Van der Pol equation

$$y'' = y'(1 - y^2) - y$$

Define two new variables, a, b such that:

$$a = y, a' = b = y', b' = y''$$

The equations then become:

$$\begin{bmatrix} a' \\ b' \end{bmatrix} = \begin{bmatrix} b \\ b(1 - a^2) - a \end{bmatrix}$$

Which is a first order ODE, but not linear in a, b. The question doesn't ask us to make it linear.

Blasius Equation

$$y''' = -yy''$$

We defined three new variables, a, b, c such that:

$$c = y$$

$$b = c' = y'$$

$$a = b' = c'' = y''$$

The equations then become:

$$\begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = \begin{bmatrix} -ca \\ a \\ b \end{bmatrix}$$

Newton's Second Law of Motion for two body problem

$$y_1'' = -\frac{GMy_1}{(y_1^2 + y_2^2)^{3/2}}$$

$$y_2'' = -\frac{GMy_2}{(y_1^2 + y_2^2)^{3/2}}$$

So many variables ☹

We define 4 variables, a, b, c, d such that:

$$b = y_1$$

$$a = b' = y_1'$$

$$d = y_2$$

$$c = d' = y_2'$$

The system of equations then becomes:

$$\begin{bmatrix} a' \\ b' \\ c' \\ d' \end{bmatrix} = \begin{bmatrix} -\frac{GMb}{(b^2 + d^2)^{3/2}} \\ a \\ -\frac{GMd}{(b^2 + d^2)^{3/2}} \\ c \end{bmatrix}$$