1.

a).

$$E[t] = \int_0^{\infty} t \, \frac{1}{\tau} e^{-\frac{t}{\tau}} dt$$

$$= \frac{1}{\tau}\left\{ \left[-\tau t e^{-\frac{t}{\tau}}\right]\Big|_0^{\infty} + \int_0^{\infty} \tau e^{-\frac{t}{\tau}} dt \right\}$$

$$= \frac{1}{\tau}\left\{ 0 + \tau \int_0^{\infty} e^{-\frac{t}{\tau}} dt \right\}$$

$$= -\tau(0-1) = \tau$$

b). $Var[t] = E[t^2] - E[t]^2$

$$E[t^2] = \int_0^{\infty} t^2 \frac{1}{\tau} e^{-\frac{t}{\tau}} dt$$

$$= 2\tau^2$$

$$\therefore Var[t] = 2\tau^2 - \tau^2 = \tau^2$$

c). $\mathcal{L}(\tau) = \prod_{i=1}^{n} f(t_i; \tau)$

$$= \frac{1}{\tau^n} \exp\left(-\sum_{i=1}^{n} \frac{t_i}{\tau}\right)$$

$$\mathcal{L}'(\tau) = \log \mathcal{L}(\tau)$$

$$= -n \log \tau - \frac{1}{\tau} \sum_{i=1}^{n} t_i$$

$$\frac{d}{d\tau} \mathcal{L}'(\tau) = -\frac{n}{\tau} + \frac{1}{\tau^2} \sum_{i=1}^{n} t_i = 0$$

$$\therefore n\tau = \sum_{i=1}^{n} t_i$$

$$\hat{\tau}_e = \frac{1}{n} \sum_{i=1}^{n} t_i$$

i.e. the mean of the sample.

d). please refer to the attached notebook.

2, 3 arein notebook aswell.

4. $Var(f) = Var(x+y)$

$$= Var(X) + Var(Y) + Cov(X,Y) + Cov(Y,X)$$

$$= V_{xx} + V_{xy} + V_{yx} + V_{yy}$$

$\because V_{xy} = V_{yx}$ $\therefore$ $Var(f) = V_{xx} + V_{yy} + 2V_{xy}$.