Chapter no. 13

Complex Numbers!

Exercise 13.1

$x = \pm\sqrt{-1}$

$x^2 = 1.$

$Z \to (x, y) \Rightarrow x + yi$ — ①

$\downarrow$ real $\quad \downarrow$ imaginary.

Complex conjugate :

Using equ ①

example:

$\bar{Z} = \overline{(x+yi)}$

$\bar{Z} = (x - yi)$

also:-

$z\bar{z}$ (multiplication of a complex no. and its conjugate is a real no.)

example:

$z\bar{z} = (x+yi)(x-yi)$

$= x^2 + y^2 \Rightarrow$ Real no.

example:

$\bar{z}_1 + \bar{z}_2 \Rightarrow \overline{(8+3i)} + \overline{(9+2i)}$

$= (8-03i) + (9-2i)$

$= (18-1) - 5i$

$\Rightarrow 17 - 5i$ Ans!

ordinary plotting ; only ... and x as
y as imaginary axis real axis.

$$\frac{z}{Z} = \frac{8+3i}{9+2i} \times \frac{9-2i}{9-2i}$$

$$\frac{(8+3i)(9-2i)}{(9)^2-(2i)^2} \Rightarrow \frac{72-16i+27i-6i^2}{81-4i^2}$$

$$\Rightarrow \frac{72+11i+6}{81+4}$$

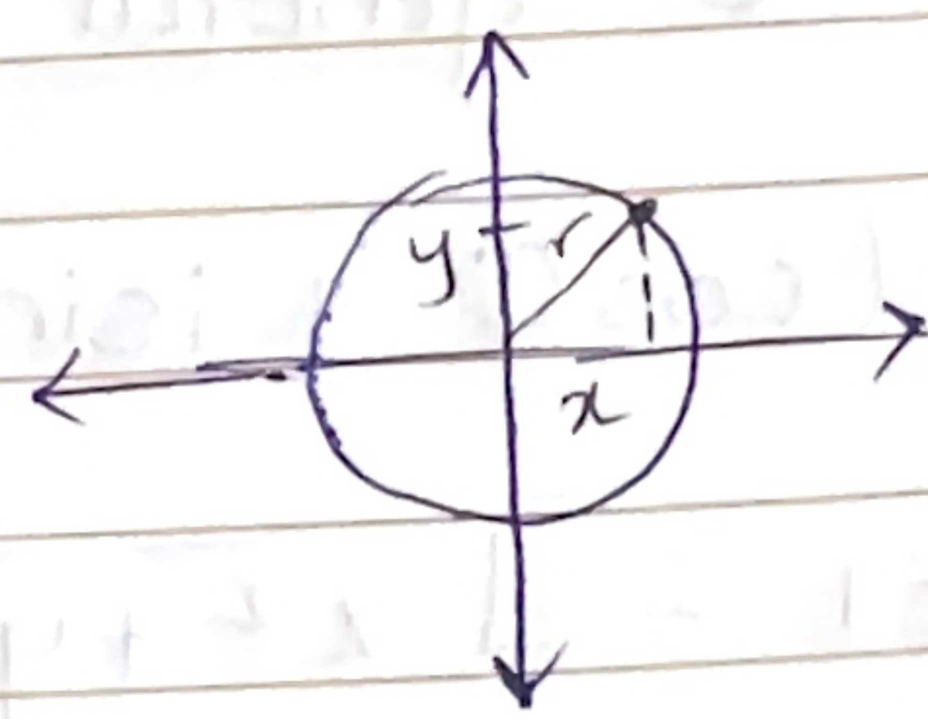
$$\Rightarrow \frac{72+6+11i}{85} \Rightarrow \frac{78}{85} + \frac{11i}{85}$$

$x = \mathrm{Re}\, z$ $\quad$ $y = \mathrm{Im}\, z$

Exercise 13.2.

Polar form of Complex Numbers.

Consider a Circle on a co-ordinate system.



Now by trignometric formulas.

$$\sin \varphi = \frac{y}{r}$$

$$\cos \varphi = \frac{x}{r}.$$

$$\boxed{y = r\sin\varphi}$$

$$\boxed{x = r\cos\varphi}$$

$\Rightarrow$ So now the complex no. in polar form can be represented as.

$$z = x + iy = r\cos\varphi + i r\sin\varphi.$$

$$\boxed{\Rightarrow r(\cos\varphi + i\sin\varphi) = re^{i\theta}} \quad \text{— } (*)$$

↳ basis of differential

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Now if radius is required.

$$|z| = \sqrt{x^2+y^2} = r. \quad — (*)$$

Now if ϕ is required.

ϕ is called argument of z or Principal value denoted by Arg z

$$\tan\phi = \frac{\sin\phi}{\cos\phi} \Rightarrow \left(\frac{y}{x}\right) \quad — (*)$$

Example:-

$z = 1+i$ find polar form.

As per the general polar form.

$$z = r(\cos\phi + i\sin\phi)$$

$$r = |z| = \sqrt{x^2+y^2}$$

$$= \sqrt{(1)^2+(1)^2}$$

$$= \sqrt{2}$$

$$\phi = \tan^{-1}\left(\frac{1}{1}\right)$$

$$= 45^\circ$$

$$z \Rightarrow \sqrt{2}\,(\cos(45) + i\sin(45))$$ Ans

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EXERCISE 13.1.

LAWS FOR CONJUGATES:- (Q no. 4 done).

① $\overline{(z_1 + z_2)} = \overline{z_1} + \overline{z_2}$

② $\overline{(z_1 - z_2)} = \overline{z_1} \cancel{+} \overline{z_2}$ imp

③ $\overline{(z_1 z_2)} = \overline{z_1} . \overline{z_2}$

④ $\left(\frac{\overline{z_1}}{\overline{z_2}}\right) \Rightarrow \overline{\left(\frac{z_1}{z_2}\right)}$

LAW TO SOLVE COMPLEX FRACTIONS! (Q no. 3 done!)

$$\frac{x_1 + yi_1}{x_2 + yi_2} \times \left[\frac{x_2(-yi_2)}{(x_2 - yi_2)}\right]$$

Multiply & divide by conjugate of the denominator.

—x—

Question no. 1

show that

$i^2 = -1$

$i = \sqrt{-1}$

$i^2 = -1$ proved!

$i^3 = -i$

$i^2 = -1$

$i^2 . i = -1 . i$

$i^3 = -i$ proved!

$\arg(z_1 \times z_2) = \arg z_1 + \arg z_2$

$\arg(z_1/z_2) = \arg(z_1) - \arg(z_1)$

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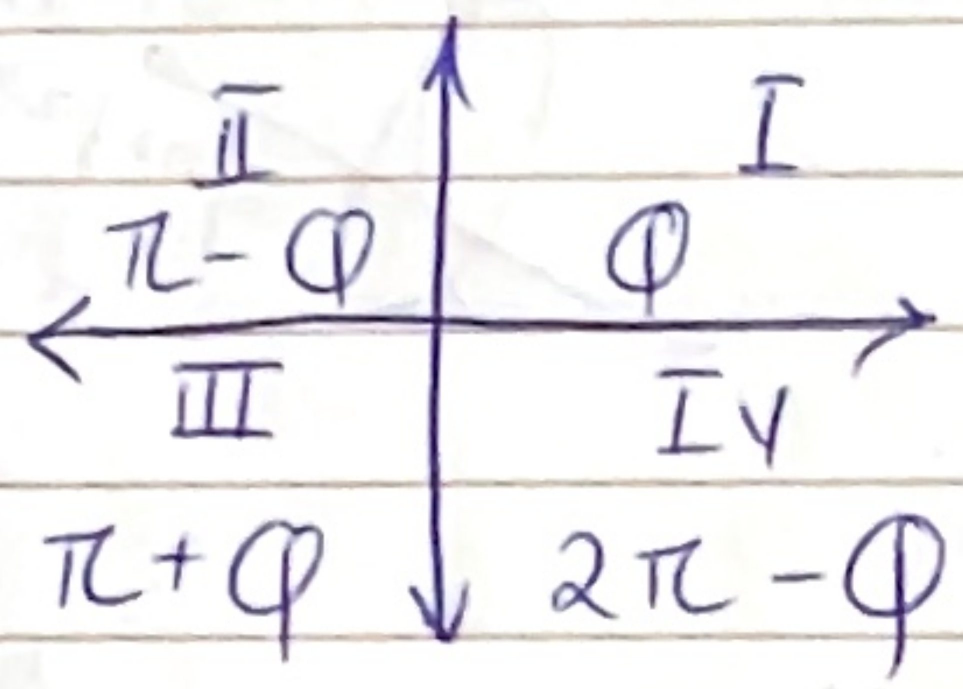
CS - Lab Tasks.

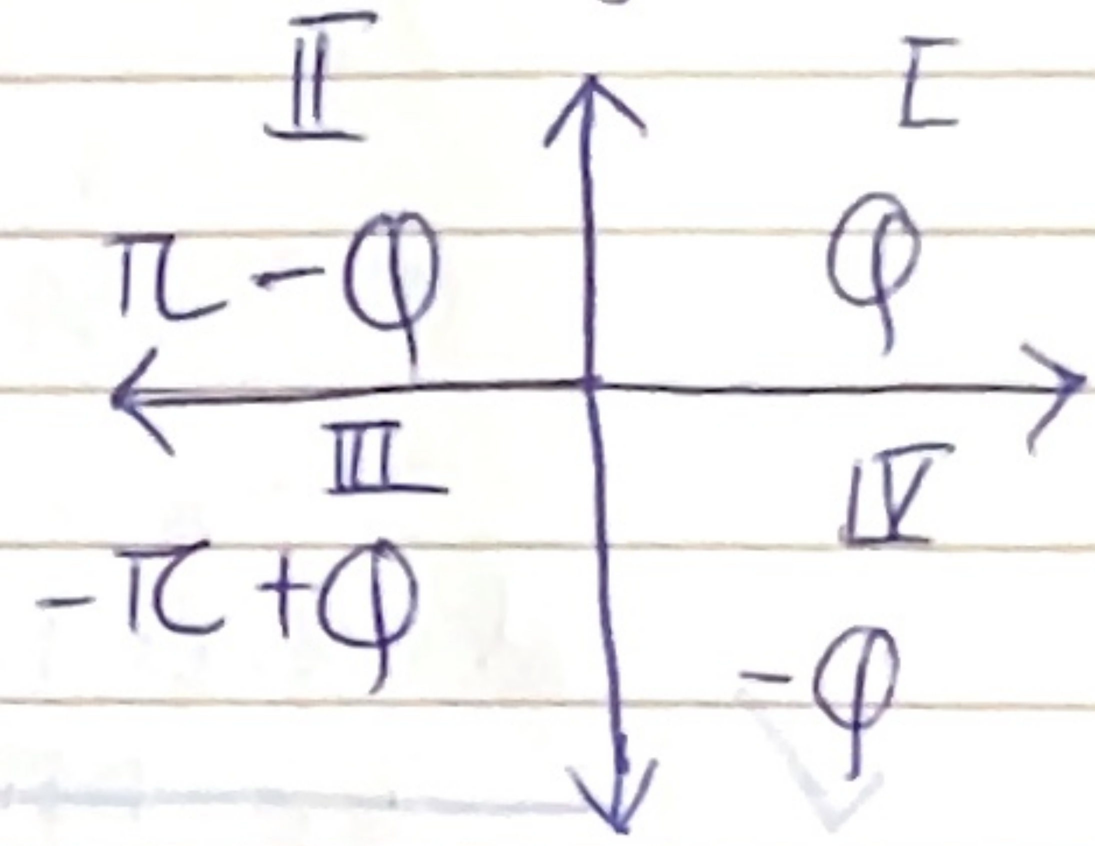
✓ MT - Chapter no. 13.

✓ Graphs & 2nd Quad prob 13.2 => take angle mod.

DIFFERENCE BETWEEN argument of z (arg z) and principle argument (Arg z)

arg (z)	Arg (z)
$-2\pi < \arg(z) < 2\pi$	$-\pi < \text{Arg } z \le \pi$





$\text{Arg}(z) \in (-\pi, \pi]$

$\arg(z) = \text{Arg}(z) + 2n\pi$

$\text{Arg}(z) = \tan^{-1}(y/x)$

Short cut :-

$i, \dots ni \Rightarrow \text{Arg } z \Rightarrow (\pi/2)$

$-i \dots -ni = \text{Arg} \Rightarrow (-\pi/2)$

$z = +100 \dots +n \Rightarrow \text{Arg } z \Rightarrow 0$

$z = -100 \dots -n \Rightarrow +\pi$

While find Arg z we do not use signs to calculate φ or we use mod.

Example:-

① $-3x + 4y$

$\varphi = \tan^{-1}(4/3) = 53.130°$

(however as in 2nd Quadrant)

$\text{Arg}(z) \mid \varphi \Rightarrow \pi - \varphi \Rightarrow 126°$

$\Rightarrow 126°$