Simulation of SHM

Haroon Sakhi

March 15, 2025

1 Simple Harmonic Oscillator

SHM can be found throughout nature in many di erent forms, But the key characteristic of SHM is that the system follows a predictable repeating paths. The most common example of SHM is of the spring mass system. We will use Newtons laws to simulate this system under di erent conditions.

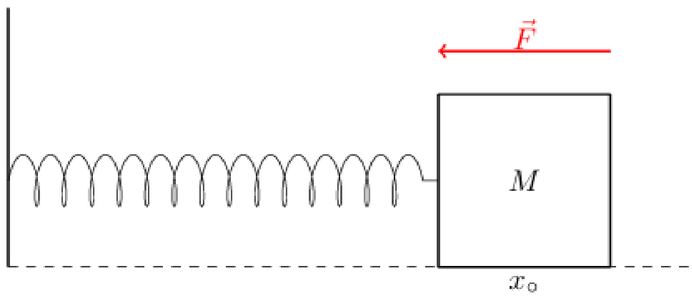


Figure 1: Simple Spring mass system

2 Theory

According to Newton's second law;

$$\vec{F} = m\vec{a}$$

We can write $\vec{a}$ as a second derivative of position

$$\vec{F} = m\ddot{x}$$

To find the force we will use the relation;

$$\vec{F} = -\nabla U(x)$$

Where $U(x)$ is the potential energy of the system. For a spring, the potential energy is defined as;

$$U(x) = \frac{1}{2}kx^2$$

Hence, the equation of motion for a spring-mass system under ideal conditions is defined as;

$$-kx = m\ddot{x}$$

Where k is the spring constant. Under realistic conditions, there will always be a force opposing the motion to slow the system down. This force must be velocity-dependent because it must only exist when the mass is in motion. Hence, the complete equation for the system will be;

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

Solving the equation we get;

$$x(t) = Ae^{-bv}\cos(\omega t)$$

Analyzing the equation, we can clearly see that if b is zero, the system will oscillate forever as predicted in a periodic motion. Now, to simulate the system, we would compute the position of the mass at each instant of time. For each dt, we would need to calculate the position at t_o and $t_o + dt$.
The velocity during the time interval and the acceleration in the time interval.
We can easily find the acceleration by using Newton's third law,

$$a = \frac{F}{m}$$

We know that the distance x can be calculated by the formula;

$$x_{n+1} = x_n + vdt + a\frac{dt^2}{2!}$$

Where the velocity is given by;

$$v_{n+1} = v_n + adt$$

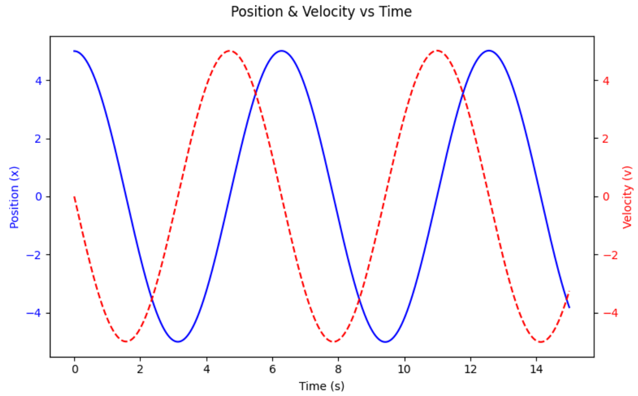
Using these equations, we can compute the position of the mass at each point in time.

3 Algorithm

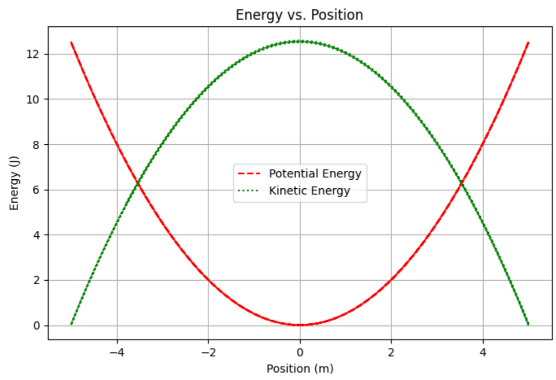
1. Calculate Potential Energy at (x_o, t_o).
2. Calculate Force using the relation
$$\vec{F} = -\nabla U(x)$$
3. Calculate acceleration using the equation
$$a = \frac{F}{m}$$
4. Calculate the velocity.
$$v_{n+1} = v_n + adt$$
5. Calculate position using the equation.
$$x_{n+1} = x_n + vdt + a\frac{dt^2}{2!}$$
6. Increment the value of t.

4 Results

For $b = 0$, $x_0 = 5.00$, $m = 1.00$, $k = 1.00$;



(a) Position vs Time



(b) Energy vs Position

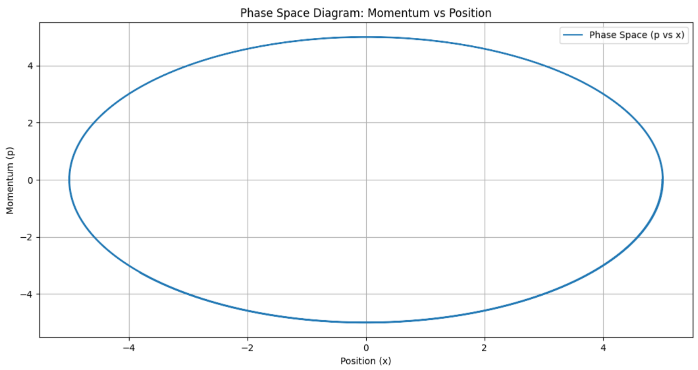
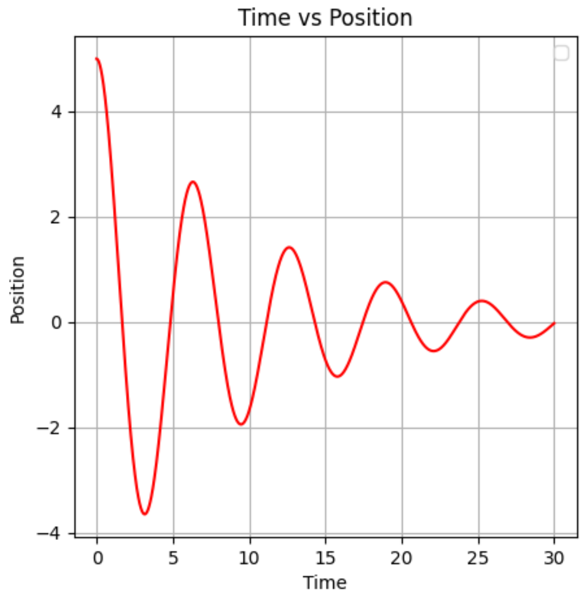


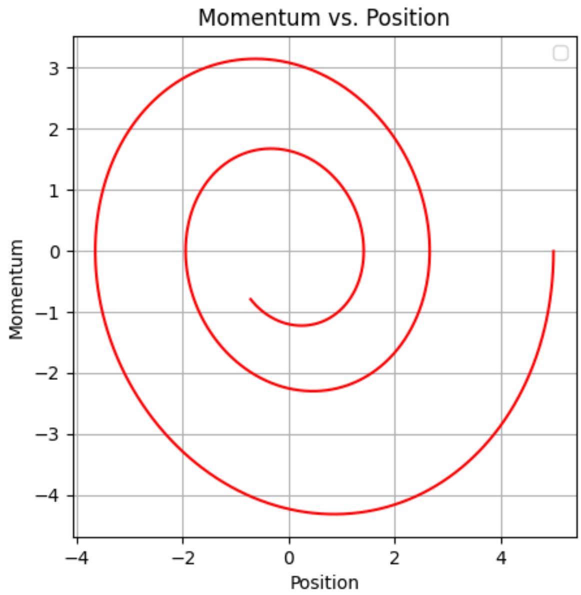
Figure 2: (c) Momemtum vs Position

5 Results

For $b = 0.20$, $x_0 = 5.00$, $m = 1.00$, $k = 1.00$;



(a) Position vs Time



(b) Momemtum vs Position

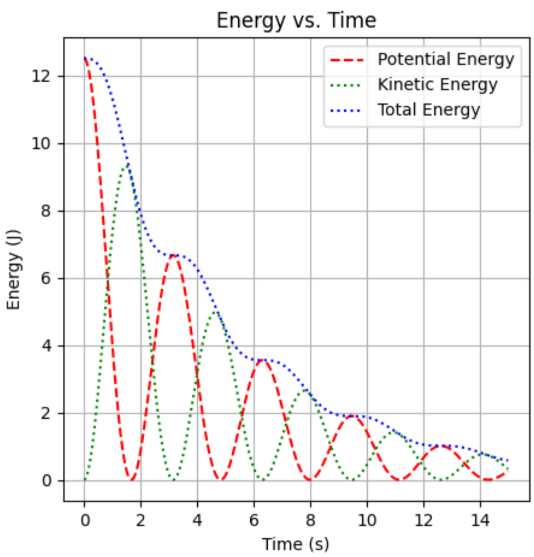


Figure 3: (c) Energy vs Time