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PES UNIVERSITY	**PES University, Bengaluru** (Established under Karnataka Act No. 16 of 2013)	**UE20CS904**

AUG 2021 : END SEMESTER ASSESSMENT (ESA)
M TECH DATA SCIENCE AND MACHINE LEARNING_ SEMESTER I

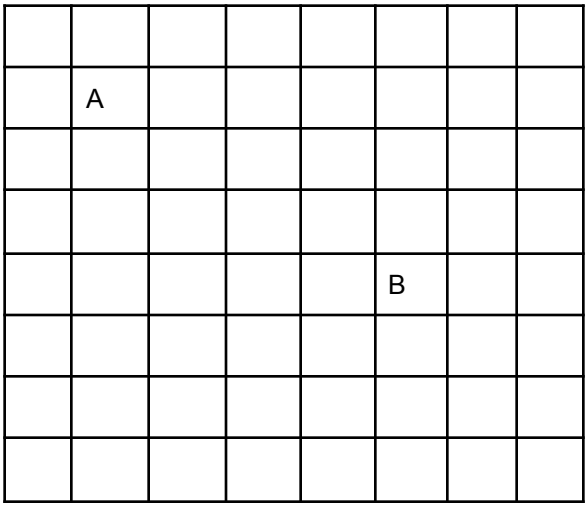
UE20CS904 - Mathematical Foundation

Time: 3 Hrs	Answer All Questions	Max Marks: 100

Section A (20 marks)

1 a) Find the determinant of the matrix A : $\begin{bmatrix} 2 & 4 & 5 \\ 6 & 1 & 3 \\ 4 & 0 & 7 \end{bmatrix}$ — 2

b) Which distance metric is suitable for calculating the least number of squares moved between the starting position(A) and ending position (B) on the chessboard (each square of unit length) for the Queen (Queen can move either diagonally or vertically or horizontally)? Give formula for the same. — 2



c) Calculate the angel between two given vectors. The two vectors are, — 2

$a = \vec{i} + 2\vec{j}$ and

$b = 9\vec{i} + 3\vec{j}$

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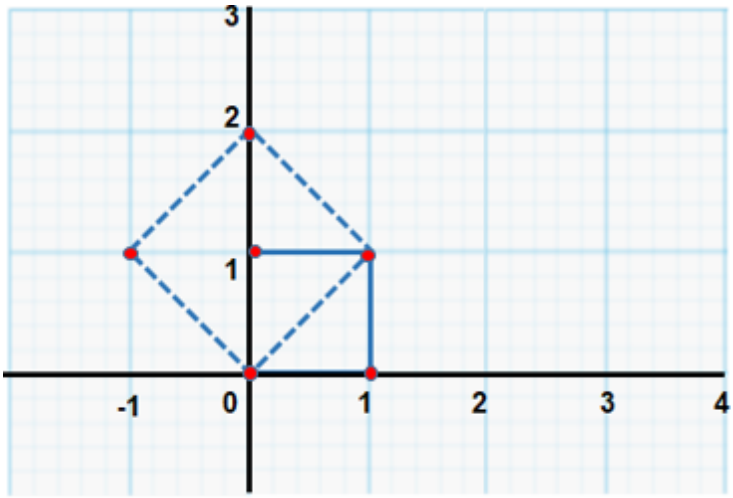
	d)	We have an rgb image saved as img. An RGB image has length and width 63.We are creating a new image by concatenating img[:,:63,1] , img[:,63:126,:2] & img[:,126:,0]. Wha kind of changes can we observe in the new image as compared to the original image (img).	2
	e)	In the plot shown below the un-dotted box portion represents the original coordinates of an object and the same after transformation is represented by the dotted box object. Write the coordinates, the transformation matrix and the coordinates after transformation.  	2
2	a)	Find out if the following is a concave function or convex function for the interval (-5, -2) $f(x) = -x^2 - 7x$	2
	b)	Find the point of inflection, local maxima & minima for the following graph in the interval, (-2, 3) $f(x) = 5x^3 + 2x^2 - 3x$	2
	c)	Statement : For any orthogonal matrix, inverse of a matrix is same as transpose of the matrix. Check whether the following matrix is orthogonal or not. Verify whether the above statement is true or not. $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$	2
	d)	Calculate the Jacobian matrix for the following function $f_1(x,y) = x^3y$ $f_2(x,y)$ $\frac{x^2}{y} + y^2$	2
	e)	Find the minimum value of $f(x)$ when x < 5. Where , $f(x) = x4 + x2 + 1$	2

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		Section B (40 marks)	
3	a)	Find out the inverse of the following matrix. A = [1 5 7 2 6 0 3 5 1]	5
	b)	Find the number of independent vectors in the following matrix. A = $\begin{bmatrix} 1 & 3 & 5 & 6 \\ 3 & 5 & 0 & 7 \\ 2 & 6 & 2 & 0 \\ 7 & 5 & 1 & 0 \end{bmatrix}$	5
	c)	Find out the derivative of the following function using chain rule. Perform step-wise operation. $f(x) = \cos\left(\frac{1}{\sqrt{1+x^2}}\right)$ Find out the Hessian Matrix of the following function $f(x,y) = x^2y^2 + \frac{x}{y^2}$	7
	d)	Mr. Johns sells Mango, Apple and Peach. The price of a kg of Mango, 3 kgs of Apple , and a kg of Peach is Rs 145. The price of 3 kgs of Mango, 4 kgs of Apple , and a kg of Peach is Rs 280. The price of 2 kgs of Apple , and a kg of Peach is Rs 65. Find out the price of a kg of each fruit.	7
	e)	Find the covariance for the following set of vectors. $\begin{bmatrix} -1 & 2 \\ 3 & 5 \\ 0 & 1 \\ 4 & 2 \\ 6 & 1 \end{bmatrix}$	8
	f)	Find the Singular value decomposition of the following matrix A = $\begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$ The $\sum$ or the D matrix is given as $\begin{bmatrix} \sqrt{40} & 0 \\ 0 & \sqrt{10} \end{bmatrix}$	8
		Section C (40 marks)	
4	a)	Find out the Eigen values and the Eigen vector for the corresponding Eigen values for the following matrix. $\begin{bmatrix} 1 & 5 & 7 \\ 2 & 6 & 0 \\ 3 & 5 & 1 \end{bmatrix}$	15
	b)	Transform the following basis into orthogonal basis using Gram-Schmidt Process. U1 = (2,1,0) U2 = (3,2,1)	10

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		U3 = (4,1,2)	
	c)	We have recorded the weekly average conversion rate of Dolar for over 6 consecutive weeks. Y shows the weekly average conversion rate and x shows the number of the week. Try to fit the best possible function 'f' to stablish the relationship between the number of the day and conversion rate.(Applying Gradient descent) where f(x) = y = a + b * x . x | y 1 | 10 2 | 14 3 | 18 4 | 22 5 | 25 6 | 33 The initial values of a & b are, a= 4.9 & b=4.401.The learning rate is mentioned as .05. The error rate of a & b should be less than .01. Plot the predicted and actual data in a graph.	15