Keplers laws of Planetary Motion

Haroon Sakhi

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1 Keplers laws

In the *17^th^* century the German astronomer Johannes Kepler proposed three laws that described the cosmic dance of our universe. The laws that he described were as follows;

1^st^ Law

All planets will orbit in an elliptical path. With the the star on one of the focal points of the orbit. The reason for this behaviour was explained long after Keplers rst proposed it. The

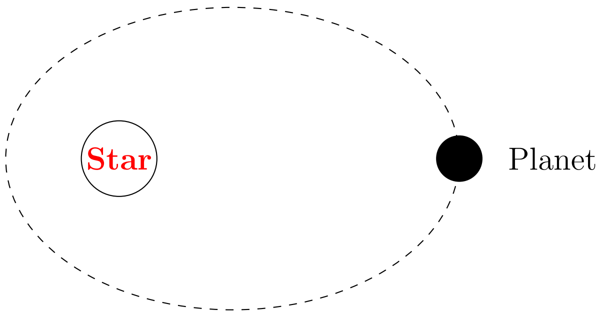


Figure 1: Kepler's First Law

explanation comes from how gravity behaves, according to newton all masses attract each other, and this attraction can be felt on very long distances. This causes the planets to veer of their orbits slightly and results in elliptical paths. The cause is the rarity of the conditions for circular paths.The orbital velocity of a an object in space is given by the following equation.

$$v = \sqrt{\frac{GM}{r}}$$

For a perfect circular orbit the tangential velocity v must be exactly equal to the factor $\sqrt{\frac{GM}{r}}$which one can clearly see is statistically very rare

2^nd^ Law

The area swept by a planet in a specific time will always remain constant. This is due to the change in velocity of the planet along its orbit,

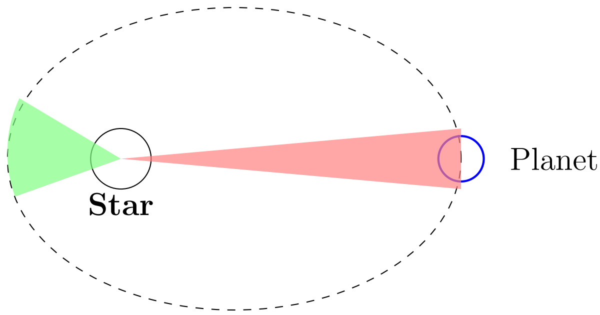


Figure 2: Red area and Green area are the same according to Kepler's second law

3rd Law

Kepler's third law states that the time period squared of a planet is proportional to the radius cubed. Mathematically this can be written as:

$$T^2 \propto kr^3$$

Kepler originally did not know the value of k; it was only after Newton's law of gravitation that we know:

$$k = \frac{4\pi^2}{GM}$$

2 Theory

Gravitational force between two bodies is given by:

$$F_G = \frac{GMm}{r^2}$$

While the centripetal force is given by:

$$F_c = \frac{mv^2}{r}$$

We know that for any body in orbit, the gravitational force must be equal to the centripetal force:

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$
$$v^2 = \frac{GM}{r}$$

This is the condition that determines if an object will stay in orbit or not. This equation can be further used to get the constant in Kepler's third law by using the definition of tangential velocity ($v = r\omega$):

$$r^2\omega^2 = \frac{GM}{r}$$
$$r^2 4\pi^2 T^{-2} = \frac{GM}{r}$$
$$T^2 = \frac{4\pi^2}{GM}r^3$$

3 Algorithm

1. Calculate the distance between the planet and the star using Pythagoras's theorem:

$$r^2 = x^2 + y^2$$

2. Calculate the gravitational force using Newton's law of gravity:

$$F_G = \frac{GMm}{r^2}$$

3. Calculate the acceleration due to the gravitational force:

$$a = \frac{F_G}{m}$$

4. Divide the acceleration into its x and y components:

$$a_x = a\cos(\theta) = a\left(\frac{x}{r}\right)$$

$$a_y = a\sin(\theta) = a\left(\frac{y}{r}\right)$$

5. Calculate the velocity at each step using the acceleration components:

$$v_x = v_x + a_x dt$$

$$v_y = v_y + a_y dt$$

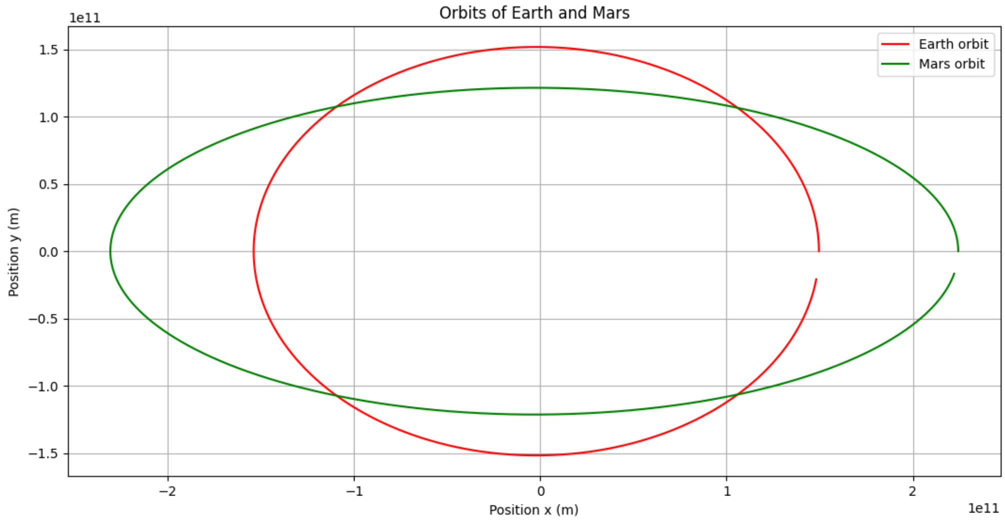
6. Calculate the position at each step using the velocity and acceleration components:

$$x = x + v_x t + \frac{1}{2}a_x dt^2$$

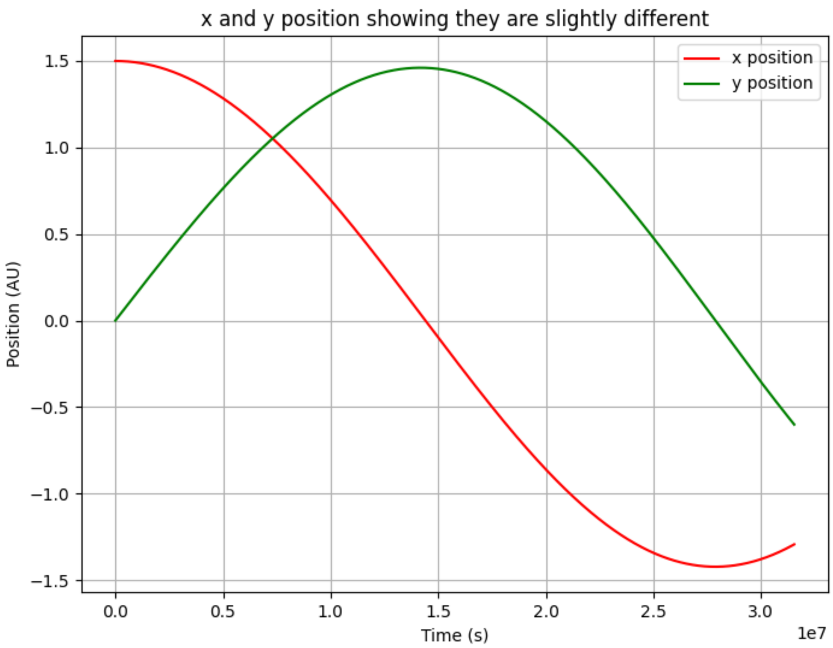
$$y = y + v_y t + \frac{1}{2}a_y dt^2$$

7. Increment the value of t.

4 Results



The orbits of planets although are elliptical, the eccentricity is incredibly low with earth having an eccentricity of 0.017 while Mars the most eccentric planet has an eccentricity of 0.097. This can clearly be seen in above figure which shows the orbit of the two planets over an year.


x and y position showing they are slightly different
x position
y position
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Position (AU)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
1e7
Time (s)