y = AK^{\beta}$. This question asks you to derive this aggregate production function from aggregating individual firms.

Suppose that there are J firms, each indexed with $j = 1, 2, \cdots, J$. Each firm j produces output y_j according to the following production function

$$y_j = \bar{A} k_j^{\alpha} N_j^{1-\alpha},$$

where $\bar{A}$ is the constant productivity, k_j is the capital input and N_j is the labour input used by firm j, and $\alpha \in (0, 1)$.

The aggregate productivity depends upon the total amount of capital that has been accumulated by all firms:

$$\bar{A} = A_0 \left(\sum_{j=1}^{J} k_j \right)^{\eta},$$

where $\eta \in (0, 1)$.

Normalize $N_j = 1$ for all j. Let K be the aggregate capital such that $K = \sum_{j=1}^{J} k_j$ and Y be the aggregate output such that $Y = \sum_{j=1}^{J} y_j$. Assume that $k_j = \frac{K}{J}$.

Question: Show that the aggregate output Y can be expressed as

$$Y = A_0 J^{1-\alpha} K^{\alpha+\eta}.$$

Set $A = A_0 J^{1-\alpha}$ and $\beta = \alpha + \eta$. Now, we get the AK^{β} form as in question 4.

******** END OF THE EXAM ********