MATHEMATICS (INTEGRATION).

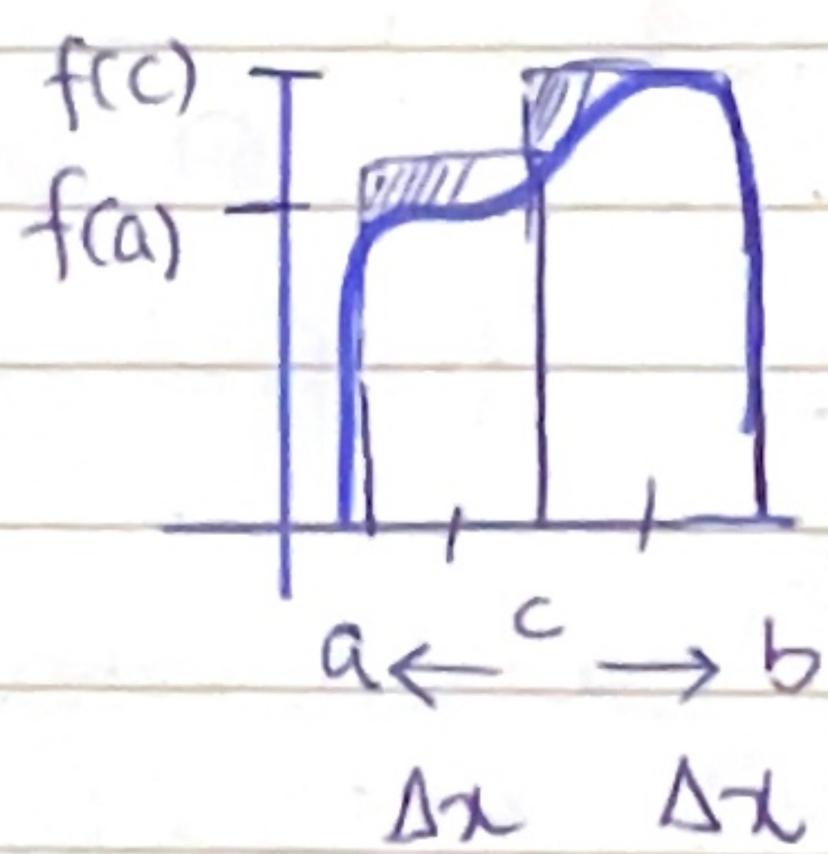
Chapter no. 5 (app. of integration).

- For a function $f(x)$ consider;
  (i) $f'(x)$ derivative.
  (ii) $F(x)$ antiderivative / integration.

$\therefore$ $F'(x) = f(x)$

Note:- The derivative of the antiderivative returns the orignal function.

- Definite integrals are used to find the area of irregular shaped bodies which are not well defined by formulas.



$\Rightarrow$ Total Area under the curve is now equal to the area covered by the (~~triangle~~) rectangles drawn.

$\Rightarrow$ The shadad region has a chance of error as it is not a part of the original area of the curve. We must eliminate the error by reducing the size and hence increasing the no. of individual rectangles drawn.

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$n =$ intervals.

From the figure.

(length x breath + length x breath)

$$A \approx f(a)\Delta x + f(c)\Delta x.$$

⇒ This phenomena is known as the Rieman solution.

Now; increasing □ to reduce the error.

$$A \approx f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \dots n$$

$$A \approx \sum_{k=1}^{n} f(x_k)\,\underbrace{\Delta x}_{\text{const.}}$$

$$\therefore A \approx \Delta x \sum_{k=1}^{n} f(x_k)$$

$$\Delta x \text{ (change / average)} = \frac{b-a}{n}.$$

$$A \approx \frac{b-a}{n} \sum_{k=1}^{n} f(x_k)$$

Consider $\lim_{n \to \infty}$ (when infinite many rectangles / intervals are drawn).

$$A \Rightarrow \lim_{n \to \infty} \frac{b-a}{n} \sum_{k=1}^{n} f(x_k)$$

⇒ as we were summing continuous values

$$A \Rightarrow \int_a^b f(x)\,dx.$$

~x~

→ $\int$ was introduced by lebaniz

$\int$ (summation of continuous values)

Σ (summation of discrete value).

i) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$

$\Rightarrow \Delta x$ changes from (greator - less) to (less - greator)

ii) $\int_a^b f(x)\,dx = 0$ (if $b = a$)

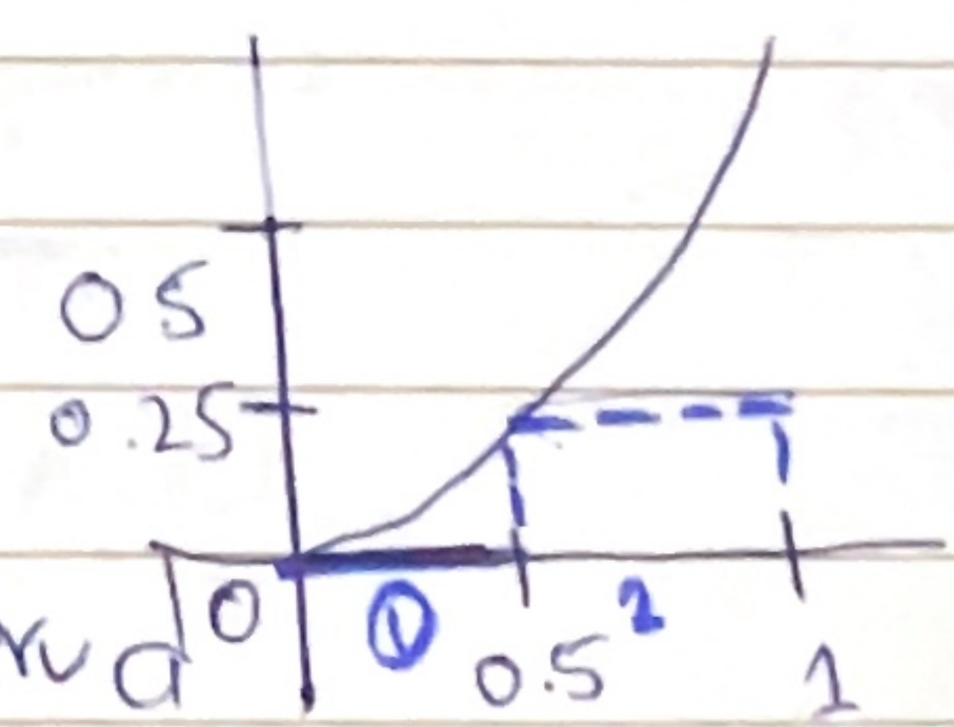
$\Rightarrow$ As there is no area under a singel point.

iii) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$

EXERCISE 5.1:-

- Use finite approx to estimate area;
  a. lower sum with 2 ▭ of equal width.
  b. lower sum with 4 ▭ of equal width.
  c. upper " " 2 ▭ of equal width.
  d. upper sum with 4 ▭ " " .

Q no.1 :- $y = x^2$ $[0, 1]$



we have considere the 2 rectangels in the given interval and thus will now approx the area.

$A \approx (0)(0.5) + (0.5)(0.25)$

$A \approx 0.125\ m^2$

Galaxy

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NOTES:-

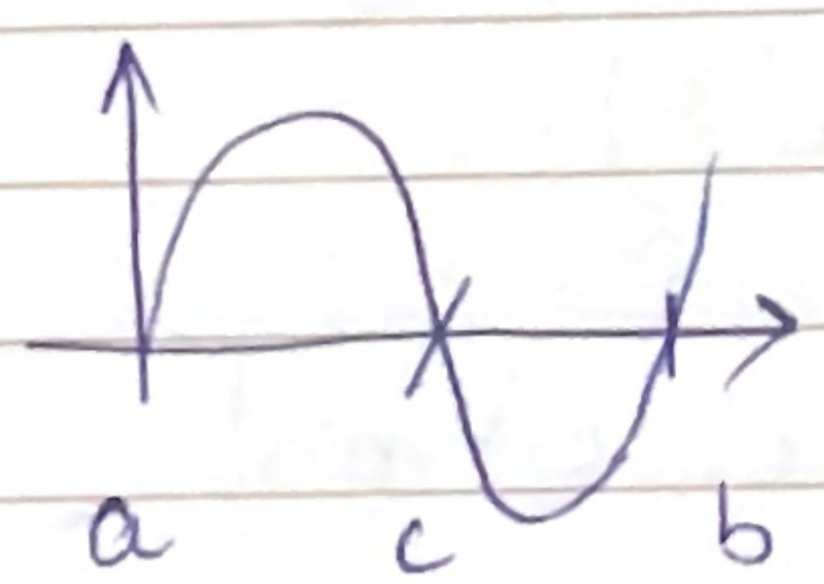
Solving integration for Area!

- Take mod if the Area is negative.
- Always check the function for roots;

  (i) if the roots donot lie on the interval solve it conventionally.

  (ii) if the roots lie on the interval given, then break the roots
  (↳ interval on those ↑)

Example:-
Consider point 'c' that lies in the interval [a, b] of a funct / integrand whose Area is required.



then now we break the intervals as;

[a, c] & [c, b].

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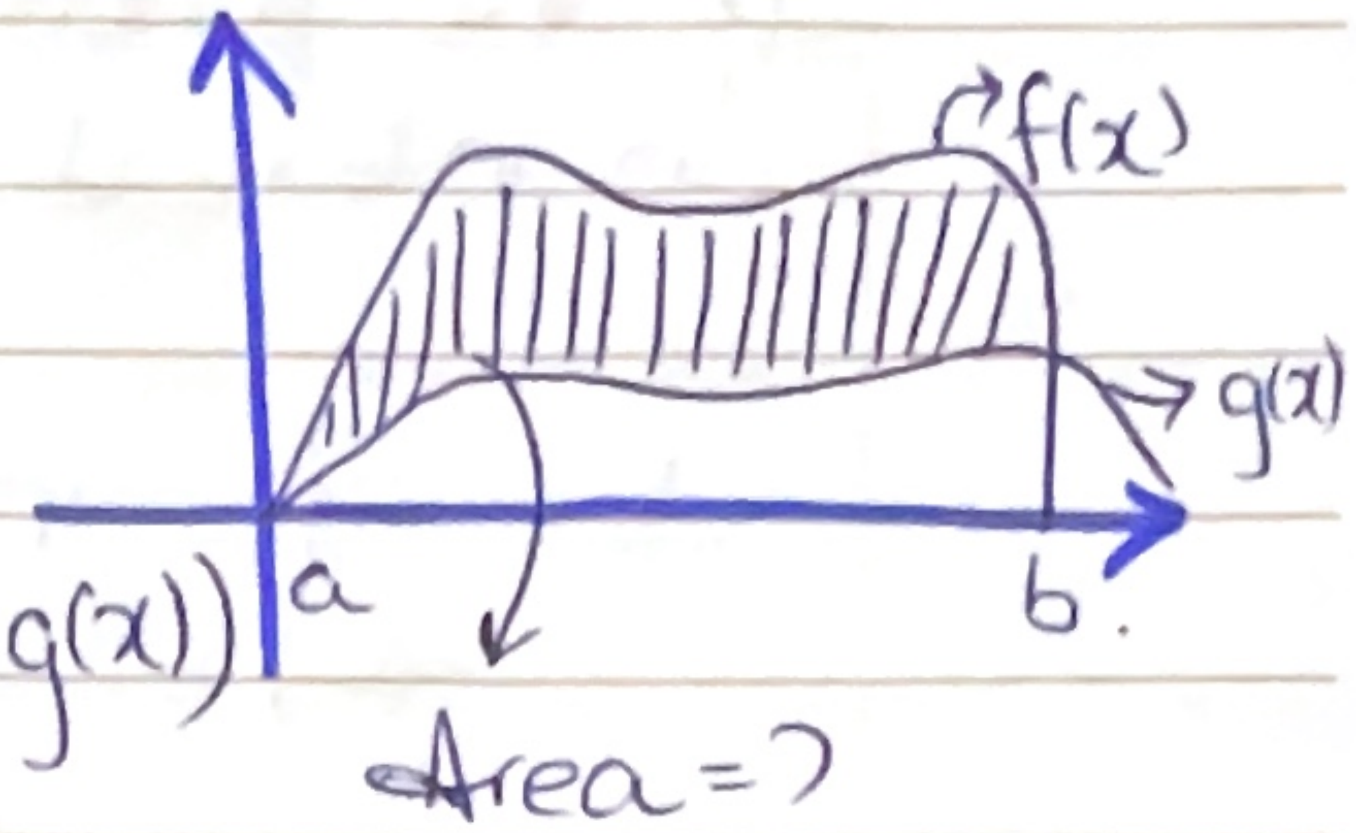
NOTES:-

★ "AREA ENCLOSED BETWEEN CURVES"

case no.1.



Now the areabetween the regions/graphs of $g(x)$ & $f(x)$ is (if $f(x) \geq g(x)$)

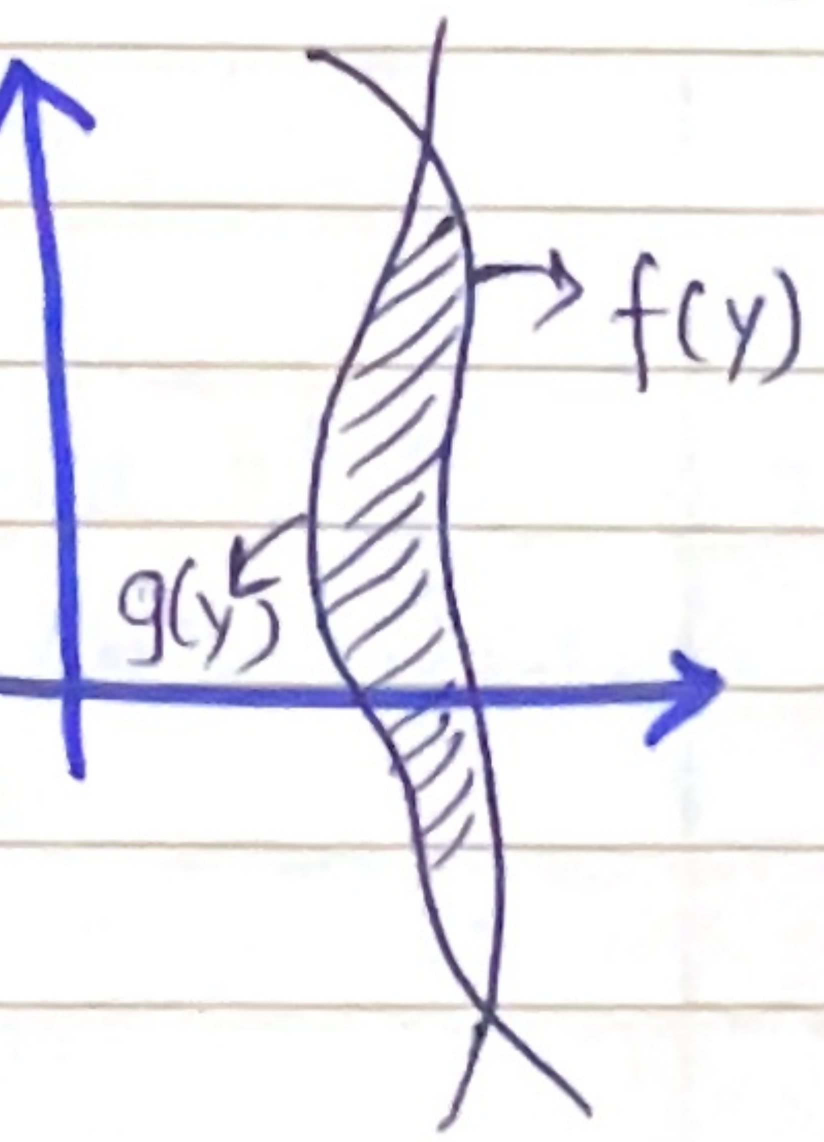
$$\int_a^b f(x) - g(x)\, dx.$$

[a,b]

★ where a & b are the limits or the values of x wherethe graph is plotted; if the info. of limits is not given find by solving them simultaneously (i.e $f(x) = g(x)$)

★ $f(x) \geq g(x)$ =>? the funct is greater or smaller depends upon the values of y axis or range in this case.

Case no.2



The second case is similar however to see if $f(y) \geq g(y)$ depends upon the values on x-axis in this case.

Test at $\left[\int_{-a}^{a} f(x)\,dx\right]$ shape/type

EVEN & ODD FUNCTIONS :-

EVEN FUNCT:- (symmetrical about y-axis)

If the function is even & defined at an interval [a, a].

$$\therefore \int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx.$$

ODD FUNCT:- (symmetrical about origin.

If the funct is odd (no area exisits.

$$\int_{-a}^{a} f(x)\,dx = 0.$$

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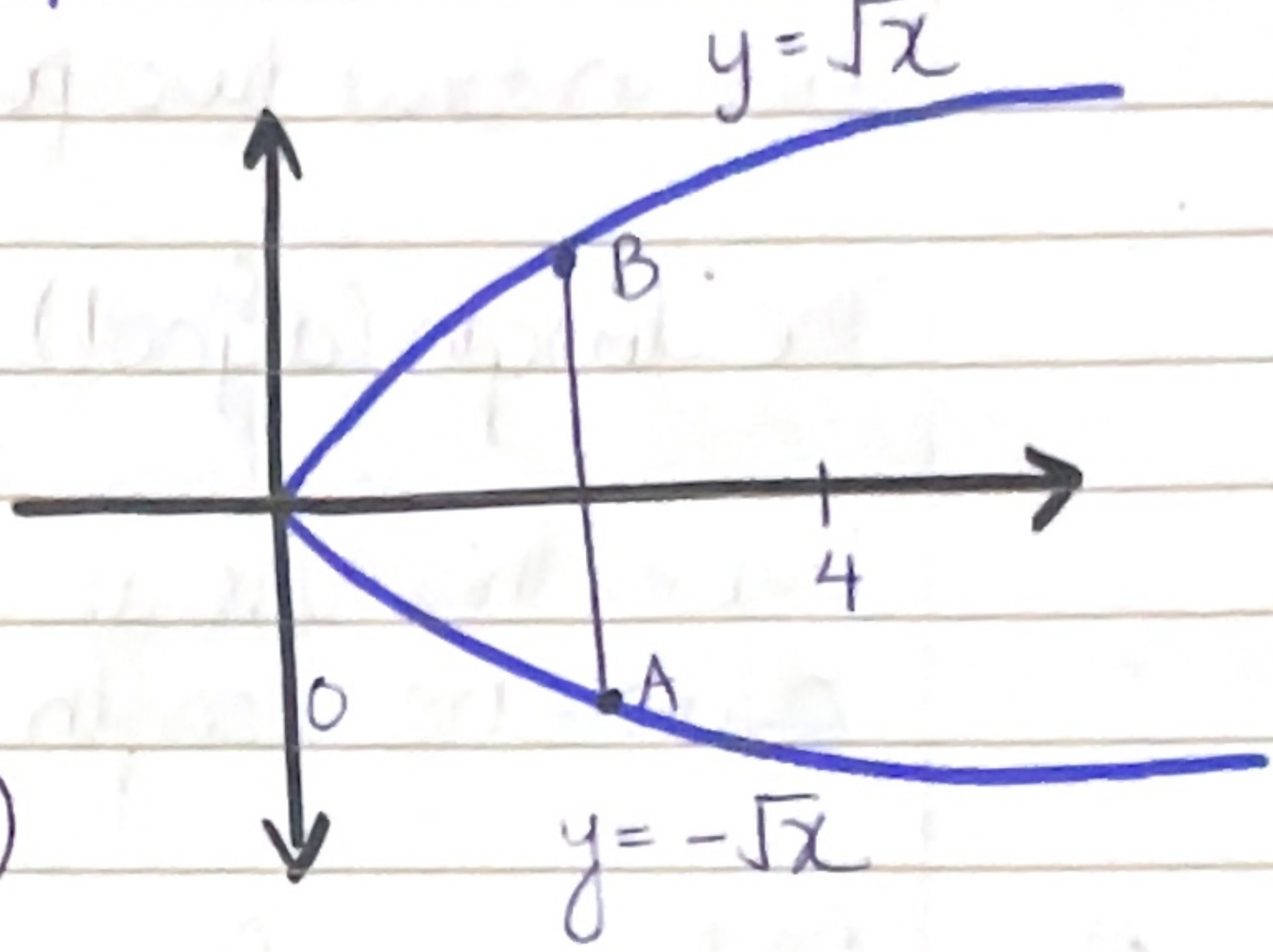
Chapter 6

EXERCISE 6.1

FIND THE VOLUME ;

1.



① Now as given the solid lies between $0 \leq x \leq 4$ and its diagnol length $\Rightarrow \sqrt{x} - (-\sqrt{x})$
$\Rightarrow 2\sqrt{x}$

② Now from this diagnol we need the side of square; eventually to find its volume.

$\sqrt{2}$ sidelength = diagnal length $\Rightarrow$ formula.

side length = $\frac{2\sqrt{x}}{\sqrt{2}} \Rightarrow \sqrt{2x}$

③ Now volume is always equal to the integral of area in an interval. (our shape is square).

$$\int_0^4 (\sqrt{2}\sqrt{x})^2 dx$$

$$\int_0^4 2x \Rightarrow 16 \text{units}^3 \text{ Ans!}$$

Chap 10 → sem-1

Part of both sem 1 & 2

Chapter no. 6

Exercise 6.1 } 1-10 ; 23-36 → sem-2

simply applicable to power series ← Ratio test.

Criteria

$\lim_{n\to\infty} \left| \frac{n+1 \text{ term}}{n \text{ term}} \right|$

Now if

✓ $L < 1$ converges

$L > 1$ diverges

$L = 1$ test not applicable → apply another test.

Remember

(i) $\lim_{n\to\infty} \frac{1}{n} \Rightarrow 0$.

(ii) $\sum_{n=1}^{\infty} \frac{1}{n}$ is a harmonic series i.e divergent

▸ Alternating Series test.

→ this test applies on alternating series.

Alternating Series e.g:

generally indicates if we can use this test

(i) $\sum_{n=1}^{\infty} (-1)^n a_n$

(ii) $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$.

Divergence test / n^{th} term test.

✓ if $\lim_{n\to\infty} a_n \neq 0$ diverges

elsewise it may or may not converge/diverge

Now alternating series test has set conditions which should be met; if met the series converges; else diverges.

(i) $\lim_{n\to\infty} a_n = 0$ [fails divergence test]

(ii) The squence is decreasing; i.e $a_{n+1} < a_n$

Example: $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n}\right)$; check this by alternating series test if it converges or diverges.

Check conditions.

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓ first condition met.

$1/_{n+1} < 1/_n \Rightarrow 1/_2 < 1$ ✓

∴ the 2 condition meet and it converges.

Example; Now check this term.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{5n+3}{2n-7}$$

$\lim_{n \to \infty} \frac{(5n+3)}{(2n-7)} \Rightarrow \frac{5}{2} \neq 0$ $\therefore$ failed the first condition and this is a divergent series.

P-series test!

If a series is in the form of

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

then this power 'p' can tell us.

if:

(i) $p > 1$ it will converge.

(ii) $p \leq 1$ it will diverge.

Abosulte Convergence test!

If we take the an term of a series and take its moduls with a limit → say this convergence.

Then we conclude than the original series convergence as well.

if $\sum_{n=1}^{\infty} |a_n|$ converges. then $\sum_{n=1}^{\infty} a_n$ converges as wel

Geometric series test

$$\sum_{n=1}^{\infty} ar^{n-1}$$

now if $|r| < 1$

↳ converges

$|r| \geq 1$ → diverges.

6. Find interval of Conv of power series & its radius.

$\sum_{k=0}^{\infty} k! (x-1)^k$.

As the power series converges;
by the not nth term test; ratio test.

$$\lim_{k \to \infty} \left| \frac{(k+1)! (x-1)^{k+1}}{k! (x-1)^k} \right| \Rightarrow \frac{(k+1) \cancel{k!} (x-1)^{k+1-k}}{\cancel{k!} (x-1)^0}$$

$$\Rightarrow \frac{(k+1)(x-1)}{1} \Rightarrow |x-1| \lim_{k \to \infty} (k+1) < 1$$

Now as the limit $k \to \infty$ we need to stop it and only $x = 1$ can do it to let our power series be convergent.

$\therefore$ the interval of convergence is reduced to only one value $x = 1$

and the radius of interval is 0; as their is only one point.

1.

$\Rightarrow \sum_{k=1}^{\infty} (-1)^k \frac{1}{k(k+1)}$ ✓

$\Rightarrow \sum_{k=1} 1 = \infty$

here we can split the series into two to check the convergence.

$\sum_{k=1}^{\infty} (-1)^k \frac{1}{k}$, $\sum_{k=1}^{\infty} (-1)^k \frac{1}{k+1}$ ✓

$\Rightarrow$ Both solved by alternating series test.

$\Rightarrow \sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

$\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right)$ $\rightarrow$ Partial fraction ✓

Expanding this series will give a telescoping series and if that telescoping series yeild a finite answer, it is convergent

~x ~

To cater negativity of the the derived terms x-variable we may adjust the values of our summation. ✓

$\rightarrow$ however x^0 is good

$$\left(1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \cdots\right)$$ → coSinh x's Maclaurin series

$$\left(x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots\right)$$ → sinh x's Maclaurin series

Now the same series with alternating signs are.

$$\left(1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots\right)$$ cos x's Maclaurin series.

$$\left(x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 \cdots\right)$$ sin x's Maclaurin series.

$$\sum_{k=0}^{\infty} \frac{x^k}{k!} \quad \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}\right)$$ e^x Maclaurin Series!

* in the question were we put a defined series just like above ones, we immediately expand and take x^power variables common afterwards!

Pre Finals report.

Physic:- 17^{th} position (avg of class ⇒ 36.35/60)
(my scoring ⇒ 48.5 (+ 18 above avg.)

Chapter no. 5

5.1:-

let a funct f(x) such that

① $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

$f'(x)$ ⇒ derivative &
$F(x)$ ⇒ antiderivative.

② $\int_b^a f(x)\,dx = 0$ such that $a=b$

$F'(x) = f(x)$.

③ $\int_b^a f(x)\,dx = \int_b^c f(x) + \int_c^a f(x)\,dx$

* to get the lower. we take sum of rectangle (with height the lower value of x as the base interval; in the lower value of x is used in the funct to get the value of y.

* As the Area in 5.1 is calculated using rectangles,
in case 1: we were given info of lower & upper sum to find the values of y.

in case 2: when no. such info is given, we use the midpoint of set intervals (from the no. of $\square$) as the value of 'y' ⇒ Midpoint Rule.

5.2:-

* forming summation series.

→ $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}$.

$\sum_{k=1}^{5} (-1)^{k+1} \left(\frac{1}{k}\right)$ Ans!

this factor gives -1 to even & +1 to odd.

→ $-\frac{1}{5} + \frac{2}{5} - \frac{3}{5} + \frac{4}{5} - \frac{5}{5}$

$\sum_{k=1}^{5} (-1)^{k} \left(\frac{k}{5}\right)$

& this gives -1 to odd & 1 to even.

① $\sum_{k=1}^{500} 7 \Rightarrow 500 \cdot 7 = 3500$ Ans!

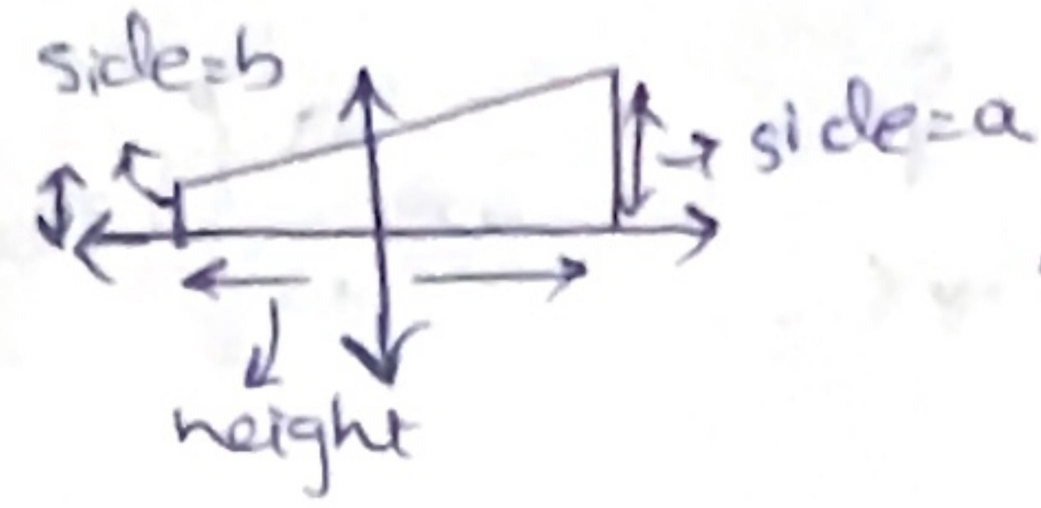
③ $\sum_{k=3}^{17} k^2$

$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{n}$.

② $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

* conversion changes k & n also (important).

* area of trapizum



$A = \left[\frac{a+b}{2} \cdot h\right]$

5.4:-

① $F(x) = \int_b^a f(x)\,dx = F(a) - F(b)$. where a>b.

② Average through integral $\Rightarrow \frac{1}{b-a}\int_a^b f(x)\,dx$ where b>a.

③ $F'(x) = \frac{d}{dx}\left(\int_0^x f(x)\,dx\right) = f(x)$. where x should not be in any other form and lower limit should be const.

else use chain Rule.

$F'(x) = \frac{d}{dx}\int_a^{g(x)} f(t)\,dt$

$f(g(x)) \cdot g'(x)$

$\Rightarrow$ very imp result. for derivative of integral.

Chain Rule (changing limits is imp).

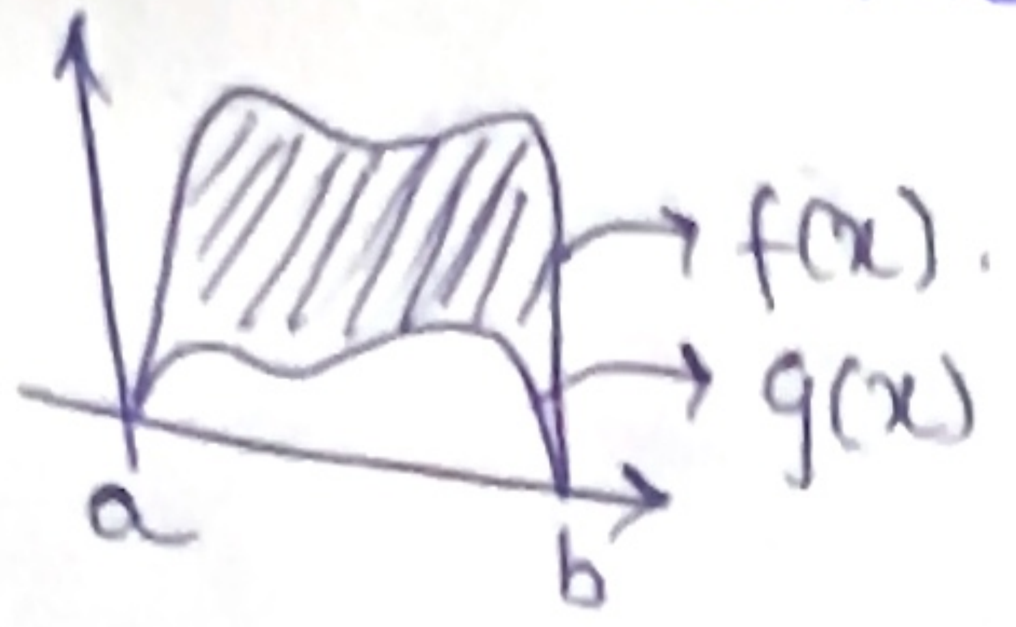
Break limit to satify modulus signs of each its terms.

5.4 Area:-

Break the given interval through the roots of funct and find defined integrals.

·6 Area b/w the curves.

(i)

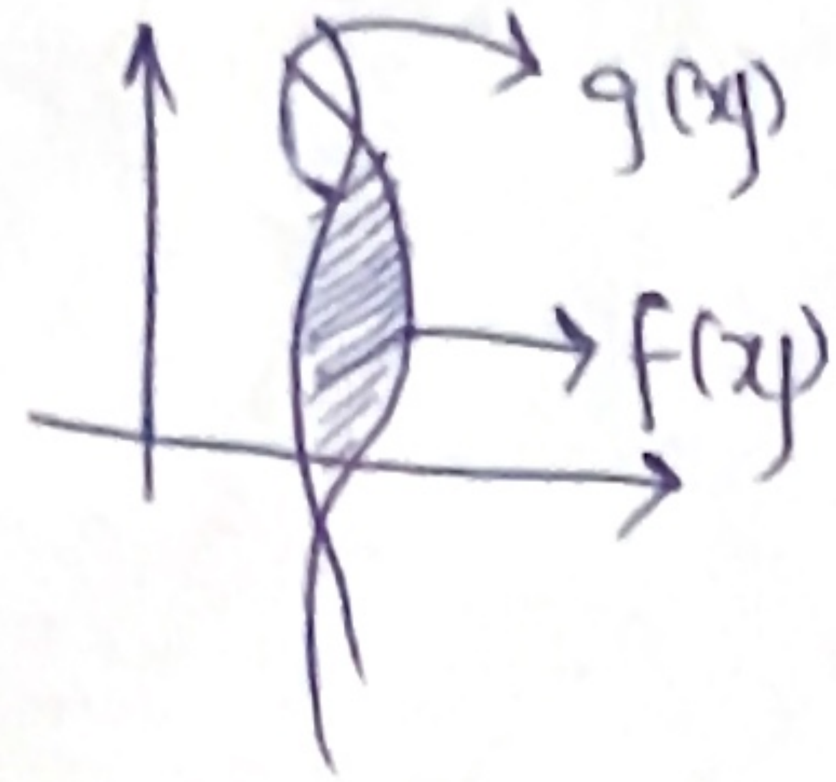


depends upon the value of y ↑

$$\int_a^b f(x)-g(x)\,dx. \quad \because f(x) > g(x)$$

where if the intervals a to b are not given; could be found by solving simultaneously.

(ii)



$$\int_a^b f(y)-g(y)\,dy.$$

depends up on the values of x.

$f(y) \geq g(y)$

Even & odd functs.

not odd.

For the funct that is even at $[-a, a]$.

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx.$$

symmetrical about the origin.

* Definition of Modulus.

$$|x^2-4| = \begin{cases} x^2-4 & x \geq 0 \\ -(x^2-4). & x < 0 \end{cases}$$

Chapter no.6.

6.3:-

equ. of circle $x^2+y^2=r^2$

$x^2+y^2=s^2$

length of curves => arc length =>

integral for length =>

$$\int_a^b \sqrt{1+(y')^2}\,dx.$$