Eigenvalues of projection matrices:

We know:

$$P^2 = P$$

Let λ be an eigenvalue. Hence, $Px = \lambda x$ where x is an eigenvector.

$$\Rightarrow P^2 x = \lambda^2 x$$

$$\Rightarrow Px = \lambda^2 x$$

Infact, $P^k x = \lambda^k x \Rightarrow Px = \lambda^k x = \lambda x \; \forall k \in \mathbb{Z}^+$. This is satisfied by $\boldsymbol{\lambda} \in \{\mathbf{0}, \mathbf{1}\}$

Householder matrix eigenvalue:

Recall that H is orthogonal and symmetric. Hence, $H^T H = H^2 = I$.

Let λ be an eigenvalue and x be the corresponding eigenvector.

$$Hx = \lambda x$$

However, $H^2 x = Ix = \lambda^2 x \Rightarrow (\lambda^2 - 1)x = 0 \Rightarrow \lambda = \pm 1$ since x is a non-zero vector.

We can see that $Hv = -v$ (simple multiplication), hence -1 is an eigenvalue.

$Hu = u$ for any u perpendicular to v ($v^T u = 0$). Hence +1 is also an eigenvalue