

Day / Date

WEEK 8 :-

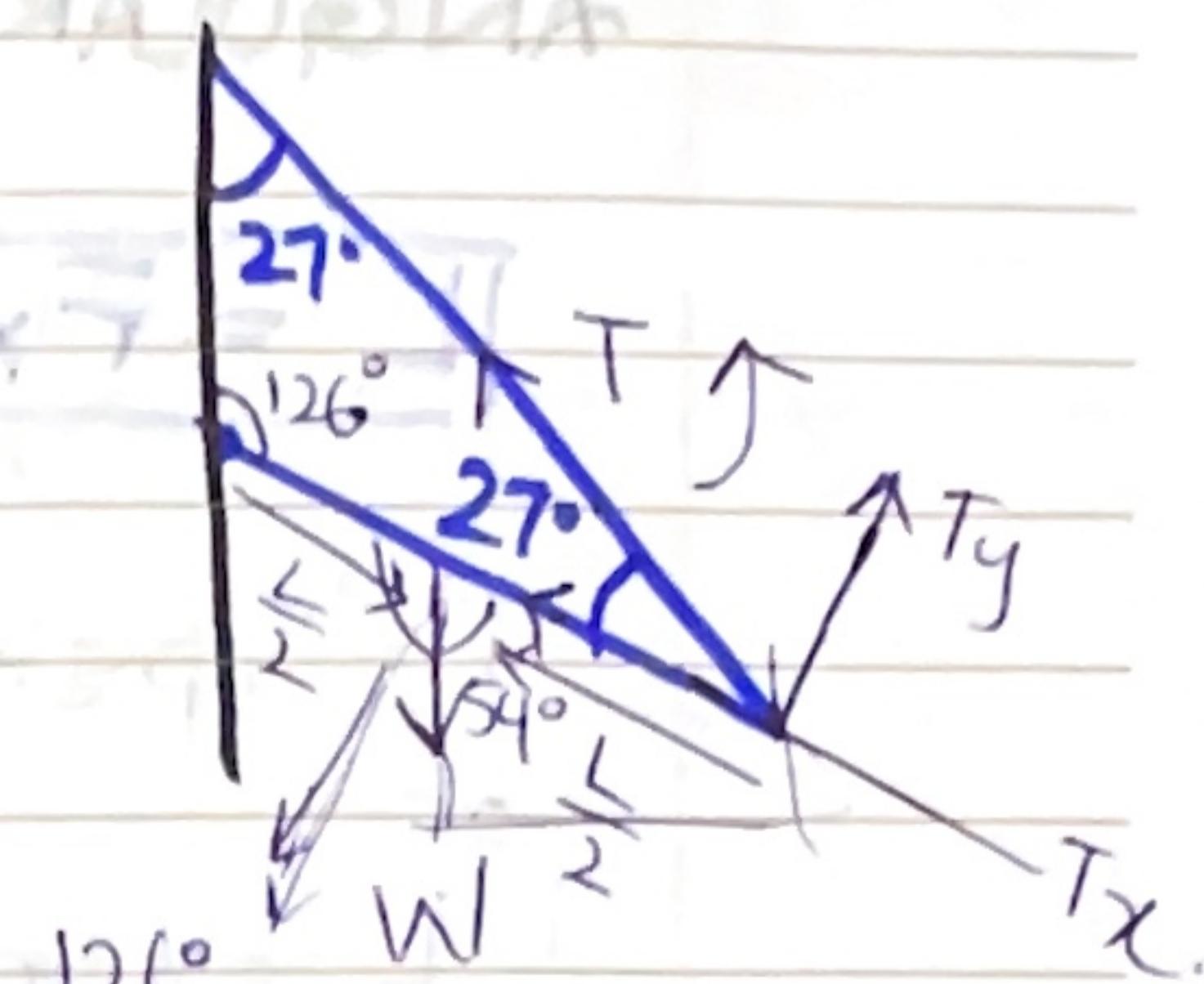
Prob:-

$$\text{Weight} = 52.7 \text{ N}$$

$$L = 3.2 \text{ m}$$

$$T = ?$$

F_x & F_y on the hinge.



As law of cons. / equilibrium

$$\sum T = 0$$

$$- W \left(\frac{L}{2} \right) \sin 54^\circ + (T) (L) \sin \phi = 0$$

$$(T) L \sin \phi = W \times \frac{L}{2} \sin 54^\circ$$

$$(T) \cancel{L} \sin 27^\circ = W \cancel{L} \sin 54^\circ$$

$$T = \frac{52.7 \sin 54^\circ}{2} \times \frac{1}{\sin 27^\circ}$$

$$T = 46.95 \text{ N} \Rightarrow 47 \text{ N}$$

$$T_x = T \cos \phi \Rightarrow 47 \cos 27^\circ = 21.33 \text{ N}$$

$$T_y = T \sin \phi \Rightarrow 47 \sin 27^\circ = 41.877 \text{ N}$$

$$\sum F_x = 0$$

$$F_H - T_x = 0$$

$$F_H = T_x$$

$$F_H = 21.33 \text{ N}$$

$$\sum F_y = 0$$

$$F_y H - T_y - W = 0$$

$$= 52 - 41.877$$

$$F_y \Rightarrow 10.8 \text{ N}$$

Galaxy

ANGULAR MOMENTUM:-

$$\boxed{\mathbf{L} = \bar{r} \times \bar{P}}$$

P = Linear momentum
 \bar{r} = position vector /
 position vector.

$$\text{or } L = r \sin \theta p.$$

$$\boxed{\epsilon \bar{L} = \bar{F} \times \bar{r}}$$

also

$$\boxed{\epsilon \bar{L} = \frac{d \bar{L}}{dt}}$$

Net torque acting on
 body is equal to
 the rate of change of
 its angular momentum.

$$\left. \begin{aligned} \epsilon \bar{T}_x &= \frac{d L_x}{dt} \\ \epsilon \bar{T}_y &= \frac{d L_y}{dt} \\ \epsilon \bar{T}_z &= \frac{d L_z}{dt} \end{aligned} \right\}$$

PROBLEM:-

Angular momentum = ? also:

$$T = F \times r$$

$$= mg(b)$$

$$\bar{r} \times \bar{p}$$

$$(\bar{r} \times m)(v)$$

$$(\bar{r} \times \bar{v})m.$$

$$\text{or } r \sin \theta p.$$

PROBLEM:-

Initially there is only one disk rotating however at final position there are three

according to the law of cons. of momentum

$$I_i \omega_i = I_f \omega_f \Rightarrow \omega_f = \frac{I_i \omega_i}{I_f}$$

$$\text{(Ax)} \quad I_i \omega_i = I_f \omega_f \quad \text{or } \omega_f = 0.28 \frac{v_{galaxy}}{r_{galaxy}}$$

also ~~Galaxy~~ ~~disk~~ ~~Blowout~~ ~~rotation~~

$$S = v_i t + \frac{1}{2} a t^2$$

ANGULAR

LAW OF CONS. OF MOMENTUM :-

At this scenario no torque is observed.

$$\frac{d \bar{L}}{dt} = \frac{d \bar{L}}{dt}$$

L = constant.

$$L_i = L_f \quad (\text{initial} = \text{final}).$$

also

$$\boxed{\bar{L} = I \omega}$$

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

PROBLEM:- (always add when coupling).

$$I_1 = 1.27 \text{ kgm}^2$$

$$\omega_{1i} = 824 \text{ rev/min} \times \left(\frac{1 \text{ min}}{60 \text{ sec}} \right)$$

$$I_2 = 4.85 \text{ kgm}^2$$

$$\omega_{2i} = \text{?} \text{ ms}^{-1}$$

$$\omega_f = \omega_{2f} = ?$$

$$I_1 \omega_i + I_2 \omega_i = I_f \omega_f$$

$$I_1 \omega_i + I_2 \omega_i = I_1 \omega_f + I_2 \omega_f$$

$$(1.27)(824) + 0 = \omega_f (1.27 + 4.85)$$

$$\omega_f = 171 \text{ rev/min Ans!}$$

ANSWER

$$\omega = 1.22 \text{ rev/s.}$$

$$I_i = 6.13 \text{ kgm}^2$$

$$I_f = 1.97 \text{ kgm}^2$$

$$\omega_f = \frac{I_i \omega_i}{I_f} \Rightarrow \frac{(6.13)(1.22)}{1.97} = 3.86 \text{ rev/sec.}$$

Date / Date

IDEA pale juice
ratio 2x biscuit
cakerush coffee/tea

Date / Date

Initially at 180m

$$(50m)$$

$$F_x = \frac{mv^2}{r}$$

$$v_f = \sqrt{\frac{v_i^2 + 0}{2}}$$

$$\bar{F} = \frac{(120)(2.5)^2}{180}$$

$$F_C = 4.2N$$

$$50m \quad F_C = \frac{(120)(2.5)^2(180)^2}{(50)^3} \Rightarrow 19N \text{ Ans}$$

$$F_C = \frac{120(9)^2}{50} = 194N$$

$\therefore \text{P.OI}$

10.5 :- (already) :- done:

Unit vector:-

The vector which is used to specify the position of a vector is known as unit vector,
magnitude = 1, dimensionless.

$$\begin{aligned} \hat{F} &= \frac{F}{|F|} \\ \hat{F} &= \frac{F}{|F|} \quad \text{a vector divided by its magnitude.} \end{aligned}$$

For any three dimensional vector $A =$

$$A = A_x i + A_y j + A_z k$$

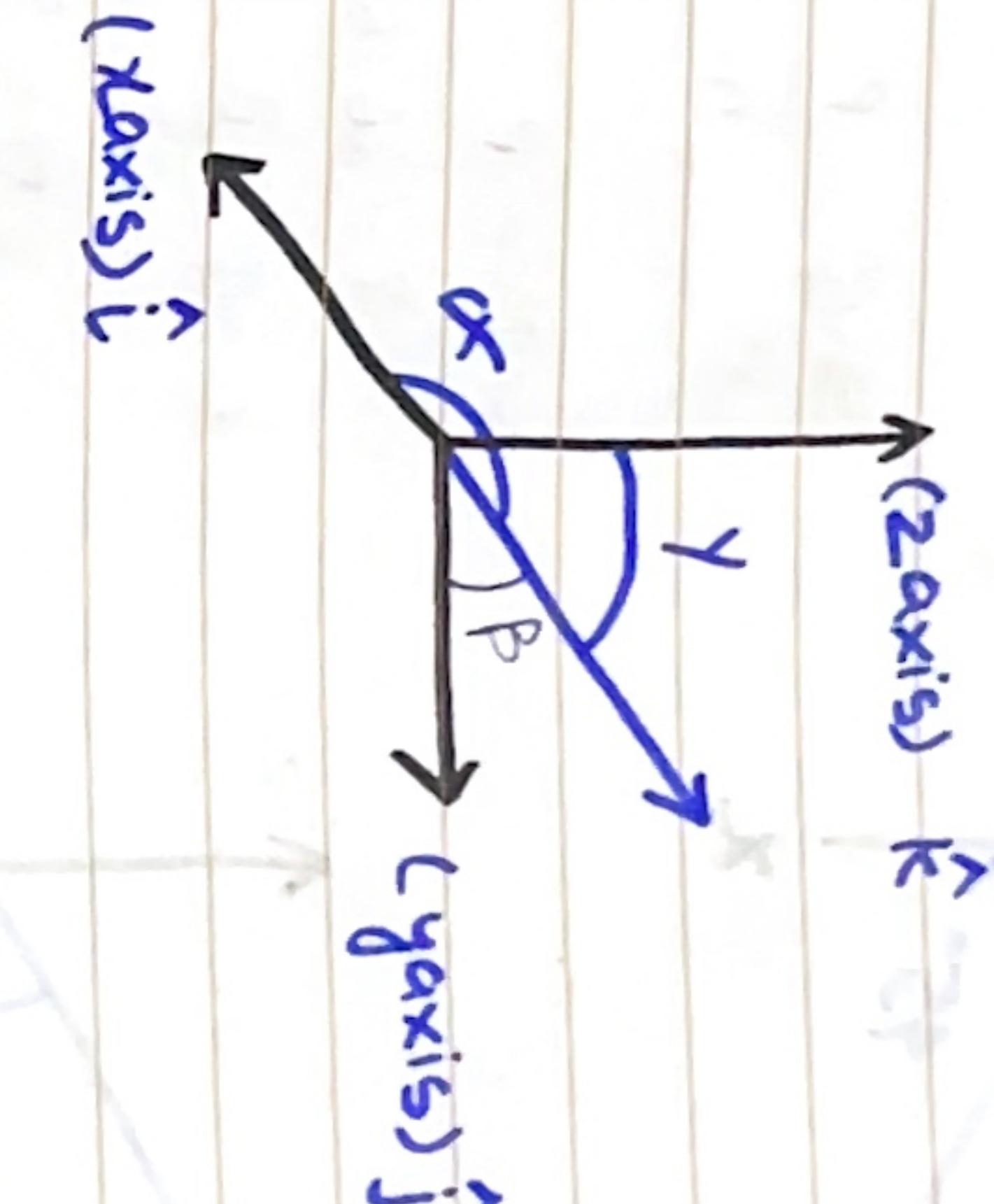
$$|A| = \sqrt{(Ax)^2 + (Ay)^2 + (Az)^2} \quad (\text{Magnitude of } A)$$

$$0 \leq \alpha, \beta, \gamma \leq 180^\circ$$

$$\cos\alpha = \frac{Ax}{|A|}$$

$$\cos\beta = \frac{Ay}{|A|}$$

$$\cos\gamma = \frac{Az}{|A|}$$



(iii) Now if the three dimensional unit vector $\hat{A} \Rightarrow$

$$\hat{A} = \left(\frac{Ax}{|A|} \right) i + \left(\frac{Ay}{|A|} \right) j + \left(\frac{Az}{|A|} \right) k$$

$$\therefore \hat{A} = \cos\alpha i + \cos\beta j + \cos\gamma k$$

$$\therefore 1 = \cos\alpha i + \cos\beta j + \cos\gamma k$$

(iv) Concurrent force (arise from the same origin) and resultant force is the sum of all the forces app

$$(A_1 + B_1)k$$

$$\begin{aligned} \bar{A} &= A_x i + A_y j + A_z k \\ \bar{B} &= B_x i + B_y j + B_z k \\ \bar{R} &= \bar{A} + \bar{B} = (A_x + B_x)i + (A_y + B_y)j + (A_z + B_z)k \end{aligned}$$

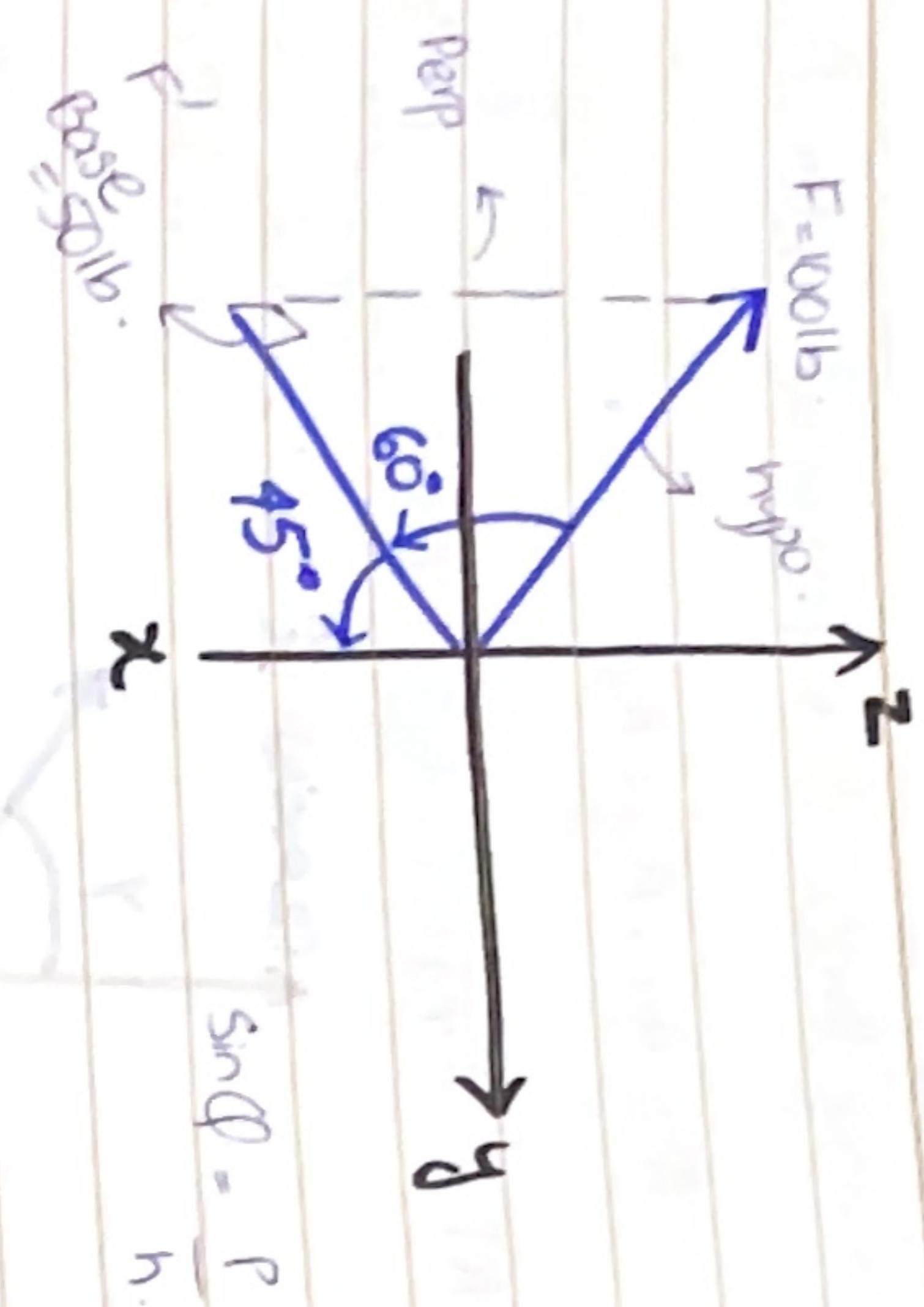
Galaxy

Galaxy

QUESTION - II - REPRESENT $\vec{P}Q$ AS A CARTESIAN VECTOR.

四百

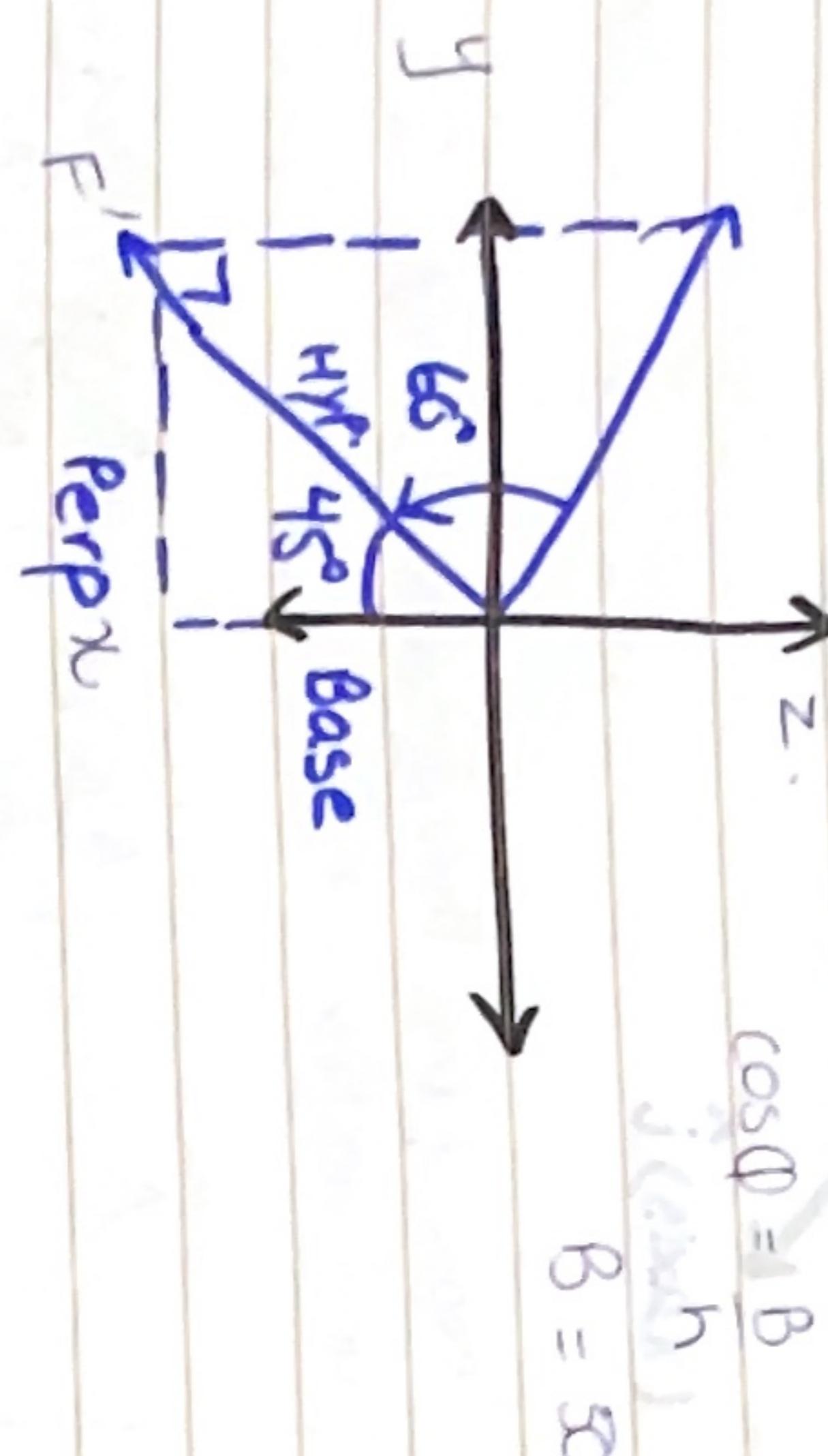
四



$$(H)^2 = (B)^2 + (\beta)^2$$

$$P = 86.60 \text{ lb.}$$

$$\cos \theta =$$



$$F_2x = ?; \quad F_2y = ?; \quad F_3z = ?$$

Using Law of cosine

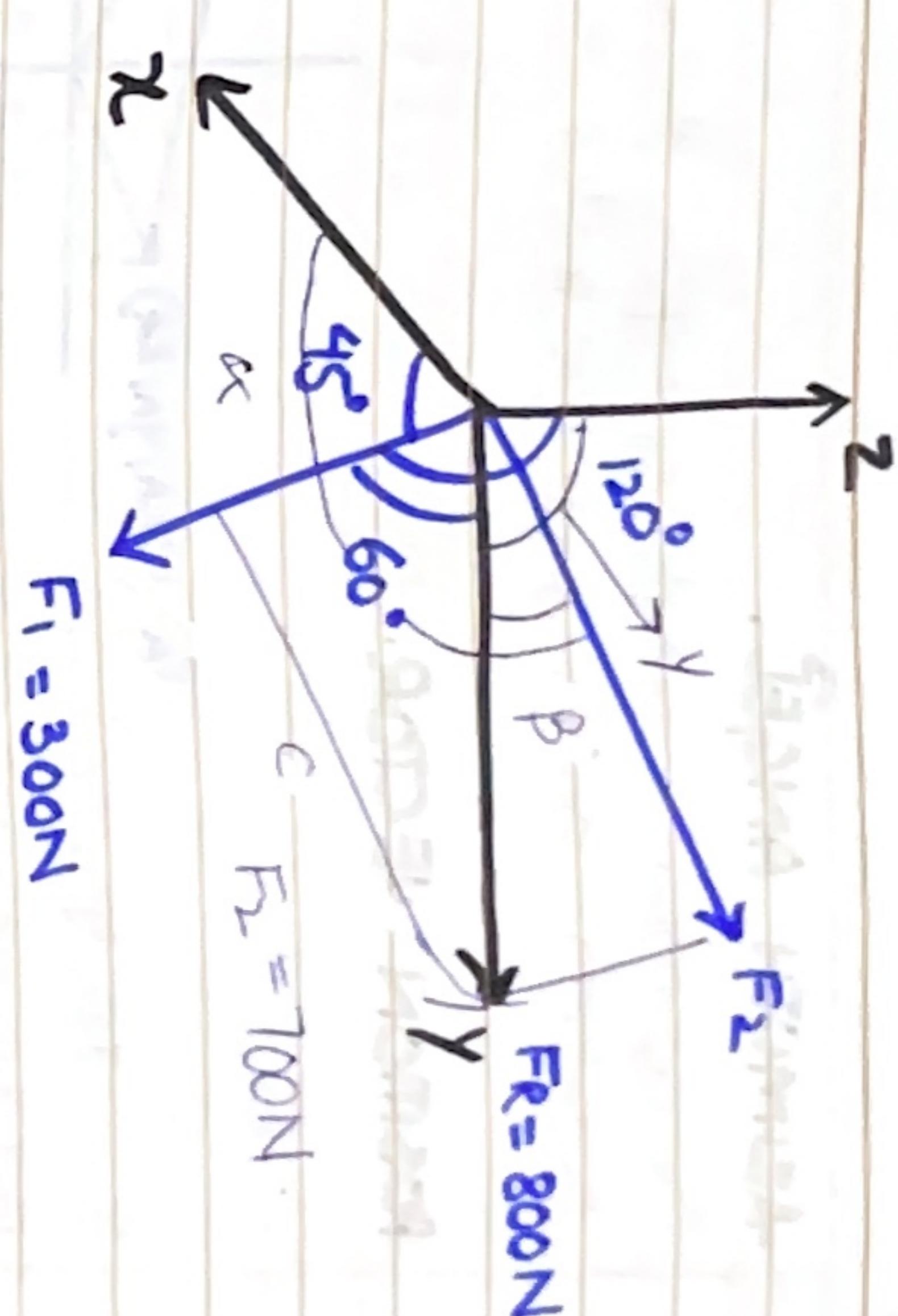
$$c^2 = a^2 + b^2 - 2ab \cos y$$

$$c = \sqrt{(800)^2 + (300)^2 - 2(800)(300) \cos(60)} \\ = \sqrt{1100 - 2400 \times 10^{-4}} = 700N.$$

$$F_1 = \sqrt{300\cos^2 y + 300\cos^2 y} \cdot$$

(not using pythagoras)

$$\Rightarrow \left\{ \begin{array}{l} 212i + 150j + 150k \end{array} \right.$$



Now

$$F_x = 50 \cos 45^\circ = 35.55 \text{ lb.}$$

$$F'_y = 50 \sin 45^\circ = 35.55 \text{ N}$$

$$\cos \alpha + \cos \beta + \cos \gamma = 1. \quad (\text{Ansatz})$$

$$\cos(180^\circ) + \cos(60^\circ) + \cos y = 1.$$

A

$$\cos(45)^2 + \cos(60)^2 + \cos^2 y = 1. \quad \text{A}$$

$$T = k \cos(\theta) + \frac{h}{I}$$

$$k_{SO} = k_1 k_2 \gamma_1 = k_2 \gamma_0$$

$$F_{\text{cosy}} = F_z'$$

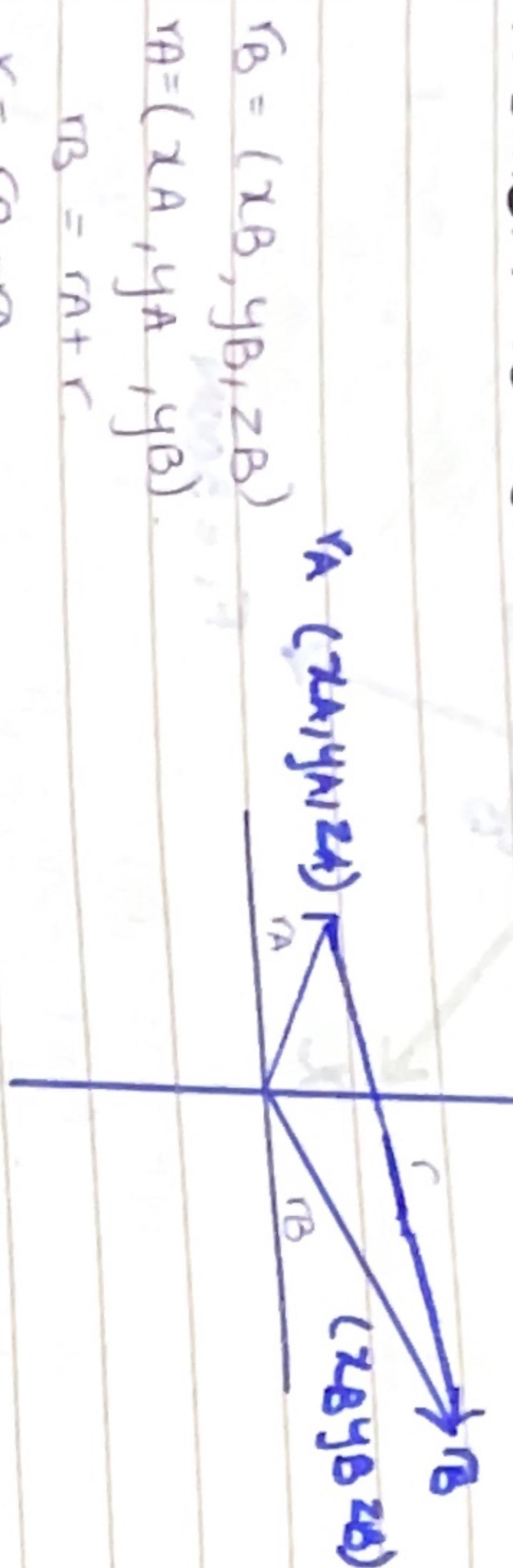
$$T = \frac{F_1}{F_1 + F_2} \cos y = \frac{F_1}{F_1 + F_2}$$

-Galaxy-

AZIMUTH ANGLE.

The azimuth angle specifies the angle between z axis and the vector.

POSITION VECTOR.



also

$$|AB| = \sqrt{(12)^2 + (-8)^2 + (-24)^2}$$

$$|AB| = 28$$

$$\vec{F} = |F| \hat{r}$$

$$= 70 \left(\frac{12}{28} i + \frac{-8}{28} j + \frac{-24}{28} k \right)$$

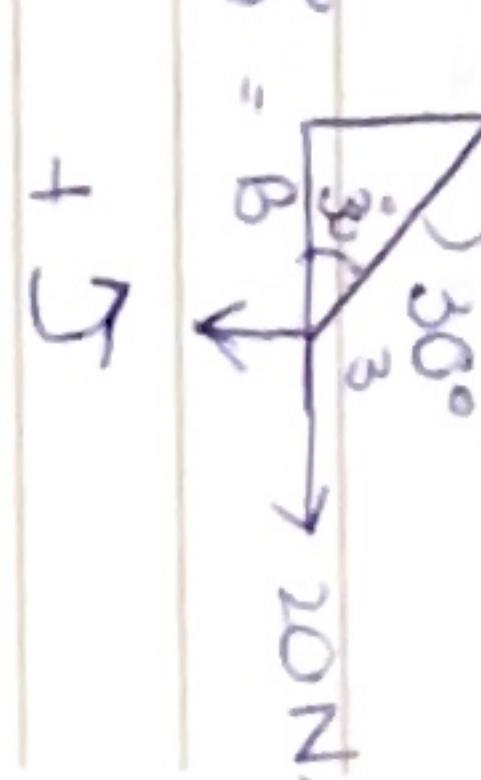
Now if we require the angles.

$$\alpha = \cos^{-1} \left(\frac{12}{28} \right) ; \beta = \cos^{-1} \left(\frac{-8}{28} \right) ; \gamma = \cos^{-1} \left(\frac{-24}{28} \right)$$

$$\alpha = 64.6^\circ ; \beta = 106.6^\circ ; \gamma = 148.9^\circ$$

MOMENT OF FORCE / TORQUE :-

Scalar formulation moments don't change and is required when and is required

$$\text{MOMENT OF FORCE} / \text{TORQUE} = (F)(d)$$


Scalar formulation moments don't change and is required

PROBLEM :-

Force applied = 70 lbs (magnitude).
position vector = ?

$$\begin{matrix} A & (0, 0, 30) \\ B & (12, -8, 6) \end{matrix}$$

$$\begin{aligned} \overrightarrow{AB} &= \vec{B} - \vec{A} \\ &= (12, -8, 6) - (0, 0, 30) \\ &\Rightarrow (12, -8, -24) \end{aligned}$$

Now

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$

$$= (12, -8, 6) - (0, 0, 30)$$

$$\Rightarrow (12, -8, -24)$$

$$M_o = r_x F$$

$$M_o = r F \sin \theta$$

$$M_o = r \times F =$$

$$\begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= 7(r_y F_z - r_z F_y) i - (r_x F_z - r_z F_x) j + (r_x F_y - r_y F_x) k . \text{ Ans!}$$

$$\text{Resultant} = (M_o)_R = \sum (r \times F)$$

r_{AB}

$$M_{o(B)} = \begin{vmatrix} i & j & k \\ 4 & 0 & 12 \\ 0.458 & 0.458 & +1.37 - 1.376 \end{vmatrix} \Rightarrow \{-1.65 i + 5.5 j\} k$$

or

$$M_{o(A)} = \begin{vmatrix} i & j & k \\ 0 & 0 & 12 \\ 0.458 & 1.37 & -1.376 \end{vmatrix} \Rightarrow \{-16.5 i + 55 j\} k$$

PROBLEM:-

If the moment of force is to be determined and positions are given with coordinates in 3 dimensions use

$$(M_o)_R = \frac{1}{2} (r \times F)$$

$$A = (0, 5, 0) \quad F_1 = (-6, 4, 2) \\ B = (4, 5, -2) \quad F_2 = (8, 4, -3)$$

$$r_A = (0, 0, 12) \\ r_B = (4, 12, 0) \\ F = 2kN$$

however

$$F_x = ?, \quad F_y = ?, \quad F_z = ?$$

$$\vec{AB} = B - A = (4, 12, 12)$$

$$\vec{F} = F_x i + F_y j + F_z k$$

$$\vec{F}(UAB) = 2 \left(\sqrt{(4)^2 + (12)^2 + (-12)^2} \right) \text{ N}$$

$$= \{0.458 i + 1.376 j - 1.376 k\} \text{ kN}$$

$$\Rightarrow [3i - 4j + 6k] \text{ kN.m} \\ \text{we can also determine } \alpha, \beta, \gamma \Rightarrow ?$$

VARIGNON'S THEOREM:- OR. PRINCIPLE OF MOMENTS.

PROBLEM:-

$$M_o = r \times F \quad \left. \begin{array}{l} M_{o_{net}} = r \times (F_1 + F_2) \end{array} \right\}$$

$$M_{o_2} = r \times F_2$$

$$\downarrow$$

$$F_x = (5 \text{ kN}) \cos 45^\circ$$

$$\begin{aligned} M_o &= d \sin \theta = 3 \sin 30^\circ \\ d \sin \theta &= d \cos \theta = 3 \cos 30^\circ \quad (\text{Force} = \bar{F}) \\ F_y &= 5 \text{ kN} \sin 45^\circ \end{aligned}$$

$$\downarrow$$

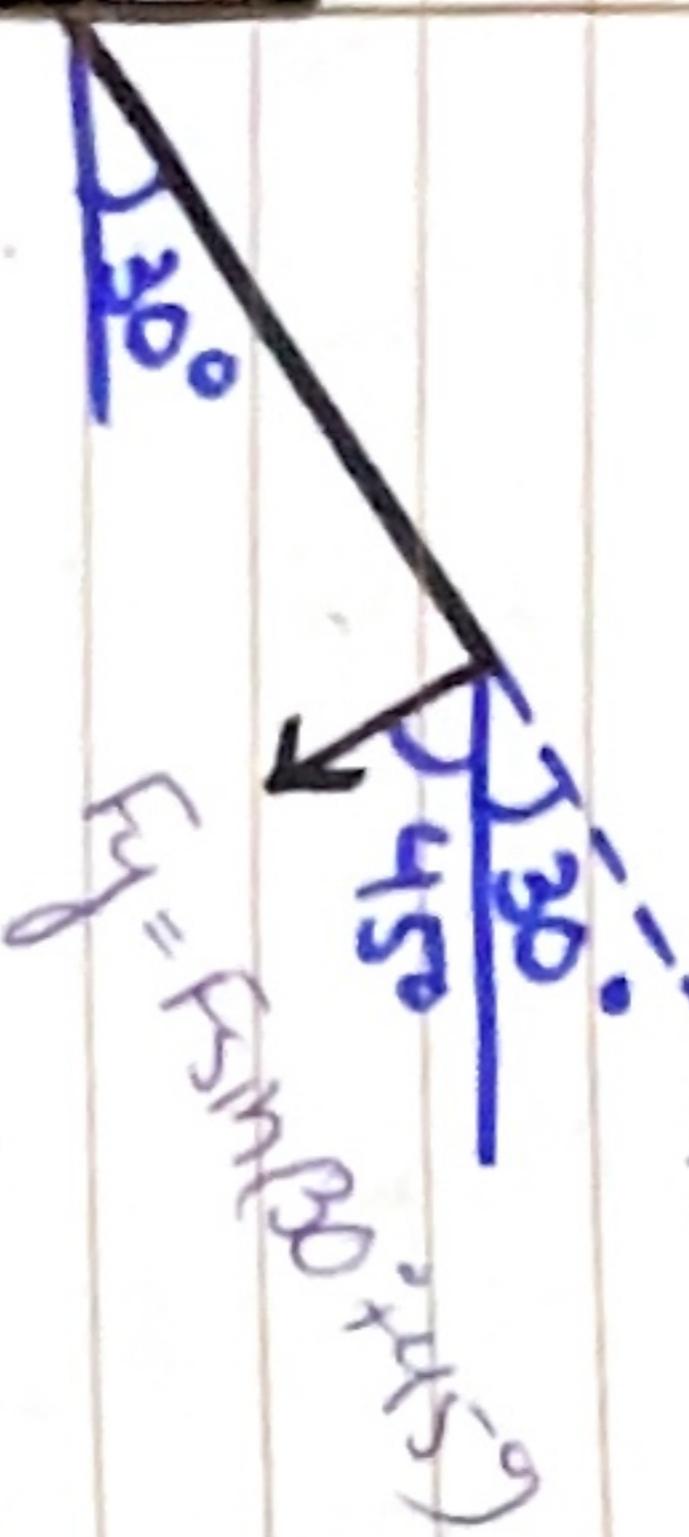
There are two approaches;

- determine the force \perp to the moment arm

or vice versa.

WORK - POWER - ENERGY:-

PROBLEM:-



$$\Rightarrow -98.6 \text{ kN m. Ans!}$$

Now F_x is not involved making any rotation.
 Now as the coordinates
 of the moment arm are
 given, it is better to
 solve by cross product.

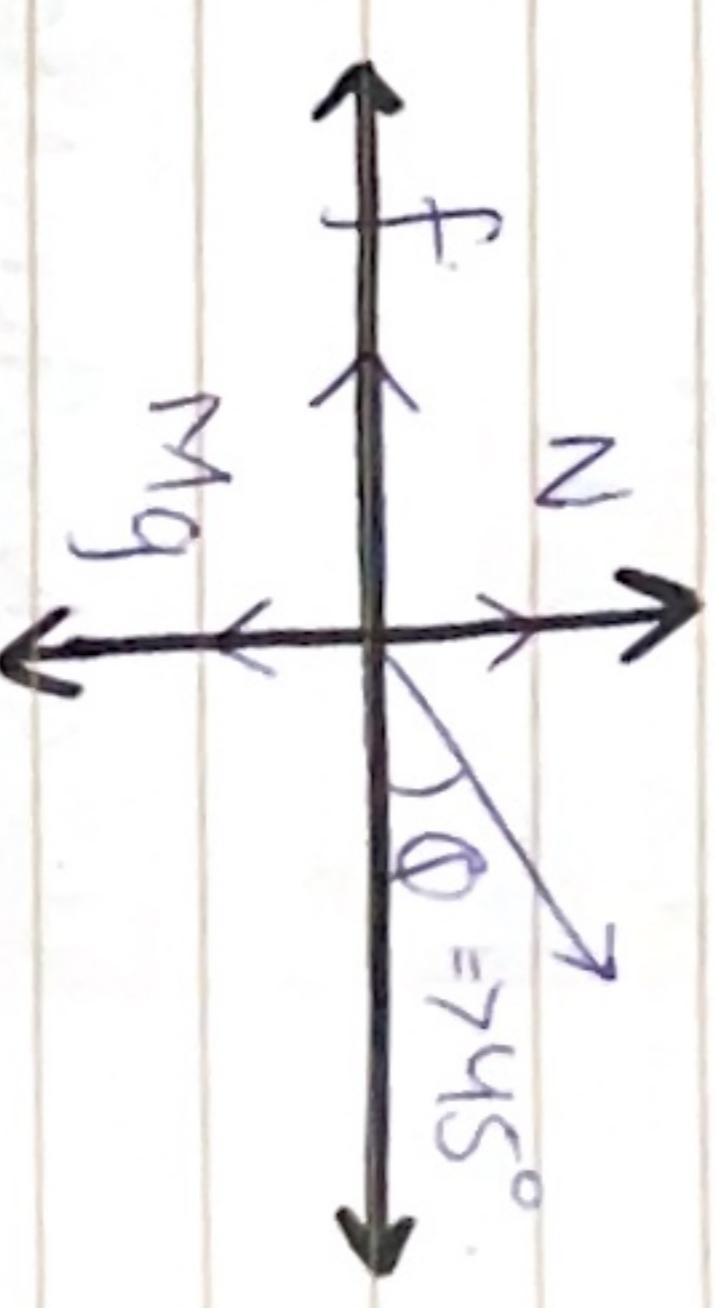
$$\begin{aligned} F \cos \theta &\quad F_x = F \cos \theta \sin \theta \\ y \text{ axis} &= 400 \cos 30 = 200 \\ F \sin \theta &= 400 \sin 30 = 200 \\ F_y &= 400 \sin 45^\circ = 282.8 \text{ N} \\ \cos & \downarrow \\ 0.4 & \quad \text{downwards.} \end{aligned}$$

PROBLEM:-

$$\begin{aligned} M_o &= r \times F \sin \theta \\ &= (3)(4820) \\ &= 14488.8 \text{ Ans!} \end{aligned}$$

Using Newton's law of motion.

$$\sum F_x = m a_f \quad (\text{const speed}) \quad \sum F_y = 0$$



Ques

$$W_{\text{work}} = ? \Rightarrow (\vec{F} \cdot \vec{d})(\cos 45)$$

$$F \cos \phi - f = 0$$

$$F \sin \phi + F \cos \phi - mg = 0$$

$$f = \mu_k N$$

$$N = \frac{F \cos \phi}{\mu_k} \quad F \sin \phi + \frac{F \cos \phi}{\mu_k} - mg = 0$$

$$F - f_k - W \sin \phi = 0$$

$$F - \mu_k N - W \sin \phi = 0$$

$$N - W \cos \phi = 0$$

$$\sum F_x = ma$$

$$\sum F_y = ma$$

Sol

$$F - f_k - W \sin \phi = 0$$

$$N - W \cos \phi = 0$$

$$f = \mu_k N$$

$$N = \frac{F \cos \phi}{\mu_k}$$

$$F \sin \phi + \frac{F \cos \phi}{\mu_k} - mg = 0$$

$$F = 10.976$$

$$0.1414 + 0.707$$

Now

$$W = (12.9)(12) \cos 45$$

$$= 155 \text{ Nm} \Rightarrow 10.976$$

$$\cos 45 = 10 \text{ Nm Ans}$$

$$P = FV$$

$$F = \frac{7545}{12257} = 6.17 \text{ N}$$

$$P = \frac{k}{t} = \frac{F \cdot d}{t} = F \cdot V$$

$$= (7545)(1.34) = 10210 \text{ Watt}$$

$$1.6 \times 10^4$$

$$m = 1380 \text{ kg}$$

$$v = 134 \text{ ms}^{-1}$$

$$\mu_k = 0.4$$

$$P = ?$$

WORK DONE BY A VARIABLE FORCE:-

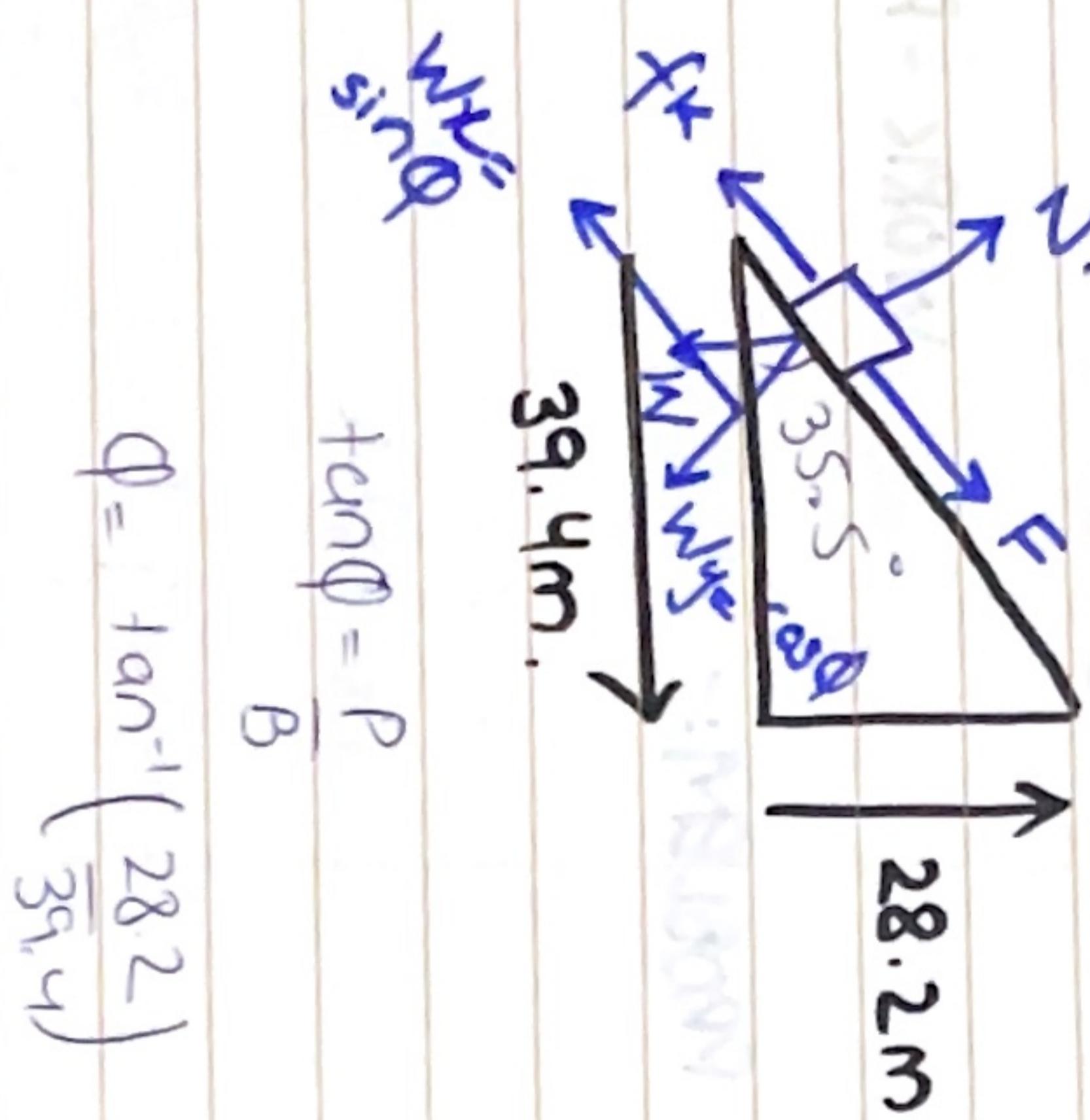
Work done by a variable force can be given by.

$$\sum W_{\text{net}} = W_1 + W_2 + \dots + W_n$$

$$W = \lim_{n \rightarrow \infty} \sum_{i=1}^n F(x_i) dx$$

Using Newton's law of motion.

$$P = FV = \frac{F \cdot d}{t} = \frac{\mu_k N \cdot d}{t} = \frac{\mu_k m g \cdot d}{t}$$



Ques

$$\sum F_x = ma$$

$$\sum F_y = ma$$

Sol

$$F - f_k - W \sin \phi = 0$$

$$N - W \cos \phi = 0$$

$$N = \frac{F \cos \phi}{\mu_k}$$

$$F = \frac{f_k + W \sin \phi}{\cos \phi}$$

$$F = \frac{\mu_k N + W \sin \phi}{\cos \phi}$$

$$F = \frac{\mu_k \frac{F \cos \phi}{\mu_k} + W \sin \phi}{\cos \phi}$$

$$F = \frac{W \sin \phi + \mu_k W \cos \phi}{\cos \phi}$$

$$F = W \tan \phi$$

$$F = \frac{W}{\cos \phi}$$

$$F = \frac{W}{\sqrt{1 + \tan^2 \phi}}$$

$$F = \frac{W}{\sqrt{1 + \frac{\sin^2 \phi}{\cos^2 \phi}}} = \frac{W \cos \phi}{\sqrt{\cos^2 \phi + \sin^2 \phi}} = W$$

$$F = W$$

WORK DONE BY SPRING FORCE:-

$$W_s = -\frac{1}{2} k x^2$$

$$\begin{aligned} F_{\text{ext}} &= F_{\text{spring}} \\ &= -kx \\ &\text{N/m} \end{aligned}$$

$$W_{\text{net}} = \Delta k$$

Form 1:- Work energy theorem i (no friction)

$$W_{\text{net}} = \Delta k \Rightarrow k_f - k_i$$

PROBLEM:-

$$\text{mass} = 0.1 \text{ kg}$$

$$k = 20 \text{ N/m}$$

$$v_i = 1.5 \text{ m s}^{-1}$$

$$d = ? (\text{max}) \text{ i.e } v_f = 0 \text{ m s}^{-1}$$

$$\mu k = 0.47$$

Now,

$$(a) \quad W = \Delta k$$

$$\cancel{\frac{1}{2} k x^2} = \frac{1}{2} m k^2 + \cancel{\frac{1}{2} m v_i^2}$$

Now we can simply use the quadratic formula.

$$\mu k N d + \frac{1}{2} k d^2 = \frac{1}{2} m v^2$$

$$-f_k d - \frac{1}{2} k d^2 = -\frac{1}{2} m v_i^2$$

$$\mu k N d + \frac{1}{2} k d^2 = \frac{1}{2} m v^2$$

$$(\mu k mg) d + \left(\frac{1}{2} k\right) d^2 = \frac{1}{2} m v^2$$

$$k x^2 = m v_i^2$$

$$x = \sqrt{\frac{m}{k}} \cdot v_i$$

$$x = \sqrt{\frac{0.1}{20}} \times 1.5 = 0.106 \text{ m Ans!}$$

(b)

Now we cannot use work energy theorem as friction is present, we have to modify it.

$$W_f + W_{\text{spring}} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$W_{\text{net}} = \Delta k$$

Example

PROBLEM:-

$$k = 15 \text{ N/cm}$$

$$F_a = ?$$

$$W_b = ?$$

$$x_1 = 7.60 \text{ mm}$$

$$x_2 = ?$$

$$F_2 = ?$$

$$\boxed{W = \frac{1}{2} (F \cdot x) x}$$

PROBLEM:-

$$m = 3.63 \text{ kg} \\ v_i = 1.22 \text{ m/s}^{-1}, v_f = 0 \\ k = 135 \text{ N/m}$$

$$W = \Delta K.$$

$$\frac{1}{2} k x^2 = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

0.

$$x = \sqrt{\frac{m}{k}} \cdot v_i = \sqrt{\frac{3.63}{135}} \times (1.22)$$

$$= 0.20 \text{ m.}$$

PROBLEM:-

$$f = GON \\ v_f = 0? \\ F_{avg} = ?$$

$$v_f =?$$

$$\Delta W_{net} = \Delta K.$$

$$\Delta W_{net} = k_f - k_i \\ = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$(M_f - M_g)d = k_f.$$

$$60(-3) - (mg)(-3) = \frac{1}{2} (200)v_f^2. \\ v_f = 7.5 \text{ m/s}^{-1}$$

There are two work done by friction and gravity

PROBLEM:-

-180 -

Now avg. forces on the I beam = ?
this force would be equal to the normal reaction force.

Using work energy theorem,
for the I beam, the final velocity of hammer head

is the initial velocity.

$$\therefore \Delta W_{net} = \Delta K \\ (N + f - w) = \Delta (\frac{1}{2} k_f - k_i)$$

$$N = w - \frac{1}{2} m v_i^2 - f_0$$

$$N = 1960 - \frac{1}{2} (200)(7.5)^2 - 60.$$

$$(N)_d = 1900 \left(-\frac{1}{2} \frac{(5625)}{d} \right)$$

\downarrow

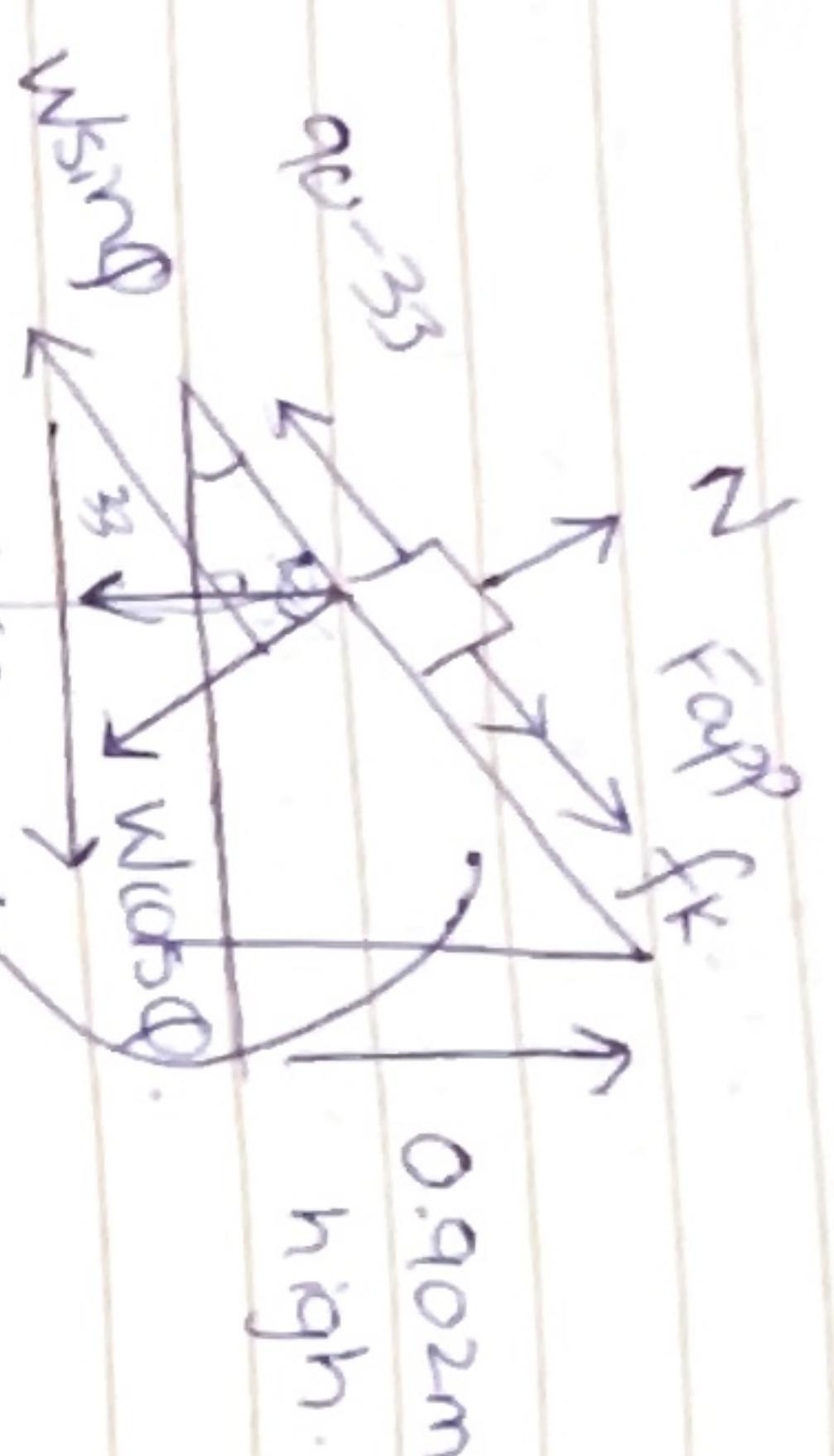
$$77613 \text{ N ans!}$$

exerted by the hammer head

PROBLEM:-

const speed
speed $\propto \alpha = 0$.

$$\mu_k = 0.110.$$



$$F_{\text{app}} = ?$$

$$W = ?$$

$$\tan \phi = \frac{P}{B}$$

Using Newton's 2nd law;

$$\phi = \tan^{-1} \left(\frac{0.902}{1.62} \right)$$

$$\phi = 29.10^\circ \text{ (wrong)}$$

$$\sin \phi = \frac{P}{H}$$

$$\sum F_x = mg \frac{L}{R}$$

$$\sin \phi = P/H$$

$$F_{\text{app}} + f_k - W \sin \phi = 0. \quad \phi = 33^\circ \checkmark$$

$$F_{\text{app}} = -f_k + W \sin \phi.$$

$$F_{\text{app}} = -\mu_k N + W \sin \phi$$

$$= -\mu_k (W \cos \phi) + W \sin \phi.$$

$$= -(0.11) (47.2) (9.8) \cos 33 + (47.2)(9.8)$$

$$\sin 33$$

$$= -42.6 + 251.96$$

$$= 209.32 \text{ N.}$$

(a) W by the worker. (imp)

$$W = F d \cos \phi \\ = (209.3) (1.69) \cos(180) \\ = -354 \text{ N} \approx -360 \text{ N}$$

(b) W by the gravity.

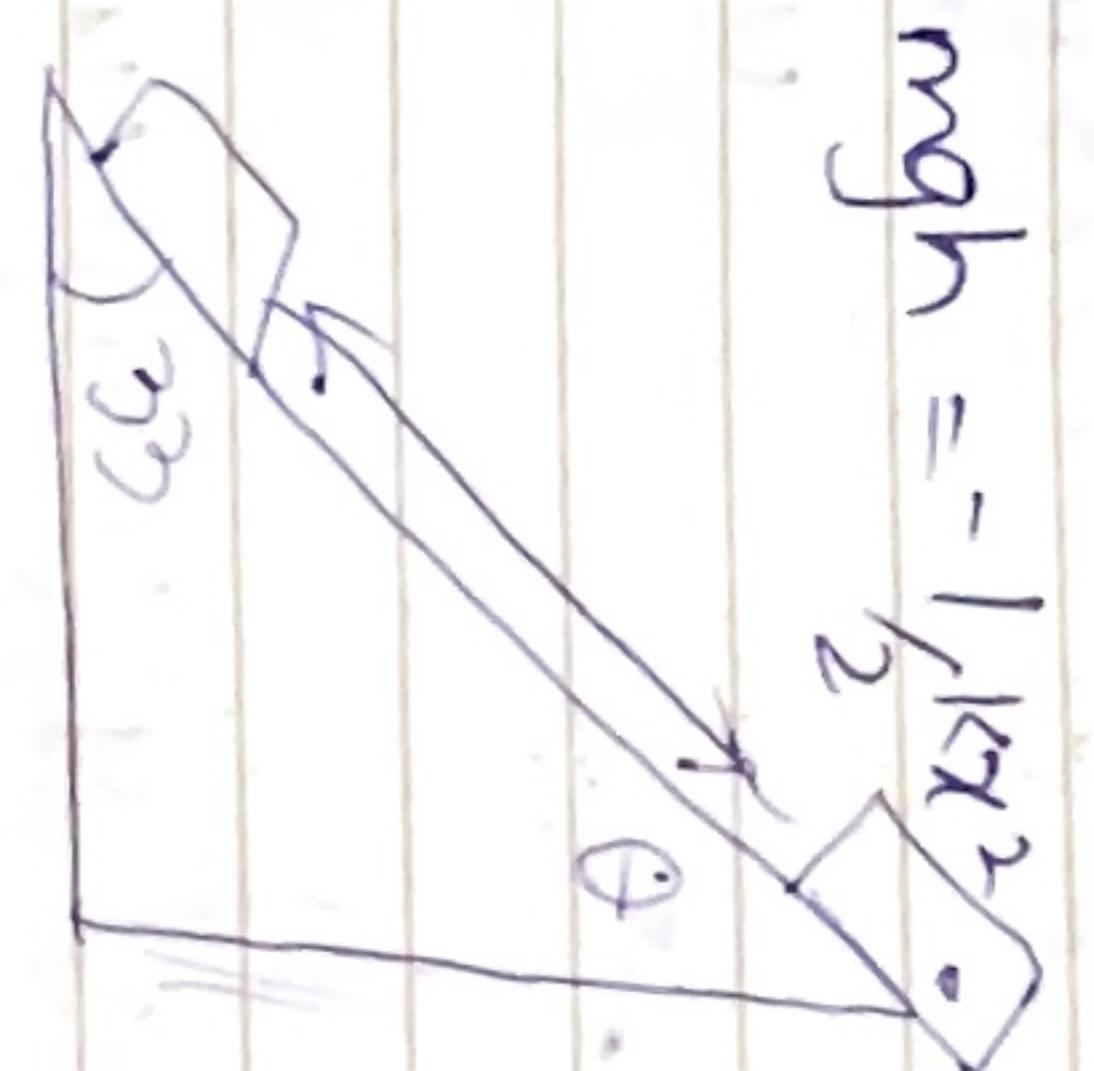
$$\bar{F}_g = mg.$$

$$W = \bar{F} d \cos \phi \\ = (mg) (1.69) \cos(90 - 33) \\ = 417 \text{ J Ans!}$$

$$W_{\text{net}} = 417 - 354 = 63 \text{ J Ans!}$$

$$\frac{1}{2} mv^2 + mgh = -\frac{1}{2} kx^2$$

$$\text{N} \times \frac{1 \text{ cm}}{10^{-2} \text{ m}}$$



$$\rightarrow x_{\text{min}}$$

challenging without accepting it.

Chapter 11:- WORK & KINETIC ENERGY:-

$$S = r\varphi$$

$$\begin{aligned} \text{Rotational K.E.} &:= \frac{1}{2} I v^2 \omega^2 \\ &\Rightarrow \frac{1}{2} I \omega^2. \end{aligned}$$

$$\text{Work energy theorem: } \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 = \Delta W$$

PROBLEM:-

Data given:- thin hoop ($I = MR^2$)

$$\begin{aligned} m &= 31.4 \text{ kg} \\ r &= 1.21 \text{ m} \\ \omega &= 283 \text{ rev/min} \Rightarrow 283 \frac{\text{rev}}{\text{min}} \times \frac{100\pi}{60\text{sec}} \\ \omega_f &= 0 \text{ ms}^{-1} \\ t &= 14.8 \text{ s.} \\ P_{av} &=? = \frac{W}{t}. \end{aligned}$$

$$- \frac{1}{2} (31.4)(1.21)^2 (4.71)^2 + 0 = \Delta W.$$

$$-101.95 = \Delta W.$$

$$\text{and if in rev/s not revs.} \Rightarrow -201.39.75$$

$$1 \text{ rev} = 2\pi \text{ rad.}$$

$$P_{av} = -\frac{201.39.7}{14.8} \Rightarrow -1360 \text{ W Ans!}$$

Problem:-

$$\begin{aligned} \omega_i &= 0 \text{ rad/s.} \\ \omega_f &= 624 \text{ rad/s.} \\ m &= 512 \text{ kg.} \\ R &= 97.6 \text{ cm.} \end{aligned}$$

$$I = \frac{1}{2} MR^2.$$

- (a) $\text{K.E.} = ?$
 (b) $T = ?$ if $P = 8.13 \text{ kW.}$

$$\text{(a) K.E.} = \frac{1}{2} (MR^2) (\omega^2)$$

$$\begin{aligned} &= \frac{1}{2} (512)(97.6 \times 10^{-2})^2 (624)^2 \\ &= 9.4 \times 10^7 \text{ J.} \end{aligned}$$

$$= 9.7 \times 10^7 \text{ J.}$$

$$\text{(b) } \Delta W = K_f - K_i$$

$$\Delta W = 9.7 \times 10^7 \text{ J.}$$

$$\begin{aligned} P &= \frac{\Delta W}{t} \Rightarrow t = \frac{\Delta W}{P} \Rightarrow \frac{9.7 \times 10^7}{8.13 \times 10^3} \\ t &= 5839.6 \text{ s.} \Rightarrow 9.7 \text{ minutes.} \end{aligned}$$

→ workdone by a conservative force in a closed path is zero.

→ workdone by a conservative force b/w two paths is dependent on the path taken.

Chapter no. 12 Potential Energy!

PROBLEM:-

$$m = 1350 \text{ kg}$$

$$\theta = 10^\circ$$

$$\text{initial } v = 10 \text{ km/h} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s.}}$$

$$t = 5 \text{ seconds.}$$

$$(i) S = ?$$

$$v_f = 0 \text{ ms}^{-1}$$

$$(ii) P.E_i = mgh_i \\ = (1350)(9.8)(\sin 10 \times 50) \\ = 114868 \text{ J.}$$

$$P.E_f = mgh_f \\ = (1350)(9.8)(\sin 10 \times 50)$$

$$\sin \theta = \frac{h}{d}$$

$$\text{as formula } d = 50 \text{ m.}$$



B

$$\omega = \frac{v}{r} = \frac{20}{100 \times 10^{-2}} = 40 \text{ rad/sec.}$$

$$\sin \theta = \frac{h}{d}$$

"CONSERVATION OF MECH. ENERGY"

$$E_T = \Delta K + \Delta U.$$

$$\Delta E_T = 0$$

$$\text{or } E_f = E_i$$

$$K_i + U_i = K_f + U_f.$$

$$(iii) K_i = \frac{1}{2} m v_i^2$$

$$= \frac{1}{2} (1350) (20)^2 = 0 \text{ ms}^{-2}$$

$$= 2.7 \times 10^5 \text{ J}$$

$$(iv) P.E_i = mgh_i$$

Let $h = 0$ at the start of the incline.

$$P.E_i = 0 \text{ J.}$$

Galaxy

Galaxy

Date / Date

PROBLEM:-

$$m = 12g.$$

$$v_f^2 = ?$$

$$v_i = 0 \text{ m s}^{-1}$$

$$k = 7.5N/\times 10^{-2} \text{ m.}$$

$$E_i = E_f.$$

$$\frac{1}{2}kd^2 + v_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}mv_i^2$$

$$\frac{1}{2} \cdot 7.5 \times 10 \cdot d^2 + v_i^2 = v_f^2$$

$$0.$$

$$\frac{1}{2}kd^2 = \frac{1}{2}mv_f^2$$

$$\frac{1}{2} \cdot 7.5 \times 10 \cdot d^2 = v_f^2$$

$$0.012 \text{ kg}$$

$$v_f = \sqrt{(7.5 \times 10) \cdot (0.032)} \Rightarrow 8 \text{ m/s}$$

Ans!

$$27000 + 2940 = \frac{1}{2}mv_f^2$$

$$\sqrt{\frac{2(27000 + 2940)}{2.4}} = v_f \Rightarrow 158 \text{ ms}^{-1} \text{ Ans!}$$

$$(c) E_i = E_f.$$

$$k \cdot E_i + v_i^2 = k \cdot E_f + v_f^2$$

$$0.$$

Problem:-

A 1.93kg block is placed against a compressed spring of an archimedes 27° incline. The spring whose const. is 20.8 N/cm is compressed 18.7 cm after which the block is released. How far up the incline will the block go before coming to rest? Measure the final position of the block with respect to its position just before being released.

$$(a) K_i = \frac{1}{2}mv_i^2$$

$$= \frac{1}{2} (2.4)(150)^2 = 54000 \text{ J} \Rightarrow 2.7 \times 10^4 \text{ J}$$

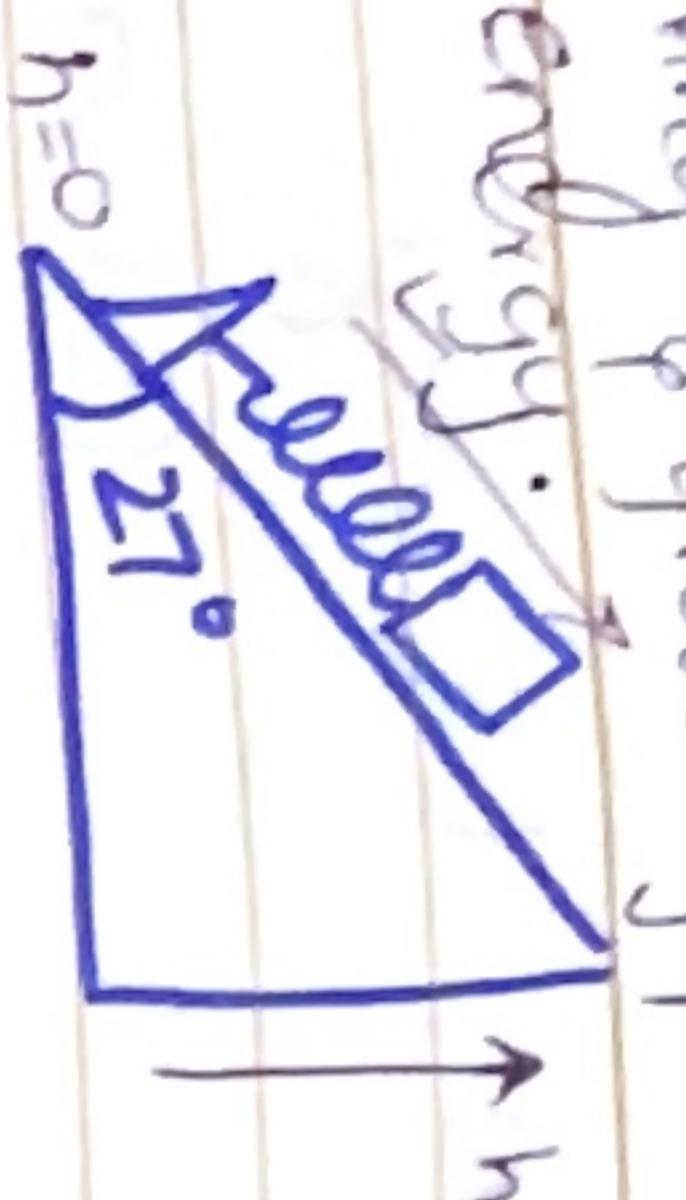
Date / Date

(b) P.E. = mgh.
 $= (2.4)(9.8)(125)$
 $\Rightarrow 2940 \text{ J.}$

The body has both spring & gravity potential energy.

Ques

$$E_i = E_f.$$



$$k_i + U_i = k_f + U_f.$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kd_i^2 + mgh_i = \frac{1}{2}mv_f^2 + \frac{1}{2}kd_f^2 + mgh_f.$$

O

O

$$\frac{1}{2}kd_i^2 = mgh_f$$

$$\frac{1}{2}(2080)(18.7 \times 10^{-2})^2 = (1.93)(9.8)h_f.$$

$$hf = 1.92m.$$

$$\tan\phi = \frac{\text{Perpendicular}}{\text{Base}} \Rightarrow \sin\phi = \frac{h}{L}$$

$$\left[F_c = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2} \right]$$

$$k = \frac{1}{4\pi\epsilon_0} \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$$

$$F = \frac{kq_1q_2}{r^2}$$

(formula of a quantized charge).

Coulomb's law;

$$F = \frac{q_1q_2}{r^2} \quad q = ne.$$

net electric charge.

- **Electric charge,**
Matter can develop charges due to imbalance of protons and electrons; or by ionization.

Ques

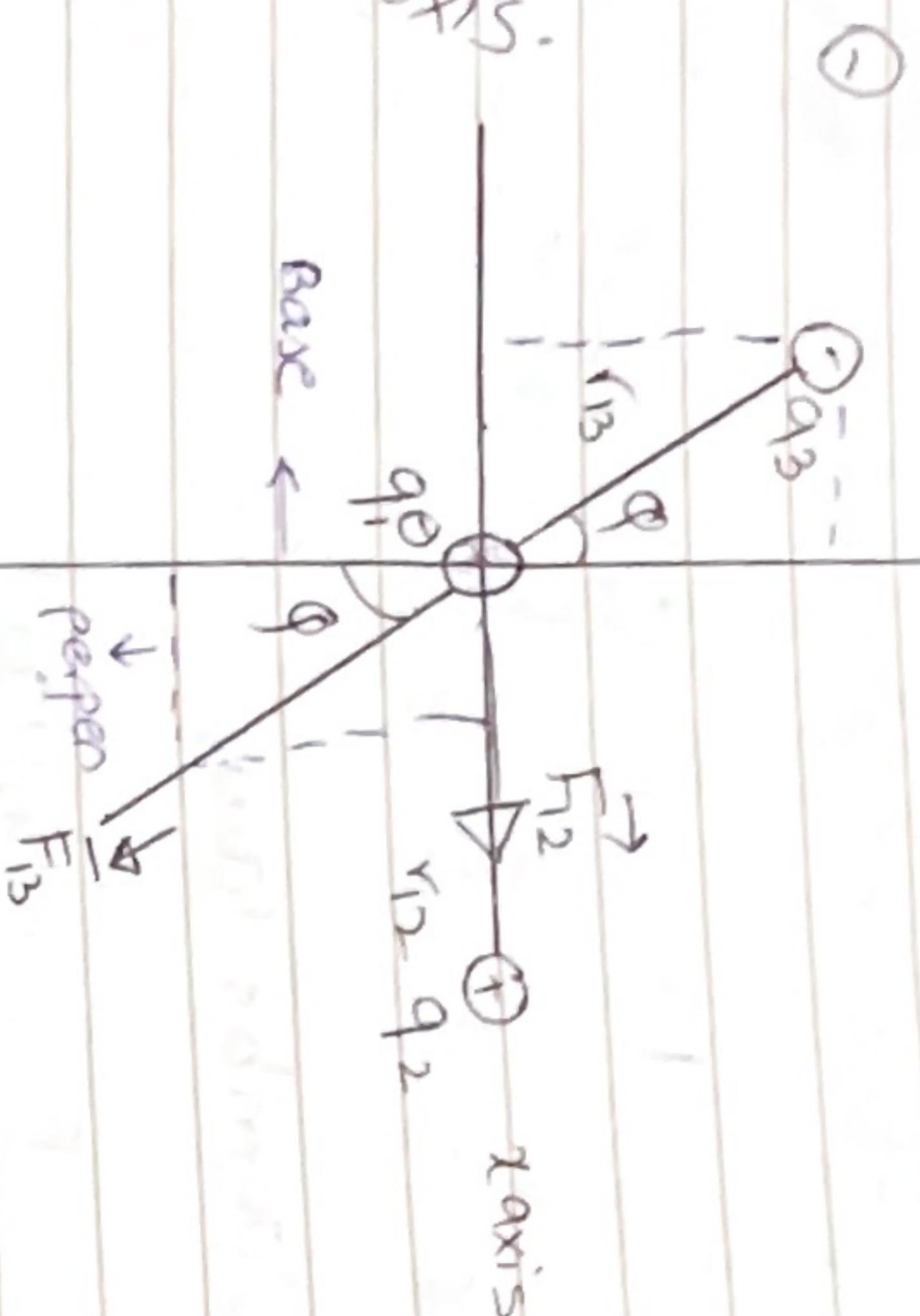
Electricity / Electric Current & field.

$$l = 4.229m.$$

(2) F_{12} \hat{y}

y -axis. \hat{y}

Take modulus in
Coulomb's law and
find signs through
axis.



Using Coulomb's law to find the force on the charge q_1 .

As the charge q_3 exerts a force F_{13} which is making some angle θ . resolve the force into 2 components.

$$F_{13} = k \frac{q_1 q_3}{r^2} \cos\theta \quad F_{13} = k \frac{q_1 q_3}{r^2} \sin\theta$$

$$F_y = k \frac{(1.2 \times 10^{-6})(3.7 \times 10^{-6})}{(1.5 \times 10^{-2})^2} = 1.774 \text{ N}$$

$$F_x = \frac{k \frac{(1.2 \times 10^{-6})(3.7 \times 10^{-6})}{(1.5 \times 10^{-2})^2}}{\sin 32^\circ} \times \sin 32^\circ = 1.34 \text{ N}$$

The force exerted by q_2 is only at a single direction using Coulomb's law.

$$F_x = k \frac{(-1.2 \times 10^{-6})(3.7 \times 10^{-6})}{(1.5 \times 10^{-2})^2} = -1.774 \text{ N}$$

$$F_{\text{net}} = \sqrt{(F_x)^2 + (F_y)^2} = \sqrt{(0.988)^2 + (2.104)^2} = 2.42 \text{ N Ans!}$$

$$F_{\text{net}} = \sqrt{(F_x)^2 + (F_y)^2}$$

$$= \sqrt{(0.988)^2 + (2.104)^2} = 2.42 \text{ N Ans!}$$

Linear charge density $\lambda = \frac{q}{L}$

Uniform line of charge.

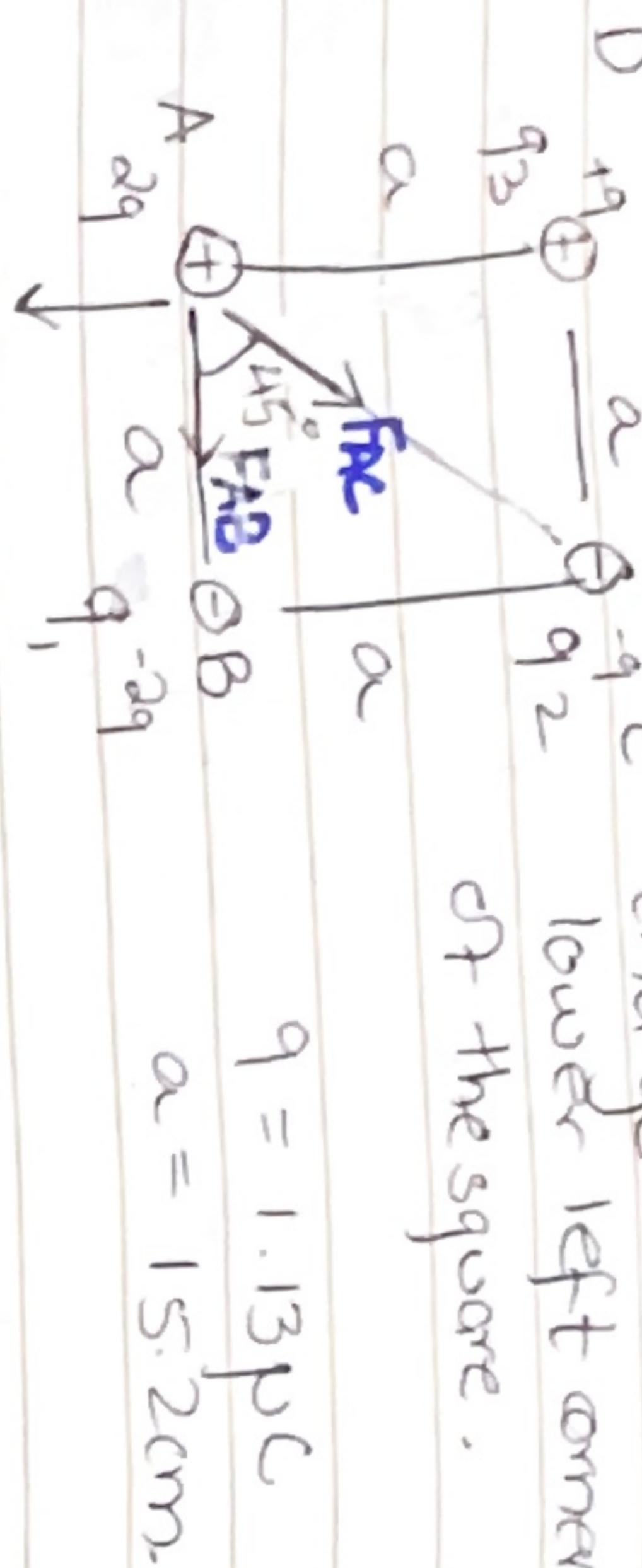
$$\lambda = \frac{q}{L}$$

$$F_y = k \frac{q_1 q_l}{L^2} \sin \theta$$

charge
along
the
line.

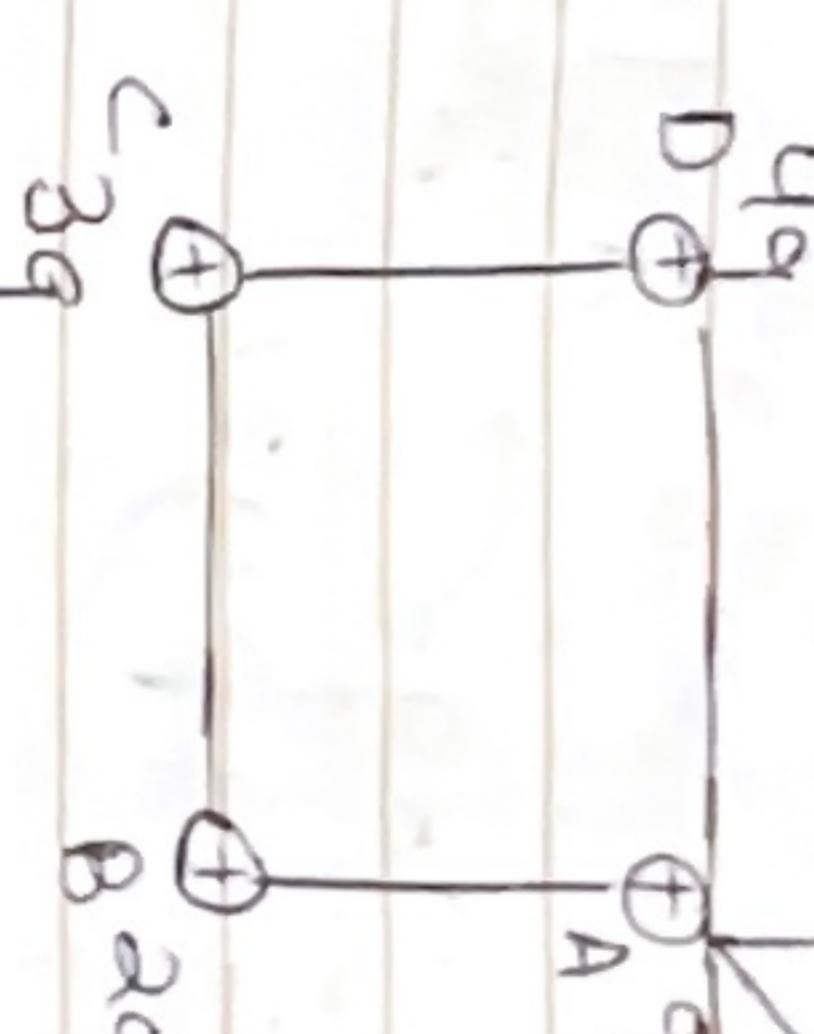
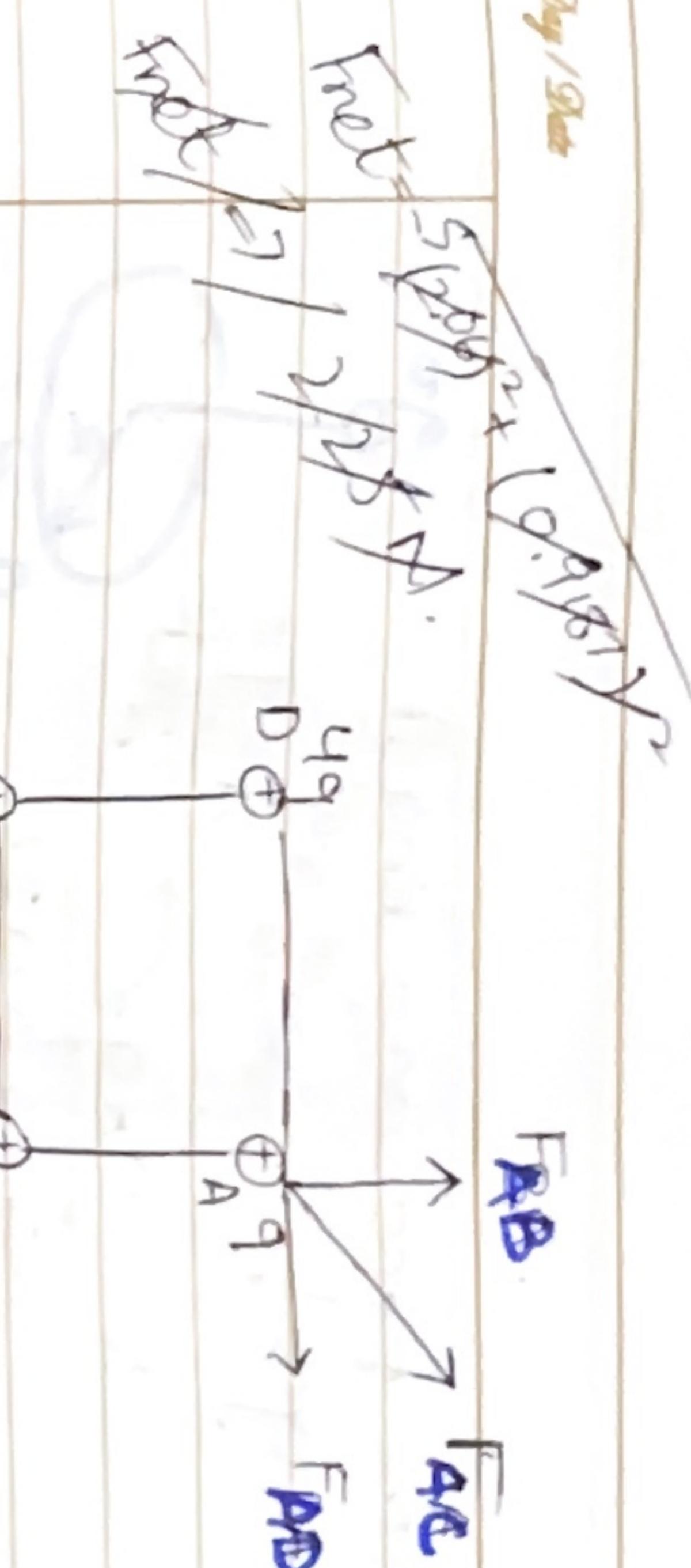
Ques

Find the horizontal components and vertical components of resultant electric force on the charge in the lower left corner of the square.



$$q = 1.13 \mu C$$

$$a = 15.2 \text{ cm}$$



$$F_{AC} \Rightarrow k q_A q_C \cos(45^\circ)$$

$$\sqrt{(15.2)^2 + (15.2 \times 10^{-2})^2} = 0.515$$

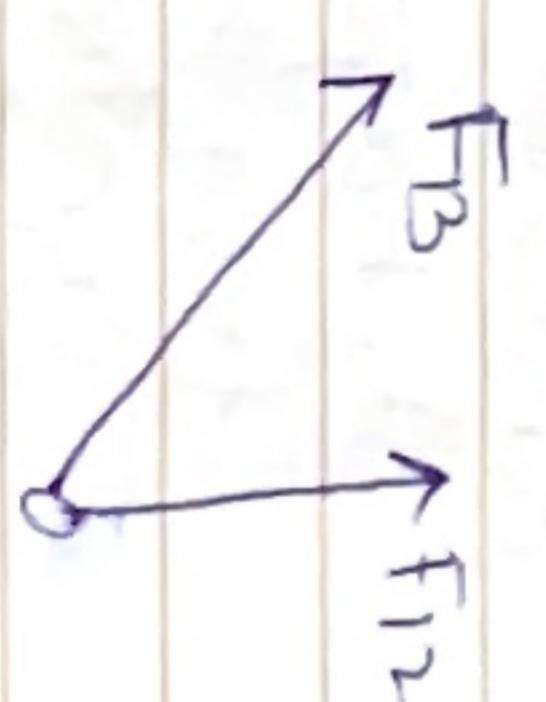
$$F_{AC} \Rightarrow k q_A q_C \sin(45^\circ) = 10.075$$

$$F_{12} = k |q_1| |q_2|$$

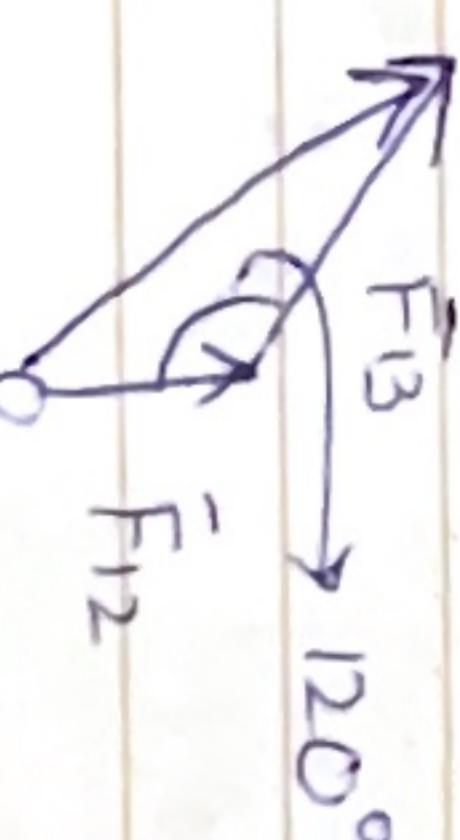
Using head to tail Rule.

$$F_{12} = 1.76 \text{ N}$$

$$F_{13} = 1.76 \text{ N}$$



also



$$F_{net} = \{ +0.075 - 0.9937 \} = 0.9187$$

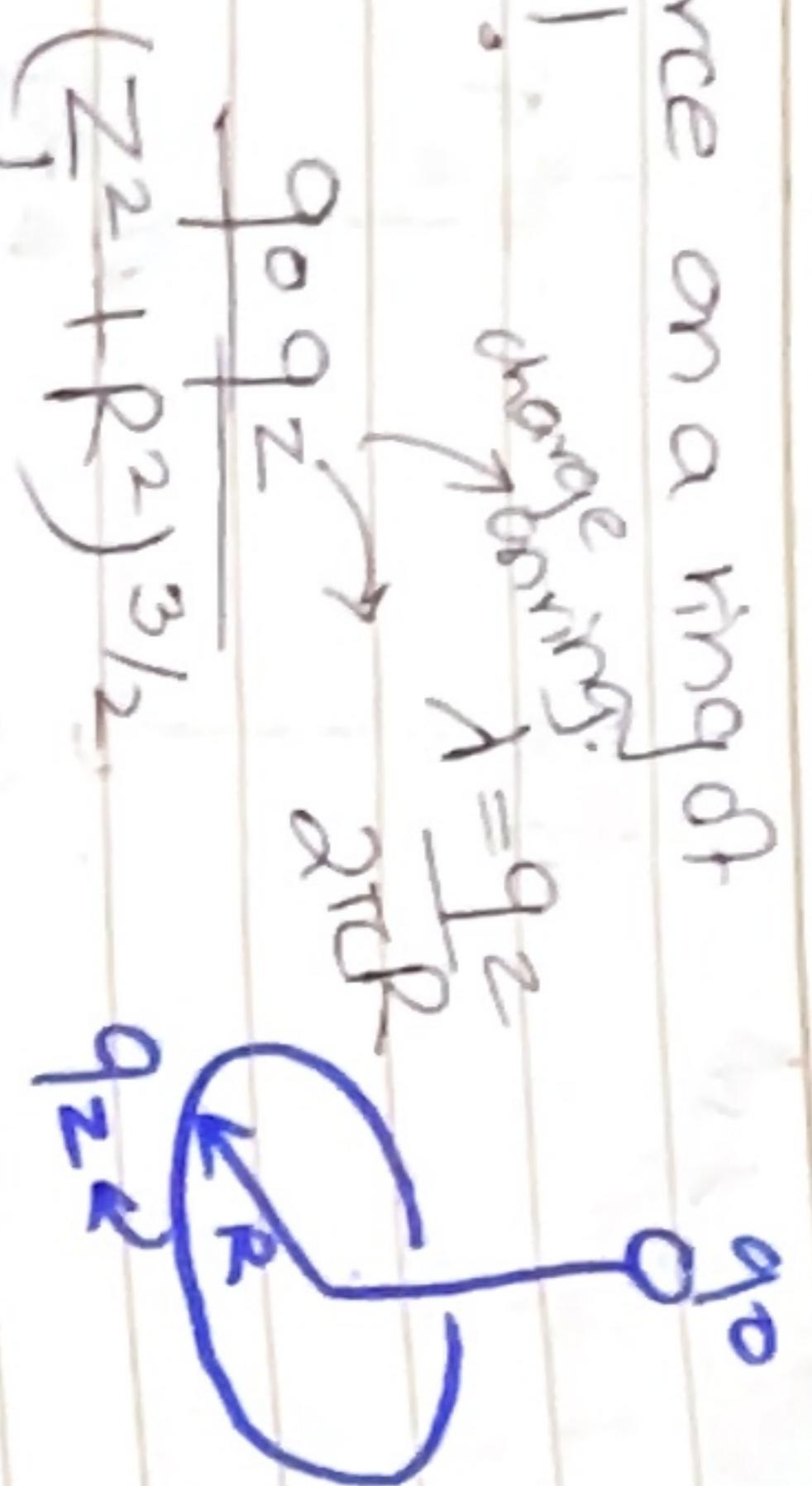
$$|R| = \sqrt{(F_{12})^2 + (F_{13})^2 - 2(F_{12})(F_{13}) \cos 120^\circ}$$

$$3.04 \text{ N} \quad |R| = \sqrt{(1.76)^2 + (1.76)^2 - 2(1.76)(1.76) \cos 120^\circ}$$

Galaxy

Electric force on a ring of charges!

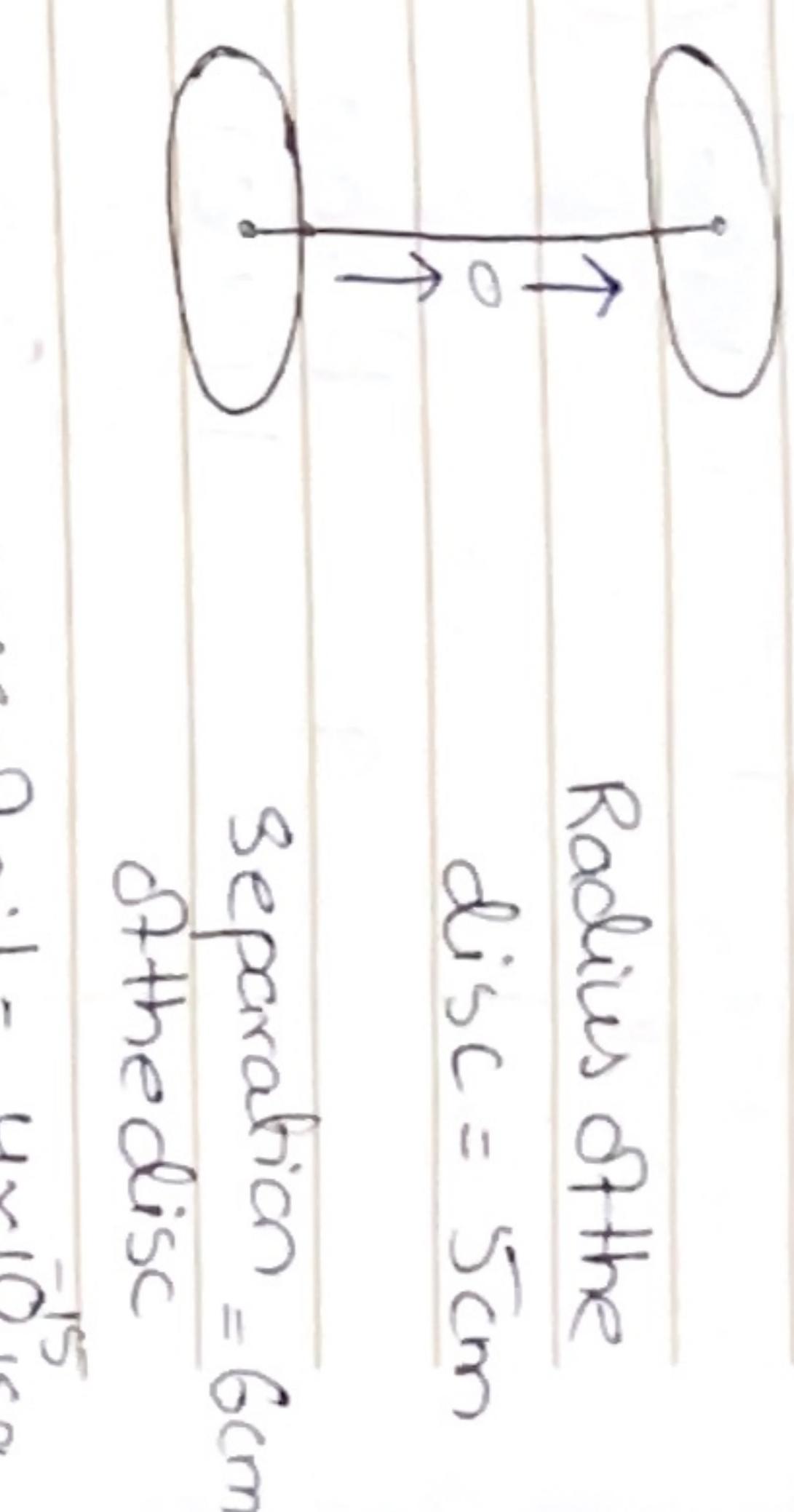
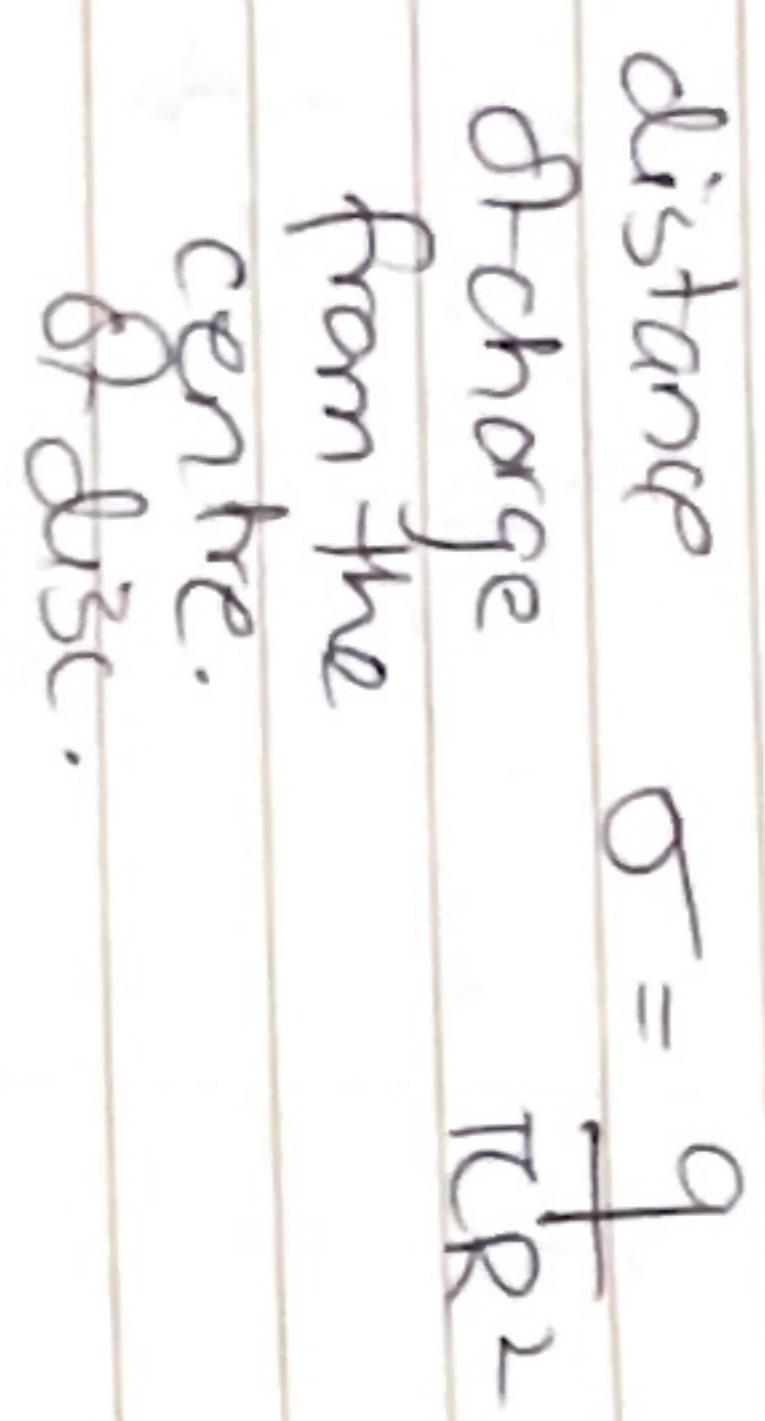
$$F = k \frac{q_0 q_z}{(z^2 + R^2)^{3/2}}$$



distance of the charge from the centre
of the Ring.

Electric force on a disc of charges.

$$F = k \frac{2q_0 q}{R^2} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$



Formula.

$$F = k \frac{2q_0 q}{R^2} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

Let that upper plate = positive
lower plate = negative

The drop is midway between the plates
and the upper plate is attracting -e & lower
plate repelling and still we maintain it
at the midway. $\sum F_y = 0$

gravitational force = $\left(\frac{\text{net electrostatic force}}{\text{force}} \right) 2$

$$mg = F_{\text{net}}$$

$$mg = 2 \left(\frac{k^2 q^2 e}{R^2} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right) \right)$$

$$mg = k \frac{4q^2 e}{R^2} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

$$q = \frac{mg}{k(4\pi e)} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

$$q = \frac{R^2 mg}{k(4\pi e)} \left(\frac{1}{\sqrt{z^2 + R^2}} - z \right)$$

$$q = 9.8 \times 10^{-17} \left(0.058 \right) \\ 5.76 \times 10^{-9} \left(0.028 \right)$$

$$q = \frac{5.684 \times 10^{-18}}{1.6128 \times 10^{-10}}$$

$$q = 35 \text{ nC. Ans!}$$

Electric force :-

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \times \frac{q_0 q}{r^2}$$

where $q_0 \Rightarrow$ positive test charge.
 $q \Rightarrow$ point charge.

$$\frac{q_2}{q_1} = \frac{v_1}{(L-v)}$$

$$\frac{q_2}{q_1} = \frac{L-v_1}{v_1} \Rightarrow -\frac{Q_2}{Q_1} = \frac{L-v_1}{v_1} \Rightarrow -Q_2 v_1 = Q_1 L - Q_1 v_1$$

$$\left[\begin{array}{l} -Q_2 = \frac{L-v_1}{v_1} \\ Q_2 = \frac{L}{v_1} - 1 \end{array} \right] \Rightarrow Q_1 v_1 - Q_2 v_1 = Q_1 L \\ \Rightarrow Q_1 v_1 - Q_1 L = Q_1 L \\ \Rightarrow v_1 (Q_1 - Q_2) = Q_1 L \\ \Rightarrow v_1 = \frac{Q_1 L}{Q_1 - Q_2} \approx 0.295 \text{ m.}$$

$q_1 = 1.5 \mu\text{C}$ $q_2 = 12.3 \mu\text{C}$. The first charge is at the axis origin of x axis and the 2nd at a distance of 13 cm from q_1 . At what distance of Point P $E = 0$.

As there are two electric fields by 2 charges on point P.

$$\bar{E}_{\text{net}} = \bar{E}_1 + \bar{E}_2$$

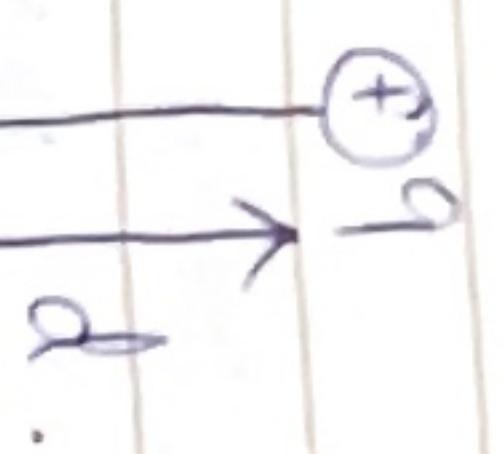
$$\bar{E} = k \frac{q_1}{r_1^2} + k \frac{q_2}{r_2^2}$$

Electric Dipole:-

Two equal & opposite charges separated by a distance 'd' is called as electric dipoles.

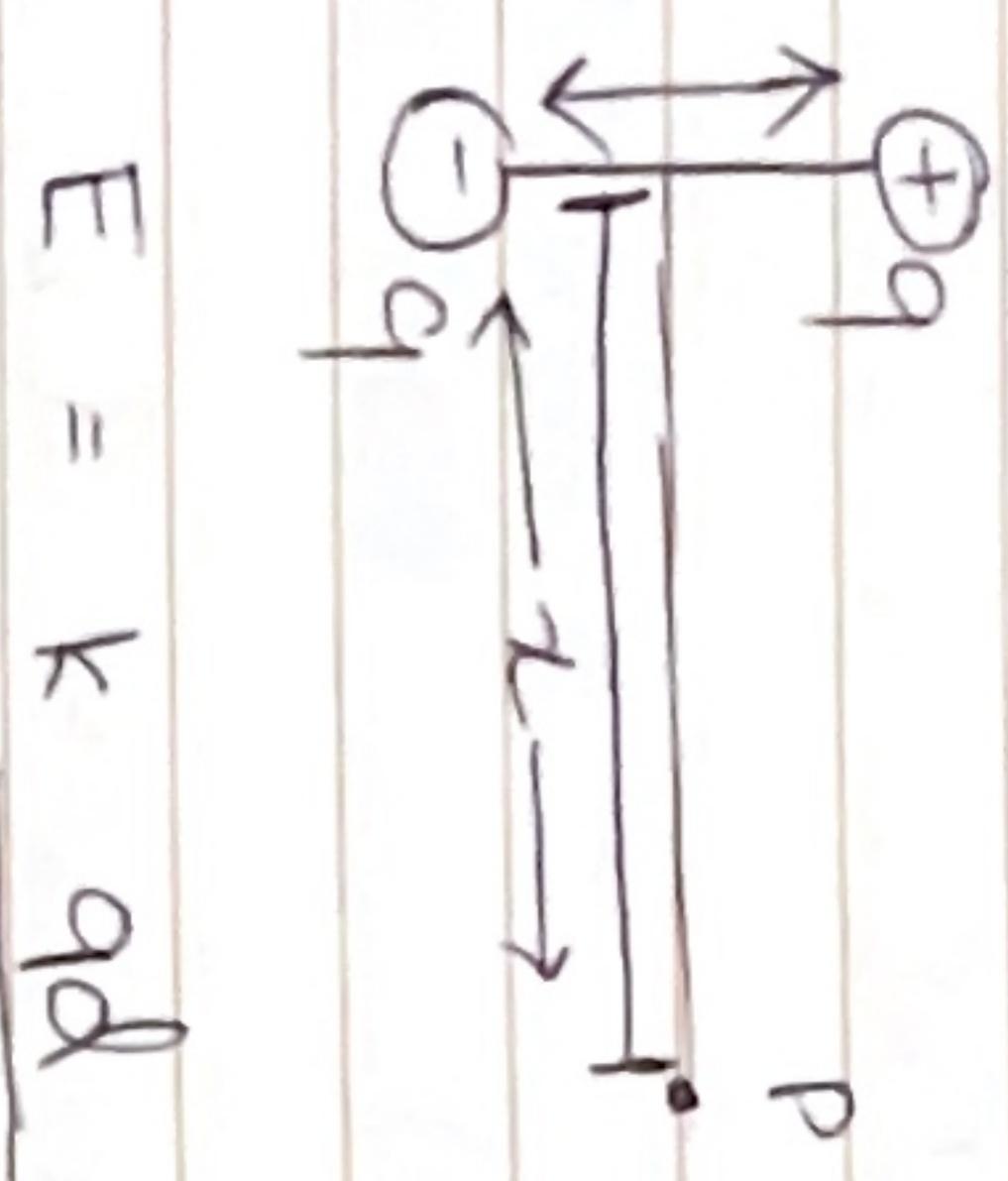
$$\mathbf{P} = q\mathbf{d}$$

- * charges that are separated should be of equal magnitude.



(ii) Electric Field b/w two charges through Electric Dipole.

Finding 'E' at a point P.



$$\therefore E = k \frac{qd}{r^2}$$

$$[x^2 + (d/2)^2]^{3/2}$$

distance separation
of the point between
from the centre of
the charges.

when

$$(i) E = \frac{k}{2\pi\epsilon_0 y} \quad (\text{impl.}) \quad \text{For line uniform Galaxy}$$

$$(ii) \text{ when } y \rightarrow \infty \quad E = k \frac{q}{y^2}$$

LINE OF UNIFORM CHARGES CASE .

$$E = k \frac{\lambda L}{\sqrt{y^2 + L^2/4}}$$

$$\lambda = \frac{q}{2\pi R}$$

(iii) RING OF CHARGES :-

$$E = k \frac{\lambda (2\pi R)}{(z^2 + R^2)^{3/2}}$$

(iv) DISK OF CHARGE :-

$$E = k \frac{\sigma \pi R^2}{R^2} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{2\pi R^2 \sigma}{R^2} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

$$\sigma = \frac{q}{A}$$

1 charges

$$2\pi\epsilon_0 y \quad \text{Galaxy}$$

Dy/Date

A plastic Rod

$$L = 220\text{cm}$$

$$R = 3.6\text{mm} = y$$

with $q = -3.8 \times 10^{-7}\text{C}$.

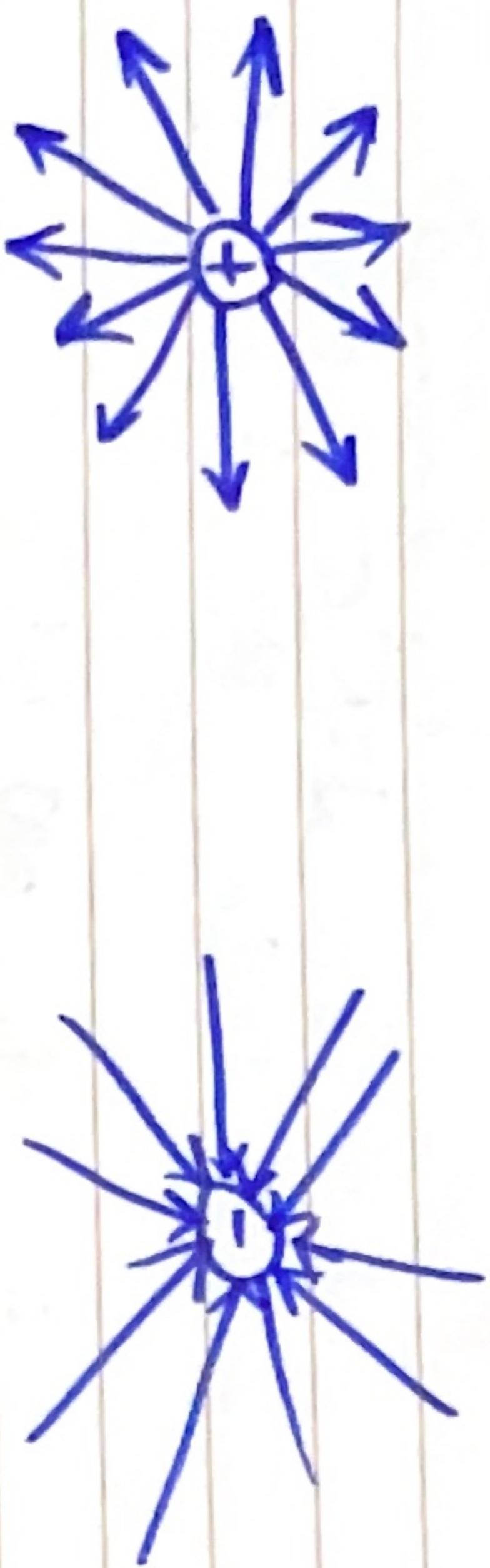
now $L \gg y$.

$$\lambda = q = -3.8 \times 10^{-7}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 y} = -1.73 \times 10^{-7}$$

$$2\pi\epsilon_0 (3.6 \times 10^{-3})$$

$$E = -8.6 \times 10^5 \text{ N/C} \quad (\text{Ans})$$



Magnitude of the electric field is proportional to the no. of electric field lines per unit cross-sectional area perpendicular to those lines.

Dy/Date

Charged drop of oil

$$R = 2.76 \mu\text{m}$$

$$\text{density} = 918 \text{ kg/m}^3$$

$$E = 1.65 \times 10^6 \text{ N/C}$$

$$V = \frac{m}{d}$$

(a) According to the Newton's law of motion:-

$$m = Vd$$

$$mg = qE$$

$$q = \frac{mg}{E} = [(4.3 \times 10^{-6})^3][918](98) / 1.65 \times 10^6$$

$$q \Rightarrow 4.8 \times 10^{-19}$$

(b)

$$\sum F = ma$$

$$qE - mg = ma$$

$$q'E = ma + mg$$

$$q'E = m(a + g)$$

$$-g + \frac{qE}{m} = a$$

$$q' = 2e$$

Dy/Date

Galaxy

Galaxy

3x slide question

Assignment.

$$\begin{cases} \phi_E = 2\pi \text{ if no source/sink} \\ \phi_E = +\pi \text{ if contain source} \\ \phi_E = -\pi \text{ if contain sink.} \end{cases}$$

$$\phi_E = EA$$

$$= E \cdot \frac{A}{r} \cos \theta$$

\rightarrow arc vector

TORQUE W.R.T ELECTRIC FIELD.

$$\tau = F \left(\frac{d}{2} \sin \phi \right) + F \left(\frac{d}{2} \right) \sin \phi$$

$$\tau = \vec{P} \cdot \vec{E} \sin \phi$$

$$\tau = (6.2 \times 10^{-30})(1.5 \times 10^4) \sin 30^\circ$$

$$(6.2 \times 10^{-30})(1.5 \times 10^4) \sin 30^\circ$$

Also

$$\tau = qE \left(\frac{d}{2} \right) \sin \phi$$

$$\vec{\tau} = \vec{p} \vec{E} \sin \phi$$

$$\begin{aligned} U &= -p \cdot E \\ U &= -p E \cos \phi. \end{aligned}$$

An H₂O neutral molecule with $p = 6.2 \times 10^{-30} \text{ Cm}$ and distance b/w positive & negative centres $\Rightarrow ?$

- (a) As total positive charge $\Rightarrow +10q$
total negative charge $\Rightarrow -10q$

$$\therefore p = qd.$$

$$\frac{6.2 \times 10^{-30}}{10(1.6 \times 10^{-19})} = d$$

$$d = 4 \mu\text{m} \Rightarrow 4\mu\text{m}.$$

$$\begin{aligned} (a) \quad p &= qd = ? \\ p &= (1.48 \times 10^{-9})(6.23 \times 10^{-6}) \\ p &= 9.2 \times 10^{-15} \text{ Cm.} \end{aligned}$$

- (b) Diff potential energy;

parallel antiparallel.

$$\begin{aligned} U &= -p \cdot E \cos \phi & -p \cdot E \cos \phi \\ &= -p \cdot E \cos(0^\circ) & -p \cdot E \sin(180^\circ) \\ &= -p \cdot E & -p \cdot E \end{aligned}$$

$$(b) \quad \text{Torque} = ?$$

$$(c) =$$

$$\Delta W = -U$$

$$\Delta U = qE \cos \phi.$$

as the workdone is to

$$\begin{aligned} \text{be found from two angles.} \\ W &= pE (\cos(-180^\circ) - \cos(0)) \\ &= -2pE \end{aligned}$$

L