Compressive Sensing – A 25 Minute Tour

Emmanuel Candès

First EU-US Frontiers of Engineering Symposium, Cambridge, September 2010

Dedication

Dennis Healy, 1957 – 2009

Compressive sensing today

Enormous field spanning

- mathematics
- applied mathematics
- computer science
- information theory
- signal processing
- circuit design
- optical engineering
- biomedical imaging
- ...

Many contributors; e. g. R. Baraniuk

`http://nuit-blanche.blogspot.com/`

`http://dsp.rice.edu/cs`

Our focus: short and biased overview

A contemporary paradox

Raw: 15MB

JPEG: 150KB

- Massive data acquisition
- Most of the data is redundant and can be thrown away
- Seems enormously wasteful

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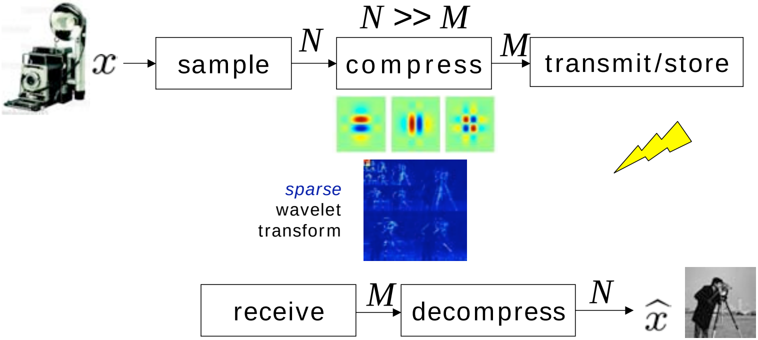
> *One can regard the possibility of digital compression as a failure of sensor design. If it is possible to compress measured data, one might argue that too many measurements were taken.*
>
> David Brady

Going against a long established tradition?

- Acquire/Sample (A-to-D converter, digital camera)
- Compress (signal dependent, nonlinear)



Fundamental question

Can we directly acquire just the useful part of the signal?

What Is Compressive Sensing?

In a nutshell...

- Can obtain super-resolved signals from just a few sensors
- Sensing is *nonadaptive*: no effort to understand the signal
- Simple acquisition process followed by numerical optimization

First papers

- Candès, Romberg and Tao, 2006
- Candès and Tao, 2006
- Donoho, 2006

By now, very rich mathematical theory

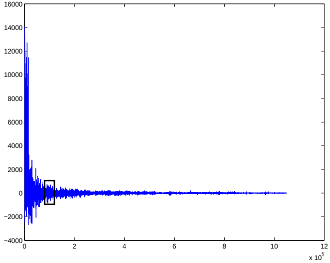
Sparsity: wavelets and images

1 megapixel image

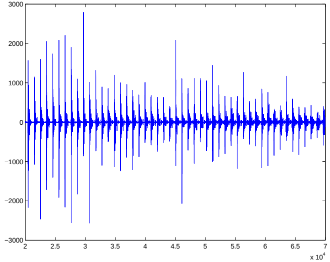
Sparsity: wavelets and images

1 megapixel image

wavelet coefficients



$\Downarrow$



zoom in

Implication of sparsity: image "compression"

1. Compute 1,000,000 wavelet coefficients of mega-pixel image
2. Set to zero all but the 25,000 largest coefficients
3. Invert the wavelet transform

original image

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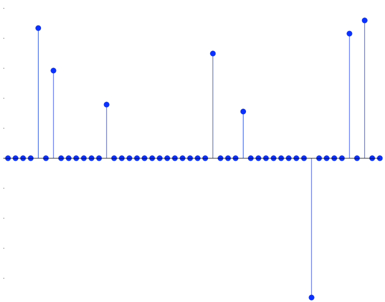
original image

after zeroing out smallest coefficients

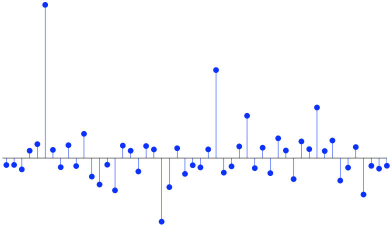
This principle underlies modern lossy coders (sound, still-picture, video)

Idealized sampling

- x: signal coefficients in our convenient representation
- collect information by measuring largest components of x



sparse x



nearly sparse x

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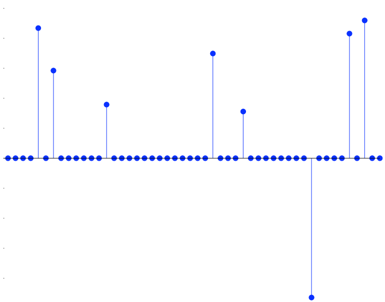
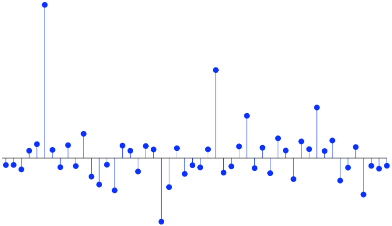


sparse x



nearly sparse x

What if these positions are not known in advance?

- what should we measure?
- how should we reconstruct?

Incoherent/random sensing

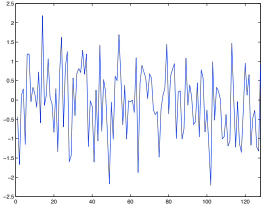
$$y = \langle a_k, x \rangle, \quad k = 1, \ldots, m$$

- Want sensing waveforms as spread out/"incoherent" as possible
- Span of $\{a_k\}$ should be as random as possible (general orientation)

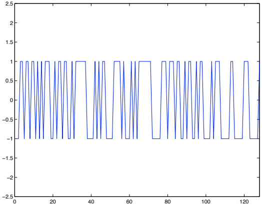
$$a_k \overset{\text{i.i.d.}}{\sim} F$$

$\mathbb{E}\, a_k a_k^* = I$ and a_k spread out

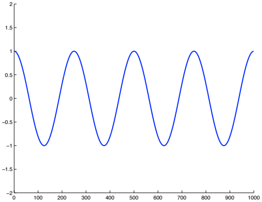
- a_k i.i.d. $\mathcal{N}(0,1)$ (white noise)
- a_k i.i.d. ± 1
- $a_k = \exp(i 2\pi \omega_k t)$ with i.i.d. frequencies ω_k
- ...



random waveform $N(0,1)$

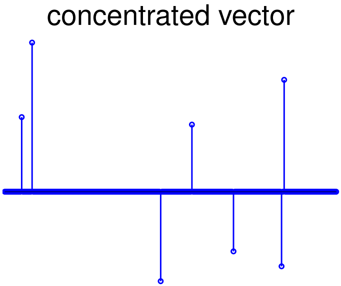


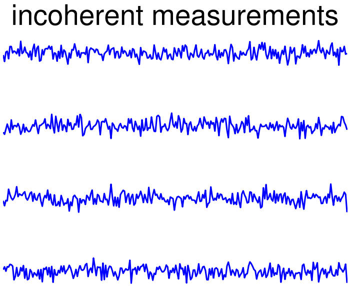
random waveform ± 1



random sinusoid

Incoherence





- Signal is local, measurements are global
- Each measurement picks up a little information about each component

Example of foundational result

Classical viewpoint

- Measure everything (all the pixels, all the coefficients)
- Keep d largest coefficients: distortion is $\|x - x_d\|$

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Compressed sensing viewpoint

- Take m random measurements: $y_k = \langle x, a_k \rangle$
- Reconstruct by *linear programming*: ($\|x\|_{\ell_1} = \sum_i |x_i|$)

$$x^\star = \text{arg min } \|\tilde{x}\|_{\ell_1} \quad \text{subject to} \quad y_k = \langle \tilde{x}, a_k \rangle, \; k = 1, \ldots, m$$

Among all the objects consistent with data, pick min ℓ_1

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Same performance with about $m = d \log n/d$ (sketch)

$$\|x^\star - x\|_{\ell_2} \leq \|x - x_d\|_{\ell_2}$$

Example

- Take 96K incoherent measurements of "compressed" image
- Compressed image is perfectly sparse (25K nonzero wavelet coeffs)
- Solve ℓ_1

original (25k wavelets)

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perfect recovery

What is compressive sensing?

Possibility of compressed data acquisition protocols which directly acquire just the important information

- Incoherent/random measurements $\longrightarrow$ compressed description
- Simultaneous signal acquisition and compression!

All we need is to decompress...

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Three surprises

- Sensing is ultra efficient and *nonadaptive*
- Recovery is possible by tractable optimization
- Sensing/recovery is robust to noise (and other imperfections)

Applications and Opportunities

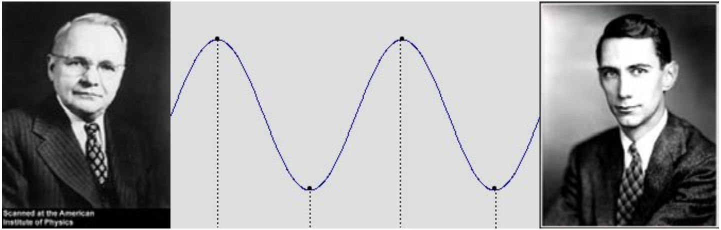
Three potentially impacted areas

- Analog-to-digital conversion
- Optical systems
- Magnetic Resonance Imaging

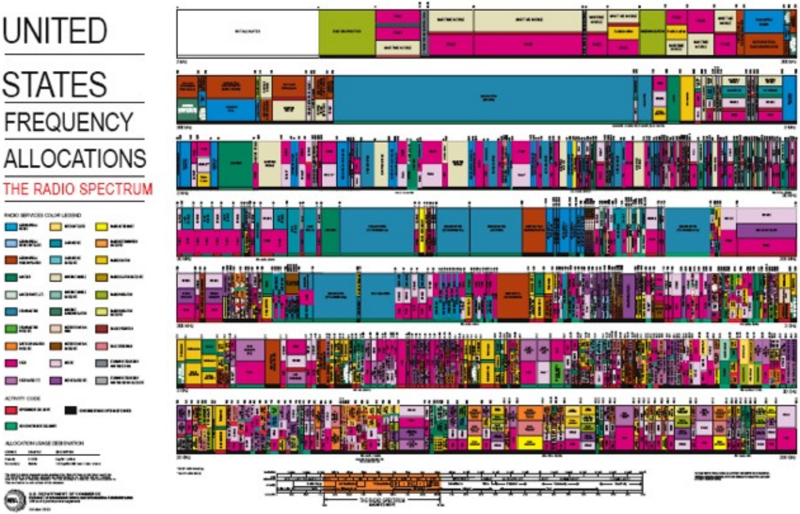
Time or space sampling

- Analog-to-digital converters, receivers, ...
- Space: cameras, medical imaging devices, ...

Nyquist/Shannon foundation: must sample at twice the highest frequency



Sampling of ultra wideband radio frequency signals



Hardware brick wall

- Signals are wider and wider band
- "Moore's law:" factor 2 improvement every 6 to 8 years

Extremely fast/high-resolution samplers are decades away

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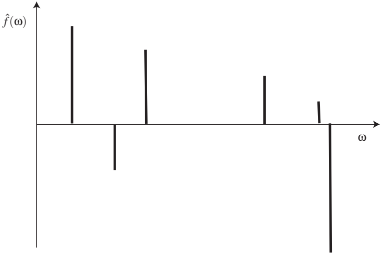
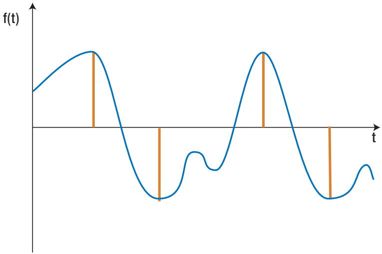
Way out? Analog-to-information (DARPA)

What have we learned?

If 'information bandwidth' less than total bandwidth, then should be able to

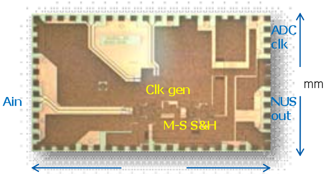
- sample below Nyquist without information loss
- recover missing samples by convex optimization



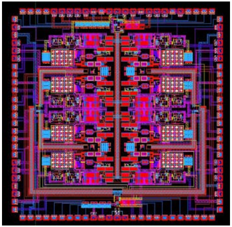


New analog-to-digital converters

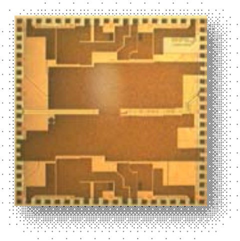
Joint with Caltech and Northrop Grumman



NUS (InP)



RMPI (CMOS)



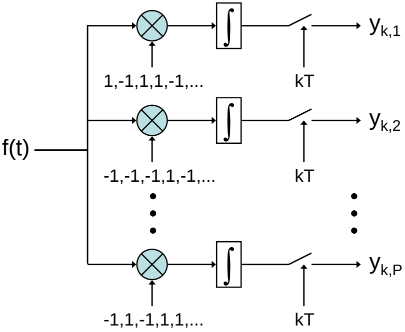
RMPI (InP)

Manufactured three chips

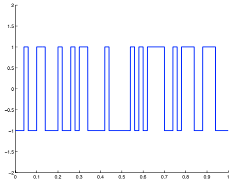
- Nonuniform sampler (InP)
- Random modulator pre-integration
  - 4 channels (InP)
  - 8 channels (CMOS)

Random modulator pre-integrator (RMPI)

Joint with Becker, Emami and Yoo (Caltech)



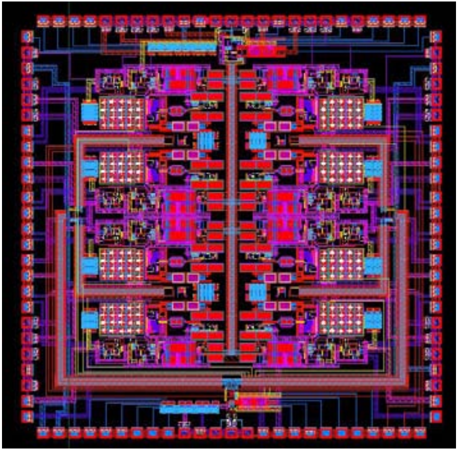
RMPI Basic architecture



random modulation

- Nyquist rate 5GHz
- 8 channels at 50MHz $\rightarrow$ 400MHz
- $12.5\times$ undersampled

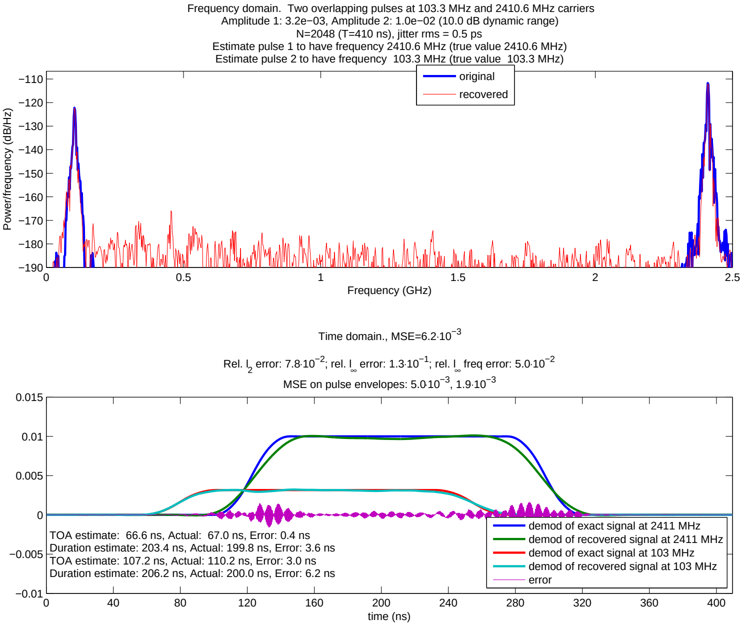
RMPI chip v.2



Process:
IBM 90nm CMOS9SF 06_02_00_LB

- Power Consumption: 830mW
- Bandwidth: 2.5GHz
- Dynamic Range: 50dB
- Sampling rate: 400 MS/s
- Die Area: $2\text{mm} \times 2\text{mm}$

Pulse recovery from full system simulations



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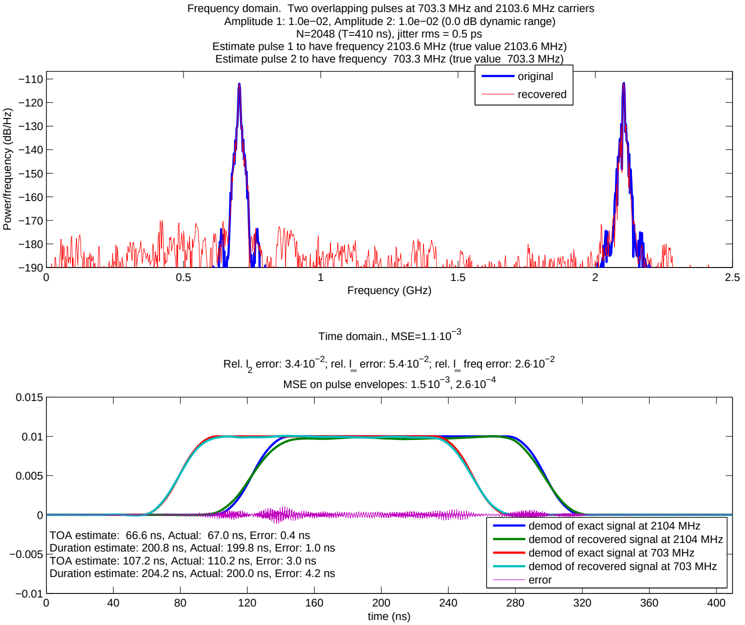
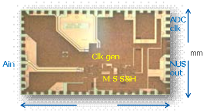
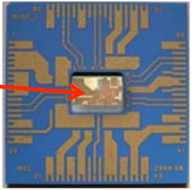


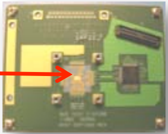
Real testing: NGC-Caltech A-to-I Receiver Test

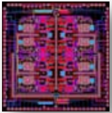
(Die at similar scale)

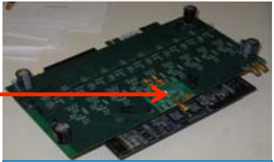




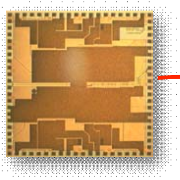
LTCC Pallet 0.7" x 0.7" & NUS Die 4mm x 2.6 mm

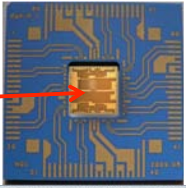




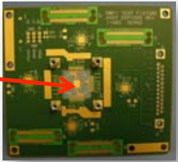


Caltech CMOS RMPI test hardware
(RMPI, ADC, and FPGA/DAQ boards)





LTCC Pallet 0.7" x 0.7" & RMPI die 4 x 4.4 mm



Real results

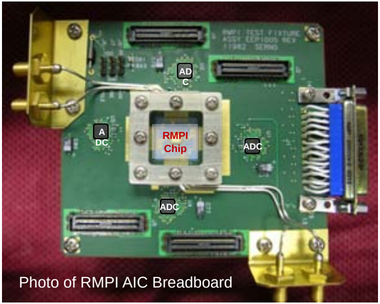


Photo of RMPI AIC Breadboard

RMPI (InP) behaves as simulated

- 2.5GHz of bandwidth
- 50-60dB of dynamic range
- $\sim$ 3W of power consumption

Other ADC efforts

- Eldar et al. (Technion)
- Fudge et al. (L3 & Wisconsin)
- Baraniuk et al. (Rice)

Three potentially impacted areas

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- Optical systems
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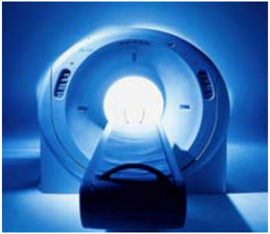
What do we measure?

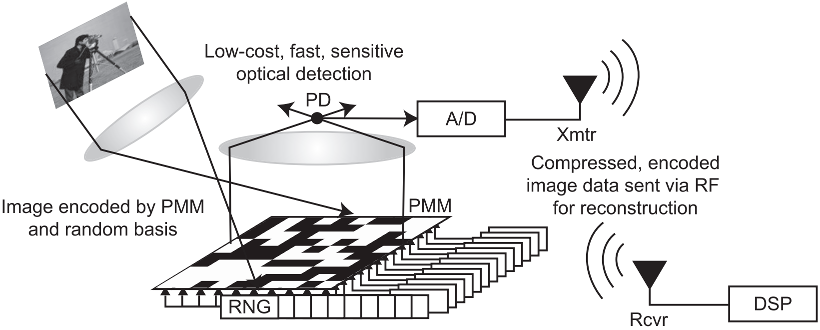
- *Direct sampling:* analog/digital photography, mid 19th century
- *Indirect sampling:* acquisition in a transformed domain, second half of 20th century; e.g. CT, MRI
- *Compressive sampling:* acquisition in an incoherent domain
  - Design incoherent analog sensors rather than usual pixels
  - Pay-off: need far fewer sensors

The first photograph?



CT scanner

One pixel camera



Richard Baraniuk, Kevin Kelly, Yehia Massoud, Don Johnson
Rice University, dsp.rice.edu/CS
MIT Tech review: top 10 emerging technologies for 2007

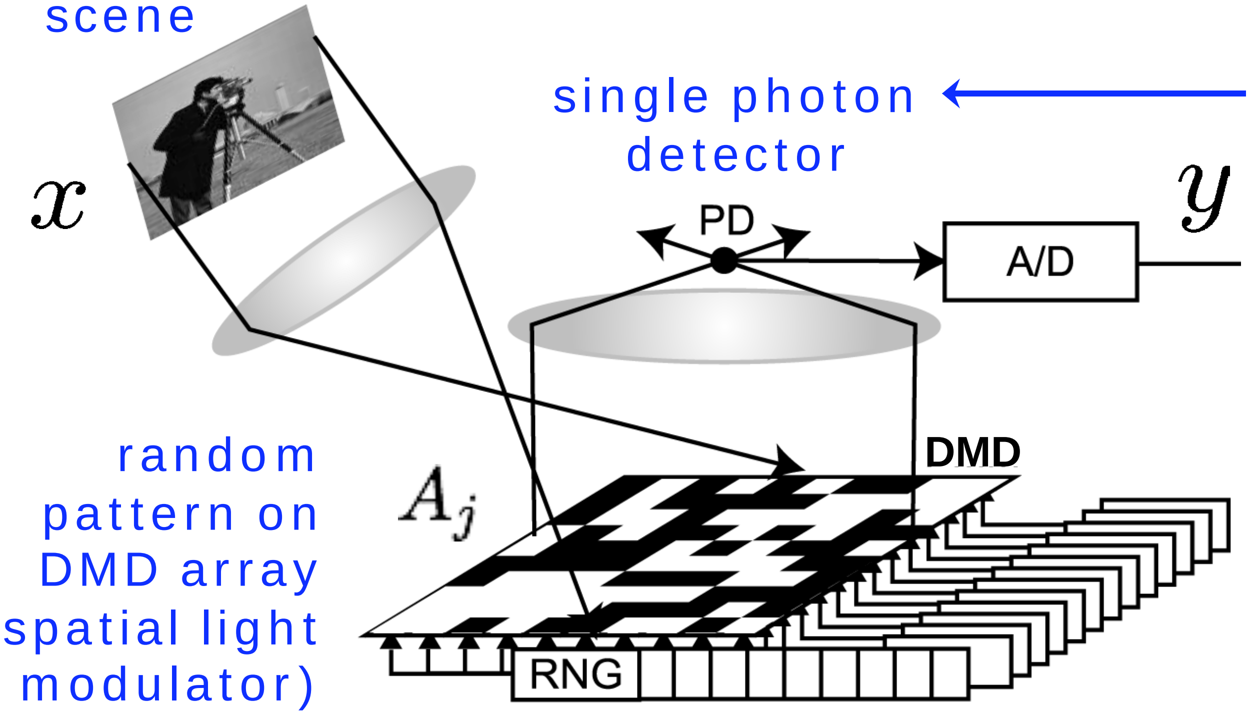
Other works: Brady et al., Freeman et al., Wagner et al., Coifman et al.

“Single-Pixel” CS Camera

Kevin Kelly
Richard Baraniuk
Rice University

scene

single photon detector

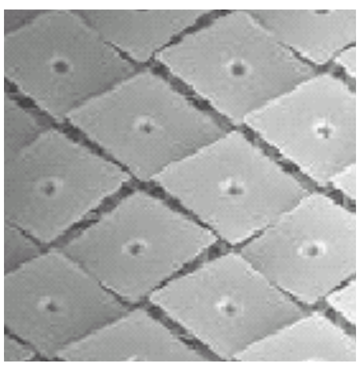


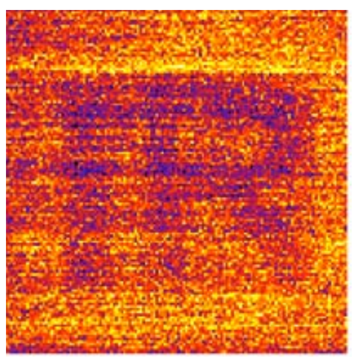
random pattern on DMD array (spatial light modulator)

can be exotic

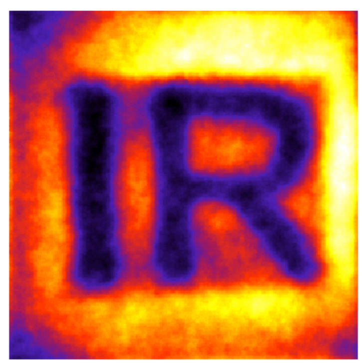
- IR, UV, THz, PMT, spectrometer, …

color target

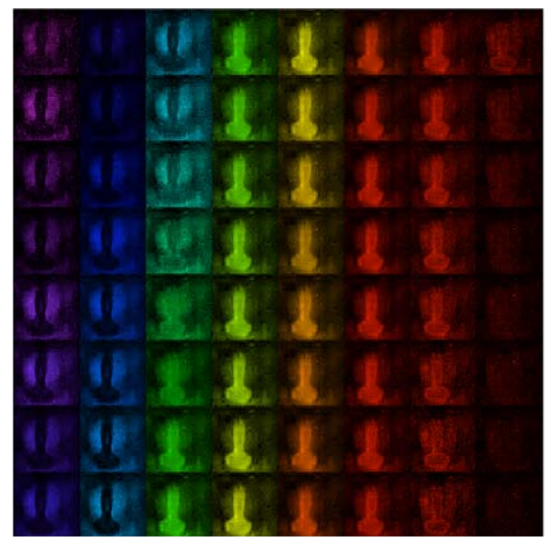




raster scan IR



CS IR



hyperspectral data cube

Three potentially impacted areas

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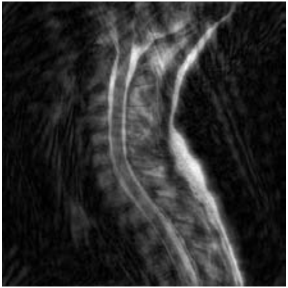
Fast Magnetic Resonance Imaging

Goal: sample less to speed up MR imaging process

Fully sampled

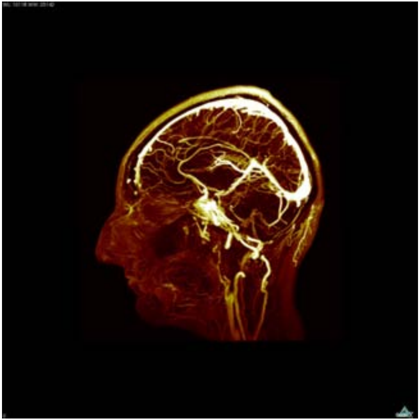


6 × undersampled classical

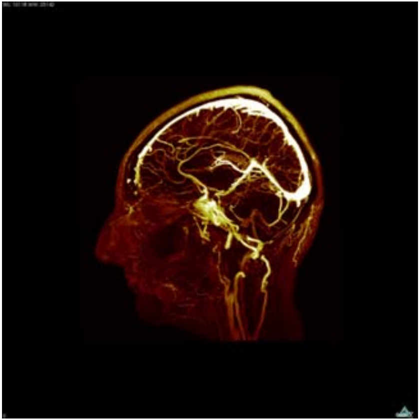
6 × undersampled CS

Trzasko, Manduca, Borisch (Mayo Clinic)

MR angiography

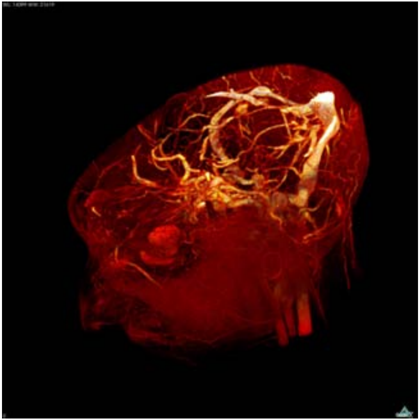


Fully sampled

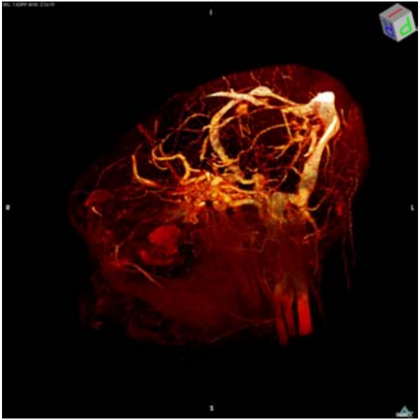


6 $\times$ undersampled

Trzasko, Manduca, Borisch



Fully sampled



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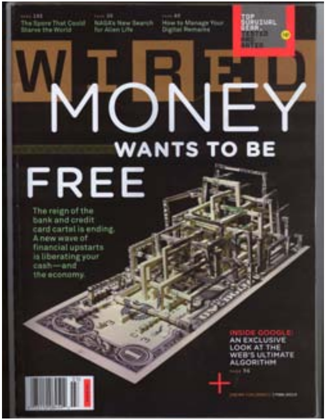
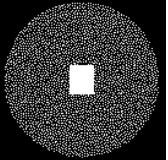
Trzasko, Manduca, Borisch

Compressive sensing in the news



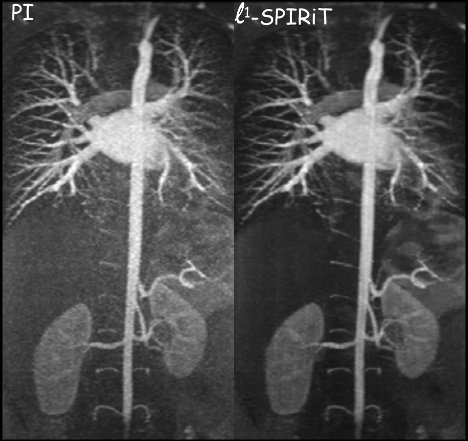
Wired, March 2010

ℓ^1-SPIRiT, 1st Pass T1 3D SPGR

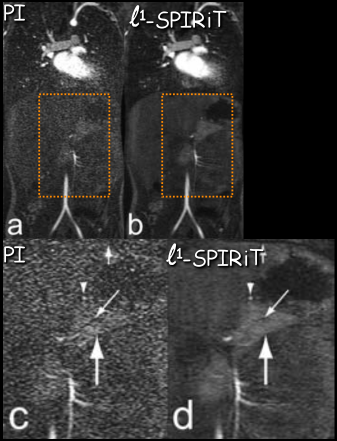


Breath-hold post-gadolinium MRA in a 9 year old male with hypertension using 4X acceleration at 1.2 mm3 resolution. Left images (a, c) are with ARC and right (b, d) are with L1-SPIRIT compressed sensing. Note improved delineation of pancreas (big arrow), pancreatic duct (middle arrow), and diaphragm (small arrow) with ℓ^1-SPIRiT. Left gastric artery (arrowhead) emerges from the noise.

Maximum Intensity Projection (MIP)



Slices



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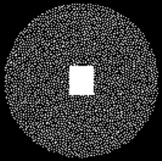
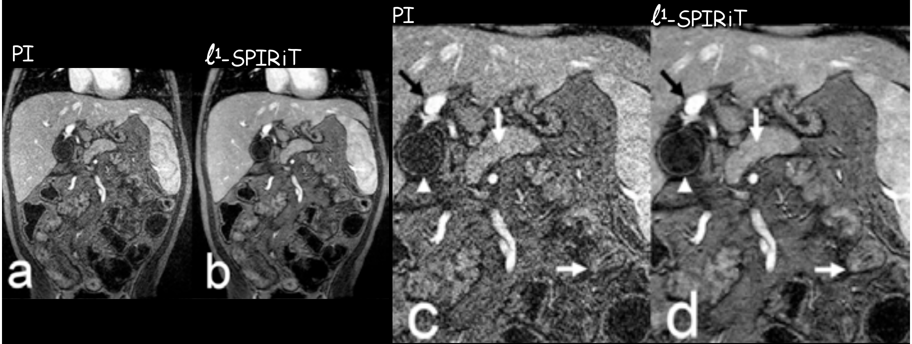
ℓ^1-SPIRiT, T1 3D SPGR



Submillimeter near-isotropic resolution MRI in an 8-year-old male. Post-contrast T1 imaging with an acceleration of 4. Standard (a, c) and compressed sensing reconstruction (b, d) show improved delineation of the pancreatic duct (vertical arrow), bowel (horizontal arrow), and gallbladder wall (arrowhead) with L1-SPIRiT reconstruction, and equivalent definition of the portal vein (black arrow)



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Summary

- Ultra efficient acquisition protocol:

  automatically translates analog data into already compressed digital form
- Change the way engineers think about data acquisition
- Already many applications
- More applications to come