Objective

Given the vanishing line of the ground plane $\mathbf{l}$ and the vertical vanishing point $\mathbf{v}$ and the top $(\mathbf{t}_1, \mathbf{t}_2)$ and base $(\mathbf{b}_1, \mathbf{b}_2)$ points of two line segments as in figure 8.20, compute the ratio of lengths of the line segments in the scene.

Algorithm

(i) Compute the vanishing point $\mathbf{u} = (\mathbf{b}_1 \times \mathbf{b}_2) \times \mathbf{l}$.

(ii) Compute the transferred point $\tilde{\mathbf{t}}_1 = (\mathbf{t}_1 \times \mathbf{u}) \times \mathbf{l}_2$ (where $\mathbf{l}_2 = \mathbf{v} \times \mathbf{b}_2$).

(iii) Represent the four points $\mathbf{b}_2, \tilde{\mathbf{t}}_1, \mathbf{t}_2$ and $\mathbf{v}$ on the image line $\mathbf{l}_1$ by their distance from $\mathbf{b}_2$, as $0, \tilde{t}_1, t_2$ and v respectively.

(iv) Compute a 1D projective transformation $\mathtt{H}_{2\times2}$ mapping homogeneous coordinates $(0, 1) \mapsto (0, 1)$ and $(v, 1) \mapsto (1, 0)$ (which maps the vanishing point $\mathbf{v}$ to infinity). A suitable matrix is given by

$$\mathtt{H}_{2\times2} = \begin{bmatrix} 1 & 0 \\ 1 & -v \end{bmatrix}.$$

(v) The (scaled) distance of the scene points $\tilde{\mathbf{T}}_1$ and $\mathbf{T}_2$ from $\mathbf{B}_2$ on $\mathbf{L}_2$ may then be obtained from the position of the points $\mathtt{H}_{2\times2}(\tilde{t}_1, 1)^\mathsf{T}$ and $\mathtt{H}_{2\times2}(t_2, 1)^\mathsf{T}$. Their distance ratio is then given by

$$\frac{d_1}{d_2} = \frac{\tilde{t}_1(v - t_2)}{t_2(v - \tilde{t}_1)}$$

Algorithm 8.1. *Computing scene length ratios from a single image.*

mation to the image line which maps $\mathbf{v}$ to infinity. A geometric construction of this projectivity is shown in figure 8.20(d) (see example 2.20(*p*51)).

Details of the algorithm to carry out these two steps are given in algorithm 8.1.

Note, no knowledge of the camera calibration $\mathtt{K}$ or pose is necessary to apply the algorithm. In fact, the position of the camera centre relative to the ground plane can also be computed. The algorithm is well conditioned even when the vanishing point and/or line are at infinity in the image. For example, under affine image conditionings, or if the image plane is parallel to the vertical scene direction (so that $\mathbf{v}$ is at infinity). In these cases the distance ratio simplifies to $\frac{d_1}{d_2} = \frac{\tilde{t}_1}{t_2}$.

Example 8.25. Measuring a person's height in a single image

Suppose we have an image which contains sufficient information to compute the ground plane vanishing line and the vertical vanishing point, and also one object of known height for which the top and base are imaged. Then the height of a person standing on the ground plane can be measured anywhere in the scene provided that their head and feet are both visible. Figure 8.21(a) shows an example. The scene contains plenty of horizontal lines from which to compute a horizontal vanishing point. Two such vanishing points determine the vanishing line of the floor (which is the horizon for this image). The scene also contains plenty of vertical lines from which to compute a vertical vanishing point (figure 8.21(c)). Assuming that the two people are standing vertically, then their relative height may be computed directly from their length ratio using algorithm 8.1. Their absolute height may be determined by comput-

Fig. 8.21. **Height measurements using affine properties.** *(a) The original image. We wish to measure the height of the two people. (b) The image after radial distortion correction (see section 7.4(*p*189)). (c) The vanishing line (shown) is computed from two vanishing points corresponding to horizontal directions. The lines used to compute the vertical vanishing points are also shown. The vertical vanishing point is not shown since it lies well below the image. (d) Using the known height of the filing cabinet on the left of the image, the absolute height of the two people are measured as described in algorithm 8.1. The measured heights are within 2cm of ground truth. The computation of the uncertainty is described in [Criminisi-00].*

ing their height relative to an object on the ground plane with known height. Here the known height is provided by the filing cabinet. The result is shown in figure 8.21(d).

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8.8 Determining camera calibration K from a single view

We have seen that once $\boldsymbol{\omega}$ is known the angle between rays can be measured. Conversely if the angle between rays is known then a constraint is placed on $\boldsymbol{\omega}$. Each known angle between two rays gives a constraint of the form (8.13) on $\boldsymbol{\omega}$. Unfortunately, for arbitrary angles, and known $\mathbf{v}_1$ and $\mathbf{v}_2$, this gives a quadratic constraint on the entries of $\boldsymbol{\omega}$. If the lines are perpendicular, however, (8.13) reduces to (8.16) $\mathbf{v}_1^{\mathsf{T}}\boldsymbol{\omega}\mathbf{v}_2 = 0$, and the constraint on $\boldsymbol{\omega}$ is linear.

A linear constraint on $\boldsymbol{\omega}$ also results from a vanishing point and vanishing line arising from a line and its orthogonal plane. A common example is a vertical direction and horizontal plane as in figure 8.19. From (8.17) $\mathbf{l} = \boldsymbol{\omega}\mathbf{v}$. Writing this as $\mathbf{l} \times (\boldsymbol{\omega}\mathbf{v}) = \mathbf{0}$