$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

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ES 111	Spring 24	Final Exam	Weight 40%	24 May 24	14:30 – 17:00

CLO 1: Calculate probabilities of events, joint probabilities, conditional probabilities using set operations and definition of probability. Justify valid and invalid probability assignments and independence of events. (C3-Applying, PLO2-Problem Analysis)

CLO 2: Calculate probability mass/density function parameters, moments and functions of random variables. (C3-Applying, PLO2-Problem Analysis)

CLO 3: Draw inferences about population and sample data using techniques of "Inferential Statistics". (C4-Analyzing, PLO2-Problem Analysis)

CLO 4: Solve problems related to basic estimators and design of experiments. (C3-Applying, PLO2-Problem Analysis)

Problem 1 [CLO 1] [1 + 2 + 2 Points]

$P(A)$ $P(B)$

In a high school graduating class of 100 students, 54 studied mathematics, 69 studied physics, and 35 studied both mathematics and physics. If one of these students is selected at random, $P(A \cap B)$

(a) Find the probability that the student took mathematics or physics. $P(A \cup B)$

(b) Find the probability that the student did not take either of these subjects. $1 - P(A \cap B)$

(c) Find the probability that the student took physics but not mathematics. $P(B \cup A')$

Problem 2 [CLO 2] [2 + 3 Points]

A soft-drink machine is regulated so that it discharges an average of 200 ml per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 ml,

a) what fraction of the cups will contain more than 224 ml?
b) what is the probability that a cup contains between 191 and 209 ml?

Problem 3 [CLO 3] [3 + 4 Points]

You independently draw 100 data points from a normal distribution.

a) Suppose you know the distribution is normally distributed with $N(\mu, 4)$, with $\sigma^2 = 4$, and you want to test the null hypothesis $H_0 : \mu = 3$, against the alternative hypothesis $H_1 : \mu \neq 3$. If you want a significance level of $\alpha = 0.05$, what will be the critical region (define where the null hypothesis will be rejected)?
b) Suppose the 100 data points have sample mean 5. Should you reject H_0? Why?

Problem 4 [CLO 3] [4 + 4 Points]

Data is collected on the time between arrivals of consecutive taxis at a downtown hotel. We collect a data set of size 30 with sample mean $\bar{x} = 5.0$ and sample standard deviation $s = 4.0$. Assume the data follows a normal random variable.

a) Find a 90% confidence interval for the mean μ of X.
b) Find a 90% confidence interval for the variance?

Problem 5 [CLO 2] [2 + 4 Points]

A manufacturer has developed a new fishing line, which the company claims has a mean breaking strength of 15 kilograms with a standard deviation of 0.5 kilogram. To test the hypothesis that $\mu = 15$ kilograms against the alternative that $\mu < 15$ kilograms, a random sample of 50 lines will be tested. The critical region is defined to be $\bar{x} < 14.9$.

a) Find the probability of committing a type I error when H_o is true.
b) Evaluate β for the alternatives μ = 14.8 and μ = 14.9 kilograms.

Problem 6 [CLO 4] [6 + 3 Points]

a) A study was made by a retail merchant to determine the relation between weekly advertising expenditures and sales.

X	**Advertising Cost ($)**	40	20	25	20	30	50	40	20	50	40	25	50
y	**Sales ($)**	385	400	395	365	475	440	490	420	560	525	480	510

1) Find the equation of the regression line to predict weekly sales from advertising expenditures.
2) Estimate the weekly sales when advertising costs are $35.

410·

5445

15650

191325

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with mean $E[X_i] = \theta$, and variance $V[X_i] = \sigma^2$. Consider the following estimator for θ.

?

$$\hat{\theta} = \frac{X_1 + X_2 + \cdots + X_n}{n} = \bar{X}$$

biased estimater unbaised estimator

?

1) Find the bias and variance of the estimator $\hat{\theta}$.

Hint: You may use the general result $V[aX + bY] = a^2V[X] + b^2V[Y] + 2abCov[X, Y]$, and the fact that by definition of random sampling, $X_1, X_2, \ldots, X_n$ are independent of each other.

H_0	**Value of Test Statistic**	H_1	**Critical Region**
$\mu = \mu_0$	$z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$, σ known	$\mu < \mu_0$ $\mu > \mu_0$ $\mu \neq \mu_0$	$z < -z_\alpha$ $z > z_\alpha$ $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$
$\mu = \mu_0$	$t = \dfrac{\bar{x} - \mu_0}{S\,\cancel{\sigma} / \sqrt{n}}$, $v = n - 1$ σ unknown	$\mu < \mu_0$ $\mu > \mu_0$ $\mu \neq \mu_0$	$t < -t_\alpha$ $t > t_\alpha$ $t < -t_{\alpha/2}$ or $t > t_{\alpha/2}$
$p = p_0$	$z = \dfrac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$	$p < p_0$ $p > p_0$ $p \neq p_0$	$z < -z_\alpha$ $z > z_\alpha$ $z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$
$\sigma^2 = \sigma_0^2$	$\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$, $v = n - 1$	$\sigma^2 < \sigma_0^2$ $\sigma^2 > \sigma_0^2$ $\sigma^2 \neq \sigma_0^2$	$\chi^2 < \chi^2_{1-\alpha}$ $\chi^2 > \chi^2_\alpha$ $\chi^2 < \chi^2_{1-\alpha/2}$ or $\chi^2 < \chi^2_{\alpha/2}$

$$Y = a + bX \qquad a = \frac{\sum_{i=1}^{n} y_i - b\sum_{i=1}^{n} x_i}{n} = \bar{y} - b\bar{x}$$

$$b = \frac{n\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$