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chapter no. 8

Concept:-

Consider the following multiplication of vector.

Case I

$$\begin{bmatrix} 6 & 3 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 33 \\ 27 \end{bmatrix}$$

case II

$$\begin{bmatrix} 6 & 3 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \end{bmatrix}$$

The case I has no interest / importance however case II we can see that the vector produced is same as the original, only with a scaled const.

$$10\,[30 \quad 40]^T$$

$\lambda \Rightarrow$ eigen value / scalar.

- latent root
- characteristic vale

Generalize

square matrix

$$Ax = \lambda x$$

$x \neq 0$ (eigen vectors)

Task : Find a nonzero vector x that has the same effect on A when multiplyed by a scalar λ.

The vector λx may be with same or opposite sign / direction as x.

Set of all eigen values for a matrix A are called as the spectrum (of eigen value) of A.

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The largest of the absolute values of eigen values of A is called spectal radius of A.

let

$$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$$

step 1.- $A - \lambda I$

$$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{bmatrix}$$

step 2: $|A - \lambda I|$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{vmatrix}$$

$$\Rightarrow (1-\lambda)^2 - (4)$$

$$\Rightarrow 1 - 2\lambda + \lambda^2 - 4$$

$$\Rightarrow \lambda^2 - 2\lambda - 3$$

$\lambda = 3$, $\lambda = -1$ (our eigen values)

Now eigen vectors for each eigenvalue $\Rightarrow$

$$(A - \lambda I)\vec{x} = 0 \quad — \ *$$

Using $\lambda = 3$.

$$A - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}$$

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Now using row operations for echelon form.

$$(A - \lambda I) = \begin{bmatrix} -2 & 1 \\ +4 & -2 \end{bmatrix} = 0$$

$$R_2 + 2R_1 = \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}.$$

Now completing form of equ (*)

$$(A - \lambda I)\vec{x} = 0.$$

$$\begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0.$$

$$-2x_1 + 1x_2 = 0$$

$$x_2 = 2x_1$$

let $x_1 = (1)$ where $x_1 \in \mathbb{R} \in \mathbb{C}$

$x_2 = (2)$.

$\therefore$ eigen vecter $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$; $\begin{bmatrix} c \\ 2c \end{bmatrix}$

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Using $\lambda = -1$

$$(A - \lambda I) = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - (-1)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} + 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$R_2 - 2R_1 = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} = (A - \lambda I)$$

Now completing the equ (*) form

$$(A - \lambda I)\vec{x} = 0.$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$2x_1 + x_2 = 0$$

$$x_2 = -2x_1$$

$$x_2 = -\tfrac{4}{2}x_1 \Rightarrow -2x_1$$

let $x_1 = 1$ $\therefore$ eigen vecter $\Rightarrow \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} c \\ -2c \end{bmatrix}$ for $c \in \mathbb{R}$.

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