IF A and B are independent events

$P(A|B) = P(A)$

$P(B|A) = P(B)$

P(sample space) = 1 (always).

	CS	NCS	
M	10	15	25
F	8	7	15
	18	22	40

$$P(CS|M) = \frac{10}{25} = \frac{P(L \cap M)}{P(M)} = \frac{10/40}{25/40} = \frac{10}{25}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A|B) \cdot P(B)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = P(B|A) \cdot P(A)$$

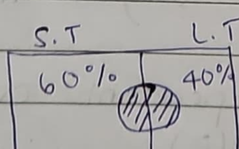
(BAYES' THEOREM)

$$\underbrace{P(A|B)}_{\text{posterior probability}} = \frac{\overbrace{P(B|A)}^{\text{likelihood}} \cdot \overbrace{P(A)}^{\text{prior probability}}}{\underbrace{P(B)}_{\text{evidence}}}$$

posterior probability : needs to determined (evaluated)

I VENN DIAG. APPROACH

① recovery = 95%



Recovery = 90%

$P(LT) = 40\% = 0.4$ } prior probabilities

$P(ST) = 60\% = 0.6$

$P(\text{Recovered}|LT) = 0.9$ } likelihood

$P(\text{Recovered}|ST) = 0.95$

S.T → Short term loan
L.T. → long term loan

$P(\text{Recovery} \cap ST) = P(\text{Recovery}|ST) \cdot P(ST)$
$= 0.95 \times 0.6 = 0.57$

$P(\text{Recovered} \cap LT) = P(\text{Recovery}|LT) \cdot P(LT)$
$= 0.9 \times 0.4 = 0.36$

$P(\text{Recovery}) = P(\text{Recovery} \cap LT) + P(\text{Recovery} \cap ST)$
$= 0.57 + 0.36 = 0.93$

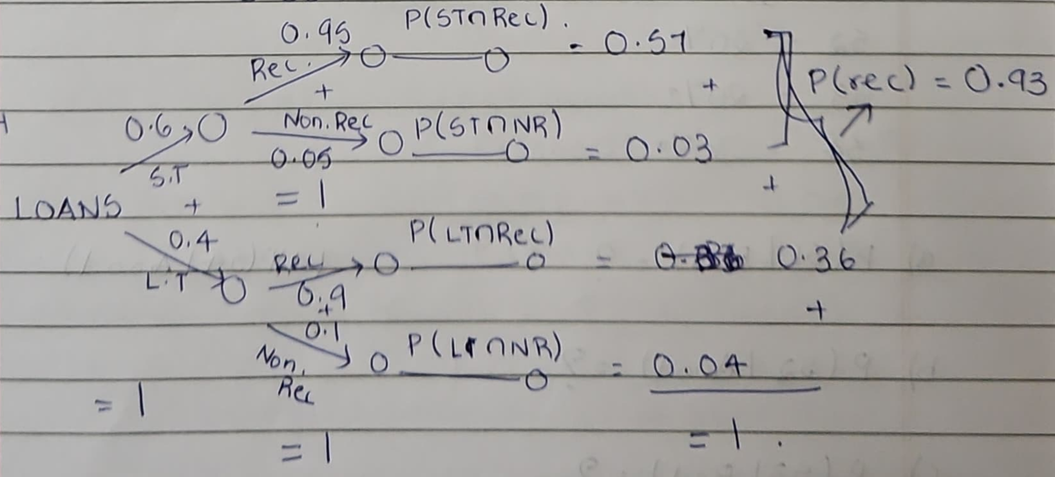
$P(LT|\text{Recovered}) = \dfrac{P(LT \cap \text{Recovery})}{P(\text{Recovery})}$
$= \dfrac{0.36}{0.93} = 0.387$

$P(ST|\text{Recovered}) = \dfrac{0.57}{0.93} = 0.613$

P(LT | Recovered) → Probability of LT given that it is recovered
Among recovered loans if LT is selected.

$P(LT|\text{Recovered}) + P(ST|\text{Recovered}) = 1.$
$0.387 + 0.613 = 1$

(2) TREE APPROACH



classmate

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$P(LT \mid \cancel{Recovered}) = \frac{\cancel{P(LT)/A}}{P(LT) \cap P}$

$$P(LT \mid Recovered) = \frac{P(LT \cap Recovered)}{P(Recovered)}$$

$$= \frac{0.36}{0.93}$$

$$P(L.T \mid NR) = \frac{P(LT \cap NR)}{P(NR)} = \frac{0.04}{0.07} = 0.57$$

$$P(ST \mid NR) = \frac{P(ST \cap NR)}{P(NR)} = \frac{0.03}{0.07} = 0.43$$

$$P(ST \mid Recovered) = \frac{P(ST \cap Recovered)}{P(Recovered)} = \frac{0.57}{0.93}$$

$$= 0.613$$

II

	(Accepted) Good (%)	(Rejected) Bad (%)
S1	90	10
S2	80	20
S3	75	25
S4	95	5

S1 - 30%

S2 - 20%

S3 - 20%

S4 - 30%

a) $P(S1 \mid Bad) = ?$

(d) $P(S4 \mid Good) = ?$

b) $P(S2 \mid Good) = ?$

c) $P(S3 \mid Bad) = ?$

S1 0.3 → Good 0.9 → $P(S1 \cap Good) = 0.9 \times 0.3 = 0.27$

S1 0.3 → Bad 0.1 → $P(S1 \cap Bad) = 0.3 \times 0.1 = 0.03$

S2 0.2 → Good 0.8 → $P(S2 \cap Good) = 0.8 \times 0.2 = 0.16$

S2 0.2 → Bad 0.2 → $P(S2 \cap Bad) = 0.2 \times 0.2 = 0.04$

S3 0.2 → Good 0.75 → $P(S3 \cap Good) = 0.75 \times 0.2 = 0.15$

S3 0.2 → Bad 0.25 → $P(S3 \cap Bad) = 0.2 \times 0.25 = 0.05$

S4 0.3 → Good 0.95 → $P(S4 \cap Good) = 0.3 \times 0.95 = 0.285$

S4 0.3 → Bad 0.5 → $P(S4 \cap Bad) = 0.3 \times 0.5 = 0.15$

~~S1 (Good/S1) = 0.9~~

$$P(S1 \mid Good) = \frac{P(S1 \cap Good)}{P(Good)} = \frac{0.27}{0.865} =$$

$$P(Good) = \cancel{0.9} + \cancel{0.8} + \cancel{0.75} + \cancel{0.95}$$
$$= 0.27 + 0.16 + 0.15 + 0.285$$
$$= 0.865$$

$$P(Bad) = 0.03 + 0.04 + 0.05 + 0.15$$
$$= 0.135$$

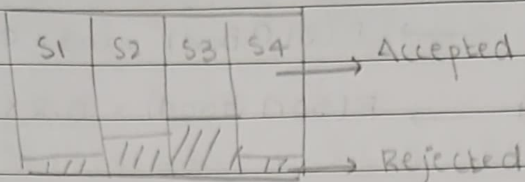
$$P(S1 \mid Bad) = \frac{P(S1 \cap Bad)}{P(Bad)} = \frac{0.03}{0.135}$$

$$P(S2 \mid Good) = \frac{P(S2 \cap Good)}{P(Good)} = \frac{0.16}{0.865}$$

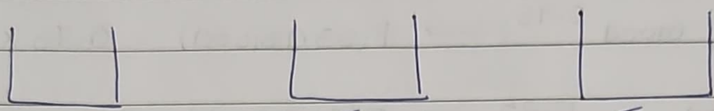
$$P(S3 \mid Bad) = \frac{P(S3 \cap Bad)}{P(Bad)} = \frac{0.05}{0.135}$$

~~P(S4~~ $P(S4 \mid Good) = \frac{P(S4 \cap Good)}{P(Good)}$

$$= \frac{0.285}{0.865}$$



III



I
2 Gold coins
1

II
2 Silver coins
0

III
1 Gold coin / 1 Silver coin
1/2

If the coin is of gold, what is the probability that the other coin in the box is also gold.

$P(\text{Box-1}) = P(\text{Box-2}) = P(\text{Box-3}) = \frac{1}{3}$

~~no. of possibilities = 2~~

Toss coin 3 times
total no. of outcome $= 2^3 = 8$.

H H H
H T H
H T T
H H T
T T T
T T H
T H T
T H H

X = no. of heads

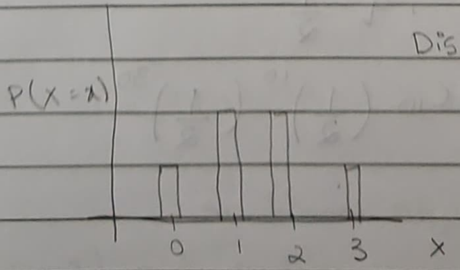
x	$P(X=x)$
0	1/8
1	3/8
2	3/8
3	1/8

$$\sum_{i=0}^{3} P(X = x_i) = 1$$

Random variable (X)

Discrete
(Discrete Probability distribution)

Continous
(Continous Prob. Distribution)



Disc. Prob. Dist.

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→ Probability of x atleast 2.

$P(x \geq 2) = P(x=2) + P(x=3)$

$= \frac{3}{8} + \frac{1}{8} = \frac{4}{8}.$

→ probability of X atmost 2.

$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

→ BINOMIAL THEOREM :

p → probability of success

q → probability of failures

$(p+q)^n = \underbrace{{}^nC_0 \cdot p^n q^0}_{P(X=0)} + \underbrace{{}^nC_1 p^{n-1} q^1}_{P(X=1)} + \dots + \underbrace{{}^nC_n p^0 q^n}_{P(X=n)}$

① $^nP_r = \frac{n!}{(n-r)!}$

② $^nC_r = \frac{n!}{(n-r)!\,r!}$

③ $^nP_r = {}^nC_r \cdot r!$

Binomial : where the outcome depends on either success or Failure.

ex : probability of getting H

probability of getting 4 in Dice.

$P(x=x) = nC_x\, p^x q^{n-x}$

$n = 40, \quad p = \frac{1}{2}, \quad q = \frac{1}{2}$

$P(X=10) = 40C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{30}$

$n = 3 \quad p = \frac{1}{2}, \; q = \frac{1}{2}$

$P(X = 0) = {}^3C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3$

$P(X = 1) = {}^3C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2$

$P(X = 2) = {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1$

$P(X = 3) = {}^3C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0$

pmf → probability mass function.

scipy.stats (scientific python)

(alias) → st.

st.binom.pmf valid for only discrete

cdf → cumulative distribution function.

sf → survival function.

V(x)	x	$P(X=x)$ pmf	$\sum_{x=1}^{n} P(X=x_i)$ cdf	sf.	E(x)
$(0-3/2)^2 \cdot 1/8$	0	1/8	1/8	8/8	$0 \times 1/8 = 1/8$
$(1-3/2)^2 \cdot 3/8$	1	3/8	4/8 (1/8 + 4/8)	7/8	$1 \times 3/8 = 3/8$
$(2-3/2)^2 \cdot 3/8$	2	3/8	7/8 (1/8 + 4/8 + 7/8)	4/8	$2 \times 3/8 = 6/8$
$(3-3/2)^2 \cdot 1/8$	3	1/8	8/8	1/8	$3 \times 1/8 = 3/8$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= \text{cdf}(2)$ st.binom.cdf (2, n, p)

$P(X \geq 2) = P(X=3) + P(X=2) = \frac{4}{8}$

st.binom.sf (1, n, p)

$P(X = x) = F(n, p)$

```
import pyForest
import scipy.stats as st

# toss a coin 3 times Find the probabilities?
n = 3
p = 0.5

x = np.arange(0,4)

pmF = st.binom.pmf(x,n,p)
pmf
```

~~plt.plot~~ plt.bar(x, pmF)

E(X) = Expected Value

$$E(X) = \sum_{i=1}^{n} x_i \, P(X = x_i) = \cancel{1.5}\ np.$$

(1.5)

$$V(X) = \sum_{i=1}^{n} (x_i - E(X))^2 \, P(X = x_i)$$

$$= npq$$

(3/4)

V(X)

0	$9/4 \cdot 1/8$	
1	$1/4 \cdot 3/8$	3/4
2	$1/4 \cdot 3/8$	
3	$9/4 \cdot 1/8$	

→ POISSON'S DISTRIBUTION :

IF $n \to \infty$ (very large)

$$P(X = x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$e = 2.72$ (constant)

λ = no. of objects per $\begin{cases} \text{time} \\ \text{length} \\ \text{area} \\ \text{volume} \end{cases}$