

Fig. 8.20. Computing length ratios of parallel scene lines. (a) 3D geometry: The vertical line segments  $\mathbf{L}_1 = \langle \mathbf{B}_1, \mathbf{T}_1 \rangle$  and  $\mathbf{L}_2 = \langle \mathbf{B}_2, \mathbf{T}_2 \rangle$  have length  $d_1$  and  $d_2$  respectively. The base points  $\mathbf{B}_1, \mathbf{B}_2$  are on the ground plane. We wish to compute the scene length ratio  $d_1:d_2$  from the imaged configuration. (b) In the scene the length of the line segment  $\mathbf{L}_1$  may be transferred to  $\mathbf{L}_2$  by constructing a line parallel to the ground plane to generate the point  $\widetilde{\mathbf{T}}_1$ . (c) Image geometry: 1 is the ground plane vanishing line, and  $\mathbf{v}$  the vertical vanishing point. A corresponding parallel line construction in the image requires first determining the vanishing point  $\mathbf{u}$  from the images  $\mathbf{b}_i$  of  $\mathbf{B}_i$ , and then determining  $\widetilde{\mathbf{t}}_1$  (the image of  $\widetilde{\mathbf{T}}_1$ ) by the intersection of  $\mathbf{l}_2$  and the line  $\langle \mathbf{t}_1, \mathbf{u} \rangle$ . (d) The line  $\mathbf{l}_3$  is parallel to  $\mathbf{l}_1$  in the image. The points  $\widehat{\mathbf{t}}_1$  and  $\widehat{\mathbf{t}}_2$  are constructed by intersecting  $\mathbf{l}_3$  with the lines  $\langle \mathbf{t}_1, \widetilde{\mathbf{t}}_1 \rangle$  and  $\langle \mathbf{t}_1, \mathbf{t}_2 \rangle$  respectively. The distance ratio  $d(\mathbf{b}_2, \widehat{\mathbf{t}}_1): d(\mathbf{b}_2, \widehat{\mathbf{t}}_2)$  is the computed estimate of  $d_1:d_2$ .

Step 1: Map the length of one line segment onto the other. In 3D the length of  $L_1$  may be compared to  $L_2$  by constructing a line parallel to the ground plane in the direction  $\langle B_1, B_2 \rangle$  that transfers  $T_1$  onto  $L_2$ . This transferred point will be denoted  $\widetilde{T}_1$  (see figure 8.20(b)). In the image a corresponding construction is carried out by first determining the vanishing point u which is the intersection of  $\langle b_1, b_2 \rangle$  with l. Now any scene line parallel to  $\langle B_1, B_2 \rangle$  is imaged as a line through u, so in particular the image of the line through  $T_1$  parallel to  $\langle B_1, B_2 \rangle$  is the line through  $t_1$  and  $t_2$ . The intersection of the line  $t_1$  with  $t_2$  defines the image  $t_1$  of the transferred point  $t_2$  (see figure 8.20(c)).

Step 2: Determine the ratio of lengths on the scene line. We now have four collinear points on an imaged scene line and wish to determine the actual length ratio in the scene. The four collinear image points are  $\mathbf{b}_2$ ,  $\tilde{\mathbf{t}}_1$ ,  $\mathbf{t}_2$  and  $\mathbf{v}$ . These may be treated as images of scene points at distances  $0, d_1, d_2$  and  $\infty$ , respectively, along the scene line. The affine ratio  $d_1: d_2$  may be obtained by applying a projective transfor-