

Project “Topic: Eigenvalues and Eigenvectors in Principal Component Analysis”

Title

IB3702 Mathematics for Machine Learning

Harman Singh

15 November, 2025

1 Introduction

Purpose: Introduce the motivation and general context of the topic.

Content:

- Machine learning relies heavily on mathematics to model, analyse, and interpret data.
- High-dimensional datasets are common in modern applications, making dimensionality reduction a crucial step for improving performance and interpretability.
- Principal Component Analysis (PCA) is one of the most widely used techniques for this purpose.
- The report focuses on the mathematical foundations of PCA, particularly the role of eigenvalues and eigenvectors in identifying directions of maximum variance.
- The aim is to connect theoretical concepts from linear algebra to their practical use in machine learning.

2 Preliminaries

Purpose: Provide the mathematical background required to understand PCA

Content:

- Review of key linear algebra concepts relevant to PCA:
 - A square matrix A acting on a vector \mathbf{v} satisfies the eigenvalue equation $A\mathbf{v} = \lambda\mathbf{v}$, where λ is an eigenvalue and \mathbf{v} is the corresponding eigenvector.
 - Geometrically, eigenvectors indicate directions that remain unchanged under the transformation A , and eigenvalues represent the scaling factor along those directions.
- Definition of the covariance matrix as a measure of how variables in a dataset vary together:

$$\Sigma = \frac{1}{n-1}(X - \bar{X})^T(X - \bar{X})$$

where X is the data matrix and \bar{X} is the mean vector.

- PCA involves the eigen-decomposition of this covariance matrix to determine the principal components.
- These eigenvectors form a new orthogonal basis for the data space, ordered by decreasing eigenvalues corresponding to decreasing variance captured.

3 Methods

Purpose: Explain how PCA works step-by-step, highlighting the role of eigenvalues and eigenvectors.

Content:

- Describe the goal of PCA (to reduce dimensionality while retaining as much information (variance) as possible)
- Outline the PCA algorithm briefly:
 1. Standardise the dataset (zero mean, unit variance).
 2. Compute the covariance matrix of the data.
 3. Perform eigen-decomposition of the covariance matrix.
 4. Sort eigenvalues (descending) and select the top k eigenvectors (principal components).
 5. Project data onto the new subspace.
- Explain the mathematical interpretation:

- Eigenvectors represent directions of maximum variance.
- Eigenvalues indicate how much variance is captured along each direction.
- Optionally, relate this to Singular Value Decomposition (SVD), which can also be used for PCA

4 Numerical Examples

Purpose: Provide a small illustrative example to demonstrate PCA in practice.

Content:

- Use a simple 2D or 3D dataset (e.g., two correlated variables like height and weight).
- Show (conceptually, not full code) how to:
 1. Compute the covariance matrix.
 2. Find its eigenvalues and eigenvectors.
 3. Project data onto the first principal component.
- Briefly interpret results:
 - First principal component captures most of the variance.
 - Dimensionality reduced from 2D to 1D (example result).
- You could include a small diagram or table if allowed.

5 Collaboration

Purpose: Explain how the group collaborated and divided tasks.

Content:

- Describe who worked on what:
 - Tobias handled ...
 - Harman handled ...
- Mention the communication method (e.g., shared documents, group meetings, GitHub, etc.).
- Reflect briefly on teamwork effectiveness.

6 Reflection

Purpose: Personal reflections on the learning experience.

Content for person:

6.1 Student a: Tobias Hungwe

Text...

6.2 Student b: Harman Singh

- Reflect on the practical side: implementing PCA, interpreting eigenvalues, and visualising results.
- Mention insights on how mathematics enables dimensionality reduction and data interpretation.
- Optionally note any challenges
 - understanding covariance or eigen-decomposition intuitively

References