COL726: Minor-2 Take Home

Haroun Habeeb **2013CS10225**

We know that $A = X^t X$. Let the $svd(X) = U\Sigma V^*$ where U, V and unitary matrices and Σ is a diagonal matrix. Hence,

$$A = V^{*t}\Sigma^t U^t U \Sigma V^* = V^{*t}\Sigma^t I \Sigma V^* = V^{*t}\Sigma^t \Sigma V^* = V^{*t}\Sigma^2 V^*$$

Where Σ^2 is a diagonal matrix with only positive entries (since Σ is real.) Notice that V^* is unitary, hence $V^* = V^{*^{t^{-1}}}$. Hence, $A = Q\Sigma^2 Q^{-1}$ with $Q = V^{*^t}$.

Then, $A = (\Sigma V^*)^t(\Sigma V^*)$, which is of the form $X^t X$.

Hence, we can simply compute the Eigenvalue decomposition of A, and get both V^* and $\Sigma = sqrt(\Sigma^2)$.

(a) I'm not sure what n-dimensional vector means here… But I suppose that we could look at the entries of Σ. If there are n non-zero diagonal entries, then we have n vectors.

(b) If (ΣV^*) has $rank = 2$, then X could be transformed into a 2xn matrix.

(c) Read them from ΣV^*.

(d) Let's say our approximation for X is $\hat{X}$. We know $A = X^t X$ and can estimate $\hat{A} = \hat{X}^t\hat{X}$. The residual error would be $X - \hat{X}$. However, we could also make $norm(A - \hat{A})$ the residual error.

Since V^* is already orthonormal, really all we must do is to consider only the first two singular values of Σ.

Extra Light:

1.

2.

Positive definite condition is that $\sum_{i,j} z_i z_j A_{ij} > 0$

which means that $\sum_{i,j} z_i z_j (D_{ij}^2 - S_i - S_j + T) < 0$

3.

4.

5.