Accuracy of trapezoid method

The trapezoid method simply adds backward and forward euler to get:

$$y_{k+1} = y_k + \frac{h_k \lambda (y_k + y_{k+1})}{2}$$

$$\Rightarrow y_{k+1}\left(1 - \frac{h_k \lambda}{2}\right) = y_k(1 + \frac{h_k \lambda}{2})$$

$$\Rightarrow y_{k+1} = y_k \frac{1+\delta}{1-\delta}$$

Here, h_k is Δt. The growth factor is $\frac{1+\frac{\lambda \Delta t}{2}}{1-\frac{\lambda \Delta t}{2}}$. As $\lambda \Delta t \to -\infty$, $G \to -1^-$. This can be seen easily using the L'Hospital rule.

The analytical growth factor is $\frac{e^{\lambda(t+\Delta t)}}{e^{\lambda t}} = e^{\lambda \Delta t}$ which goes to 0 as $\lambda \Delta t \to 0$