**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

**Ans:-** [Tap to View](https://colab.research.google.com/drive/1HmNhy-Sior9jYjWrCUPpkP43Oyj5JSHM#scrollTo=6nOK3lRooynC)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:-**

**A:-** We have a normal distribution with  µ = 38 and *s*   = 6.

 Let X be the number of employees.

a)      Probability of employees greater than age of 44= P(X>44)

          P(X > 44) = 1 - P(X ≤ 44).

         Z = (X -  )/  = (X - 38)/6 = 1

        P(X ≤ 44) = P(Z ≤ (44 - 38)/6) = P(Z ≤ 1)=84.1345%

Probability that the employee will be greater than age of 44 = 100-84.1345=15.86%

 So the probability of number of employees between 38-44 years of age

= P(X<44)-P(X<38)

  =84.1345-50.00

  = 34.1345%

                                                                                                                                                                 Therefore, the statement that “More employees at the processing center are older than 44 than between 38 and 44” is **FALSE.**

**ANS B:**Probability of employees less than age of 30 =  P(X<30).

Z = (X - µ )/ *s* = (30 - 38)/6

Thus, the question can be answered by using the normal table to find

P(X ≤ 30) = P(Z ≤ (30 - 38)/6) = P(Z ≤ -1.333)=9.12%

So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48(or 36 employees).

Therefore, the statement B of the question is also **TRUE.**

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1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:-**

X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2)

* 2X1 ~ N(2 μ, 2σ2)
* X1+X2 ~ N(μ, σ2)+ N(μ, σ2) ~ N(2 μ , 2 σ2)
* 2X1 – (X1+X2) ~ N(2 μ-2 μ, 2σ2 **+**2σ2 ) ~ (0, 4σ2)

**In the difference between 2X1 and X1+X2 mean getting zero and variance getting doubled.**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans.**

[Tap to view](https://colab.research.google.com/drive/1HmNhy-Sior9jYjWrCUPpkP43Oyj5JSHM#scrollTo=6nOK3lRooynC)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans.**

[Tap to view](https://colab.research.google.com/drive/1HmNhy-Sior9jYjWrCUPpkP43Oyj5JSHM#scrollTo=6nOK3lRooynC)