# Space Complexity

Dr Kamal Bentahar

School of Engineering, Environment and Computing
Coventry University

12/12/2016

# Space complexity

## Space complexity

The **space complexity** of a TM that always halts is the maximum number of tape cells $f(n)$ that a TM $\mathcal{M}$ scans on any input of length $n$.
We say that $\mathcal{M}$ runs in space $f(n)$ if its complexity is $f(n)$

# SPACE and NSPACE

Let $f : \mathbb{N} \to \mathbb{R}^+$ be a function

$SPACE(f(n)) = \{L \mid L$ is a language decided by an $O(f(n))$ space deterministic TM$\}$

$NSPACE(f(n)) = \{L | L$ is a language decided by an $O(f(n))$ space nondeterministic TM$\}$

# Savitch's Theorem



## Savitch's Theorem

For any function $f : \mathbb{N} \to \mathbb{R}^+$, where$f(n) \geq n$,

$$NSPACE(f(n)) \subset SPACE(f^2(n))$$

A nondeterministic TM that uses $f(n)$ space can be converted to a deterministic TM that uses only $f^2(n)$!

# PSPASE vs NPSPACE



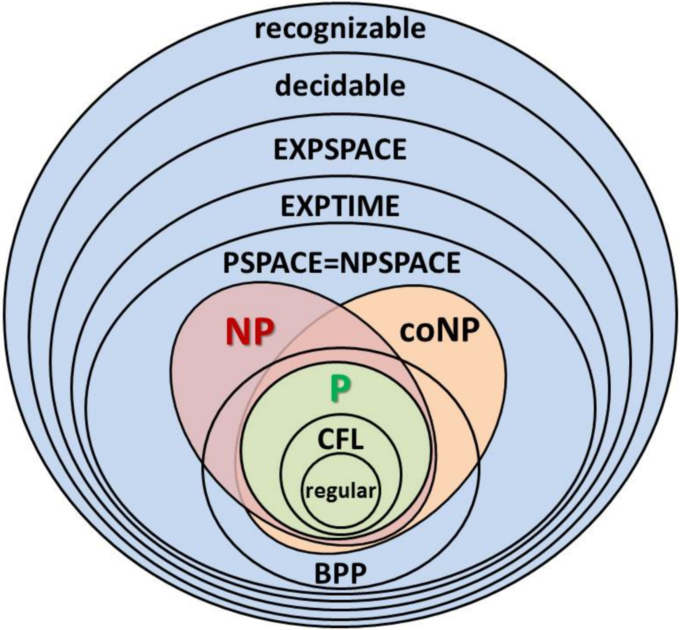
## PSPACE

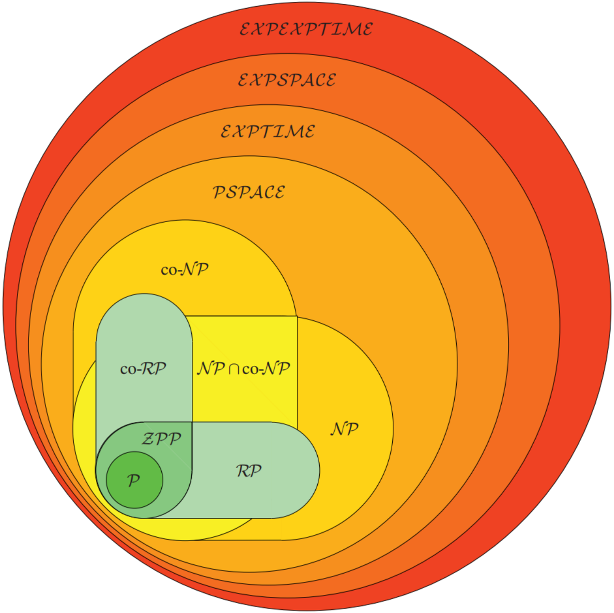
PSPACE is the class of languages that are decidable in polynomial space on a deterministic TM

$$PSPACE = SPACE(1) \cup SPACE(n) \cup SPACE(n^2) \cup \cdots$$

By Savitch's theorem we have that

$$PSPACE = NSPACE$$


recognizable
decidable
EXPSPACE
EXPTIME
PSPACE=NPSPACE
NP
coNP
P
CFL
regular
BPP


EXPEXPTIME
EXPSPACE
EXPTIME
PSPACE
co-NP
co-RP
NP ∩ co-NP
NP
ZPP
RP
P

# The Extended Chomsky Hierarchy