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$$\int_2^5 8\,dx = 8x\Big]_2^5 =$$

$= [8(5)] - [8(2)]$

$= 40 - 16 \Rightarrow 24$ Ans!

$$\int_1^4 (5x - 4)\,dx.$$

integrate generally.

$$= \frac{5x^2}{2} - 4x + c\Big]_1^4$$

$$= \left[\frac{5(4)^2}{2} - 4(4) + c\right] - \left[\frac{5(1)^2}{2} - 4(1) + c\right]$$

$= [40 - 16 + c] - [5/2 + 4]$

$= 28 \cdot \frac{5}{2} \Rightarrow 26 - \frac{1}{2} \Rightarrow 25.5.$

$$\int_{-3}^4 \frac{8}{x^3}\,dx.$$

$$\Rightarrow \int_{-3}^4 8x^{-3}\,dx.$$

$$\Rightarrow \frac{8x^{-2}}{-2}\Big]_{-3}^4 \Rightarrow -4x^{-2}\Big]_{-3}^4 \Rightarrow \frac{-4}{(4)^2} - \left(\frac{-4}{(-3)^2}\right)$$

$\Rightarrow -4/16 + 4/9 \Rightarrow -1/4 + 4/9 \Rightarrow \frac{-9+16}{36} \Rightarrow 7/36.$ Ans!

—— x ——

CLASS : 1 :-

Real no.s :- ($\mathbb{R}$)

A real no. is a number that can be used to measure a continuous but one dimensional quantity such as distance, duration or temp.

OR.

Union of both rational & irrational no.s.

Any no. that can be placed on a no. line.

RATIONAL NO.S ($p/q = a/b$) | **IRRATIONAL NOS** ($\neq p/q \neq a/b$)

Integers = -3, -2, -1, 0, 1, 2.

Whole = 0, 1, 2 ... (terminating recurring)

Natural = 1, 2, 3.

Square root = $\sqrt{2}$, $\sqrt{3}$.

(non-recurring non-terminating) π, e

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zero is rational, integer and an whole no.

$\mathbb{R} \subseteq \mathbb{C}$

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BRACKET REPRESENTATION:-

Examples:-

(i) $S = \{1, 3\} \Rightarrow x = 1$ or $x = 3$

solution set.

(ii) $A = (1, 3)$ $1 < x < 3$

(iii) $B = [1, 3)$ $1 \leq x < 3$

(iv) $C = (1, 3]$ $1 < x \leq 3$

(v) $D = [1, 3]$ $1 \leq x \leq 3$.

"FUNCTIONS"

An operation (mathematical) that describes relationship betweeen two variables. (input gives output).

$y = f(x)$

dependant variable. (output). independant variable. (input)

even.

$\sqrt[2]{-16} = 4i$

$\sqrt[3]{-27} = -3$

odd.

* a single input should always give a single output only then it forms a function. If not; we restrict either the domain / Range.

* In a real-valued function, a real input gives a real output.

Example:- $y = \sqrt{x}$ if $x = 4$

$y = \sqrt{x}$

$y = \pm 2$. (restrict Range).

DOMAIN:-

All the "INPUT VALUES" that give a defined output are known as domain.

RANGE:-

" All the outputs obtained from a specified function's specified/adjusted domain is known as the range."

(i) $f(x) = x+1$.

$D = \mathbb{R}$
$R = \mathbb{R}$

(ii) $f(x) = 1/x$.

$D = \mathbb{R} - \{0\}$.
$R = \mathbb{R} - \{0\}$.

(iii) $f(x) = x^2$

$D = \mathbb{R}$
$R = \mathbb{R}^+$

* Calculus II holds multi-variable functions.

11/09/23.

GREATEST INTEGER FUNCT:- (FLOOR FUNCT):

We chose the lowest integer or the nearest low integer.

E.g => $\lfloor 0.1 \rfloor = 0$ Ans| $\qquad \lfloor 1.1 \rfloor = 1$ Ans|.

LOWEST INTEGER FUNCT:- (CEILING FUNCT):-

We chose the highest integer or the nearest high integer.

E.g $\lceil 0.1 \rceil = 1$ Ans| $\qquad \lceil 1.1 \rceil = 2$ Ans|

INCREASING AND DECREASING FUNCT:-

Increasing:

$x_2 > x_1$
$f(x_2) > f(x_1)$
& vice versa

Decreasing:

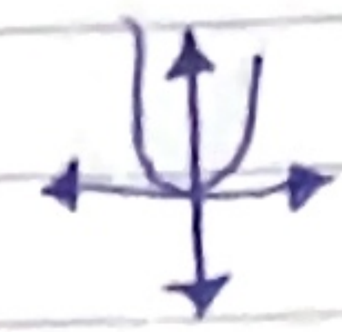
$x_2 > x_1$
$f(x_2) < f(x_1)$
& vice versa.

* For a complete set of domain it is not necessary for a single funct to be always increasing or decreasing.

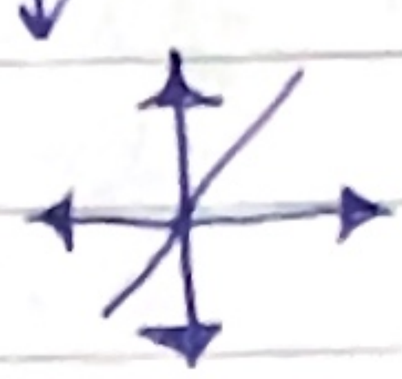
* absolute valued funct is an example of a piece-wise function.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

SYMMETRY OF FUNCTION :-

$y = x^2 \Rightarrow$ 

(i) Even funct is symmetrical about Y-axis.
(ii) Odd funct is symmetrical about origin.
(iii) Some non-functs are symmetrical about x-axis eg parabolic $y = \sqrt{x}$ funct (not) a funct

$y = x$ 

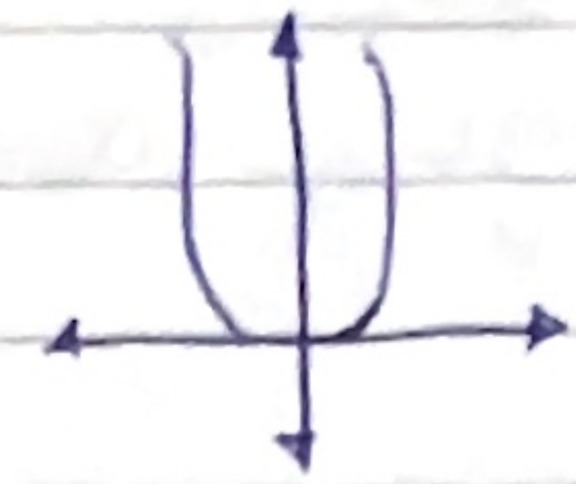
TYPES OF FUNCT.

(i) Algebric Funct ↗ Polynomial
↘ Non polynomial

(ii) Trignometric funct
(iii) Inverse trigno funct.
(iv) Hyper bolic funct.
(v) Inverse hyperbolic funct.
(vi) log funct.
(vii) exponential funct.

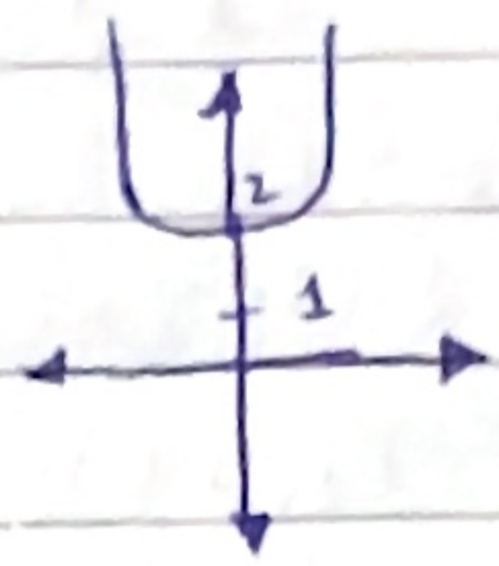
* The new funct obtained by addition of two functions would have a domain that is a subsets of the intersection of the original.

Shifting of functions.



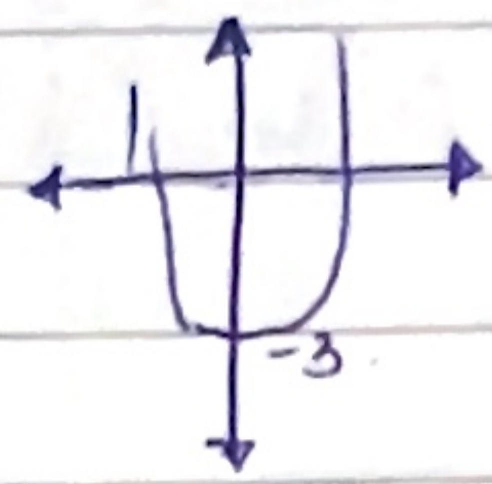
Example.

$f(x) = x^2$

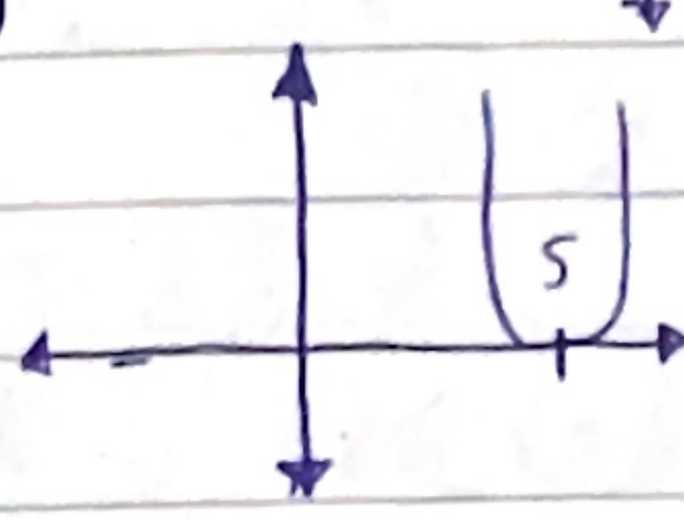


if (i) $f(x) = x^2 + 2$

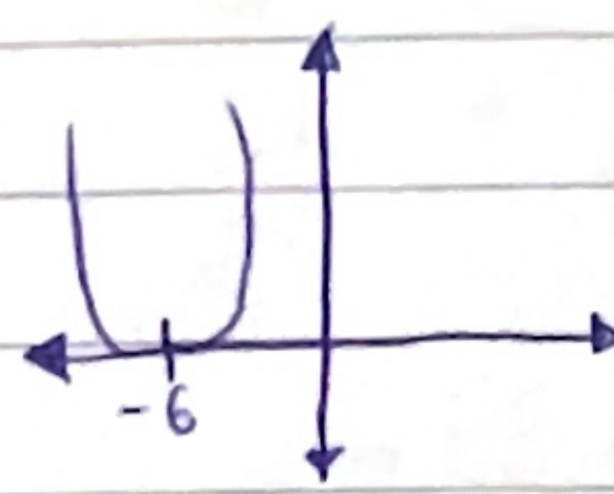
if $f(x) = x^2 - 3$



if $f(x) = (x-5)^2$



if $f(x) = (x+6)^2$



CHECK DESMOS.COM FOR SCALING OF A FUNCTION.

EXERCISE 1.1

DOMAIN & RANGES :- domain $\xrightarrow{}$ function $\xrightarrow{}$ Range.
input output

$f(x) = 1 + x^2$

$D = \mathbb{R}, (-\infty, +\infty), \infty < x < +\infty$

R = (as the question holds a square, neg. no(s) will excluded i.e $0 < x^2 < \infty$ now add one on bothside as given in the question). $0+1 \leq x^2 \leq \infty$

$1 \leq x_i^2 \leq \infty \Rightarrow$ Range $\Rightarrow [1, \infty)$

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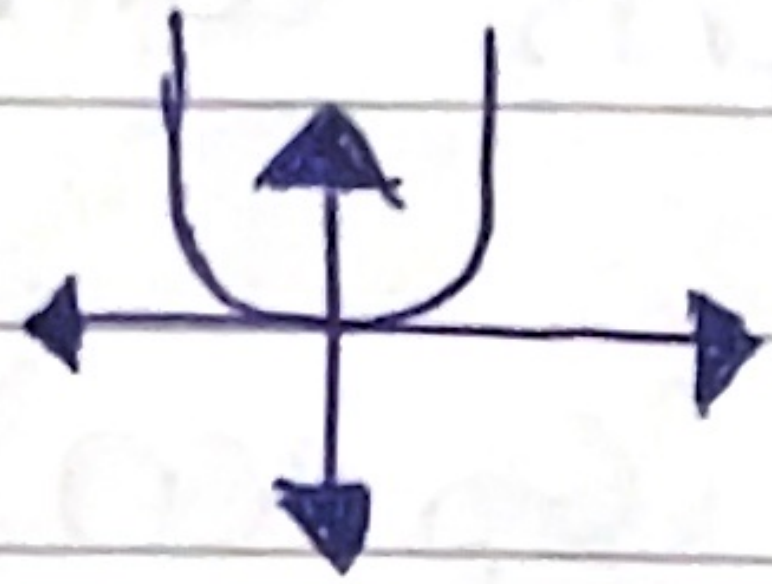
NOTES

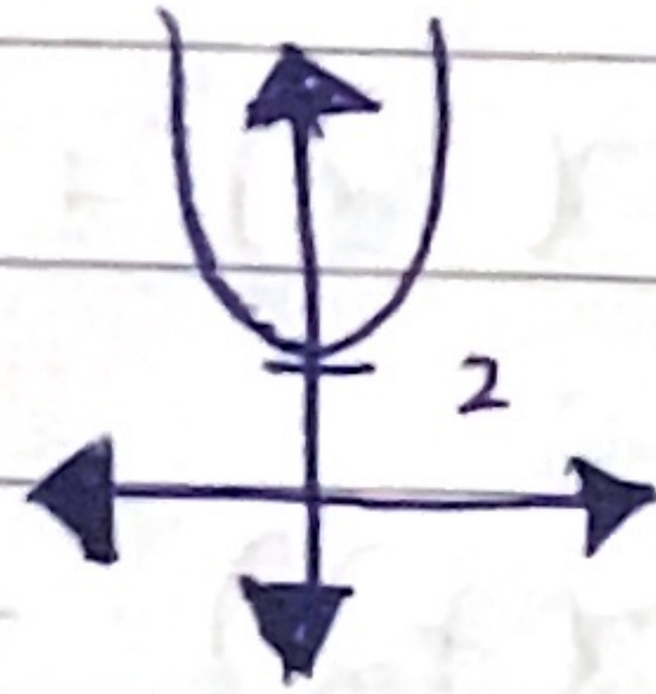
SHIFTING OF FUNCTIONS

(i) $f(x) = x^2$



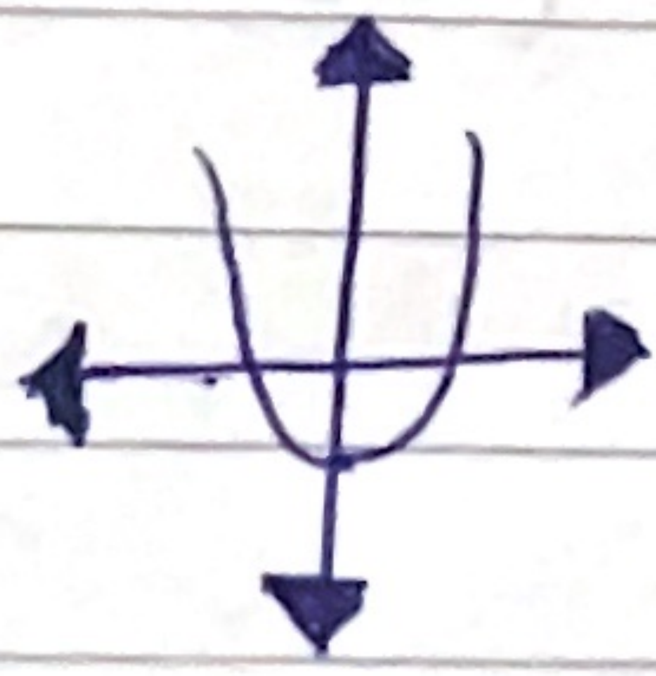
(ii) $f(x) = x^2+2$

(addition with not x variable shifts upwards).



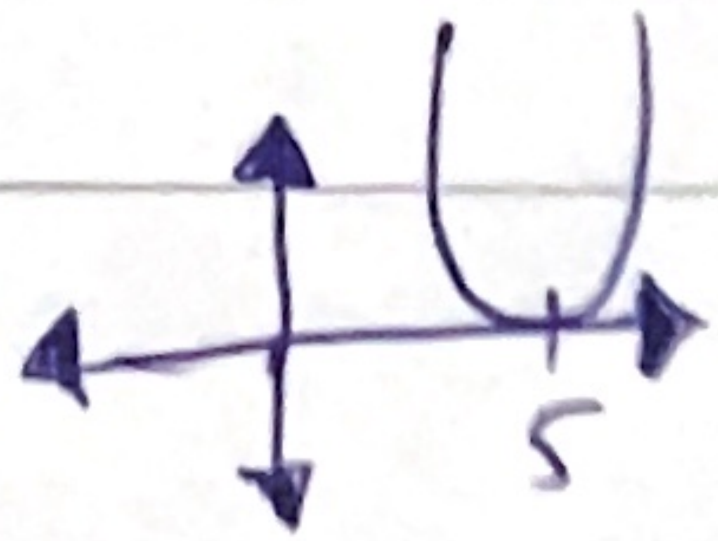
(iii) $f(x) = x^2-3$

(substraction with not x variable shift downwards)



(iv) $f(x) = (x-5)^2$

✓ Sub / addition with variable x shifts to right



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As log is inverse of exponential
∴ Domain $= \mathbb{R}^+$
Range $= \mathbb{R}$

growth rate function:-

$P = P_0 e^{kt}$

P → Current Population
P_0 → initial Population
k → growth rate.
t → time taken

EXERCISE 1.5

Inverse function.

It exisits of any function; (that obeys)

(i) One - One correspondance (Bijective)
↳ every input gives output

(ii) Horizontal line test.

* Composition of a 'function' & its 'inverse' is an identity function.

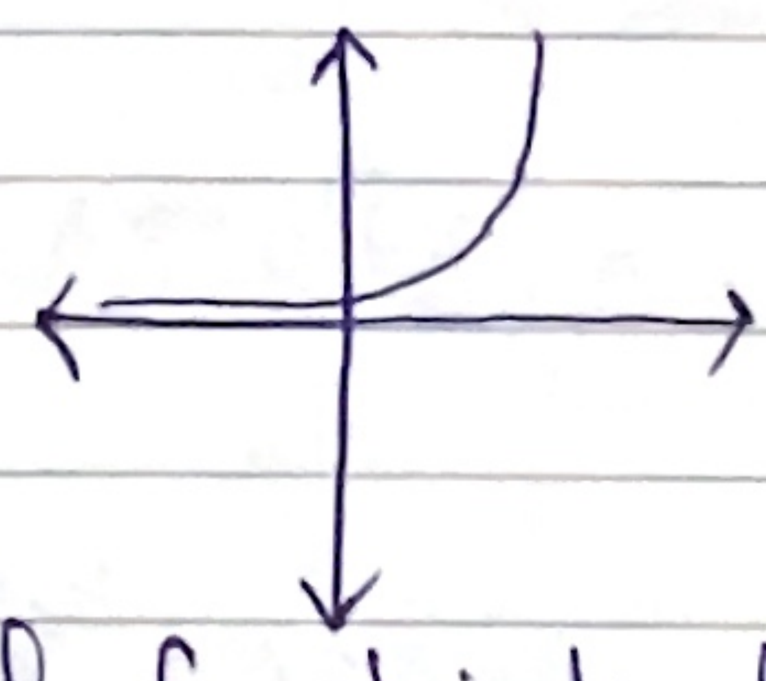
$$f(x) \ \& \ f^{-1}(x)$$
$$f \circ f^{-1}(x) = I.$$

Exponential Function:-

(conditions
$a \in \mathbb{R}$, $a > 0$
$a \neq 1$

Example:- $Y = a^x$

Domain $= \mathbb{R}$
Range $= \mathbb{R}^+$



Converting exponential funct into log (it's inverse).

$Y = a^x$
$X = \log_a Y$

$Y = e^x$
$x = \log_e Y$
$x = \ln y$

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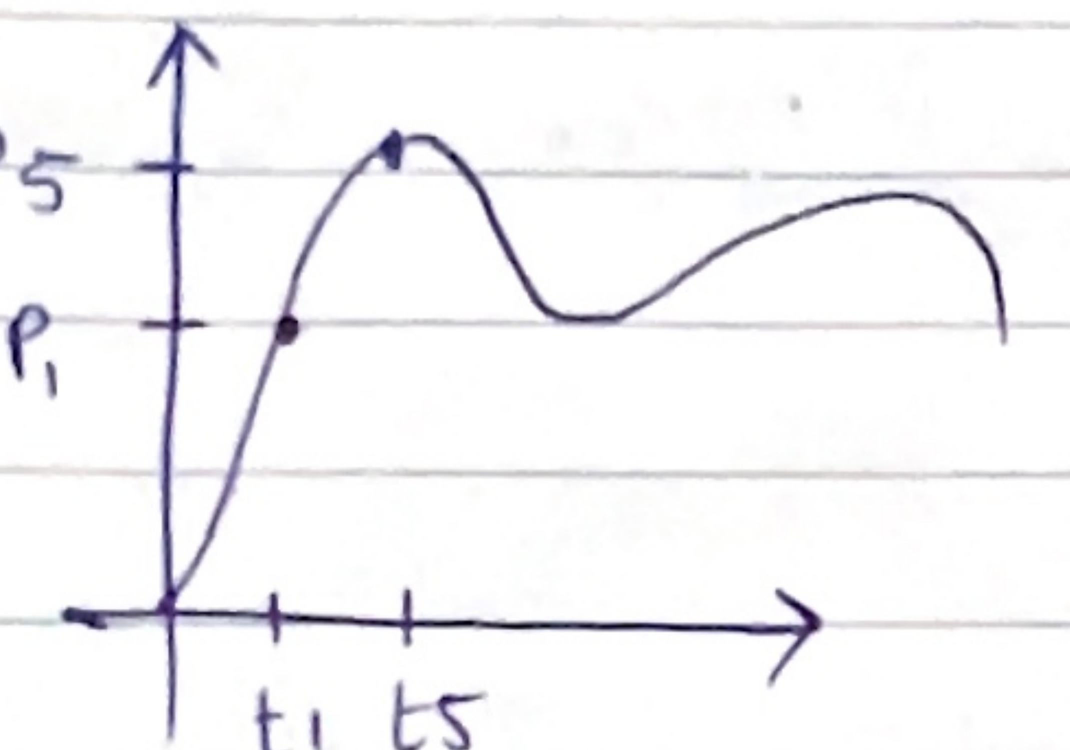
$\lim_{\Delta h \to 0}$
↳ approaches to zero but not exactly zero.

Chapter no. 2 "LIMITS & CONTINUITY"

to find the instanteous rate of change. ↲

Average rate of change can be found by specifing distance and time at 2 specific points.



— fig(i)

$$\frac{\Delta P}{\Delta t} = \frac{P_5 - P_1}{t_5 - t_1}$$

The avg. speed by this method = slope of the secant line.

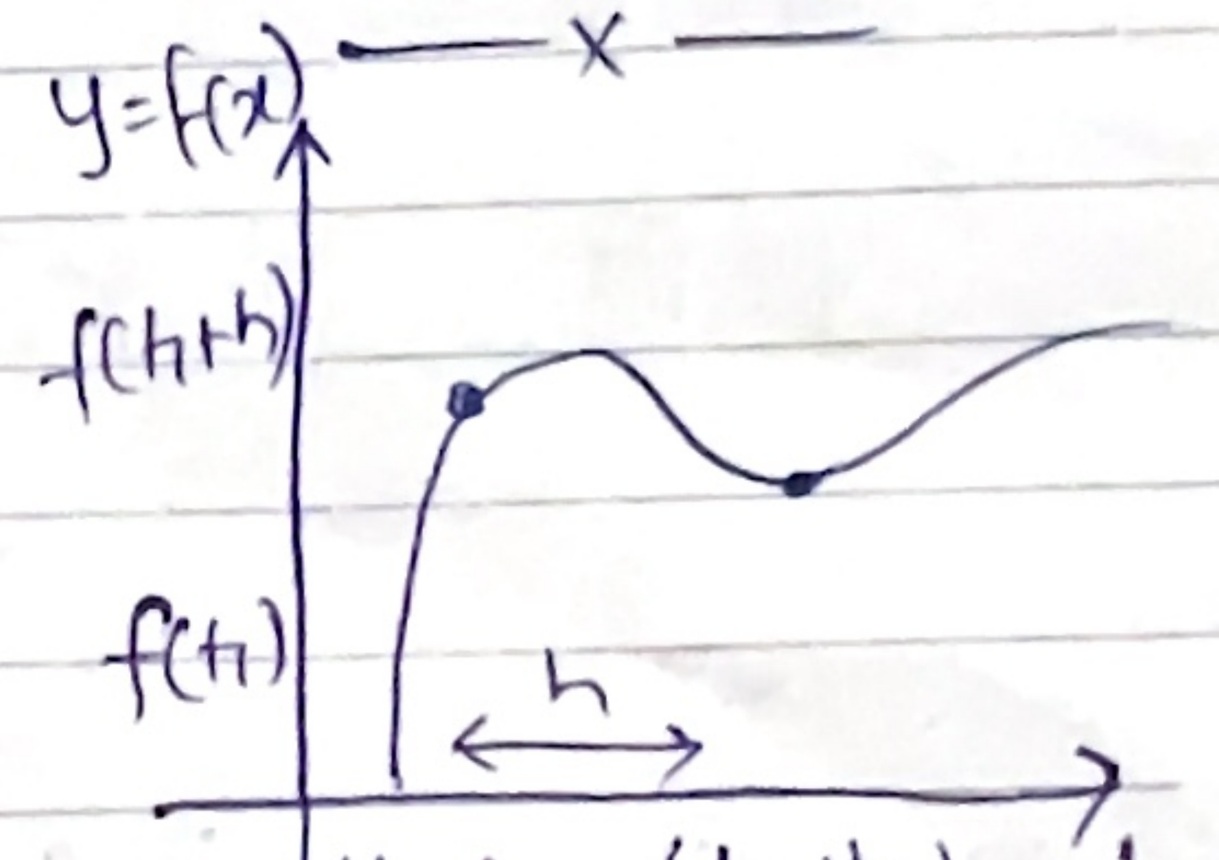
— x —

↳ The avg rate of change of speed can be converted into instanteous rate of change by limiting / reducing the interval intial & final point i.e bringing them closer.

$$\lim_{\Delta s \to t_1} \frac{\Delta P}{\Delta t}$$] inst change of speed.

This is now equal to the slope of the tangent line.

— x —



— fig(ii)

Now by fig(ii)

$$\frac{\Delta f}{\Delta x} = \frac{f(t_1+h) - f(t_1)}{t_1+h-t_1} \Rightarrow \text{Avg } \Delta \text{ of speed.}$$

$$\lim_{\Delta h \to 0} \frac{\Delta f}{\Delta x} = \frac{f(t_1+h - f(t_1)}{h} \Rightarrow \text{instanteous rate of change}$$

Ans! ↗ increasing change = Positive
↘ Neg " = Negative / decrease
const = no change ↙

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EXERCISE 2.2:-

If the Right hand limit and the left hand limit are not equal, limits doesnot exist.

- R.H.L(s) & L.H.L(s) are individually called as the one sided limits.

- Limits are linear operator.

- Before solving the limits undefining factor is removed.

- Quadratic functs = parabolic graphs.

Example:-

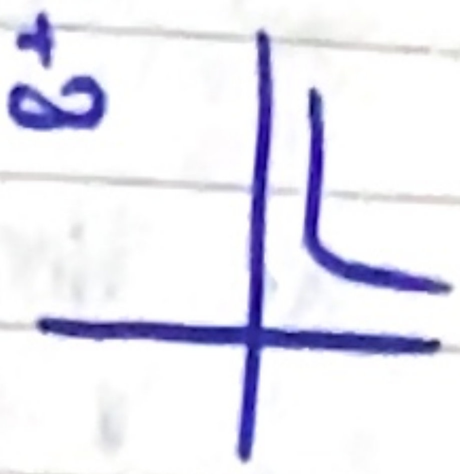
(i) a. $\lim\limits_{x\to 2} \dfrac{x^2-4}{x-2}$ (ii) b. $= \lim\limits_{x\to 2} x+2$.

undefining criteria retain

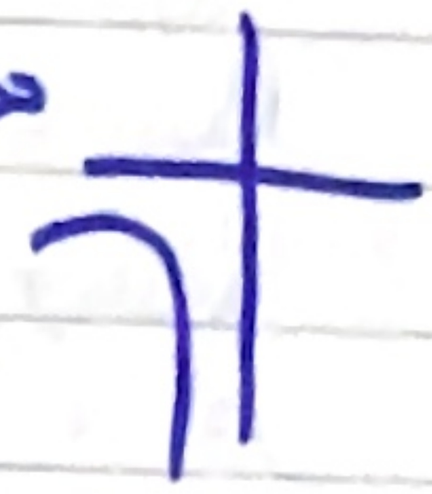
undefining Criteria removed.

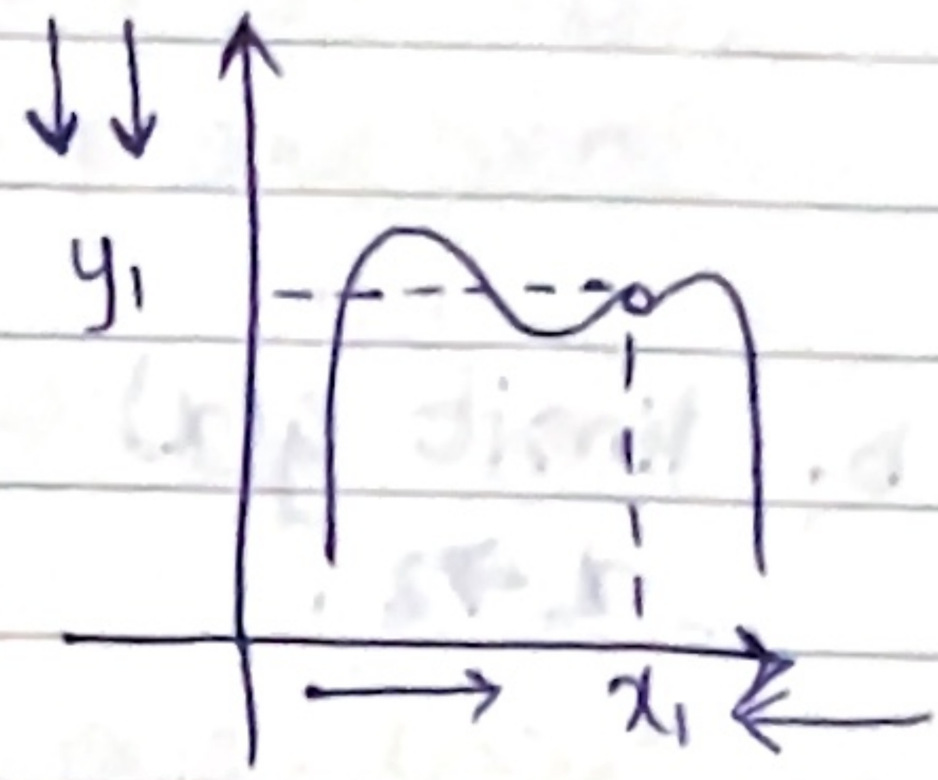
(i) a & (ii) b are different as the first would be undefined at a certain point.

a. $\lim\limits_{x\to 0^+} \dfrac{1}{x} = +\infty$



b. $\lim\limits_{x\to \bar{0}} = -\infty$





"the not in the graph describes that the funct. is undefined at that point".

$\therefore \lim\limits_{x\to x_1^+} f(x) = y_1$

$\therefore \lim\limits_{x\to x_1^-} f(x) = y_1$

This example explains that approach from left and right have the same eff.

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NOTES:-

In the vertical streching or compressing the integer is multiplied outside x.

e.g $f(x) = x^2$ $\Rightarrow$ $2x^2$ (streched vertically).

In horizontal strechting / compressing it multiples inside x.

$$f(x) = (2x)^2$$ (compressed horizontally)

CLASS NOTES!

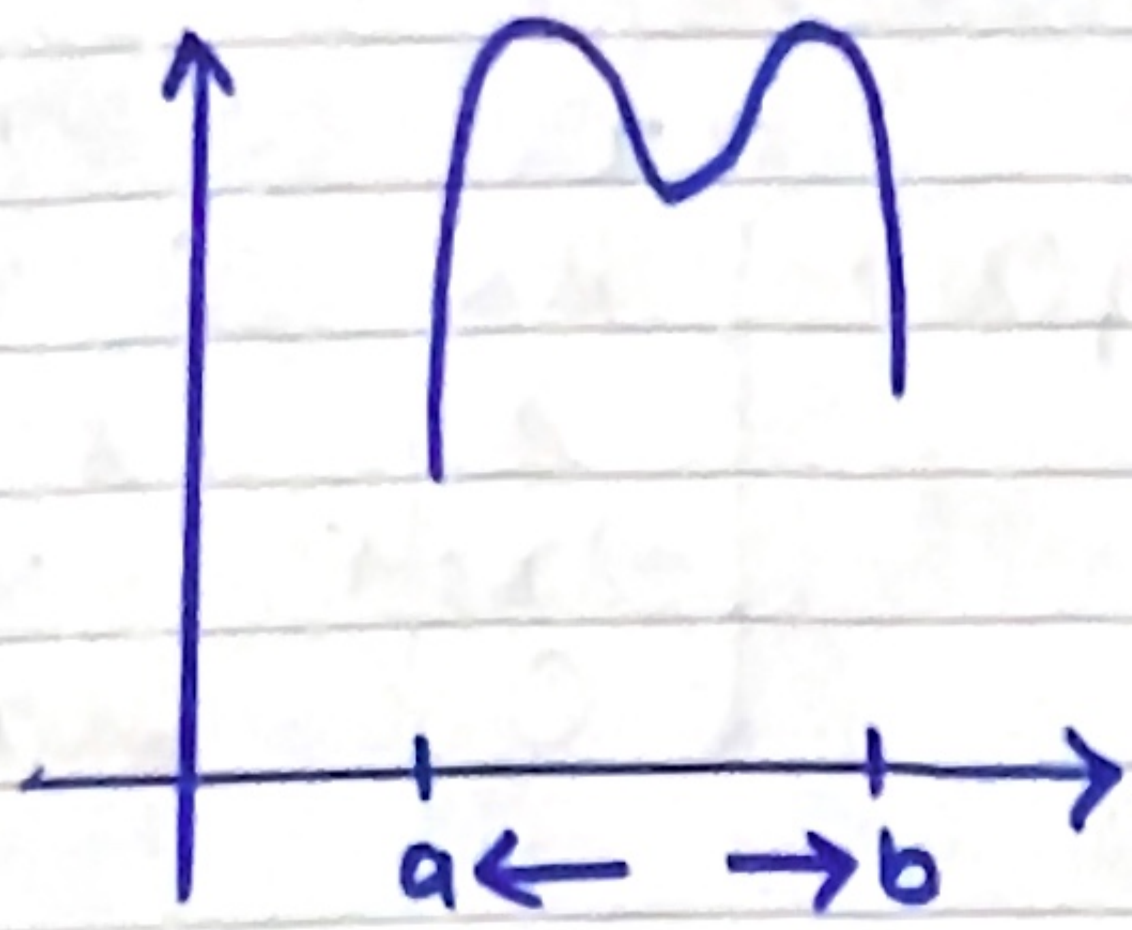


Fig (i)

As the limits in the fig (i) can only be drawn from one side, such are called as one sided limits.

CHECKING CONTINUITY OF A FUNCT :-

Following conditions must satisfy.

(i) $f(x_0) = y_0$ where y_0 should be defined value.

(ii) $\lim_{x \to x_0} f(x_0)$ = exisit (even one sided limit existence questions are feasible).

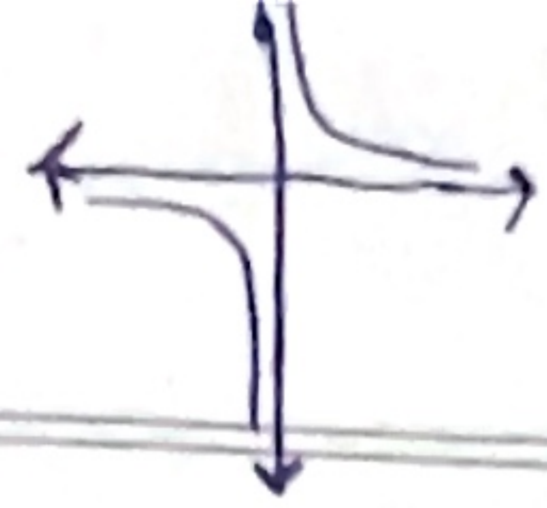
(iii) $f(x_0) = \lim_{x \to x_0} f(x_0)$

* In a piece wise funct a discontinuity can not be removed i.e non-removeable discontinuity.

TWO FUNCTS If continuous at point (c) are :-

(i) $f(x) + g(x)$
(ii) $f(x) \times g(x)$.
(iii) $f(x) \div g(x)$
(IV) $[f(x) \text{ or } g(x)]^n$
(V) $fog(x)$ or $gof(x)$.
(VI) $f^{-1}(x)$ or $g^{-1}(x$

are all continuous.

Graph of $\frac{1}{x}$ 

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CLASS NOTES!

(i) LIMIT AT INFINITY:-

(i) $\lim\limits_{x \to +\infty} f(x)$.

(ii) $\lim\limits_{x \to -\infty} f(x)$.

Example of Limit at infinity:-

$\lim\limits_{x \to +\infty} \frac{1}{x} = 0$.

$\lim\limits_{x \to -\infty} \frac{1}{x} = 0$

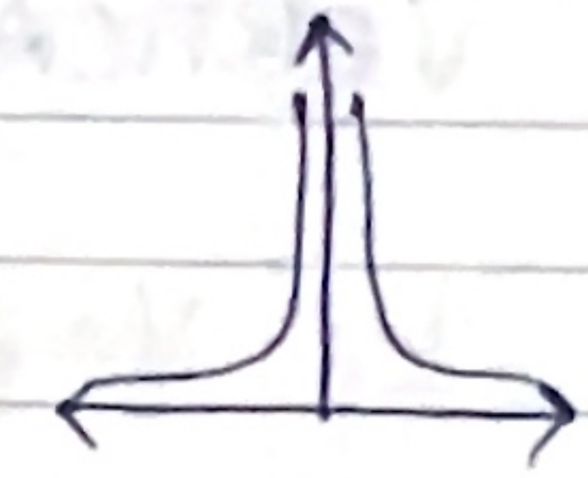
(ii) INFINITE LIMITS:-

Example.

(i) $\lim\limits_{x \to 0^+} \frac{1}{x} = \overset{+}{\infty}$

(ii) $\lim\limits_{x \to 0^-} \frac{1}{x} = \bar{\infty}$

Example:-



(i) $\lim\limits_{x \to 0^+} \frac{1}{x^2} = +\infty$

(ii) $\lim\limits_{x \to 0^-} \frac{1}{x^2} = -\infty$

Rational Functions:- For solving limits:-

(i) Take the highest power variable and divide on both numerator and denominator.

and then;

if NUM < DENOM (Degree) = 0 Ans!

if NUM > DENOM (Degree) = ∞ Ans!

Example:- $\lim\limits_{x \to +\infty} \frac{x^2+1}{x^2+2} \Rightarrow \frac{1 + 1/x^2}{1 + 2/x^2} \Rightarrow \frac{1+0}{1+0} \Rightarrow 1$ Ans!

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"ASYMPTOTES"

HORIZONTAL AYSMPTOTE :-

let $y = f(x)$ be the funct and let $\lim_{x \to \infty} f(x) = a$

or $\lim_{x \to -\infty} = a$ (be the limits 'at' infinity) the $y = a$ is

known as the Horizontal Aysmptote.

VERTICAL AYSMPTOTE :-

let $y = f(x)$ be a funct and $\lim_{x \to a^{+}} = \pm\infty$ or

$\lim_{x \to a^{-}} = -\infty$ (be the infinite limits) $\therefore$ $x \Rightarrow a$ are

known as vertical aysmptotes.

OBLIQUE AYSMPTUTE :-

$f(x) = \frac{P(x)}{Q(x)}$ where $P(x)\uparrow$ & $Q(x)\downarrow$ there an improper fraction.

$Q(x) \overline{\big) P(x)}$ with quotient $z(x)$, $\vdots$, remainder $r(x)$ $\Rightarrow z(x) + \frac{r(x)}{Q(x)}$

$\therefore$ $z(x)$ is a linear term and an oblique aysmptote.

$\Downarrow$

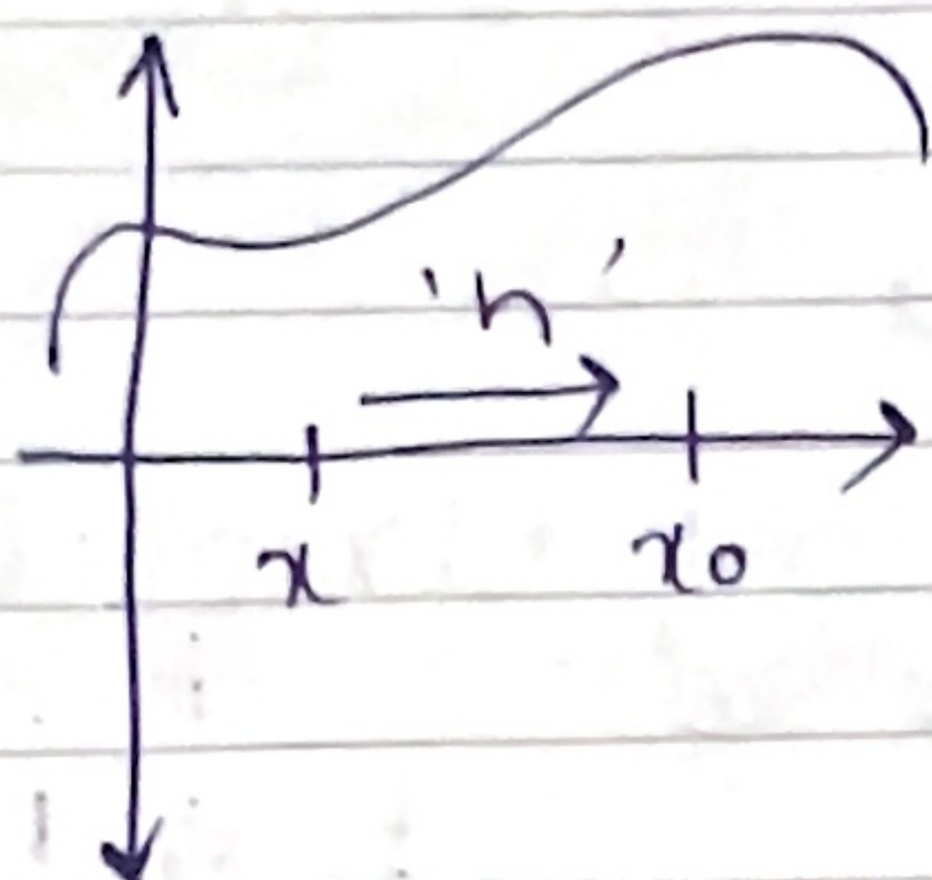
this usually gives v. aysmptotes and often horizontal.

"Differentiability implies that a funct is continuous but vice versa is not possible."

Chapter no. 3

"Anti INTEGRATION" (DIFFERENTIATION).

$$\lim_{h \to 0} \frac{f(x+h) - f(x_0)}{h} \Rightarrow \text{slope} \Rightarrow \lim_{x \to x_0} \frac{f(x_0) - f(x)}{x_0 - x} = f'(x)$$



(The two point interval form).

Therefore we conclude; that the phenomena where the limit shortens and in the interval b/w 2 points;

(i) Instanteous rate of change
(ii) slope of a tangent line.
(iii) first derivative of a funct

} same.

$f'(x) \Rightarrow$ Prime notation.

$\left.\frac{dy}{dx}\right|_{x=?} \Rightarrow$ lebaniz notation.

THE MOD FUNCTION:-

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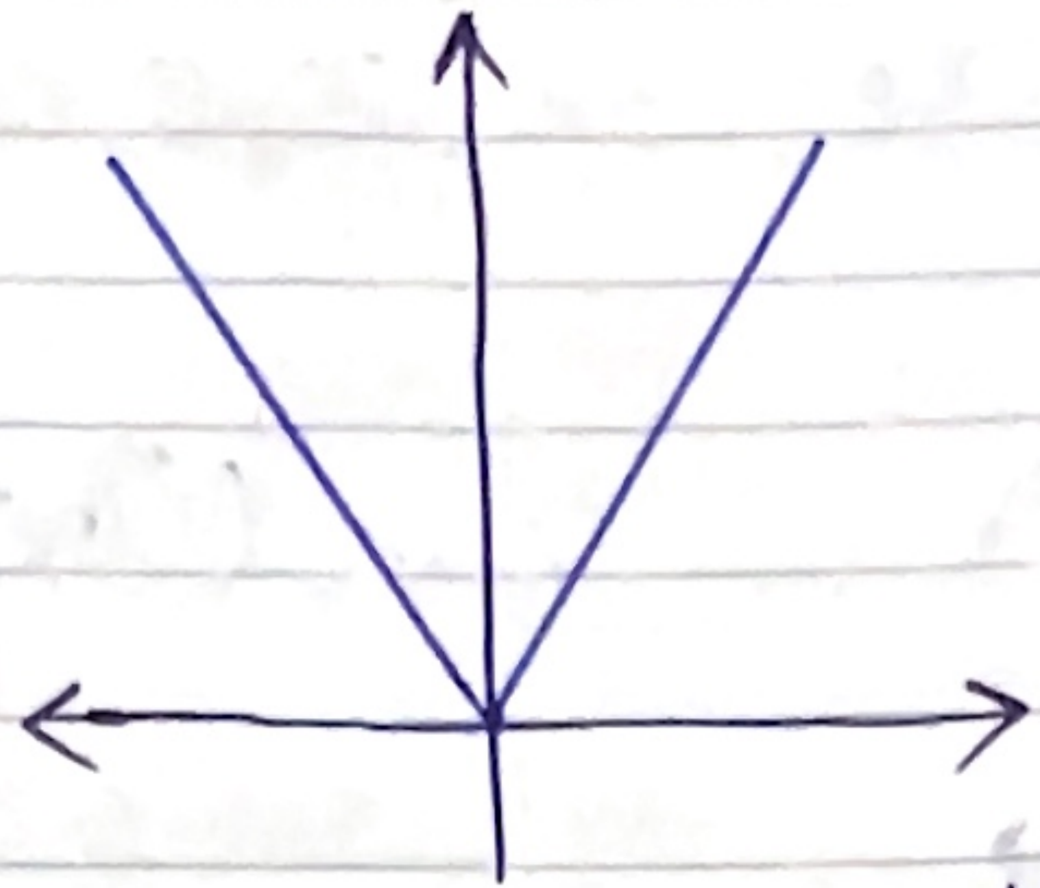
The vertex of mod function $y = |x|$ can plot many tangent lines, therefore the derivative doesnot exisit at that point.



Proof:-

$f'(x) = |x|$

$\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\lim\limits_{h \to 0} \dfrac{|x+h| - |x|}{h}$

R.H.L $\neq$ L.H.L

$\Rightarrow \lim\limits_{h/x \to 0^+} \dfrac{+(x+h) - (x)}{h}$

$\dfrac{h}{h} \Rightarrow 1$

$\Rightarrow \lim\limits_{h \to 0^-} \dfrac{-(x+h) - (x)}{h}$

$\Rightarrow -\dfrac{h}{h} \Rightarrow -1.$

"EXERCISE 3.1"

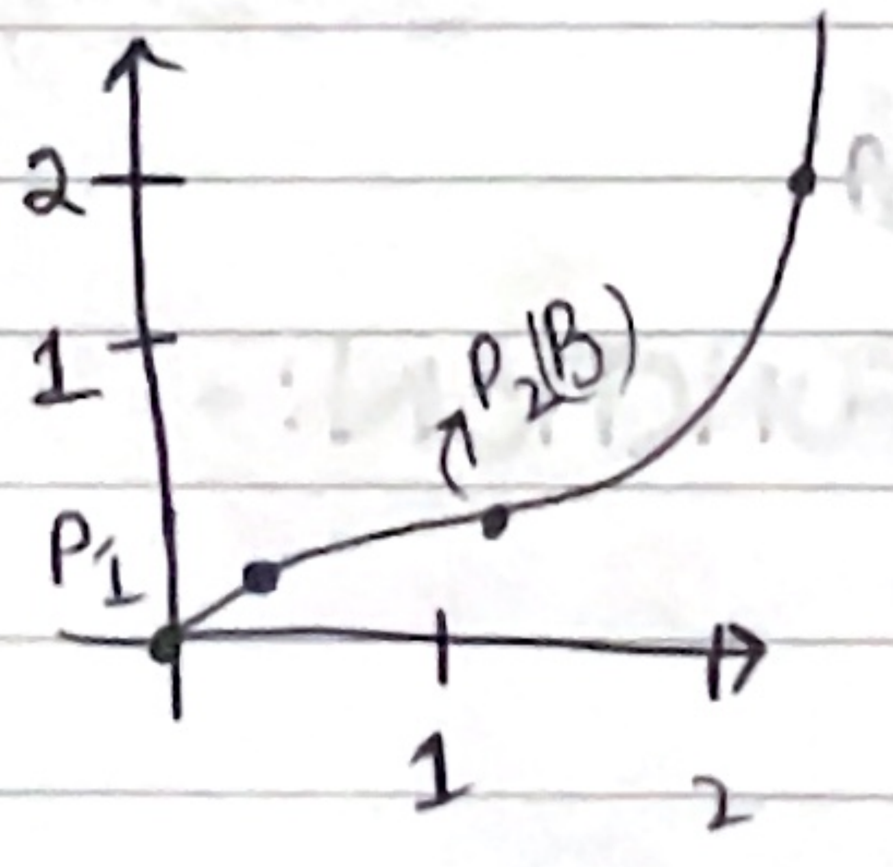
Exercise 1-4, show the tangent points by rough estimation.

(1)

For P_1 $x_0 = 0$

let point A

such that $h = 0.2$.



NOTES!

General form of Polynomial.

$$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^2 \ldots$$

General form of derivative of monomial.

$$\frac{dx^n}{dx} = n(x^{n-1})$$ Power Rule.

$$\frac{dc}{dx} = 0$$ where c is const.

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Chapter no. 14.
Partial Derivatives

(i) Multivariable functions

a. $y = f(x,y)$. double variable function.

b. $y = f(x,y,z)$ triple variable funct.

c. $y = f(x_1, x_2, x_3 \dots x_n)$ nth time of funct.

In multivariable funct :-

* a linear function is always a plane structure.
* Non-linear " give a surface/3D object.

(partial)
Taking derivatives of multivariable funct.

(i) w.r.t x

a. $z = f(x,y)$

$$\frac{\partial f}{\partial x} / f_x(x,y) = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}.$$

(ii) w.r.t y.

a. $z = f(x,y$.

$$\frac{\partial f}{\partial x} / f_y(x,y) \lim_{h \to 0} \frac{(x, y+h) - f(x,y)}{h}$$

$$Z = f(x,y) = xy^2 + x + y.$$

$$\frac{\partial f}{\partial x} = f_{(x)} = \frac{\partial}{\partial x}(xy^2) + \frac{\partial x}{\partial x} + \frac{\partial y}{\partial x}.$$

$\because$ Treat 'y' as a const.

$$f_x = y^2 + 1 + 0$$

$$f_x = y^2 + 1 \text{ Ans!}$$

$$\frac{\partial f}{\partial y} = f_y = \frac{\partial}{\partial y}(xy^2) + \frac{\partial(x)}{\partial y} + \frac{\partial(y)}{\partial y}.$$

$\because$ Treat x as a const.

$$f_y = x(2y) + 0 + 1$$

$$f_y = 2xy + 1. \text{ Ans!}$$

* when the extreme values are described at a subdomain they are called the local minima & local maxima
* When they are described at the whole ~~domain they are called as global~~ domain they are called as global Minima & maxima.

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Chapter no-4
EXTREME VALUES OF FUNCT:-

Ex 14.1

let $y = f(x)$ (a,b)

Minima:-

If $(x_0) \in (a,b)$ & $(a,b) \in D$ and $f(x_0) \leq f(x)$ $\forall x \in (a,b)$.
then it is Minima.

Maxima:-

If $(x_0) \in (a,b)$ & $(a,b) \in D$ & $f(x_0) \geq f(x)$ $\forall x \in (a,b)$
then it is maxima.

STEPS TO FIND THE EXTREME VALUES:-

let $f(x)$ be a funct defined at $[a,b]$.

(i) find $f(a)$ & $f(b)$ endpoints

(ii) find $f'(x) = 0$ critical point

finally compare $f(a), f(b), f'(x)=0$

$f'(x)=0$ will give us another value of x then find $f(x)$.

* It is not necessary for a funct to have (extreme) values that exisit globally

e.g $f(x) = (x+1)$.

However the extreme values can be found by sepcifying the interval or Domain.

* When the funct is defined everywhere the Critical value will not exisit in such domain.

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NOTES:-

Ex. 4.3:-
FIND THE INTERVAL AT WHICH THE FUNCT IS INCREASING & DECREASING.

a. $f'(x) = (x)(x-1)$

Critical point

$0 = (x)(x-1)$.
$x = 0$ & $x = 1$.

$-\infty$ ... 0 ... 1 ... $+\infty$

(1) Now checking at $(-\infty, 0)$
let in this interval a point -1

$f'(-1) = (-1)(-1-1)$
$\Rightarrow 2 > 0$
$\therefore$ The funct is increasing at this interval.

(2) Now checking. $(0, 1)$

$f'(0.5) = (0.5)(0.5-1)$
$= -\frac{1}{4} < 0$

$\therefore$ hence the funct is decreasing at this interval.

(3) Now checking. $(1, +\infty)$

$f'(2) = (2)(2-1)$
$= 2 > 0$
hence the funct is increasing.

To check increasing & decreasing of the funct and; we find the critical points & set intervals to check for increasing & decreasing.

We can find local Minima & local Maxima by using critical point.

For undescribed / Natural / unrestricted domain absolute Minima & maxima cannot be found.

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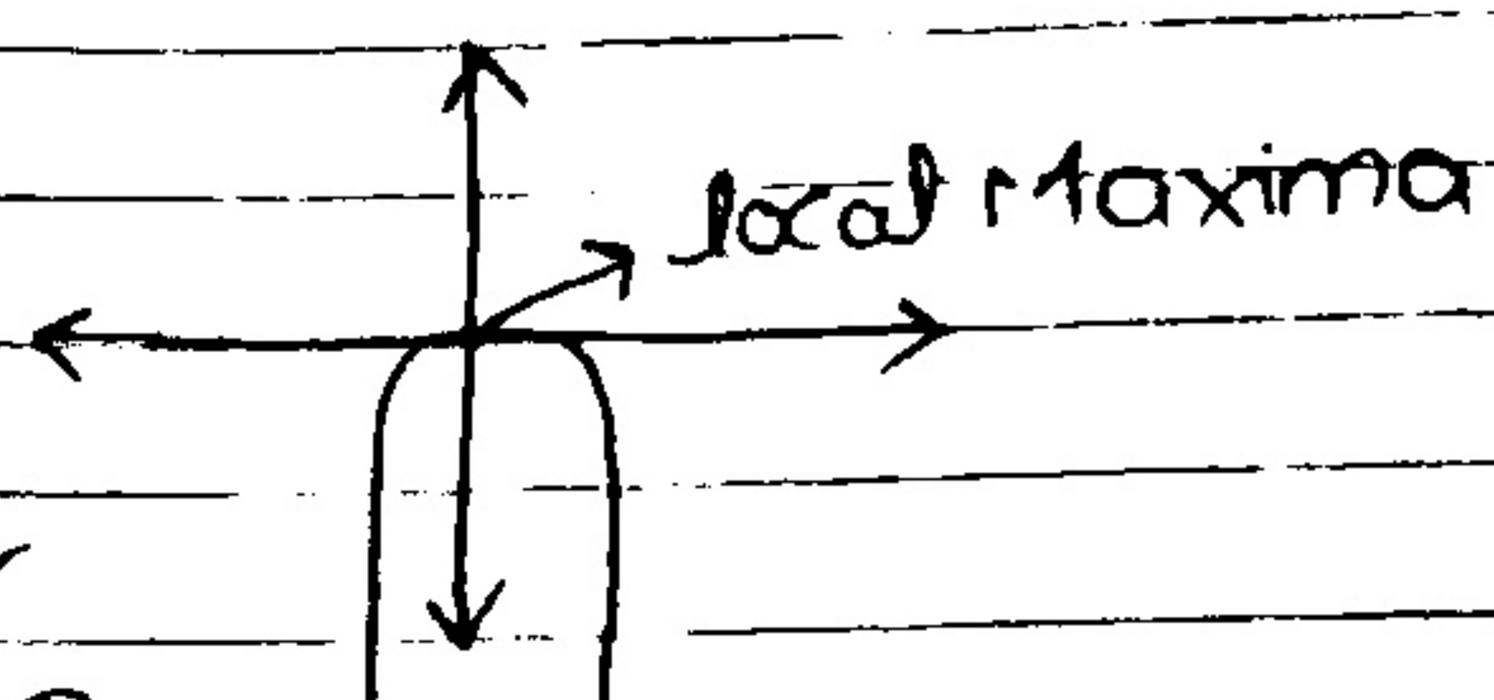
NOTES 4.4:-

CONCAVE UP:-

For a defined domain of concave up condition.
⇒ 2nd order derivative is always positive.
⇒ The first order derivative at the point where it is known is zero. called as the local/global Minima.

CONCAVE DOWN:-

For a defined domain of concave down cond.
⇒ 2nd order derivative is negative.
⇒ The first order derivative at the point where it is zero is called at the local/global Maxima.



* when the secondorder derivative is equal to zero It is known as the point of inflexion (the point where the concavity is changing and thus cannot be noted).

EXAMPLE:-

$y = x^3 - 3x + 3$. and

$y' = 3x^2 - 3$

$y' = 0$

$3x^2 - 3 = 0$

$x = \pm 1$ Critical point.

$y'' = 6x = 0$ $\boxed{x = 0}$ point of inflexion!

Now using 2nd order derivative test for minima & maxima.

$y'' = 6x$

$y''(+1) = 6 > 0$ local Minima

$y''(-1) = 6 < 0$ local Maxima.

Date: ______ 20__
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NOTES:-

Ex 4.5

Shortcut phenomena of eliminating the indeterminat forms of linear derivatives and functions irrespective of Quotient or product Rule.

1^{∞} , 0^{0} , $0/0$, ∞/∞ , $\infty - \infty$

Theorem:-

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f'(x)}{g'(x)}.$$

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MATHEMATICS (INTEGRATION).

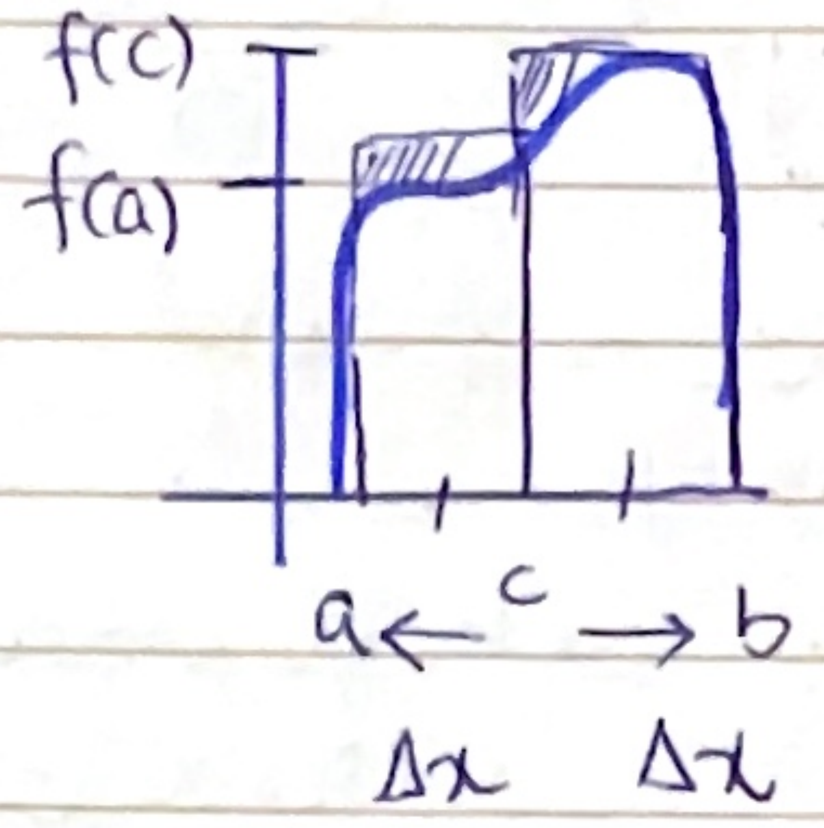
Chapter no. 5 (app. of integration).

▷ For a function $f(x)$ consider;
(i) $f'(x)$ derivative.
(ii) $F(x)$ antiderivative / integration.

$\therefore$ $F'(x) = f(x)$

Note:- The derivative of the antiderivative returns the orignal function.

▷ Definite integrals are used to find the area of irregular shaped bodies which are not well defined by formulas.



⇒ Total Area under the curve is now equal to the area covered by the (triangle^x) rectangles drawn.

⇒ The shadad region has a chance of error as it is not a part of the original area of the curve. We must eliminate the error by reducing the size and hence increasing the no. of individual rectangles drawn.

Galaxy

n = intervals.

From the figure.

(length x breath + length x breath)

$A \approx f(a)\Delta x + f(c)\Delta x.$

=> This phenomena is known as the Rieman solution.

Now; increasing ▭ to reduce the error.

$$A \approx f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \dots n$$

$$A \approx \sum_{k=1}^{n} f(x_k)\Delta x$$

↳ const.

$$\therefore A \approx \Delta x \sum_{k=1}^{n} f(x_k)$$

Δx (change/average) $= \frac{b-a}{n}$.

$$A \approx \frac{b-a}{n} \sum_{k=1}^{n} f(x_k)$$

Consider $\lim_{n \to \infty}$ (when infinite many rectangles/intervals are drawn).

$$A \Rightarrow \lim_{n \to \infty} \frac{b-a}{n} \sum_{k=1}^{n} f(x_k)$$

=> as we were summing continuous values

$$A \Rightarrow \int_a^b f(x)\,dx.$$

~x~

→ $\int$ was introduced by lebaniz

Galaxy