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MT-102 :

(Primary) DEMIS G ZILL ; 10th Edition

✓ ERWIN KREYSZIG ; 10th Edition.

Chapter 7 :-

$A = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ (coloumn matrix or coloumn vector)

$B = \begin{bmatrix} a & b & c & d \end{bmatrix}$ (row matrix or row vector)

Only square matrix's inverse is possible.

① Symmetric Martrix

If A (a square) matrix $= A^T$

$$A = A^T$$

$$A^T = \begin{bmatrix} 20 & 120 & 200 \\ 120 & 10 & 150 \\ 200 & 150 & 30 \end{bmatrix} = A$$

(a short tick to check if it is symmetric ; pick the nondiagnol or triangular terms and check if they are same)

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② Skew - symmetric :

A square matrix B is skew symmetric if $B^T = B$.

$$B = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 2 \\ -3 & -2 & 0 \end{bmatrix} = -B.$$

(to check if it is skew symmetric ; pick the non-diagnol entities that are negative of each other.

③ Upper triangular Matrix :-

$$\begin{bmatrix} 0 & 4 & 2 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix} ; \begin{bmatrix} 1 & 4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \begin{bmatrix} 1 & 4 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

(square matrix that can have non-zero entities only on & above the main diagnol)

* & vice versa for the lower diagnol matrix.

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④ **Diagnol Matrix :-** (square)

Non zero entities only on the main diagnol.

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

⑤ **Identity Matrix :-** (square)

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$(A.I = I.A = A)$

property of a identity during multiplication of/with a given matrix.

"APPLICATIONS OF MATRIX MULTIPLIPY"

PROBLEM :-

$$A = \begin{array}{c} \begin{array}{cc} PC1086 & PC1186 \end{array} \\ \begin{bmatrix} 12 & 16 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \end{array} \begin{array}{l} \text{Raw Components.} \\ \text{Labour} \\ \text{Miscellenous} \end{array}$$

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$$B = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 3 & 8 & 6 & 9 \\ 6 & 2 & 4 & 3 \end{bmatrix} \end{array} \begin{array}{l} \text{PC 1086} \\ \text{PC 1186.} \end{array}$$

'A' shows cost per computer & 'B' shows production figures for a year.

Find: Matrix that shows the cost per quater for raw materials, labour, & miscalleneous

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$$A \times B \Rightarrow \begin{array}{c} \begin{array}{cccc} ① & ② & ③ & ④ \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} \end{array} \begin{array}{l} \text{Raw components.} \\ \text{Labour} \\ \text{Miscellenous.} \end{array}$$

hence if we multiply $A \times B$ we will get AB of (3x4) that will satisfy the demands as highlighted above

Result :-

$$A \times B = \begin{bmatrix} (12\times3)+(16\times6) & 128 & 136 & 156 \\ 33 & 32 & 34 & 39 \\ 51 & 52 & 54 & 63 \end{bmatrix}$$

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singular matrix $|A| = 0$; no solution. $A^{-1} = \frac{adj A}{|A| \neq 0}$

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7.3: Linear System of Equations.

m equations with n unknowns.

(1) $a_{11} x_1 + a_{12} x_2 \dots a_{1n} x_n = b_1$

(2) $a_{21} x_1 + a_{22} x_2 \quad a_{2n} x_n = b_2$

(m) $a_{m1} x_1 + a_{m2} x_2 \dots a_{mn} x_n = b_m$

Conversion into a matrix.

$$\underset{m \times n}{\underbrace{\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}}_{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{X \; n \times 1} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}}_{B \; m \times 1}$$

FOR Solving a rectangular system of matrices obtained from a linear system of equation cannot be solved by

$AX = B$ Inverse method.

$X = A^{-1}B$ (not possible → as not square)

The only method left to solve these (rectangular system) of equations is through "**Gaussian Elimination**".

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Steps:-

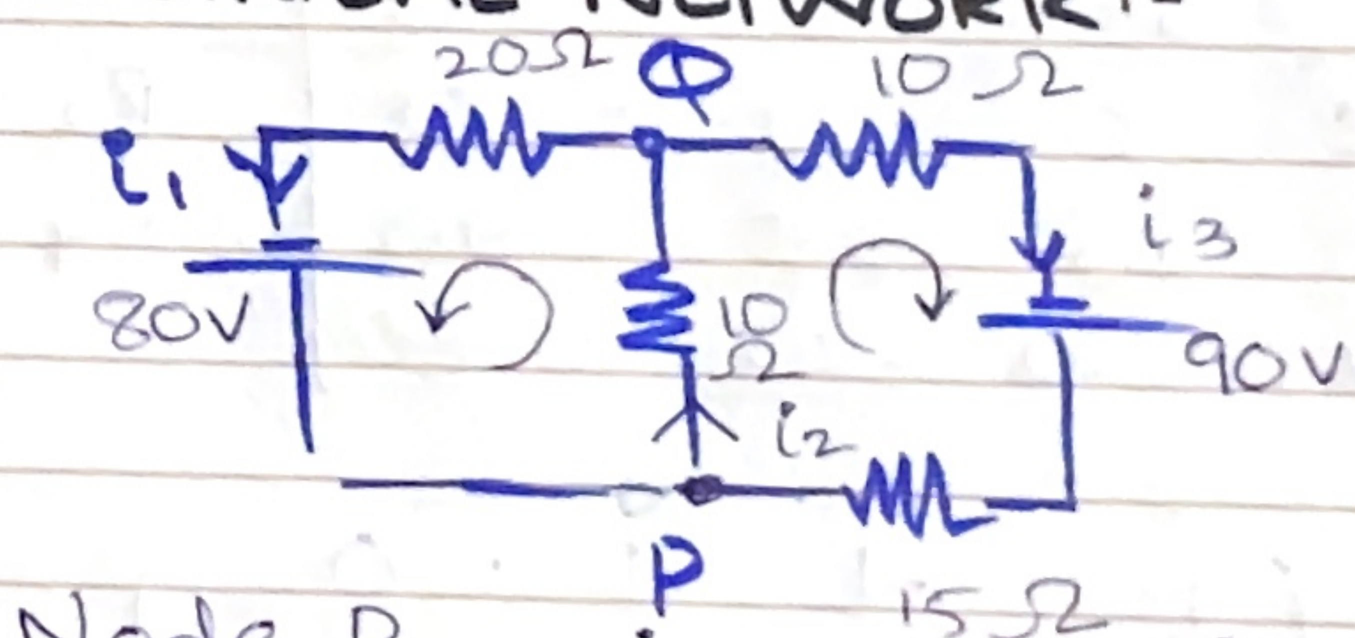
Create a augmented Matrix.

$\tilde{A} = [A : B]$

$$\Rightarrow \left[\begin{array}{ccc:c} a_{11} & a_{12} \dots & a_{1n} & b_1 \\ a_{21} & a_{22} \dots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} \dots & a_{mn} & b_m \end{array}\right]$$

Perform row operations to make an upper triangle matrix.

EXP:- **ELECTRICAL NETWORK:-**



Node P. $-i_2 + i_3 + i_1 = 0$. — (1)

Node Q: $-i_1 + i_2 - i_3 = 0$. — (2)

$10i_2 + 10i_3 + 15i_3 = 90V$

$10i_2 + 25i_3 = 90$ — (3)

$10i_2 + 20i_1 = 80V$ —

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Making equation matrix system.

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 0 & 10 & 25 \\ 20 & 10 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 90 \\ 80 \end{bmatrix}, \quad X = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

* The pivoted entry (row) will have no change ever.

$$\tilde{A} = \left[\begin{array}{ccc:c} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 10 & 25 & 90 \\ 20 & 10 & 0 & 80 \end{array}\right]$$

first value of diagonal = first pivot

move through the column down and make zeros

$$\left[\begin{array}{ccc:c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \end{array}\right] \quad \begin{array}{l} R_2 + R_1 \\ R_4 - 20R_1 \end{array}$$

zeros have been converted

Now make 2nd diagonal entry the pivot

$$\left[\begin{array}{ccc:c} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \end{array}\right] \quad R_2 \leftrightarrow R_4$$

Zero in a diagonal entry is not helpful; change it somehow

$$\left[\begin{array}{ccc:c} 1 & -1 & 1 & 0 \\ 0 & 30 & -20 & 80 \\ 0 & 10 & 25 & 90 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad 3R_3 - R_2$$

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7.2, 7.3, 7.4, 7.5, 7.7, 7.8,

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Now '10' in the highlighted portion is the only value that needs to be zero in order to be a upper tangle.

$$\left[\begin{array}{ccc:c} 1 & -1 & 1 & 0 \\ 0 & 30 & -20 & 20 \\ 0 & 0 & 95 & 190 \\ 0 & 0 & 0 & 0 \end{array}\right] \checkmark$$

Back Substitution

$0 = 0$ * ✓

$95 i_3 = 190 \Rightarrow i_3 = 2$

$30 i_2 - 20 i_3 = 80$

$i_3 = \frac{190}{95} \Rightarrow i_3 = 2$ Ans!

$30 i_2 - 20(2) = 80$

$30 i_2 - 40 = 80 \Rightarrow i_2 = \frac{120}{30} = 4$ Ans!

$i_1 - i_2 + i_3 = 0$

$i_1 - (4) + 2 = 0$

$i_1 - 2 = 0$

$i_1 = 2$ Ans!

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Stochastic Matrix:-

Stochastic Matrix is a square matrix with all the entries non-negative and the coloumn entries sum up to become 1.

this can also occur by summing a row.

MARKOV'S PROCESS:-

a process for which the probability of a newer state / entering a newer state depends upon the last state occupied.

For a markov process a matrix must be a stochastic matrix. If X_0 is the last/ intial stage then any later stage (X_f) is given by.

$$X_f = P\,X_0$$

X_f → stage achieved after change

P → stochastic matrix (of probability)

X_0 → intial / last state / stage.

CASES :

COLOUM STOCHASTIC MATRIX

In it the square matrices has entries in a coloum summing equal to one

Such case has a coloumn matrix for X_0

ROW STOCHASTIC MATRIX

In this the square matrix has row entries summing equal to one.

Such case has row matrix for X_0.

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Example 13:-

Suppose that 2004 state of land used in a city of 60 mi² built up area of

C. commerical area 20% + 5 = 25%.

I. Industrial " 20%.

R. Residential " 55%. =>

Find the states in 2009, 2014, 2019 assuming that the transistion probabilities for 5 years intervals are given by matrix A.

$$A = \begin{array}{c|c} & \text{From} \\ & \begin{array}{ccc} C & I & R \end{array} \\ \text{To} & \left[\begin{array}{ccc} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{array}\right] \begin{array}{c} C \\ I \\ R \end{array} \end{array}$$

Now as the probability of newer state is required we use the markov's process. The matrix A is stochastic (coloumn wise), thus the X_0 is given as.

$$X_0 \begin{bmatrix} 25\% \\ 20\% \\ 55\% \end{bmatrix}$$; By method $X_f = P X_0$

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$$\Rightarrow \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix} \times \begin{bmatrix} 25 \\ 20 \\ 55 \end{bmatrix}$$

There fore at 2014 is

$$\Rightarrow \begin{bmatrix} 19.5 \\ 34.0 \\ 46.5 \end{bmatrix} \% \quad \text{Ans} \quad \text{— (a)}$$

similarly using (a) again 2009, then 2014, 2019 can be predicted.

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system with variable power atmost 1.

linear system of equ(s) in matrix form.

$a_{11}x_1 + \dots + a_{1n}x_n = b_1$

$a_{21}x_1 + \dots + a_{2n}x_n = b_2$

$\vdots$

$a_{m1}x_1 + \dots + a_{mn}x_n = b_m$.

* the equations are given in the staight line;
* terms variating at 'a_{mn}' are coefficients.
* '$b_1 \dots b_m$' are the numbers given on the righ
  - if all 'b_j' => 0 it is homogeneous system
  - if atleast one 'b_j' $\neq 0$ it is nonhomogeneous

* For a homogeneous system the trivial solution exisit (atleast)

$$AX = 0$$
$$X = 0$$

thus

* any non-zero solution can be termed as a 'non-trivial' solution.

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \dots & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \quad \& \quad X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad \& \quad b = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

COEEFFICIENT MATRIX.

$$\tilde{A} = \left[\begin{array}{ccc:c} a_{11} & \dots & a_{1n} & b_1 \\ \vdots & \dots & \vdots & \vdots \\ a_{m1} & & a_{mn} & b_m \end{array}\right]$$

AUGMENTED MATRIX.

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CASE STUDY:-

If two equation and two unknown variable are given.

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

If we interpret x_1, x_2 in coordinates in $x_1 x_2$-plane then each of the two equations represent a straight line and (x_1, x_2) solution if & only if the point P with coordinates x_1, x_2 lie on both the line.

Hence there are 3 possibilities:

(i) Precisely one solution (lines intersect)
(ii) Infinitely many solution (lines coincide)
(iii) no solution (line parallel.)

* if the system is homogeneous i.e (b_1 & $b_2 = 0$) then case (iii) is not possible because for a homogeneous system; trivial solution always exisits.

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CONCEPT :-

TYPES OF LINEAR SYSTEM:-

Pivot = no. of equ.

overdetermined no. of equation > no. of unknows.

Determined no. of equation = no. of unknowns

under determined no. of equation < no. of unknowns.

CONSISTENT :- If a system has atleast one sol. (i.e one or infinitely many solutions).

INCONSISTENT:-

If a system has no solution.

* homogeneous system can never be inconsistent.

CASES :- (OF Gauss Elimination)

(I) Gauss Elimination if infinetly many solution exist

Usually seen when under-determined form exisits (& when case II does not exisit)

↓ occurance of arbitrary values.

check Q7, Q9, Q11

(II) Guass E; if no solution exisits

The condition appears when after achieving Δ augmented matrix a contradiction occurs

e.g Q12 $5 \neq 15$

III SOLUTION EXISITS UNIQUELY.

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NOTES:-

Elementary Matrices:-

The elementary matrices can be obtained by matrix multiplication.

If A is a matrix of (m×n) order on which we want to perform operations (elementary) then there is a matrix 'E' such that 'EA' is the new matrix after the operation.

Such E is called as the elementary matrix.

—x—

however generally elementary matrices are those matrices that can form an identity by only a single / elementary row operation

Example

Q24 (a)+(b)

$$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \Leftrightarrow R_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

hence elementary matrix.

~~(b)~~

$$E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 + 5R_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

hence elementary matrix

$$E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

$$\frac{R_4}{8} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \cancel{8}1 \end{bmatrix}$$ hence elementary matrix

We can examine the operation we performed in the matrix to obtain the concept required in the sol.

→ tranferming into systems of equ / matrix

(i) $$E_1 M \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a \\ c \\ b \\ d \end{bmatrix}$$

(ii) $$E_2 M \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a \\ b \\ -5a+c \\ d \end{bmatrix}$$

(iii) $$E_3 M \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ 8d \end{bmatrix}$$

7.4 Linear Independance & dependance of vectors.

Given any set of m vectors $V_1, V_2 \dots V_m$ with same number of components, a linear combination of these vector is

$$a_1 V_1 + a_2 V_2 + \dots a_m V_m ; \quad a_1, a_2 \dots a_m \in \mathbb{R}$$

The equation:-

$$a_1 V_1 + a_2 V_2 + \dots a_m V_m = 0$$

hold if we chose all a_j's zero since atleast one of the vectors is non-zero. As a result the vectors $V_1, V_2 \dots V_m$ are linearly independant. & vice versa = linearly dependant.

EXAMPLE:

$V_1 = [\,0 \quad 1 \quad 1\,]$; $V_2 = [\,1 \quad 1 \quad 1\,]$
$V_3 = [\,0 \quad 0 \quad 1\,]$

Check if the vectors are linearly independant or not.

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$a_1V_1 + a_2V_2 + a_3V_3 = 0$

$a_1[0\ 1\ 1] + a_2[1\ 1\ 1] + a_3[0\ 0\ 1] = 0$

$[0\ a_1\ a_1] + [a_2\ a_2\ a_2] + [0\ 0\ a_3] = 0$

$[a_2,\ a_1 + a_2,\ a_1 + a_2 + a_3] = [0\ 0\ 0]$

$a_2 = 0$
$a_3 = 0$
$a_1 + a_2 = 0$
$a_1 = 0$

thus in this questions the components of vectors cannot be written in term of one another.

$\Rightarrow$ linear independance.

EXAMPLE:-

$V_1 = [3\ \ 0\ \ 2\ \ 2]$
$V_2 = [-6\ \ 42\ \ 24\ \ 54]$
$V_3 = [21\ \ -21\ \ 0\ \ -15]$

Now to check if vecters are dependant or independant linearly.

$a_1V_1 + a_2V_2 + a_3V_3 = 0$

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$a_1[3\ \ 0\ \ 2\ \ 2] + a_2[-6\ \ 42\ \ 24\ \ 54]$
$a_3[21\ \ -21\ \ 0\ \ -15] = 0$

$[3a_1\ \ 0\ \ 2a_1\ \ 2a_1] + [-6a_2\ \ 42a_2\ \ 24a_2\ \ 54a_2]$
$[21a_3\ \ -21a_3\ \ 0\ \ -15a_3] = [0\ \ 0\ \ 0\ \ 0]$

$[3a_1 - 6a_2 + 21a_3,\ 0 + 42a_2 - 21a_3,$
$2a_1 + 24a_2 + 0,\ 2a_1 + 54a_2 - 15a_3]$
$= [0,\ 0,\ 0]$

Now drawing equations;

$3a_1 - 6a_2 + 21a_3 = 0$ — (1), $42a_2 - 21a_3 = 0$ — (2)

$2a_1 + 24a_2 = 0$ — (3) $2a_1 + 54a_2 - 15a_3 = 0$ — (4)

Using $42a_2 = 21a_3$
$[a_2 = \frac{1}{21} a_3]$

using $2a_1 + 24a_2 = 0$
$[a_1 = -12a_2]$

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Rank of the matrix:-

The rank of the matrislx A is the maximum number of linearly independant row vectors of A.

OR.

Rank of the matrix is the number of non-zero rows in echelon form.

Example:-

$$A = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 98 & 58 \\ 0 & -21 & -14 & -29 \end{bmatrix} \quad \begin{matrix} R_2 + 2R_1 \\ R_3 - 7R_1 \end{matrix}$$

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 98 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad 2R_3 + \cancel{2}R_2$$

R.W

$-14 + 2(46)$

$-29 + 2(58)$

Thus rank of the matrix $= 2$.

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7.4

$Q_1 - 10$

$7 - 26$.

First two rows of the matrices corresponding to V_1, V_2 are non-zero suggesting that V_1 & V_2 are linearly independant. but due to V_1, V_2, V_3 altogether they are dependant.

Check:-

$$a_1 V_1 + a_2 V_2 = 0$$

$$a_1 [3 \quad 0 \quad 2 \quad 2] + a_2 [-6 \quad 42 \quad 24 \quad 54] = [0 \quad 0 \quad 0 \quad 0]$$

$$[3a_1 - 6a_2, \quad 42a_2, \quad 2a_1 + 24a_2, \quad 2a_1 + 54a_2] = [0 \quad 0 \quad 0 \quad 0]$$

7.5 * HOMOGENEOUS LINEAR SYSTEM:-

A homogeneous system of m equations in n unknows ; total no. of / total no. of columns.

$$a_{11} x_1 + a_{12} x_2 \ldots + a_{1n} x_n = 0$$

$$a_{21} x_1 + a_{22} x_2 \ldots + a_{2n} x_n = 0$$

$$\vdots$$

$$a_{m1} x_1 + a_{m2} x_2 + a_{mn} x_n = 0$$

It always have a trivial solution and non-trivial

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EXAMPLE :-

let $\tilde{A} = \left[\begin{array}{ccc|c} 2 & 4 & 1 & 0 \\ -1 & 1 & -2 & 0 \\ 4 & 0 & 6 & 0 \end{array}\right]$

Echelon form $\tilde{A}$ $\left[\begin{array}{ccc|c} 2 & 4 & 1 & 0 \\ 0 & 6 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Non augmented matrix's rank.

$$\begin{bmatrix} 2 & 4 & 1 \\ 0 & 6 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

Now

Rank of the matrix = 2

no. of unknown variables = 3

hence only trivial solution exisits. but also nontrivial

as Rank $A < n$.

EXISTENCE & UNIQUENESS OF SOLUTION OF LINEAR SYSTEM.

NON-HOMO. **HOMOGENEOUS.**

A linear system of m equations in n unknowns has;

① A unique solution if rank A = rank $\tilde{A} = n$.

Example :- electrical Network.

Echelon form $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 30 & -20 & 80 \\ 0 & 0 & 95 & 190 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$n = 3$

Rank $\tilde{A}$ = 3

Rank of A = 3

hence unique solution exisit.

② Infinite many solutions if

rank A = rank $\tilde{A} < n$.

Example :

$$\left[\begin{array}{cccc|c} 3 & 2 & 2 & -5 & 8 \\ 0.6 & 1.5 & 1.5 & -5.4 & 2.7 \\ 1.2 & -0.3 & -0.3 & 2.4 & 2.1 \end{array}\right]$$

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Echelon form $\begin{bmatrix} 3 & 2 & 2 & -5 & : & 18 \\ 0 & 1 & 1 & -4 & : & 1 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$

rank $A = 2$

rank $\tilde{A} = 2$

$n = 4$

rank $A =$ rank $\tilde{A} < n$.

③ System has no solution. if A & $\tilde{A}$ have different ~~solved~~ ranks.

Example:

$$\tilde{A} = \begin{bmatrix} 3 & 2 & 1 & : & 3 \\ 2 & 1 & 1 & : & 0 \\ 0 & 2 & 4 & : & 6 \end{bmatrix}$$

echelon form $= \begin{bmatrix} 3 & 2 & 1 & : & 3 \\ 0 & -1 & -1 & : & -6 \\ 0 & 0 & 0 & : & 12 \end{bmatrix}$

rank $\tilde{A} \Rightarrow 3$ $\therefore$ no solution.

rank $A \Rightarrow 2$

"DETERMINANTS"

A determinant of order n is always associated with square matrix $A = [a_{ij}]$

$$D = \det A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

Determinants :-

Determinant of a triangular matrix is the is product of its diagonal matrix's / triangular matrix's diagonal entries

Example:-

$$A = \begin{bmatrix} -3 & 0 & 0 \\ 6 & 4 & 0 \\ -1 & 2 & 5 \end{bmatrix}$$

$$|A| = (-3)(4)(5) = -60 \text{ Ans!}$$

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Apply's when guass elimination has failed.

CRAMMER'S RULE :- ↳ happen when the pivoted element=0 and zero cannot be removed.

If a linear system of n equations in n unknowns i.e $x_1, x_2, x_3 \dots x_n$ has a non-zero determinant of coefficient matrix. i.e $\det A \neq 0$ than the system has unique solution. → non singular matrix.

given ; $x_1 = \frac{D_1}{D}$.

$x_2 = \frac{D_2}{D}$ ... $x_n = \frac{D_n}{D}$

* if D is => zero (Crammer's Rule is not applicable switch to another.)

7.8

Inverse of a matrix.

The inverse of a matrix A is defined as

$$A^{-1} = \frac{\text{adj}\,A}{|A|} = \frac{[C_{ij}]^T}{\det A}$$

cofactors ← $C_{ij} = (-1)^{i+j} M_{ij}$
where $i = 1, 2, 3 \dots n$
where $j = 1, 2, 3 \dots n$.

where M_{ij} is found by eliminating ith row & jth coloumn in det A.

Example:-

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$ find $A^{-1} = \frac{\text{Adj}\,A}{|A|}$

$$|A| = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{vmatrix}$$

$= [-1(-4-3) - 1(12+1) + 2(9-1)]$

$\Rightarrow (7) - 13 + 16 \Rightarrow 10$ Ans!

=> Finding adjoint through cofacter method.

$C_{11} = (-1)^{1+1} M_{11} \Rightarrow \begin{vmatrix} -1 & 1 \\ 3 & 4 \end{vmatrix} \Rightarrow -7$

$C_{12} = (-1)^{1+2} M_{12} \Rightarrow \begin{vmatrix} 3 & 1 \\ -1 & 4 \end{vmatrix} \Rightarrow -13$

$C_{13} = (-1)^{1+3} M_{13} \Rightarrow \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} \Rightarrow 8$

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$$C_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \Rightarrow 2$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} -01 & 2 \\ -1 & 4 \end{vmatrix} \Rightarrow -2$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} -1 & 1 \\ -1 & 3 \end{vmatrix} \Rightarrow 2.$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} \Rightarrow 3$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} -1 & 2 \\ 3 & 1 \end{vmatrix} \Rightarrow 7$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} -1 & 1 \\ 3 & -1 \end{vmatrix} \Rightarrow -2$$

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \Rightarrow \begin{bmatrix} -7 & -13 & 8 \\ 2 & -2 & 2 \\ 3 & 7 & -2 \end{bmatrix}$$

Now $(C_{ij})^T = \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$

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$C_{ij} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$ $(C_{ij})^T$

Reduced echelon :- has both upper & lower Δs.

$$A^{-1} = \frac{\text{adj} A}{|A|} = \frac{\begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}}{10} \Rightarrow \begin{bmatrix} 0.7 & 0.2 & 0.3 \\ -1.3 & -0.2 & 0.7 \\ 0.8 & 0.2 & -0.2 \end{bmatrix}$$

★ Inverse of a diagonal matrix exisits iff all the elements in diagonal are non-zero and is given by.

let

$$A = \begin{bmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ 0 & a_{22} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & a_{nn} \end{bmatrix}$$

$$A^{-1} \begin{bmatrix} 1/a_{11} & 0 & 0 & \cdots & 0 \\ 0 & 1/a_{22} & 0 & & 0 \\ 0 & 0 & 0 & & 1/a_{nn} \end{bmatrix}$$

7.8 Inverse of a matrix by Gauss-Jordon Method.

Extension of Gaussian Elimination.
Augmented matrix has a form.

$$\tilde{A} = [A \mid I] \xrightarrow[\text{operations}]{\text{after row}} \tilde{A} = [I \mid A^{-1}]$$

we require!

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$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

$$\tilde{A} = \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 & 1 & 0 \\ -1 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} (-)R_1 \\ \frac{R_2}{3} \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & -2 & -1 & 0 & 0 \\ 1 & -1/3 & 1/3 & 0 & 1/3 & 0 \\ -1 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} R_2-R_1 \\ R_3+R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & -2 & -1 & 0 & 0 \\ 0 & 2/3 & 7/3 & 1 & 1/3 & 0 \\ 0 & 2 & 2 & -1 & 0 & 1 \end{array}\right]$$

make identity

$$\frac{3}{2}R_2 \left[\begin{array}{ccc|ccc} 1 & -1 & -2 & -1 & 0 & 0 \\ 0 & 1 & 7/2 & 3/2 & 1/2 & 0 \\ 0 & 2 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$R_3-2R_2 \left[\begin{array}{ccc|ccc} 1 & -1 & -2 & -1 & 0 & 0 \\ 0 & 1 & 7/2 & 3/2 & 1/2 & 0 \\ 0 & 0 & -5 & -4 & -1 & 1 \end{array}\right]$$

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$$R_1+R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 3/2 & 1/2 & 1/2 & 0 \\ 0 & 1 & 7/2 & 3/2 & 1/2 & 0 \\ 0 & 0 & -5 & -4 & -1 & 1 \end{array}\right]$$

$$\frac{R_3}{-5} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 3/2 & 1/2 & 1/2 & 0 \\ 0 & 1 & 7/2 & 3/2 & 1/2 & 0 \\ 0 & 0 & 1 & 4/5 & 1/5 & -1/5 \end{array}\right]$$

$$\begin{array}{c} R_2-\frac{7}{2}R_3 \\ R_1-\frac{3}{2}R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7/10 & 1/5 & 3/10 \\ 0 & 1 & 0 & -13/10 & -1/5 & 7/10 \\ 0 & 0 & 1 & 4/5 & 1/5 & -1/5 \end{array}\right]$$

hence now the inverse is:-

$$\begin{bmatrix} -7/10 & 1/5 & 3/10 \\ -13/10 & -1/5 & 7/10 \\ 4/5 & 1/5 & -1/5 \end{bmatrix}$$

Galaxy

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