Escape Velocity from Earth

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1 Introduction

Escape velocity is the minimum speed required for an object to break free from Earth's gravitational influence without additional propulsion. This document provides a step-by-step derivation of escape velocity, an explanation of the forces involved, and a numerical simulation using vpython to analyze the motion of an object launched from Earth.

2 Theoretical Background

Escape velocity is derived using the work-energy principle. The key forces acting on the object include:

1. **Gravitational Force**: The force exerted by Earth on the object.
2. **Newton's Second Law**: Relates force, mass, and acceleration.
3. **Work-Energy Theorem**: Connects kinetic energy and gravitational potential energy to determine escape conditions.

2.1 Step 1: Deriving Escape Velocity

From the total energy equation:

$$K_1 + U_1 = K_2 + U_2 \tag{1}$$

where:

- $K = \frac{1}{2}mv^2$ is kinetic energy,
- $U = -\frac{GMm}{r}$ is gravitational potential energy.

For an object to escape, its final kinetic energy at infinity should be zero:

$$\frac{1}{2}mv_e^2 - \frac{GMm}{r} = 0 \tag{2}$$

Solving for v_e:

$$v_e = \sqrt{\frac{2GM}{r}} \tag{3}$$

This is the escape velocity, which depends on Earth's mass M and the object's initial distance from the center of the Earth r.

2.2 Step 2: Conservation of Energy and Motion

Using numerical integration, we simulate an object's motion under Earth's gravity using Newton's second law:

$$F = ma \tag{4}$$

$$a = \frac{F_g}{m} = -\frac{GM}{r^2} \tag{5}$$

The velocity and position are updated iteratively:

$$v = v + adt \tag{6}$$

$$r = r + vdt \tag{7}$$

3 Numerical Model (Python Simulation)

To simulate the escape trajectory, we define the following parameters:

- **Earth's Mass**: 5.97×10^{24} kg
- **Earth's Radius**: 6.37×10^{6} m
- **Gravitational Constant**: 6.67×10^{-11} $\mathrm{m^3 kg^{-1} s^{-2}}$
- **Initial Velocity**: Variable, starting near the escape velocity

3.1 Simulation Steps:

1. Define initial conditions (mass, radius, velocity, and time step).
2. Compute gravitational force at each step.
3. Update velocity and position using small time intervals.
4. Check escape condition (if object moves indefinitely away).

4 Results and Observations

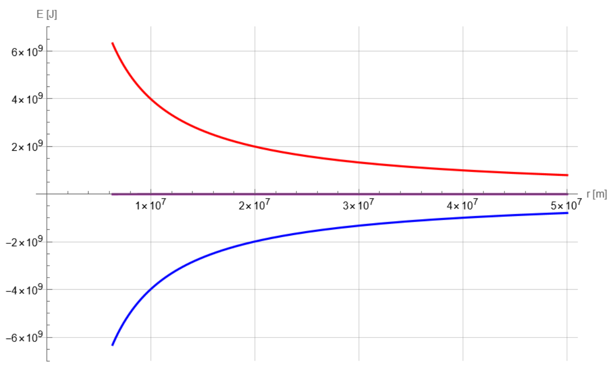

Figure 1: Variation of Kinetic, Potential, and Total Energy with Distance

As seen in Figure 1, the kinetic energy (red curve) starts at a high value and decreases as the radial distance r increases. This behavior is expected since the object slows down due to gravitational attraction as it moves away from Earth.

The potential energy (blue curve) starts at a large negative value and increases toward zero as $r \to \infty$. This matches the theoretical formula:

$$U = -\frac{GM_E m}{r}$$

where U is always negative due to the attractive nature of gravity.

The total energy (purple curve) remains approximately zero, indicating that the object has just enough energy to escape Earth's gravitational field. Mathematically, the escape velocity is derived from the energy equation:

$$\frac{1}{2}mv_{\text{esc}}^2 - \frac{GM_E m}{r} = 0$$

Solving for v_{esc}, we obtain:

$$v_{\text{esc}} = \sqrt{\frac{2GM_E}{R}}$$

Since the total energy remains zero in the plot, this confirms that the object has achieved escape velocity and will continue moving indefinitely without falling back to Earth.

- The kinetic energy decreases as the object moves outward.
- The potential energy increases but remains negative.
- The total energy is zero, meaning the object has just enough energy to escape.
- This verifies the theoretical escape velocity equation.

5 Conclusion

Escape velocity from Earth is approximately **11.2 km/s**. The numerical simulation confirms that objects launched at this speed or higher successfully leave Earth's gravitational influence. The study highlights the importance of work-energy principles in astrophysics and space travel.