Duality principle :-
in an identity if union is replaced by intersection and intersection by union and also if we replace universal set (S) with an empty set (ϕ) and empty set with universal set, it then identity remains

Duality principle :-

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Index

Date

set(s) of all possible outcomes = universal set.

Probability =>

$P(A)$:- $\frac{\#\text{ of ways } A \text{ can occur}}{\text{total no. of outcomes}}$

Steps of assigning a probability.

① Define an experiment.

② Define all the possible outcomes. $(S) = \{1,2,3,4\}$

③ Define what outcomes you require. $(A) \Rightarrow$ even no $\Rightarrow \{2,4\}$

④ Assign as probability $\hookrightarrow \frac{\text{no. of ways A occur}}{\text{total ways}} \Rightarrow \frac{2}{4}$

⑤ should make sense!

Probability operations.

(4) $P(A') = 1 - P(A)$

① union :-

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

② union :- (mutally exclusive)

$P(A \cup B) = P(A) + P(B)$

③ $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Probability conditions.

① $0 \le P(E) \le 1$, event

② $P(S) = 1$, samplespace

③ probability assigned to mutally exclusive events should make sense.

$E_1 \cap E_2 = \phi$

$P(E_1 \cup E_2) = P(E_1) + P(E_2)$

Day / Date

python Certification (Before Mid 1 week)

software Evaluation test (1 week after Mid)

PROBILITY :-

lack of knowledge; in which we use the available information to estimate the best guess; (to plan, design & take steps to control the situation better).

SET OF POSSIBILITIES :-

For any given event, the universal set or sample space of any event is known as the set of possibilities.

Example:

Roll a dice [1, 2, 3, 4, 5, 6] → set of possibilities.

however the objects '1', '2', '3', '4' .... are called the elements of the set

① $B - C \neq C - B$ (for any two sets B & C)

Noncommutative

those present in B but not in C

those present in B but not in C

Galaxy

Associative property:- $(A \cup B) \cup C = A \cup (B \cup C) = A \cup B \cup C$

$(A \cap B) \cap C = A \cap (B \cap C) = A \cap B \cap C$

Day / Date

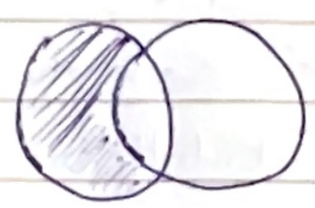
② $B \cup C = C \cup B$ (commutative)

③ $B \cap C = C \cap B$ (commutative)

④ $B - C \neq C - B$ (non-commutative)

⑤ $A - (B \cup C) = \phi$ (empty set).

Note :- $A - B$ or $A \setminus B$ is the set of all values present in A that are not members of B.



A B

* Two sets are 'mutally disjoint / exclusive' if they have no common element

$A \cap B \quad \phi$

* A universal set is the largest set encompassing (all) set of objects under discussion. (denoted as S or U)

* The complement of set A is a set of all elements not in A.

$\bar{A} = U - A.$

Galaxy

Distributive property: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Day / Date

* Two sets are equal if they both are subsets of eachother.

if $A \subseteq B$ and $B \subseteq A$ then $A = B$.

METHODS OF DEFINING A SET:-

(list all element) **TABULAR**

(set a certain Rule) **RULE**

OPERATION.

$A = \{$Abdullah, Haris, Ahmad ... $\}$

$A = \{$all students of ES112$\}$

$C = B - A.$

* Union can be denoted as; example.

$$C = A_1 \cup A_2 \cup \ldots \cup A_n = \bigcup_{n=1}^{N} A_n.$$

* Intersection can be denoted as.

$$D = A_1 \cap A_2 \cap \ldots \cap A_N = \bigcap_{n=1}^{N} A_n.$$

De-Morgan's law:-

$$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$$

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$

Galaxy

① Range of Probability $0 \le x \le 1$.

AXIOMS OF PROBILITY

Day / Date

② Probability of sample space should be 1 $P(S) = 1$.

(Error handling) Result of probability is always $0 \le x \le 1$

③ $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

MULTIPLE EVENTS:-

JOINT PROBABILITY:-

Consider, "two event occuring at the same time".

let A_1 & A_2 be two events,

A_1 = Ali is sleeping.
A_2 = Ali is in lecture.
$(A_1 \cap A_2)$ = Ali is sleeping in lecture.

CONDITIONAL PROBABILITY:-

DEPENDANT EVENT:-

A certain condition reduces the given sample space.

let A & B;

required event

reduced sample space

$P(A|B) = P(A \text{ given } B) = \frac{P(A \cap B)}{P(B)}$

$P(B) \neq 0$

for these formulas always use original sample space.

Example:

Sample space $S = \{1, 2, 3, 4\}$
condition : only odd numbers.
$B \Rightarrow \{1, 3\}$.
required event $\Rightarrow A = 3$
$P(A \cap B) = P(3)/P(B)$

$P(A)\,P(B) = P(A \cap B)$

$P(A \cup B) = P(A) + P(B)$ or $P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Day / Date

$\frac{P(3)}{P(1,3)}$ $\Rightarrow$ $\frac{\text{no of way 3 can occur}}{\text{orig total sample space}}$ $\frac{\text{no. of ways 1, 3 can occur}}{\text{orig total sample space}}$

$\Rightarrow \frac{1/4}{2/4} \rightarrow \frac{1}{2}$ Ans!

Independant Event:-

knowledge of one event doesnot effect the other event

let A & B are two events.

test 2
$P(A \cap B) = P(A)P(B)$

$P(A|B) = P(A) = \frac{P(A \cap B)}{P(B)}$ → test 1.

PROSTIER OR PROBABILITY:-

System (α, β) → components.
↳ fails (if α & β fails)

$P(\alpha) = P(\alpha \text{ fails}) = 0.01$
$P(\beta) = P(\beta \text{ fails}) = 0.005$

$P(\beta/\alpha) = P(\beta \text{ fails if } \alpha \text{ has failed})$
$\Rightarrow 0.015$

Q. What is the probability that α will fail given that β has failed?

i.e $P(\alpha|\beta) \Rightarrow ?$

Day / Date

$$P(\alpha|\beta) = \frac{P(\alpha \cap \beta)}{P(B)}$$

$$P(\beta)\ P(\alpha|\beta) = P(A \cap B) \quad \text{—①}$$

Similary ;

based on (i)

$$P(A)\ P(\beta/\alpha) = P(\alpha \cap \beta) \quad \text{—②}$$

∴ By equ ② & ①

$$P(\beta \cap \alpha) = P(\alpha \cap \beta)$$

$$P(A)\ P(\beta/\alpha) = P(B)\ (P(\alpha|B)$$

FORMULA:-
BAYÉS THEOREM

$$P(\alpha|\beta) = \frac{P(A)\ P(\beta/\alpha)}{P(\beta)} \quad \text{—①}$$

Posterier. Prior

Input values & solve using ①.

Day / Date

* most no. of possibilities when order matters and repitition is allowed.

Total Probability:-

FORMULA:- $P(B) = \sum_{\text{for all } k} P(B/A_k)\ P(A_k)$

CONDITIONS.

① If A_k are multally exclusive.
② Union of all A_k = sample sets.

How to solve conditional probabilities?

Example:-
Conditional probability.

let

$A = \{1, 2 \text{ or } 3\}$

$B = \{2, 3 \text{ or } 4\}$

$S = \{1, 2, 3, 4, 5, 6\}$

$P(A|B) = ?$
$= \frac{P(A \cap B)}{P(B)}$.

$P(A \cap B) \Rightarrow \frac{\text{size of } (A \cap B)}{\text{size of } P(S)} \Rightarrow \frac{02}{06} \Rightarrow \frac{1}{3}$.

$P(B) \Rightarrow \frac{\text{size of } B}{\text{size of } S} \Rightarrow \frac{3}{6} \Rightarrow \frac{1}{2}$.

$\Rightarrow 1/3 \,/\, 1/2 \Rightarrow 2/3$. Ans!

Day / Date

UNDERSTANDING ORDER & REPITITION.

Order and Repitition matters.

n = total no. of objects & r = number of place to fill

Order matters ?	Repetition Allowed ?	Formula.
Yes	yes.	n^r
No	No	${}^nC_r = \frac{n!}{(n-r)!\,r!}$
Yes	No	${}^nP_r = \frac{n!}{(n-r)!}$

Day / Date

$0 \to 1$ $\quad 1 \to \infty$

Problem no. 1.

Consider the type of electric/gas dryer bought by 5 different customer; at a certain store.

a. If the probability that at most one of these purchases an electric dryer is .428. What is the probability that at least two purchases, this electric dryer?

→ it means either one or none

→ minimum limit (covers all set after limit).

let E) represent that at most one purchases an electric dryer ; P(E) = 0.428.

let (E)' represent that atleast (one purchases an electric dryer ; P(E)' = 1 − P(E)

↑ +1 ⇒ two)

as Formula $P(E) + P(E)' = 1$ → P(S).

$P(E)' = 1 - P(E)$

$= 1 - 0.428$

→ ____ Ans!

b. If Probability all purchases gas) = 0.116 and probability (all five purchase electric) = 0.05. What is the probability that atleast one of each is purchased.

$P(S) = 1$

$P(A) = 0.116$ ⇒ $P(S) = P(A) + P(B) + P(U) \to 1$

$P(B) = 0.05$

$P(U) = 1 - (0.116 + 0.05)$

$P(U) \to ?$ (unknown; to find)

$P(U)$ Ans!

Galaxy

Galaxy

Day / Date

Module no. 2
Collecting Probabilities.

Random event $\xrightarrow{\text{draw}}$ Random Variable.

discrete (0, 1, 2 ...) | Continuous (range certain)

(i) Random variables are drawn from a random event. (ii) They have 2 types
(iii) They obey their certain conditions.
e.g $P(X = \infty) = 0$. (etc)

Concept.
let X be a discrete Random Variable
let Y be a continuous Random Variable.

diff out comes let 'x'

X = no. of mobile a students has
Y = height of students in cm.

different outcome let 'y'

To represent the different values of X and Y we can use:

(i) Table (ii) graph (iii) Formula.

Representations through Formula
Discrete Random Variable.

① **Probability Mass Function:- (PMF) $f(x)$**

let X be a discrete random variable with many possible outcomes x.

$f(x) = P(X = x)$

$\boxed{\text{condition}}$

① $\sum_{i=1}^{n} f(x_i) = 1$

② $f(x_i) \geq 0$

③ $f(x_i) = P(X = x_i)$

② **CUMMULATIVE DISTRIBUTION FUNCTION (CDF) $F(x)$**

Sometime we talk about the range of random values (discrete) of a random variable.

let;
What is the probability that the next student entering the room has 2 mobiles or less?

$P(X \leq 2)$

$F(x) = P(X \leq x)$. Possibility that X takes a value x or less.

CDF in terms of PMF.

$$F(x) = \sum_{x_i \leq x} f(x_i)$$

$\boxed{\text{conditions}}$

① $F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$

② $0 \leq F(x) \leq 1$

③ If $x \leq y$ then $F(x) \leq F(y)$

$\boxed{\text{Note: } F(2) = P(X \leq 2) = f(1) + f(2)}$!

③ **JOINT PROBABILITY DISTRIBUTION (JPD) $f(x,y)$**

Sometimes we want to find the probability of two things happening at the same time.

let, What is the probability that the next student entering the room has two mobiles and no pen?

X = number of mobiles he has.
Y = number of pens he has.
$P(X = 2 \text{ and } Y = 0)$

JP = 3x graph
(MP = 20 graph.

Probability that X takes the value 'x' and Y take 'y'

$$f(x,y) = P(X = x, Y = y)$$

$\boxed{\text{condition}}$

① $f(x,y) \geq 0$

② $f(x,y) = P(X=x, Y=y)$.

③ $\sum_x \sum_y f(x,y) = 1$.

— x —

Application of JDP.

For any region A in an xy plane,
$P[(X,Y) \in A] = \sum\sum_A f(x,y)$.

Q. let $A = \{x > 1, y < 3\}$

Then we can find area by using conditions of A in JDP.

a 2 balls drawn has no specifics

let a table of sample set.

according to our condition

x, y	1	2	3
1	(1,1)	(1,2)	(1,3)
2	(2,1)	(2,2)	(2,3)
3	(3,1)	(3,2)	(3,3)

$P[(X,Y) \in A] = f(2,1) + f(3,1)$
$f(2,2) + f(3,2)$

$\Rightarrow \frac{1}{6} + \frac{1}{6} + 0 + \frac{1}{6} \Rightarrow \frac{1}{2}$

Galaxy

④ MARGINAL DISTRIBUTION :-

In senerios where we are interested in question such as given that joint distribution of X and Y, find the distribution of X.

→ This may be the case, for instance we have an easy access to the joint distribution.

Marginal distribution of X $\Rightarrow g(x) = \sum_y f(x,y)$

Marginal distribution Y $\Rightarrow g(y) = \sum_x f(x,y)$

↓

sum of $f(x,y)$ for all possible values of X.

⑤ CONDITIONAL DISTRIBUTION :-

$f(x|y)$

The probability of an event given that another event has taken place.

Conditional distribution of random variable(s) Y given that X = x

$$f(y|x) = \frac{f(x,y)}{g(x)}$$

marginal dist $g(x) > 0$

Galaxy

The conditional probability of X given that $Y = y$ is

$$f(x|y) = \frac{f(x,y)}{h(y)} \qquad h(y) > 0$$

$h(y)$ → marginal distribution

statistically independant.

↳ to show ; prove.

$f(x|y) = g(x)$

$f(y|x) = h(y)$

$f(x,y) = g(x)h(y)$

} if proved they would be statistically independant.

— x —

Converting the distributions for a continuous Random Variable.

① PMF $\xrightarrow[\text{variable}]{\text{Continous}}$ PDF $\quad f(x) \neq P(X = x)$

$P(X = x) = 0$

Because probability over a continuous range of a single point is equal to zero.

let $P(a < x \leq b) = P(a < x < b) + \cancel{P(b)}^{0}$

$= P(a < x < b)$

PDF MI

JDF CD

if the range is given it forms the limits of integration or else we take $-\infty$ to ∞

② CDF (continuous variable)

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

gives complete picture of probability ranging from 0 to 1.

also

$$F(x) = \int_{-\infty}^{\infty} f(x) = 1.$$

$f(x,y) =$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = P(A|B)$$

$$f(x|y) = \frac{f(x,y)}{f(y)}$$ → MD of Y.

$f(x,y) = f(y)f(x|y)$ Not ID.

$f(x,y) = f(x)f(y)$: $X \perp\!\!\!\perp Y$ ID

if $E[X]$ of two distributions are same we use $V[X]$ to distinguish.

Day / Date 4/03/24.

Module : 3 Informations of distributions.

(1st Moment)

(i) **Expected value : (μ_x or $E[X]$)**

For $X \Rightarrow$ discrete Random Variable.

$$E[X] = \sum_{\text{for all } x} x f(x) \qquad E[c] = c$$

For $X \Rightarrow$ continuous Random Variable

$$E[X] = \int_{-\infty}^{\infty} x f(x).$$

Expected value holds linearity

$E[aX+bY] \Rightarrow aE[X] + bE[Y]$

$E[g(x)] = \sum_{\text{for all } x} g(x) f(x)$

(2nd Moment)

(ii) **Variance : (σ_x^2 or $V[X]$)** $V[c] = 0$

↳ degree of spread of RV about Mean value.

↳ measure of variations in values of distribution

DISCRETE RV.

$$V[X] = E[(X - \mu_x)^2]$$

$$= \sum_{x_i} (x_i - \mu_x)^2 f(x_i)$$

CONTINUOUS R.V.

$$V[X] = \int_{-\infty}^{\infty} (X - \mu_x)^2 f(x_i)$$

Day / Date

Chapter no. 4

Mean of the random variable.
Mean of the probability of distribution of X.

Let X be a random variable with probability distribution $f(x)$. The mean / expected value of X is.

where X is discrete. $\mu = E(X) = \sum_{x} x f(x)$.

where X is continuous $\mu = E(X) = \int_{-\infty}^{+\infty} x f(x)\, dx$.

Example:-

$$f(x) = \frac{\binom{4}{x}\binom{3}{3-x}}{\binom{7}{3}}$$

$x = 0, 1, 2, 3$ no of components selected.

$$f(0) = \frac{\binom{4}{0}\binom{3}{3}}{\binom{7}{3}} = 1/35$$

$f(2) = 18/35$

$f(3) = 4/35$.

$$f(1) = \frac{\binom{4}{1}\binom{3}{2}}{\binom{7}{3}} = \frac{12}{35}$$

P(something did not occur in 2 events) = P(first event failed) x P(2nd event also failed) [note both events are independent]

Day / Date

$$\mu = (0)f(0) + (1)f(1) + 2(f(2)) + 3f(3)$$
$$= 1.7$$

Expected value of good components can be drawn.

Example 4.2:-

The salesman has 4-2 => 2 appointment each with 70% & 40% probability which can result in 4 outcomes

0$, 1000$, 1500$, 2500$.

$f(0\$) = (1-0.7)(1-0.4)$
$= 0.18$

$f(1000\$) = (0.7)(1-0.4) = 0.42$
$f(1500\$) = (0.4)(1-0.7) = 0.12$
$f(2500\$) = (0.4)(0.7) = 0.28$.

$\mu = E(X) = 0(0.18) + 1000(0.42) + 1500(0.12) + 2500)(0.28$
$\Rightarrow 1300\$$ Ans!

Example 4.3:-

$$f(x) = \begin{cases} \frac{20{,}000}{x^3}, & x > 100 \\ 0, & \text{elsewise!} \end{cases}$$

$$\mu = E(X) \int_{100}^{\infty} x \frac{20000}{x^3} dx.$$

$$20000 \int_{100}^{\infty} \frac{1}{x^2} dx = 200 \text{ hours}$$ Ans!

New case! * Consider a new random variable $g(X)$. which depends upon X.

$[g(X) = X^2]$ where $X = [x = -1, 0, 1, 2]$.

then

$P[g(X) = 0] = P(X = 0) \Rightarrow f(0)$

$P[g(X) = 1] = P(X = -1) + P(X = 1) \Rightarrow f(-1) + f(1)$

$P[g(X) = 4] = P(X = 2) \Rightarrow f(2)$.

$g(x)$	0	1	4
$P[g(X) = g(x)]$	$f(0)$	$f(-1)+f(1)$	$f(2)$.

To solve this senario!

$$\mu_{gx} = E(g(x)) = \sum_x (gx) f(x).$$

$$(-1)^2 f(-1) + (0)^2 f(0) + (1)^2 f(1) + (2)^2 f(2).$$

The same phenomena is expressed in continuous random variable as:

$$\mu_{g(x)} = E(g(x)) = \int_{-\infty}^{+\infty} g(x) f(x) dx.$$

Ex. 4.4:-

$g(X) = 2X - 1$.

$$E(X) = \sum_4^9 g(x) f(x) dx$$

$$= \sum_4^9 (2X-1) f(x) dx$$

$$= (2\underset{4}{X}-1) f(4) + (2\underset{5}{X}-1) f(5) + (2\underset{6}{X}-1) f(6) \cdots (2(9)-1) f(9)$$

Expected value from joint probability distribution.

discrete. $\mu_{g(x,y)} = E[g(X,Y)] = \sum_x \sum_y g(x,y) f(x,y)$.

continuous $\mu_g(x,y) = E(g(X,Y)) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(x,y) f(x,y) dx\, dy$

Day / Date

Example:-

$g(x,y) = xy$

$$\sum_{x=0}^{2} \sum_{y=0}^{2} g(x,y) f(x,y)$$

$$\iint$$

$$\sum_{x=0}^{2} \sum_{y=0}^{2} (xy) f(x,y).$$

$\Rightarrow$ $0 f(0,0) + 0 f(1,0) + 0 f(0,1)$

$\Rightarrow$ $2 f(2,0) + 2 f(0,2) + 2 f(2,2).$

$1 f(1,1) + 2 f(1,2).$

$\Rightarrow$ $1 f(1,1) \Rightarrow 3/14$ Ans!

Example

$E(Y/X)$ if

$$f(x,y) = \begin{cases} x(1+3y^2)/4 & 0<x<2,\ 0<y<1 \\ 0 & \text{elsewise} \end{cases}$$

if we convert it into singular variable in 2nd step

1st step $\int_0^1 \int_0^2 x(1+3y^2)/4 \, dx\, dy.$

2nd step $\int_0^1 y\left(1+\frac{3y^2}{4}\right) dy \Rightarrow 5/8$

Galaxy

Day / Date

4.22.

Random variable $\Rightarrow$ $Y = X+4$

$f(x) \Rightarrow \frac{32}{(x+4)^3}.$

$\int_{0-x}^{\infty} (\cancel{x+4}) \cdot \frac{32}{(x+4)^{\cancel{3}\,2}}$

$\int_0^{\infty} \frac{32}{(x+4)^2} dx$

Using geometric series

$32 \int_{-\infty}^{\infty} \frac{1}{(x+4)^2} dx$ $\quad r = \frac{32}{(x+4)^2}$

letting a result from calculus II to solve improper integral. $(-2+1)$.

$\lim_{t\to\infty} \int_0^t \frac{1 \times 32}{(x+4)^2} dx.$

$\lim_{t\to\infty} \left[-\frac{32}{x+4} \Big|_0^t \right] \Rightarrow 8$ Ans!

Galaxy

Day / Date

Variance of probability of distribution of X.
The random variable X.

Consider

$E(X) \Rightarrow E(g(X))$

then

$$E(g(X)) = \sum_{x=0}^{n} g(x) f(x)$$

Now for variance let (Variance σ^2)

$$g\,(\underbrace{X-\mu}_{\text{deviation of an observation}})^2$$

then variance σ^2 is;

discrete $\sigma^2 = E[(X-\mu)^2] = \sum_{x} (x-\mu)^2 f(x).$

continuous $\sigma^2 = E(X-\mu)^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)$

* The positive square root of variance is called as standard deviation!

Variance of a random variable (X) is.

$$\sigma^2 = E(X^2) - \mu^2.$$

Day / Date

Case: When the random variable X is written as! $g(X)$.

Normally.

$$\sigma^2 = E(X-\mu)^2 = \sum_{x=0}^{x=n} (X-\mu)^2 f(x)$$

then if $g(x) = X$.

$$\sigma^2 = E(g(x)-\mu)^2 = \sum_{0}^{n} (g(x)-\mu)^2 f(x)$$

also $\sigma^2 = E(g(x)-\mu)^2 = \int (g(x)-\mu)^2 f(x\, dx$

Case: Variance for a joint probability funct.

Normally.

$$\sigma^2 = E[(X-\mu)(Y-\mu)] = \sum_{x}\sum_{y} (x-\mu_x)(Y-\mu_y) f(x,y).$$

Continuous.

$$\sigma = E[(X-\mu)(Y-\mu)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (x-\mu_x)(y-\mu_y)\,dx\,dy.$$

1st moment : mean.
2nd moment : variance

Day / Date

Covariance :-

Covariance is the measure of association between two statistically dependant Random variables.

∴ if independant Random variable ⇒ covariance = 0.

Explanation

(if) large values of X result in large values of Y ;

$+(X-\mu_X)$ will result in $+(Y-\mu_Y)$
$-(X-\mu_X)$ will result in $-(Y-\mu_Y)$

it means that $(X-\mu_X)(Y-\mu_Y)$ would be positive in this case.

2nd (if) large values of X result in small values of Y

the product : $(X-\mu_X)(Y-\mu_Y)$ would be negative!

★ Covariance is a linear relationship between variable.

X & Y with non linear relation ⇒ covariance = 0.

Galaxy

theorem 4.5
theorem 4.6

Day / Date

$$\sigma_{XY} = E(XY) - \mu_X\mu_Y.$$

$$\downarrow$$

$$\sum_{i=0}^{Y}\sum_{i=0}^{n}(XY)f(x,y) - \mu_X\mu_Y$$

Theorem 4.5 :-

If a & b are two consts. then

$$E(aX+b) = aE(X) + b.$$

also

$$E(ax+b) = \int_{-\infty}^{\infty}(ax+b)\,f(x)\,dx.$$

$$a\int_{-\infty}^{\infty}xf(x)dx + b\int f(x)dx.$$

$$\downarrow$$

$$a\,E(X) + 1$$

Corollary $a=0$

$$E(ax+b) = E(b) = b.$$

Corollary $b=0$

$$E(ax+b) = aE(X)$$

Galaxy

Day / Date

Example 4.17.

$f(X) = 2X - 1.$

$E(X) = E(2X-1)$
$= 2E(X) - 1.$

as per the data of example 4.11.

$\mu = E(X) = 2\left[\sum_{4}^{9} x f(x)\right] - 1. \Rightarrow \$12.67$

Theorem 4.6

$E[g(X) \pm h(X)] = E[g(X)] \pm E[h(X)].$

$\int_{-\infty}^{+\infty} g(x) f(x) \pm \int_{-\infty}^{+\infty} h(x) f(x)\, dx.$

Example 4.18.

$Y = (X-1)^2$

$E(Y) = E(X-1)^2$
$= E(X^2 - 2X + 1)$
$= E(X^2) - 2E(X) + E(1)$
$= E(X^2) - 2E(X) + 1.$

Galaxy

Day / Date

$\int_{0}^{3} x^2 f(x)\, dx - 2\int_{0}^{3} x f(x) + 1.$

Solve to get answers!

Chapter no. 5!

The Bernoulli's process possesses the following properties:-

(i) Experiment consists of repeated trials
(ii) Each trial results in an outcome that may be classified as a success or failure!
(iii) The probability of success remains const, denoted by p from trial to trial.
(iv) The repeated trials are independant

Binomial Distribution:-

$b(x; n; p)$

$f(x) = {}^{n}C_{x}\, p^{x} (1-p)^{n-x}$

keyword ⇒ replacement
independant events.
Bernoulli trial with 2 outcomes
P(s) remains const
success.

n = no. of repetition / trials
x = no. of success
p = probability of success.

μ_x (mean) $= np$
$V(x)$ variance $= np(1-p)$

Galaxy

Day / Date

Example 5.1:-

no. of success => x => 2.
no of trials => n => 4
probability of success => 3/4

$b(n, x; p)$

$$^nC_x (p)^x (1-p)^{n-x}.$$

$$^4C_2 \left(\tfrac{3}{4}\right)^2 \left(1-\tfrac{3}{4}\right)^{4-2}.$$

$$\binom{4}{2}\left(\tfrac{3}{4}\right)\left(\tfrac{1}{4}\right)^2 \Rightarrow \frac{27}{128}.$$

Example 5.2:-

$$P(X \geq 10) = 1 - P(X < 10)$$

$$= 1 - \sum_{0}^{9} b(x, n, p)$$

$$= 1 - \sum_{0}^{9} b(x, 15, 0.4).$$

$$= 1 - 0.9662.$$

$$\Rightarrow 0.0338.$$

$$P(3 \leq X \leq 8) \quad \sum_{x=3}^{8} (x, 15, 0.4).$$

$$\Rightarrow \sum_{x=0}^{8} (x, 15, 0.4) - \sum_{x=2}^{2} (x, 15, 0.4)$$

Galaxy

Day / Date

$$P(X=5) \Rightarrow \sum_{0}^{5} (x, 15, 0.4) - \sum_{0}^{4} (x, 15, 0.4).$$

Example 5.3:-

(a)

$n = 20$
$x = ?$
$p = 0.3$.
$P(X \geq 1)$

$1 - \quad P(X=0)$

$1 - b(0, 20, 0.3) \Rightarrow 0.4562$

10 shipments / month $\rightarrow X$
test of 20 devices / month $\rightarrow Y$

$P(X = 3 \mid Y = 1)$

$(3, 10, 0.4562)$
$x \quad n \quad p$

$$^nC_x \, p^x (1-p)^{n-x}.$$

$$\binom{10}{3}(0.4562)^3(1-0.4562)^7 \Rightarrow 0.160$$

Galaxy

Day / Date

HYPERGEOMETRIC DISTRIBUTION.

Hyper geometric experiment possesses two of the following properties.

(I) A random sample of size n is selected without replacement from N items.

(II) Of the N items, k may be classified as successes & N-k classified as failures.

$h(x; N; n, k)$

no. of success from N items.

# of success set of 'n' items N.

selected items making samplespace.

$$\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$\mu = \frac{nk}{N}$$

$$\sigma^2 = \frac{N-n}{N-1} \cdot n \cdot \frac{k}{N}\left(1-\frac{k}{N}\right)$$