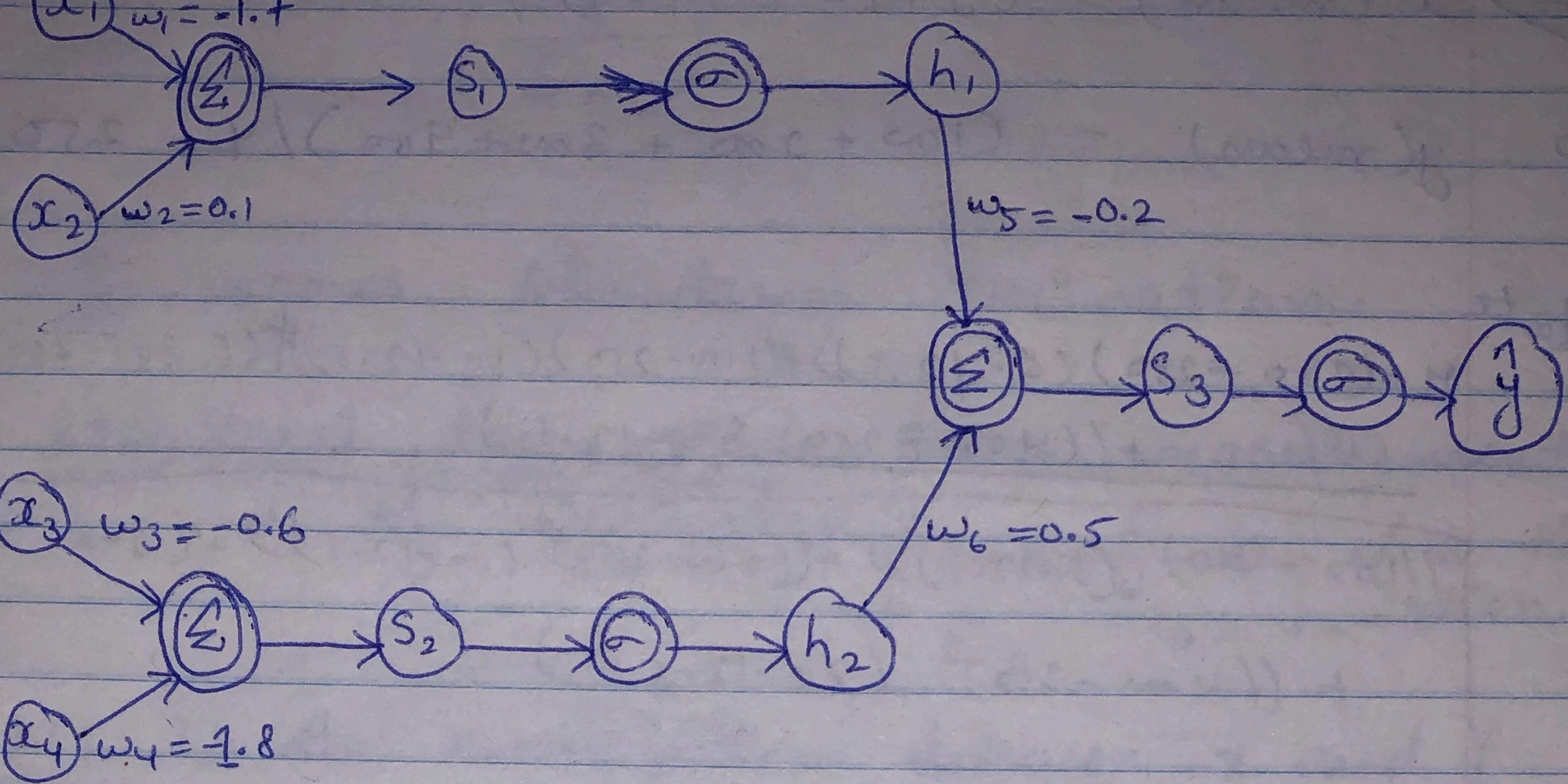


Deep Learning Assignment 2

Given :-

Q.1.



* $\sigma(z) = \frac{1}{1 + e^{-x}}$

* $h_1 = \frac{1}{1 + e^{-(w_1x_1 + w_2x_2)}}$

* L2 Loss $L(y, \hat{y}) = \| \hat{y} - y \|^2$

* Inputs :- $x_1 = 0.7, x_2 = 1.2, x_3 = 1.1, x_4 = 2$

* weights :- $w_1 = -1.7, w_2 = 0.1, w_3 = -0.6, w_4 = -1.8, w_5 = 0.2, w_6 = 0.5$

* $y = 0.5$

$$S_1 = x_1 w_1 + x_2 w_2$$

$$S_1 = (0.7)(-1.7) + (1.2)(0.1)$$

$$S_1 = (-1.19) + (0.12)$$

$$\boxed{S_1 = -1.07}$$

$$S_2 = x_3 w_3 + x_4 w_4$$

$$S_2 = (1.1)(-0.6) + (2)(-1.8)$$

$$S_2 = (0.66) + (-3.6)$$

$$\boxed{S_2 = -4.26}$$

Now, calculate:- $h_1 = \frac{1}{1 + e^{-w_1 x_1 - w_2 x_2}}$

$$h_1 = \frac{1}{1 + e^{-(-1.7)(0.7) - (0.1)(1.2)}}$$

$$h_1 = \frac{1}{1 + e^{(1.19 - 0.12)}}$$

$$h_1 = \frac{1}{1 + e^{1.07}}$$

$$h_1 = \frac{1}{3.915}$$

$$\therefore \boxed{h_1 = 0.255}$$

$$h_2 = \frac{1}{1 + e^{-w_3 x_3 - w_4 x_4}}$$

$$h_2 = \frac{1}{1 + e^{-(-0.6)(1.1) - (-1.8)(2)}}$$

Hilroy

$$h_2 = \frac{1}{1 + e^{0.66 + 3.6}}$$

$$h_2 = \frac{1}{1 + e^{-4.26}}$$

~~$$h_2 = \frac{1}{14}$$~~

$$h_2 = 0.0139$$

Now, $s_3 = h_1 w_5 + h_2 w_6$

$$s_3 = (0.255(-0.2)) + (0.0139)(0.5)$$

$$s_3 = -0.051 + 0.0065$$

$$s_3 = -0.0445$$

Now, $\hat{y} = \frac{1}{1 + e^{-h_1 w_5 - h_2 w_6}}$

$$\hat{y} = \frac{1}{1 + e^{-(0.255)(-0.2) - (0.013)(0.5)}}$$

$$\hat{y} = \frac{1}{1 + e^{-(0.051) - (0.0065)}}$$

$$\hat{y} = 0.4889$$

Now, the gradient of an L2 loss function

$$\|\hat{y} - y\|_2^2 \Rightarrow 2\|\hat{y} - y\| = \frac{\delta E}{\delta \hat{y}}$$

By using Backward propagation :-

$$\frac{\delta E}{\delta w_i} = \frac{\delta E}{\delta \hat{y}} \times \frac{\delta \hat{y}}{\delta s_3} \times \frac{\delta s_3}{\delta h_i} \times \frac{\delta h_i}{\delta s_i} \times \frac{\delta s_i}{\delta w_i} \quad \} - \textcircled{A}$$

$$\frac{\delta E}{\delta \hat{y}} = 2\|\hat{y} - y\| \quad - \textcircled{1}$$

$$\sigma'(x) = \sigma(x)[1 - \sigma(x)] \quad - \textcircled{2}$$

$$\frac{\delta s_3}{\delta h_i} = w_5, \quad \frac{\delta s_i}{\delta w_i} = x_i \quad \} - \textcircled{3}$$

Now, substituting $\textcircled{1}, \textcircled{2}, \textcircled{3}$ in \textcircled{A}

$$\frac{\delta E}{\delta w_i} = 2\|\hat{y} - y\| \times \sigma'(s_3) \times w_5 \times \sigma'(s_i) \times x_i$$

$$= 2[||0.4889 - 0.5||] \times \sigma(s_3)[1 - \sigma(s_3)] \times (-0.2) \times \sigma(s_i)[1 - \sigma(s_i)] \times 0.7$$

$$\sigma(s_3) = \frac{1}{1 + e^{-(0.4889)}} =$$

$$\boxed{\sigma(s_3) = 0.4889}$$

$$\sigma_{CS_1} = \frac{0.107}{1 + e^{-(1.07)}}$$

$$\boxed{\sigma_{CS_1} = 0.2554}$$

$$\frac{\delta E}{\delta \omega_1} = 2 \left[\cancel{0.107} \right] \times [0.4889)(1-0.4889)] \times (0.2) \times \\ [0.2554)(1-0.2554)] \times (0.7) \\ = [2(0.107)] \times (0.4889)(0.5110)$$

$$\boxed{\frac{\delta E}{\delta \omega_1} = 0.0014}$$