SRN | | | | | | | | | | | |

PES UNIVERSITY	**PES University, Bengaluru** (Established under Karnataka Act No. 16 of 2013)	**UE20CS904**

Aug 2023 : END SEMESTER ASSESSMENT (ESA)

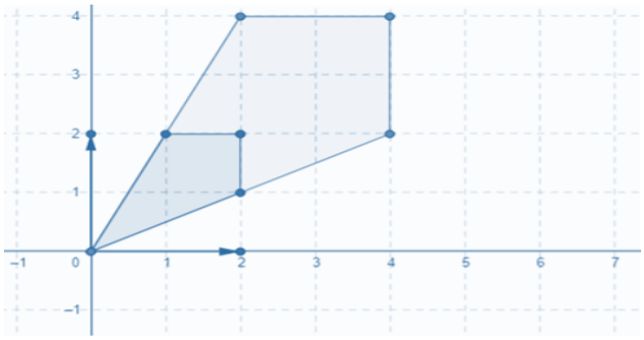
M TECH DATA SCIENCE AND MACHINE LEARNING_ SEMESTER I

UE20CS904 - Mathematical Foundation

Time: 3 Hrs	Answer All Questions	Max Marks: 100

		Section A (20 marks)	
1	a)	Find X if $\det\begin{pmatrix}2 & 4\\5 & 1\end{pmatrix} = det\begin{pmatrix}2x & 4\\6 & x\end{pmatrix}$	2
	b)	Calculate the angle between two given vectors. The two vectors are, a = $\vec{\imath} + 2\vec{\jmath}$ and b = $9\,\vec{\imath} + 3\,\vec{\jmath}$	2
	c)	Find out whether the function is concave or convex , $f(x) = -8x^2 + 15$	2
	d)	Find the vector projection of the vector a = {5; 5} on b = {8; 2}	2
	e)	What is the effect of higher learning rate in Gradient descent algorithm?	2
2	a)	Write the transformation matrix rotation of a 2d image.	2
	b)	What will happen when eigenvalues are roughly equal? I. PCA will perform outstandingly II. PCA will perform badly III. Can't Say	2
	c)	Calculate the Jacobian matrix for the following function $f_1(x,y) = x^3y$ $f_2(x,y) = \frac{x^2}{y} + y^2$	2
	d)	In simple linear regression for a single data point (x_1, y_1) we define loss as $L(w_0, w_1) = (\widehat{y_1} - (w_0 + w_1x_1))^2$ where $\widehat{y_1}$ is predicted value for y_1 find $\frac{\partial L}{\partial w_0}$ and $\frac{\partial L}{\partial w_1}$	2
	e)	We have an rgb image saved as img. An RGB image has length and width 63.We are creating a new image by concatenating img[:,:63,1] , img[:,63:126,:2] & img[:,126:,0]. What kind of changes can we observe in the new image as compared to the original image (img) ?	2
		Section B (40 marks)	
3	a	Mr X is an investor. His portfolio primarily tracks the performance of the Nifty index and he wants to add the stock of company 'A'. Before adding the stock to his portfolio, he wants to assess if there exists a relationship between Nifty and Stock A. Year Nifty Stock A 2015 1692 68 2016 1978 102 2017 1884 110 2018 2151 112	7

SRN | | | | | | | | | | | |

		2019 2519 154 Help Mr X to assess the same.	
	b	A stone is dropped into a quiet lake and waves move in circles at a speed of 5cm per second. At the instant, when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?	7
	c)	Compute the following convolution for the middle pixel with intensity 99. What visual effect will the following convolution have on the image? $\begin{bmatrix} 138 & 134 & 101 \\ 119 & 99 & 83 \\ 84 & 80 & 79 \end{bmatrix} * \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	7
	d)	In the plot shown below the dark shaded portion represents the original coordinates of an object and the same after transformation is represented by the lightly shaded object. Write the coordinates, the transformation matrix and the coordinates after transformation. 	7
	e)	The Following table lists the weight and heights of 5 boys Find the covariance matrix for the data. Boy | 1 | 2 | 3 | 4 | 5 Weight(lb) | 120 | 125 | 125 | 135 | 145 Height(in.) | 61 | 60 | 64 | 68 | 72	6
	f)	Find the Eigen values of A and A^2: $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$	6
		Section C (40 marks)	
4	a)	A headphone manufacturer determines that in order to sell x units of a new headphone, the price per unit, in dollars, must be p(x) = 1000 - x. The manufacturer also determines that the total cost of producing x units is given by C(x)= 3000 + 20x. i) Find the total revenue R(x) ii) Find the total profit P(x). iii) How many units must the company produce and sell in order to maximize profit? iv) What is the maximum profit? v) What price per unit must be charged in order to make this maximum profit?	15
	b)	Find singular Value decomposition of $A = \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$	12
	c)	Consider the data given below and fit a linear regression line y = ax+b using gradient descent. X 0 0.4 0.6 1 Y 0 1 0.48 0.95 Initialize the weights a and b to 0.8, 0.2 respectively. Update the weights such that the error is minimum using gradient descent. Use the function sum of squared errors$\sum$ $[\![(y-y\hat{})]\!]^2$ where $y\hat{}$ is the y-predicted value and y is the actual given y. Plot the linear regression line after updating.	13