$$f(\boldsymbol{x}) = \frac{1}{2}(x_1^2 - x_2)^2 + \frac{1}{2}(1 - x_1)^2$$

$$\nabla f = \begin{bmatrix} 2(x_1^2 - x_2)x_1 - (1 - x_1) \\ -(x_1^2 - x_2) \end{bmatrix}$$

$$H = \begin{bmatrix} 6x_1^2 - 2x_2 + 1 & -2x_1 \\ -2x_1 & 1 \end{bmatrix}$$

Since $f(x)$ is the sum of 2 non-negative numbers, its minima is attained when both numbers are 0.

Hence, the minima is at $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, where it takes on the value 0.

$$\boldsymbol{x}^1 = \boldsymbol{x}^0 - H^{-1}\nabla f$$

At $\boldsymbol{x}^0 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$,

$$\nabla f = \begin{bmatrix} 9 \\ -2 \end{bmatrix}$$

$$H = \begin{bmatrix} 21 & -4 \\ -4 & 1 \end{bmatrix} \Rightarrow H^{-1} = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 4.2 \end{bmatrix}$$

$$H^{-1}\nabla f = \begin{bmatrix} 0.2 \\ -1.2 \end{bmatrix}$$

Hence,

$$\boldsymbol{x}^1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 0.2 \\ -1.2 \end{bmatrix} = \begin{bmatrix} 1.8 \\ 3.2 \end{bmatrix}$$

The step is good because it reduces the value that we are trying to minimize.

A quick calculation shows that $f(\boldsymbol{x}^0) = 2.5$, while $f(\boldsymbol{x}^1) = 0.3208$.

However, the step also moves us further away from the global minima.