$$(t, y) \in \{(-1,1), (0,0), (1,1)\}$$

Monomial basis

Let the interpolating polynomial be:

$$p_M(t) = \lambda_1 + \lambda_2 t + \lambda_3 t^2$$

Vandermonde matrix A is:

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Therefore, λ can be obtained by solving the system $A\lambda = y$. The solutions turns out to be $\lambda_3 = 1, \lambda_1 = \lambda_2 = 0$.

Hence, the polynomial is t^2

Lagrange basis

$$l_1(t) = \frac{t(t-1)}{2}$$

$$l_2(t) = -(t+1)(t-1)$$

$$l_3(t) = \frac{t(t+1)}{2}$$

$$p_L(t) = l_1(t) + 0 * l_2(t) + t_3(t) = t^2$$

Newton basis

$$p_N(t) = x_1 + x_2(t+1) + x_3 t(t+1)$$

Solving the system of equations, we get that:

$$x_1 = 1$$

$$x_2 = -1$$

$$x_3 = 1$$

Substituting into and simplifying $p_N(t)$ gives us that $p_N(t) = t^2$

Clearly, all three equations are the same.