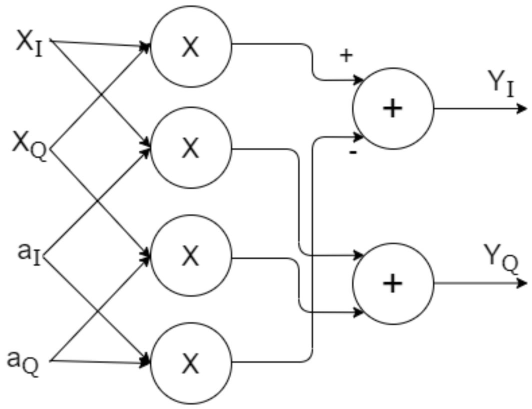
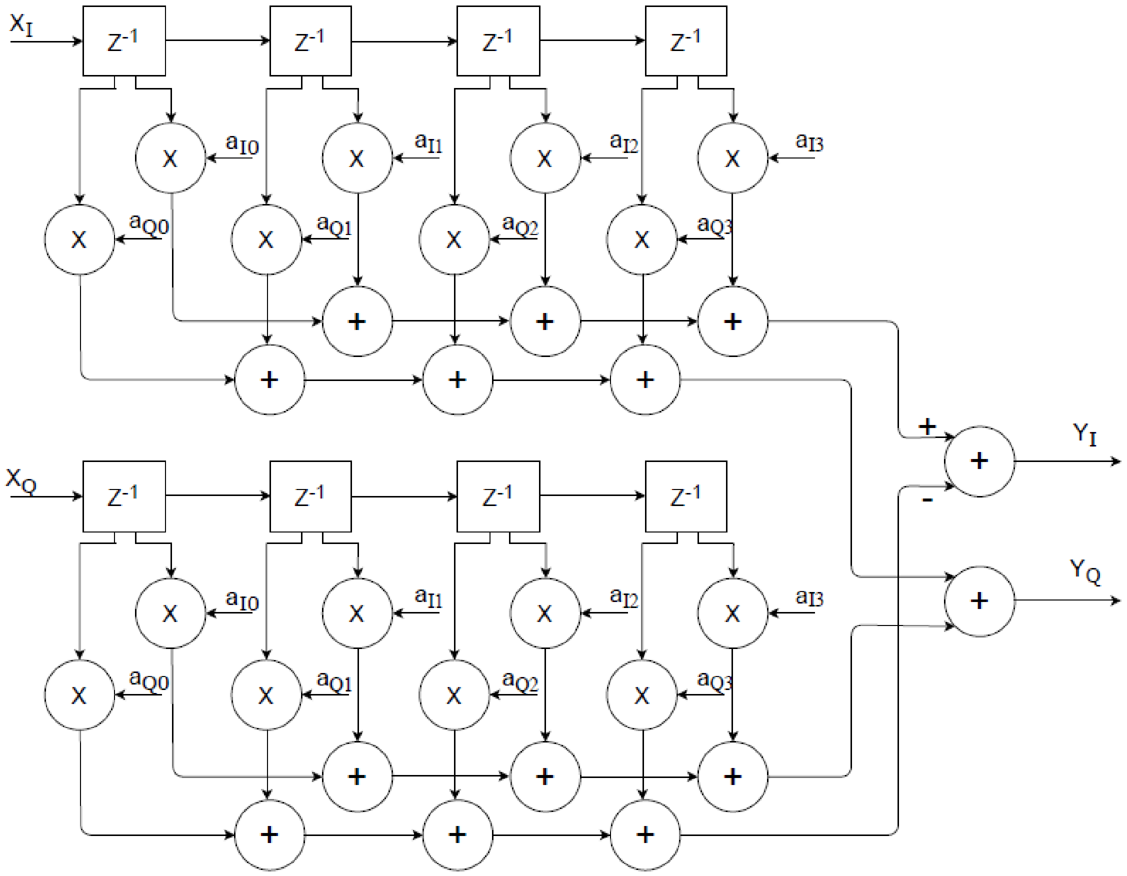
A complex FIR (finite impulse response) filter, takes the digital input date, manipulates it, and outputs the digital data. In this example, a simple convolution operation course between the complex input data (X_I and X_Q), the FIR filter coefficients (a_I and a_Q) to give an output (Y_I and Y_Q) which is the convolution between $X_I + jX_Q$ and $a_I + ja_Q$.

The design of a complex multiplier can be created through 4 multipliers and two adders as shown:

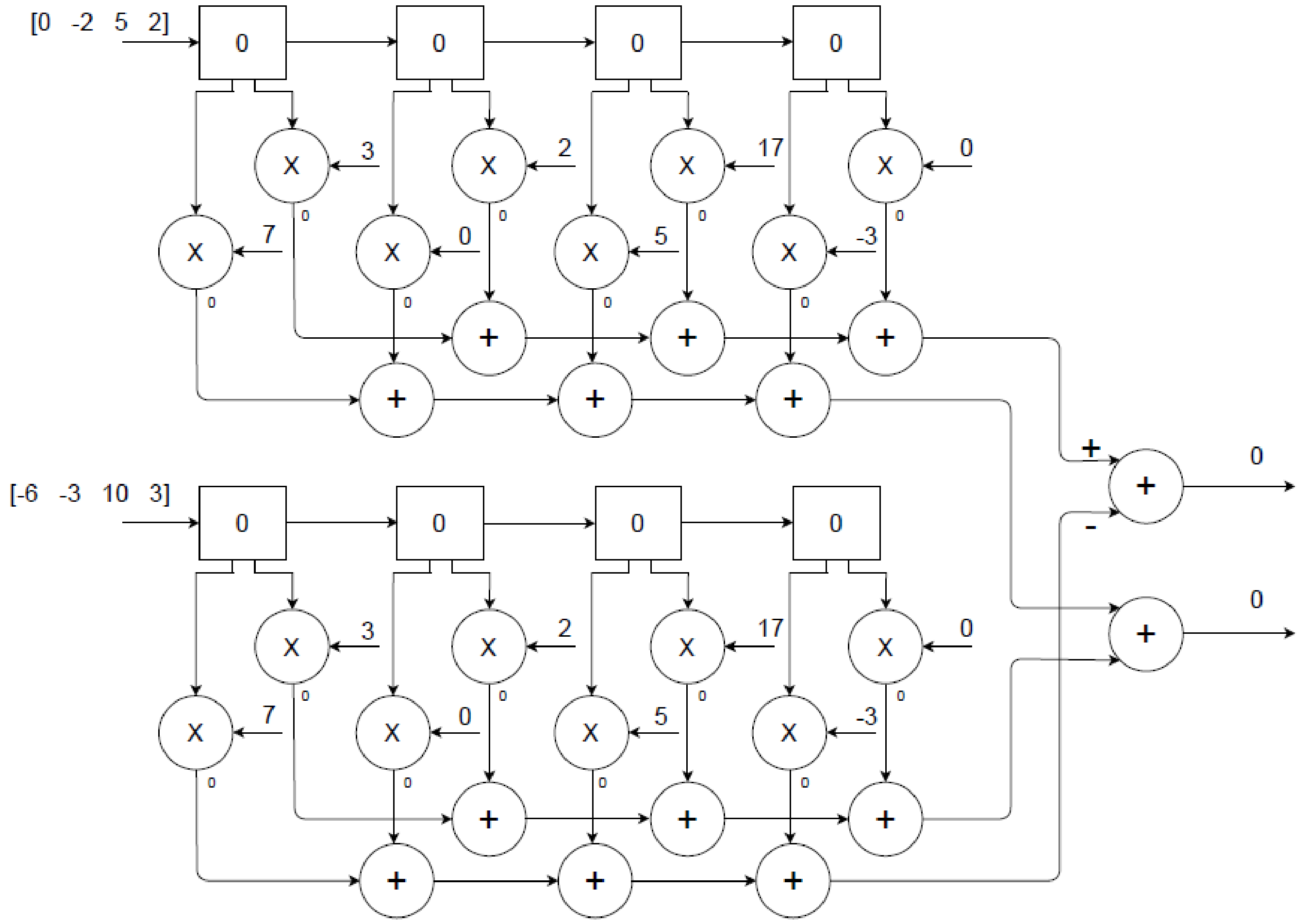
$$(X_I + jX_Q)(a_I + ja_Q) = (X_I a_I - X_Q a_Q) + j(X_I a_Q + X_Q a_I)$$



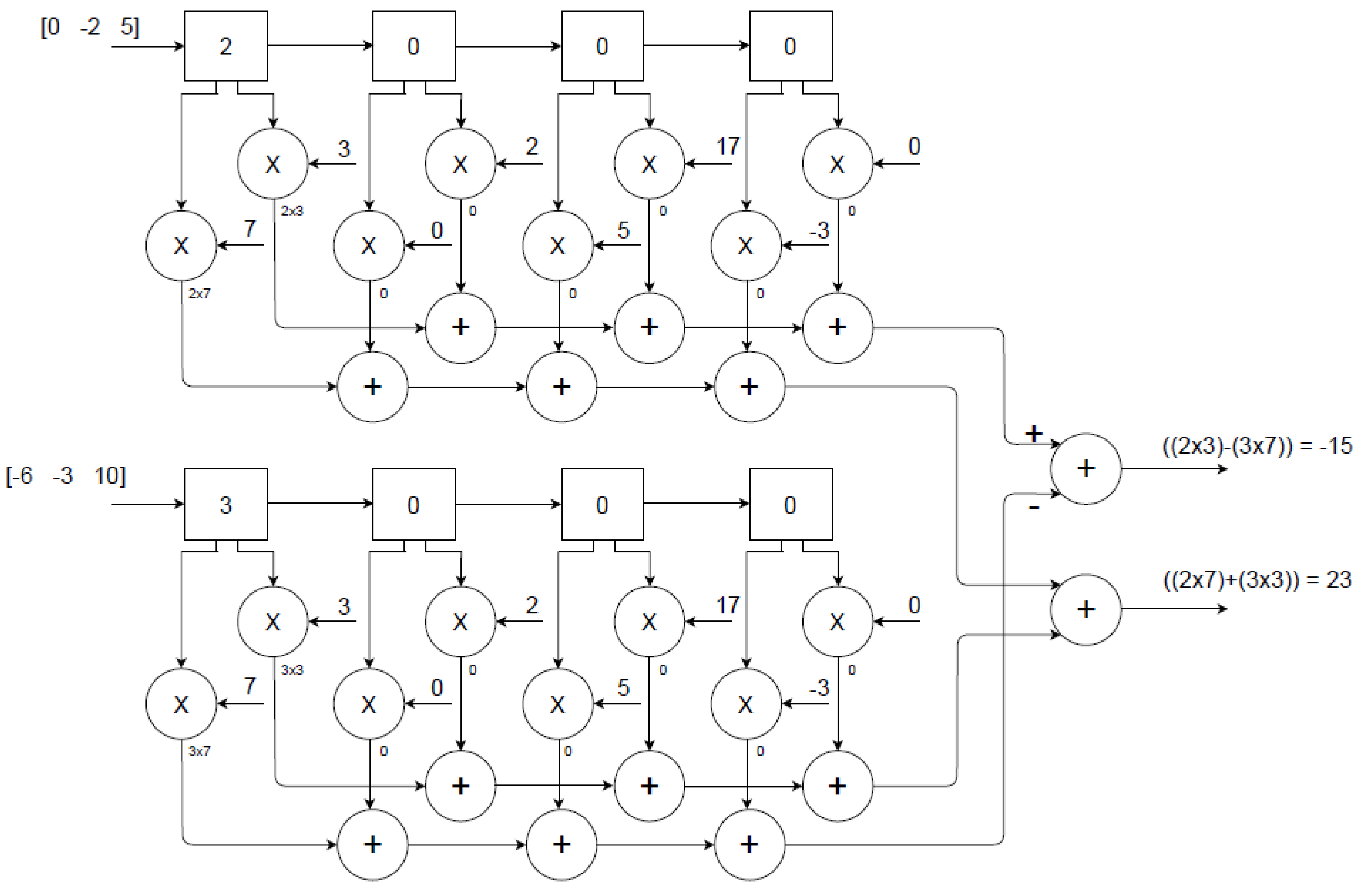
Shown below is a generic design of a 4 tap complex FIR filter. As we can see, a FIR filter tap consists of delay block (Z^{-1}), a multiplication block (⊗), and the addition block (⊕).A similar principle can be applied for an n tap complex FIR filter.



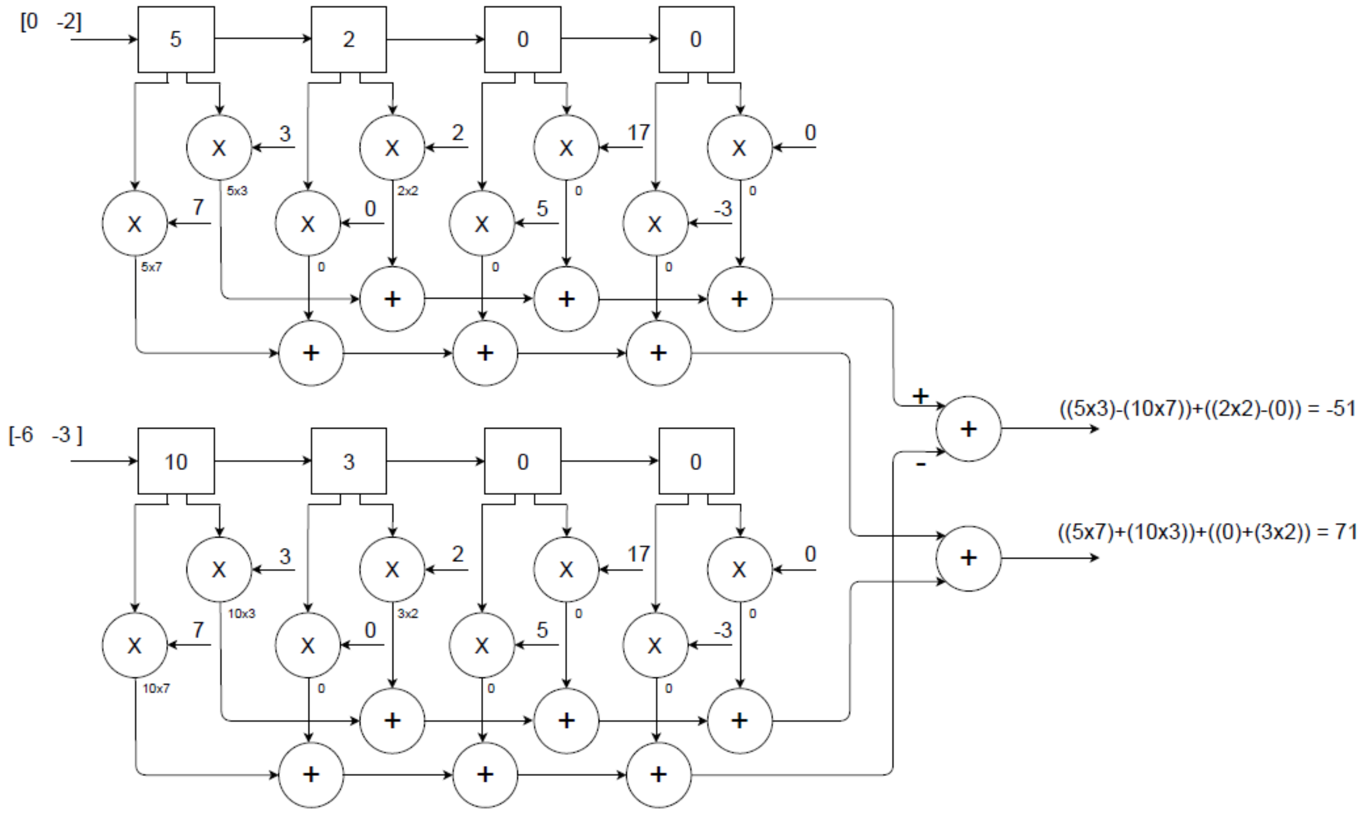
1)


[0 -2 5 2]
[-6 -3 10 3]
0
0
3
2
17
0
7
0
5
-3
+
-
0
0

2)

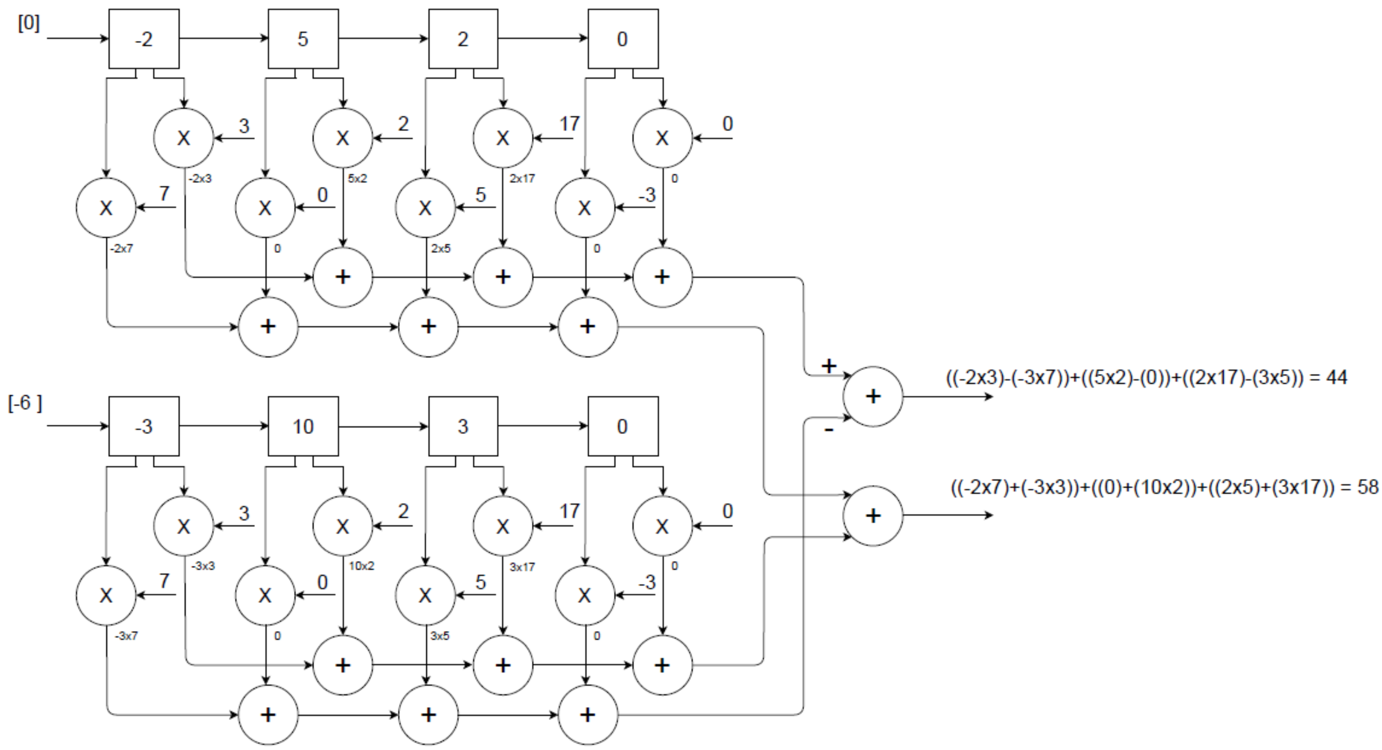

[0 -2 5]
[-6 -3 10]
2
3
0
2x3
2x7
3x3
3x7
((2x3)-(3x7)) = -15
((2x7)+(3x3)) = 23

3)



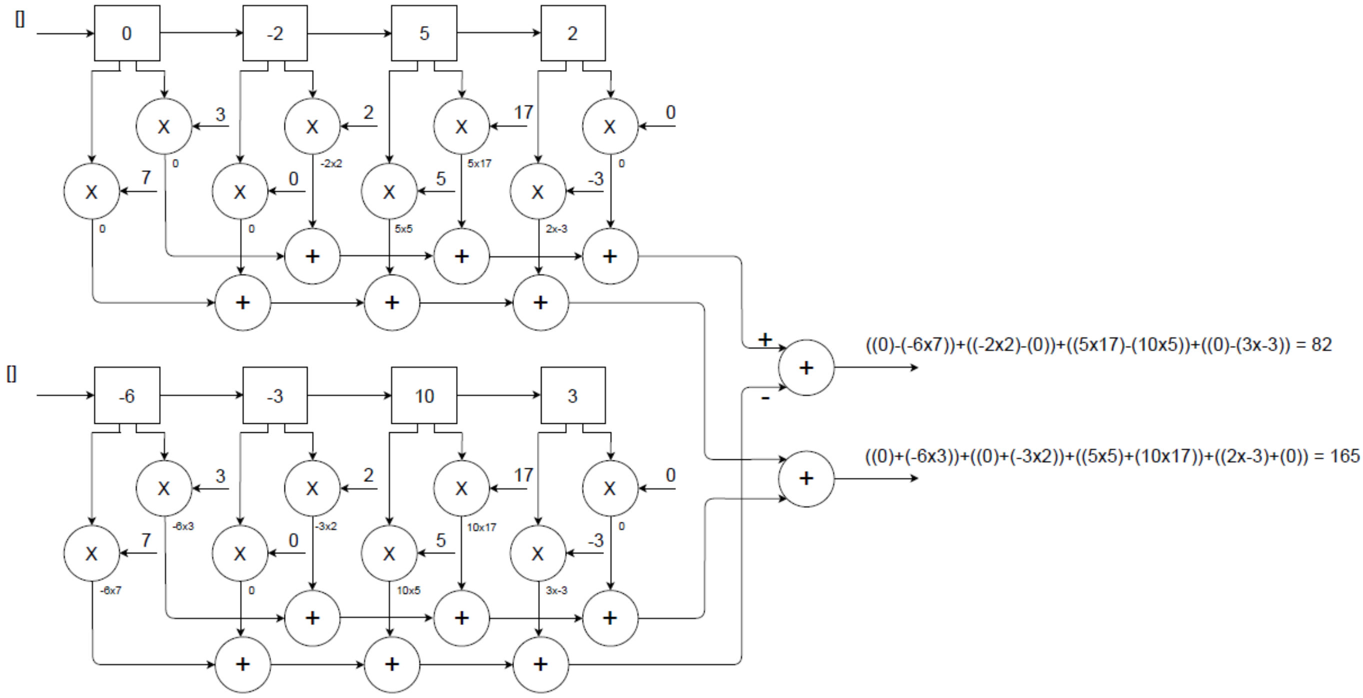
[0 -2]
5
2
0
0
3
2
17
0
7
0
5
-3
5x3
2x2
5x7
[-6 -3]
10
3
0
0
10x3
3x2
10x7
+
-
((5x3)-(10x7))+((2x2)-(0)) = -51
((5x7)+(10x3))+((0)+(3x2)) = 71

4)

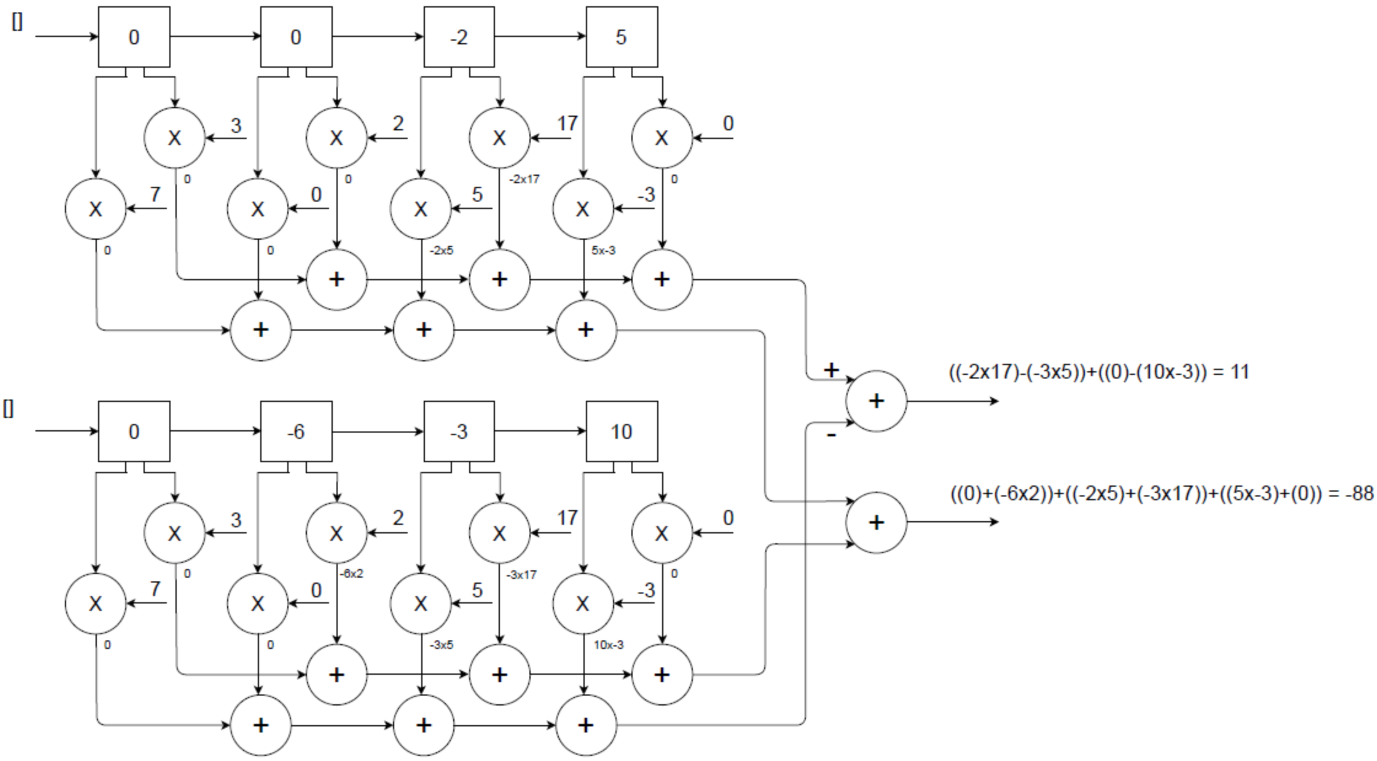


[0]
-2
5
2
0
[-6]
-3
10
3
0
-2x3
5x2
2x17
-2x7
2x5
-3x3
10x2
3x17
-3x7
3x5
+
-
((-2x3)-(-3x7))+((5x2)-(0))+((2x17)-(3x5)) = 44
((-2x7)+(-3x3))+((0)+(10x2))+((2x5)+(3x17)) = 58

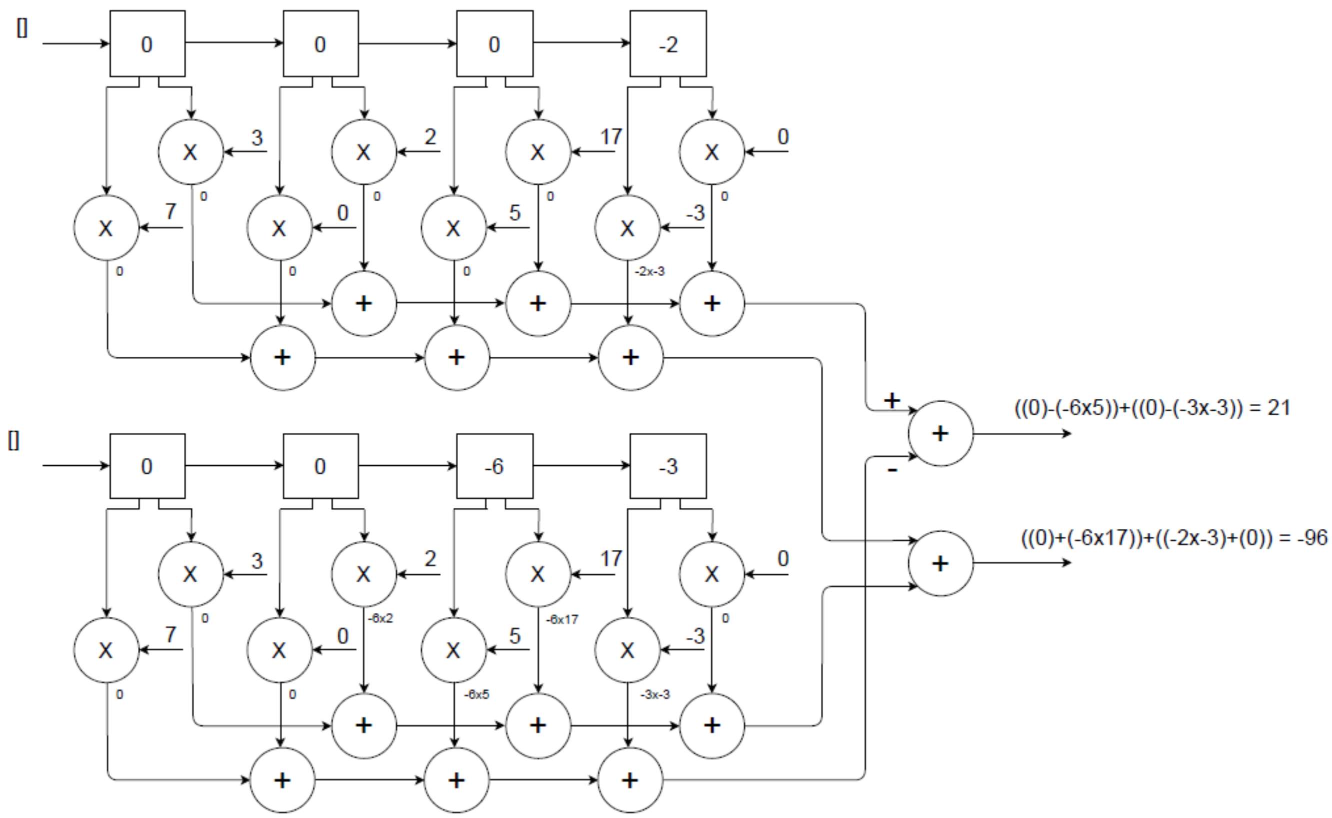
5)


((0)-(-6x7))+((-2x2)-(0))+((5x17)-(10x5))+((0)-(3x-3)) = 82
((0)+(-6x3))+((0)+(-3x2))+((5x5)+(10x17))+((2x-3)+(0)) = 165

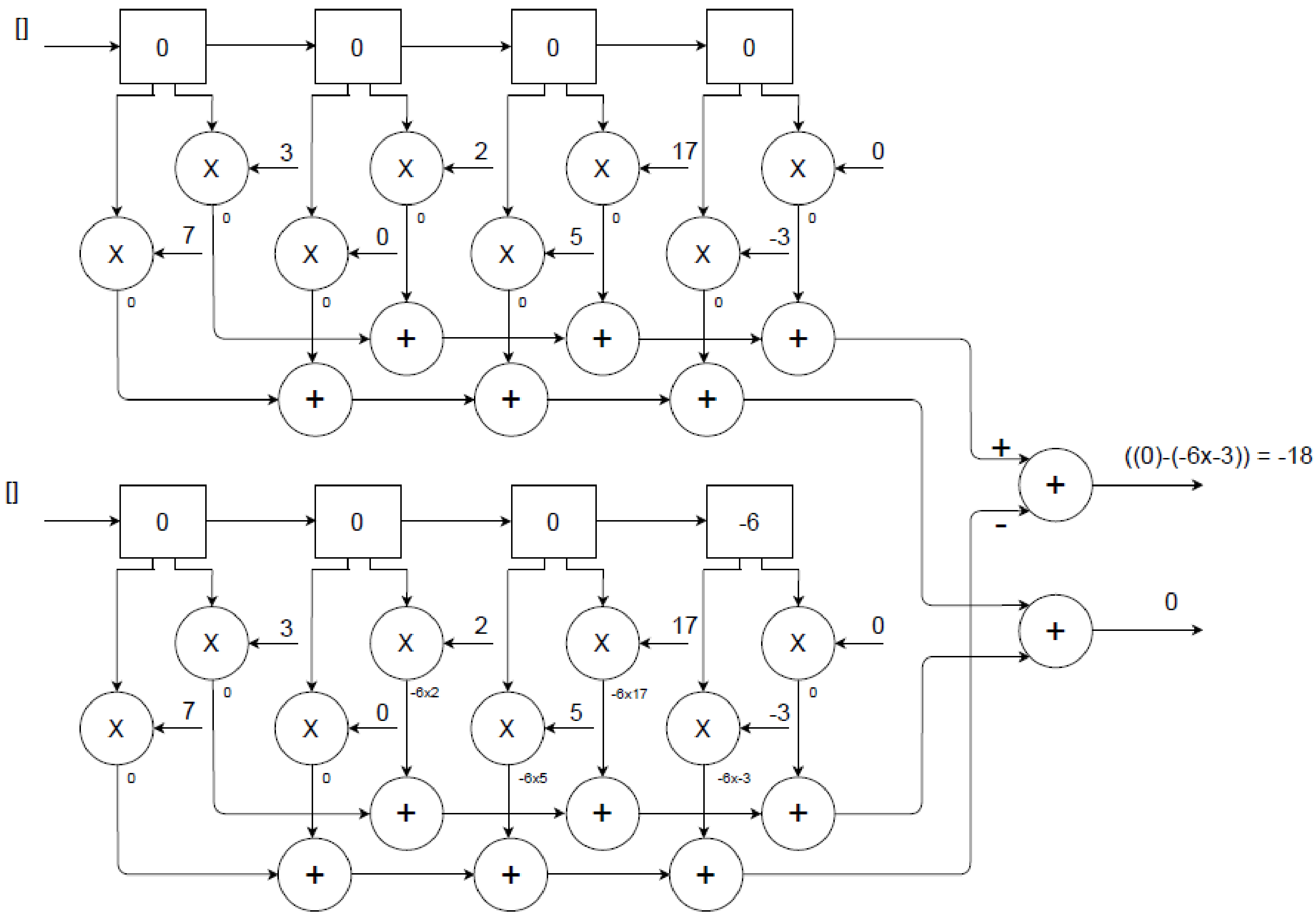
6)


((-2x17)-(-3x5))+((0)-(10x-3)) = 11
((0)+(-6x2))+((-2x5)+(-3x17))+((5x-3)+(0)) = -88

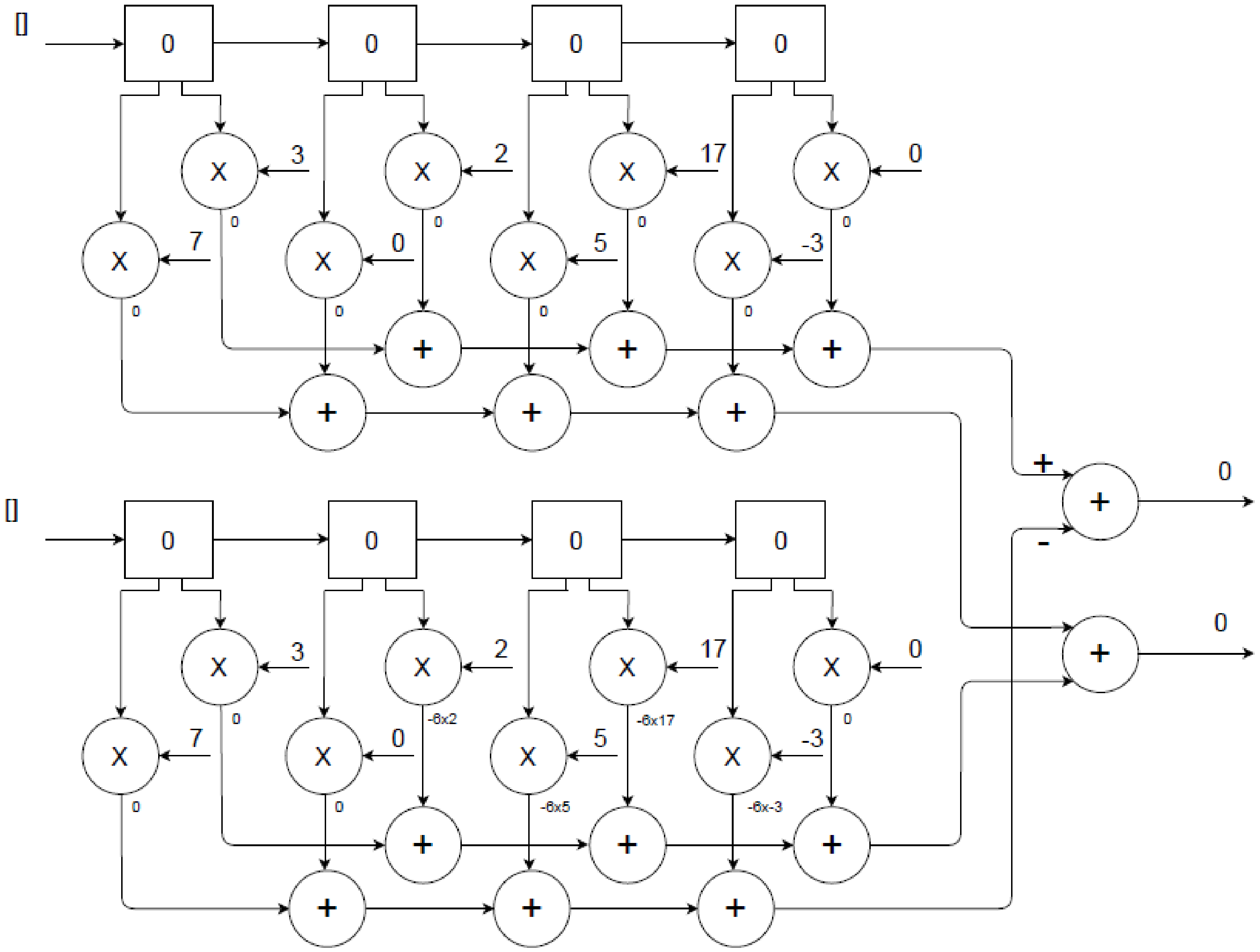
7)



8)



9)



The following convolution process between two complex numbers can be confirmed to be correct through by checking the corresponding operation in MATLAB, as shown below:

```
>> x = [2+3i 5+10i -2-3i 0-6i];
>> a = [3+7i 2+0i 17+5i 0-3i];
>> conv(x,a)

ans =

   1.0e+02 *

  Columns 1 through 6

  -0.1500 + 0.2300i  -0.5100 + 0.7100i   0.4400 + 0.5800i   0.8200 + 1.6500i   0.1100 - 0.8800i   0.2100 - 0.9600i

  Column 7

  -0.1800 + 0.0000i
```