Given data points, we have quadratics. Each of these quadratics has 3 parameters, hence we have a total of original parameters. Concretely, Let the knots be at . Let the quadratic between be . Each has 3 parameters.

Let the corresponding values of the data at be .

We know that our splines have to fit the data. Hence, we have equations as follows:

**Left side:**

**Right side:**

There are now parameters that are unaccounted for.

**Once continuously differentiable:**

Since each is continuously differentiable, if the derivative of match at , then the entire system is once continuously differentiable.

Requiring that adds Equations.

This leaves us with 1 free parameter.

Hence, **yes, it is possible to create a once continuously differentiable interpolant**.

**Twice continuously differentiable:**

A twice continuously differentiable function is also once continuously differentiable.

To ensure twice continuously differentiability when the interpolant is already once continuously differentiable is the same as saying

This add equations again. Leading to a total of equations for a total of parameters.

The maximum value of for which creating such an interpolant is possible can be obtained by noticing that must be satisfied. Hence, **.**

However, sometimes, it may be possible even if **IF** some of the equations are redundant.