Chap 10 → sem-1

simply applicable to power series ← Part of both ... Ratio test. ~~Ration~~

Criteria

$\lim_{n\to\infty} \left| \frac{n+1 \text{ term}}{n \text{ term}} \right|$

Chapter no. 6

Exercise 6.1. } 1-10; 23-36 → sem-2

Now if

$L < 1$ converges

$L > 1$ diverges

$L = 1$ test not applicable ↓ apply another test.

Remember

(i) $\lim_{n\to\infty} \frac{1}{n} \Rightarrow 0$.

(ii) $\sum_{n=1}^{\infty} \frac{1}{n}$ is a harmonic series i.e divergent

▷ Alternating Series test. → this test applies on alternating series.

Alternating Series e.g:

generally indicates if we can use this test {

(i) $\sum_{n=1}^{\infty} (-1)^n a_n$

(ii) $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$.

Divergence test ~ nth term test.

if $\lim_{n\to\infty} a_n \neq 0$ diverges

elsewise it may or may not converge/diverge

Now alternating series test has set conditions which should be met; if met the series converges; else diverges.

(i) $\lim_{n\to\infty} a_n = 0$ [fails divergence test]

(ii) The squence is decreasing; i.e

$a_{n+1} < a_n$

Example: $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n}\right)$; check this by alternating series tes[t] if it converges or diverges.

Check conditions.

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓ first condition met.

$1/_{n+1} < 1/n \Rightarrow 1/2 < 1$ ✓

∴ the 2 condition meet and it converges.

Example; Now check this term.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{5n+3}{2n-7}.$$

$$\lim_{n \to \infty} \frac{(5n+3)'}{(2n-7)'} \Rightarrow \frac{5}{2} \neq 0$$

$\therefore$ failed the first condition and this is a divergent series.

P-series test!

If a series is in the form of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ then this power 'p' can tell us.

if:

(i) $p > 1$ it will converge.

(ii) $p \leq 1$ it will diverge.

Abosulte Convergence test!

If we take the an term of a series and take its modulus with a limit → say this convergence!

Then we conclude than the original series convergence as well.

if $\sum_{n=1}^{\infty} |a_n|$ converges. then $\sum_{n=1}^{\infty} a_n$ converges as well

Geometric series test

$\sum_{n=1}^{\infty} ar^{n-1}$ now if $|r| < 1$.

↳ converges

$|r| \geq 1$ → diverges.

Q. Find interval of Convg of power series & its radius.

6. $\sum_{K=0}^{\infty} K! \, (x-1)^K.$

As the power series converge s;
by the n^{th} term test not; ratio
test.

$$\lim_{k \to \infty} \left| \frac{(K+1)! \, (x-1)^{k+1}}{K! \, (x-1)^k} \right| \Rightarrow \frac{(K+1) \cancel{K!} \, (x-1)^{K+1-K}}{\cancel{K!} \, (x-1)^0}$$

$$\Rightarrow \frac{(K+1)(x-1)}{1} \Rightarrow |x-1| \lim_{K \to \infty} (K+1) < 1$$

Now as the limit $k \to \infty$ we need to stop it and only $x = 1$
can do it to let our power series be convergent.
$\therefore$ the interval of convergence is reduced to only one
value $x = 1$
and the radius of interval is 0, as their is
only one point.

$\Rightarrow \sum_{k=1}^{\infty} 1 = \infty$

$\Rightarrow \sum_{k=1}^{\infty} (-1)^k \frac{1}{k(k+1)}$

✓

here we can split the series into two to check the convergence.

$\sum_{k=1}^{\infty} (-1)^k \frac{1}{k}$, $\sum_{k=1}^{\infty} (-1)^k \frac{1}{k+1}$ ✓

$\Rightarrow$ Both solved by alternating series test.

$\Rightarrow \sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

$\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right) \rightarrow$ Partial fraction ✓

Expanding this series will give a telescoping series and if that telescoping series yeild a finite answer, it is convergent

~x~

To cater negativity of the the derived terms x-variable we may adjust the values of our summation.

✓

→however x^0 is good

$$\left(1+\frac{1}{2!}x^2+\frac{1}{4!}x^4+\ldots\right)$$ ⇒ $\cancel{\cos}$ Sinh x's Maclaurin Series ✓

$$\left(x+\frac{1}{3!}x^3+\frac{1}{5!}x^5+\ldots\right)$$ ⇒ sinh x's Maclaurin Series. ✓

Now the same series with alternating signs are.

$$\left(1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\ldots\right)$$ cos x's Maclaurin series. ✓

$$\left(x-\frac{1}{3!}x^3+\frac{1}{5!}x^5\ldots\right)$$ sin x's Maclaurin Series. ✓

$$\sum_{k=0}^{\infty}\frac{x^k}{k!}\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}\right)$$ e^x Maclaurin Series!

* in the question were we put a defined series just like above ones, we immediately expand and take x ^power variables common afterwards! ✓