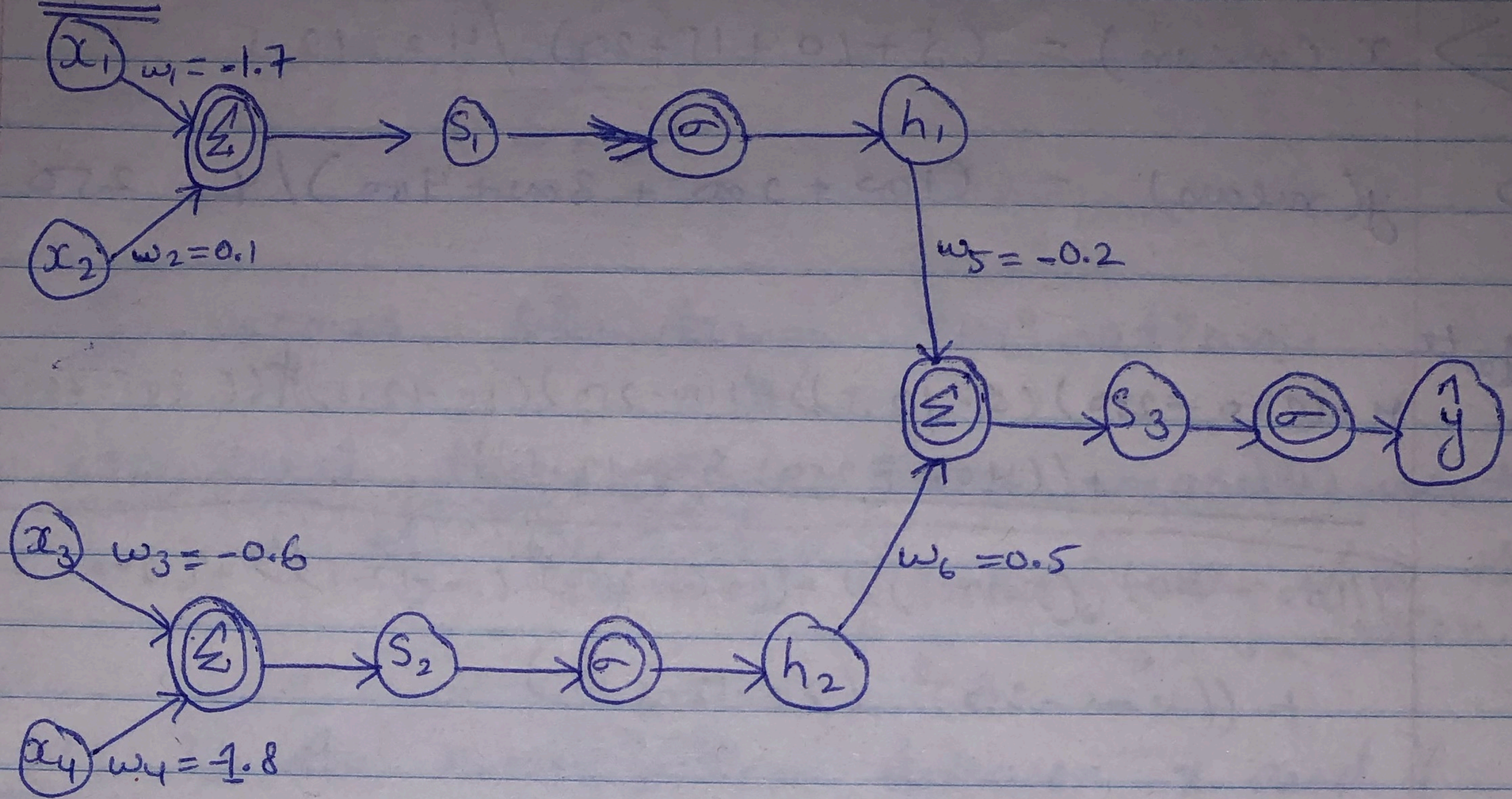
Deep Learning Assignment 2

Given :-

Q.1.



★ $\sigma(z) = \dfrac{1}{1+e^{-x}}$

★ $h_1 = \dfrac{1}{1+e^{-w_1x_1 - w_2x_2}}$

★ L2 Loss $L(y,\hat{y}) = \|\hat{y} - y\|^2$

★ Inputs :- $x_1 = 0.7$, $x_2 = 1.2$, $x_3 = 1.1$, $x_4 = 2$

★ weights :- $w_1 = -1.7$, $w_2 = 0.1$, $w_3 = -0.6$, $w_4 = 1.8$, $w_5 = 0.2$, $w_6 = 0.5$

★ $y = 0.5$

$S_1 = x_1 w_1 + x_2 w_2$

$S_1 = (0.7)(-1.7) + (1.2)(0.1)$

$S_1 = (-1.19) + (0.12)$

$\boxed{S_1 = -1.07}$

$S_2 = x_3 w_3 + x_4 w_4$

$S_2 = (1.1)(-0.6) + (2)(-1.8)$

$S_2 = (0.66) + (-3.6)$

$\boxed{S_2 = -4.26}$

Now, calculate :- $h_1 = \dfrac{1}{1+e^{-w_1 x_1 - w_2 x_2}}$

$h_1 = \dfrac{1}{1+e^{-(-1.7)(0.7) - (0.1)(1.2)}}$

$h_1 = \dfrac{1}{1+e^{(1.19 - 0.12)}}$

$h_1 = \dfrac{1}{1+e^{1.07}}$

$h_1 = \dfrac{1}{3.915}$

$\therefore \boxed{h_1 = 0.255}$

$h_2 = \dfrac{1}{1+e^{-w_3 x_3 - w_4 x_4}}$

$h_2 = \dfrac{1}{1+e^{-(-0.6)(1.1) - (-1.8)(2)}}$

Hilroy

$$h_2 = \frac{1}{1+e^{(0.66+3.6)}}$$

$$h_2 = \frac{1}{1+e^{4.26}}$$

$$\cancel{h_2} = \cancel{\frac{1}{14}}$$

$$\boxed{h_2 = 0.0139}$$

Now, $S_3 = h_1 w_5 + h_2 w_6$

$$S_3 = (0.255(-0.2)) + ((0.0139)(0.5))$$

$$S_3 = -0.051 + 0.0065$$

$$\boxed{S_3 = -0.0445}$$

Now, $\hat{y} = \frac{1}{1+e^{-h_1 w_5 - h_2 w_6}}$

$$\hat{y} = \frac{1}{1+e^{-(0.255)(-0.2)-(0.013)(0.5)}}$$

$$\hat{y} = \frac{1}{1+e^{-(-0.051)-(0.0065)}}$$

$$\boxed{\hat{y} = 0.4889}$$

Now, the gradient of an L2 loss function

$\| \hat{y} - y \|_2^2$ is $2\| \hat{y} - y \| = \frac{\delta E}{\delta \hat{y}}$

By using, backward propagation :-

$$\frac{\delta E}{\delta w_1} = \frac{\delta E}{\delta \hat{y}} \times \frac{\delta \hat{y}}{\delta S_3} \times \frac{\delta S_3}{\delta h_1} \times \frac{\delta h_1}{\delta S_1} \times \frac{\delta S_1}{\delta w_1} \Big\} \text{ — (A)}$$

$$\frac{\delta E}{d \hat{y}} = 2\| \hat{y} - y \| \text{ — (1)}$$

$$\sigma'(x) = \sigma(x)[1 - \sigma(x)] \text{ — (2)}$$

$$\frac{\delta S_3}{\delta h_1} = w_5, \quad \frac{\delta S_1}{\delta w_1} = x_1 \Big\} \text{ — (3)}$$

Now, Substituting (1), (2), (3) in (A)

$$\frac{\delta E}{\delta w_1} = 2\| \hat{y} - y \| \times \sigma'(S_3) \times w_5 \times \sigma'(S_1) \times x_1$$

$$= 2[\| 0.4889 - 0.5111 \|] \times \sigma(S_3)[1 - \sigma(S_3)] \times (-0.2) \times \sigma(S_1)[1 - \sigma(S_1)] \times 0.7$$

$$\sigma(S_3) = \frac{1}{1 + e^{-(-0.04405)}}$$

$$\boxed{\sigma(S_3) = 0.4889}$$

Hilroy

$$\sigma(S_1) = \frac{1}{1+e^{-(-1.07)}}$$

$$\boxed{\sigma(S_1) = 0.2554}$$

$$\frac{\delta E}{\delta w_1} = 2\,[\overset{0.107}{\cancel{\quad}}] \times [(0.4889)(1-0.4889)] \times (0.2) \times [(0.2554)(1-0.2554)] \times (0.7)$$

$$\cancel{= [2(0.107)] \times (0.4889)(0.5110)}$$

$$\boxed{\frac{\delta E}{\delta w_1} = -0.0014}$$