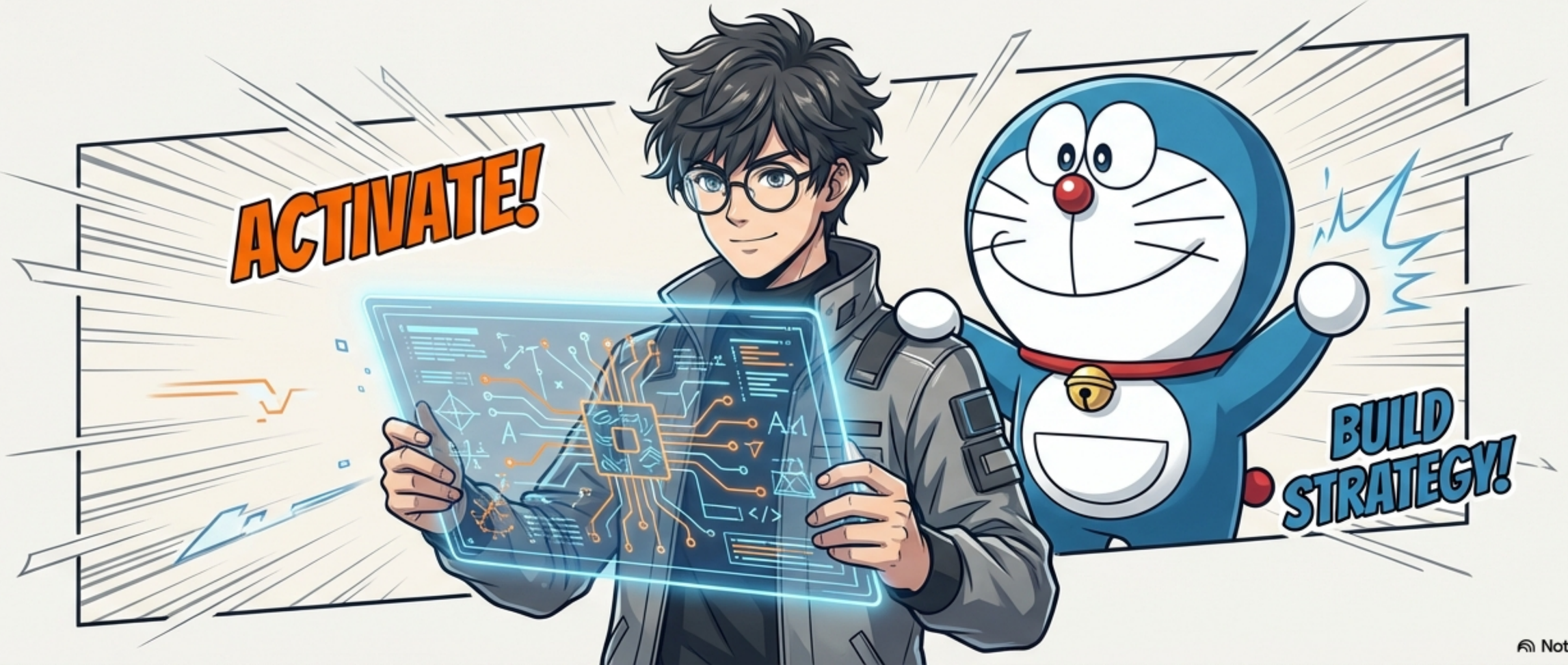
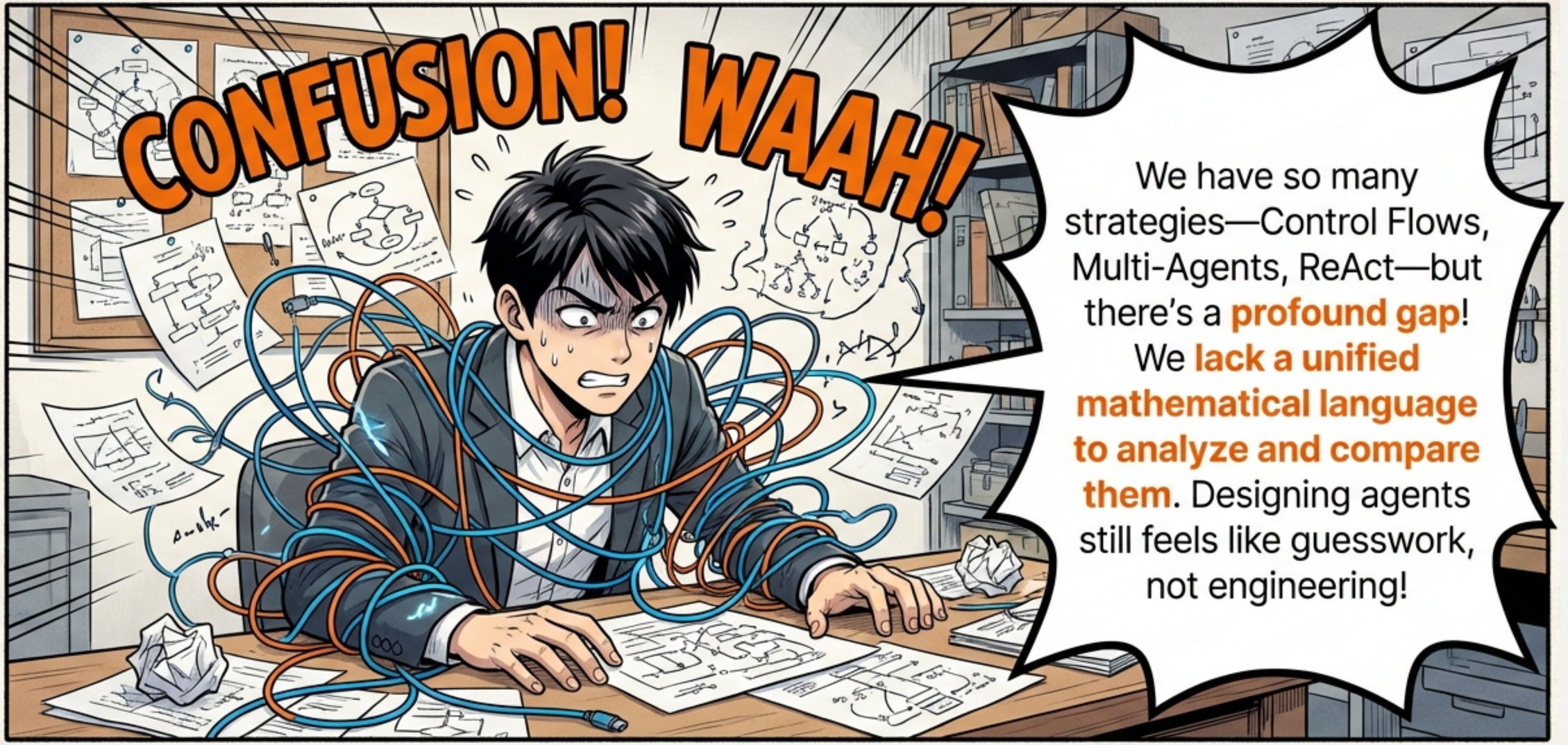
AI Agent Adventure: The Mathematical Map of LLM Strategies

Moving from Empirical Art to Systematic Engineering



The “High-Dimensional Art” of Agent Design

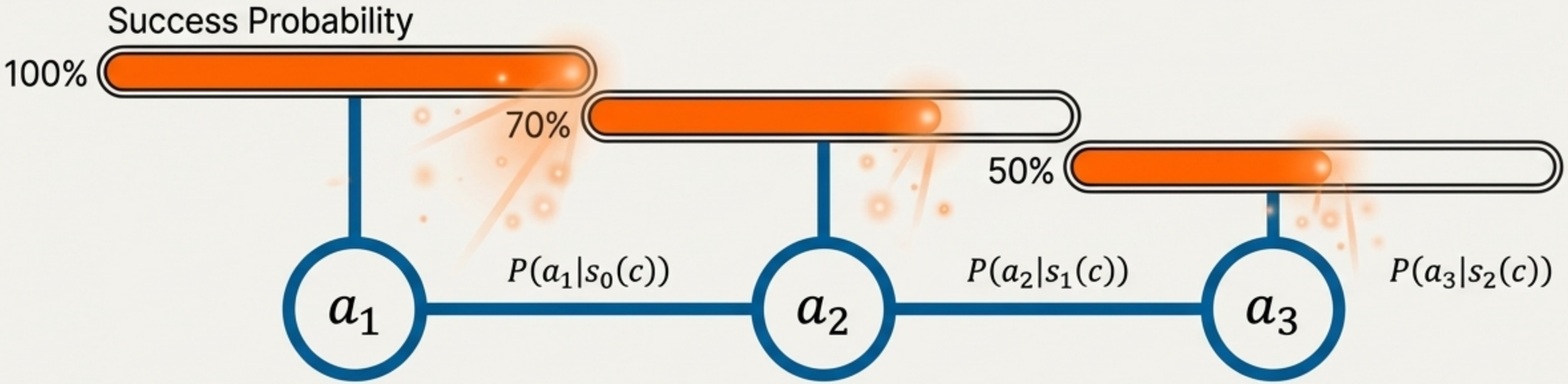


A Unified Language for All Agent Strategies

This paper introduces a **unified mathematical framework**! It allows us to see *every* agent architecture as a **probabilistic process** aiming to maximize the chance of success.

Important: This is a **modeling framework** for analysis, not a new algorithm for training the underlying LLM!

Every Agent Operation is a “Chain of Probabilities”



The probability of success for an action sequence is the product of individual probabilities:

$$P(a|c) = P(a_n|s_{n-1}, c) * \ldots * P(a_1|s_0, c).$$

Since each $P < 1$, the total probability **inevitably shrinks** as the chain lengthens.

The goal of ***all*** agent design is to fight this decay! “Thoughts” (t_i) are the mechanism to **boost the probability** of choosing the ***correct*** next action a_i!

Level 1: The Degrees of Freedom in a ReAct Loop

In a basic ReAct loop, which is fundamentally a "random walk," we have only **two main levers** to optimize the probability chain:

1. **Static Prompt (s_0):** The initial prompt engineering that defines the starting state `$s_0(c)$`.
2. **State Update (u):** How we append history to the context, typically `$u(a_i, s_{i-1}) = concat(s_{i-1}, t_i, o_i)$`.

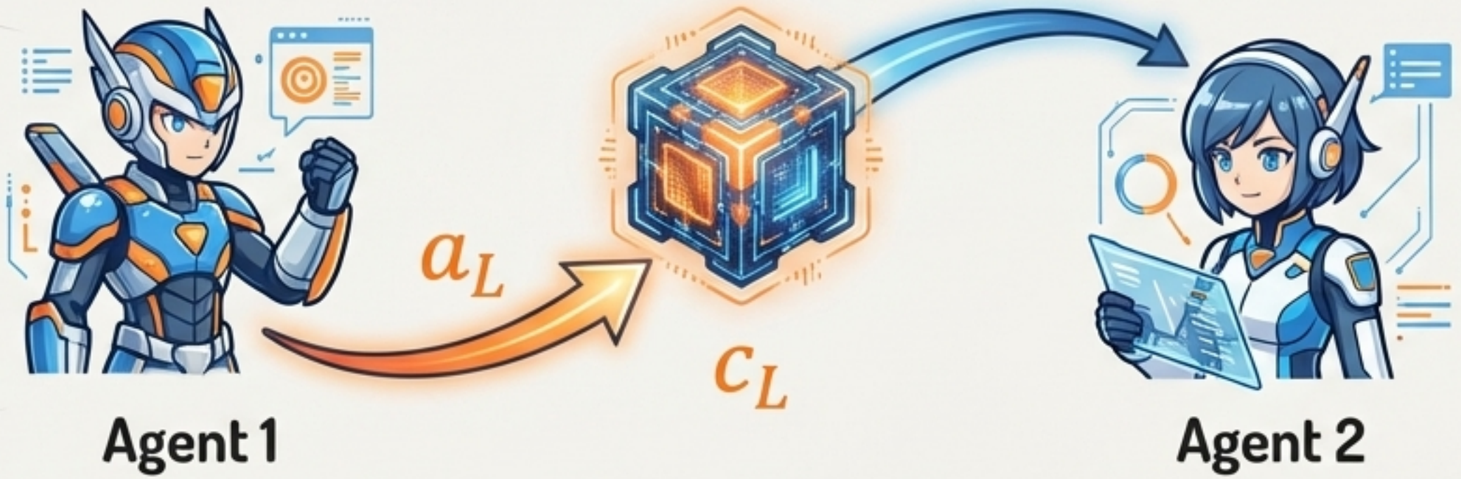
So it's a "random walk" with very few controls. We set it up and hope it doesn't fall into hallucinatory loops.

Level 2: Control Flow Exposes Dynamic Levers

Control Flow architectures provide more powerful levers by allowing us to manipulate `s`, `u`, and `F` at each step:

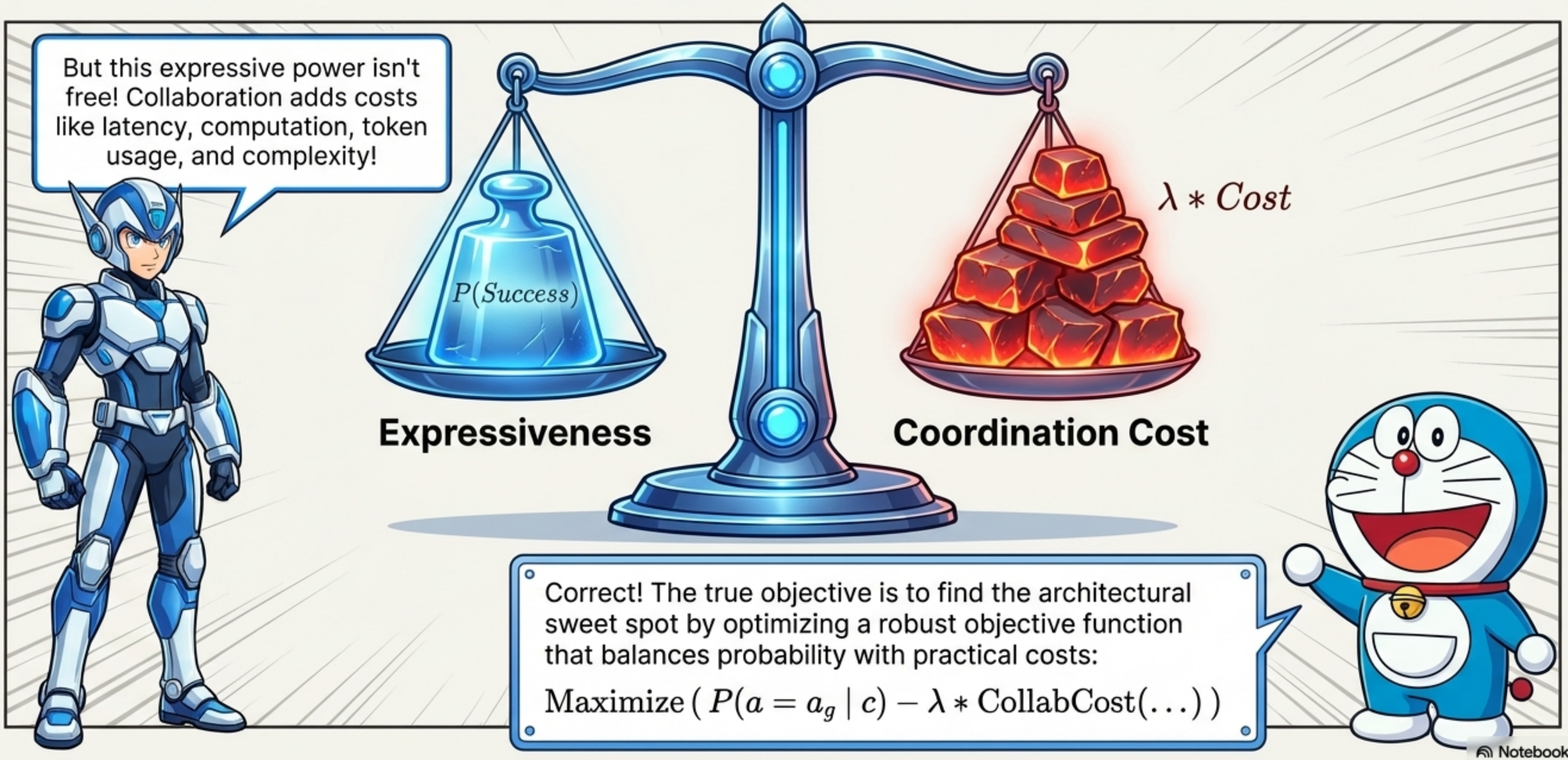
- **Dynamic Prompts**: Use a different, specialized prompt **`s(c)`** at each node in a graph.
- **Partitioned Actions**: Make only a subset of tools (α_i) available at each step.
- **Custom Updates**: Strategically prune or reset memory (**u**) at key moments.

The Ultimate Lever: Multi-Agent Collaboration



The multi-agent paradigm introduces a fundamentally **new** Degree of Freedom, modeled by the probability $P(c_L \mid a_L)$.

Balancing Expressiveness with Coordination Cost



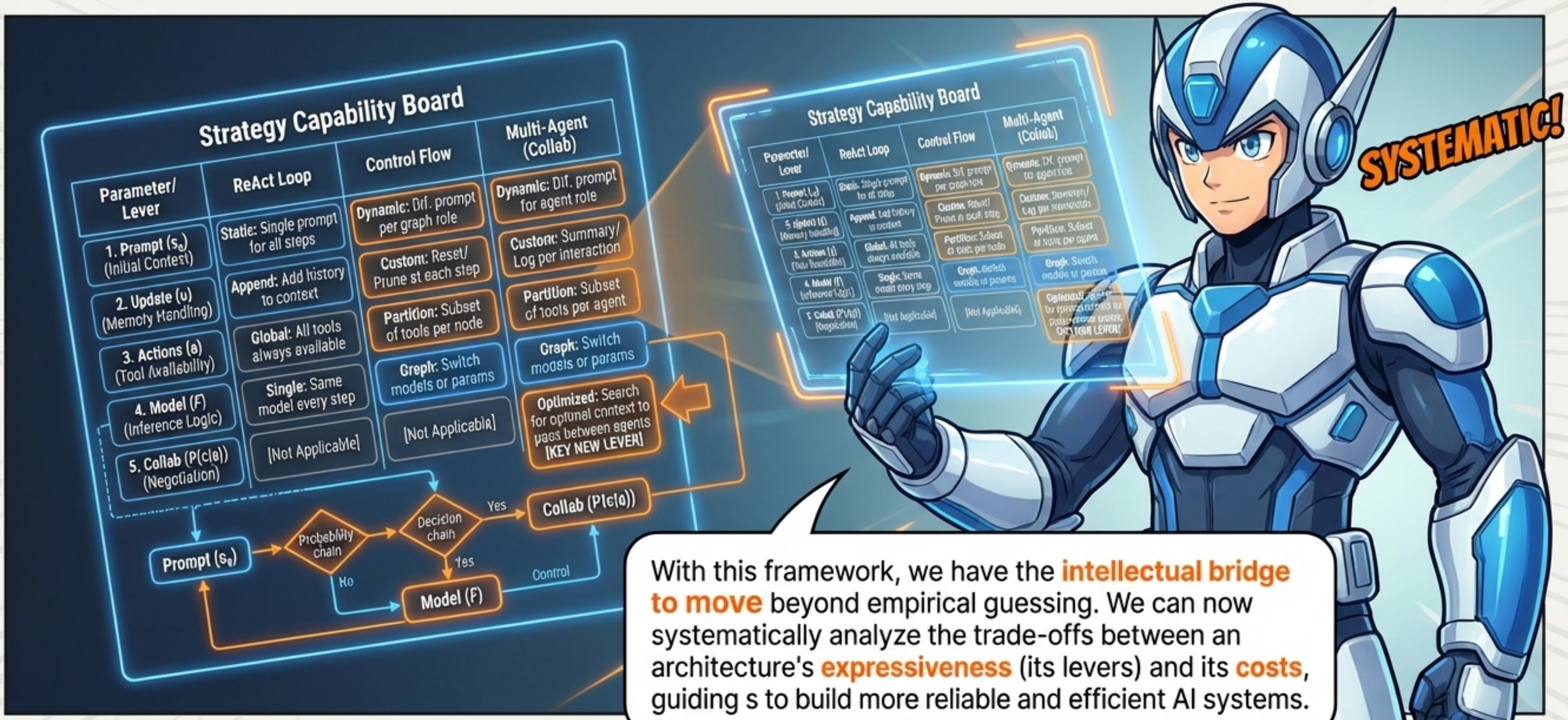
A Unified Map of Agent Capabilities

Parameter / Lever	ReAct Loop	Control Flow	Multi-Agent (Collab)
1. Prompt (s_0) (Initial Context)	**Static:** Single prompt for all steps	**Dynamic:** Dif. prompt per graph node	**Dynamic:** Dif. prompt for agent role
2. Update (u) (Memory Handling)	**Append:** Add history to context	**Custom:** Reset/ Prune at each step	**Custom:** Summary/ Log per interaction
3. Actions (a) (Tool Availability)	**Global:** All tools always available	**Partition:** Subset of tools per node	**Partition:** Subset of tools per agent
4. Model (*F*) (Inference Logic)	**Single:** Same model every step	**Graph:** Switch models or params	**Graph:** Switch models or params
5. Collab (P(c|a)) (Negotiation)	[Not Applicable]	[Not Applicable]	**Optimized:** Search for optimal context to pass between agents **[KEY NEW LEVER]**

This framework provides a clear way to compare architectures by the **'Degrees of Freedom'** they expose.

- **ReAct**: Static prompt, simple append memory.
- **Control Flow**: Unlocks dynamic, per-step levers for prompts, memory, and actions.
- **Multi-Agent**: Introduces the unique 'Collaboration' lever, turning inter-agent communication into an optimizable search space.

From Guesswork to Guided Engineering



Your Mathematical Map: Key Takeaways

Model Everything as a Probability Chain: Success probability naturally decays. Your job is to bolster it at every step.

Think in "Degrees of Freedom": Identify the levers your architecture gives you (`$s_0$`, `u`, `a`, `F`) to optimize that chain.

Unlock Levers with Architecture: Control Flow unlocks dynamic optimization. Multi-Agent systems unlock the powerful 'Collaboration' lever.

Always Balance Expressiveness with Cost: The goal isn't just maximizing probability, but optimizing the equation: `P(Success) - Cost`.

The Intellectual Bridge to Systematic Agent Design

This work provides the intellectual bridge connecting high-level agent concepts with precise, quantifiable principles. It offers designers a clear roadmap for dissecting and optimizing architectural trade-offs, enabling the systematic maximization of success probability within any complex agentic system.

To explore the full formalism, read the paper:
"Mathematical Framing for Different Agent Strategies."