

# Accuracy of trapezoid method

The trapezoid method simply adds backward and forward euler to get:

$$\begin{aligned}y_{k+1} &= y_k + \frac{h_k \lambda (y_k + y_{k+1})}{2} \\ \Rightarrow y_{k+1} \left(1 - \frac{h_k \lambda}{2}\right) &= y_k \left(1 + \frac{h_k \lambda}{2}\right) \\ \Rightarrow y_{k+1} &= y_k \frac{1 + \delta}{1 - \delta}\end{aligned}$$

Here,  $h_k$  is  $\Delta t$ . The growth factor is  $\frac{1 + \frac{\lambda \Delta t}{2}}{1 - \frac{\lambda \Delta t}{2}}$ . As  $\lambda \Delta t \rightarrow -\infty$ ,  $G \rightarrow -1^-$ . This can be seen easily using the L'Hospital rule.