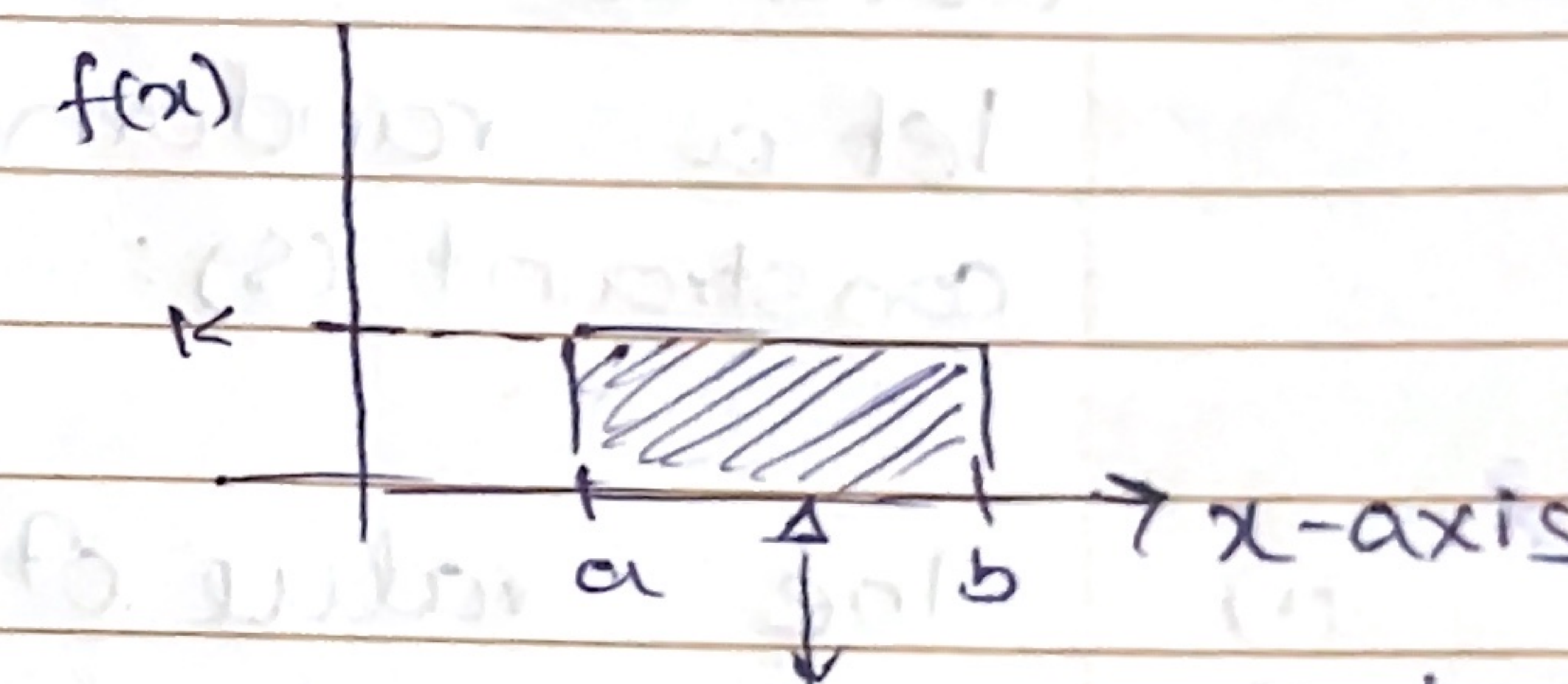
Post Mid-term!

26-03-24.

Continuous (→ uniform DIST. case).

The type of distribution in which the probability of all the outcomes of an event are equally likely.

As by the definition of probability



$$\int_a^b k\,dx = 1$$

where k is a const.

mean: a point where a & b are balanced.

$$k\int_a^b dx = 1$$

$$k = \frac{1}{(b-a)}$$ } uniform's case)

Day / Date

(→ probability density funct)

Now writing PDF of uniform distribution (continuous).

$$f(x) \begin{cases} 1/_{b-a} & a<x<b \\ 0 & \text{otherwise} \end{cases}$$

(expected value) ★ Mean :- $\mu = \frac{a+b}{2}$

★ variance :- $\sigma^2 = \frac{(b-a)^2}{12}$.

IDENTIFICATION :- $X \sim$ uniform (in Q statement).
$X \sim U[a,b)$ (").

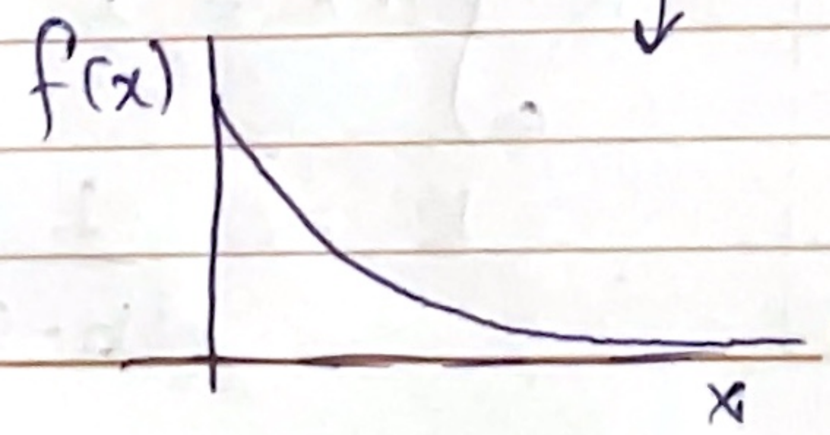
→ EXPONENTIAL :-

DIST. CASE

let a random variable X that has(a) constraint (s):

(i) The value of X is never negative.

(ii) Larger values of X have relative probability that drops exponentially.



Galaxy

Day / Date

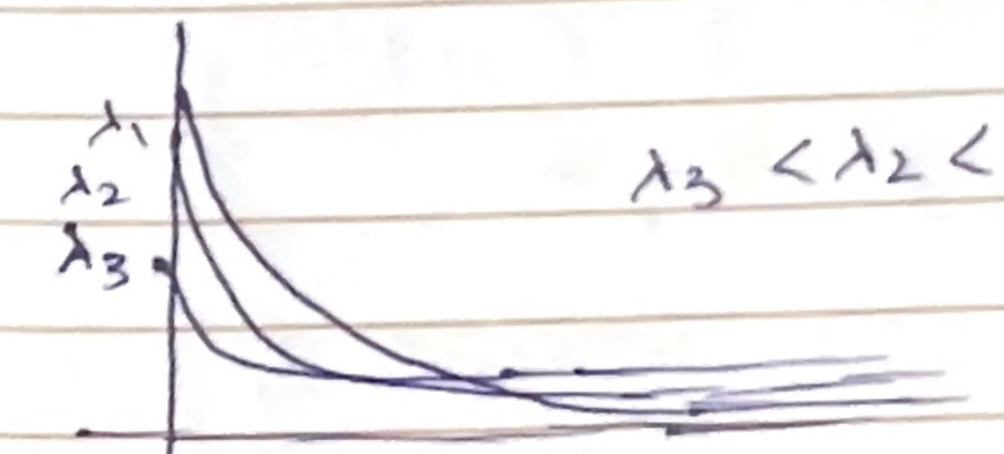
Designing probability ~~Mass~~ Density function.

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

also note $\lambda \Rightarrow$ rate of decay.
$\lambda = \frac{1}{\beta}$

$\therefore$

$$f(x) = \begin{cases} \lambda e^{-x\lambda} & x \geq 0 \\ 0 & x < 0 \end{cases}$$



$\lambda_3 < \lambda_2 < \lambda_1$ "The rate of decay of λ_1 is the greatest (great steep/drop").

★ expected value / Mean :- $\mu / E(x) = 1/_\lambda \Rightarrow \beta$

Variance :- $\sigma^2 / V[X] = \frac{1}{\lambda^2}$

Galaxy

Chapter no. 11

بسم الله الرحمن الرحيم

Simple linear Regression and Correlation.

$y = mx + c$.

$y = b_1 x + b_0$. slope

*

$$b_1 = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{n(\sum x_i^2) - (\sum x_i)}$$

$\sum_{i=1}^{33} x_i = 1104$, $\sum_{i=1}^{33} y_i = 1124$ $\sum_{i=1}^{33} x_i y_i = 41355$ $\sum_{i=1}^{33} x_i^2 = 41086$

first square then sum.

$$B_1 = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{n(\sum x_i^2) - (\sum x_i)^2}$$

$$b_1 = \frac{(33)(41355) - (1104)(1124)}{33(41086) - (1104)^2} \Rightarrow 0.903643.$$

$$b_0 = \frac{\sum y_i - b_1 \sum x_i}{n} \Rightarrow \frac{1124 - (0.903643)(1104)}{33.}$$

$\Rightarrow 3.829633$

$\therefore$ $y = b_1 x + b_0$.

$y = 0.903643x + 3.829633$ Ans!

بسم الله الرحمن الرحیم

hapter no. 2

yle no 2.1.

If an operation is performed in n_1 ways and for each of these ways another operation can be performed in n_2 ways. The total no. of ways (an) operation can be performed is:

$n_1 \times n_2$. (and so on).

Permutations:-

Arrangement of all or some parts of a set of objects.

→ In general n distinct objects can be arranged as;

$n(n-1)(n-2)$ ---- ways. or $n!$

→ No. of permutations of n distinct object take r at a time is given by.

$$^nP_r = \frac{n!}{(n-r)!}$$

→ no. of permutation of n things where n_1 are of one kind, n_2 are of the second kind .... n_k of the k^{th} kind.

$$\frac{n!}{n_1!\, n_2! \cdots n_k!}$$

→ the no. of ways of partioning a set of n objects into r cells with n_1 elements in one cell, n_2 in the 2^{nd} and so on ......

⇒ example 2.23

$$\frac{n!}{n_1!\, n_2! \cdots n_r!}$$

Combination.

The number of selection from n objects of r taken at a time, without the trf resition of order and selection without replacement.

example 2.22

$$ {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!} $$

Probability of an event.

(i) $0 \le P(A) \le 1$ — probability of an event lies b/w 0 and 1.

(ii) $P(S) = 1$ — probability of a sample space = 1.

Union of two events :-

3 event's union.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

overlapping sets. A & B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

mutually exclusive or disjoint sets. A & B.

$$P(A \cup B) = P(A) + P(B).$$

Theorem: The sum of Probability of an event and its complement is alway equal 1.

$$P(A) + P'(A) = 1.$$

Conditional Probability *

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \Rightarrow \frac{P(A \cap B)}{P(A)}$$

When one event is independant of the other.

$P(B|A) = P(B)$ → independant of $P(A)$

$P(A|B) = P(A)$ → independant of $P(B)$

Independance Criteria of two events!

* Two events are said to be independant

if $P(A \cap B) = P(A).P(B)$.

[$P(A \cap B) = P(A|B) \times P(B)$
then shows dependance of both events on one another.]

Total probability.

A probability of an event predicted through probilities of the all the possible outcomes of an event being summed.

always check for first event probability for the second event if using combination.

Chapter no.9. of μ.

(i) Confidence interval of known μ & σ^2

If $\bar{x}$ is the mean of a random sample of size n from a population of known σ^2, $100(1-\alpha)\%$ interval of μ is given by.

not lower bound. not upper bound

$$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}.$$

$$n = \left(\frac{Z_{\alpha/2}\,\sigma}{e}\right)^2.$$ → predicting sample size for μ estimate.

upper bound $\Rightarrow \bar{x} + Z_\alpha \frac{\sigma}{\sqrt{n}}$

lower bound $\Rightarrow \bar{x} - Z_\alpha \frac{\sigma}{\sqrt{n}}$.

take value of Z_α positive

(ii) Confidence interval of μ when μ, σ^2 are unknown

If $\bar{x}$ & s are the mean and variance of random sample of size n then the confidence interval $(1-\alpha)100\%$ for μ is given as.

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}.$$

(iii) Confidence interval of σ^2

$$\frac{(n-1)^2 s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)^2 s^2}{\chi^2_{1-\alpha/2}}.$$

Sample mean

Chapter 8. true mean $= \mu$

$$\bar{X} \Rightarrow \frac{1}{n} \sum_{i=1}^{n} X_i$$

Sample variance.

true variance $= \sigma^2$.

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \Rightarrow \checkmark \; \frac{n \sum X_i^2 - (\sum X_i)^2}{n(n-1)}$$

95% confidence.

$\chi^2_{0.975} - \chi^2_{0.025}$

$0.484 - 11.413$.

$P(Z < \_)$

means to find area under the curve.

χ^2 distribution.

$$\chi^2 = \frac{(n-1) S^2}{\sigma^2}$$

$P(\chi^2 = \chi^2_\alpha) = 0.99 \quad \nu = 4$

$\alpha = p$

$\chi^2_{0.99}$ with ν4 is used!

t-distribution.

$$T = \frac{X - \mu}{S/\sqrt{n}}$$

confidence interval of 95%.

$-t_{0.025} \leftrightarrow t_{0.025}$.

By def $P(T > t_\alpha) = \alpha$.

Chap 6:-

Uniform density funct. Dist.

Density funct of a continuous random variable on the interval [A,B] is given as.

$$f(x; A,B) = \begin{cases} \frac{1}{B-A} & A \le x \le B \\ 0 & \text{elsewhere.} \end{cases}$$

$$\mu = \frac{A+B}{2} \qquad \sigma^2 = \frac{(B-A)^2}{12}$$

Exponential Dist function.

The density function of a continuous random variable X with parametre β

$$f(x, B) = \begin{cases} 1/B \, e^{-x/B} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

$\mu = \beta$ & $\sigma^2 = \beta^2$.

can be transformed into decay By $\lambda = \frac{1}{\beta}$.

Normal Distribution.

The density of a normally distributed random variable is;

$$n; (x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-1/2\sigma^2 (x-\mu)^2} \qquad -\infty < x < \infty$$

it is σ^2 of normal.

$$Z = \frac{X-\mu}{\sigma}$$

Chapter no.10.

	NULL Hypothesis True	NULL Hypothesis False.
Reject H_0	False positive	True positive.
Fail to Reject H_0	True negative.	False negative.

Type I error :- false positive.
Null hypothesis is true but we rejected it.

Type II error : False negative.
Null hypothesis is false but we reject it.

Z-test :- (two tail test).

- Check if the necessary data is complete.

(i) Set H_0 & H_1

(ii) Set confidence interval from $-Z_{\alpha/2} - +Z_{\alpha/2}$

(iii) find Z-value from given data using $Z = \frac{X-\mu}{\sigma}$

(iv) evaluate 'Z' in the confidence interval.

→ if it lies in the critical region H_0 accept

→ elsewise H_0 reject.

* same for t-test if 'S' is given & $n<30$.

Z/t -test (one tail):-

for $n < 30$ use t-

ibu

ue

(i) State H_0 & H_1

(ii) Find the rejection & critical region but remember to use α now (either Z_α or T_α) as a one tail test.

(iii) Find the Z or t value by.

$$Z = \frac{X - \mu}{\sigma/\sqrt{n}} \quad \text{or} \quad T = \frac{X - \mu}{s/\sqrt{n}}.$$

(iv) compare ~~with~~ the drawn regions.

Calculating probability of type I error / type II error.

Problem 8.

$H_0 = 68 \Rightarrow \mu$

$H_1 \Rightarrow \mu \neq 68$.

$67 < \bar{x} < 68$

$\sigma = 3.6$

$n = 36$.

$\alpha \Rightarrow$ probability of the type I error.
not accepting the true hypothesis.

$\alpha = P(\bar{X} < 67 \text{ when } \mu = 68) + P(\bar{X} > 69 \text{ when } \mu = 68)$

$P(Z < -1.67) + P(Z > 1.67)$

$2P(Z < -1.67) \Rightarrow 0.095$.

low

$H_0 \geq 70$

$H_1 \neq 70$

$67 < \bar{x} < 69$

β = Probability of accepting a false hypothesis.

$P(67 < \bar{x} < 69$ when $\mu = 70)$

$z_1 = \frac{67-70}{0.45} = 6.67$ $z_2 = \frac{69-70}{0.45} = -2.22$

$P(-6.67 < z < -2.22)$

$P(z < -2.22) - P(z < -6.67) \Rightarrow 0.0132$.

Change is usually in a alternate hypothesis.
Alway identify what we are checking for in the question.

$P(z > -a)$ or $P(z < -a)$

$P(z < a)$ $P(z > a)$.

also $P(a < z < b)$

$\boxed{P(z < b) - P(z < a)}$