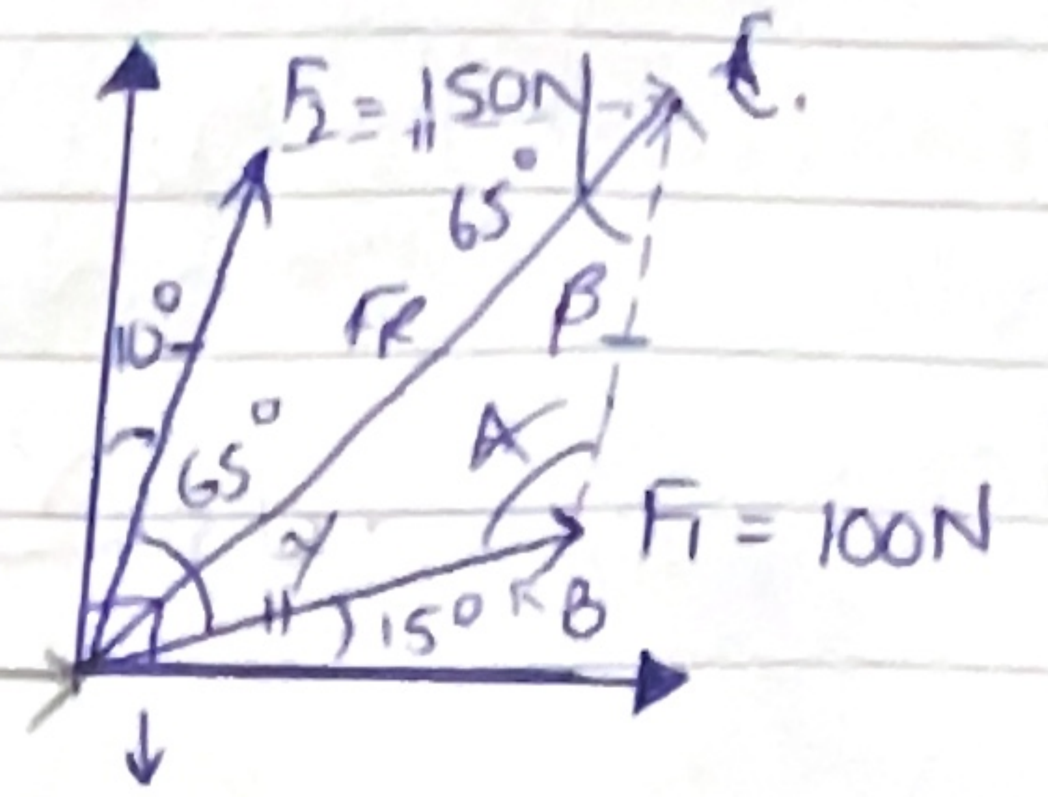
(14th Edition solution Manual)

Date: 5/9 2023 M T W T F S S

RC HIBBLER ENG MECHANICS - STATICS

PHYSICS:-

VECTORS:

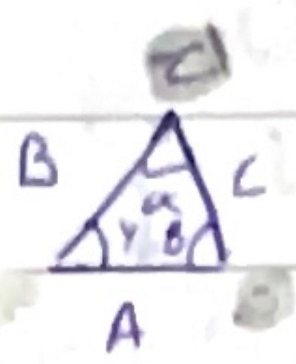


EXAMPLE 2.1:-

Resultant force & direction = ?

law of sine:-

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$



$90° = 10° + 15° + Z$

$Z = 65°$.

★ in a closed four sided figure the sum of angles = 360°. —(i)

$\Rightarrow 65° + 65° + A + A' = 360°$

$130 + 2A = 360°$

$A = 115°$

Now α is known by (i).

law of cosine.

$A^2 = B^2 + C^2 + 2BC\cos\alpha$.

$A^2 = (100)^2 + (150)^2 + 2(100)(150)\cos 115°$.

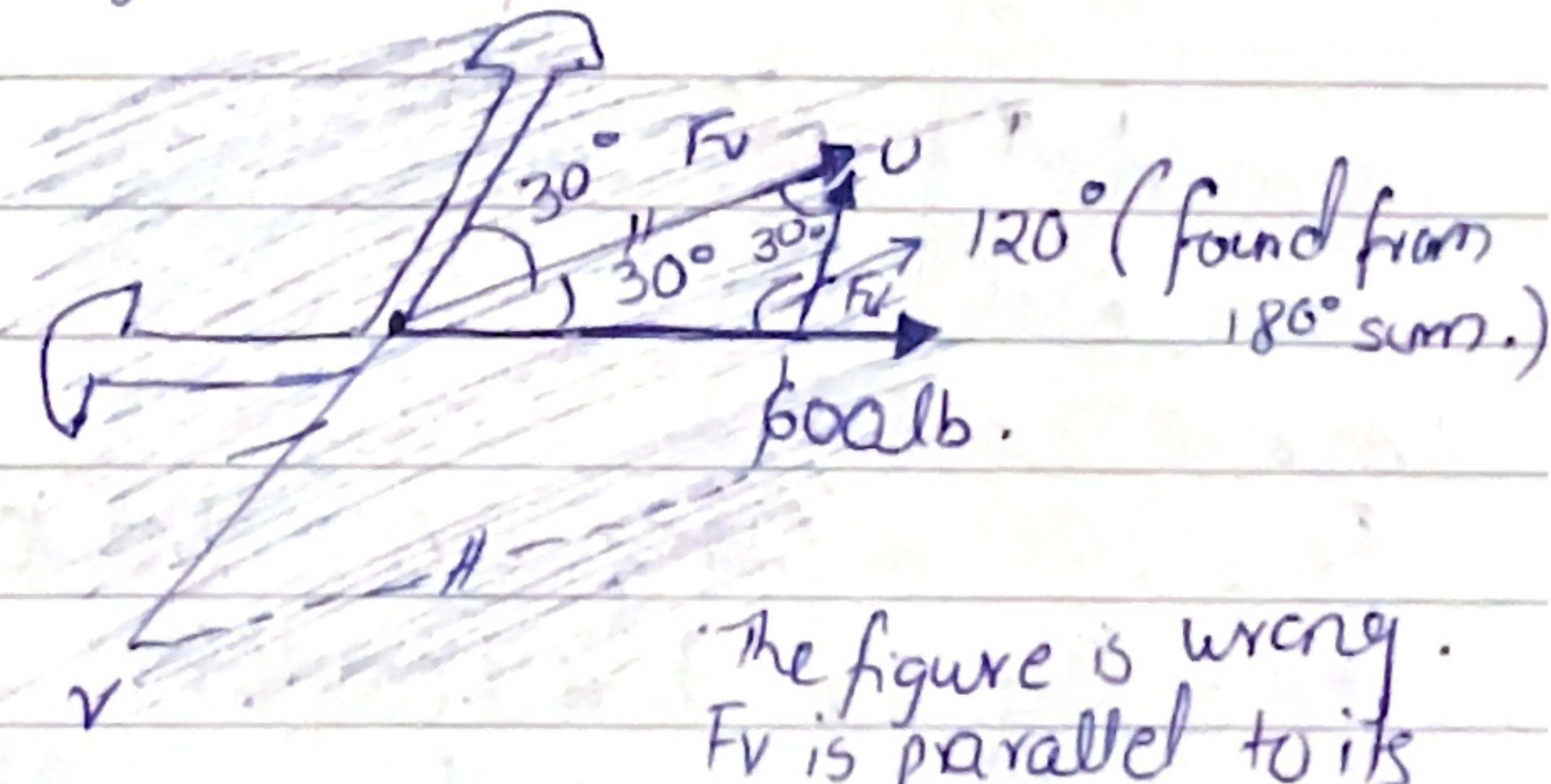
$F_R / A = 213N$.

Now for direction.

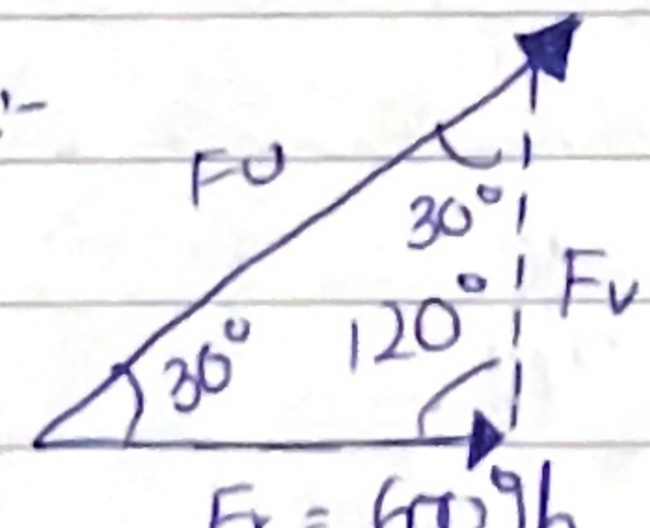
$$\frac{A}{\sin\alpha} = \frac{C}{\sin\gamma} \Rightarrow \frac{215}{\sin 115°} = \frac{150}{\sin\gamma} \Rightarrow \gamma = 39.8°$$

So the Resultant makes an angle of $39.8° + 15°$ with the horizontal.

Example 2.2 :-



Final figure :-



Now simply use law of sine twice

The figure is wrong. Fv is parallel to its another that starts above x-axis.

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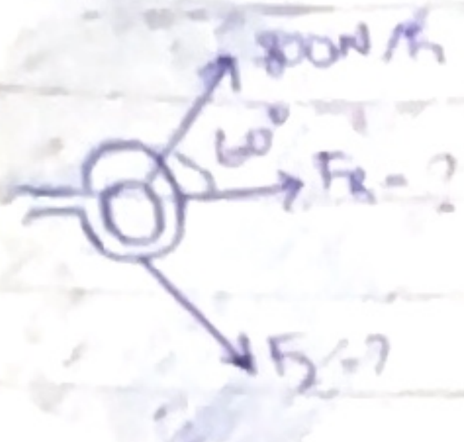
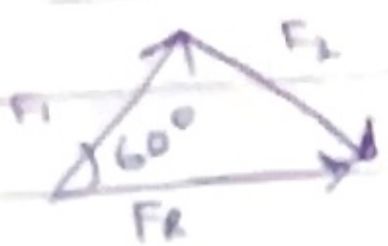
EXAMPLE 2.4 :-

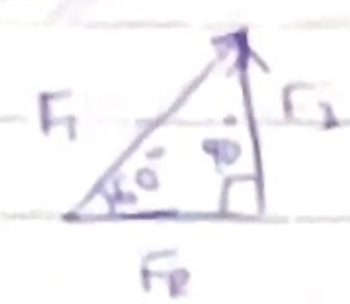
In this case we see that resultant force F_2 & its component i.e F_2 force itself are not specified. therefore we can arrange F_2 in accordance to our desire.



→



→ maybe the force F_2 is 90° to the resultant as it is not specified.



now F_2 acts as a resultant connecting both F_R & F_1 by head to tail Rule.

F_R = ? = $F \cos\phi$ = $800 \cos 60$ = 400N

F_2 = ? = $F \sin\phi$ = $800 \sin 60$ = 693N

FUNDAMENTAL PROBLEMS:-

F:2-1

$60° + 45° + Z = 180°$

$105 = 180° - Z$

$Z = 75°$

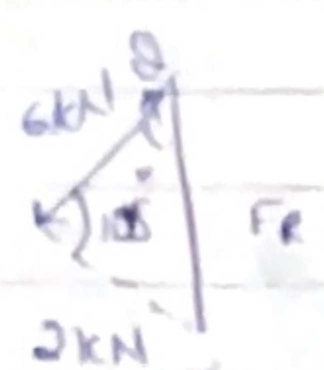
$2X + 75 + 75 = 360°$

$360 - 150° = 2X$

$\frac{210}{2} = X$

$X = 105$

→



LAW OF COSINE:-

$F_R = (6)^2 + (2)^2 - 2(6)(2)\cos 105°$

$F_R = 36 + 4 - 24\cos 105°$

$F_R = 40 - 24\cos 105°$ ⇒ 8.7N.

LAW OF SINE

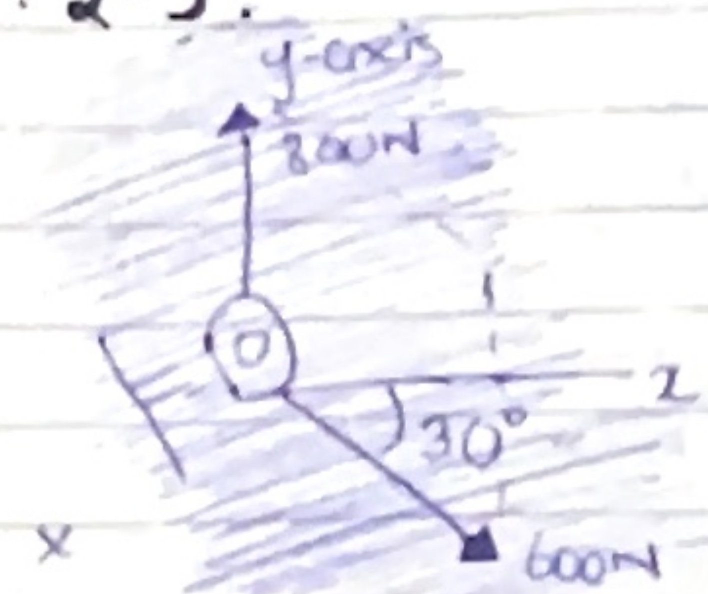
$\frac{F_R}{\sin 105°} = \frac{2kN}{\sin\phi}$ ⇒ $\sin\phi = \frac{2kN \times \sin 105°}{8.7kN}$

$\sin\phi = \frac{1.931}{8.7} = 0.289°$

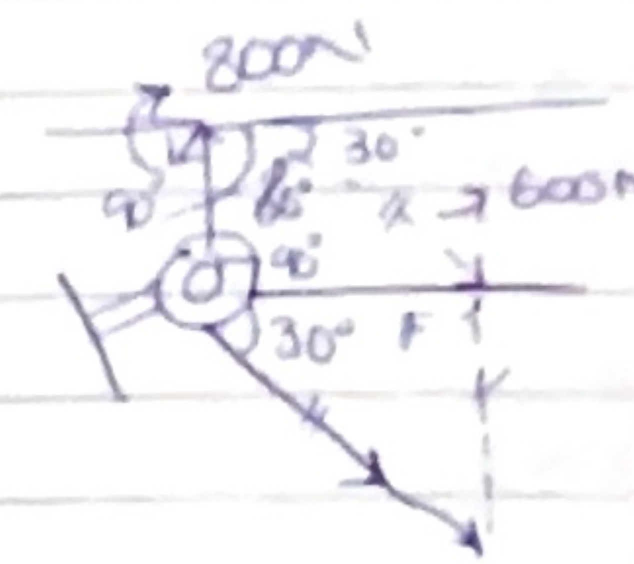
$\phi = 58.5°$

hence the direction of the required resultant is $45° + 58.5$ ⇒ $103°$ Ans!

F: 2-3.



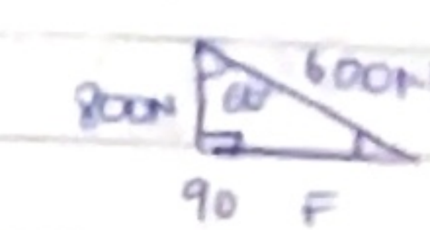
A little wrong diagram



$\cos\phi = \frac{B}{H}$

$\sin\phi = \frac{P}{H}$

$\sin\phi = \frac{800}{600}$ ⇒ $\frac{4}{3}$



$\frac{F}{\sin\phi} = \frac{600}{\sin 90°}$

$\frac{F}{\sin 60°} = \frac{600}{1}$

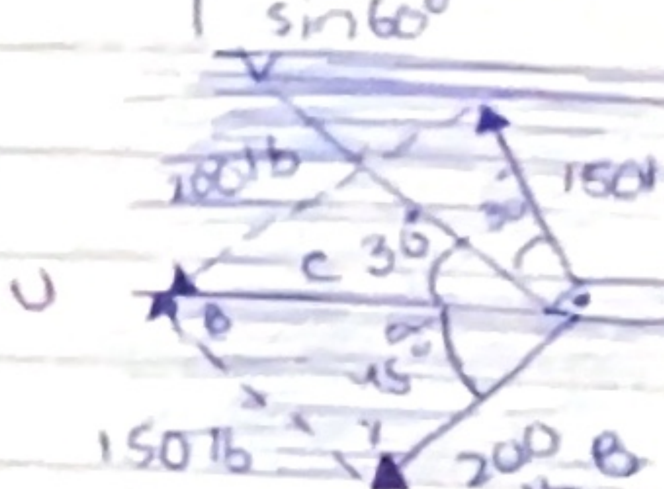
$F = \sqrt{A^2 + B^2 - 2AB\cos\phi}$

$F = \sqrt{(800)^2 + (600)^2 - 2(800)(600)\cos(60)}$

$F = 712N$. Ans!

$F = \sin 60°$

PROBLEMS:- 2-4.



$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta}$ ⇒ $\frac{150}{\sin 45} = \frac{200}{\sin\beta}$.

$\sin\beta = \frac{200}{150} \times \sin 45$ ⇒ $\sin^{-1}\left(\frac{4}{3}\sqrt{\frac{1}{2}}\right)$ ⇒ $70.5° = \beta$

$\gamma = 64.5°$

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$\gamma = 64.5$ (By triangular property).

$\frac{B}{\sin\beta} = \frac{C}{\sin\gamma} = \frac{A}{\sin\alpha}$.

$\frac{A}{\sin\alpha} = \frac{C}{\sin\gamma}$

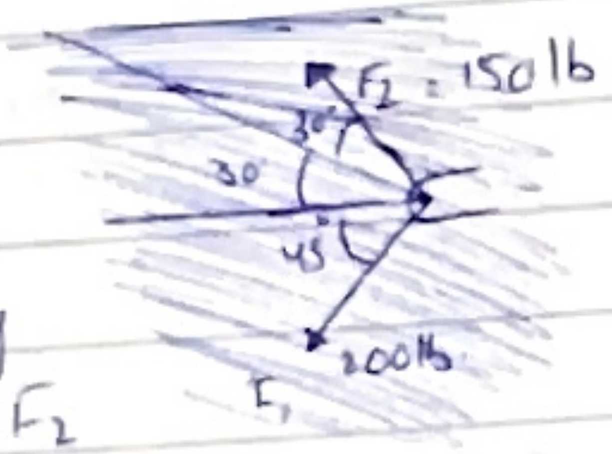
$\frac{150}{\sin 45} = \frac{C}{\sin(65)}$

$C = \frac{150}{\sin 45} \times \sin(65)$

$C = 192.25$ Answer! N

2-5 ★

we can simply resolve F_1 & F_2 as the angles are already given.



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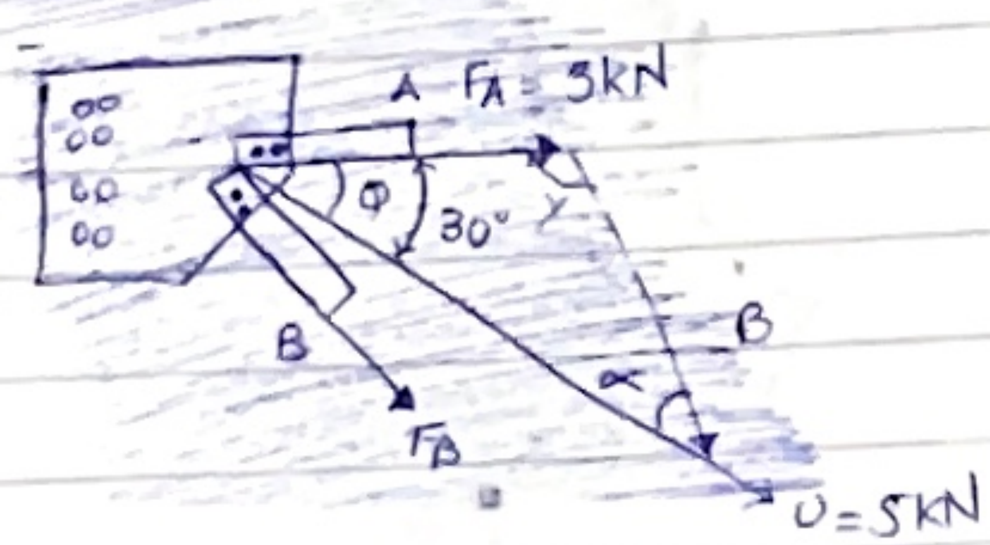


$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} \Rightarrow \frac{A\sin\beta}{B} = \sin\alpha$. or A, B, C / α, β, γ

$\alpha/\beta = \sin^{-1}\left(\frac{B}{A}\right)$ or $\frac{A}{\sin\alpha} = \frac{C}{\sin\gamma} \Rightarrow \sin\gamma = \frac{C\sin\alpha}{A}$

$\alpha =$

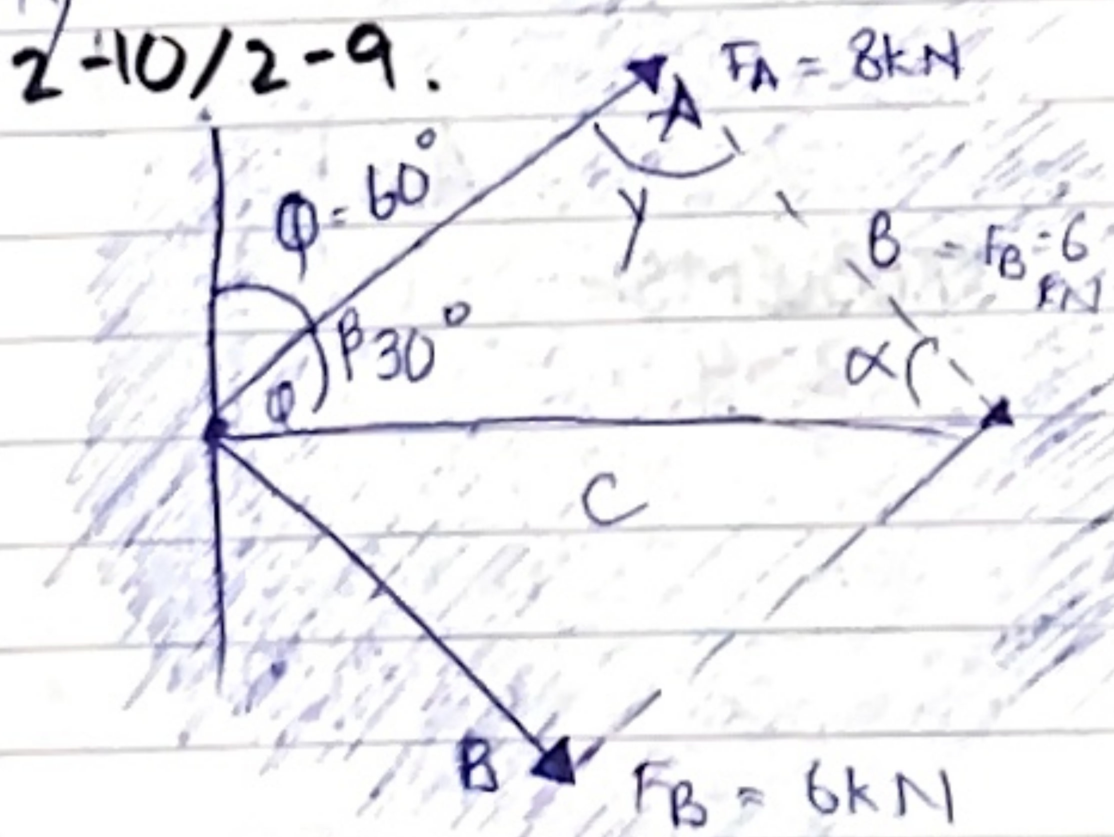
$B^2 = A^2 + C^2 - 2AC\cos\beta$.

$B^2 = (3)^2 + (5)^2 - 2(3)(5)\cos 30$

$B^2 = 9 + 25 - 30\cos 30$

$B = 2.8$ N

2-10/2-9.



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Now using triangular properties.

$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta}$. $\sin\alpha = \frac{8}{12} \Rightarrow \frac{2}{3}$

$\frac{8kN}{\sin\alpha} = \frac{8kN}{\sin(30°)}$ $\alpha = 41.8°$

$\frac{8kN}{\sin\alpha} = \frac{6kN}{1/2}$ $\alpha + \beta + \gamma = 180°$

$\frac{8}{\sin\alpha} \Rightarrow = 12$ $\gamma = 180° - 30° - 41.8°$

$\gamma = 108.1°$

—x—

$\frac{C}{\sin\gamma} = \frac{A}{\sin\alpha} \Rightarrow C = \frac{8 \times \sin(108.1)}{\sin(41.8)} = 11.4$ kN Ans|

PHYSICS:- "The mother of all sciences"

The study of the behaviour and the structure of matter, energy and interaction b/w matter and energy.

- MECHANICS
  - KINEMATICS (focus on the motion of the object)
  - KINETICS (focus is on the cause of motion)
- THERMO DYNAMICS
- ELECTRICITY

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* unit vectors are dimensionless quantities with magnitude 1.

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/Division

- Multiplication of a vector by a scalar can increase or decrease its magnitude respectively.

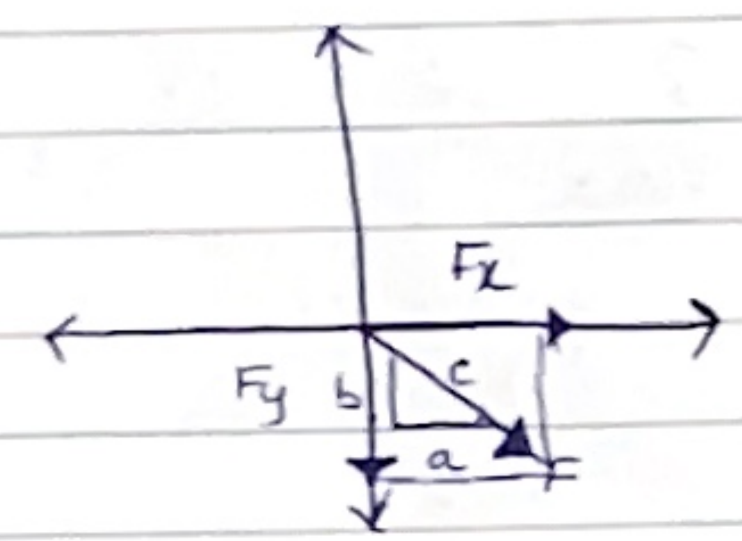
RESOLUTION OF FORCES!

- Resolution of vector along x-axis and y-axis is known as the resolution of components.

$F_x = F\cos\phi \qquad F_y = F\sin\phi$

SIMILAR TRIANGLE METHOD:-

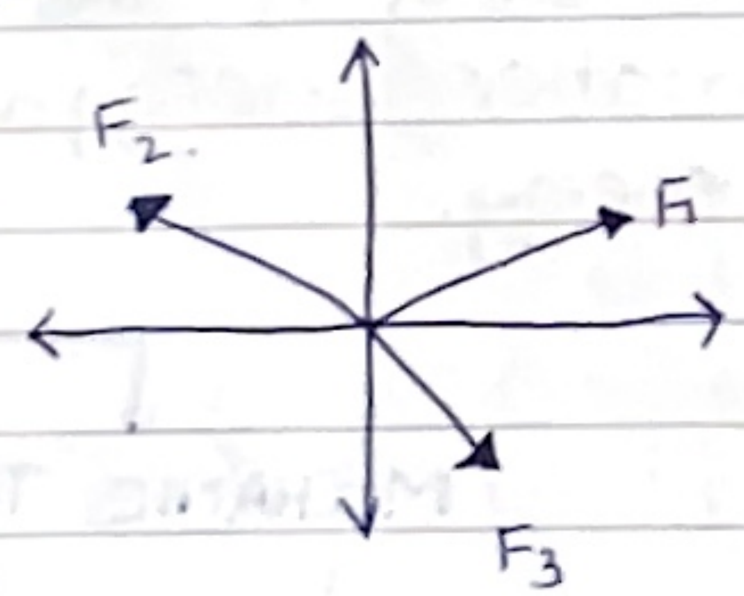
- By the similar triangle method we can omit the use of theta in a question.



$\frac{F_x}{F} = \frac{a}{c} \qquad \frac{F_y}{F} = \frac{b}{c}$

$F_x = F\left(\frac{a}{c}\right) \qquad F_y = -F\left(\frac{b}{c}\right)$

ADDITION OF COPLANAR FORCES:-



$F_1 = F_{1x}i + F_{1y}j$
$F_2 = -F_{2x}i + F_{2y}j$
$F_3 = F_{3x}i - F_{3y}j$

$F_R = F_1 + F_2 + F_3$

$F_R = \sqrt{(F_{Rx})^2 i + (F_{Ry})^2 j}$

$F_{Rx} = F_{1x} - F_{2x} + F_{3x} = \Sigma F_x$

$F_{Ry} = F_{1y} + F_{2y} - F_{3y} = \Sigma F_y$

$\tan\phi = \frac{F_{Ry}}{F_{Rx}} \quad ; \quad \phi = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right)$

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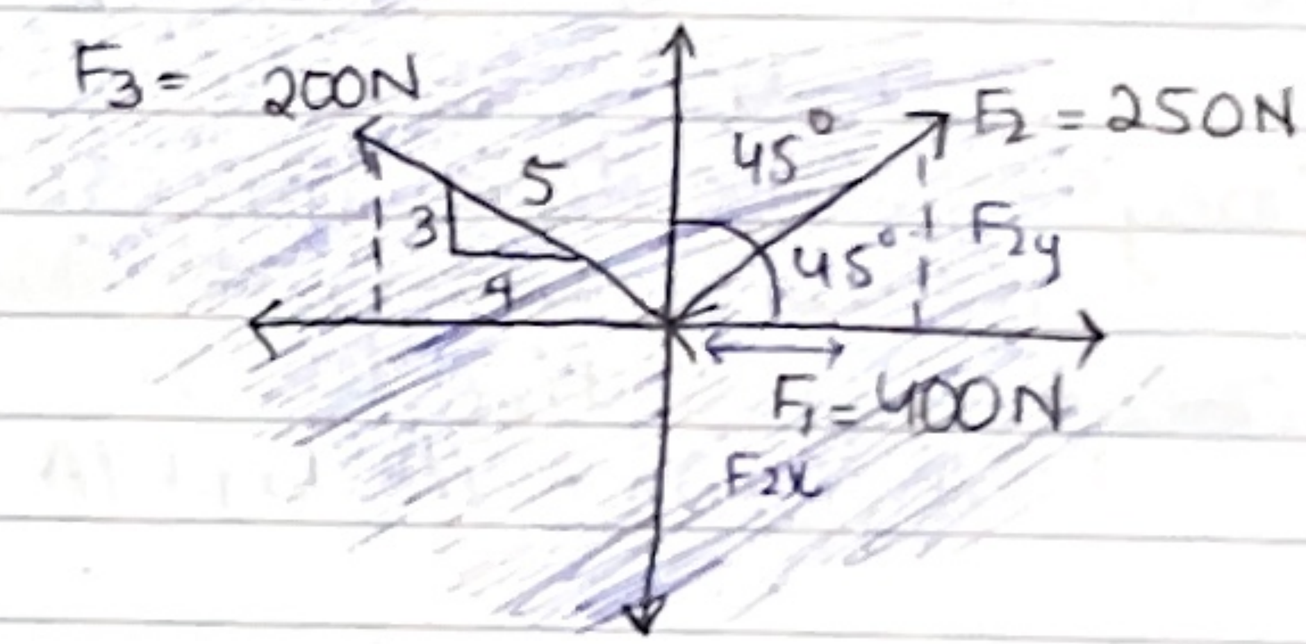
QUESTION NO (CLASS):-

The end of the boom O in Fig is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of resultant.

(Parallelogram Method can also be used.)



$F_1 = 400N$
$F_{1x} = 400N$
$F_{1y} = 0N$

$F_{2x} = F_2\cos\phi$
$= 250\cos 45$
$= +176.77N$

$F_{2y} = F_2\sin\phi$
$F_{2y} = 250\sin 45$
$= 176.77N$

$\frac{F_{3x}}{F} = \frac{4}{5} \qquad \frac{F_{3y}}{F} = \frac{3}{5}$

$F_{3x} = 250\left(\frac{4}{5}\right)$
$= -200N.$

$F_{3y} = 250\left(\frac{3}{5}\right)$
$= 150N$

$F_{Rx} = F_{1x} + F_{2x} + F_{3x}$
$= 400 + (176.7) + 200\ N$

$F_{Ry} = F_{1y} + F_{2y} + F_{3y}$
$0 + 176.77 + 150$

$F_{Rx} = 376.77N$

$F_{Ry} = 326.77$

$F_R = 498.1N.$

$\phi = \tan^{-1}\left(\frac{326.77}{376.77}\right) = 40.9°$ Ans!

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DOT PRODUCT:- (Scalar Product).

$\bar{A}.\bar{B} = AB\cos\phi$ where $0° \leq \phi \leq 180°$

(Vector).(Vector) = Scalar (it is simple Multiplication.)

(i) Obeys Law of Commutativity.

$\bar{A}.\bar{B} = \bar{B}.\bar{A}$

(ii) Obeys Multiplication with scalar.

$a(A.B) = a(A).B = A.(aB) = (A.B)a.$

(iii) Obeys distributive law.

$A.(B+D) = (A.B) + (A.D)$

Unit Vectors (w.r.t Dot product).

$i.i = (1)(1)\cos(0°)$; $j.j = 1$; $k.k = 1.$

$= 1$

$i.j = (1)(1)\cos(90)$ $\quad k.j = (1)(1)\cos 90°$

$= 0$ $\quad = 0$

$i.k = (1)(1)\cos(90)$

$= 0$

Example: $A.B = (A_x i + A_y j + A_z k).(B_x i + B_y j + B_z k)$

$A_x B_x (i.i) + A_x B_y (i.j) + A_x B_z (i.k)$
$A_y B_x (j.i) + A_y B_y (j.j) + A_y B_z (j.k)$
$A_z B_x (k.i) + A_z B_y (k.j) + A_z B_z (k.k)$

$\Rightarrow A_x B_x (1) + A_y B_y (1) + A_z B_z (1)$
Ans.

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TICK

inst acc = $\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$ inst velocity = $\lim_{\Delta t \to 0} \Delta r/\Delta t = \frac{dv}{dt}$

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POSITION VECTOR:-

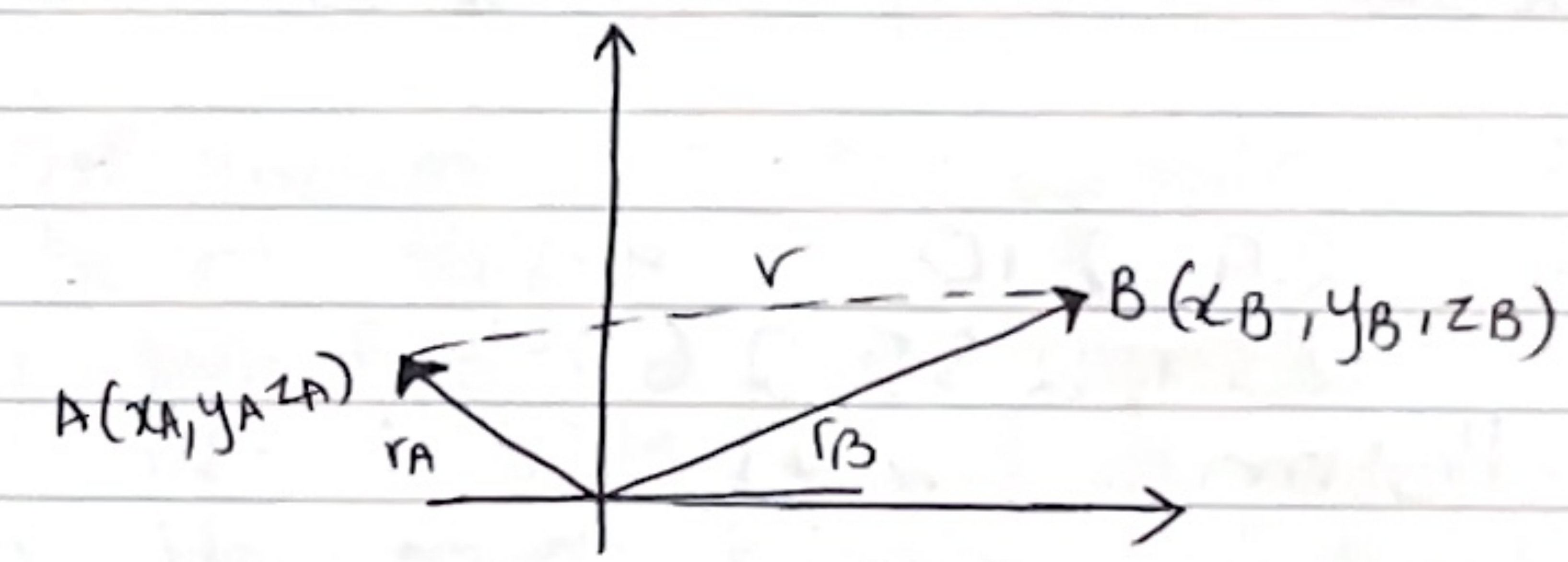
(extends from the origin of a coordinate to particle)

A position vector (r) is defined as a fixed vector which locates a point in space relative to another point.

$r = xi + yj + zk$

Example:-

Consider the following graph:-



$r = r_B - r_A$

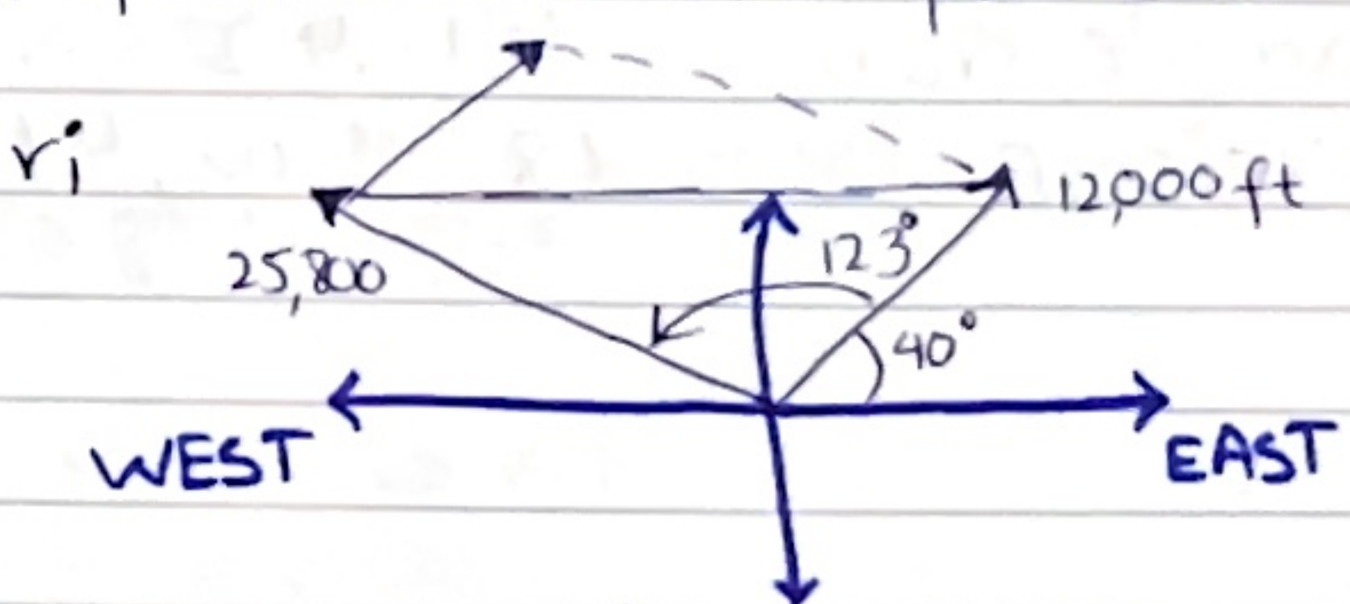
$= (x_B i + y_B j + z_B k) - (x_A i + y_A j + z_A k)$

$= (x_B - x_A)i + (y_B - y_A)j + (z_B - z_A)k$

$\Rightarrow$ Displacement from a given point can also Be give by diff of final and initial positions.

i.e $\Delta r = r_f - r_i$

QUESTION:-



SIMPLE QUESTION

QUESTION:-

speed = 1300 km/h. $\quad$ 43° $\quad x$ $\quad$ 35 y. $\quad \tan\phi = y/x$

$1300 \times \frac{1000}{3600} = 361.1$ $\quad x = \frac{y}{\tan(43)}$

$t \Rightarrow \frac{465}{361} \Rightarrow 1.28$ sec

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TICK

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• A motion of a body can be described by mathematical equations and graph.

—— x ——

R. Resnick

PENDING PRACTICE PROB(S):-

Sample prob 2.4, 2.5, 2.6, 2.7, 2.8 ✓
Exercise 2.16, 2.23, 2.24, 2.29, 2.30 ✓
2.41, 2.47, 2.50, 2.55, 2.60 ✓
End of chap Problems 2.14, 2.17, 2.21. ✓

Sample prob 2.9, 2.10 ✓
Exercise 2.54, 2.55, 2.60 ✓
End of Chap Problem 2.21. ✓

Sample prob 3.1 - 3.7 ✓
Ex: ✓3.2, 3.5, 3.9, 3.11, 3.21, 3.27, 3.29, 3.30
Problem: 3.7, 3.8, 3.10 ✓

Sample Prob 4.1, 4.2 ✓
Exercise 4.8, 4.10, 4.12 ✓

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This is placed if we are solving with some specific initial displacement. $S = Y_0 + V_0 t - \frac{1}{2} g t^2$

FREELY FALLING BODIES:-

$V_y = V_{oy} - gt$

$Y = Y_0 + V_{oy} t - \frac{1}{2} g t^2$

total displacement; initial displacement; initial velocity (Y axis).

A ball thrown straight up takes 2.25s to reach a height 36.8m.
(a) What is the initial speed.
(b) What is its speed at this height?
(c) How much higher will the ball go?

$S = V_i t + \frac{1}{2} a t^2$

$V_f \neq 0$ (not specified)
$t = 2.25$
$H = 36.8$

$S = V_i t - \frac{1}{2} g t^2$

(c) $2aS = v_f^2 - v_i^2$

$S = V_i t - \frac{1}{2}(9.8)(2.25)^2$

(b) $V_f - v_i = at$

$36.8 = V_i t - \frac{1}{2}(9.8)(2.25)^2$

$36.8 + \frac{1}{2}(9.8)(2.25)^2 \Rightarrow V_i t$.

$61.6 = V_i t$

$\frac{61.6}{2.25} = V_i$

$V_i = 27.37 m/s$

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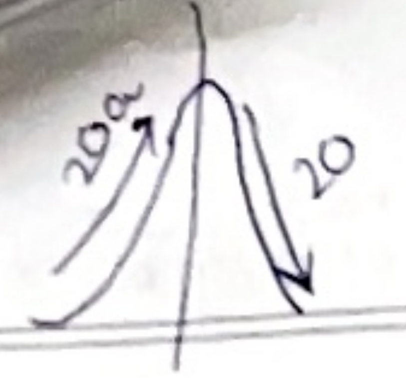

"WEEK-3."

MOTION OF 3-DIMENSIONAL CONST. ACCELERATION

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

If the particle starts at $t=0$;

intial position $\Rightarrow$ $r_0 = x_0\hat{i} + y_0\hat{j} + z_0\hat{k}$
intial velocity $\Rightarrow$ $v_0 = v_{0x}\hat{i} + v_{0y}\hat{j} + v_{0z}\hat{k}$

Now; if $\vec{a}$ is constant it implies.
a_x, a_y & a_z are all constant.

$$\left.\begin{aligned} v_x &= v_{0x} + a_x t \\ v_y &= v_{0y} + a_y t \\ v_z &= v_{0z} + a_z t \end{aligned}\right\} \text{const} \Rightarrow \vec{v} = \vec{v_0} + \vec{a}t$$

Similarly the position of the body is also const.

$$\left.\begin{aligned} x &= x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\ y &= y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \\ z &= z_0 + v_{0z}t + \frac{1}{2}a_z t^2 \end{aligned}\right\} r = r_0 + v_0 t + \frac{1}{2}at^2$$

Newton's Law in 3-dimensions:-

$\Sigma F_x = ma_x$
$\Sigma F_y = ma_y$
$\Sigma F_z = ma_z$

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$S_y = (v_{iy})(t) + \frac{1}{2}a_y t^2$
$S_y = +\frac{1}{2}(2.235)(15)^2$

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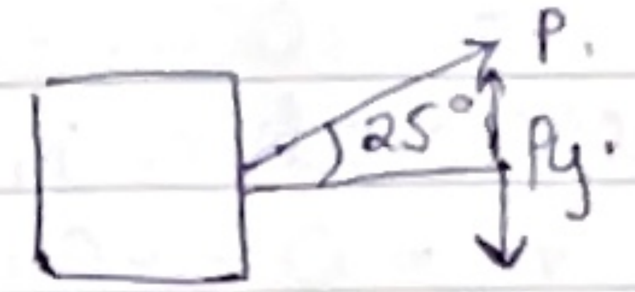
PROBLEM:- (i)

Force applied = 12N
mass = 5.1kg.
$\phi = 25°$



$P_y = P\sin\phi$
$\Rightarrow 5.07N$.

(i) a = ? (as the body is at horizontal)
$F = ma$.
$a = \frac{F_x}{m} \Rightarrow$ ~~2.35~~ ms^{-2}

$a = 0.99\,ms^{-2}$

(ii) Force at the block just before it is lifted
(at this point to lift the block the vertical component should be equal to weight) $\Rightarrow P\sin\phi = mg$.
(ii) Acceleration at the block before it is lifted.

before the block is liftedoff it is following the horizontal path and $\therefore$ it has $2.13\,ms^{-2}$ acceleration.

PROBLEM (ii)

mass $\Rightarrow$ 8.5kg.
$v_{ix} \Rightarrow 42\,ms^{-1}$
$v_{iy} \Rightarrow 0$.
$F_y = 19N$
$v_{fx} = ?$
$x/S \Rightarrow ?$
$t = 15s$.
$\phi = 90°$

$\frac{v_f - v_i}{t} = a$ $\quad v_f \Rightarrow at + v_i \Rightarrow 75.45\,ms^{-1}$.
$v_f \Rightarrow (2.23)(15) + (42)$

$F_y = F\sin\phi$

$19 = F\sin 90°$
$\frac{19}{\sin 90} = F$
$F = 19N$.

$F = ma$.
$a = \frac{F}{m} \Rightarrow \frac{19}{8.5} \Rightarrow 2.235\,ms^{-2}$

—x—

$S_x = v_{ix}t + \frac{1}{2}a_x t^2 \Rightarrow$ or $S = v_x t$
$S_x \Rightarrow (42)(15) \Rightarrow 630m$.

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$[m_1 a_1 = m_2 a_2]$
analysis relationship.

$S = \frac{1}{2}at^2$

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PROBLEM:-

$S = 100$.
acceleration is constant. $(\Delta V = 0) = 2.80ms^{-2}$
$v_i = 0 = v_f = 0$. $\Rightarrow t = ?$ (i)
$(t = 12.2s \Rightarrow S = ?)$ (ii)

$2(S) = at^2 \Rightarrow \sqrt{\frac{200}{2.8}} = t$
$\boxed{t = 8.45}$

$S = \frac{1}{2}at^2$

$S = \frac{1}{2}(2.8)(12.2)^2$

$S = 208.37$ Ans!

"NEWTON'S LAW OF MOTION"

(i) FIRST LAW:-
Everybody continues its state of rest or uniform motion in a straight, such that no external force acts on it.

body at rest, remains at rest.

if the body is moving with const. velocity, it continues to do so, no force is needed to do so.

(ii) SECOND LAW:-

$\vec{a} \propto \vec{F}$
$\vec{a} \propto \frac{1}{m}$

$[\vec{F} = m\vec{a}]$

"mass is a measure of amount of matter in an object."

"The property of a body that determines its resistence to a change in its motion."

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THIRD LAW:-
Newton's third law of motion.
"To every action there is always an equal but opposite reaction."

- Action and reaction occurs in two pairs.

PROBLEM:-

i) Weight $= 26N$
$g = g = 9.8ms^{-1}$

(a) W at $a = 4.6ms^{-2} \Rightarrow ?$
$m = ?$

$W = mg$
$\frac{W}{g} = m \Rightarrow 5.65$ Kg.

$W = mg$
$= (5.65)(4.6)$
$\Rightarrow 26 \, kg\,ms^{-2}/N$.

$F = G\frac{(m_1 m_2)}{R^2} \Rightarrow 0$.

(b) $W = 0$
$m = 5.65kg$.
$\sim x \sim$

$F = ma$
$= (0.45)(3.92)$
$\Rightarrow -1.764 \, kg\,ms^{-2}/N$.
in the negative x-axis.

(ii)
$m = 0.45kg$.
$v_{ix} = 2.8ms^{-1}$
$v_{fx} = 0ms^{-1}$.
$S = 1m$.

$2aS = v_f^2 - v_i^2$.

$a = \frac{v_f^2 - v_i^2}{2(S)} \Rightarrow \frac{(2.8)}{2(1)}$

$a = (-3.92ms^{-1})$

Page #

F_{AB} ON BY

F_{BA} ON BY

$g \to$ hovering $\to 9.8 ms$

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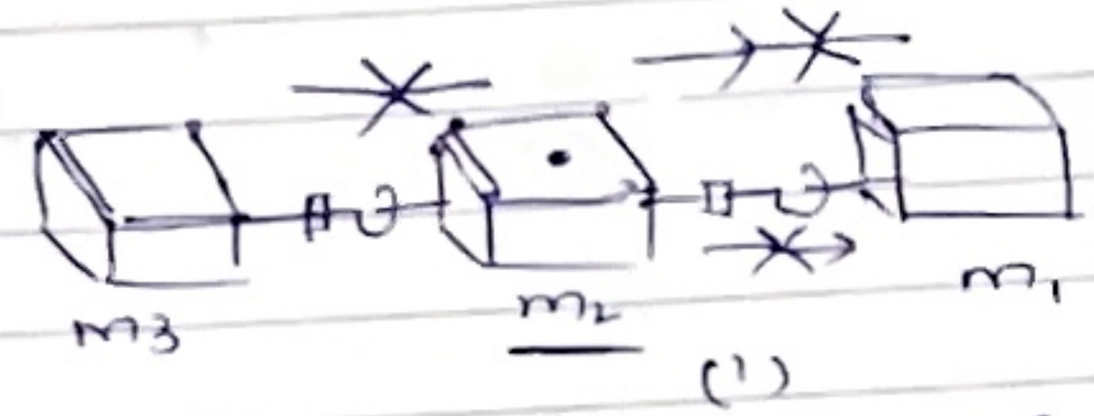
iii) $m = 51000$ kg.
$a_1 = g = 9.8 ms^{-2}$

$a_2 = 18 ms^{-2}$

$F_1 = ma_1$
$= (51,000)(9.8)$
$\Rightarrow 499,800 N$.

$F_2 = ma_2$
$= (51,000)(18)$
$= 918,000 N$

iv)



$m_1 = 3.1$ kg
$m_2 = 2.4$ kg
m_3 1.2 kg

(1)
$F = ma$
$F = (3.1 + 2.4 + 1.2)(a)$
$\frac{6.5}{6.7} = a \Rightarrow 0.97 ms^{-2}$.

(2) $F = ma$
$F = m_3 a$
$F = (1.2)(0.97)$
$F = 1.164 N$.
Ans!

(3)
$F_{net} = m_2 a$
$a = (0.97)$
$m_2 = 2.4$ kg

$2aS = v_f^2 - v_i^2$
$a = \frac{v_f^2 - v_i^2}{2S}$

$F = ma$.
$e = 9.11 \times 10^{-31}$ kg.

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Chapterno. 4.

PROJECTILE:-
A 2dimensional ideal motion, in which noresistive factor involves and is under the influence of gravity and inertia.

Trajectory:- the path followed by the projectile motion.

Velocity:-

Total velocity final $\Rightarrow \sqrt{v_f(x)^2 + v_f(y)^2} \Rightarrow$ inst velocity.

Horizontal Component x

The horizontal component of velocity remains const throughout. (V_x = const).

$\boxed{V_{ix} = v_f(x) = V_i \cos\phi \text{ or } = V_f \cos\phi}$

Why=?
$a_x = 0$ (There is no acceleration along the x-axis).

$\boxed{V_{fy} = V_{iy} - gt \quad V_{fy} = V_i \sin\phi - gt}$ final velocity at the vertical component Yaxis.

Vertical Component of velocity (V_y) x

The vertical component is variable and controlled by gravity

$\boxed{V_{iy} = V_i \sin\phi. \quad a_y = -g.}$

at the highest point the velocity is equal to the horizontal component
$V = V_x = V_i \cos\phi$.
$= V_f \cos\phi$.
- at only max height velocity and g are $\perp$.

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TICK

The velocity and gravity are never equal i.e parallel.
The K.E is $\frac{1}{2}$ of its max at $\theta = 45°$.

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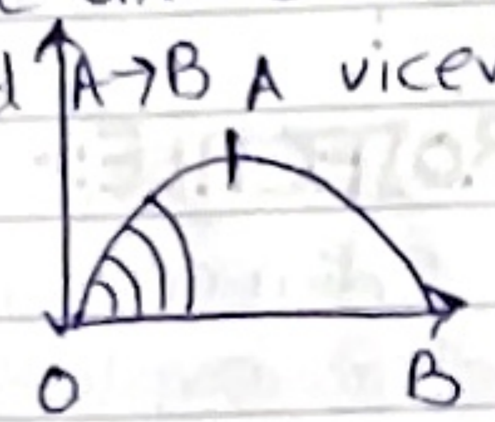
ENERGY FORMULAS:-

From O→A angle with x-axis decreases and A→B A viceversa.



$K.E = K.E_{max} \cos^2\theta$

↓ (At point A) ↓ (at point O & B)

↑ ↑

$P.E = K.E_{max} \sin^2\theta$

also; $P.E = E_T \sin^2\theta$.

HEIGHT:-

$H = \frac{v_i^2 \sin^2\theta}{2g}$.

RANGE:-

$R = \frac{v_i^2 \sin 2\theta}{g}$. $\left(= v\sqrt{gR}\right)$

TIME:-

$T = \frac{2 v_i \sin\theta}{g}$.

DISTANCE :-

HORIZONTAL COMPONENT

VERTICAL COMPONENT

$S = vt$

$X = (v_{ix})(t)$

$X = (v_i \cos\theta)t$

$X = (v_f \cos\theta)t$

$v_{ix} = v_{f(x)}$

$S = v_i t + \frac{1}{2} a t^2$

$at^2 = -gt^2$

$v_i = v_{iy}$.

$v_{iy} = v_i \sin\theta$.

$Y = (v_i \sin\theta)t + \frac{1}{2}(g)t^2$

$= (v_i \sin\theta)t - \frac{1}{2} g t^2$

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Displacement :- x-axis & y-axis.

$Y = Y_0 + V_{0y}t + \frac{1}{2} a_y t^2$

$X = X_0 + V_{0x}t + \frac{1}{2} a_x$

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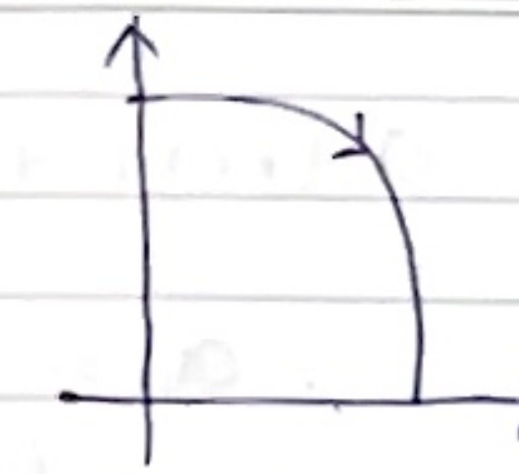
SPECIAL CASE :-



$V_x \Rightarrow V_{ix} \Rightarrow$ (constant)

$V_{iy} \Rightarrow 0$; $V_{fy} \neq 0$.

$V_{fy} - V_{iy} = gt \Rightarrow \boxed{V_{fy} = gt}$

Horizontal distant :- $\boxed{X = (V_{ix})(t)}$

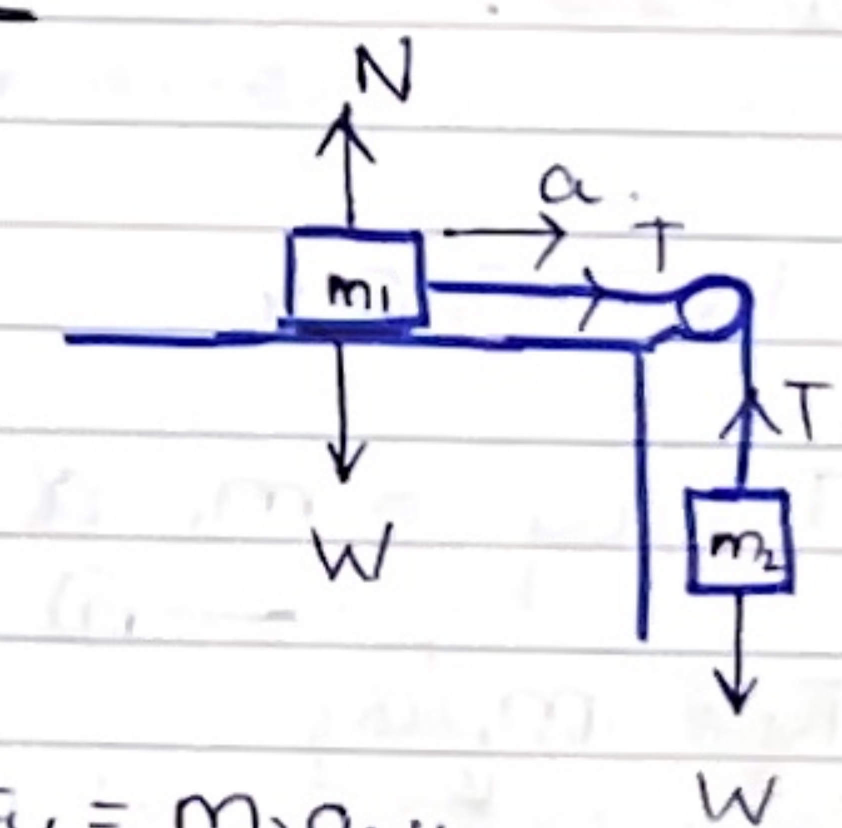
Vertical distance :- $Y = (V_{iy})t + \frac{1}{2} g t^2$

$\boxed{Y = \frac{1}{2} g t^2}$

Time :- $\boxed{\Rightarrow \sqrt{\frac{2Y}{g}} = t}$

CLASS OF CHAP 5.

PROBLEM:-



$m_1 = 2kg$.

$m_2 = 1kg$.

Frictionless.

$\Sigma F_x = m_1 a_{x_1}$

$T = m_1 a_1$ —(1)

$\Sigma F_y = m_1 a_{y_1}$, $a_{y_1} = 0$

$N - m_1 g = 0$.

$N = m_1 g$.

$\Sigma F_y = m_2 a_{2y}$.

$T - m_2 g = m_2(-a)$.

$T - m_2 g = -m_2 a$ —(2)

Solve equ (i) & (ii).

$m_1 a = T$

$-m_2 a = -T - m_2 g$

$m_1 a + m_2 a = m_2 g$.

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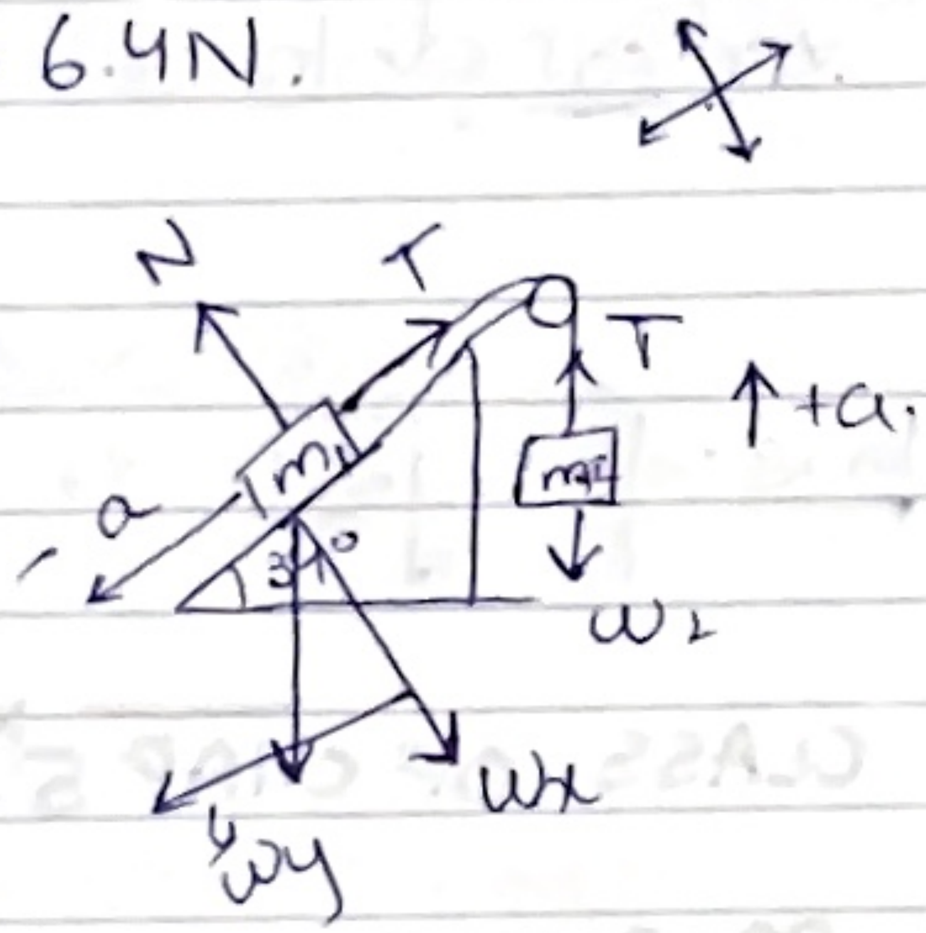
$a(m_1 + m_2) = m_2 g.$

$a = \dfrac{m_2 g}{(m_1+m_2)} \Rightarrow 3.2 ms^{-2}.$

Using equ. (i)
$T = m_1 a$

$= (4.2)(3.2) \Rightarrow 6.4N.$

PROBLEM:-



$m_1 = 9.5kg.$
$m_2 = 2.6kg.$
$\Phi = 34°.$

$\Sigma F_x = m_1 a_x$ $\qquad$ $\Sigma F_x = m_2 a_x$
$\qquad\qquad\qquad\qquad\qquad 0$

$T - w_y = m_1 a. \quad — (1)$

$\Sigma F_y = m_1 \cancel{a_y}$ $\qquad$ $\Sigma F_y = m_2 a_y.$
$\qquad\quad 0.$

$-T + W = m_2 a_y$

$N = W_x.$ $\qquad$ $T = -m_2 a_y + W$

$T = m_1 a + W_y$

$N = W\cos\Phi.$ $\quad$ $\boxed{T = m_1 a_x + m g \sin\Phi}$ — (1)

$N = mg\cos\Phi$

$77N.$ $\qquad$ $T = m_1 a_x + 52$

acceleration the system is same in x & y as connected with a common rope.

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$m_1 a + 52 = -m_2 a + 25.$

$m_1 a + m_2 a = +25 - 52.$

$a(m_1 - m_2) = 26.52.$

$a = \dfrac{26.52}{12.1} \Rightarrow 2.19 ms^{-2}$

$T = -(9.5)(2.19) + (52)$ (using equ i)
$T = 31 N$

MOMENTUM :-

(i) Quantity of motion of a body.
(ii) $\dfrac{\Delta P}{\Delta t} = F$ (with direction in terms of Force)
" LAW of momentum in terms of law of motion."

(iii) Conservation of momentum:-
$P_i = P_f.$
$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$

Problem:- $\qquad$ Using law of Momentum conservation.

$m_1 = 75.2 kg$ (man) $\qquad$ $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$
$m_2 = 38.6 kg$ (cart)
$v_{1i} = 2.33 ms^{-1}$ $\qquad$ $\dfrac{(75.2)(2.33)}{(38.6)} = v_{2f} - v_{2i}$
$v_{1f} = 0.$
$v_{1i} = 2.33$
$\Delta V_2 = V_{2f} - V_{2i}$ (cart) $\qquad$ $v_{2f} - v_{2i} = 4.5 ms^{-1}$

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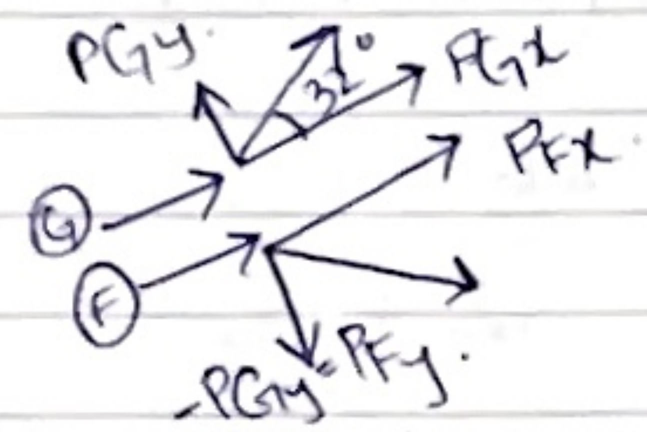
m_1

$m_2 = (1.5)m_1$

$v_{1i} = 2.48ms^{-1}$ $\quad v_{1f} = 1.59ms^{-1} (50°)$

$v_{2i} = 1.86ms^{-1} (40°)$ $\quad v_{2f} = ?$ $\quad \phi \Rightarrow ?$

SAMP PROB :- 3.6



$P_{G(x)} \Rightarrow (55kg)(3.2ms^{-1}) \Rightarrow 176kgms^{-1}$

$P_{F(x)} \Rightarrow (75kg)(3.2ms^{-1}) \Rightarrow 240kgms^{-1}$.

$\tan\phi = \frac{P_{Gy}}{P_{Gx}}$

$P_{Gx} \tan 32 = P_{Gy} \Rightarrow +110kgms^{-1}$.

As they pushed eachother perpendicularly.

$\therefore -P_{Gy} = P_{Fy}$.

$\tan\phi = \frac{P_{Fy}}{P_{Fx}}$.

$\phi = \tan^{-1}\left(\frac{P_{Fy}}{P_{Fx}}\right) \Rightarrow -25°$ Ans!

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SAMPLE PROB 6.4:-

$m_1 = 65kg$.
$v_{1i} = 4.9ms^{-1}$.

$m_2 = 88kg$.
$v_{2i} = 1.2ms^{-1}$.

$P_{m(x)(i)} \Rightarrow (65)(4.9) \Rightarrow (\cancel{69.9 kgms^{-1}})$ 318.5

$P_{b(x)(i)} \Rightarrow (88)(1.2) \Rightarrow 105.6kgms^{-1}$.

total $P_{(i)} \Rightarrow 424kgms^{-1}$.

As the man is finally in contact with the boat

$\therefore v_{1f} = v_{2f} = v_{fx}$.

$P_f = m_1(v_{f(x)}) + m_2(v_{fx})$
$= v_{f(x)}(m_1 + m_2)$

$P_f = v_{f(x)}\ 153$

$424 = 153\ v_{f(x)}$
$v_{f(x)} = \frac{424}{153} \Rightarrow 2.8ms^{-1}$

Physics → complete
Maths → complete.
CS → slides complete.

ding exercise → complete
Chem practice
Physics lab → revision

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SAMPLE PROB 6.5 :-

$m_1 = 1.25kg.$
$v_{1i} = 3.62ms^{-1}$

$m_2 = 2.30kg.$
$v_{2i} = 0.00ms^{-1}$

$v_{1f} = -1.07ms^{-1}$ (as in negative x axis).
$v_2f = ??$

$P_i = P_f$

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

$(1.25)(3.62) + (2.3)(0) = (1.25)(-1.07) + (2.3) v_{f2}$

$+1.33 + 4.5 = 2.3 v_{f2}$

$v_{f2} = \frac{3.17}{2.3} \Rightarrow$

$m_1v_{1i} - m_1v_{1f} = m_2v_{2f} - m_2v_{2i}$

$m_1(v_{1i} - v_{1f}) = m_2(v_2f)$

$v_2f = \frac{m_1}{m_2}(v_{1i} - v_{1f}) \Rightarrow \frac{1.25}{2.30}(3.62 + 1.07)$

$\Rightarrow 2.5ms^{-1}$

SAMPLE PROB 6.6 :-

As final velocity in combination.
$\therefore v_{f_1} = v_{f_2} = v_{fx}$.

$m_1 = 1.25kg$
$v_{1i} = 3.62ms^{-1}$
$m_2 = 2.3kg.$
$v_{2i} = 0ms^{-1}$

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

$m_1v_{1i} - m_1v_{1f} = m_2v_{2f} - m_2v_{2i}$

or.

$\frac{(1.25)(3.6) - (2.3)(0)}{m_1 + m_2} = v_f = ms^{-1}$

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SAMPLE PROB 6.7 :-

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}.$

$v_i(m_1 + m_2) = m_1v_1f + m_2v_2f.$ —— ①

$v_i = 8.45km/s. \Rightarrow v_{1i} \Rightarrow v_{2i}.$

$m_1 = m/4$
$m_2 = 3m/4$

Plot the answers/data in the given equation.

$v_{f1} = 11.63 km/s.$
$v_{f2} = ?$

$(8.45)\left(\frac{m}{4} + \frac{3m}{4}\right) = \frac{m}{4}(11.63) + \frac{3m_2}{4}(v_2f)$

$v_f = \frac{(8.45)(m) - \frac{m}{4}(11.63)}{3/\frac{m}{4}}$

$v_f \Rightarrow +7.36km/s.$

SAMPLE PROB 6.8 :-

$m_1 = m$
$m_2 = 1.5m.$
$v_{1i} = 2.48m/s.$
$v_{2i} = 1.86m/s.$ (40° away from the first puck).

$v_1f = 1.59m/s$ ($\phi = 50°$)
$v_2f = ?$ $\theta = \phi$?

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As the angle is given.
Find momentum in 2 dimension (x & y).
(i)

$V_{1x} = 2.48 m/s$. $V_{2x} = V\cos\theta$.

$V_{1y} = 0 ms^{-1}$ $V_{2x_i} = 1.86 \cos 80$.
$V_{2y_i} = 1.86 \sin 80$.

$V_{1x}^{f} = V\cos\theta = 1.59 \cos 50°$. $V_{2f_x} = ?$
$V_{1y_f} = 1.59 \sin 50$ $V_{2f_y} = ?$

Now as per statement (i).

$m_1(V_{1x})_i + m_2(V_{2x})_i = m_1(V_{1x})_f + m_2(V_{2x})_f$

$m(2.48) + 1.5m(1.42) = m(1.02) + 1.5m(V_{2x})_f$

$2.48m + 2.13m = 1.02m + 1.5m(V_{2x})_f$.

$\frac{3.59}{1.5} (V_{2x})_f \Rightarrow 2.39 m/s$.

Similarly.

$m_1(V_{1y})_i + m_2(V_{2y})_i = m_1 (V_{1y})_f + m_2(V_{2y})_f$

$m_1(0) + 1.5m(1.192) = m(1.21) + 1.5m(V_{2y})_f$

$1.78m = 1.21m + 1.5m(V_{2y}f)$

$(V_{2y})f = 0.38$.

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6.1
6.2-6.3.
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$(V_2)f = \sqrt{(V_{2x})_f^2 + (V_{2y})_f^2}$

$= \sqrt{(2.39)^2 + (0.38)^2}$
$= 2.42$ Ans!

$\theta = \tan^{-1} \frac{0.38}{2.39} \Rightarrow 9.03°$ Ans!

EXERCISE:-
6.17
$m_1 = 75.2 kg$.
$V_{1i} = V_{2i}$ (body in contact) $\Rightarrow 2.33 m/s$.
$m_2 = 38.6 kg$.
$V_{1f} = 0 ms^{-1}$
$V_{2f} = ?$ $(m_1v_1)_i + (m_2v_2)_i = (m_1v_1)_f + (m_2v_2)_f$.

Using law of cons. of momentum.

$(75.2)(2.33) + (38.6)(2.33) = (75.2)(0) + (38.6)(V_2f)$.

$175.216 + 89.93 = 38.6 V_2f$.

$\frac{265.146}{38.6} = V_2f \Rightarrow 6.86 m/s$.

$\Delta V_2 = V_{2f} - V_{2i} \Rightarrow 6.86 - 2.33 \Rightarrow 4.5 ms^{-1}$.

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Note:- If any object say B has a speed relative to the speed of object A then (imp)

orig obj B speed = object speed − object B speed
object A speed

6.19 :-

m_1 (command module) = m.
m_2 (rocket motor) = 4m.

$(V_1)_i = (V_2)_i \Rightarrow 3860$ km/h. (positive x axis)

$V_{2f} = 125$ km/h (negative x-axis).
→ this w.r.t the command module.
$V_{1f} = ?$ $(V_{f\,of} - V_{f\,m})$ → V_{2f} ⇒ Imp!
$\rightarrow 125$

Using law of conservation of momentum.

$(m_1v_1)_i + (m_2v_2)_i = (m_1v_1)_f + (m_2v_2)_f$

$v_i(m + 4m) = m_1v_{1f} + 4m(v_{f1} - v_{f2}m)$

$= m_1v_{1f} + 4m v_{1f} - 4m v_{2f} m$

$v_i(5) = (4+1)v_{1f} - 4(125)$

$\frac{3860 \times 5 + 4(125)}{5} = v_{1f} \Rightarrow 3960\ m/s^{-1}$
Ans!

Ex
6.30. IMP!
Data given:- elastic collision.

$m_1 = 342g$.
$v_{1i} = 1.24m/s$. $v_{1f} = 0.636m/s$.
$m_2 = ?$ $V_{2f} = ?$
$V_{2i} = 0ms^{-1}$.

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law of conservation of momentum.

$(m_1v_1)_i + \cancel{(m_2v_2)_i} = (m_1v_1)_f + (m_2v_2)_f$.
0
?? as two unknown

$m_1v_{1i} - m_1v_{1f} = m_2v_{2f}$. (donot use law of $m_1v_{1i} = m_2v_{2f} + m_1v_{1f}$ — (i) conser. of P) directly

As it is an elastic collision (use law of K.E cons. w.r.t equ i)

$(\times\ V_{2f} = V_{1i} + V_{1f})$.

$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_2v_{2f}^2 + \frac{1}{2}m_1v_{1f}^2$.

$m_1v_{1i}^2 = m_2v_{2f}^2 + m_1v_{1f}^2$ — (ii)

Using (i) & (ii)

$m_2v_{2f} = m_1v_{1i} - m_1v_{1f}$.
$= (342)(1.24) - (342)(0.636)$ → 206.568

$m_2v_{2f}^2 = m_1v_{1i}^2 - m_1v_{1f}^2$
$(342)(1.24)^2 - (342)(0.636)^2$. → 387.5

Divide equ (ii) & (i)

$\frac{\cancel{m_2}v_{2f}^{\cancel{2}}}{\cancel{m_2}\cancel{v_{2f}}} \Rightarrow \frac{387.5}{206.5} \Rightarrow 1.87ms^{-1}$

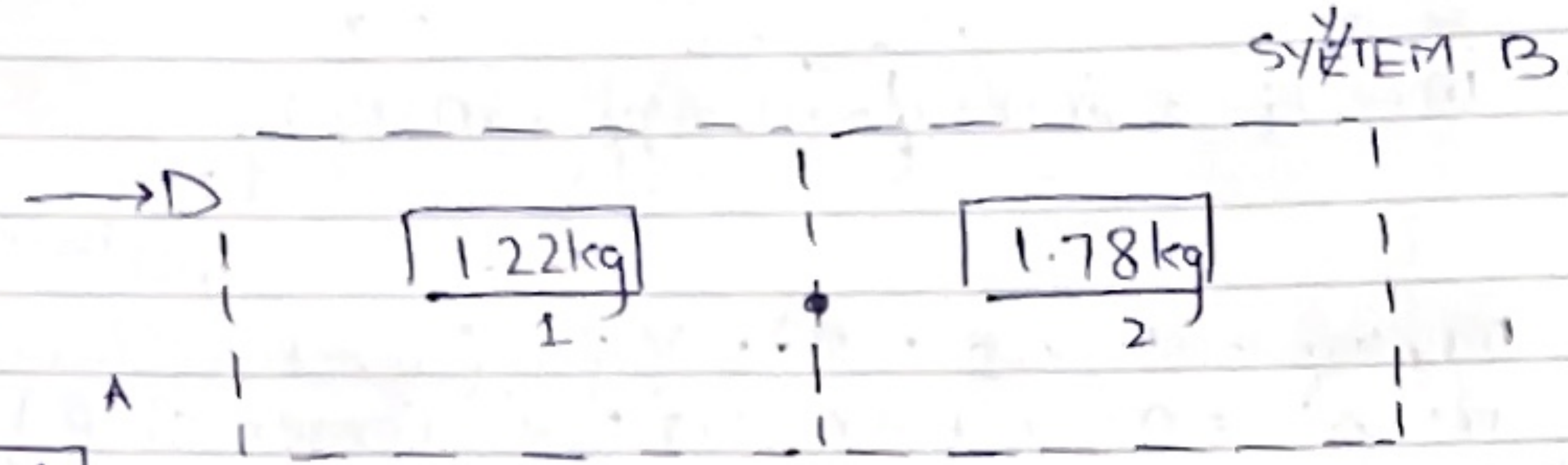
now mass is.

$m_2 = \frac{(342)(1.24) - (342)(0.636)}{1.87} \Rightarrow 0.93kg \approx 0.1kg$

$\frac{\Delta P}{\Delta t}$ (Rate of change of momentum)

Momentum: $P = mv$

CLASS PROB:-



LINE A

$m_b = 3.54kg$. $v_{bi} = ?$ $v_{bf} = ?(2.21)$

$m_{B1} = 1.22kg$. $v_{Bi} = 0ms^{-1}$; $v_{Bf} = 0.63ms^{-1}$.

SYSTEM / LINE B

$m_{B2} = 1.78kg$ $v_{Bi} = 0ms^{-1}$; $v_{Bf} = 1.48 m/s$.

$m_b = 3.54kg$ $v_{bi} = ?$; $v_{bf} = 1.48m/s$.

$m_b v_{bi} + m_{B2} v_{Bi} = m_b v_{bf} + m_{B2} v_{Bf}$

$(3.54) v_{bi} + (1.78kg)(0) = (3.54)(1.48) + (1.78)(1.48)$.

$v_{bi} = \frac{5.2 + 2.63}{3.54}$

$v_{bi} = 2.12 ms^{-1}$ (This the velocity after emerging from block one)

Now using equ(i).

$(3.54)(v_{bi}) + (1.22)(0) = (3.54)(2.2) + (1.22)(0.63)$

$V_{bi} = 9.47 ms^{-1}$.

(Note) the equations that mix linear and angular variable should always have angular quantities in radians.

CLASS PROB:- (Problem no 1)
Rotation Quantities.

$\omega_i = 48.6 rpm$ ⇒ $48.6 \frac{rev}{min} \cdot \frac{1 min}{60}$.
$\omega_f = 0 rpm$.
$t = 32$ seconds.
8.8 revs (no. of turns) (total displacement)

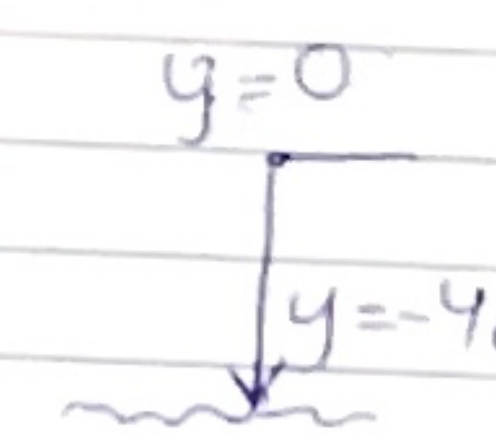
(a) $\omega_{av} = ?$ (imp)
(b) $\alpha_{av} = ?$

$\omega_{av} = \frac{\Delta\phi}{\Delta t} = \frac{8.8 revs}{32}$ ⇒ 0.275 rev/s

$\alpha_{av} = \frac{0 - 0.82}{32}$ ⇒ $-0.025 rev/s^2$.

Class Prob:-

$v_i = 0ms^{-1}$.
$\Delta\phi$ (total displacement) = 4 revs.
Consider the diver had $\omega_i = 0$



$\omega_f = ?$ (not needed)
$\Delta t / t_2 = ?$ (t after the displacement = total time)

$\omega_{av} = \frac{\Delta\phi}{\Delta t}$ ⇒ 4 revs

Using 2nd equ. of motion.
$y = y_0 + v_i t + \frac{1}{2} a t^2$.
$-40 = -\frac{1}{2} g t^2$ ⇒ 8.8 rad/se.
$\sqrt{80 \times 9.8} = t$ ⇒ 2.8 seconds

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PRACTICE PROBS.

SAMPLE PROB 8.2 :-

as the given inst angular velocity is ;

$\omega = At^2 + Bt^2$. —— (i)

(a)

also angular inst velocity is as position i.e ϕ is required

$\omega = \frac{d\phi}{dt}$

$\int \omega dt = \phi$

$\int At + Bt^2 = \phi$

$A\int t dt + B\int t^2 dt = \phi$ ∴ now as give t = 2sec.

$A\frac{t^2}{2} + \frac{Bt^3}{3} = \phi$

$A\frac{(2)^2}{2} + B\frac{(2)^3}{3} = \phi$

$A\frac{4}{2} + B\left(\frac{8}{3}\right) = \phi$

$2A + \frac{8B}{3} = \phi$

$\phi = 2(6.02) + \frac{8}{3}(8.7) = \phi$

$\phi = 35.6 rad$

(b) $\alpha = \frac{d\omega}{dt}$

$\frac{d(At + Bt^2)}{dt}$

$\alpha_{inst} = A + 2Bt$

$\alpha_{inst} = \frac{6.2 + 2(8.2)}{(0.5)^2}$

$\Rightarrow 14.9 rad/s^2$ Ans!

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EXERCISES :-

8.6.

a. $\Delta\phi = 2\pi rad$
$\Delta t = 60s$.

$\omega = 0.104 rad/s$

b. $\Delta\phi = 2\pi rad$.
$\Delta t = 60min = 3600s$

$\omega = 1.75\times10^{-3} rad/s$.

c. $\Delta\phi = 2\pi rad$
$\Delta t = 12\times 60min \times 60$

$\Rightarrow \omega \Rightarrow 1.45\times10^{-4} rad/s$.

8.7:-

$V_i = 85 mi/h \Rightarrow 125 ft/s$.

$\omega_i = 1800 rev/min \Rightarrow 1800/60 \; rev/s \Rightarrow 30 \; rev/s$

$S = 60ft$.

revalution $= \Delta\phi = ? = (\omega)(t)$.

$(30)\left(\frac{60}{125}\right) \Rightarrow 14 revs$

Ans!

8.8:-

$\Delta\phi = 2.5 revs$.

$v_i = 0$.

$\omega_{avg} = \frac{\Delta\phi}{\Delta t} \Rightarrow ?$

$y = 0$

$y = -10m$

$y = y_0 + V_i t + \frac{1}{2}(-g)t^2$

$-10 = -\frac{1}{2}(10)t^2$

$\frac{\sqrt{20}}{10} = t$

$t = \sqrt{2}$

$\Rightarrow \frac{2.5}{\sqrt{2}} \Rightarrow 1.74 revs^{-1}$

$\Rightarrow 11 rads^{-1}$

Ans!

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PROBLEM:-

$w_i = 1170$ rev/min
$w_f = 2880$ rev/min
$t = 12.6s \times 60m$
a. $\alpha = ?$ (in rev/min²)
b. $\phi = ?$ (revolutions / displacement).

a. using first equ. of motion.

$w_f - w_i = \alpha t$

$\alpha = \frac{w_f - w_i}{t} \Rightarrow \frac{2880 - 1170}{12.6} \Rightarrow 8142$ rev/min²

b. $\phi = {}_0\cancel{\phi_0} + w_i t + \frac{1}{2}\alpha t^2$

$\phi = (1170)(0.21) + \frac{1}{2}(8142)(0.21)^2$

$\phi = 425$ 25 revs!

PROBLEMS FOR PRACTICE

EXERCISE

8.3:-

$\phi = at + bt^3 - ct^4$ if a, b & c are const.

equ. of angular velocity and equ. of angular acceleration.

(a) $\frac{d(\Delta\phi)}{dt} = a + 3bt^2 - 4ct^3$ Ans!

(b) $\frac{d}{dt}\left(\frac{\Delta\phi}{dt}\right) = a + 6bt - 12ct^2$ Ans!

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8.4:-

$\omega = 250$ km/s.

$\phi = 1$ revolution, $r = 2.3 \times 10^4$ ly $\times \left[\frac{12\text{ months}}{1\text{ year}} \times \frac{365\text{ day}}{12\text{ months}} \times \frac{8760\text{ hrs}}{365\text{ days}} \times\right]$

(a) $t = ?$

(b) $n = ?$ (= 4.5×10^9

now

$\frac{\Delta\phi}{\Delta t} = \omega$

$\Delta\phi = S \Rightarrow 2\pi r$ (1 revolution i.e one circle) Donot do this conversion; Rather.

$\frac{\Delta\phi}{\omega} = t$

$2\ 3 \times 10^4$ ly $\times (3 \times 10^8 ms^{-1})$

$r \Rightarrow 6.9 \times 10^{12}$ m.y/s.

$t = \frac{2\pi 6.9 \times 10^{12}}{250 \times 10^3}\left(\frac{m.y/s}{m/s}\right) \Rightarrow 1.7 \times 10^6$

$[\phi = 2\pi r]$ no. of revolutions $= \frac{4.5 \times 10^9}{1.7 \times 10^6} = 26$ revs

8.5:- IMP!

$a_z = 4at^3 - 3bt^2$ where a & b are const.

integrating will take us rate behind per quantity. (i)

(a) $\int_0^t a_z\,dt = \int_0^t (4at^3 - 3bt^2)\,dt$

$\Delta w_z = \frac{4at^4}{4} - \frac{3bt^3}{3}$

$w_z - w_0 = at^4 - bt^3$

$w_z = w_0 + (at^4 - bt^3)$

(b) $\int_0^t w_z = \int_0^t w_0 dt + \int_0^t (at^4 - bt^3)\,dt$

$\Delta\phi = w_0 t + \frac{1}{5}at^5 - \frac{1}{4}bt^4$ Ans!

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TORQUE:-

The quantity in rotational dynamics that takes into account both magnitude of the force and direction of the location at which it is applied is called as torque.

(i) $\tau = r F_T$ (tangential component of force)

$\therefore F_T = F\sin\phi$.

(ii) $\tau = r_T F$ (tangential component of radius / position vector).

$\therefore r_T = r\sin\phi$

3 dimensional Quantities.

$\vec{r} = \vec{r_i} + \vec{r_j} + \vec{r_3}$

$\vec{F} = \hat{F_x} + \hat{F_y} + \hat{F_3}$

↘ Then torque is found by the vector/cross product

$\bar{r} \times \bar{F} = \begin{vmatrix} i & j & k \\ r_i & r_j & r_k \\ F_x & F_y & F_3 \end{vmatrix}$

$\Rightarrow [i((r_j)(F_3)-(r_k)(F_y))] - j[(r_i)(F_3)-(r_k)(F_x)] + \hat{k}[(r_i)(F_y)-r_j(F_x)]$

$\Sigma F_T = ma_T$

$F\sin\phi = ma_T$.

$a_T = r\alpha$ —— ①

①

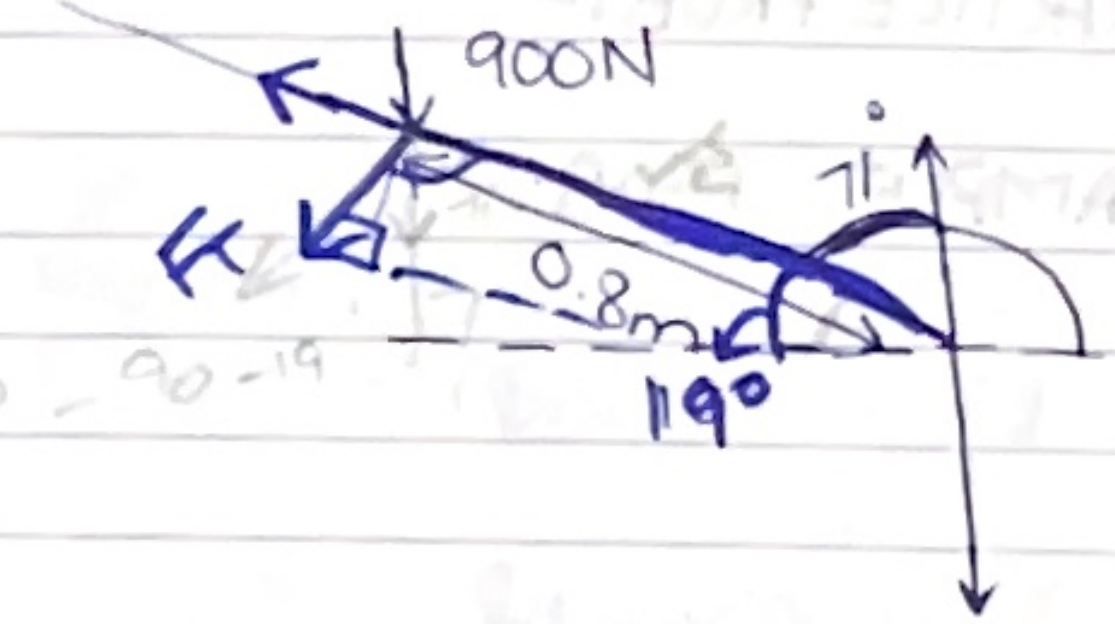
$(r)F\sin\phi = (m\alpha r)(r)$

$\Rightarrow I = mr^2$ (kgm^2)

$\tau = I\alpha_z$

$\Sigma\tau = (m_1 r^2 + m_2 r^2)\alpha_z$.

C.P (i) ?



$\phi = 180° - 90° - 19°$
$= 71°$ $= ?$

$\tau = \bar{r} \times \bar{F} \sin\phi$

$(0.8)(900)\sin 71 \Rightarrow 680\ Nm$.

C.P (ii)

$r = 0.54\ i + (-0.36)j + 0.85k$.

(a) $F = 2.6$ at positive x axis.
(b) $F = 2.6$ at negative z axis.

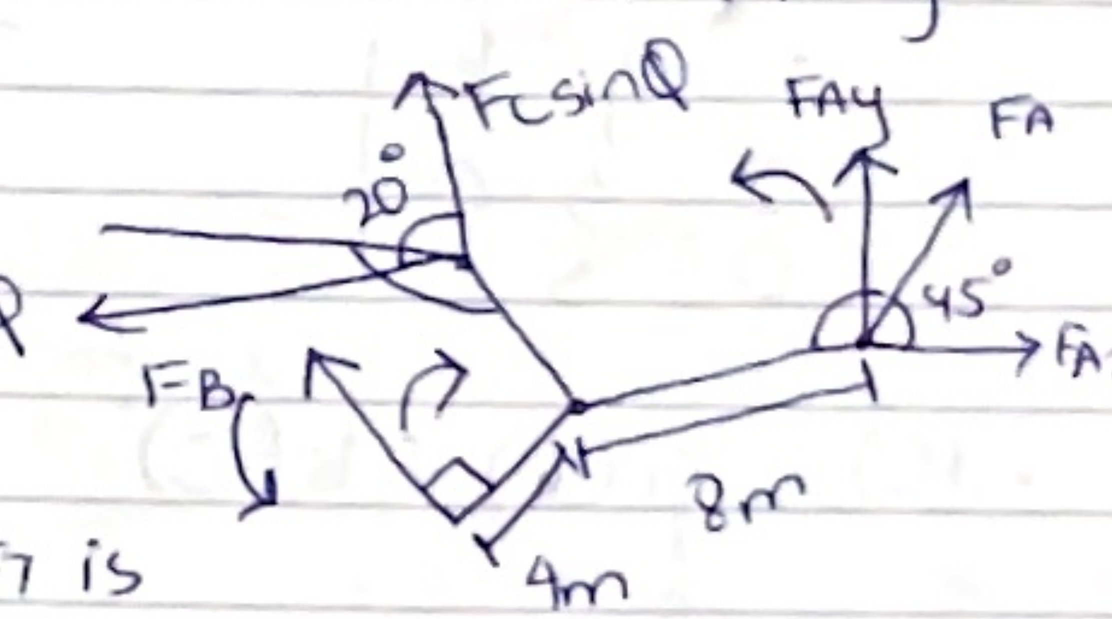
(a) $\begin{vmatrix} i & j & k \\ 0.54 & -0.36 & 0.85 \\ 2.6 & 0 & 0 \end{vmatrix}$

$\Rightarrow 2.2Nm\ j + 0.94Nm\ \hat{k}$

(b) $\begin{vmatrix} i & j & k \\ 0.54 & -0.36 & 0.85 \\ 0 & 0 & -2.6 \end{vmatrix}$

$\Rightarrow 0.94\hat{i} + 1.4\hat{j}\ Nm$

C.P (iii) :- ?



So after resolution we take the components that are exactly perpendicular so that F_T is maintained.

$\tau = (r)F_B$ $\quad (r)F_C\cos\phi \quad (r)F_A\sin\phi$

$\Rightarrow (4)(16) \quad (3)(19\cos 20) \quad (8)(10\sin 45)$

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EXERCISE 9.9:- Simple ; **EXERCISE 9.10:-** SIMPLE.

Centre of Mass:- (Balancing point of a body).

The point at which a force can apply to cause a linear acceleration without any angular acceleration.

$r_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$ —— ①

$V_{cm} = \frac{dr_{cm}}{dt} = \frac{m_1 \frac{dr_1}{dt} + m_2 \frac{dr_2}{t}}{m_1 + m_2}$

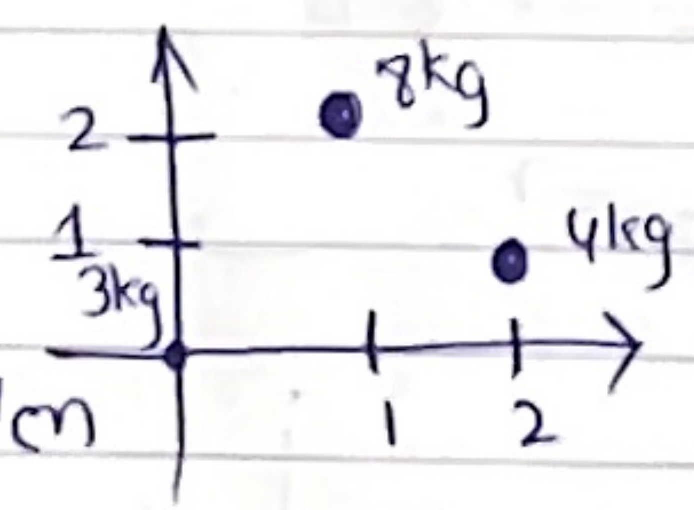
Velocity of C.O.M

$V_{cm} \Rightarrow \frac{m v_1 + m_2 v_2}{m_1 + m_2}$ —— ②

acceleration of C.O.M.

$a_{cm} \Rightarrow \frac{d V_{cm}}{dt} \Rightarrow \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$.

C.P(i) CENTRE OF MASS.



$X_{cm} = \frac{(3)(0) + 8(1) + 4(2)}{3+8+4} \Rightarrow 1.07m$

$Y_{cm} = \frac{(3)(0) + 8(2) + 4(1)}{3+8+4} \Rightarrow 1.33m$

Centre of mass is at (1.07, 1.33)
x , y.

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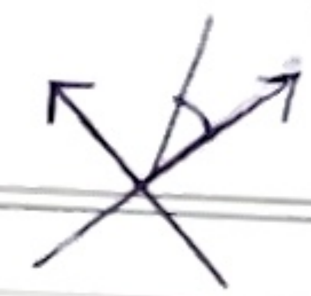
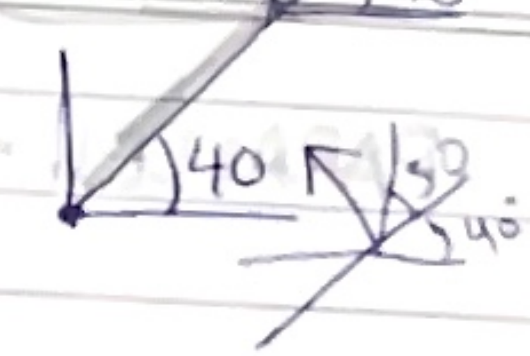


$\vec{\tau} = \vec{r} F_T$



C.P(ii):- ?

($\phi = 40°$ with positive x axis)

mass = 25g.

length = 0.74m.

F = 22N.

I = ?

α = ?

(a)

$I = mr^2$

$I = (25 \times 10^{-3})(0.74)^2$

$\Rightarrow 0.0136\ kgm^2$.

(b) $\tau = I\alpha$.

$rF_T = I\alpha$.

$\alpha = \frac{(0.74)(22 \sin 50)}{0.01369}$

PRACTICE PROBLEMS:-

S.Prob (7.2):- ?

Ex: 7.10 (already done)

Ex: 7.11 (?)

NH_3 molecule.

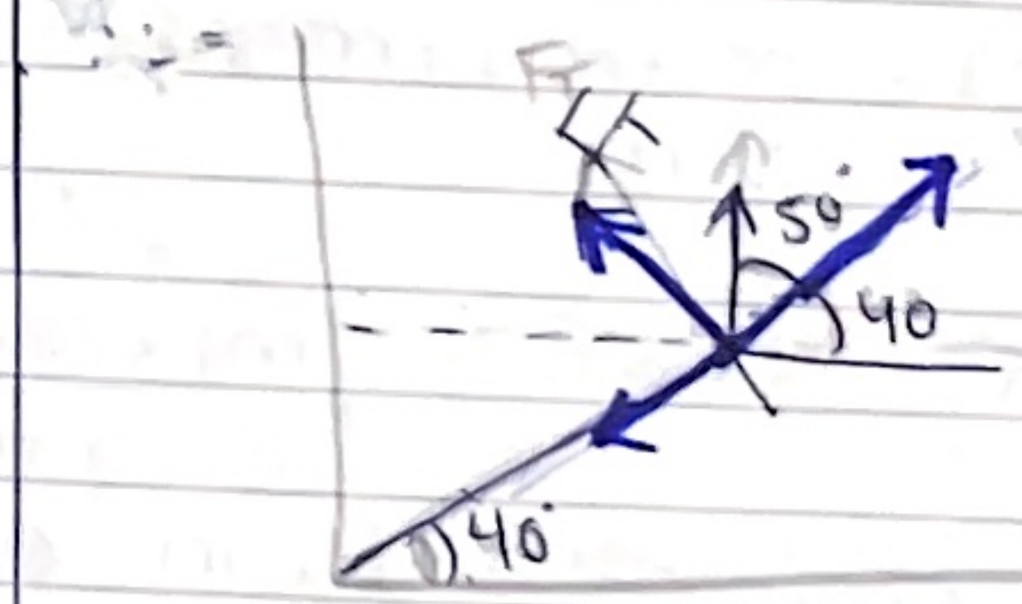
N:H (mass ratio) = 13:9.

7.12:- (?)

Velocity components at centre of mass.

V_{cm} , x = 7.3 m/s

y = 110 m/s.



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PARALLEL AXIS THEOREM:-

$I = I_{cm} + Mh^2$

I → net rotational inertia.

h → length measured from ⊥ axis to the centre of mass of the body.

C.P (ii)

$m_1 = 52 \times 10^{-3}$ kg.
$r_1 = 0.27$ m.
$m_2 = 35 \times 10^{-3}$ kg.
$r_2 = 0.45$ m.
$m_3 = 0.024$ kg.
$r_3 = 0.65$ m.

$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$

$I = (0.052)(0.27) + (0.035)(0.45) + (0.024)(0.65)$

$I = 0.021\ kgm^2$.

$I = ?$
$I_{cm} = ?$

$I_{cm} = I - Mh^2$ —— ①

$M = m_1 + m_2 + m_3$
$= 0.111$

$h^2 = (X_{cm})^2 = \left(\frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}\right)^2 = (0.41m)^2$.

Using all data in ①

$I_{cm} = 0.021 - (0.111)(0.41)^2$
$= 2.3 \times 10^{-9}$ Ans!

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PRACTICE PROBLEM:-)

S.Prob 9.2:-

$m_1 = 2.3$ kg.
$m_2 = 3.2$ kg.
$m_3 = 1.5$ kg.

(a) As we require rotational inertia about each of the three masses acting as axes. ∴
Pivot each mass separately.

$I_1 = \Sigma m_n r_n^2 = (2.3)(0)^2 + (3.2)(3)^2 + (1.5)(4)^2$
$\Rightarrow 53 kg.m^2$.

$I_2 = \Sigma m_n r_n^2 = (3.2)(0)^2 + (2.3)(3)^2 + (1.5)(5)^2$
$\Rightarrow 58 kg.m^2$

$I_3 = \Sigma m_n r_n^2 \Rightarrow (1.5)(0)^2 + (2.3)(4)^2 + (3.2)(5)^2$
$\Rightarrow 117 kg.m^2$

(b) Now as the given force rotates the body. The rotation of a body in xy plane is only possible in z axis. ∴ $\tau_z = r_\perp F$. $[\tau = I\alpha]$

We require the ⊥ component of 'r' = ?
$I = 117 kg.m^2$ (as taken at mass$_3$).

(vii) Rectangular plate :- $I = \frac{1}{12} M(a^2+b^2)$

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9.14 :- (from (9(b))).

Consider m_1 along positive x axis.
m_2 along (ne x) positive y axis.

$$\therefore \quad \bar{r} = (0.42)\hat{i} + (0.65)\hat{j}$$
$$\bar{F} = (3.6N)\hat{i} + (2.5N)\hat{j}$$

$$\bar{r} \times \bar{F} = \begin{vmatrix} i & j & k \\ 0.42 & 0.65 & 0 \\ 3.6 & 2.5 & 0. \end{vmatrix}$$

$$\tau = \hat{i}(0) - \hat{j}(0) + \hat{k}\left[(0.425)(2.5) - (3.6)(.65)\right]\hat{k}$$

$$\tau = -1.29\, N.m\, \hat{k}.$$

$$I = \frac{\tau}{\alpha} \Rightarrow \alpha = \frac{\tau}{I} = -\frac{1.29}{0.026}\, N.m\, \hat{k}$$
$$\Rightarrow 50\, rad\, s^{-2}\, \hat{k} \text{ Ans!}$$

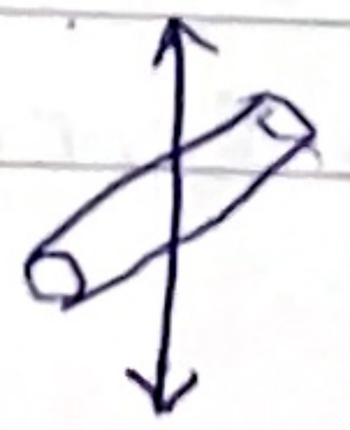
Rotational Inertia of a few solid bodies:-

(i) Hoop :- ( )
$I = MR^2$

(ii) Ring
$I = \frac{1}{2} M(R_1^2 + R_2^2)$

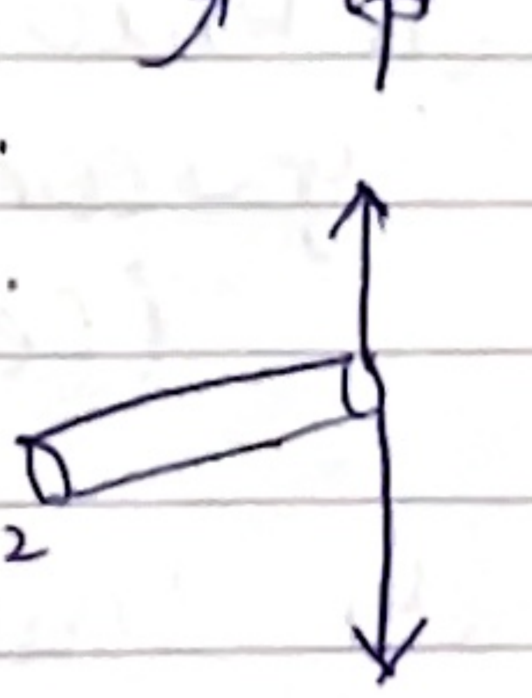
(iii) Hoop ( )
$\frac{1}{2} MR^2$.

(iv) $\frac{1}{12} ML^2$



(iii) Solid Cylinder.
$I = \frac{1}{2} MR^2$.

(iv) Thin rod
$I = \frac{1}{3} ML^2$



(v) Solid sphere
$\frac{2}{5} MR^2$.

(vi) Spherical shell
$\frac{2}{3} MR^2$.

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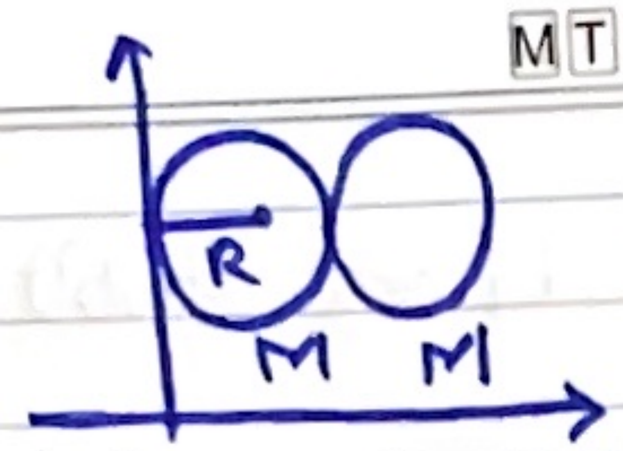
C.P (i)



$I_{cm} = \frac{2}{5} MR^2$. (as this formula has axis passing through the centre.)

$I_1 = I_{cm} + Mh^2$.

$I_1 = \frac{2}{5} MR^2 + MR^2 \Rightarrow \frac{7}{5} MR^2$.

$I_2 = \frac{2}{5} MR^2 + M(3R)^2$

$I_2 = \frac{2}{5} MR^2 + 9MR^2$

$I_2 = \frac{47MR^2}{5}$.

$\Sigma I = I_1 + I_2$.

$\left(\frac{7}{5}MR^2\right)\left(\frac{47}{5}MR^2\right) \Rightarrow 54/5 \; MR^2$.

STEPS TO SOLVE EQUILIBRIUM PROBLEM:-

(i) Draw a boundary around the system, so that you can separate (isolate the system).
(Certain Forces are neglected doin so).

(ii) Draw a free body diagram showing all external forces that act on a system.

(iii) Set up a coordinate system and axis for resolving torque into its components.

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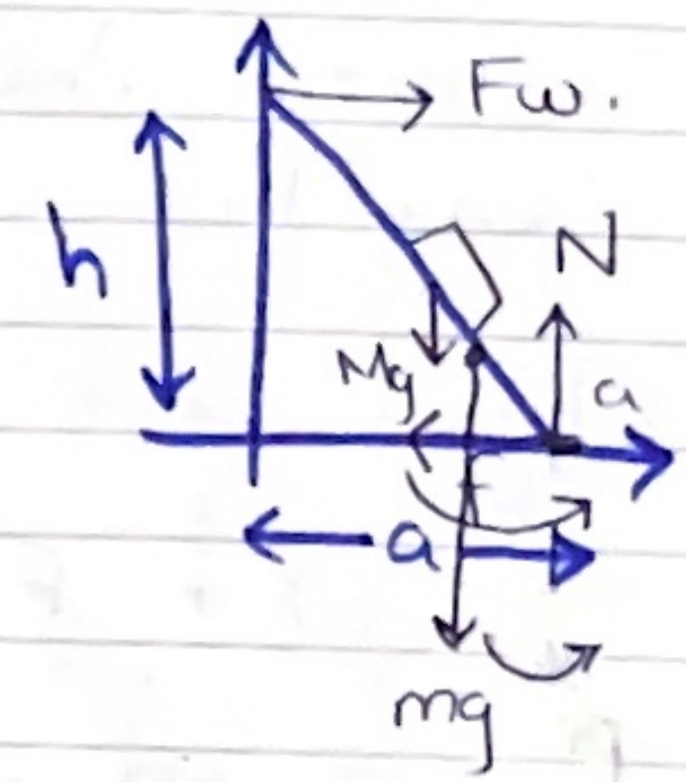
C.P (i) :- $L = 12m$, $h = 9.3m$.



$\Sigma F_x = 0$ — (i)

$F_w - f = 0$.

$F_w = f$. — (ii)

$\Sigma F_y = 0$.

$N - mg - Mg = 0$.

$N = Mg + mg$.

$= g(M + m) = (72 + 45)9.8)$

N — (iii) $\Rightarrow 1150N$.

Now using equ (iii) in (i)

$F_w = \mu_s N$ — (a)

$F_w \Rightarrow$ cannot be found from equ (a) as μ_s is not given.

Then use 2nd $\Sigma \tau_z = 0$. as equilibrium is maintained
(conside a as the pivot).

$f(\cancel{0}) + (N)\cancel{(0)} + (+)(mg)(a/3) + (+)(Mg)(a/2) - F_w(h) = 0$

$F_w = mg\left(\frac{a}{3}\right) + Mg\left(\frac{a}{2}\right) \Rightarrow F_w = f = 410N$

Solved!

$a = ?$ $a = \sqrt{L^2 - h^2}$

$a = 7.6m$.

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FORMULA SHEET.

$F = ma.$

$f_s = \mu_s N$

$(m_1v_1)_i + (m_2v_2)_i = (m_1v_1)_f + (m_2v_2)_f.$

$(\frac{1}{2}m_1v_1^2)_i + (\frac{1}{2}m_2v_2^2)_i = (\frac{1}{2}m_1v_1^2)_f + (\frac{1}{2}m_2v_2^2)_f.$

$W = mg.$

ϕ (angular displacement) $= \frac{s}{r}$ $\left(\frac{\text{arclength}}{\text{radius}}\right)$

1 rev = 360° = 2π rads.

1 rad = 57.3°

1 rad = 0.159 rev.

Rotational quantities donot hold commulative law as;

$\Delta\phi_1 + \Delta\phi_2 \neq \Delta\phi_2 + \Delta\phi_1$

also;

$d\phi_1 + d\phi_2 = d\phi_2 + d\phi_1$

(it is valid.

$W_{av} = \frac{\Delta\phi}{\Delta t} = \frac{\phi_2 - \phi_1}{t_2 - t_1}$

$W_{inst} = \text{limit } \frac{\Delta\phi}{\Delta t} = \frac{d\phi}{dt}.$

$\alpha_{av} = \frac{\Delta W}{\Delta t} = \frac{w_2 - w_1}{t_2 - t_1}.$

1mile = 5280 ft.

$\alpha_{inst} = \text{limit } \frac{\Delta W}{\Delta t} = \frac{dw}{dt}.$

$\left.\begin{array}{l} v_f = v_i + at \\ w_{fx} = w_{ix} + \alpha_x t \end{array}\right\}$ remains in the same dimensions. (angular quantities)