Project “Topic: Eigenvalues and Eigenvectors in Principal Component Analysis” Title IB3702 Mathematics for Machine Learning

Harman Singh

15 November, 2025

1 Introduction

r[1]

Purpose: Introduce the motivation and general context of the topic.

Content:

- Machine learning relies heavily on mathematics to model, analyse, and interpret data.
- High-dimensional datasets are common in modern applications, making dimensionality reduction a crucial step for improving performance and interpretability.
- Principal Component Analysis (PCA) is one of the most widely used techniques for this purpose.
- The report focuses on the mathematical foundations of PCA, particularly the role of eigenvalues and eigenvectors in identifying directions of maximum variance.
- The aim is to connect theoretical concepts from linear algebra to their practical use in machine learning.

2 Preliminaries

Purpose: Provide the mathematical background required to understand PCA

Content:

- Review of key linear algebra concepts relevant to PCA:
  - A square matrix A acting on a vector $\mathbf{v}$ satisfies the eigenvalue equation $A\mathbf{v} = \lambda\mathbf{v}$, where λ is an eigenvalue and $\mathbf{v}$ is the corresponding eigenvector.
  - Geometrically, eigenvectors indicate directions that remain unchanged under the transformation A, and eigenvalues represent the scaling factor along those directions.
- Definition of the covariance matrix as a measure of how variables in a dataset vary together:
$$\Sigma = \frac{1}{n-1}(X - \bar{X})^T(X - \bar{X})$$
where X is the data matrix and $\bar{X}$ is the mean vector.
- PCA involves the eigen-decomposition of this covariance matrix to determine the principal components.
- These eigenvectors form a new orthogonal basis for the data space, ordered by decreasing eigenvalues corresponding to decreasing variance captured.

3 Methods

Purpose: Explain how PCA works step-by-step, highlighting the role of eigenvalues and eigenvectors.

Content:

- Describe the goal of PCA (to reduce dimensionality while retaining as much information (variance) as possible)
- Outline the PCA algorithm briefly:
  1. Standardise the dataset (zero mean, unit variance).
  2. Compute the covariance matrix of the data.
  3. Perform eigen-decomposition of the covariance matrix.
  4. Sort eigenvalues (descending) and select the top k eigenvectors (principal components).
  5. Project data onto the new subspace.
- Explain the mathematical interpretation:

  - Eigenvectors represent directions of maximum variance.
  - Eigenvalues indicate how much variance is captured along each direction.
- Optionally, relate this to Singular Value Decomposition (SVD), which can also be used for PCA

4 Numerical Examples

Purpose: Provide a small illustrative example to demonstrate PCA in practice.

Content:

- Use a simple 2D or 3D dataset (e.g., two correlated variables like height and weight).
- Show (conceptually, not full code) how to:
  1. Compute the covariance matrix.
  2. Find its eigenvalues and eigenvectors.
  3. Project data onto the first principal component.
- Briefly interpret results:
  - First principal component captures most of the variance.
  - Dimensionality reduced from 2D to 1D (example result).
- You could include a small diagram or table if allowed.

5 Collaboration

Purpose: Explain how the group collaborated and divided tasks.

Content:

- Describe who worked on what:
  - Tobias handled …
  - Harman handled …
- Mention the communication method (e.g., shared documents, group meetings, GitHub, etc.).
- Reflect briefly on teamwork effectiveness.

6 Reflection

Purpose: Personal reflections on the learning experience.

Content for person:

6.1 Student a: Tobias Hungwe

Text...

6.2 Student b: Harman Singh

- Reflect on the practical side: implementing PCA, interpreting eigenvalues, and visualising results.
- Mention insights on how mathematics enables dimensionality reduction and data interpretation.
- Optionally note any challenges
  - understanding covariance or eigen-decomposition intuitively

References

[1] Ian T. Jolliffe. Principal component analysis. https://doi.org/10.1007/b98835, 2002. Accessed: 2025-11-11.