Critical points in 2D

For each problem, we compute , then find the critical points by setting . After that, we compute the hessian, at each critical point (the hessian is not explicitly computed, we only mention the general hessian.

Notice that is neither positive definite, nor negative definite. Hence, is a saddle point.



At , the hessian is neither positive definite, nor negative definite. Hence, is a saddle point

At and , the hessian is .

The inequality is because the polynomial has no roots.

Hence, and are minima.

1. For this question, I checked if matrices were positive definite by looking at the eigenvalues that numpy reports.

At , , NOT positive/negative definite Saddle point

At , , NOT positive/negative definite Saddle point

At , , positive definite Minima

At , , negative definite Maxima

1. I looked at definiteness using numpy’s eigvals.

To solve the system, we simply add the two equations, and get , which we then substitute back in either to get their values

At , which is not positive or negative definite. Hence, is a saddle point