Given n data points, we have $n-1$ quadratics. Each of these quadratics has 3 parameters, hence we have a total of $3(n-1)$ original parameters. Concretely, Let the knots be at $\{x_i\}_{i=1}^{n}$. Let the quadratic between x_i, x_{i+1} be f_i $\forall\, i \in \{1,2 \ldots (n-1)\}$. Each f_i has 3 parameters.

Let the corresponding values of the data at x_i be y_i.

We know that our splines have to fit the data. Hence, we have $2(n-1)$ equations as follows:

Left side: $f_i(x_i) = y_i \;\forall\, i \in \{1 \ldots (n-1)\}$

Right side: $f_i(x_{i+1}) = y_{i+1} \;\forall\, i \in \{1 \ldots (n-1)\}$

There are now $n-1$ parameters that are unaccounted for.

Once continuously differentiable:

Since each f_i is continuously differentiable, if the derivative of f_i, f_{i+1} match at x_i, then the entire system is once continuously differentiable.

Requiring that $f_i^{(1)}(x_{i+1}) = f_{i+1}^{(1)}(x_{i+1}) \;\forall\, i \in \{1 \ldots (n-2)\}$ adds $n-2$ Equations.

This leaves us with 1 free parameter.

Hence, **yes, it is possible to create a once continuously differentiable interpolant**.

Twice continuously differentiable:

A twice continuously differentiable function is also once continuously differentiable.

To ensure twice continuously differentiability when the interpolant is already once continuously differentiable is the same as saying $f_i^{(2)}(x_{i+1}) = f_{i+1}^{(2)}(x_{i+1}) \;\forall\, i \in \{1,2 \ldots (n-2)\}$

This add $n-2$ equations again. Leading to a total of $4n-6$ equations for a total of $3n-3$ parameters.

The maximum value of n for which creating such an interpolant is possible can be obtained by noticing that $4n-6 \leq 3n-3$ must be satisfied. Hence, $\boldsymbol{n \leq 3.}$

However, sometimes, it may be possible even if $n > 3$ **IF** some of the equations are redundant.