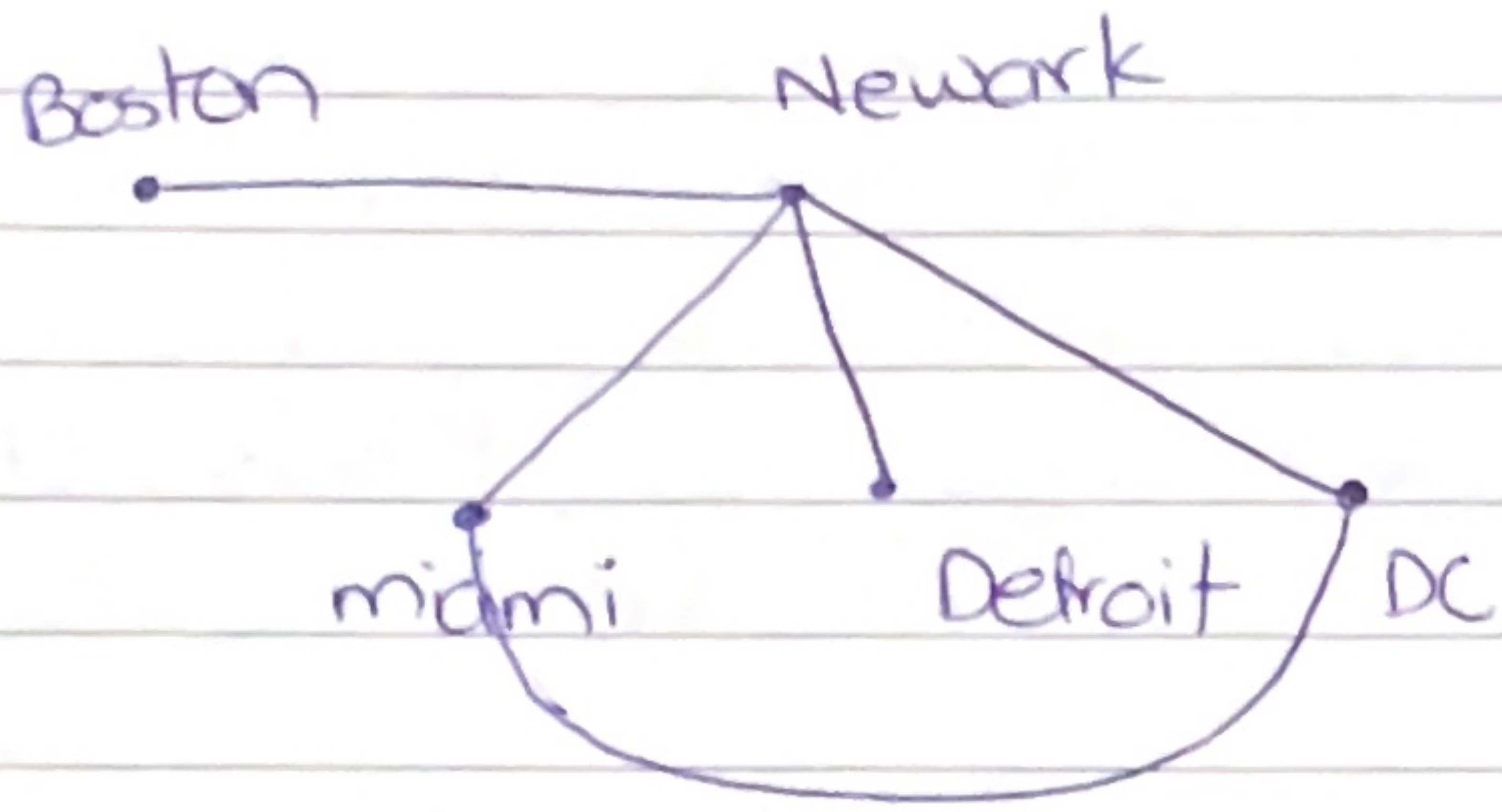
1, 3-9, 15, 16, 18, 19, 20, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34*, 35*, 37, 38.

Date

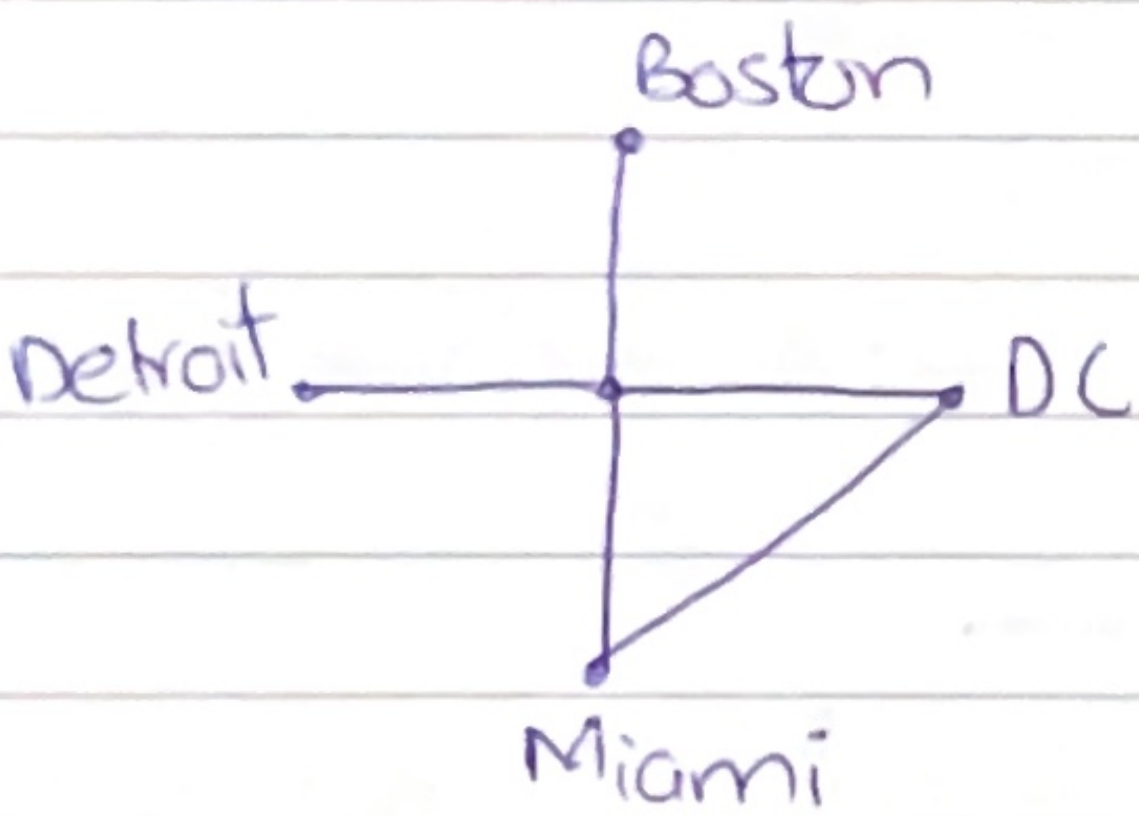
EXERCISE 10.1.

Question no 1

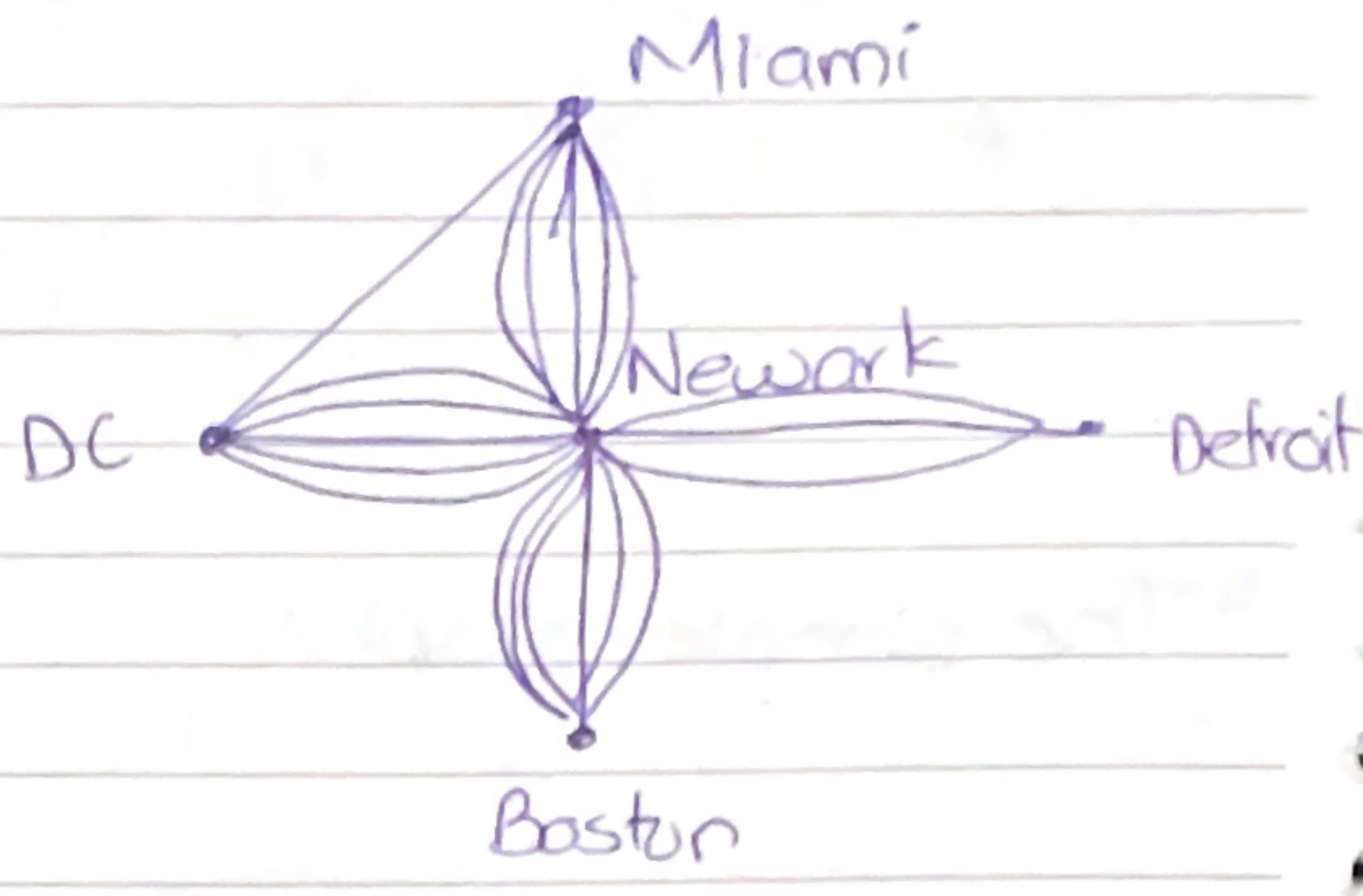
(a) one edge can represent flights in either direction.



or

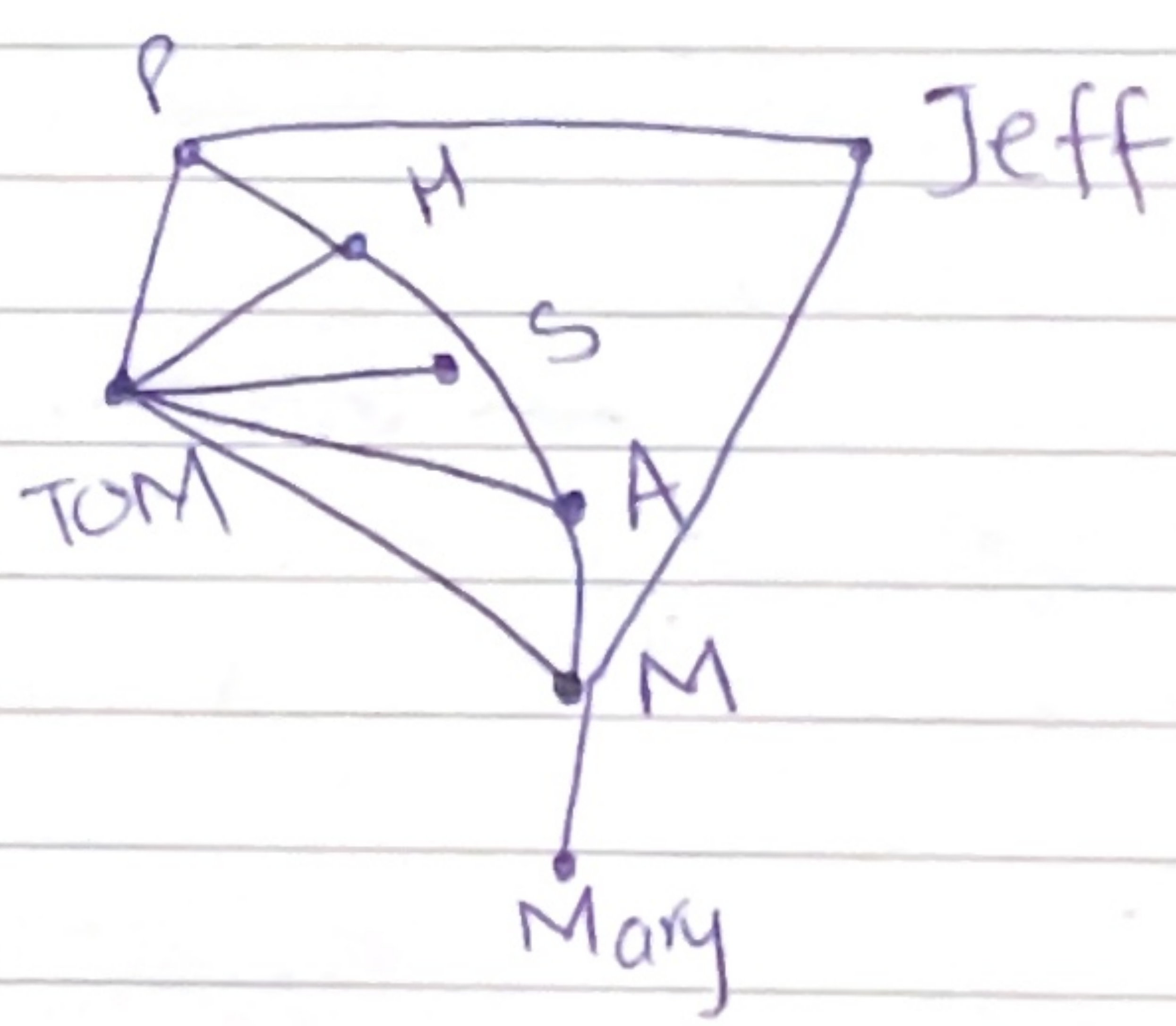


(b) now one edge represents one flight in both directions



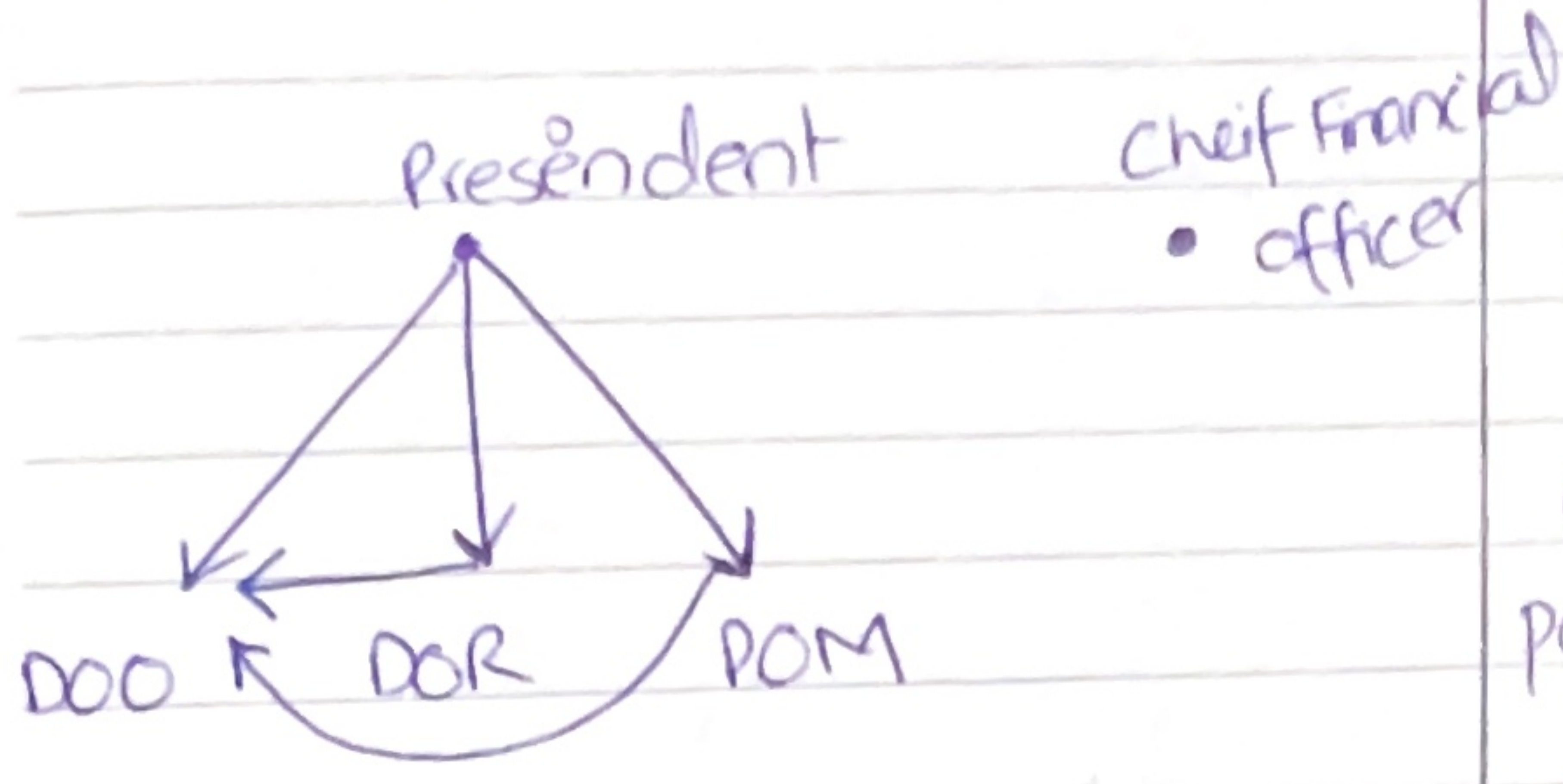
(c) easy
(d) easy
(e) easy.

Question no. 16



Date

Question no. 19



Question no. 20

Question no. 27

One way to tackle this problem would be to assign a weight to each edge representing the particular email (address used to) send it.

A new weight on an edge originating from a computer vertex would indicate a new email address being used to send email messages.

Question no. 29:-

V = vertices = set of all people in the party.

E = edges = $\{(u,v) \in V \times V \mid u \text{ knows } v\}$

The edges should be directed indicating that a person knows a certain person and its not ~~usually~~ always the other way around e.g fans of a celebrity.

Multiple edges aren't allowed because a person either knows the other person or does not.

1-~~3~~, ~~4~~, ~~5~~, ~~6~~, ~~7-9~~, ~~10~~, ~~12~~, ~~13~~, ~~16~~, 18, 19
~~20~~, 21-~~25~~, ~~26~~, ~~27~~, 28

Date

No loops in the graph as every person knows himself.

Question no. 31

we are interested in creating a graph that models the prereqs of courses at university.

V = set of all the courses at the university.

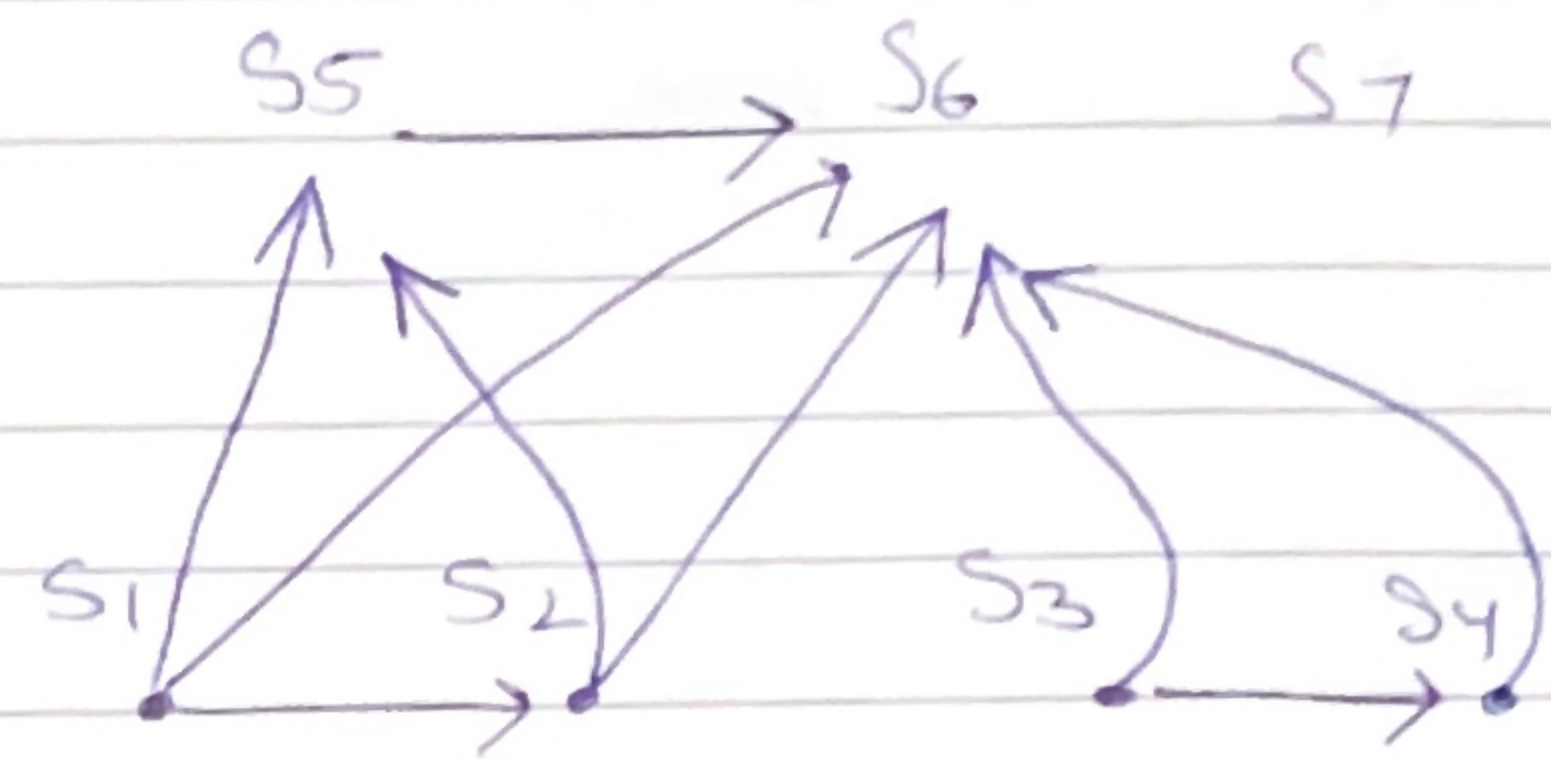
$E = \{(u,v) \in V \times V \mid$ course u is a prereq for taking v $\}$

The graph edges should be directed because if u is a pre-req of v then v cannot be the pre-req of 'u' (generally).

Multiple edges are not allowed because either u is a prereq of v or it is not.

The graph should not contain loops because a certain course cannot be a pre req of itself.

Question no. 35 *



EXERCISE 10.2

(3)

No. of vertices = 8+1 => 9.

no. of edges = 10+2 => 12.

a : 3
b : 2
c : 4
d : 0
e : 6
f : 0
g : 4
h : 2
i : 0 3

Isolated.

$2m = \Sigma \deg(v)$

$2(12) = 24$ proved.

Date

Exercise no. 5

In order to verify if a graph can have 5 degree each of 15 vertices; we will have to use the handshaking theorem.

total no. of degree = 15·5
= 75.

$75 = 2(m)$
↓
no. of edges;
however for no value of m; $2m = 75$ hence such a graph cannot exist.

Q no. 6

We are interested in the set of people at a party and in which people have shaken hands with another person.

Vertices = V = set of all people at a party.

Edges = $E = \{(u, v) \in V \times V \mid$ u has shaken hands with v$\}$

The edges of the graph should be undirected, because if u shakes hands with a person v then the other way around is also possible.

Since the degree would represent the no. of people shaken hands with, the sum of degree would be even.

$$2m = \sum_{v \in V} \deg(v)$$

Q no. 9.

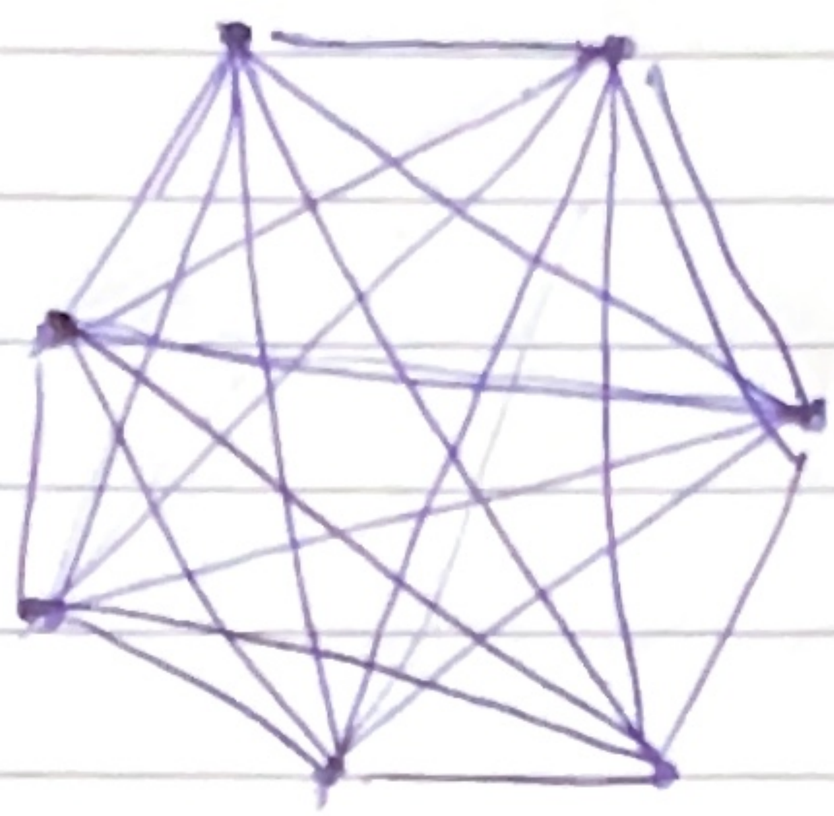
no. of vertices ⇒ 5
no. of edges ⇒ 13.

in-degree.	out degree
a : 4+2	a : 1
b : 1	b : 3+2
c : 2	c : 3+2
d : 3+1.	d : 2.
e : 0	e : 0
↓	↓
13 ⇒	13 ⇒ edges

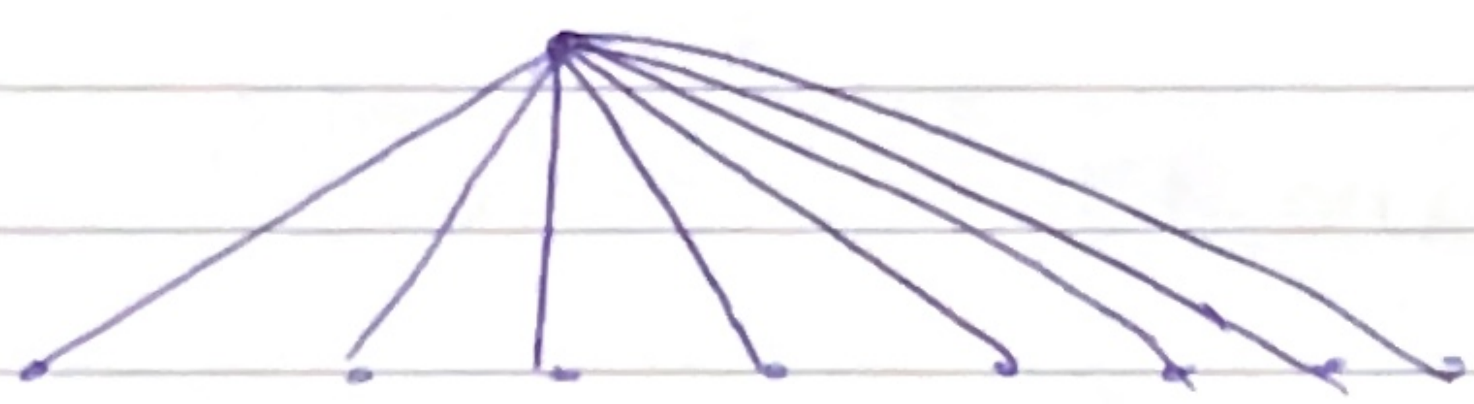
Date

Q no. 20

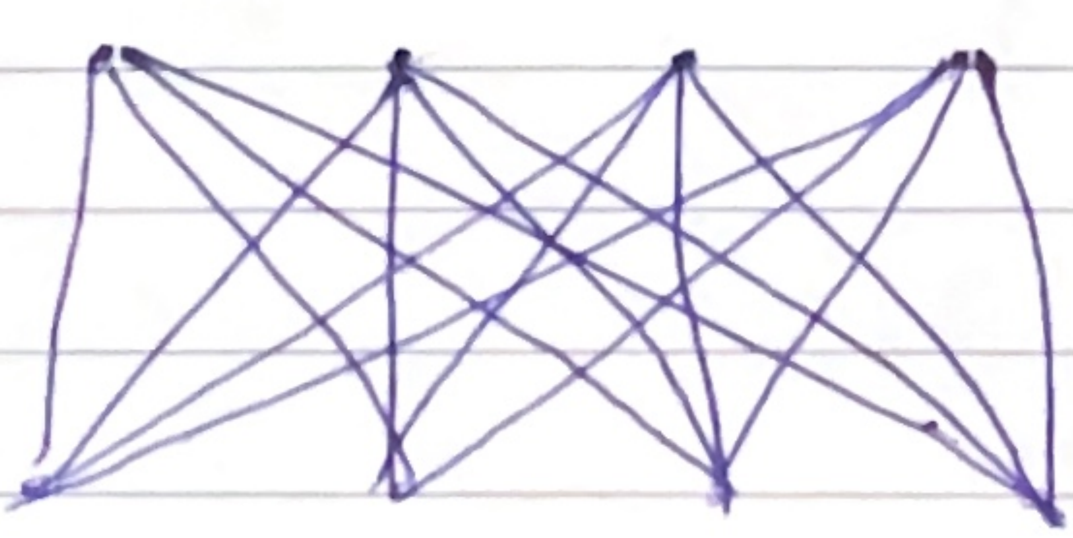
(a) K_7.



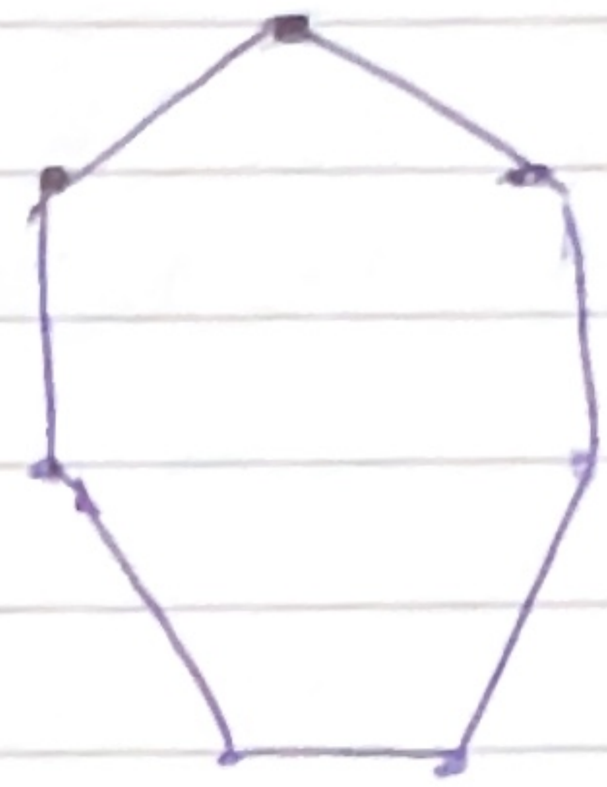
(b) $K_{1,8}$



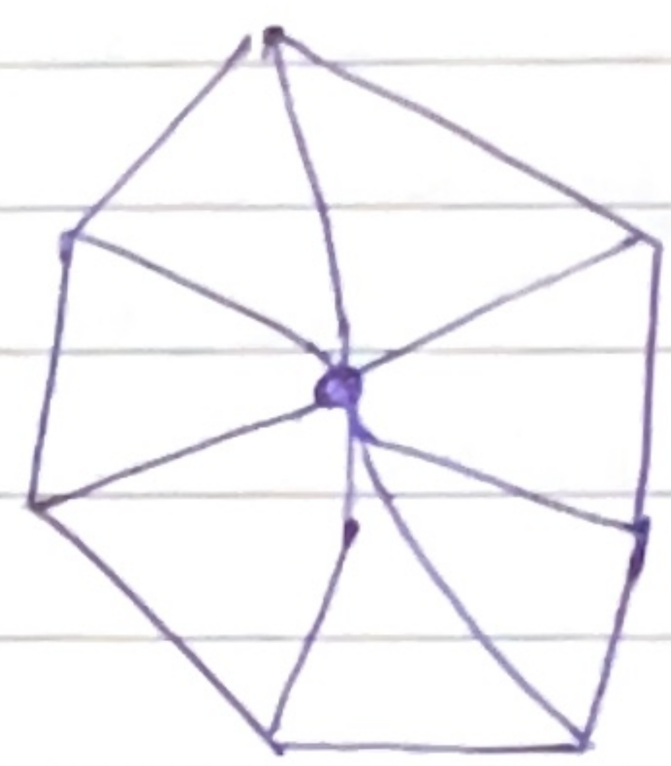
(c) $K_{4,4}$.



(d) C_7.

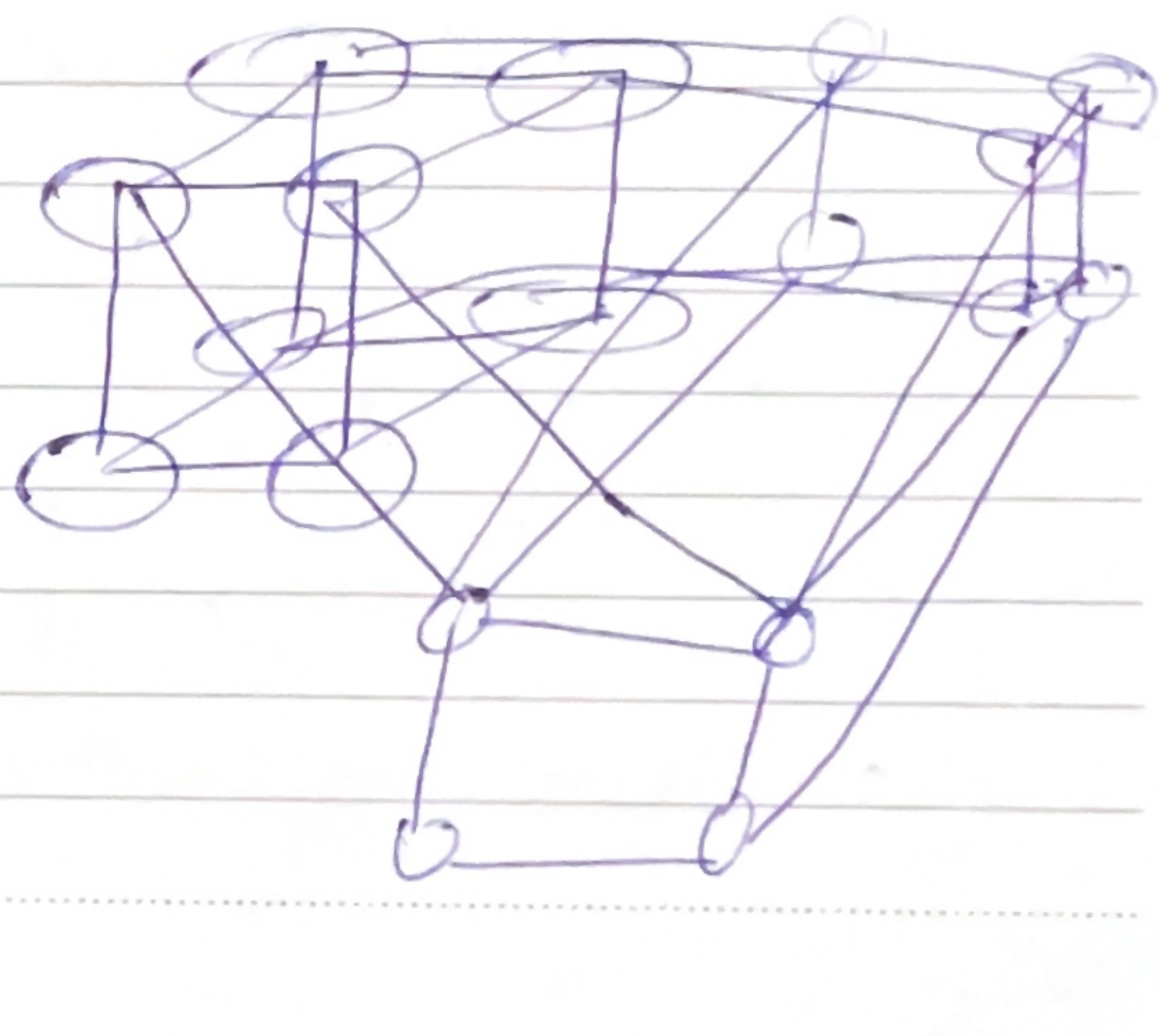


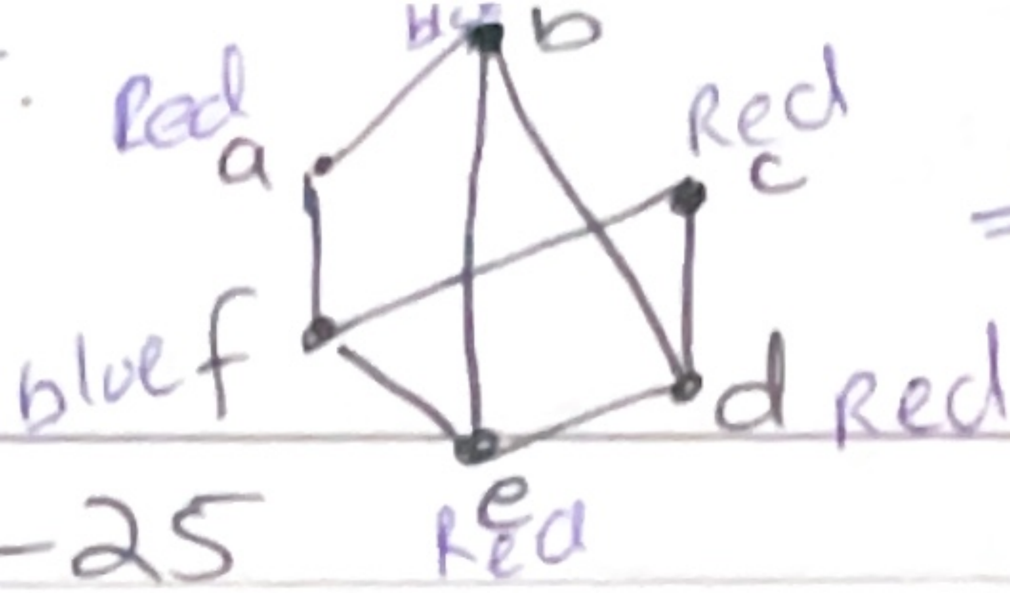
(e) W_7.



(f) Q_4

$2^4 \Rightarrow 16$ vertices

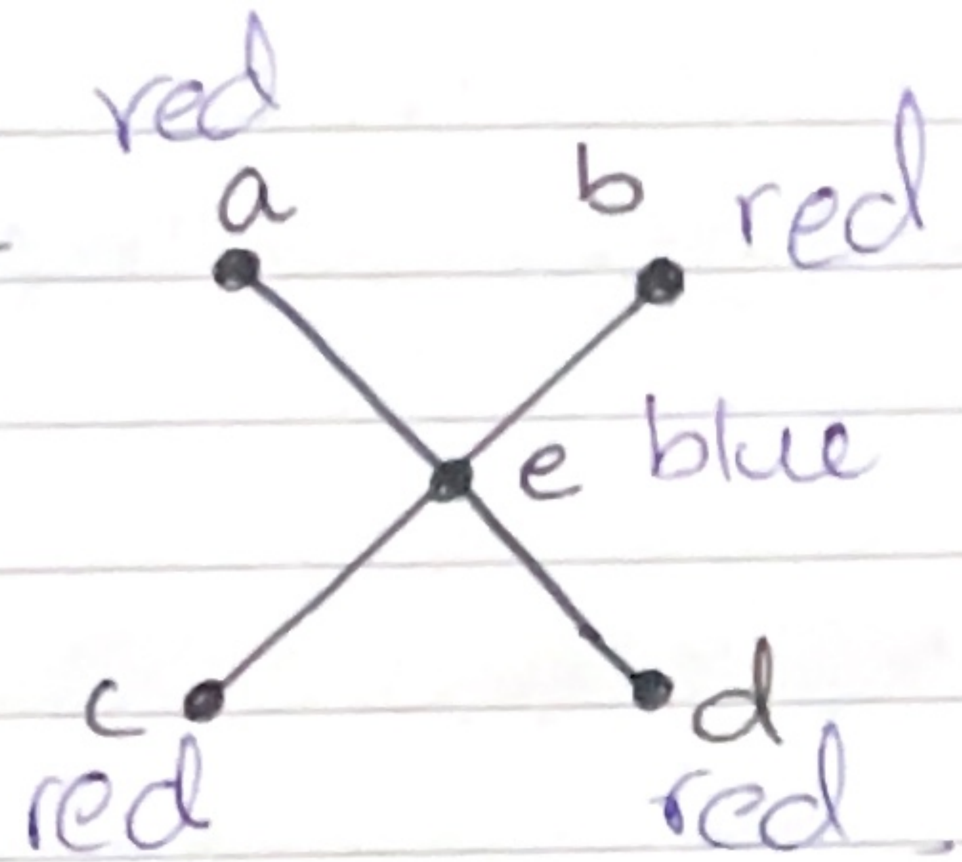


25. 

=> Not a Bipartite.

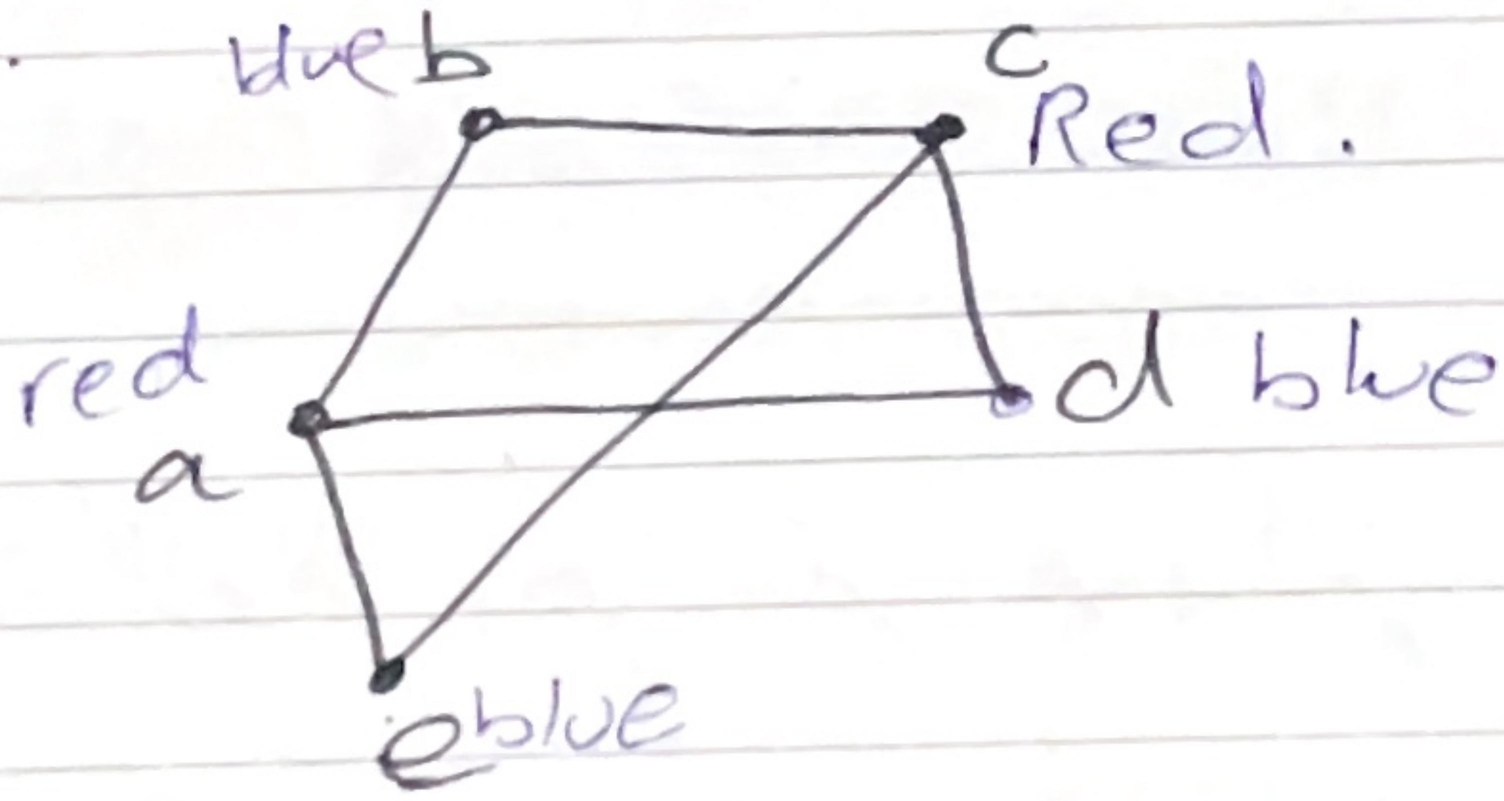
Date

Question 21-25
identify if the graph is
Bipartite.

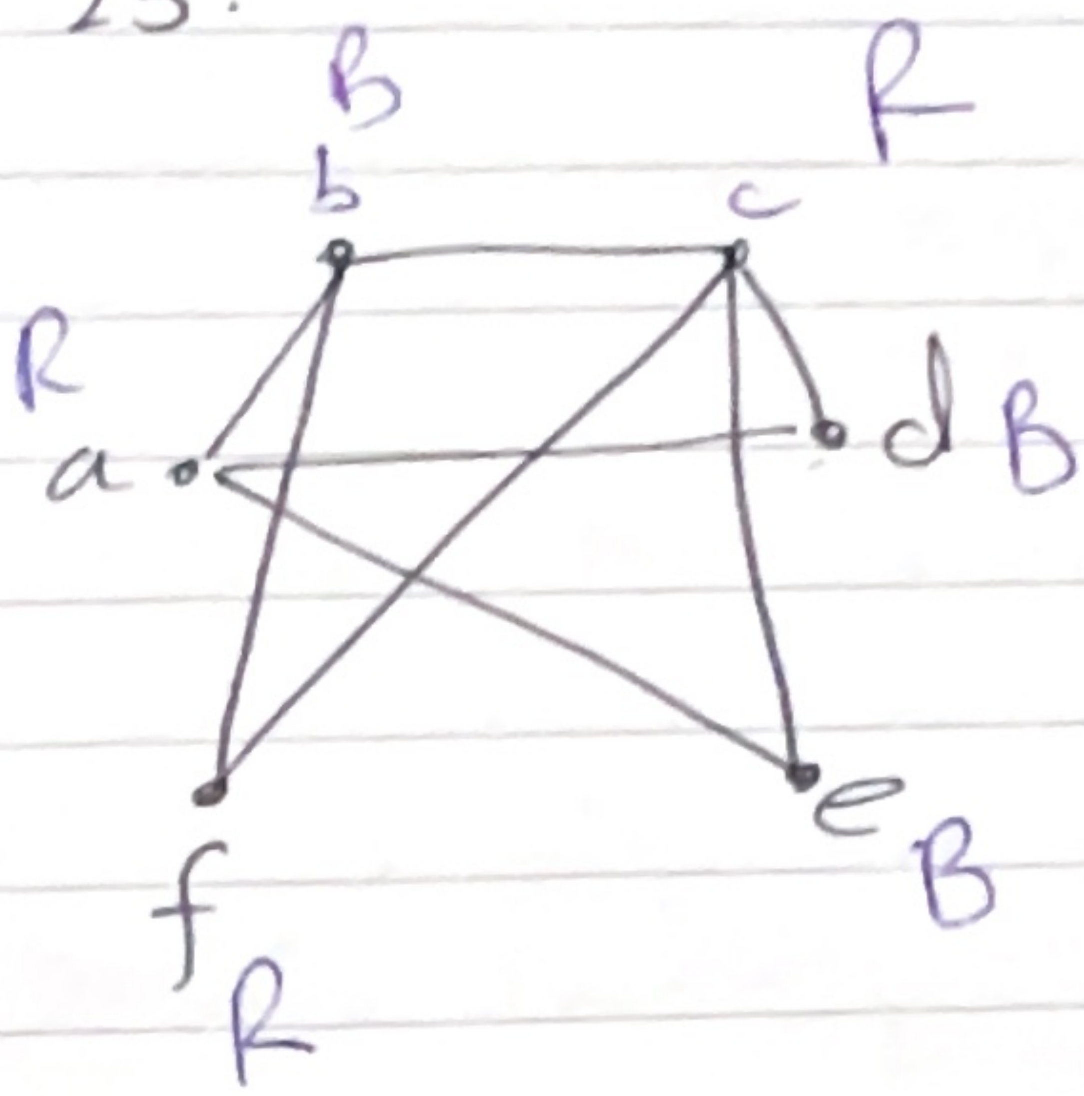
21 

let us assign red to a
and blue to its adjacents
and so on.

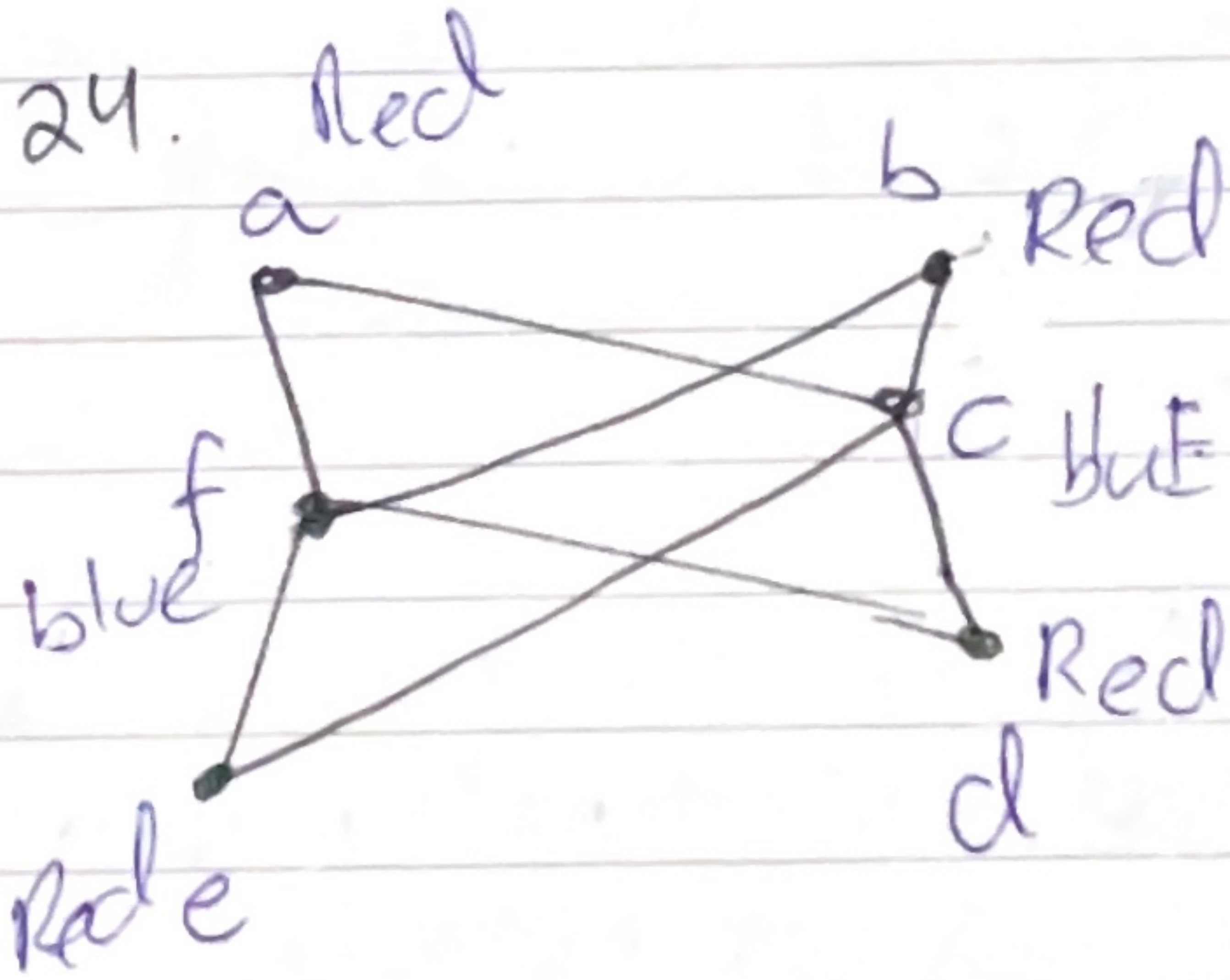
No two same colors are connecting
hence it is a bipartite graph.

22. 

a => red.
b, d, e => blue
c => red.
hence it is a bipartite graph

23. 

a => Red.
b, d, e => blue
c => Red.
f -> red.
however 'c and f'
are both red now.!
thus not a Bipartite.

24. 

Bipartite.

Q_n is Bipartite for all the value of n.
W_n is not " for all the " " "
~~C_n is bipartite for $n \in E^+$ and not for $n \in O$~~

Question no. 18

Consider a simple graph with n vertices,
these simple graphs can have n-1 degrees of vertices.

let us by contradiction prove:

let that no two vertices in a graph can have the same degree, in that case the degree of each vertex will have one-to-one correspond once to each vertex.

$V = \{v_1, v_2, v_3\}$

$D = \{0, 1, 2, \ldots n-1\}$

then

let v_1 has degree 0 and eventually some n vertex has degree n-1, these both will not be possible

because if nth vertex has n-1 degree it means it will connect with v_1 as well but we have given v_1 the degree of 0.

Thus not possible.

The proof also relies on the pigeonhole principle.

"If you have more pigeons than the holes, and you try to assign each pigeon to a hole, at least one hole will contain more than one pigeon".

—x—

~~1-4~~, ~~8~~, ~~9~~, ~~12~~, ~~15~~, ~~18~~, ~~21~~, ~~24~~

~~64 (b)~~
~~65~~
~~66~~
69
70

72
74

~~58~~
~~59~~
~~60~~
~~61~~
~~62~~
~~63~~

Date

EXERCISE 10.3.

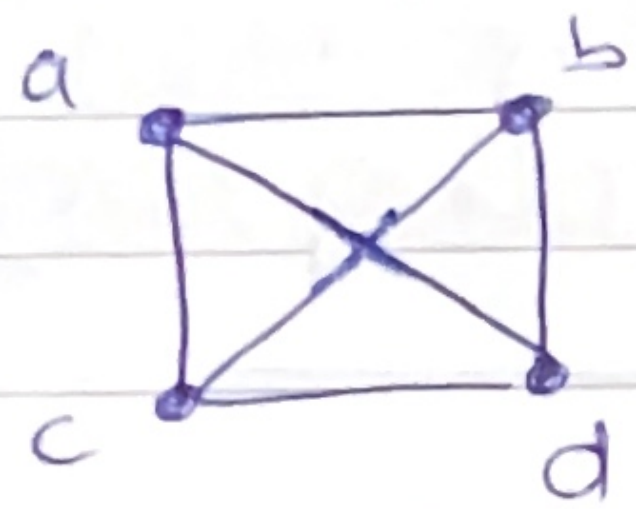
4.

a	b, d
b	a, c, e, d
c	c, b
d	a, e
e	e, c.

8.

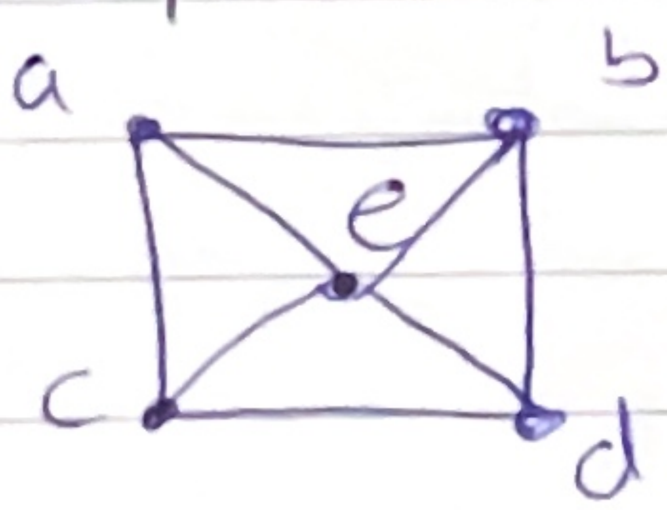
	a	b	c	d	e
a	0	1	0	1	0
b	1	0	1	1	1
c	0	1	1	0	0
d	1	0	0	0	1
e	0	0	1	0	1

9. (a) K_4.



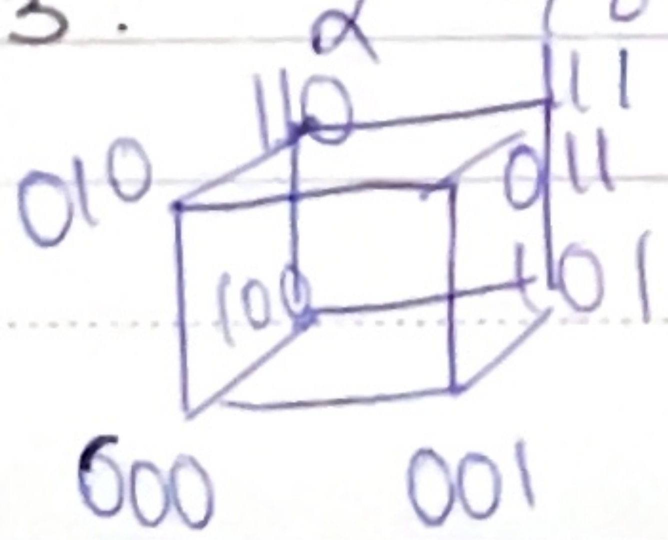
drawing adjacency matrix is simple.

(b) W_4.



"

(f) Q_3. $2^3 = 8$ vertices.

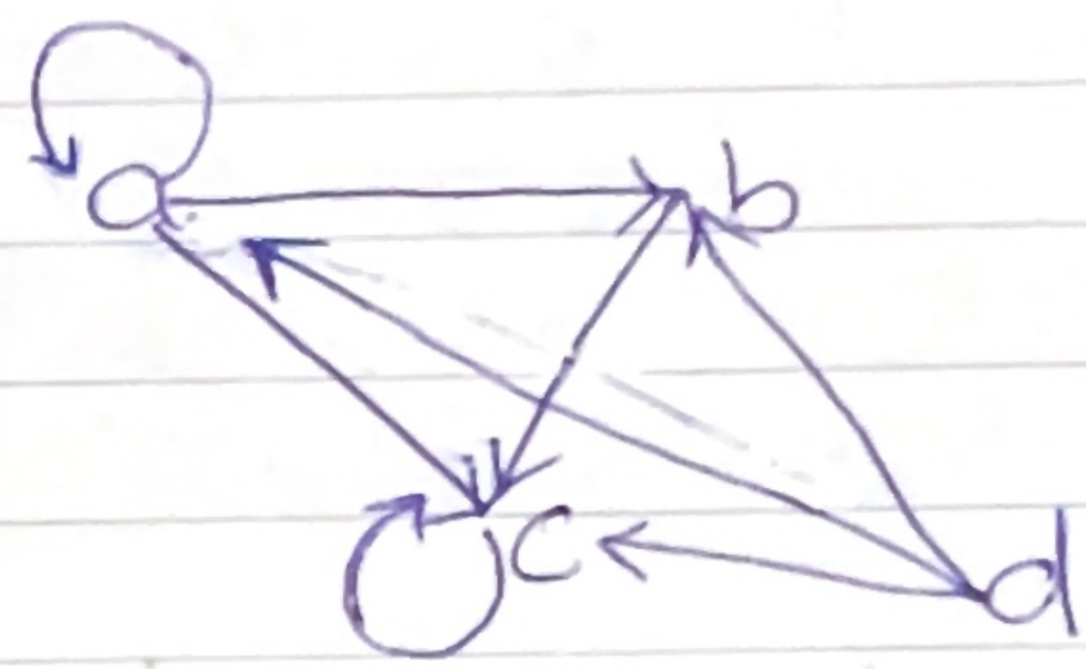


"

12. draw a graph from.

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & \multicolumn{4}{c}{\multirow{4}{*}{}} \end{array}$$

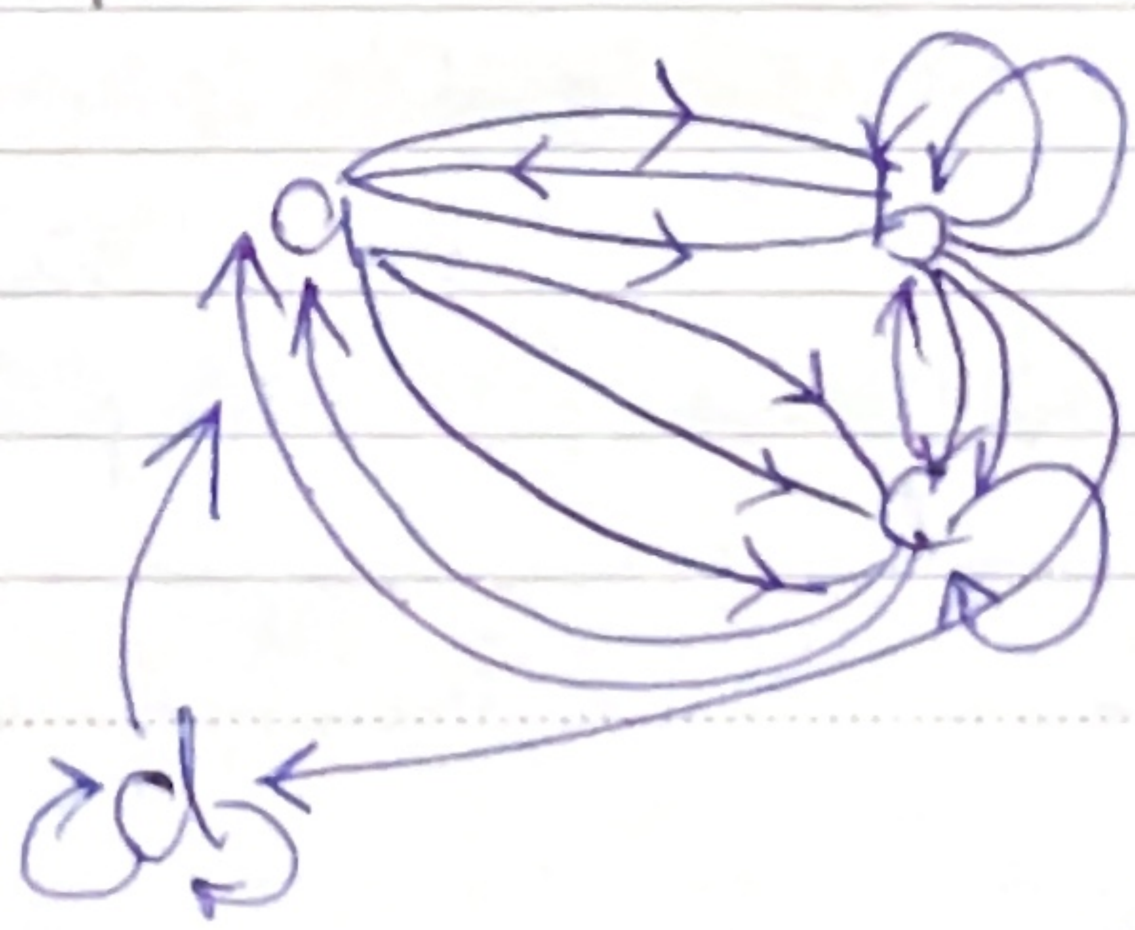
$$\begin{array}{cc} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



15.

	a	b	c	d
a	1	0	2	1
b	0	1	1	2
c	2	1	1	0
d	1	2	0	1

24.



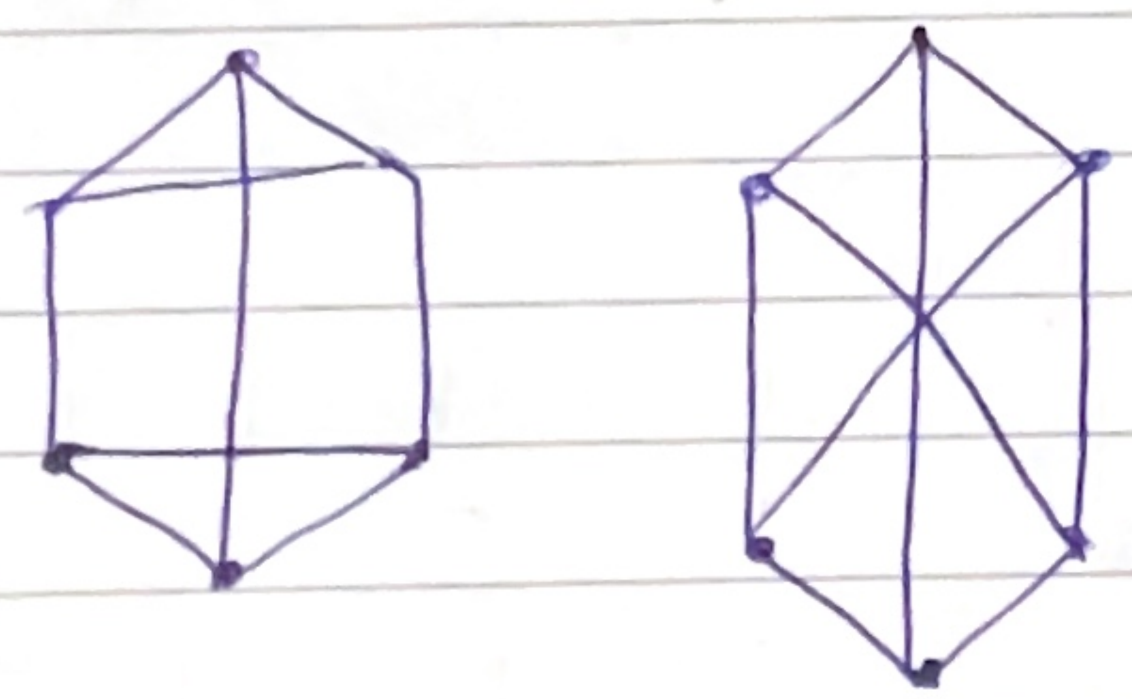
Two simple graphs G_1 & G_2 are isomorphic if there exists a one-to-one and onto function $f: V_1 \to V_2$ such that a and b are adjacent in G_1 if and only if f(a) and f(b) are adjacent in G_2.

64 (b) determine whether the graphs are isomorphics.

we note that the two groups have 4 vertices (noted from no. of rows) and 5 edges (noted from no. of columns)

(not included)

61 FIND the no. of no isomphic graphs that have 6 vertices in which each vertex has degree 3.



2 graphs possible.

Ex65 :-
Extend the definition of isomorphism of simple graphs to undirected graps containing loops and multiple edges.

let $G_1 = (V_1, E_1)$

$G_2 = (V_2, E_2)$

if there is a loop at vertex a, then a and a are adjacent in G_1 then, in G_2 f(a) and f(a) are adjacent. (remains same)

We can included multiple edges by making slight adjustment to the definition, instead of assuming that two vertices are adjacent we require that for every edge in G_1, there is a corresponding edge in G_2.

—— x ——

Date DISCRETE MATHEMATICS.

Chapter no. 10

Exercise 10.3.

(i) no. of vertices :=?

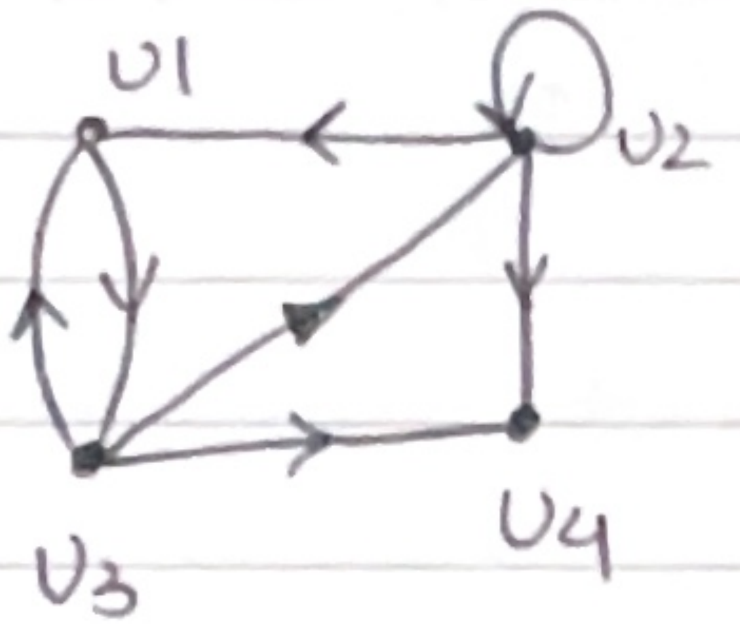
(ii) no of edge :?

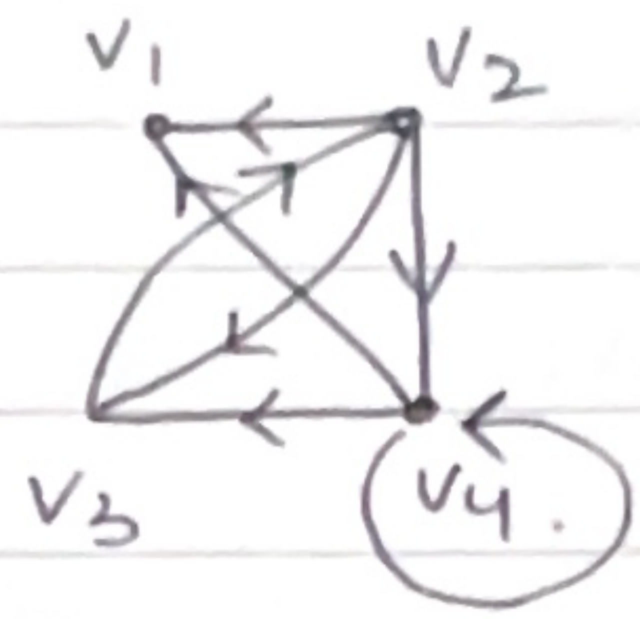
(iii) degree sequence while analyzing self degree and adjacent's degree.

(iv) Mapping

(v) edge preservance matrix.

69 - 2 => 67.





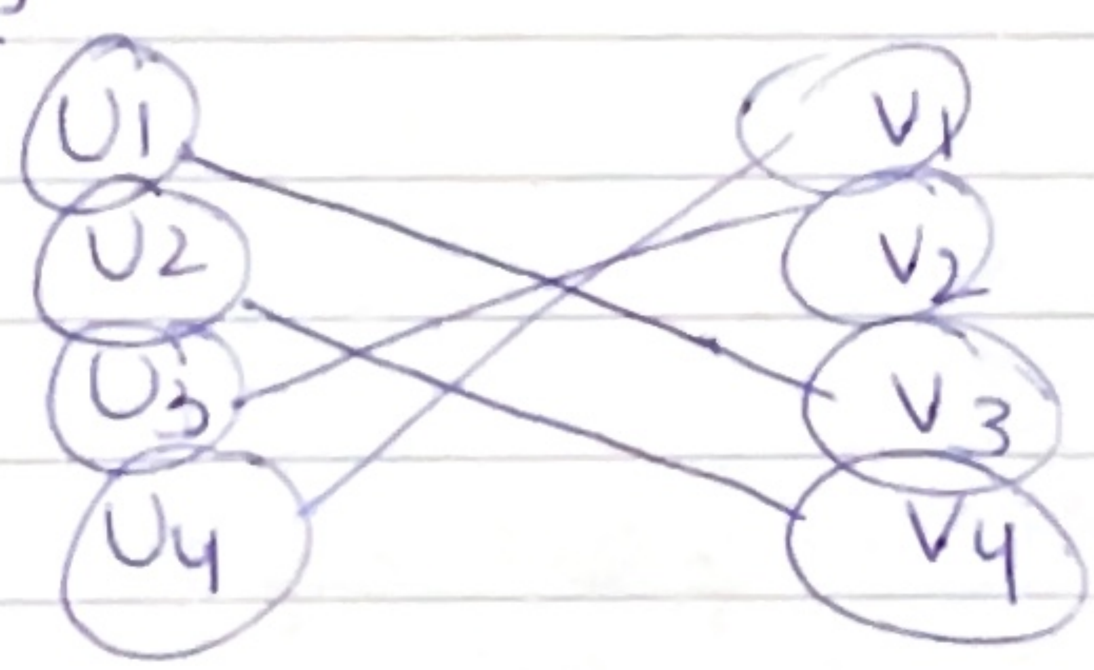
$V_1 = \{U_1, U_2, U_3, U_4\} \Rightarrow 4$

$V_2 = \{V_1, V_2, V_3, V_4\} = 4$

$E_1 = \{(U_1, U_3)(U_2, U_1)$
$(U_2, U_2)(U_2, U_4)(U_3, U_4)$
$(U_3, U_1)\ (U_3, U_2)\} \Rightarrow 7$

$E_2 = \{(V_3, V_2)(V_4, V_1)(V_4, V_3)$
$(V_4, V_4)(V_2, V_4)(V_2, V_1)\}$
$(V_2, V_3) \Rightarrow 7.$

Mapping



hence proved!
it is isomorphic.

$f(U_1) = V_3$
$f(U_2) = V_4$
$f(U_3) = V_2$
$f(U_4) = V_1$

hence

$(U_1, U_2) = (V_3, V_2)$
and so on from the graph.

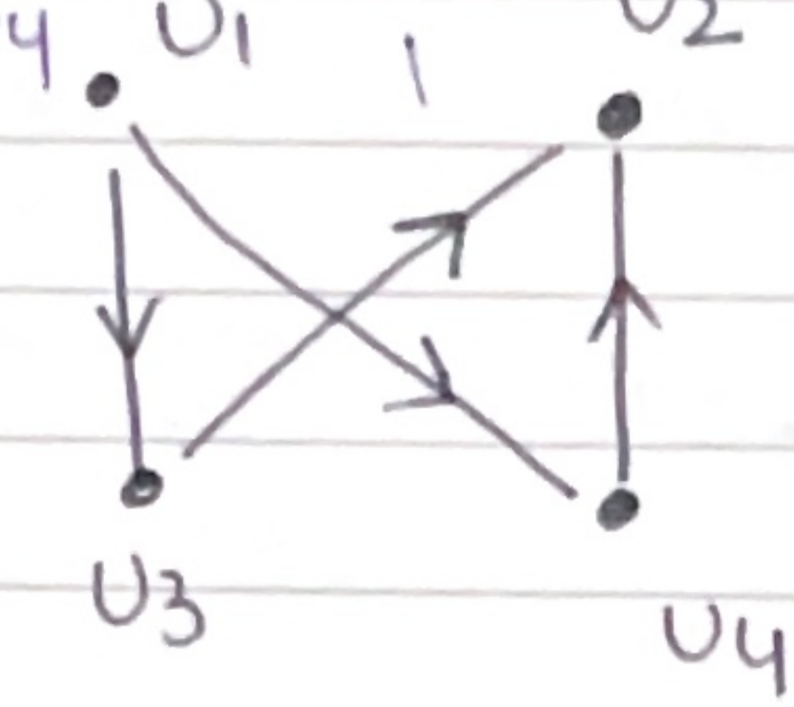
Directed Degree sequence.

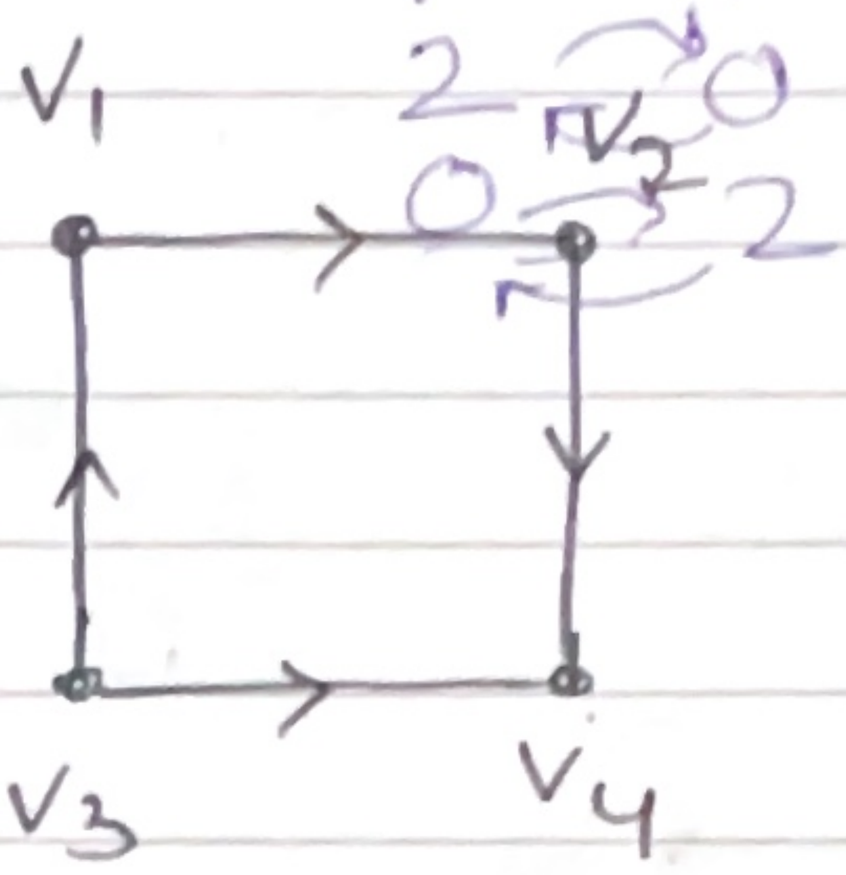
72, 74.

G1 Vertex	In	Out
U_1	0	2
U_2	2	0
U_3	1	1
U_4	1	1

G2 Vertex	In	Out
V_1	1	1
V_2	1	1
V_3	2	0
V_4	0	2

68.





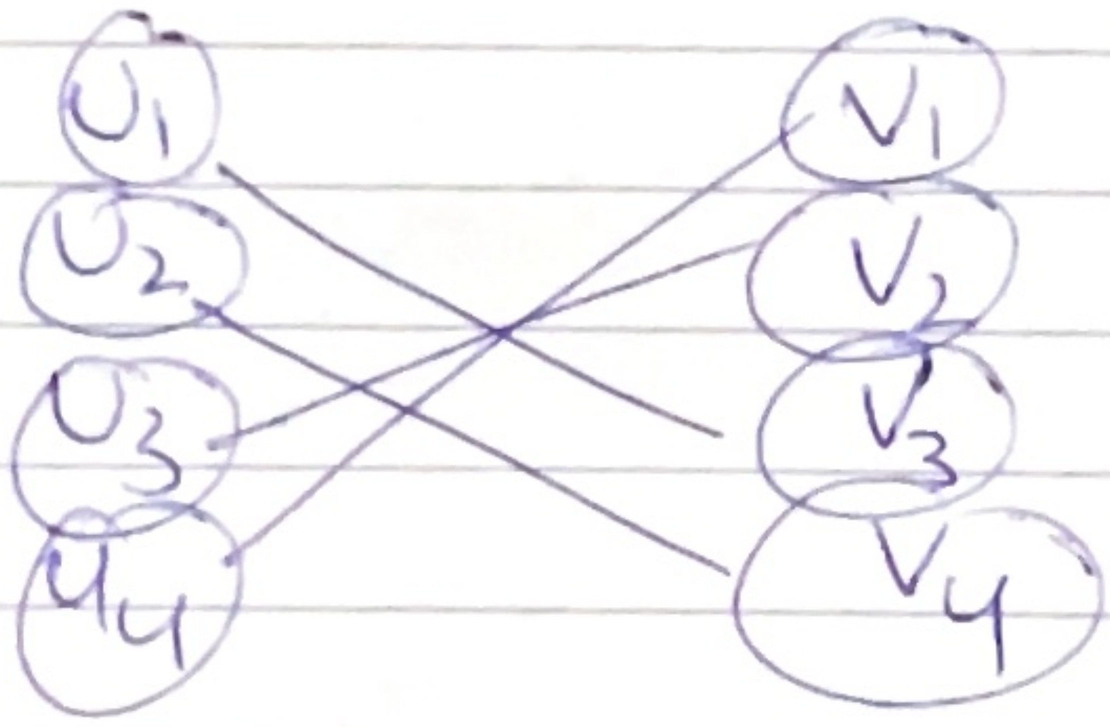
$V_1 = \{U_1, U_2, U_3, U_4\} = 4$

$E_1 \{(U_1, U_3)(U_1, U_4)$
$(U_3, U_2)(U_4, U_2)\}$

$V_2 \{V_1, V_2, V_3, V_4\} = 4$

$E_2 = (V_1, V_2)(V_2, V_4)$
$\{(V_3, V_4)(V_3, V_1)\}$
$\Rightarrow 4$

Mapping.



But we shall also consider in & out degree of the graphs.

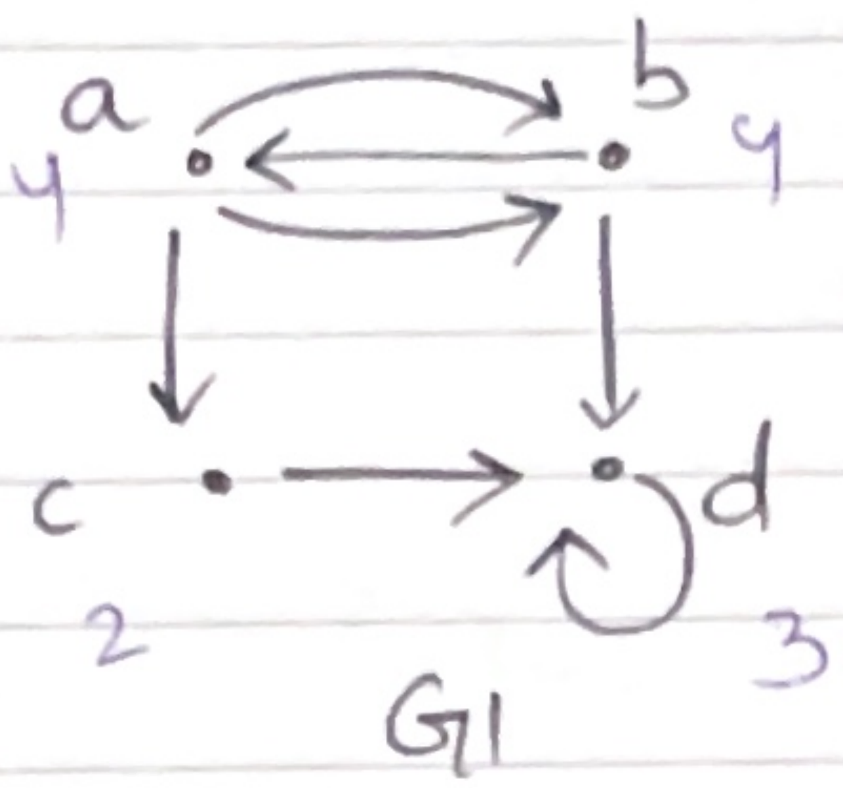
U_1 is connected to nodes that have both in & out degree of 1. while V_3 is connected to nodes having in and out degree different.

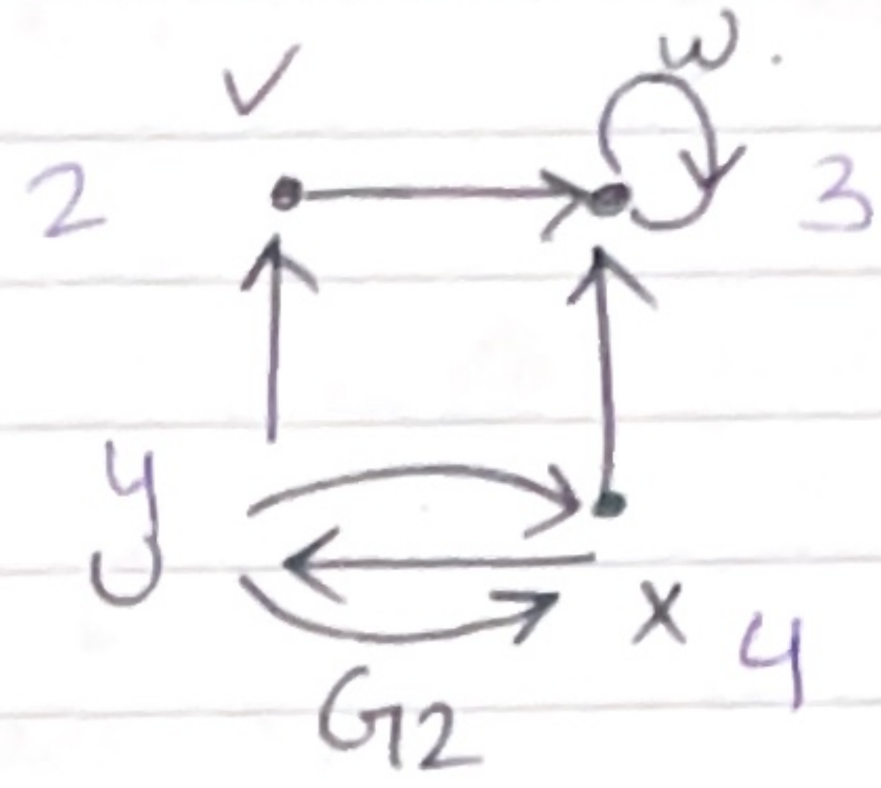
Thus not isomorphic because adjacent vertices always need to have the same in/out degrees in two graphs.

14.82. 14.14.

Date

Example.





(1) No. of Vertices = → $G_1 = 4$
→ $G_2 = 4$

(2) No. of edges = → $G_1 = 7$
→ $G_2 = 7$.

(3) Degree Sequence.

G1 {2, 3, 4, 4}

G2 {2, 3, 4, 4}.

(4) Directed sequence.

G1

Vertex	Indegree	Outdegree
a	1	3
b	2	2
c	1	1
d	3	1.

G2

Vertex	indegree	outdegree.
u	1	3
v	1	1
w	3	1
x	2	2.

$f(a) = u$
$f(b) = x$
$f(c) - v$
$f(d) = w$

hence they are isomorphic graphs.

but we will still see if the nodes adjacent, are also having the same in-degree and outdegree.

R.W
22323 32221
222

~x~

continuing analysis of question no. 68.

u_1 is connected to u_4 & u_3

	u_4		u_3	
	in d	out d	ind	out d
	1	1	1	1.

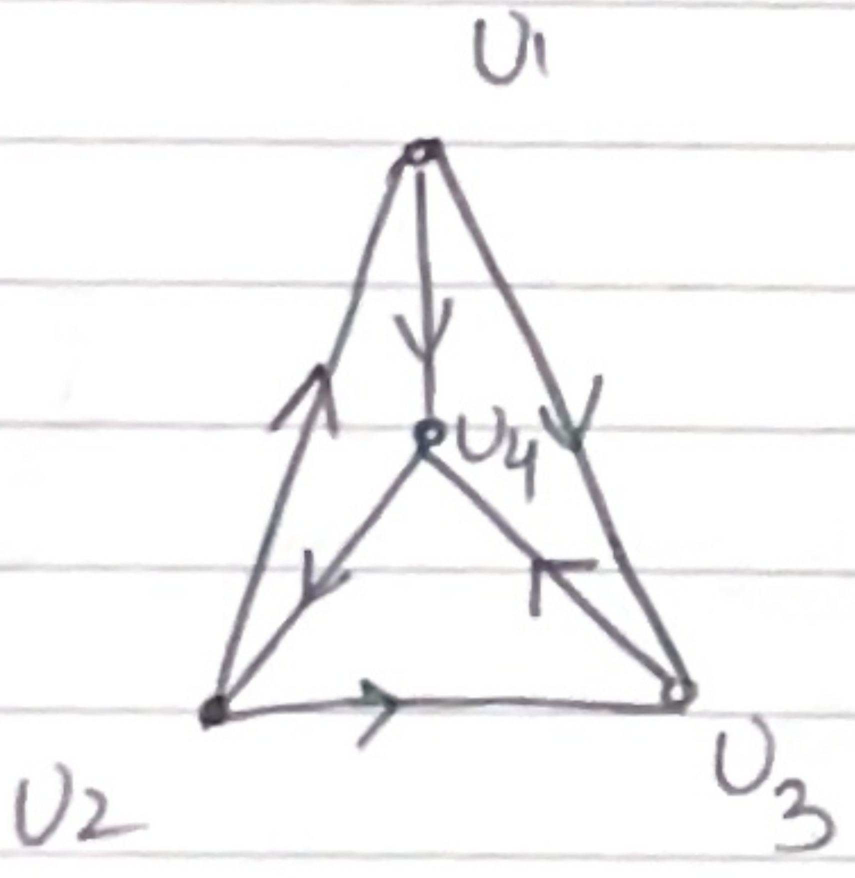
v_3 is connected to v_4 & v_1

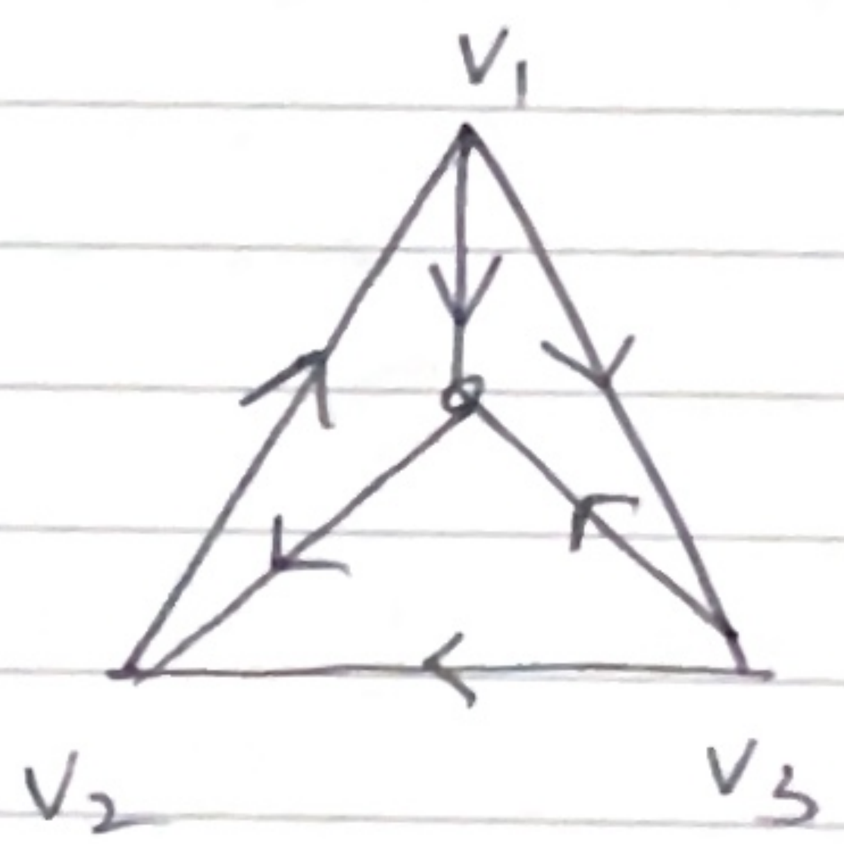
	v_4		v_1	
	2	0	ind	out d
	ind	out d	0	2.

thus no isomorphic.

~x~

69.





Same isomorphs.

Bipartite graphs are also isomorphic.

Date

70.

G1

No. of veritices = 6
No. of edges = 9

degree sequence.

$\{3, 3, 3, 3, 3, 3\}$

directed degree Sequence:

Vertex	In	Out
u_1	2	1
u_2	2	1
u_3	1	2
✓ u_4	2	1
✓ u_5	1	2
✓ u_6	1	2

G2.

No. of vertices = 6
No. of edges = 9.

degree sequence.

$\{3, 3, 3, 3, 3, 3\}$.

Directed degree sequence.

vertex	In	out
✓ V_1	1	2
V_2	2	1
✓ V_3	1	2
V_4	2	1
✓ V_5	2	1
V_6	1	2

$U_1 \rightarrow V_2$
$U_2 \rightarrow V_4$
$U_3 \rightarrow V_6$
$U_4 \rightarrow V_5$
$U_5 \rightarrow V_1$
$V_6 \rightarrow V_3$

always check the adjancy in & out degree of the adjacent vertices as well.