

Final Report - Application of Machine Learning to Aircraft Conceptual Design

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I. Introduction

Conceptual design and performance estimation for aircraft is a complex multi-disciplinary problem that involves modelling the effects of the aerodynamics, propulsion, stability and structural response of the aircraft for a specified design mission. However, for applications like simulation of the flights across the entire airspace as shown in Fig 1, it becomes necessary to model tens of thousands of aircraft simultaneously making the problem extremely computationally expensive. The goal of this project is to use aircraft performance data to build surrogate models using different regression techniques and observe which techniques are best suited for the problem at hand. Once the surrogate is built, aircraft missions can be inexpensively simulated using the surrogate thus allowing us to solve extremely large design problems involving thousands of designs inexpensively.



Figure 1. Air traffic around the world at any instant in time

II. Data and Features

The data used for this problem was obtained by performing simulations on a large number of aircraft of different sizes using a conceptual design code called the Program for Aircraft Synthesis Studies. These simulations were performed a few months ago as part of a research project with the Federal Aviation Administration (FAA). The data set contains the performance estimates of about 48,000 different aircraft configurations at different flight conditions. Each data point originally contained 24 inputs and 8 outputs. Of the outputs we are only interested in fuel burn for this project. Moreover, the input dimension is reduced to 7 as described in section A. The 7 inputs (columns) that are fed into the different regression models are the aircraft payload, the mission range, the takeoff weight, cruise Mach number, wing span, wing sweep and the wing area. Each of these variables is non-dimensionalised to ensure that they remain between 0 and 1. Table 1 shows a sample data point from the training data set. The data is used to create 6 training sets of sizes 500, 1000, 2000, 5000, 10000 and 20000 samples and 4 test sets each with 4000 samples.

A. Feature selection

Sequential feature selection is used to reduce the input space from 24 to 7 dimensions. The 'sequentialfs' function in matlab is used for this. It sequentially adds features to an empty set such that they best predict y . The sum of the squared error based on linear regression is used to evaluate the quality of the feature on the training set. Based on this the 7 input dimensions shown in the table 1 are selected.

Table 1. Aircraft Data

Payload (tonnes)	Range (km)	Takeoff(lbs) weight (lbs)	Cruise Mach	wing span(ft)	wing area (ft^2)	wing sweep (deg)	Fuel burn (kg)
10.7	2240	174200	0.8	117	1344	25	27800

III. Models

The different regression models applied to the dataset are described below.

LINEAR REGRESSION

Linear regression was the first algorithm tried. The normal equations $\theta = (X^T X)^{-1} X^T y$ are solved for the training inputs x and outputs y . θ was then used to compute the outputs for the test data using $y_{test} = \theta_0 + \theta_1 x_1 + \theta_2 x_2 \dots$. The linear regression models seems to work well on the test data. However, the results are not as accurate as required for the prediction of fuel burn. Moreover as the number of samples are increased, the error does not improve the estimate by much.

WEIGHTED LINEAR REGRESSION

Next weighted linear regression is tried on the data with a weighing function $\exp \frac{-(||x - x_{test}||)^2}{2\tau^2}$. The modified normal equations solved for this case are $\theta = (X^T W X)^{-1} X^T W^T y$. This model doesn't work much better than the linear regression and is more expensive to compute than linear regression. It also follows a trend similar to linear regression.

HIGHER ORDER REGRESSION

Now we look at quadratic regression where we use the formula $y_{test} = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1 x_2 + \theta_4 x_1^2 + \theta_5 x_2^2 \dots$. Addition of the higher dimensional features improves the fit to the training data and reduces the test error as show in Fig 2. However like the linear and weighted linear regression cases, increasing the number of samples does not significantly improve the test error.

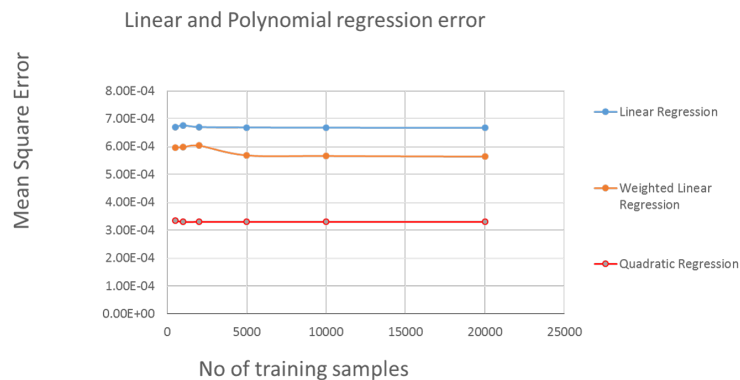


Figure 2. Comparison of linear, weighted linear and quadratic regression

K-NEAREST NEIGHBOURS REGRESSION

In this method, the k- nearest neighbours of the test point are computed and then the estimate of the fuel burn at the test point is obtained using a weighted average of the values at the k nearest points $y_{eval} = \frac{\sum_{i=1}^k W(i)y_{train}(i)}{\sum_{i=1}^k W(i)}$. We use the inverse of the euclidean distance between the training point i and evaluation point as the weight $W(i)$. The 'knnsearch' function in matlab is used to compute the k-nearest neighbours and this is fed into matlab code written for this project that performs the prediction and iterations to compute optimal k. To compute the optimal k, an iterative procedure is used and the value of k that minimises the mean square error over the training set is selected. The effect of varying k on the mean square error for the different samples is shown in Fig 3(a). It is observed that although for smaller sample sizes the k-NN algorithm does not perform very well, as the sample sizes are increased, the K-NN algorithm is able to give a fairly good estimate of the fuel burn.

GAUSSIAN PROCESS REGRESSION

Gaussian process regression is the next method that is applied to the data set. The reason for trying this is that in a different study, gaussian process regression was successfully used to build surrogate models for aircraft propulsion systems. Selection of the appropriate mean and covariance functions is a tricky task. For this study we try 3 different mean and 4 different covariance functions as shown in table 2 . The isotropic matern covariance function along with either linear or quadratic mean function are the best performing models. Figure 3(b) shows how the different Gaussian models perform on the test sets. The capabilities of 'gpml' an existing matlab code for Gaussian Process Regression supplemented by code written for this project are leveraged for this study.

Table 2. Covariance functions

Covariance functions	formula
Isotropic Matern	$k(x^p, x^q) = sf^2 * f(\sqrt{d} * r) * e^{-\sqrt{d}*r}$ where r is $\sqrt{((x^p - x^q)^T * (P)^{-1} * (x^p - x^q))}$, P is a diagonal matrix of the hyperparameters and sf^2 is the signal variance
Isotropic RQ	$k(x^p, x^q) = sf^2 * [1 + (x^p - x^q)^T * (P)^{-1} * (x^p - x^q)/(2 * \alpha)]^{-\alpha}$ where P is the diagonal matrix of the hyperparameters, sf^2 is the signal variance and α is the shape parameter
Isotropic SE	$k(x^p, x^q) = sf^2 * e^{-(x^p - x^q)^T * P^{-1} * (x^p - x^q)/2}$ where P is the diagonal matrix of the hyperparameters, sf^2 is the signal variance and x is the matrix of input data
ARD SE	$k(x^p, x^q) = sf^2 * exp(-(x^p - x^q)^T * P^{-1} * (x^p - x^q)/2)$ where P is a diagonal matrix with the ARD parameters, sf^2 is the signal variance, x is the matrix of training inputs

where RQ stands for Rational Quadratic covariance function SE stands for Squared Exponential covariance function and ARD stands for Automatic Relevance Detemination which is a distance measure

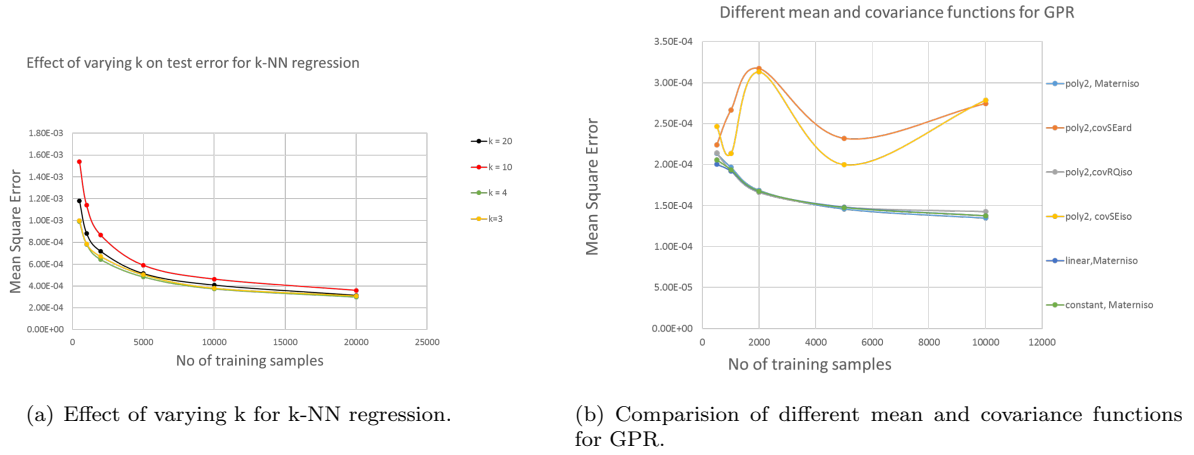


Figure 3. Plots for k-NN and GPR.

ARTIFICIAL NEURAL NETWORK - MULTILAYER PERCEPTRON NETWORK

A feed-forward Multilayer Perceptron Network is the last method that has been applied to the data. Work done by Bryan Yukto³ as part of his dissertation at MIT has shown that feed forward ANNs provide promising results for prediction of aircraft parameters. We use a 3 layer network with 7 input neuron, 7 hidden neurons and 1 output neuron. The network used was arrived at by trying out different 3 and 4 layer networks by varying the number of hidden layers and neurons in these layers. We try both the sigmoid and the hyperbolic tangent activation functions. For this study the *tanh* activation function performs better. Moreover, standard gradient descent based back propagation is unable to bring down the training error. Thus a Levenberg Marquardt back propagation error is used for this study. In this method, the gradient g is estimated using $J^T e$ where J is the jacobian matrix computed using back propagation as shown in the references¹ and e is the error vector for the n training samples. The Hessian H is approximated using $J^T J + \nu I$. The weights are then updated using $W := W - H^{-1}g$. Figure 4 shows the convergence history of the network for the case with 1000 training samples. For the current study, we stop the training at 10000 iterations (as the error is already significantly low). ANN's are promising for the current application as the training and test error can be significantly reduced even further by training for larger number of iterations as shown by the trends. A python implementation of the multi-layer perceptron network algorithm along with the different back-propagation methods was written from scratch for this project.

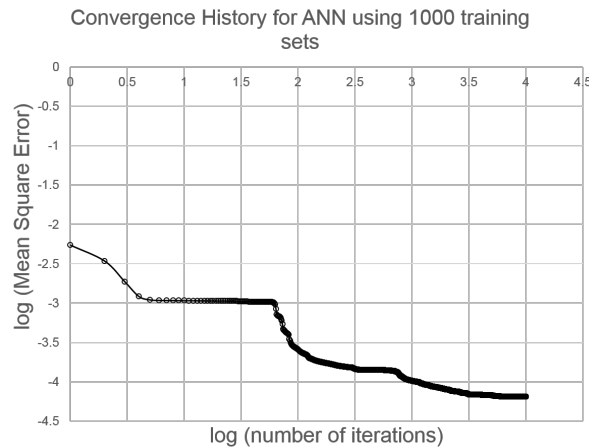


Figure 4. Convergence of the the Levenberg Marquardt based back propagation

IV. Summary of the results

We summarise the results described in the previous section by tabulating the testing and training error obtained using the different methods in table 3. The test error stated below are averaged values over the 4 test sets of 4000 samples each, for the training set of 1000 samples .

Table 3. Comparision of different regression methods

Model	Training MSE	Test MSE
Linear Regression	6.22e-4	6.76e-4
Weighted linear regression	5.42e-4	s.97e-4
Quadratic Regression	3.11e-4	3.40e-4
k-NN regression	-	7.84e-4
Gaussian Process Regression(best case)	4.19e-4	1.97e-4
Multilayer Perceptron Network	6.49e-4	3.97e-4

V. Conclusion

We see that machine learning techniques perform well in predicting aircraft performance. Gaussian Process regression and Artificial Neural Networks are the most promising methods. GPR allows us to reduce the prediction errors significantly and the approximation improves as the number of samples is increased. ANN's also perform well and importantly as the number of training iterations are increased, the prediction error can be reduced further. This is encouraging as now, millions of predictions can be performed in a matter of seconds without resorting to parallel computation. This will allow designer to simulate large scale aircraft systems like the National Airspace system accurately and perform optimizations on air transportation networks (changing flight paths and schedules) to try and minimize the overall fuel burn of the system without worrying about computational costs.

Another interesting trend that jumps out is that simple methods like linear or quadratic regression also work fairly well with estimation errors below 5%. This is encouraging as these methods can be used by people with minimal machine learning experience in cases where quick estimates are required, without having to worry about the complications of GPR or ANNs.

VI. Future Work

The studies for this case were all performed on conventional aircraft configurations. Looking to see if these methods work for unconcventional aircraft configurations like Blended wing bodies etc. will be an interesting next step. For those configurations, the interactions between the different disciplines are extremely complex and modelling them using regression methods might not work out as well as they did for this case.

References

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