— $Q_{16}, Q_{17}, Q_{18}, Q_{19}, Q_{30}, Q_{34}$.

$\frac{1}{2}A_0, t = 5730$
↓
half life of Carbon

Chapter no.3 :-

* Usually the amount of substance that is left/or exist behind is subjected to taking data. (if data also indicates). ✓

A breeder recator converts stable U-238 into Pu-239. After 15 years that 0.043% of the initial amount A_0 of 'Pu' has been disintegrated. Find the half life of this isotope if the rate of disintegration ∝ the amount remaining.

$\frac{dA}{dt} = kA$; The growth & decay function ⇒ $A(t) = A_0 e^{kt}$.

0.043% of A_0 has been dis integration after 15 years.

∴ 99.957% of A_0 are still remaining after 15 years.

$$0.99957 \overset{A_0}{} = A_0 e^{k(15)} \checkmark$$

$$k = \frac{1}{15} \ln 0.99957 \Rightarrow -0.00002867$$

half life ⇒ ? → $\frac{1}{2}A_0$

time ⇒ ?

$$\frac{1}{2}\cancel{A_0} = \cancel{A_0} e^{(-0.00002867)t}$$

$$\ln\frac{1}{2} = \ln e^{-0.00002867t}$$

$$-\ln\frac{1}{2} = 0.00002867t.$$

$$\ln 2 = 0.00002867t \Rightarrow t = 24,180 \text{ years.}$$

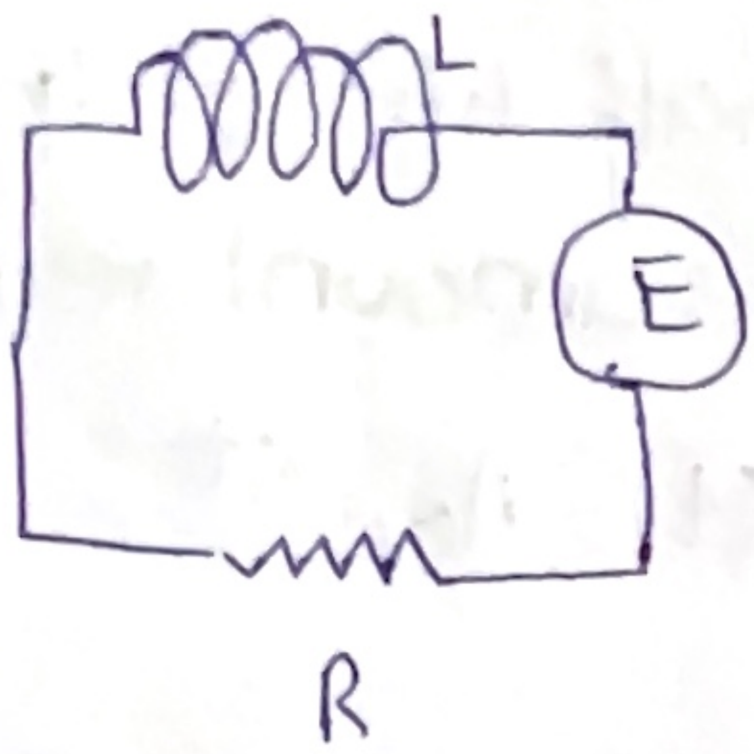
NEWTON'S LAW OF COOLING & WARMING.

$$\frac{dT}{dt} = k(T - T_m) \Rightarrow T(t) = T_m + Ce^{kt}$$

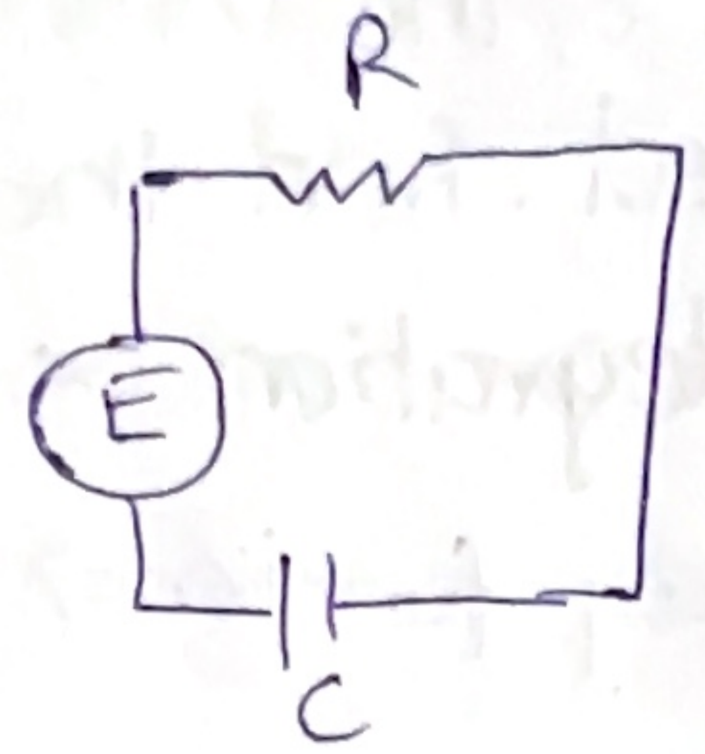
$\rightarrow \ln(0) = \text{undef}$

Example no. 4

Sometime the required sol. may not be give a finite ans to demonstrate. In that case we will give ans, based on intiuition from the nearest range of values.



LR Circuit



RC Circuit.

$\rightarrow \lim_{t\to\infty} e^{-\infty} = 0$.

$L\frac{di}{dt} + Ri = E(t)$ $\quad\quad$ $R\frac{dq}{dt} + \frac{1}{C}q = E(t)$.

* Remember that rate of increase ↑ = first derivative.

$P'(t) = (P_t)(\cancel{A})(k)$.

* In IVP if shortest term is given then $t=0$ and the longest term is $\lim_{t\to\infty}$

$$E(t) = \begin{cases} 120; & 0 \le t \le (20) \\ 0; & t > (20) \end{cases}$$

if $H = 20\,H$

$R = 2$ ohm.

$i(0) = (0)$.

Find current.

For LR series.

$$L\frac{di}{dt} + Ri = E(t)$$

$$E(t) = 20\frac{di}{dt} + 2i = Et.$$

$$\frac{E(t)}{20} = \frac{di}{dt} + \frac{1}{10}i \quad ✓$$

Finding integration factor.

$$e^{+1/10\,t} \quad ✓$$

Multiplying on b.S.

$$\frac{E(t)}{20} \times e^{+1/10\,t} = e^{+1/10\,t}\frac{di}{dt} + e^{+1/10}\frac{1}{10}$$

$$\Rightarrow \int (e^{1/10\,t}\, i)\frac{d}{dt}$$

$$\int \frac{E(t)\,e^{1/10\,t}}{20}$$

$$+ E(t)\frac{10}{20}e^{1/10\,t} + C \Rightarrow e^{1/10\,t}\, i$$

$$+ \frac{E(t)}{2} + Ce^{-1/10\,t} = i(t) \quad ✓$$

Now we have the required equ.

$$\boxed{i(t) = E(t) + Ce^{-1/10\,t}} \rightarrow (a)$$

We shall solve it for the initial condition; for value of C.

(zero)

$$i(0) \Rightarrow +\frac{1}{2}E + Ce^{-1/10\,0}$$

$$0 \Rightarrow +\frac{1}{2}E + Ce^{0}$$

$$✓\ C = -\frac{1}{2}E + (0)$$

Now replacing it in the original equ.

$$✓\ i(t) = +\frac{E(t)}{2} - \frac{1}{2}E(t)e^{-1/10\,t}$$

Now at interval

$0 \le t \le 20$.

$E(t) = 120$.

$$\therefore ✓\ i(t) = 60 + (60)e^{-1/10\,t}$$

Similarly now we need to find for the next interval.

at the $t > 20$:

"the end of $0 \le t \le 20$ will be considered the start of the $t > 20$)

where $t = 20$:

therefore using equ. of previous interval with $t = 20$:

$$i(20) = 60 + (60)e^{\frac{-20}{10}} \quad \text{—— (b)}$$

Now using (b) in (a) for the value of C in this interval. we will Resubstitute it into the original current equ. for (i).

$$60 + (60)e^{-20/10} = \frac{1}{2}E + Ce^{-1/10(20)}$$

$$60 + (60)e^{-2} = \frac{1}{2}E + Ce^{-2}$$

$$\left[C = 60e^{2} + (60) - \frac{1}{2}e^{2}E\right]$$

again equ (a) for the interval $t > 20$ becomes.

$$i(t) = \frac{1}{2}E + \left(-\frac{1}{2}Ee^{2} + 60 + (60)e^{2}\right)e^{\frac{-1}{10}t}$$

Now at this interval $E = 0$.

$$\therefore i(t) = (60e^{2} + (60))e^{\frac{-1}{10}t}$$

answer

$$i(t) = \begin{cases} 60 + 60e^{-1/10t} & 0 \le t \le 20 \\ (60e^{2} + (60))e^{-1/10t} & t > 20 \end{cases}$$

$$L(t) = \begin{cases} 1 - \frac{1}{10}t, & 0 \le t < 10 \\ 0, & t > 10 \end{cases}$$

find current $i(t)$ if the resistance $0.2\,\Omega$, $E(t) = 4$ and $(i)(0) = 0$.

$$L\frac{di}{dt} + Ri = E(t).$$

let interval $\overset{0 \le t < 10}{\uparrow}$ $1 - \frac{1}{10}t$.

$$(1 - \frac{1}{10}t)\frac{di}{dt} + Ri = E(t).$$

$$\frac{di}{dt} + \frac{Ri}{(1 - \frac{1}{10})t} = \frac{E(t)}{1 - (\frac{1}{10}t)}$$

$$\frac{di}{dt} + \frac{2}{10 - t} = \frac{10E(t)}{10 - t}.$$

$$\frac{di}{dt} + \frac{2i}{10 - t} = \frac{40}{10 - t}$$

finding the integration factor.

$$e^{\int 2/10 - t}$$

$$-2\ln|10 - t| + C$$

Multiplying integration factor on b.s.

$$-2\ln|10 - t|\frac{di}{dt} + \frac{-2\ln|10 - t| \times 2}{10 - t}$$

$$= \frac{40(-2\ln|10 - t|)}{10 - t}$$

$$\frac{d}{dt}(i + 2\ln|10 - t|i) = \frac{40}{10 - t} \times (2\ln|10 - t|)$$

Taking $\int$ on b.s.

$$-2\ln|10 - t|i = 40 \times 2 \int \frac{\ln|10 - t|}{10 - t}$$

$$-2\ln|10 - t|i = 40 \times \cancel{2}\left(-\frac{\ln^2|x - 10|}{x}\right)$$

This method of integration factor is too difficult.

Doing by another method

$$\left(1-\frac{1}{10}t\right)\frac{di}{dt} + 0.2i = 4.$$

std form

$$\boxed{\frac{di}{dt} + \frac{2}{10-t}i = \frac{40}{10-t}}$$

$$\frac{di}{dt} = \frac{40}{10-t} - \frac{2}{10-t}i$$

$$\frac{di}{dt} = \frac{40-2i}{10-t}$$

$$\int \frac{1\,di}{40-2i} = \int \frac{1}{10-t}\,dt$$

$$-\frac{1}{2}\ln|40-2i| = -\ln|10-t| + \ln c$$

$$\ln|40-2i| = 2\ln|10-t| + \ln c \rightarrow \ln k.$$

$$\cancel{\ln}|40-2i| = \cancel{\ln}(k(10-t)^2)$$

$$40-2i = k(10-t)^2$$

$$2i = 40 - k(10-t)^2$$

$$i(t) = \frac{40-k(10-t)^2}{2}$$

There we have the required equation of current as.

$$i(t) = 20 - \frac{1}{2}k(10-t)^2.$$

Now applying the condition.

$$i(0) = 0.$$

$$i(0) = 20 - \frac{1}{2}k(10-0)^2$$

$$0 = 20 - \frac{1}{2}k(10)^2$$

$$k = 2/5$$

$$\therefore \boxed{i(t) = 20 - \frac{1}{2}\times\frac{2}{5}(10-t)^2}$$

for interval $0 \le t < 10$

Similarly

$L=0$ when $t>0$

$$0\frac{di}{dt} + 0.2i = 4$$

$$\boxed{i = 20\,\text{amp}}$$