[CM7]

Here, we are given a Poisson distribution:

$$x_i \sim \text{Poisson}(\lambda) = \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

→ where X is a random variable that models a distribution over the number of people that will have positive COVID-19 tests on a given day i, in Ontario.

→ The occurrence rate λ, is unknown.

→ For MAP, we additionally assume a prior distribution for λ to be the univariate normal distribution with known mean & variance.

First, we derive the likelihood term:

$$P(x_1, \ldots x_N / \lambda) = \prod_{i=1}^{N} P(x_i / \lambda)$$

$$= \prod_{i=1}^{N} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

$$= \frac{\lambda^{\sum_{i=1}^{N} x_i} e^{-N\lambda}}{\prod_{i=1}^{N} x!}$$

Next, we note that log is a monotonically increasing function, so we can maximize the log-likelihood:

$$\ln(P(x_1, \ldots x_N | \lambda)) = \ln \lambda \sum_{i=1}^{N} x_i - N\lambda - \ln\left(\prod_{i=1}^{N} x_i!\right)$$

We take derivatives of this with respect to λ and find:

$$\frac{\partial \ln(P(x_1, \ldots x_N | \lambda))}{\partial \lambda} = \sum_{i=1}^{N} x_i \left(\frac{1}{\lambda}\right) - N$$

Setting the left hand side equal to zero, we find:

$$0 = \sum_{i=1}^{N} \frac{x_i}{\lambda} - N$$

$$N = \sum_{i=1}^{N} \frac{x_i}{\lambda}$$

$$\boxed{\hat{\lambda} = \sum_{i=1}^{N} \frac{x_i}{N}}$$