Error Analysis of Finite Difference Derivative Formulas

# Answers

1. or is being approximated.
2. From the above calculation, the leading order truncation error is

# Working out the answers:

# Part 1&2

From taylors expansion we know that

Therefore, by substitution (with and ) and simple rearrangement,

Under suitable assumptions of smoothness,

Thus, the truncation error is

# Part 3

We’ll use to represent the floating point version of .

We also know that for some

Let , , .

Thus, , ,

Thus,

1. The first term is the derivative that we want to compute.
2. Under limit , .

We can bound the second term, which is the round off error by

# Part 4

We now know that the total error is

To minimize wrt , we look at the first differential wrt :

Alas! We are forced to deal with this *terrifying* equation. But we are brave, and hopefully not stupid, so lets carry on.

**Let**

For this to be a minimizer, we must look at the second derivative of error. It is

If the second derivative is positive, we’re good. Since must be real, we have two possibilities - , we have that the second derivative at both values is .

Notice that is symmetric wrt , hence we’ll only talk about and ignore .

is the minimizer.

# Part 5

Substituting the value of into error, we get that total error is