$$y' = -5y$$

The solution is

$$y(t) = e^{-5t}$$

The condition number of this solution at $t$ is $|5t|$, which is stable if $t$ is reasonable.

From class notes, we know that Euler's method is stable if $|1 + h\lambda| < 1$, in this case, $\lambda = -5$ and $h = 0.5$. Hence, $|1 + h\lambda| = 1.5$, hence **euler's forward is NOT stable.**

Performing one iteration of euler's forward method:

$$y_1 = y_0 + hf_0$$

In this case, $f_0 = -5y(0) = -5$, and $y_0 = y(0) = 1$, and $h = 0.5$

Hence, $y_1^{(forward)} = 1 + 0.5 * (-5) = -1.5$

For the given ODE, we know that backward Euler is stable if $\left|\frac{1}{1-h\lambda}\right| < 1 \Rightarrow \left|\frac{1}{1-0.5*(-5)}\right| = \left|\frac{1}{3.5}\right| < 1$

Hence, **backward Euler is stable**.

Performing one iteration of backward Euler, we have:

$y_1 = y_0 + hf(t_1, y_1) = 1 + 0.5(-5y_1) \Rightarrow y_1(3.5) = 1 \Rightarrow y_1 = \frac{1}{3.5}$