
Summer Internship Report

June 2023 - August 2023

Harmandeep Singh

Thapar University
hsingh1_be21@thapar.edu

Sumit Singh

Indian Institute of Technology, Delhi
me1200978@iitd.ac.in

Under the guidance of:

Dr. Krishna Kant Agrawal

Department of Mechanical Engineering
IIT Delhi

Contents

1	Abstract	ii
2	Section I	iii
2.1	Introduction	iii
2.2	Types of nozzles	iv
2.3	Designing the parabolic nozzle	v
2.3.1	Formulation	vi
3	Section II	viii
3.1	Introduction	viii
3.2	Geometry	viii
3.3	Meshing	ix
3.4	Results	x

1 Abstract

Section 1 describes a short study and design method to design Rao's Parabolic nozzle without delving into the actual numerical methods used by G.V.R Rao [Rao58]. The study was not utilized in the final CFD simulation of the Rocket combustion chamber which is discussed in detail in Section 2 but served as an important cornerstone in the study of Rockets.

A brief discussion on the ideal nozzle geometry(parabolic) is discussed by referring to elementary equations in the Introduction(2.1) section along with various types of Nozzles prevalently used in rockets in the next section. The Formulation section(2.3.1) demonstrates the use of equations to find the expansion area ratio against the Initial and Final parabola angle of the nozzle.

Section 2 consists of the CFD steps and results which were performed on a model Rocket geometry. The geometry was sliced into a 60°portion in order to minimize computational cost and use the maximum number of cells that were available in the Ansys Academic version to obtain a finer mesh.

2 Section I

2.1 Introduction

A rocket engine nozzle is used to expand and accelerate exhaust products to high velocities. They are usually of De Laval type and comprise of a converging and diverging section separated by the throat.

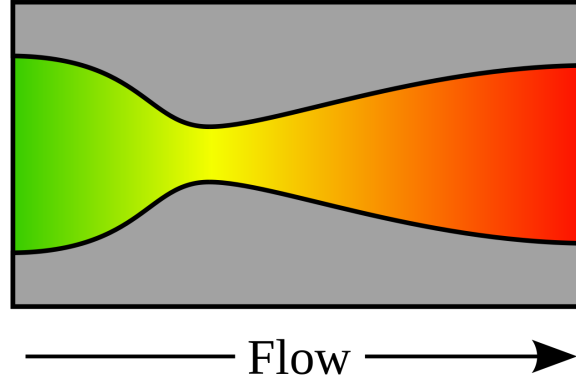


Figure 1: A de Laval nozzle, showing approximate flow velocity increasing from green to red in the direction of flow

Since a nozzle is assumed to be an adiabatic device, from the principle of conservation of energy the equation

$$h_1 - h_2 = \frac{1}{2}(v_2^2 - v_1^2) = C_p(T_1 - T_2) \quad (1)$$

States that the decrease in enthalpy equates to an increase in the velocity of the gas but a decrease in its temperature. i.e., the Higher the temperature of the gases in the combustion chamber, the higher the thrust.

Furthermore, from equation

$$\frac{A}{A^*} = \frac{1}{M} \left(\frac{1 + \frac{k-1}{2} M^2}{1 + \frac{k-1}{2}} \right)^{\frac{k+1}{2(k-1)}} \quad (2)$$

A =Nozzle Area cross-section

A^* =Critical condition when $M=1$

$k = \frac{C_p}{C_v}$ =ratio of specific heats

It can be observed that as the Mach number increases, the Area ratio first decreases till $M=1$ and then increases. The plot clearly shows that a converging-diverging passage with a section of the minimum area is required to accelerate the flow from subsonic to supersonic speed.

The critical point where the flow is at sonic velocity ($M=1$ at $A/A^*=1$) is seen to exist at the throat of the nozzle. This proves that the ideal wall geometry of a nozzle will not be a fixed angle, but rather a function that varies along the axis

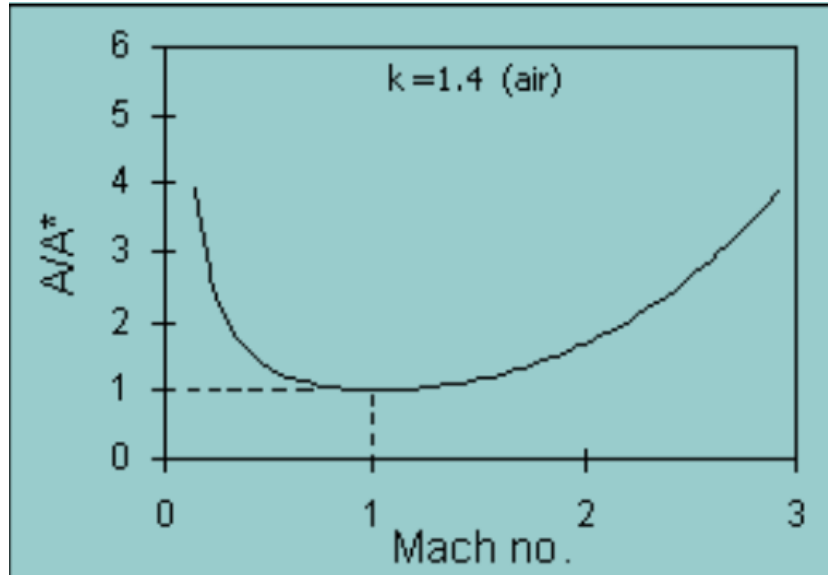


Figure 2: Plot of A/A^* versus Mach number from [Richard Nakka's Rocketry website](#)

2.2 Types of nozzles

The nozzles commonly used in rockets are as follows:

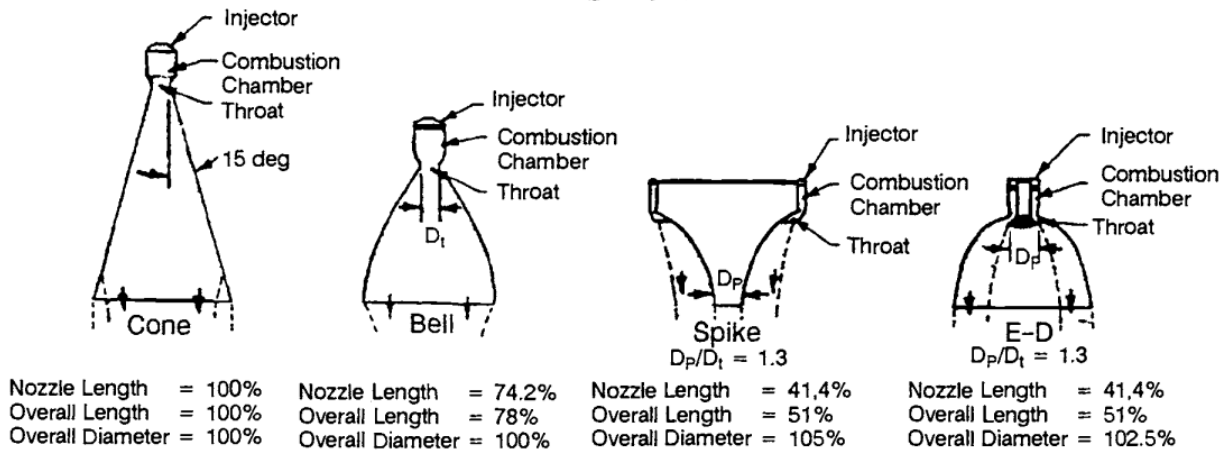


Figure 3: Types of rocket nozzles and their geometric specification ([Huz92])

The selection of an optimum nozzle shape for a given expansion area ratio is generally influenced by the following design considerations and goals:

- ° Uniform, parallel, axial gas flow at the nozzle exit for maximum momentum vector.
- ° Minimum separation and turbulence losses within the nozzle.
- ° Shortest possible nozzle length for minimum space & weight,
- ° Ease of manufacturing.

An equivalent 15-degree half-angle conical nozzle is commonly used as a standard to specify bell nozzles. For instance, the length of an 80% bell nozzle (distance between throat and exit plane) is 80% of that of a 15-degree half-angle conical nozzle having the same throat area, radius below the throat, and area expansion ratio.

Conical Nozzle:

The conical nozzle was used often in early rocket applications because of its simplicity. The cone's walls are at a constant angle. A small angle produces greater thrust, because it maximizes the axial component of exit velocity and produces a high specific impulse. Unfortunately, on the other hand large angles reduce performance at low altitude because the high ambient pressure causes flow separation, but size and weight are minimized.

Bell Nozzle:

To gain higher performance and shorter length, engineers developed the bell-shaped nozzle. It employs a fast-expansion (radial-flow) section in the initial divergent region, which leads to a uniform, axially directed flow at the nozzle exit. The wall contour is changed gradually enough to prevent oblique shocks.

The spike and E-D nozzle:

In some nozzle designs, such as annular nozzles, the gas at the throat does not necessarily flow parallel to the axis, but the exit flow is similar to that of a conical or bell nozzle and thus produces the same thrust results.

2.3 Designing the parabolic nozzle

To design the nozzle, a set of parameters have to be defined first, which are:

1.**Thrust:** The amount of force an engine can produce from the reaction forces of the combusting gases. For this case we have considered the thrust to be $20KN$.

2.**Specific Impulse:** A measure of how efficiently a reaction mass engine (a propellant engine in our case) creates thrust. A propulsion system with a higher specific impulse uses the mass of the propellant more efficiently. $I_{sp} = 250s$.

3.**Mixture Ratio:** Mixture ratio is defined as the mass flow of oxidizer divided by the mass flow of fuel. in this case, we have opted for liquid Oxygen(LOX) and liquid Methane(LCH_4) with their mixture ratio being 2.8.

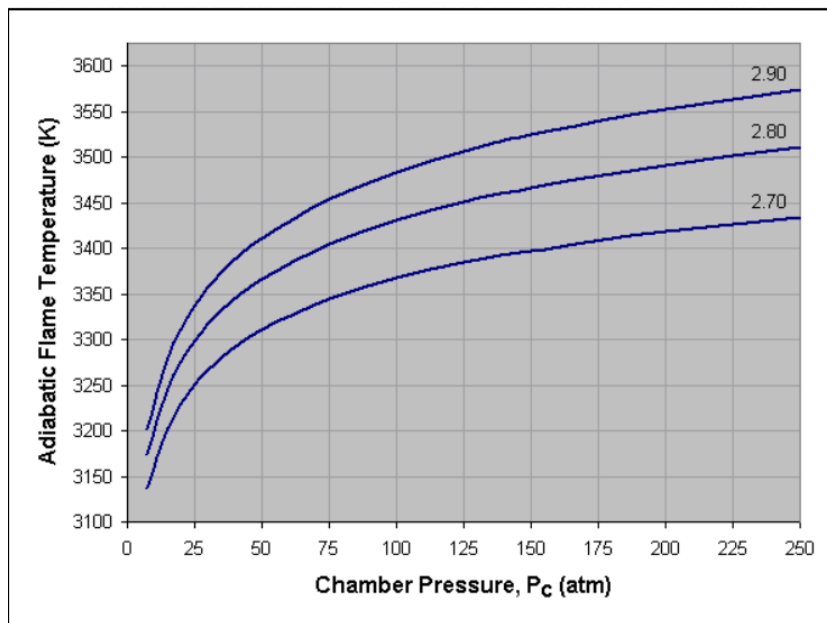


Figure 4: Adiabatic Flame Temperature graph of LOX & LCH_4 (Rocket and Space Technology)

2.3.1 Formulation

The total mass flow of the propellants (\dot{m}_{tot}) is related to the thrust force (F), the specific impulse (I_{sp}) and the gravity constant (g), as:

$$\dot{m}_{tot} = \frac{F}{I_{sp}} \quad (3)$$

The equation to obtain mass flow rate of oxidizer(\dot{m}_{ox}) as well as mass flow rate of fuel(\dot{m}_{fuel}) from Mixture ratio(MR) and mass flow rate of propellants(\dot{m}_{tot}) is :

$$\begin{aligned} \dot{m}_{ox} &= \frac{\dot{m}_{tot}}{1 + \frac{1}{MR}} \\ \dot{m}_{fuel} &= \dot{m}_{tot} - \dot{m}_{ox} \end{aligned} \quad (4)$$

At the nozzle throat, the Pressure(P_t) and Temperature(T_t) are obtained from the following equations after the chamber pressure(P_c) and chamber temperature(T_c) have been obtained from the adiabatic flame temperature:

$$\begin{aligned} P_t &= P_c \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{-\gamma}{\gamma - 1}} \\ T_t &= \frac{T_c}{1 + \frac{\gamma - 1}{2}} \end{aligned} \quad (5)$$

The Mach number is the ratio of gas velocity to the local speed of sound, The mach number at the nozzle exit can be obtained from the following equation:

$$M_n = \left(\left(\frac{2}{\gamma - 1} \right) \left[\left(\frac{P_c}{P_a} \right)^{\frac{(\gamma - 1)}{\gamma}} - 1 \right] \right)^{\frac{1}{2}} \quad (6)$$

The nozzle throat area(A_t) can be obtained from the equation:

$$A_t = \frac{\dot{m}_{tot}}{P_t} \left(\frac{RT_t}{\gamma M_n} \right)^{\frac{1}{2}} \quad (7)$$

On assuming isentropic expansion through the nozzle, the expansion ratio (ϵ) will be:

$$\epsilon = \frac{\left(\frac{P_c}{P_a} \right) \left(\frac{2}{\gamma + 1} \right)^{\frac{1}{(\gamma - 1)}}}{\left(\frac{\gamma + 1}{\gamma - 1} \left[1 - \left(\frac{P_a}{P_c} \right)^{\frac{\gamma}{(\gamma - 1)}} \right] \right)^{\frac{1}{2}}} \quad (8)$$

From Eq. 8 and Eq. 7 The nozzle exit area can be calculated as:

$$A_e = \epsilon A_t \quad (9)$$

Note: Reference for Eq. 3 – Eq. 9 [HL⁺20]

Design of a specific nozzle requires the following data: throat diameter D_t , axial length of the nozzle from throat to exit plane L_n (or the desired fractional length, L_f , based on a 15-degree conical nozzle), expansion ratio ϵ , initial wall angle of the parabola θ_n , and nozzle exit wall angle θ_e . The wall angles θ_n and θ_e are shown in Figure 5 as a function of the expansion area ratio.

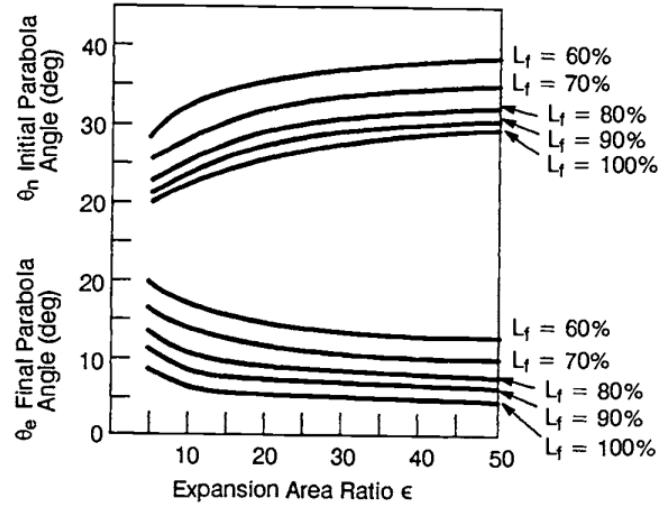


Figure 5: Expansion Area Ratio and angle relations
([Reaction Research Society](#))

Once all the above parameters have been evaluated, they can be used as inputs in Rao's nozzle:

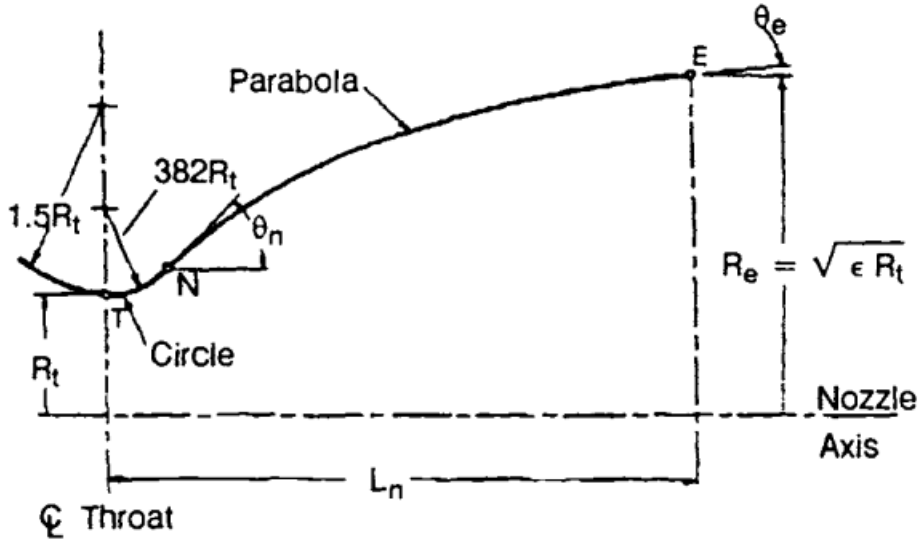


Figure 6: Nozzle design based on Rao's approximation ([[Huz92](#)])

3 Section II

3.1 Introduction

The simulation was based on the assumption that there was no heat transfer taking place across the walls(Adiabatic). Ansys Fluent(Academic License) was used for meshing as well as simulation.[Link](#) to access the files relevant to this section

3.2 Geometry

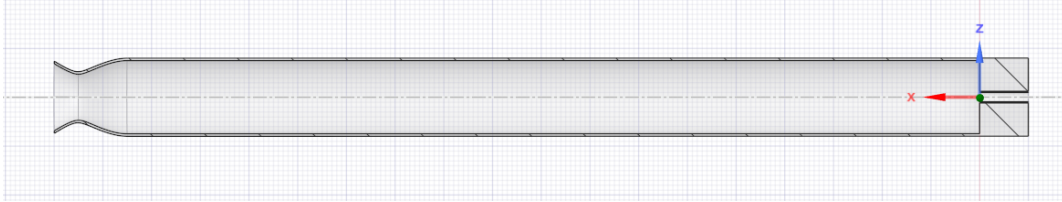


Figure 7: Cross sectional view of the setup

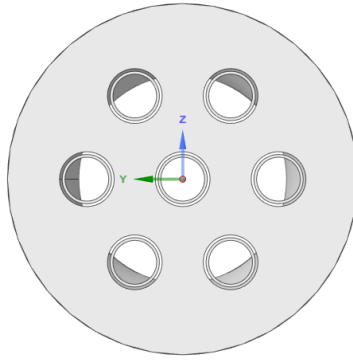


Figure 8: Injector plate

Case	OF	\dot{m}		T		J	VR
	Ratio	GOX	GCH4	GOX	GCH4		
A	2.66	44.4	16.7	280.7	275.9	0.587	1.07
B	3.48	49.1	14.1	281.7	280.5	0.347	0.83

Figure 9: Operating conditions [MPSS⁺14]

Chamber length	290mm
Chamber width	12mm
Chamber height	12mm
Throat height	4.8mm
Contraction ratio	2.5

Table 1: Combustion chamber dimensions [MPSS⁺14]

GOX diameter	4mm
GOX post wall thickness	0.5mm
GCH4 diameter	6mm
Injector area ratio AGCH4/AGOX	0.7mm

Table 2: Injector dimensions

3.3 Meshing

Local sizing option was not used since there were no particular regions where a finer mesh had to be used to capture the fluid interaction more precisely.

A growth rate of 1.1 was chosen over the default 1.2 for a more gradual increase in cell size from 0.03mm to 1mm.

Curvature and proximity size functions were opted to accurately mesh the curved regions as well as other surfaces. (Default value of 18° for curvature normal angle).

Use Custom Size Field/Control Files?	No
Minimum Size [mm]	0.03
Maximum Size [mm] ?	1
Growth Rate	1.1
Size Functions	Curvature & Proximity
Curvature Normal Angle [deg]	18
Cells Per Gap	2
Scope Proximity To	edges
<input checked="" type="checkbox"/> Draw Size Boxes	
Separate Out Boundary Zones by Angle?	No

Figure 10: Surface mesh variables

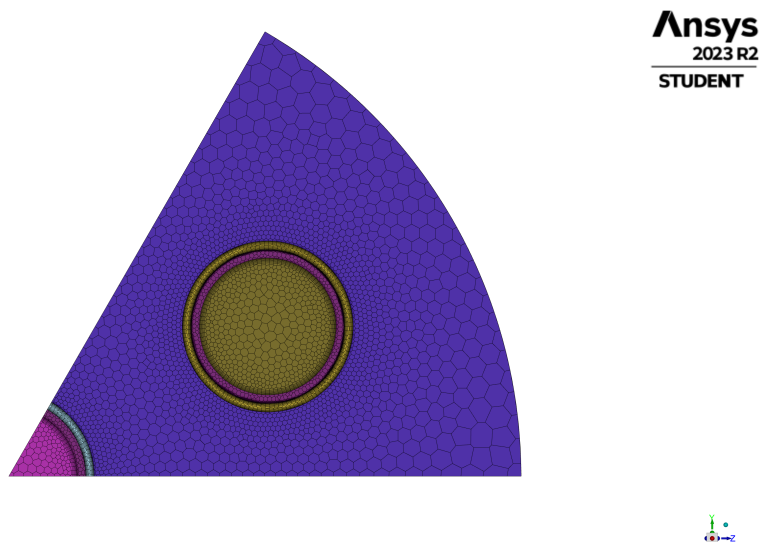


Figure 11: 60°sliced portion of the original geometry

Only the volume portion was meshed since heat transfer from the walls wasn't considered in the initial iterations.

3.4 Results

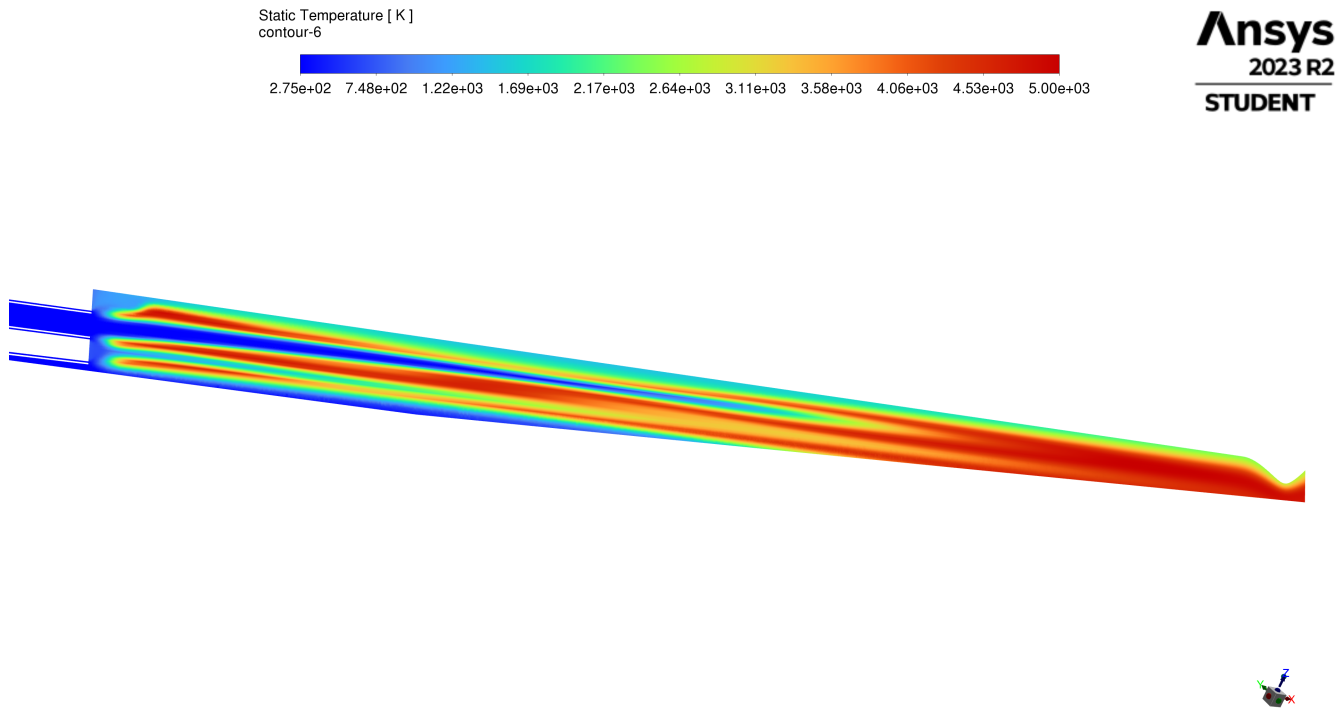


Figure 12: Temperature contours of the Combustion chamber

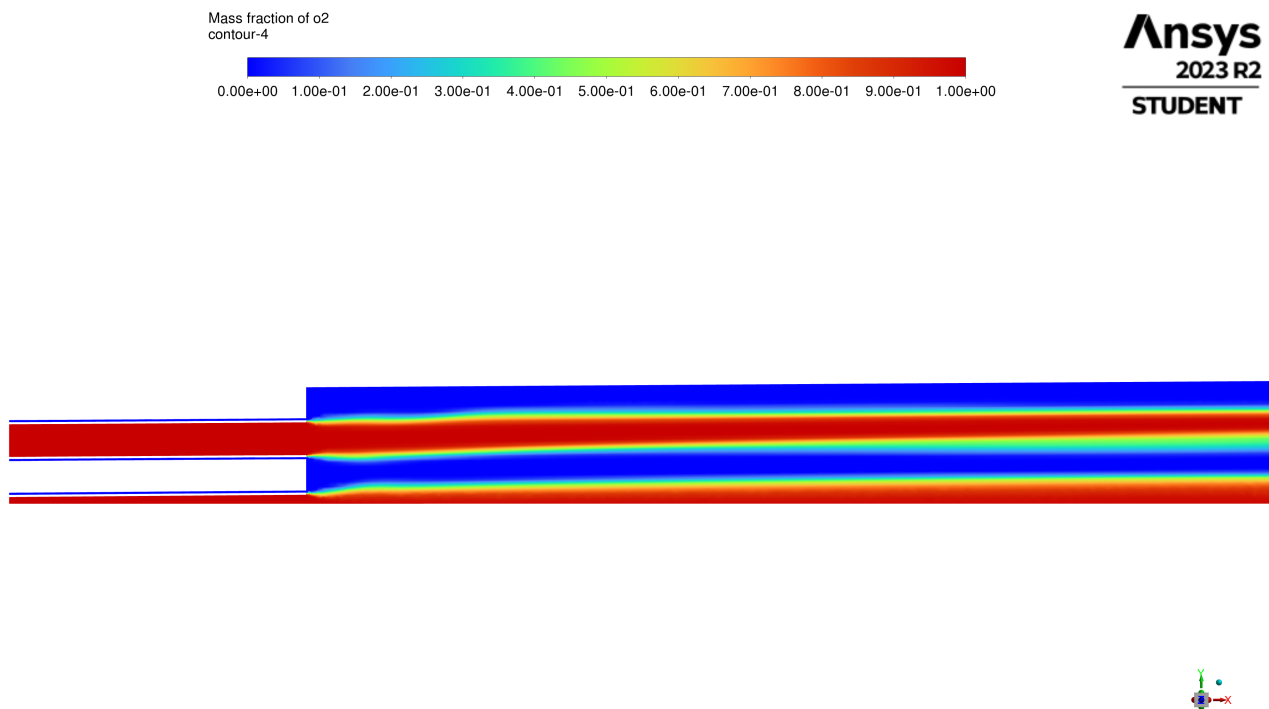


Figure 13: Oxygen mass fraction contours in the combustion chamber

The increase in temperature from 275K to 5000K indicates that combustion has taken place via the reaction between the oxidizer and the fuel

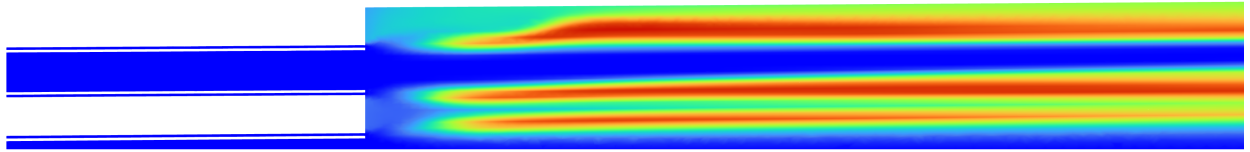
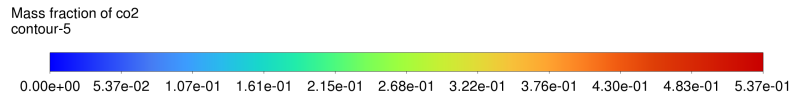


Figure 14: Mass fraction contour of CO_2

References

- [HL⁺20] Hatem Houhou, Hemza Layachi, et al. Design of the thrust chamber: dimensional analysis of the combustion chamber and the nozzle of rocket engine using lox/lch4 propellants. In *IOP Conference Series: Materials Science and Engineering*, volume 715, page 012085. IOP Publishing, 2020.
- [Huz92] Dieter K Huzel. *Modern engineering for design of liquid-propellant rocket engines*, volume 147. AIAA, 1992.
- [MPSS⁺14] Celano Maria Palma, Simona Silvestri, Gregor Schlieben, Christoph Kirchberger, Oskar Haidn, and Suresh Menon. Numerical and experimental investigation for a gox-gch4 shear-coaxial injector element. 05 2014.
- [Rao58] GVR Rao. Exhaust nozzle contour for optimum thrust. *Journal of Jet Propulsion*, 28(6):377–382, 1958.