The Existence, Nature, and Effect of Dark Energy

PHYS364: Cosmology Group Project

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Abstract

This report explores the necessity of dark energy and the evidence for its existence. Observational dating methods of the Universe are discussed, with specific focus on globular clusters. The cosmic age problem is approached by comparing this observation data with the theoretical age of the Universe derived from the Friedmann equation, assuming a cosmological constant and a Universe which is flat and matter dominated. From this data it can be concluded that there is a need for dark energy, as the observational age of the Universe stands to be at least 2.5 billions years greater than the theoretically determined value if dark energy is not considered. The future history of the universe and our galaxy, the Milky Way, is discussed under different possible situations. The possibility of a cyclic Universe solves problems in the Cosmological Standard Model such as the flatness problem, the coincidence problem, the horizon problem, and the dark energy problem. The cyclic Universe model is not definitive, however, and possesses its own shortcomings such as how to model "the bounce" and how to eradicate "monster black holes". Quintessence is outlined with an example provided and examined. No major difference to the future history of the universe is found when this example is compared to the predictions of the Cosmological Standard Model. Due to the nature of cosmology, an understanding of dark energy would not only strengthen our understanding of the Universe now, but the entire timeline of its existence.

Contents

1 Introduction

In the past century alone we have discovered more about cosmology than we have in all previous human history. To understand dark energy, however, would be to surpass this knowledge yet again, as it is currently believed that dark energy dominates the Universe. Understanding the nature of dark energy and how it interacts will help to determine the future of our Universe, and the way it came to be.

Dark energy is a hot topic of debate in modern cosmology. Although its presence is strongly suggested by current observations, the true nature of dark energy remains a mystery. This report outlines the need for dark energy, the effect it has had on forming the Universe observed today, and any contributions it may have to the future history of the Universe.

The requirement for dark energy is demonstrated as the theoretical age of the Universe and the observed age conflict. A range of methods for determining the observed age of the Universe are explored such as globular cluster dating through the main-sequence turn-off method (MSTO) and through white dwarf cooling rates.

The nature of dark energy is studied, in particular whether the energy density is a constant or a function of time, and also the precise format of its equation of state. This is done by looking at data from the Cosmic Microwave Background (CMB) using the Planck satellite, and determining how sensitive this data is to the barotropic parameter of dark energy.

Despite our lack of understanding about dark energy, the evidence suggests it will dominate as the Universe continues to evolve, and this presents a broad range of possibilities for the future history of the Universe. From this perspective, we determine how quintessence (a particular hypothetical form of dark energy) would affect the future of the Universe if at all, and we also explore the idea of a cyclic Universe.

2 Part I: The Necessity of Dark Energy

If the theoretical age of the Universe is compared to the age we determine observationally then we find that the two results differ. This is the basis of the Cosmic Age Problem (CAP). We will discuss the nature of this problem and how it is overcome throughout this section.

2.1 Theoretical Age of the Universe

Define: $\Omega_m = \frac{\rho_{m0}}{\rho_0}$, $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_0}$, $\rho_0 = \frac{3H_0^2}{8\pi G}$, $\rho_{m0} = \rho_m \left(\frac{a}{a_0}\right)^3$

Using the Friedmann equation for a flat Universe dominated by matter and cosmological constant:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda)$$
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_{m0}\left(\frac{a_0}{a}\right)^3 + \rho_\Lambda)$$
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left\{\frac{\Omega_m 3H_0^2}{8\pi G}\left(\frac{a_0}{a}\right)^3 + \frac{\Omega_\Lambda 3H_0^2}{8\pi G}\right\}$$

Cancelling the $\frac{3H_0^2}{8\pi G}$ term, taking the H_0^2 out as factor, multiplying by a^2 and square rooting leaves:

$$\dot{a} = H_0\sqrt{\Omega_m\left(\frac{a_0^3}{a}\right) + \Omega_\Lambda a^2}$$

Let $a_0 = 1$ and use the relation $\Omega_\Lambda + \Omega_m = 1$ for a flat Universe.

$$\dot{a} = H_0\sqrt{\frac{1-\Omega_\Lambda}{a} + \Omega_\Lambda \cdot a^2} = \frac{\mathrm{da}}{\mathrm{d}t}$$

From this differential equation, an integral can be formed:

$$H_0\int_0^{t_0}\mathrm{d}t = \int_0^1 \frac{\sqrt{a}\cdot \mathrm{da}}{\sqrt{1-\Omega_\Lambda + \Omega_\Lambda \cdot a^3}} \tag{1}$$

use substitution $u = \frac{\sqrt{\Omega_\Lambda \cdot a^3}}{\sqrt{1-\Omega_\Lambda}}$

So that after evaluating the left side of the equation with respect to time the integral becomes:

$$t_0 H_0 = \int_{a=1}^{0} \frac{2}{3\sqrt{\Omega_\Lambda}}\cdot\frac{\mathrm{d}u}{\sqrt{u^2+1}}$$

Using the substitution $u = \sinh x$ and the relationship $\sinh^2 x + 1 = \cosh^2 x$ the integral becomes:

$$t_0 H_0 = \frac{2}{3\sqrt{\Omega_\Lambda}}\int_0^{a=1}\mathrm{d}x$$

To determine limits, when $a = 0$, $u = 0$. When $a = 1$, $u = \frac{\sqrt{\Omega_\Lambda}}{\sqrt{1-\Omega_\Lambda}}$

$$t_0 H_0 = \frac{2}{3\sqrt{\Omega_\Lambda}}\left[\sinh^{-1}\left(\frac{\sqrt{\Omega_\Lambda}}{1-\Omega_\Lambda}\right) - \sinh^{-1}(0)\right]$$

Since $\sinh^{-1} 0 = 0$ and again using $\Omega_\Lambda + \Omega_m = 1$, we find t_0 to be:

$$t_0 = \frac{2H_0^{-1}}{3\sqrt{1-\Omega_m}}\sinh^{-1}\left(\frac{\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}}\right) \tag{2}$$

This equation can also be written in logarithmic form.

Using the definition for arsinh(x),$\sinh^{-1} x = \ln\left(x + \sqrt{1+x^2}\right)$

$$\sinh^{-1}\left(\frac{\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}}\right) = \ln\left[\frac{\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}} + \sqrt{1+\frac{1-\Omega_m}{\Omega_m}}\right]$$
$$\sinh^{-1}\left(\frac{\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}}\right) = \ln\left[\frac{\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}} + \frac{1}{\sqrt{\Omega_m}}\right]$$
$$\sinh^{-1}\left(\frac{\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}}\right) = \ln\left[\frac{1+\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}}\right]$$

Therefore $t_0 = \frac{2H_0^{-1}}{3\sqrt{1-\Omega_m}}\sinh^{-1}\left(\frac{\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}}\right)$ can be written as:

$$t_0 = \frac{2H_0^{-1}}{3\sqrt{1-\Omega_m}}\ln\left[\frac{1+\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}}\right] \tag{3}$$

The value of H_0 is taken to be $H_0^{-1} = 67.4\text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$. The error in H_0^{-1} is taken to be small. The value of Ω_m used is $\Omega_m = 1$, as is correct for a matter dominated Universe.

Using this method the value of the theoretical age of the Universe when assumed to be flat and matter dominated (That is, $\Omega = \Omega_m = 1$) to be:

$$t_0 = 9.7\text{Gyr}$$

2.2 Methods for Determining the Age of the Universe

In this section we will discuss different methods that astronomers and physicists use to attempt to determine the age of the Universe. We discuss how astronomers use globular clusters (GC) to determine the age of the Universe, and how astronomers use the Universe's expansion rate to extrapolate back to the Big Bang and its use as an age determining method.

The Age of the Universe from Globular Cluster Observations

Astronomers currently utilise GCs through multiple techniques when determining the Universe's age[12]. A GC is a large group of stars that, due to their proximity, form at the same time. This makes them incredibly useful when studying the formation of stars. When measuring the Universe's age via GC aging methods, the fact that it is impossible for the Universe to be younger than any of the objects the GC contains can be exploited[16]. As a result of this, it is possible to determine a minimum value for the age of the Universe from the maximum possible age of GCs. In this section of the report three methods for determining the age of GCs, and in turn the Universe, will be discussed: the main sequence turn-off point (MSTO), white dwarf cooling curves, and radioactive dating.

The MSTO method makes use of the fact that all of the stars are approximately the same age in a GC and have the same composition. As a result of this, in a GC only the star's masses determine their point on an Hertzsprung-Russell diagram. During the main sequence of a star, hydrogen fuses into helium in the star's core, and most of the stars in the GC will be undergoing this process. A star of slightly higher mass will have exhausted its supply of hydrogen in its core already, and this results in the star undergoing a change of composition. As a result of this, it will begin to move away from the main sequence into the main sequence turn-off region (MSTO).[1]. While a GC ages, MSTO stars will begin to decrease in mass as higher mass stars will have already reached this point. By determining the mass of the MSTO stars and using the fact that the main sequence lifetime of an MSTO star equals the age of the GC, the age of the cluster can be determined, and therefore a lower bound for the age of the Universe[3] can be found.

Another method astronomers use for determining the age of the Universe is by studing the cooling of white dwarf (WD) stars. A WD is the final remnant of a star with mass smaller than 10$\pm$2 M$\odot$, with a not yet well known upper mass limit of the progenitor. A WDs evolution can be approximated to a cooling process lasting $\sim$10Gyr, allowing information to be obtained about the age of the galaxy or cluster containing the WD and also obtain information about past stellar formation rate in their stellar neighbourhood. Due to this, WDs can be used to pose a limit on the Universe's age[8].

The luminosity of the faintest WDs in a high age GC can be used to estimate the age of the GC by comparing observed luminosities to theoretical cooling curves. These theoretical curves are subject to significant uncertainties due to detailed radiative transfer calculations that have innately large uncertainties at low temperatures, and uncertainties related to the core composition of WDs. This method is, as such, difficult to exploit effectively to determine a minimum age of the Universe.

An age of 12.7$\pm$0.7Gyr was found by Hansen et al (2002)[17] via the cooling sequence of WDs. This value is consistent with ages of GCs obtained via the MSTO method. Unfortunately this method has not been used much, due to the fact it requires long exposure observation, on the highly demanded Hubble Space Telescope. The errors quoted do not take into account any uncertainties in the theoretical cooling calculations, and so the value is likely inaccurate.

The radioactive dating method involves studying radioactive element abundance to determine the age of the oldest stars in a GC. This is done by studying their stellar spectra. Other radioactive elements, for instance ^{176}Lu, ^{40}K and ^{87}Rb, could also be useful for this method, but we do not currently use them as we do not know enough about the history of their nucleosynthesis. The main issue with using radioactive dating as a method for age determining is that very few measurements have been taken, and the measurements that have been taken have larger errors than both the MSTO or WD cooling methods[4].

Convection is a poorly understood concept in this area of stellar astrophysics, and is a large cause for uncertainty in observationally determining the age of GCs, and in turn the Universe. In regions such as the cores of low mass main sequence stars that form the constituents of GCs, the lack of knowledge about their convection is not important as the cores are not convective, but the outer layers of the model for these stars are, and as a result brings significant uncertainties. As the cores are not convective, however, uncertainties due to convection are considered to not affect the stellar lifetimes and luminosities of the model. The MSTO method is deemed the most accurate when it comes to determining the age of these GCs, due to the fact that it relies on these two factors. The MSTO method also has unresolved uncertainties, particularly in the determination of the distance to the GCs and their chemical composition, with these uncertainties leading to changes of order $\sim$22%[5, 10]. B. Chaboyer states that, through a combination of errors when determining the age of these GCs, their absolute age is found to fall within the range of 11-21 Gyr. In another paper by Chaboyer[7], following the numerical analysis of 18 of the oldest GCs, the age of the oldest GCs were found to be 14.6$\pm$1.7 Gyr, and produced a lower bound on the age of these GCs to be 12.07 Gyr, with a 95% (2-σ) confidence level.

The formation time of GCs also needs to be taken into account when calculating the age of the Universe. Due to their composition being of primarily light elements (GCs contain a large amount of Population II, metal-poor, stars) we are able to confidently assume that their formation occured soon after the collapse of gases within early protogalaxies, before stellar evolution allowed for the production of heavier elements. Research on the formation time of early protogalaxies predicts that these objects formed around 100 million years (0.1 Gyr) after the Big Bang[2]. We therefore have a value for the age of the Universe as calculated by observation of GCs to be 12.17 Gyr.

2.3 Comparing Theoretical Age to Observed Age

By knowing the age of the oldest objects in the Universe we are able to impose a minimum age constraint on the Universe. This means that by calculating the minimum age of the oldest objects and combining that with their minimum formation time, a minimum age of the Universe itself can be determined. As seen in Section 2.2 the age of the Universe that calculated through the observation of GCs is approximately 12.2 Gyr, which is $\approx$ 125% of the value derived theoretically in Section 2.1 of 9.7 Gyr using a value of $\Omega_m = 1$ (that is that the Universe is formed entirely of pressureless matter). From this it is apparent that the age of the Universe derived from GCs is actually significantly larger than that which was derived theoretically. This inconsistency is known as the Cosmic Age Problem[13].

By considering the cosmological constant, Λ, a solution is provided to this age problem[14, 15]. Λ is the value of the energy density of the vacuum of space. As such, for any value $\Lambda > 0$ the corresponding vacuum density has negative pressure which leads to the Universe undergoing accelerated expansion[11]. It is this accelerated expansion of the Universe which accounts for the difference between the theoretically obtained, and observed age of the Universe. A value for the effective mass density which must be accounted for by the cosmological constant can be deduced, Ω_Λ, by plotting the theoretically derived age over a range of values of Ω_m. The relationship for a flat universe between these is given by:

$$\Omega = \Omega_m + \Omega_\Lambda = 1$$
$$\Omega_\Lambda = 1 - \Omega_m \tag{4}$$

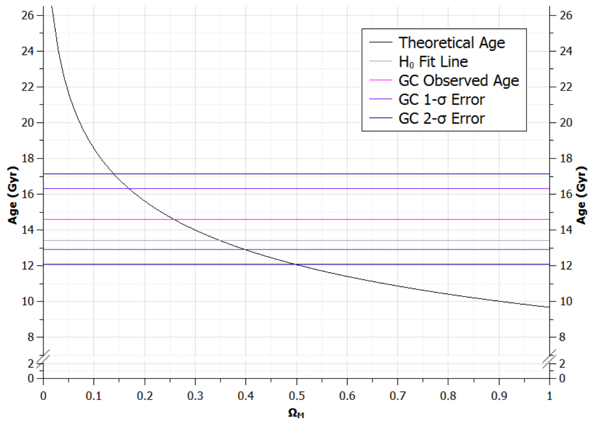

Figure 1: Graph displaying the theoretically derived age of the Universe over the range of possible values of Ω_m including indication of the observed age of the Universe which is determined by the ageing of GCs. These values do not take into account the length of time taken for the GCs to form. A fit line of t_0 for a given value for H_0 is shown in grey.

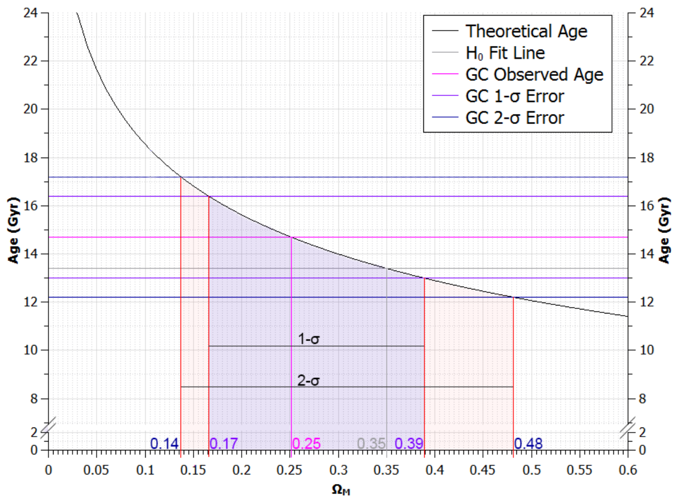

Figure 2: Graph displaying the theoretically derived age of the Universe over the range of possible values of Ω_m including the maximum and minimum ages of the Universe allowed by the analysis of GCs. The 2-σ errors are used here, with the upper and lower limits for the formation time of the GCs included in the values of the upper and lower limits respectively. The values of Ω_m which correspond to the observed and theoretical age aligning are indicated in red, while the value for the given value of H_0 is shown in grey.

The values we are using here are quoted with an indication of the amount of error in the values, that is 1-σ and 2-σ error values. These errors are related to standard deviation, which is a measure of the dispersion of a set of data values. A value given with 1-σ errors would indicate that $\approx 68.3\%$ of the values of the age lie within one standard deviation of the mean, for 2-σ this value is $\approx 95.5\%$. This would mean that we would expect the true value to be within the 2-σ error with a certainty of approximately 21 in 22.

The maximum value of Ω_m possible produced by the data from GC observation which is consistent with the theoretically calculated age is $\Omega_m = 0.39$ (1-σ max), or $\Omega_m = 0.48$ (2-σ max). This shows that $\Omega_m \neq 1$ and that the majority of the energy density of the Universe must be from another source than normal, baryonic matter. This is consistent with our current understanding of the age problem, that the flat Universe is dominated by Ω_Λ.

The direct observational constraints on Ω_m from galaxy cluster dynamics and BBN give a 2-σ range of $\Omega_m = 0.35 \pm 0.1$ [9], and from the GC data gives a 2-σ range of $\Omega_m = 0.34 \pm 0.2$. As such, it can be seen that these values are consistent in that the Universe is not matter dominated, however the GC data carries much larger uncertainties and the weightings of the upper and lower bounds of the uncertainties are not known. The fact these are consistent allows for a value for the age of the Universe to be produced by considering a Λ dominated Universe.

The Age of the Universe from Measuring the Universe's Expansion

So far it has been determined that the Universe is constantly expanding. The rate of this expansion can be measured and extrapolated to determine the age of the Universe. As a result, finding the expansion rate, given by the Hubble constant, H, is required. There are different factors that determine the value of the Hubble constant, namely the composition of the Universe given by the density of the matter (either ρ_m, or ρ_Λ). H is given by the Friedmann equation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda) \tag{5}$$

Missions such as NASA's Wilkinson Microwave Anisotropy Probe (WMAP) and the European Space Agency's Planck spacecraft have both mapped the cosmic microwave background radiation (the left over thermal radiation from the Big Bang) of the Universe, and from this have determined the Universe's composition, and so, expansion rate. The best estimate for the H obtained so far is 72km/s/Mpc [23].

Once the Hubble constant is known, it can be used to extrapolate back to the Big Bang. If the Universe is flat and matter dominated (k = 0, $\Omega_m = 1$), the age of the Universe can be given by

$$t_0 = \frac{2}{3H_0} \tag{6}$$

where H_0 is the Hubble constant and T_0 is the age of the Universe.

However, if the Universe has a very low matter density, then the age determined by extrapolation is bigger, given by

$$t_0 = \frac{1}{H_0} \tag{7}$$

This age can be even larger if the Universe contains a matter form similar to the cosmological constant[23].

The mapping of the CMB that WMAP obtained makes it possible to determine the composition of the Universe, H and the density of the Universe to a 1.5% accuracy in 2012, thus providing an estimate of the Universe's age to within 0.4% accuracy. The estimate from the WMAP data given on the NASA website is 13.77 ± 0.059 billion years, while in 2013 the Planck spacecraft measured the age to be 13.82 billion years.

3 Part II: The Nature of Dark Energy

3.1 The Sensitivity of the Planck Satellite

For the case of a cosmological constant, the results determined by Planck observations give a range of ages of the Universe. Uncertainties in the determination of m and H_0 account for the range of t_0 that is found. To determine whether the Planck range of t_0 is found, and the shift of t_0 due to deviation of the dark energy from a cosmological constant, the shift of the mean value of t_0 due to w_X should be larger than the Planck error on t_0. We can estimate how large the deviation of w_X from -1 has to be by solving the equation for the age of the Universe computationally or graphically over different values of w_x until they fall within the constraint of t_0 determined by Planck, at a value of $\Omega_m = 0.314$. The equation used to plot a graph of this information was derived from the Friedmann equation as used in Part 2.1 (The full derivation can be found in Appendix A).

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_X\right)$$
$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0\left(\Omega_m\left(\frac{a_0}{a}\right)^3 + \Omega_X\left(\frac{a_0}{a}\right)^{3(1+\omega_X)}\right)$$

$$t_0 = \frac{1}{H_0}\int_0^1 \frac{da}{\sqrt{\Omega_m a^{-1} + (1-\Omega_m)a^{-1-3\omega_X}}} \tag{8}$$

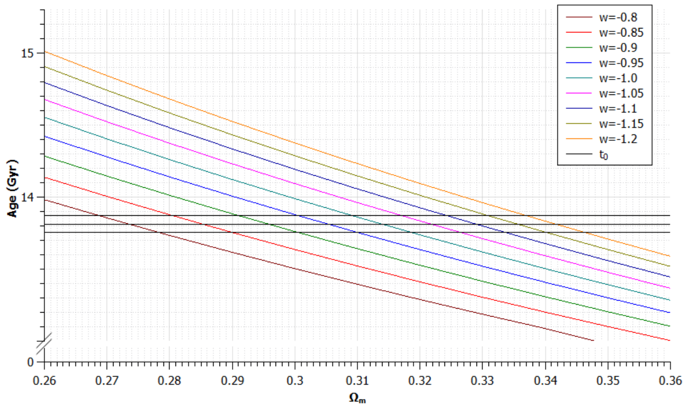

Figure 3: A section of the data produced by evaluating the integral in equation 8 for the age of the Universe for a range of values of w_X from $w_X = -0.8$ to $w_X = -1.2$, against Ω_m. The data is centered about $\Omega_m = 0.314$, the value determined by Planck observations, and the horizontal black bars represent the 1-σ errors on t_0, the current age of the Universe, as determined by Planck. The deviation of w_X from -1 can be determined by extrapolating where the w_X line meets the maximum and minimum allowed values of t_0 at $\Omega_m = 0.314$. The values of w_X can also be calculated computationally; the method for this is shown in Appendix A, Section 5 along with a complete view of the output data.

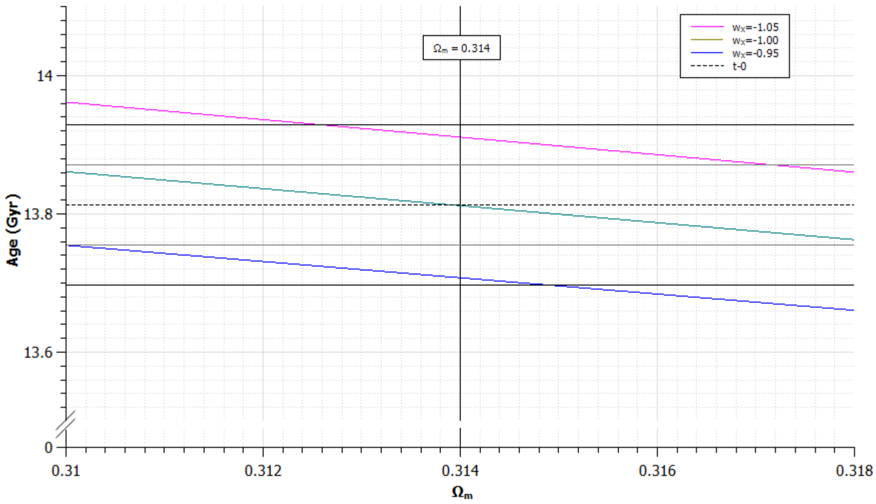

Figure 4: A closeup of figure 3.1 concentrating on data around t_0 and its uncertainties, and $\Omega_m = 0.314$. It can be seen that the values of $w_X = -0.95, -1.00, -1.05$ are within the range of ages given by the Planck data, and so any value outside of this range indicates the fluctuation is not due to uncertainty in the results with 95% confidence.

From figure 3.1 it is apparent that the 1-σ error accounts for a deviation in w_X of around ± 0.025, and the 2-σ error of t_0 encompasses a deviation of w_X from -1 of over ± 0.05. This means that if the value of w_X deviates by a larger amount than this, it can be said with 95% confidence that this difference is due to deviation of the dark energy from a cosmological constant, and not uncertainties in the data. The value can be determined more precisely by using computational techniques. The values determined computationally for w_X are as follows:

$$1-\sigma \qquad w_X = -0.972821, -1.029862$$
$$2-\sigma \qquad w_X = -0.945669, -1.059849$$

This means that in the 1-σ range the values deviate by -0.027179/+0.029862, and in the 2-σ range the values deviate by -0.054331/+0.059849. This is a more precise constraint on the 2-σ error than given before, although the meaning of the results remains the same. If the value determined by Planck deviates w_X from -1 by more than these 1-σ values there is a 68% certainty that the difference is due to non-inherent sources, and 95% confidence for the 2-σ results.

3.2 The EUCLID Satellite and Dark Energy Equation of State

3.2.1 The EUCLID Satellite

The EUCLID mission is a six year mission that is part of ESAs "cosmic vision" programme, a system that allocates budget to interesting space science objectives. It is a medium-class mission, with a budget of approximately €500 million[18]. Using a space telescope with a 1.2m diameter aperture that covers an area of 0.5 deg^2, the mission aims to map the distribution of dark matter within our Universe and to measure the strength and time-dependence of dark energy.

The mission will make use of three observational instruments to achieve this: a CCD based optical imaging channel (VIS), a near infrared imaging photometry channel (NIP), and a near infrared spectrometric channel (NIS)[19]. By using the spectrometer to measure the redshifts of 70 million galaxies, a huge amount of data may be gathered on their apparent separation as a function of redshift. By comparing this distribution to the known baryon acoustic oscillation (BAO) length, the expansion of the Universe may be analysed in sufficient detail to determine if dark energy is time-dependent.

Away from the light pollution of our own galaxy, dark matter may be detected by using the CCD arrays to image almost the entire extragalactic sky, making it a survey of 20,000 square degrees. The mission will measure galaxies out to redshifts of z$\sim$2. This is effectively the same as looking back 10 billion years in time, therefore obtaining data for the period over which dark energy accelerated the Universe's expansion. Weak gravitational lensing (WL) will use extremely high image quality to map the gravitational pull of dark matter as it bends the light that passes it[20]. This can be mapped as it affects the images of the distant galaxies. This gravitational shear will be measured for almost three billion galaxies, providing a detailed picture of the distribution of dark matter in the Universe. Together, these techniques will reveal much about the nature of our Universe, and provide huge quantities of data to test models against.

Although dark matter and dark energy have been measured before, the EUCLID mission promises to dramatically improve on the best of the current knowledge in these areas. By measuring the constant and time-dependent dark energy parameters w_0 and w_x to within 2% and 10% respectively, the mission will achieve a dark energy figure of merit of 500. This is the reciprocal of the product of the fractional uncertainties in w_0 and w_x. The current best figure for the dark energy figure of merit is 10. This figure is derived from data obtained by the Planck spacecraft and its maps of the cosmic microwave background. EUCLID thus stands to improve 50-fold on the accuracy of our knowledge of the magnitude and time-dependence of dark energy[19].

3.2.2 The Physics of Baryonic Acoustic Oscillations

Since the birth of modern cosmology in the early 20th century, astronomers have been trying to make sense of the Universe: its history and evolution over time. At the same time, they have been observing numerous celestial objects, not just the planets in our solar system but also stars and galaxies much further away from our Sun. For individual galaxies and groups of galaxies, it is now known that they are not randomly distributed in space but instead follow a pattern called Baryon Acoustic Oscillation (BAO). BAO falls under the region of Statistical Standard Ruler: the measurement of angular distance and expansion rate diameter as a function of redshift[21]. BAO is also an ancient relic from pre-decoupling Universe and its effects can still be observed by astronomers today.

400,000 years before the Big Bang[22], at a redshift of $z = 1100$, baryons (electrons and free nuclei) and photons were constantly in a fluid-like plasma that tightly coupled with each other via Thomson scattering. Oscillating patterns then arose where there were high density of baryons, held together by gravity and other areas of high photon density, heat pressure dominated. Pressure differences from these two different sources created "sound waves" analogous to sound waves in air where you have rarefaction (low pressure) areas and compression (high pressure) areas. Then when recombination occured and temperature dropped below 1 eV, coupling stopped leading to the free electrons and free nuclei combining to form neutral atoms. The photons' mean free path length grew rapidly and became longer than the horizon distance. Hence, the Universe became transparent and the free travelling photons now can be observed as CMB. This phenomenon can also be observed in the temperature fluctuations of CMB from the WMAP image:

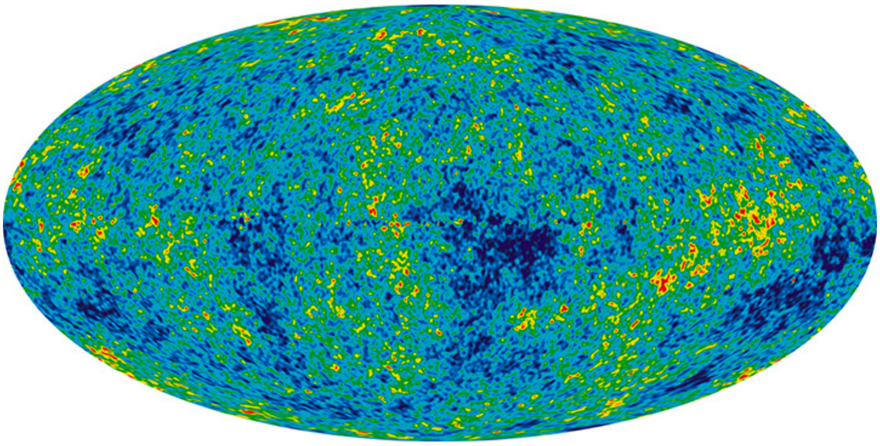

Figure 5: Colour differences in this image from Wilkinson Microwave Anisotropy Probe (WMAP) reveal 13.77 billion year old temperature fluctuations that correspond to where galaxies would grow. Credit: NASA/WMAP Science Team[6]

3.2.2.1 Sound Horizon At Decoupling

Consider a spherical baryon-photon plasma before decoupling. The pressure differences will create acoustic waves that propagate outwards. When photons decoupled from baryons and travelled freely along with the expansion of the Universe, traces of baryons were left in the middle of the sphere as they are heavy and do not have sufficient energy to be dispersed. Dark matter then interacted and clumped with the heavy baryons through gravity and this created the seed for future galaxies. It also produced BAO scale which is the characteristic size of the sound wave. The BAO scale or sound horizon, measured to be 146.8 ± 1.8 Mpc[24], is the distance sound waves could have travelled in the time before recombination[25] based on conditions of the early Universe. In addition, It shows how long these sound waves persisted as the effects can still be observed until now. Therefore, this type of measurement is important in modern cosmology as it can measure a distance to approximately 11 billion light years away to better than 2% precision[26].

3.2.2.2 Distribution of Galaxies At Later Times

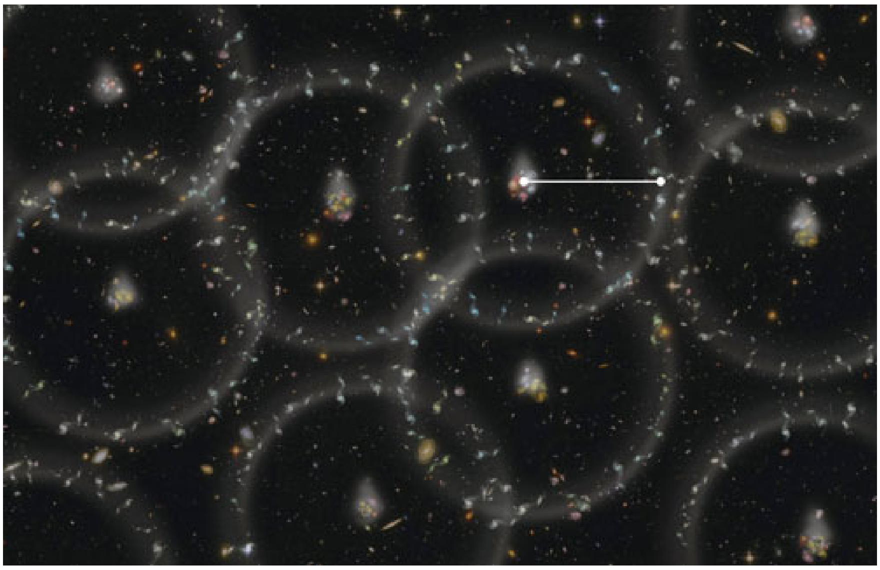

Figure 6: A simulation produced by the BOSS project[27] showing the spheres of baryons and dark matter clumps. The radius of sphere is the sound horizon.

As seen in Figure 2 above, this is the ancient relic that the decoupling period left behind. At the centre of the sphere, dark matter and heavy baryonic matter clumped together as they were not able to travel as the Universe expands. Free photons however travelled along the the Universe's expansion giving a final radius of shell of around 500 million light years which is equal to the sound horizon.[28] Thus from this phenomenon astronomers found that it is more likely to find galaxies separated by 500 million light years than any other distances, a pattern emerging from BAO.

3.2.3 The Angular Size of Baryonic Acoustic Oscillations

First, the angular size of a BAO as a function of redshift and the parameters of the dark energy equation of state w_p and w_a must be found. In comoving coordinates, light behaves perfectly normally and travels distance D in comoving time τ at speed c.

$$D = a_0 c\tau$$

To move to physical coordinates:

$$dt = a(t)d\tau$$
$$\tau = \int_{t_i}^{t_0} \frac{dt}{a(t)}$$
$$D = a_0 \int_{t_i}^{t_0} c\frac{dt}{a(t)}$$

Now using the continuity equation to find ρ_X:

$$\begin{aligned}
\frac{d\rho_X}{dt} + \frac{3}{a}\frac{da}{dt}\left(\rho_X + \frac{P_X}{c^2}\right) &= 0 \\
\int_{\rho_{X_0}}^{\rho_X} \frac{1}{\rho_X} d\rho_X &= \int_{a_0}^{a} -\frac{3}{a}(1+w_X)da \\
ln\left(\frac{\rho_X}{\rho_{X_0}}\right) &= -3\left[(1+w_p+w_a)ln\left(\frac{a}{a_0}\right) + \frac{w_a a_0}{a_0} - \frac{w_a a}{a_0}\right] \\
\rho_X &= \rho_{X_0}\left(\frac{a}{a_0}\right)^{-3(1+w_p+w_a)} e^{3w_a\left(\frac{a}{a_0}-1\right)}
\end{aligned} \tag{9}$$

Then using the Friedmann equation: $D(z)$

$$\begin{aligned}
H^2 &= \frac{8\pi G}{3}\left(\rho_m + \rho_X\right) \\
H^2 &= \frac{8\pi G}{3}\left(\Omega_m \rho_{c_0}\left(\frac{a}{a_0}\right)^3 + \Omega_X \rho_{c_0}\left(\frac{a}{a_0}\right)^{-3(w_p+w_a)} e^{3w_a\left(\frac{a}{a_0}-1\right)}\right) \\
D(z) &= a_0 c \int_a^{a_0} \frac{da}{a^2\sqrt{H_0^2\left(\frac{a_0}{a}\right)^3\left(\Omega_m + \Omega_X\left(\frac{a}{a_0}\right)^{-3(w_p+w_a)} e^{3w_a\left(\frac{a}{a_0}-1\right)}\right)}} \\
D(z) &= \frac{c}{H_0\sqrt{a_0}} \int_a^{a_0} \frac{da}{a^{-\frac{1}{2}}\sqrt{\Omega m + \Omega_X\left(\frac{a}{a_0}\right)^{-3(w_p+w_a)} e^{3\left(\frac{a}{a_0}-1\right)}}}
\end{aligned} \tag{10}$$

Now the distance to the BAO is known, so its theoretical length and angular size can be related. With $D(z)$ and the BAO length as the long and short sides respectively of a very slim triangle, the formula is

$$\tan(\theta_{BAO}) = \frac{l_{BAO}}{D(z)}$$

Applying the small angle approximation, since $\theta_{BAO} \ll 1$, gives us

$$\theta_{BAO} = \frac{l_{BAO}}{D(z)} \tag{11}$$

3.2.4 Analysing Simulated EUCLID Data

Using the simulated EUCLID data provided (which can be found in Appendix A), computational analysis was performed to produce values for w_p and w_a. Code was written first to numerically integrate the expression for $D(z)$ using the midpoint method. This method evaluates the integral by taking the midpoint of the intervals, as these points are most likely to be close to the average[29].

Then a hillclimbing algorithm was used to find the values of w_p and w_a which best fit our data. This works by repeatedly integrating this expression to calculate θ from z, and comparing to the actual θ / z pairs in the data sets. Of a tight grouping of such points on the graph of possible w_p and w_a values, one point would have the lowest distance from the target point, and so a new cluster was generated around this point to move closer to the true values. The code converges on values of $w_p = -1.1304$ and $w_a = -0.088715$, so the simulated dark energy is not a cosmological constant as the time-dependent parameter of its equation of state (w_a) is non-zero. The real EUCLID could have also detected this divergence from a cosmological constant, since their stated capability is to measure w_a to 10%

Errors in BAO data could be dealt with in another brute-force computational manner; a monte carlo approach. By making many runs, each with the data 'fuzzed' (randomly altered) by a normal distribution with the same σ as the error of the data, the distribution of output values of w_p and w_a can be observed and the the standard deviation can therefore be measured. While obviously taking more computer time, this removes the need to attempt to analytically carry errors through a numerical process.

4 Part III: The Future History of the Universe

4.1 Cyclic Universe

A cyclical Universe is one which oscillates between expansion and contraction. There are many different models, for example some models predict that the dimensionless constants of nature vary from cycle to cycle [30].

One such model[31] gives the Friedmann equation as:

$$H^2 = \frac{1}{3}\rho + \gamma\rho^2 + \frac{\Lambda}{3}$$

When solved, this model describes a bounce in replacement of the initial singularity in the Cosmological Standard Model (CSM). For $\Lambda \leq 0$ and $\upsilon \leq 0$, this bounce becomes cyclic behaviour.

A cyclic model is proposed due to problems in the current CSM. These cracks appear in the form of the flatness problem, the horizon problem, the coincidence problem, as well as there being no explanation for the beginning of time and the initial conditions of the Universe, or the "end" of the Universe.

A cyclic Universe has no need to account for initial conditions or an end and so bypasses these two issues instantaneously. It can be described in the same way as an inflationary Universe, that is, as a scalar field ϕ in a potential $V(\phi)$. In order to satisfy the oscillatory model, the potential must tend to 0 when the scalar field tends to $-\infty$, have a positive potential as the scalar field tends towards $+\infty$, and be less than 0 in between. This can be described by the following equation:

$$V(\phi) = V_0(1 - e^{-c\phi})F(\phi)$$

The most commonly accepted model proposes the Universe is infinite and flat. The homogeneity and flatness in our Universe can be explained by periods of slow accelerated expansion as observed today, and periods of slow contraction. Rather than the curvature then, the contraction is governed by a negative potential energy. Like the CSM the Universe begins each big bang with a period of matter and radiation domination, and dark energy causes the accelerated expansion. This expansion dilutes black holes and is important because it prevents "monster black holes" from forming in the contraction period.

Such Universes have been explored in an attempt to solve current problems in the standard cosmological model, and indeed it is a beautiful solution, as Lematre states, "The solutions where the Universe successively expands and contracts, periodically reducing to an atomic system with the dimensions of the solar system, have an incontestable poetic charm, and bring to mind the Phoenix of the legend" [32].

4.1.1 Successes of the Cyclic Universe Model

Flatness Problem: Present data indicates that Ω_{tot} is close to 1, however the expression $|\Omega_{tot}(t) - 1|$ grows as a power of t. This implies that Ω_{tot} must have been very close to 1 in the early Universe. This is a problem because there is not yet any concrete reason why Ω_{tot} should have initially been so close to 1. It is a coincidence which leaves much unanswered.

In a cyclic Universe, Ω_{tot} only begins to deviate from 1 when the scale factor, a, approaches a maximum value. This maximum value of a increases as the number of cycles do. In an old Universe which has had many cycles, Ω_{tot} would take a long time to deviate from 1. A 'long time' in a oscillatory Universe is considered to be of the order of a trillion years, and since our current observations indicate that the age of the current cycle of our Universe is on the order of 10 billion years, it can be considered very young and thus could explain why present data indicates that Ω_{tot} is so close to 1.

The coincidence problem: Another coincidence occurring in the standard cosmological model is the reason why the Universe started accelerated expansion just as we are able to measure it. The cyclical Universe model solves this problem, because in many models [33] the amount of time that a Universe spends in the so called 'coincidence state' is similar to the time period of the oscillation itself and so it is no suprise that we find the Universe at the beginning of accelerated expansion.

The horizon problem: The Universe has a high degree of homogeneity. Since light signals travel a finite distance, there is no way all the light can be in causal connection, but the degree of homogeneity observed suggests interaction between the light.

To calculate the age of the Universe for a cyclic Universe, one must add all the ages of the previous cycles. This gives the light extra time to propagate and thus could be in causal connection.

Dark energy problem: In the CSM there is no explanation for dark energy. In

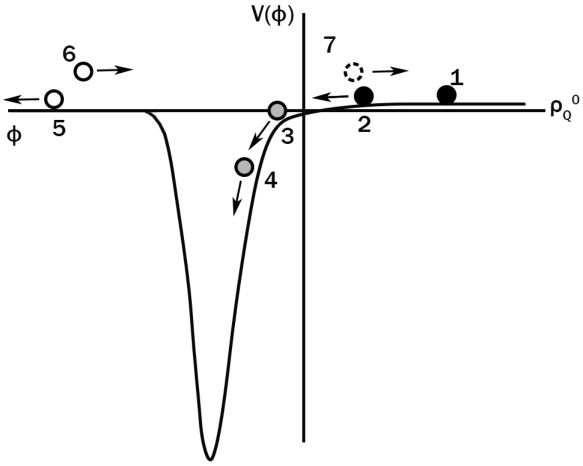

Figure 7: The potential as a function of the scalar field showing various points in the Universe's future history

an oscillatory Universe, however, dark matter (and therefore exponential expansion) is actually a mandatory component in order for the Universe to dilute entropy and keep the Universe homogeneous by suppressing density and energy fluctuations

4.1.2 Shortcomings of the Cyclic Universe Model

Monster Black Holes: When the Universe collapses back in on itself, the scale factor gets smaller and so all matter becomes closer and more compact, including black holes. These black holes have the potential to merge and create a monster black hole which would disrupt the homogeneity currently observed in our Universe. Therefore, when expanding the Universe must reach an energy density such that the black holes are torn apart and the Universe becomes smooth before it can collapse. Such an energy density is presented below[34].

$$\rho = M_P^4 \left(\frac{M_P}{M}\right)^2 \frac{3}{32\pi} \frac{1}{|1+3\omega|}$$

The Bounce: In previous cyclical models, as the Universe contracts and the scale factor tends to 0, the density becomes a singularity. To combat this, the model presented is that the singularity in 4-dimensions is non-singular in a higher dimensional model. Therefore the density remains finite and the Universe can traverse a smooth transition from big crunch to big bang, and the temperature and density can remain finite at each transition during the bounce.

Figure 4.1.2 shows the potential and explains the future history of the Universe. At point 1 the Universe it at its current state where it has finished the radiation and matter dominated eras and is beginning to enter the dark energy dominated era. This creates accelerated expansion and this expansion lasts for trillions of years, solving the "coincidence problem" previously mentioned. The accelerated expansion causes the Universe to become diluted and smooth, where the density of particles can be as little as one per Hubble volume, $n = 10^{-79}\text{m}^{-3}$. The slight slope in potential causes ψ to slowly roll back towards, see point 2. This continues until point 3 where the potential reaches 0. At point 4, gravitational potential energy is converted into the kinetic energy of the scalar field and the field moves past the minimum to $-\infty$ (point 5). By now the Universe is very small and so a tends towards 0. The Universe bounces back, and initially the Universe is dominated by its scalar kinetic energy density which is proportional to a^{-6}, point 6. By stage 7, the Universe is radiation dominated, and remains at this near maximum value of potential for the majority of the next cycle.

4.1.3 Impact on the Milky Way

As dark energy begins to increasingly dominate during the long expansion period, the Milky Way will drift apart from all but its nearest neighbouring galaxies. Although a lonely existence for the Milky Way, life could continue to prosper. This is due to the binding of interstellar space through gravity, which resists the impact of an exponentially growing scale factor. This binding keeps the Earth at the same distance from the Sun, and stars at the same distance from the galactic core.

At some time the rate of growth will become too large for even gravity to resist. The Milky Way will be torn apart, and shortly after, the galactic core (the black hole at the centre) will follow. Eventually, the exponential expansion "restores" the universal homogeneity and smoothness needed for a successful big crunch.

4.1.4 Comparing the Cyclic Universe Model to the Cosmological Standard Model

CSM requires two periods of cosmic acceleration. One of which happened in the very early Universe, and the other which exists and can be measured today. The cyclic model of the Universe only relies on the latter, and thus this part of the model can be explained purely by observation. According to A. Mazumdar, 'current observational data strongly favours primordial inflation'[35] and so provides evidence for an inflationary Universe.

In the cyclic model, the Universe we observe is representative of the Universe as a whole. In the CSM, however, most of the Universe is still inflating and our region is one which has stopped. What we can measure is therefore not typical of the rest of the Universe and so in this respect lacks some physical evidence.

4.2 Quintessence

4.2.0.1 Background

Both the cosmological constant and quintessence are theories that aim to provide the answer to the overwhelming amount of observational evidence suggesting the expansion of the Universe is accelerating. The cosmological constant has a constant equation of state, given by[36]:

$$\omega_\Lambda = \frac{P}{\rho} = -1 \tag{12}$$

Scalar fields arise in string theory and particle physics alike, for example the Higgs field, and this is the basis of quintessence. Quintessence is defined by a scalar field which satisfies the continuity equation[37]:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0 \tag{13}$$

Its equation of state is given as such[38]:

$$\omega_\phi = \frac{P}{\rho} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \tag{14}$$

Unlike the cosmological constant, the equation of state is dependent on the scalar field potential. This means that it can evolve in both time and space, and so quintessence is dynamic and as such can be both attractive and repulsive depending on its kinetic and potential energy.

One key issue in the CSM is the coincidence problem. The matter density and the missing energy density both decrease as a function of a, and so the initial ratio between energy density and the matter density must be infinitesimally small, in order that they coincide today. This can be solved by a form of quintessence called a “tracker field”. A tracker field is very insensitive to initial conditions, and so any choice of initial ratio should result in today’s observations. Any initial conditions of the field ϕ converge in $V(\phi)$ quickly and are ‘funnelled’ to a similar path. The models proposed by Zlatev et al.[39] are in excellent agreement (within 1-σ error) with current CMB measurements.

Here is one particular example[39]:

$$V(\phi) = \alpha^4 \left[e^{M_p/\phi} - 1\right] \tag{15}$$

Here, M_p is the plank mass, and there is only one free parameter, α. The value of α is dependent on the value of Ω_m. The exact dependence is characterised by a smaller value of Ω_m resulting in ω_ϕ being closer to -1. Since Ω_m decreases with time, the Universe would be expected to eventually be dominated by the equation of state, $\omega = -1$

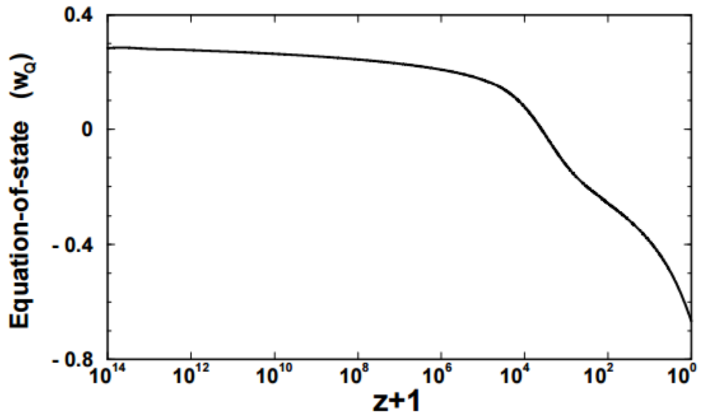

Figure 8: The equation of state vs redshift highlights the variability of the baratropic parameter[41].

During each epoch, the equation of state abruptly moves to different values. During radiation domination, $\omega_\phi \sim \frac{1}{3}$, during matter domination $(Z+1 = 10^4)$, $\omega_\phi \sim -0.2$ and then as dark energy begins to dominate, ω_ϕ decreases again, this time below $-\frac{1}{3}$ and so the Universe begins to accelerate its expansion. The equation of state will eventually tend towards the value of -1.

Based on this model, the future of the Universe should proceed as though there is a cosmological constant, but the difference between quintessence and the cosmological constant is in the way it reaches this point.

4.2.0.2 Problems with the Quintessence Model

As can be seen in figure 4.2.0.1, today's value gives $\omega_\phi \sim -0.8$, but the observed baratropic parameter of dark energy from a cosmological constant is -1. This difference cannot yet be explained unless manufactured parameters and other complications are added.

The main problem with the idea of quintessence, however, is that the field couples so weakly to matter, approximately 14 orders of magnitude below the electroweak scale[40]. The solution to this lies in searching for enough supersymmetry breaking so that the mass difference between scalars and fermions can be accounted for.

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5 Appendix A: Derivations, Raw Data, and Code

Using the following equation, the values of w_X can be compared to the values from the Planck data as discussed in Part II(i). The full derivation starting from Section 2.1 is shown:

$$
\begin{aligned}
H^2 &= \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_X\right) \\
H^2 &= \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m \left(\frac{a_0}{a}\right)^3 + \rho_X \left(\frac{a_0}{a}\right)^3\right) \\
H^2 &= \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 \left(\Omega_m \left(\frac{a_0}{a}\right)^3 + \Omega_X \left(\frac{a_0}{a}\right)^{3(1+\omega_X)}\right) \\
H^2 &= \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left(\Omega_m \left(\frac{a_0}{a}\right)^3 + \Omega_X \left(\frac{a_0}{a}\right)^{3(1+\omega_X)}\right) \\
\frac{da}{dt} &= \dot{a} = H_0 a \sqrt{\Omega_m \left(\frac{a_0}{a}\right)^3 + \Omega_X \left(\frac{a_0}{a}\right)^{3(1+\omega_X)}}
\end{aligned}
$$

$$
\int_0^a \frac{da}{a\sqrt{\Omega_m \left(\frac{a_0}{a}\right)^3 + \Omega_X \left(\frac{a_0}{a}\right)^{3(1+\omega_X)}}} = \int_t^0 H_0 dt = H_0 t
$$

$$
\begin{aligned}
t_0 &= \frac{1}{H_0}\int_0^1 \frac{da}{a\sqrt{\Omega_m a^{-3} + \Omega_X a^{-3(1+\omega_X)}}} \\
t_0 &= \frac{1}{H_0}\int_0^1 \frac{da}{a\sqrt{\Omega_m a^{-3} + (1 - \Omega_m a^{-3(1+\omega_X)})}} \\
t_0 &= \frac{1}{H_0}\int_0^1 \frac{da}{\sqrt{\Omega_m a^{-1} + (1 - \Omega_m) a^{-1-3\omega_X}}}
\end{aligned}
$$

```python
from math import sqrt
samples = 1000 # sample points per integration
datapoints = 100 # points for plotting on omegam vs t0
#hubbleTime = 4.578 * 10 ** 17 #seconds
hubbleTime = 14.51 #billion years
def integrand(a):
    return 1/(sqrt(omegam/a+(1-omegam)*a**(-1-3*wx)))

def integrate(f, start, end, samples): # midpoint method numerical integration
    spacing = (end - start) / samples
    x = start + spacing / 2 # set our first point in the middle of the first segment
    runningsum = 0
    for i in range(samples):
        runningsum += f(x)
        x += spacing # this overruns after the last sample, but we never sample there
    return runningsum * spacing

wx = -1.2 # these are down here, not at the top, because they are starting values for iteration
omegam = 0 # not canonical ones. mass density parameter is not really 0, but it's where we start
for i in range(9):
    with open("wx={0:.2f}".format(wx),'w') as outfile: # one file per line on the graph
        outfile.write("omegam age(billion_years)\n") # column headings with units
        for j in range(datapoints):
            outfile.write("{} {}\n".format(omegam, hubbleTime * integrate(integrand, 0, 1, samples)))
            omegam += 1/datapoints # we always go from 0 to 1 so this works
    wx += 0.05 # move to the next wx value
    omegam = 0 # reset to plot the line again

def hillclimb(target):
    global wx # clunky, should have done this more functionally
    wx = -1
    lastwx, increment = -1, 0.1
    moves = [0, 1, -1] # either forward, back, or close to here. 1D hillclimbing
    for i in range(100):
        errors = [] # should end up with three numbers in for three moves. We want the lowest
        for move in moves:
            wx = lastwx + move * increment # try this point
            errors.append(abs(target - hubbleTime * integrate(integrand, 0, 1, samples))) # calculate how close
        nextmove = moves[errors.index(min(errors))] # index of the lowest error corresponds to index of the best move
        lastwx += nextmove * increment
        if nextmove == 0: # tighten the net if we're getting close
            increment *= 0.5
    print(nextmove, increment, lastwx, min(errors)) # prints to screen, we only want the most recent result

omegam = 0.314 # planck data; set to the observed value for
planckt0 = 13.813 # billion years
planckt0sigma = 0.058 # 1 sigma radius. 2 sigma is twice this.
hillclimb(planckt0 - planckt0sigma) #gives-0.9728205548126232
hillclimb(planckt0 + planckt0sigma) #gives -1.0298616555795008
hillclimb(planckt0 - 2 * planckt0sigma) #-0.9456685840473908
hillclimb(planckt0 + 2 * planckt0sigma) #-1.0598488844025145
```

Figure 9: Python code used to plot curves of t_0 against Ω_m for different w_X for Part IIi. It then uses a hillclimbing algorithm to find the smallest deviations of w_X from -1 planck can detect, to 1σ and 2σ certainty.

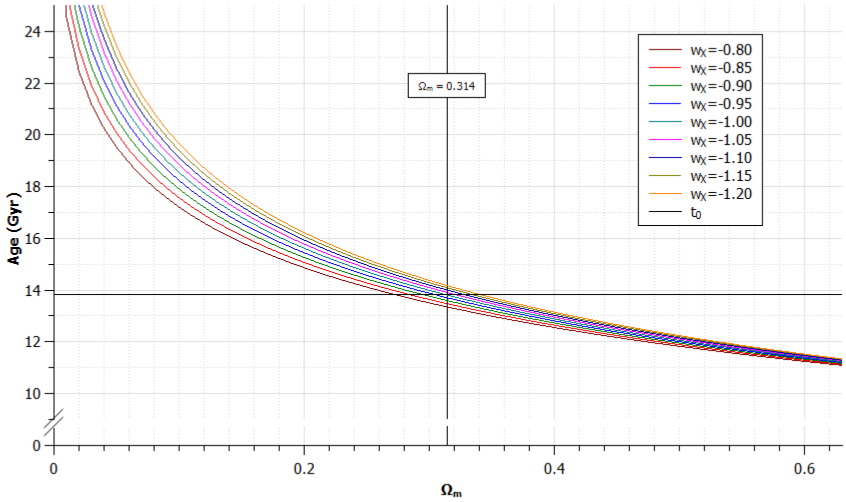

Figure 10: Python code output showing the asymptotic nature of the curve across a range of values of Ω_m. Gives greater context to the small sections shown in part IIi. t_0 is shown without uncertainties here, as they are too small to have any significance at this scale.

```
from math import sqrt,exp
samples = 1000 # sample points per integration
a0 = 1
numerator = 0.03298 # H0 * BAO length / c
omegam = 0.3175 # current observed density parameters
omegax = 0.6825 # best-fit planck values

def integrand(a):
    return 1/(sqrt(a)*sqrt(omegam+omegax*(a0/a)**(3*(wp+wa))*exp(3*wa*(a/a0-1))))

def integrate(f, start, end, samples): # midpoint method numerical integration
    spacing = (end - start) / samples
    x = start + spacing / 2 # set our first point in the middle of the first segment
    runningsum = 0
    for i in range(samples):
        runningsum += f(x)
        x += spacing # this overruns after the last sample, but we never sample there
    return runningsum * spacing

data = []
with open('data','r') as datafile: # read in the data from a space-separated file
    for line in datafile:
        data += [[float(x) for x in line.split(' ')]] # to a list of z, theta pairs

wp,wa,lastwp,lastwa,increment = 0,0,0,0,0.1 # set these for the hillclimb algorithm
for i in range(1000): # number of iterations (1000 takes a while but definitely converges)
    errors = [] # for keeping track of our closest position to the target
    moves = [[0,0],[0,increment],[increment,0],[0,-increment],[-increment,0]] # quincunx of positions to evaluate
    for move in moves:
        error=0
        wp = lastwp + move[0]
        wa = lastwa + move[1]
        for point in data: # sum the error for each position over our 8 points
            z, theta = point
            error += abs(theta - sqrt(a0) * numerator / integrate(integrand, a0/(1+z), a0, samples))
        errors.append(error)
    nextmove = moves[errors.index(min(errors))] # errors and moves lists are in the same order, so just use the index
    lastwp += nextmove[0] # set the values to the closest position
    lastwa += nextmove[1]
    if nextmove == [0,0]: # tighten the net if the value is between our points
        increment *= 0.5
    # converges to wp = -1.1304466235388642 wa = -0.08871547500975167
```

Figure 12: Python code used to find values of w_p and w_a that best fit the simulated EUCLID data (given later in this appendix)

The following is the full derivation for $D(z)$ used when calculating the angular size of BAOs in part IIii.
To move to physical coordinates:

$$
\begin{aligned}
dt &= a(t)d\tau \\
d\tau &= \frac{dt}{a(t)} \\
\tau &= \int_{t_i}^{t_0} \frac{dt}{a(t)} \\
D &= a_0 c \int_{t_i}^{t_0} \frac{dt}{a(t)} \\
D &= a_0 \int_{t_i}^{t_0} c\frac{dt}{a(t)}
\end{aligned}
$$

Now we use the continuity equation to find ρ_X

$$
\begin{aligned}
\frac{d\rho_X}{dt} + \frac{3}{a}\frac{da}{dt}\left(\rho_X + \frac{P_X}{c^2}\right) &= 0 \\
\frac{d\rho_X}{dt} &= -\frac{3}{a}\frac{da}{dt}\rho_X(1+w_X) \\
\int_{\rho_{X_0}}^{\rho_X} \frac{1}{\rho_X} d\rho_X &= \int_{a_0}^{a} -\frac{3}{a}(1+w_X)da \\
ln\left(\frac{\rho_X}{\rho_{X_0}}\right) &= -3\int_{a_0}^{a} \frac{1+w_p+\left(1-\frac{a}{a_0}\right)w_a}{a}da \\
ln\left(\frac{\rho_X}{\rho_{X_0}}\right) &= -3\int_{a_0}^{a} \left(\frac{1+w_p+w_a}{a} - \frac{w_a}{a_0}\right)da \\
ln\left(\frac{\rho_X}{\rho_{X_0}}\right) &= -3\left[(1+w_p+w_a)ln\left(\frac{a}{a_0}\right) + \frac{w_a a_0}{a_0} - \frac{w_a a}{a_0}\right] \\
\rho_X &= \rho_{X_0} e^{-3\left[(1+w_p+w_a)ln\left(\frac{a}{a_0}\right)+w_a\left(1-\frac{a}{a_0}\right)\right]} \\
\rho_X &= \rho_{X_0}\left(\frac{a}{a_0}\right)^{-3(1+w_p+w_a)} e^{3w_a\left(\frac{a}{a_0}-1\right)}
\end{aligned}
$$

Then using the Friedmann equation, we can find $D(z)$

$$\begin{aligned}
H^2 &= \frac{8\pi G}{3}\left(\rho_m + \rho_X\right)\\
\rho_{c_0} &= \frac{3H_0^2}{8\pi G}\\
\Omega_X &= \frac{\rho_{X_0}}{\rho_{c_0}}\\
\Omega_m &= \frac{\rho_{m_0}}{\rho_{c_0}}\\
\rho_{X_0} &= \Omega_X \rho_{c_0}\\
\rho_{m_0} &= \Omega_m \rho_{c_0}\\
\rho_m &= \rho_{m_0}\left(\frac{a_0}{a}\right)^3 = \Omega_m \rho_{c_0}\left(\frac{a_0}{a}\right)\\
H^2 &= \frac{8\pi G}{3}\left(\Omega_m \rho_{c_0}\left(\frac{a}{a_0}\right)^3 + \Omega_X \rho_{c_0}\left(\frac{a}{a_0}\right)^{-3(w_p+w_a)} e^{3w_a\left(\frac{a}{a_0}-1\right)}\right)\\
H^2 &= H_0^2\left(\frac{a_0}{a}\right)^3\left(\Omega_m + \Omega_X\left(\frac{a}{a_0}\right)^{-3(w_p+w_a)} e^{3w_a\left(\frac{a}{a_0}-1\right)}\right)\\
H &= \frac{a}{a_0}\frac{da}{dt}\\
dt &= \frac{1}{aH}da\\
D(z) &= a_0\int_{t_i}^{t_0}\frac{cdt}{a}\\
D(z) &= a_0 c\int_a^{a_0}\frac{1}{a^2H}da\\
D(z) &= a_0 c\int_a^{a_0}\frac{da}{a^2\sqrt{H_0^2\left(\frac{a_0}{a}\right)^3\left(\Omega_m + \Omega_X\left(\frac{a}{a_0}\right)^{-3(w_p+w_a)} e^{3w_a\left(\frac{a}{a_0}-1\right)}\right)}}\\
D(z) &= \frac{a_0 c}{H_0}\int_a^{a_0}\frac{da}{a^2 a^{-\frac{3}{2}} a_0^{-\frac{3}{2}}\sqrt{\Omega m + \Omega_X\left(\frac{a}{a_0}\right)^{-3(w_p+w_a)} e^{3\left(\frac{a}{a_0}-1\right)}}}\\
D(z) &= \frac{c}{H_0\sqrt{a_0}}\int_a^{a_0}\frac{da}{a^{-\frac{1}{2}}\sqrt{\Omega m + \Omega_X\left(\frac{a}{a_0}\right)^{-3(w_p+w_a)} e^{3\left(\frac{a}{a_0}-1\right)}}}
\end{aligned}$$

6 Appendix B: β Collaboration Documentation and Evidence

β Collaboration Meeting

14:00 Friday 27$^{\text{th}}$ January 2017
C07 Pod Library
Lancaster University

AGENDA 1

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Patrick Richardson, Anwar Abdullah

2. Minutes of Previous Meetings

No previous minutes.

3. Matters Arising

No previous minutes.

4. Matters for Discussion

- Work delegation
- Rough time frames
- Questions for John (Do the derivations have to be done the way they advise? Who is the project aimed at?)
- Prepare presentation

5. Any Other Business

N/A

β **Collaboration Meeting**

14:00 Friday 27th January 2017
C07 Pod Library
Lancaster University

MINUTES 1

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Patrick Richardson, Anwar Abdullah

2. Matters Discussed

I. Work delegation

We agreed to choose Shaun May as the Project Manager and Harry Spearing as Administrator. Then we divided our works for next week: Harry, Shaun and Jordan will work on majority of Part 1, Anwar on physics of BAO and Patrick on EUCLID. We will try to read as much as possible before next Tuesday (31st January 2017).

II. Rough time frames

We discussed our rough time frame for the next 8 weeks until the deadline of this project. We also came up with a Gantt chart of our time frame.

III. Questions for John

Shaun: Will have a meeting with John to ask the following questions:

- Do the derivations have to be done the way they advise?
- Who is the project aimed at?
- Check derivation?
- Find hints for Part III?

IV. Prepare presentation

We spent the last hour of our meeting preparing a 5 minutes presentation for next Tuesday about our research plan and how we are going to finish the project.

3. Any Other Business

- **Shaun**: Make cloud sharing file to upload and share files and documents
- **Harry**: Upload the presentation files

- **All**: Review of everyone's progress next Tuesday
- **All**: Remind everyone to get lab books from Louise
- **Harry**: Book rooms for future meetings in advance

β Collaboration Meeting

12:00 Tuesday 7$^{\text{th}}$ February 2017
C27 Physics Building
Lancaster University

AGENDA 2

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Patrick Richardson, Anwar Abdullah

2. Minutes of Previous Meeting

Previous minutes are confirmed as true record.

3. Matters Arising All actions were executed - no matters arising.

4. Matters for Discussion

- Previous weeks workload
- Work control
- Review long term progress
- Ideas for Part III

5. Any Other Business

Rearrange time for next meeting.

β **Collaboration Meeting**

12:00 Tuesday 7th February 2017
C27 Physics Building
Lancaster University

MINUTES 2

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Patrick Richardson, Anwar Abdullah

2. Minutes of Previous Meeting

Confirmed as a true record

3. Matters Arising

- Review of our Gantt Chart
- Review of our works for the past week
- Review of previous minutes from first formal meeting

4. Matters Discussed

I. Work control and everyone's feedback:

So far, everyone is happy with their assigned tasks and everyone is on track as far as our Gantt chart is concerned.

II. Review of our long term process:

Again referring to our Gantt chart, research for Part 1(theory, derivations and calculations) should be done by Friday, 10th February. After that, next tasks allocations were made:

- **Harry**: Error calculation in Part 1
- **Anwar**: Start researching Part II (i) Plancks sensitivity
- **Patrick**: Start researching Part II (ii) and Theory Part III
- **Jordan & Shaun**: Review their work in Part I (Theory, derivations and calculations)

5. Any Other Business

I. Ideas for Part III, referring to John's replies:

- Issues and problems concerning phantom dark energy
- Future of life, science fiction - something that would stand out
- Other options: Big Rip, Heat Death etc.
- Literature review using INSPIRE

II. Preparing LATEX documents

Jordan already prepared a LATEX template and uploaded it on Box for other people to put in their notes and findings so it can be a rough draft of the final report.

III. Discussion of findings

Jordan calculated the theoretical age of the Universe to be 13.81/13.82 Gigayears from Planck data which happens to be the most reliable data to date.

IV. Start to look into theory Part II (i)

V. Discussion on Plancks sensitivity

Start researching on how much can we deviate from -1?

VI. Planning to finish initial research of Part II by next week or next 2 weeks.

β Collaboration Meeting

15:00 Friday 10^{th} February 2017
C02 Pod Library
Lancaster University

AGENDA 3

1. Apologies for Absence

N/A

2. Minutes of Previous Meeting

All actions were executed - no matters arising.

3. Matters Arising

Ideas for Part III had not been developed. We will focus on part III ideas from Tuesday 14^{th} until Friday 17^{th}.

4. Matters for Discussion

- Previous weeks workload
- Work control
  - Beginning write up
  - Comparing theoretical and observed age of Universe
  - Allocating the rest of Part II
- Ideas for Part III

5. Any Other Business How to deal with error calculations.

β **Collaboration Meeting**

15:00 Friday 10th February 2017
C02 Pod Library
Lancaster University

MINUTES 3

1. Attendance list

Shaun May, Harry Spearing, Jordan Aubin, Patrick Richardson, Anwar Abdullah

2. Minutes of Previous Meeting

Ideas for Part III had not been developed. We will focus on part III this from Tuesday 14th till Friday 17th

Other than that, confirmed as a true record.

3. Matters Arising

- Ideas for Part III
- Tasks allocation for Part II

4. Matters Discussed

I. Review of Part I:

Harry came up with theoretical age of universe to be: 13.4$\pm$0.2 Gigayears compared to Jordans value from Plancks data: 13.81/13.82 Gigayears. He is going to check with John next Tuesday, 14th February. Shaun is going to plot age graphs. Then, we plan to have the graphs and rough write up of Part 1 by next Tuesday if possible. This write up would include comparison of theoretical and observational data.

II. Initial research of Part II:

We plan to finish the initial research of Part II by next Friday, 17th February:

- **Harry**: Part (i)
- **Anwar**: Second half of (ii)(4)
- **Patrick**: Part (ii)(2) and first half of (ii)(4)
- **Jordan**: Ideas for Part III
- **Shaun**: Part (ii)(5)

III. LATEX files:

Everyone is encouraged to keep separate files of their findings and write ups to avoid confusion.

IV. Uploading rough write up of Part I by next Friday, 17th February.

5. Any Other Business

I. **Jordan**: Revise Part I

II. **Everyone**: Encouraged to read on ideas for Part III

III. **Harry**: Book rooms for future meetings

β **Collaboration Meeting**

15:00 Friday 17$^{\text{th}}$ February 2017
C06 Library Pod
Lancaster University

AGENDA 4

1. Attendance List

Shaun May, Harry Spearing, Patrick Richardson, Anwar Abdullah

Apologies of Absence

Jordan - Hospital appointment.

3. Minutes of Previous Meeting

Previous minutes are confirmed as true record.

4. Matters Arising

We haven't yet made the graph, but all preliminary steps have been completed.

5. Matters for Discussion

- Previous weeks workload
- Work control
  - Beginning write up
  - Part II(i)
- Ideas for Part III continued

6. Any Other Business

Review long term plan.

β Collaboration Meeting

15:00 Friday 17$^{\text{th}}$ February 2017
C06 Library Pod
Lancaster University

MINUTES 4

1. Attendance list

Shaun May, Harry Spearing, Patrick Richardson, Anwar Abdullah

2.Apologies for Absence

Jordan Aubin (medical appointment)

3. Minutes of Previous Meeting

Confirmed as a true record

4. Matters Arising

- Review of previous week workload
- Begin write up for finished tasks
- Discussion of Part II (i)
- Continue discussing ideas for Part III

5. Matters Discussed

I. Review of previous week workload:

Harry: For Theory I, Harry is not required to include his error bounds as Jordans error are more important. Then, his derivation of Part I Theory has been uploaded onto Box.

Regarding Theory II (i), no progress has been made as we found it to be difficult so were going to ask John about it on Tuesday, 21st February. Harry has uploaded Theory III documents on cyclic universe (his initiative was commended by the coordinator).

Jordan: Jordan is expected to finish her Theory I report and is going to upload the documents by Monday, 20th February.

Shaun: In addition to Jordans part in Theory I, Shaun is going to include discussion on Ω_M, comparison of theoretical and observational data as well as graph plots. Then, he is going to look at Theory II (ii).

Patrick: Patrick's is going to write Python code to evaluate numerical integral for first half of Theory II (ii) (4) if possible. Then, he is going to look into Theory III.

Anwar: Anwar is to upload formal meeting minutes and wait for Patrick to finish first half of Theory II (ii) (4).

<u>II. Discussion of Part II</u>

Patrick: By Tuesday 21st February, if possible finish first 2 sentences of Part II (ii) (4).

Jordan: By Monday 20th February, finish up Part I and upload to Box.

Harry: By Tuesday 21st February, look into ideas of cyclic universe and comparison of universe model.

Anwar: By Tuesday 21st February, write up Part II (ii) (3) on LATEX and upload to Box.

Shaun: By Friday, finish researching Part II (ii) (5).

Everyone: Encouragement to look into Theory II (i) and find more ideas for Part III.

<u>III. Continue discussing ideas for Part III</u>

Harry already looked into idea of cyclic universe and uploaded 4.5 pages of pdf document to Box. Also, Shaun spoke to Kostas in his academic meeting and found out that we should give Part III a big focus as it carries a heavy weight in the marking. So, he suggested that we condense Part I and II in the final report to make room for Part III discussion.

Ideas put forward: fate of galaxies like Milky Way and Andromeda (possibly in laymen terms?), superclusters, gravitational waves and harvesting dark energy. To be continued in the following week.

6. Any Other Business

N/A

β Collaboration Meeting

12:00 Tuesday 21st February 2017
A8 Science & Technology Lab
Lancaster University

AGENDA 5

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Anwar Abdullah, Patrick Richardson

2.Minutes of Previous Meeting

Previous minutes are confirmed as true record.

3. Matters Arising

Anwar hasn't yet uploaded document for Part II(ii)(3)(BAO). This will be done by Friday.
Theory Part II has not been figured out as of yet. Jordan and Shaun are set to look into this.
All other actions have been completed.

4. Matters for Discussion

- Work control
  - Part II(i)

5. Any Other Business

N/A

β Collaboration Meeting

12:00 Tuesday 21st February 2017
A8 Science & Technology Lab
Lancaster University

MINUTES 5

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Anwar Abdullah, Patrcik Richardson

2. Minutes of Previous Meeting

Anwar hasnt yet uploaded document for part II(ii)(3)(BAO). This will be done by Friday.

Theory Part II has not been figured out as of yet. Jordan and Shaun are set to look into this.

Other than that, confirmed as a true record.

3. Matters Arising

- Review of previous weeks work
- Tasks assignments
- Issues and feedbacks for steering committee meeting

4. Matters Discussed

I. Review of previous weeks work:

Jordan: Theory 1 tasks will be uploaded on Sunday 26th February

Shaun: All of his tasks will be uploaded on Tuesday, 21st February except for Part I (4).

Harry: Cyclic universe documents for Part III are done and have been uploaded on Box. Comparison of universe models are still ongoing.

Patrick: Numerical solution for ω_A and ω_P values have been found using Python although the values are not Λ (this was commended by our coordinator). Then he is going to continue with the rest of Part II (ii) (4) and Part II (ii) (5).

Anwar: BAO documents are to be uploaded on Box.

II. Tasks assignments:

Everyone is encouraged to read on Part III ideas. Harry is interested to start researching on fate of Milky Way galaxy.

Also, everyone is encouraged to help Jordan and Harry on Part II (i).

III. Issues and feedbacks:

Jordan: She found the topic given is interesting and got to learn new ideas in modern cosmology.

Anwar: No complaints.

Harry: The overall project has the right amount of work to be divided among 5 people.

Shaun: Same with Harry.

Patrick: Same with Harry.

5. Any Other Business

I. Deadline for assigned tasks: Next Tuesday, 28th February.

II. **Harry**: To upload agenda on Box

III. Patrick will not be available next Tuesday so he is going to leave his lab book with the group for it to be signed and he will be informed of any new updates.

β Collaboration Meeting

12:00 Tuesday 28th February 2017
A8 Science & Technology Lab
Lancaster University

AGENDA 6

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Anwar Abdullah

2. Apologies for Absence

Patrick - Lectures

3. Minutes of Previous Meeting

Previous minutes are confirmed as true record.

4. Matters Arising

Anwar - in process of uploading document for Part II(ii)(3)(BAO).

5. Matters for Discussion

- Work control
  - Part II(i)
- Steering committee

6. Any Other Business

N/A

β **Collaboration Meeting**

12:00 Tuesday 28th February 2017
A8 Science & Technology Lab
Lancaster University

MINUTES 6

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Anwar Abdullah

2. Apologies for Absence

Patrick Richardson - Lectures

3. Minutes of Previous Meeting

Anwar in process of uploading document for part II(ii)(3)(Physics of BAO)

Other than that, confirmed as a true record.

4. Matters Arising

- Work control
- Steering committee points

5. Matters Discussed

I. Review of previous weeks work:

Harry: Part III needs uploading onto Box but is now mostly completed. Agendas are too to be updated.

Anwar: Physics of BAO documents and minutes to be uploaded.

Shaun: Finish errors for Part I and review it with Jordan.

Jordan: Going to look into Theory II (i) and review Part I works and compare it to checklist.

Patrick: Request update later and upload his works on Part II.

Everyone: Upload every relevant document before Wednesday night (1st March) so we can prepare a rough draft for next Tuesday meeting (5th March). Also, everyone is encouraged to read through Harrys work on Part III.

II. Part II (i):

Jordan is going to take a look at Part II (i) today and let Patrick know and can discuss about it with him. If were still stuck with it by next week, Shaun will have a discussion with John next Tuesday meeting (5th March).

III. Part III:

So far, Harry has prepared 5 pages for Part III. Were going to put more emphasis on this Part compared to the other two.

6. Any Other Business

I. Amalgamate all uploaded documents into a rough draft

II. Ideas for next meeting: Delegation of tasks for final report layout, production etc

β **Collaboration Meeting**

15:00 Tuesday 7$^{\text{th}}$ March 2017
A25 Physics Building
Lancaster University

AGENDA 7

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Patrick Richardson, Anwar Abdullah

2. Minutes of Previous Meeting

Previous minutes are confirmed as true record.

3. Matters Arising

Anwar still to upload BAO research.

4. Matters for Discussion

- Individual updates
- Make checklist together
- Report breakdown (who's doing what)
- Any parts left to do

5. Any Other Business

N/A

β Collaboration Meeting

15:00 Tuesday 7$^{\text{th}}$ March 2017
A25 Physics Building
Lancaster University

MINUTES 7

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Patrick Richardson, Anwar Abdullah

2. Minutes of Previous Meeting

Anwar is still to upload BAO research.

Other than that, confirmed as a true record.

3. Matters Arising

- Work control
- Make checklist
- Report breakdown
- Any parts left to do

4. Matters Discussed

I. Work control:

Everyone seems to be on track with what they're supposed to do. Although Anwar has to finish his tasks by next Friday, 10th March 2017.

II. Make checklist:

Harry: Make a checklist of everything to keep everyone on track for final report.
III. Report breakdown:

Details of the report breakdown and who is doing what will be discussed next Friday, 10th March 2017.

5. Any Other Business

I. Meeting next Friday, 10th March 2017 to discuss everyone's works so far. Everyone is encouraged to upload separate files on Box to avoid confusion.

β Collaboration Meeting

15:00 Tuesday 14$^{\text{th}}$ March 2017
A25 Physics Building
Lancaster University

AGENDA 8

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Patrick Richardson, Anwar Abdullah

2. Minutes of Previous Meeting

Previous minutes are confirmed as true record.

3. Matters Arising

Theory Part II not done - ask John for help.

4. Matters for Discussion

- Individual updates/Look at online uploads
- Peer assessment
- Review checklist
- Report breakdown
- Cyclic - graph

5. Any Other Business

N/A

β Collaboration Meeting

15:00 Tuesday 14$^{\text{th}}$ March 2017
A25 Physics Building
Lancaster University

MINUTES 8

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Patrick Richardson, Anwar Abdullah

2. Minutes of Previous Meeting

Confirmed as a true record

3. Matters Arising

- Individual updates on Box
- Peer assessment
- Report breakdown
- Cyclic graph

4. Matters Discussed

I. Individual updates:

Patrick: Still not sure about Theory Part II(i) so may need to ask John to clear things up. All documents then need to be LATEX-ified. Solution of numerical integrals gave 11.67 billion years as age of Universe. Also, need to check with coordinator and those who work in Part I.

Shaun: For Part I, big uncertainties from big range of values and also need to check 2σ values. All Part I documents will have to be uploaded by Friday, 17th March 2017. He also volunteered to do report abstract by next Thursday, 23rd March 2017.

Jordan: All assigned tasks are done and already uploaded on Box. She can help Patrick with Part II tasks.

Anwar: BAO documents already uploaded on Box although they may need editing and refining. Minutes also need to be LATEX-ified by Friday, 17th March 2017.

Harry: Finish cyclic graph for Part III. Also, he volunteered to do report introduction due by Thursday, 23rd March 2017.

II. Peer assessment

Reminder for everyone to finish project peer assessment which is due on Friday, 17th March 2017 at 3 pm.

III. Report breakdown

As mentioned above, Harry volunteered to do report introduction and Shaun volunteered to do report abstract for Thursday, 23rd March 2017. Also, our coordinator made Tuesday, 21st March 2017 the final deadline for all individual files to be uploaded on Box. On Thursday, 23rd March 2017, everyone is going to meet up for half a day to put everything together.

5. Any Other Business

I. Arrange group meeting with John to discuss Part II(i)

II. **Harry**: Book a room for writing up on Thursday, 23rd March 2017.

β Collaboration Meeting

16:00 Tuesday 7$^{\text{th}}$ March 2017
A25 Physics Building
Lancaster University

AGENDA 9

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Patrick Richardson, Anwar Abdullah

Minutes of Previous Meeting

Previous minutes are confirmed as true record.

3. Matters Arising

Shaun is still in the process of editing Part I.
Anwar still has a couple of Minutes documents to upload to LaTeX.

4. Matters for Discussion

- Individual updates
- Review checklist
- Agenda/Conclusion
- Page count

5. Any Other Business

N/A

β **Collaboration Meeting**

16:00 Tuesday 7th March 2017
A25 Physics Building
Lancaster University

MINUTES 9

1. Attendance List

Shaun May, Harry Spearing, Jordan Aubin, Patrick Richardson, Anwar Abdullah

2. Minutes of Previous Meeting

Shaun is still finishing up Part I and Anwar hasn't uploaded all complete minuted on Box.

Other than that, confirmed as a true record.

3. Matters Arising

- Individual updates
- Review checklist
- Agenda/conclusion
- Page count

4. Matters Discussed

I. Individual updates:

Patrick: Part II are all finished and uploaded on Box. Code for Part II can be put in final report's Appendix A.

Shaun: Majority of Part I are finished. Some bits still need editing.

Jordan: All assigned tasks are done and already uploaded on Box. She helped Patrick with Part II and helped convert agendas to LATEX for the report's Appendix B.

Anwar: BAO documents already uploaded on Box although they may need editing and refining. Minutes also need to be LATEX-ified.

Harry: Finished all assigned tasks. Can help anyone that needs help.

II. Final report breakdown

Reminder for everyone to finish project peer assessment which is due on Friday, 17th March 2017 at 3 pm.

III. Report breakdown

- Sort out all cites and references
- Put everything together in one single final report
- Sort out page counts for final report

5. Any Other Business

N/A