1

Date:

EXERCISE 1.1:-

2.

(a) Donot pass go. (No) ✓
(b) What time is it? (No) ✓
(c) There are no black flies in Maine. (Yes) ✓ F
(d) $4+x=5$ (No) ✓
(e) The moon is made of green cheese. (Yes) ✓ F
(f) $2^n \geq 100$. (No) ✓

4.

(a) Janice does not have more Facebook friends than Juan.
(b) Quincy is not smarter than Venkat.
(c) Zelda doesnot drives more miles to school than Paola.
(d) Briana does not sleep longer than Gloria.

7.

(c) $7 \cdot 11 \cdot 13 \neq 999$ ✓

14.

p : You have the flu, q: You miss the final examination.

r: You pass the course.

$p \to q$: If you have flu you will miss the final examination. ✓

Date:

$\neg q \leftrightarrow r$:
You won't miss the final examination if and only if you pass the course.

& so on .....

imp! * 16. —

(a) $r \wedge \neg q$
(b) $p \wedge q \wedge r$
(c) $r \rightarrow p \Rightarrow$ (necessary condition for r is p).
(d) $r \wedge \neg q \wedge r$
(e) $p \wedge q \rightarrow r$ (a sufficient condition for r is $p \wedge q$)
(f) $r \leftrightarrow p \vee q$

20.
imp! *

(a) if 1+1=3, then unicorn exist.

as both p and q are false then the conditional statement is not false rather True

(b) if 1+1=3, then dogs can fly

not False = True

(c) if 1+1=2, then dogs can fly

False

(d) if 2+2=4 then 1+2=3.
True.

Date:

22.

impl *

OR :- True when either one is true.
XOR:- True when only one is true.

enough/necessary:

OR (a) $\neg$XOR $\xrightarrow{}$ not. because experience is one would be = OR
XOR (b) $\neg$OR $\xrightarrow{}$ not. because lunch with soup or salad but never both.
OR (c) $\neg$XOR $\xrightarrow{}$ not. because atleast one is required to be true.
(d) XOR.

28. p is necessary and sufficient q => if and only if 'p' 'q'

impl *

(a) You can get an A in this course if and only if you learn how to solve discrete mathematics questions.

if 'pq,' and conversely => 'p' if and only if 'q'

You read the newspaper everyday if and only if you will be informed.

p if q and q if p

It rains if and only if it is a weekday.

p only if q and q only if p.

You can see the wizard if and only if the wizard is not in.

p exactly when q

My airplane flight is late if and only if I have to catch a connecting flight.

imp ✗

32.

Number of rows can be obtained by

$N = 2^n$ where n is the no. of propositional variables.

a negated propositional variable is not evaluated as a new variable if its stand alone is already prougth up.

a; $n = 2$
$2^2 = 4$ rows

b; $n = 3$; $2^3 = 8$ rows

c; $n = 6$; $2^6 = 64$ rows

d; $n = 5$; $2^5 = 32$ rows

41. $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$

p	q	$p \leftrightarrow q$ (A)	r	s	$r \leftrightarrow s$ (B)	$A \leftrightarrow B$
0	0	T	0	0	T	T
0	0	T	0	1	F	F
0	0	T	1	0	F	F
0	0	T	1	1	T	T
0	1	F	0	0	T	F
0	1	F	0	1	F	T
0	1	F	1	0	F	T
0	1	F	1	1	T	F
1	0	F	0	0	T	F
1	0	F	0	1	F	F
1	0	F	1	0	F	[illegible]
1	0	T	1	1	[illegible]	[illegible]
1	[illegible]	T	0	0	[illegible]	[illegible]
1	1	T	0	1	T	[illegible]

43.

$(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true.

Cp/

The or operation would return true whenever one value is true. as we are operating and only the complements / negation of two/three same terms then we would first or them give that

→ they are true

→ one is false

⇒ it would always result in true.

44.

impl *

very important

$$\bigwedge_{i=1}^{n-1} \bigwedge_{j=i+1}^{n} (\neg p_i \vee \neg p_j)$$

If we write the expended version form, we have

$$(\neg p_1 \vee \neg p_2) \wedge (\neg p_1 \vee \neg p_3) \wedge (\neg p_1 \vee \neg p_4) \ldots$$

$$\wedge (\neg p_1 \vee \neg p_n) \wedge (\neg p_2 \vee \neg p_3) \wedge (\neg p_2 \vee \neg p_4) \wedge$$

$$\ldots (\neg p_2 \vee \neg p_n) \wedge \neg (p_{(n-1)} \vee p_n) \equiv I.$$

we know that

$$(\neg p_i \vee \neg p_j) = \neg (p_i \wedge p_j)$$

we also know that I is true when/for every i and j meaning that $p_i \wedge p_j$ should be false for every i and j……

Date:

46. (a) $x = 2$.
(b) $x = 1$.
(c) $x = 2$.
(d) $x = 1$.
(e) $x = 2$.

47. BITWISE (NOTDLD RULES!)

(a) 101 1110, 0100001

	1	0	1	1	1	1	0	AND: 0000000
OR	0	1	0	0	0	0	1	XOR: 1111111
	1	1	1	1	1	1	1	

(b) 1111 0000, 10101010

	1111 0000	AND 10100000
OR	1010 1010	XOR 01011010
	1111 1010	

(c) 000 111 0001, 1001001000

	000 111 0001	AND ~~00~~0000000000
OR	100 100 1000	XOR 011011~~0~~1001
	100 111 1001	

(d) OR: 1111111111 XOR: 1111111111
AND: 0000000000

EXERCISE 1.2.

4.

W: You can use the wireless network in the airport.

d: You pay the daily fee.

s: You are a subscriber to the service.

$w \to (d \vee s)$

6. u: You can upgrade your operating system

b_{32}: You have a 32 bit processer.

b_{64}: You have a 64 bit processor.

g_1 : Your processor runs at 1 GHz or faster.

g_2 : Your processor runs at 2 GHz or faster.

r_1 : Your processor has atleast 1GB ram.

r_2 : Your processor has at least 2GB ram.

h_{16} : You have at least 16GB free HDD space

h_{32}: " " " 32 " " ".

$u \to (b32 \wedge g1 \wedge r_1 \wedge h16) \vee (b64 \wedge g2 \wedge r_2 \wedge h32)$

14.
imp! *

To look for hiking in West Virginia:-

HIKING AND (WEST AND VIRGINIA).

and if west virginia is excluded.

HIKING AND VIRGINIA.

Date:

16.

MENS AND (SHOES OR BOOTS) AND NOT WORK.

36.

(a) Smith is the killer?

Smith can not be the killer because that would mean that smith is lying. — (a)

As Smith stated that;

" Cooper was a friend with Jones and that william disliked him".

if smith is lying then it would mean that jones was not the friend of cooper
∴ it would make both jones and smith a liar which cannot be possible.

∴ — (a).

(b) Jones the killer?

Jones is the only person stating that he did not know cooper which statements of william and smith reflect him as a friend of cooper.

∴ Jones is a lier. → killer.

(c) William is telling the truth.

40. Carlos & Diana both cannot be lying. Either one of them is saying the truth.

And Alice is supporting either one of them which can be true or false.

∴ only john stands alone that is why he is the criminal in the case.

If exactly one is lying then all fingers of supporting evidence point at Carlos that he did it.

44.

(a) $\neg p \vee \neg q$

(b) $\neg((\neg p \wedge q) \vee p)$

45.

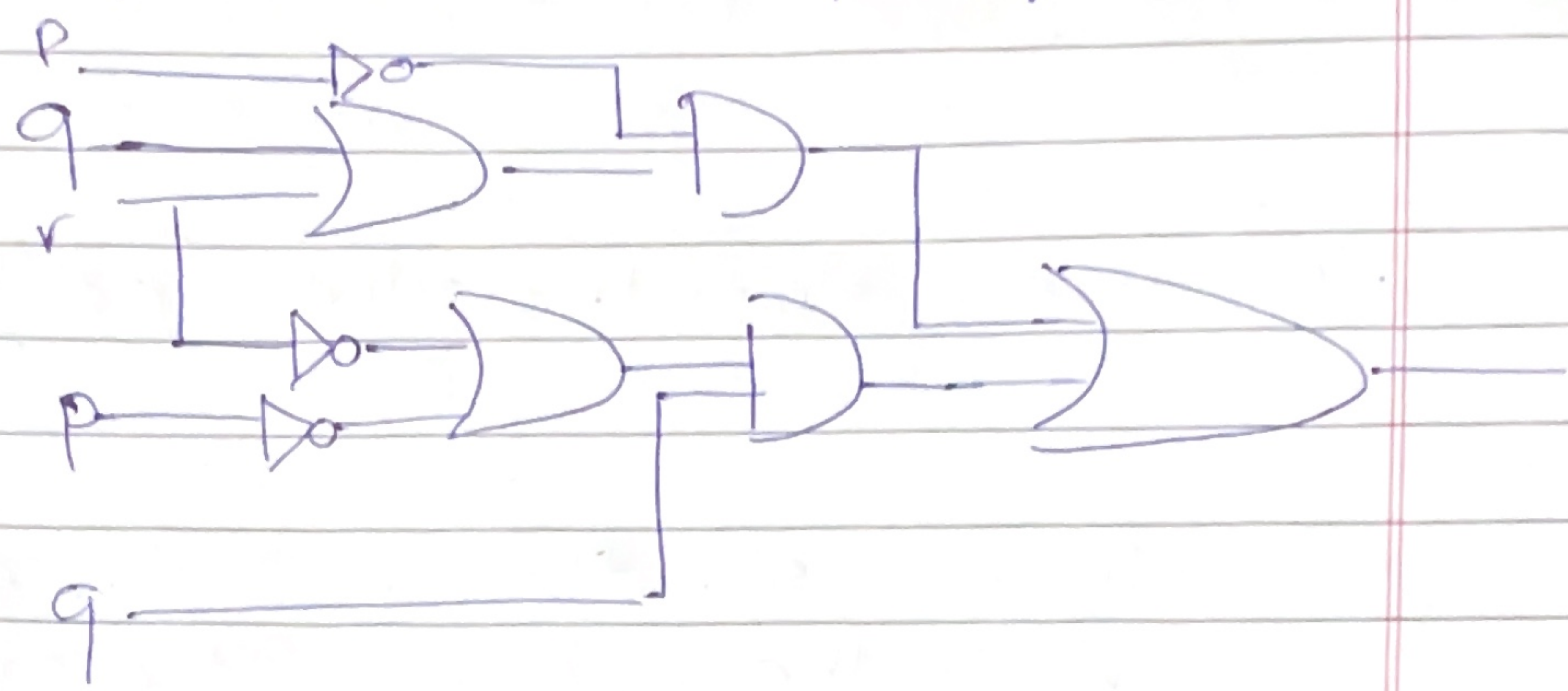
(a) $\neg(p \wedge (q \vee \neg r))$

(b) $(\neg p \wedge \neg q) \vee (p \wedge r)$

46. $((p \wedge \neg r) \vee (\neg q \wedge r))$

Date: ______

47. $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$



EXERCISE 1.3 :-

Demorgan's law:-

7. $\neg(p \wedge q) = \neg p \vee \neg q$

(a) $\neg(p \vee q) = \neg p \wedge \neg q$.

Jan is notrich and not happy.

(b)

Carlos will not Bicycle and not run tommorrow.

(c) Meri doesnot walk and doesnot take the bus to class.

(d) Ibrahim is not smart and not hard working.

9. conditional disjuction equivalence.

(a) $p \rightarrow \neg q$ $\qquad p \rightarrow q \equiv \neg p \vee q$.

$\neg p \vee \neg q \Rightarrow \neg(p \wedge q)$

Date:

(b) $(p \to q) \to r$

$\neg(p \to q) \vee r$

$\neg(\neg p \vee q) \vee r$

$\neg(\neg p) \wedge \neg q \vee r$

$(p \wedge \neg q) \vee r$

(c)

$(\neg q \to p) \to (p \to \neg q)$

$(\neg(\neg q) \vee p) \to (\neg p \vee \neg q)$

$(q \vee p) \to (\neg(p \wedge q))$

$\neg(q \vee p) \vee \neg(p \wedge q)$

$\neg((q \vee p) \wedge (p \wedge q))$ Dermorgan's law

B.

given: $(p \wedge q) \to p$.

a conditional statement $p \to q$ is false when p is true and q is false.
lets assume that $p \wedge q$ is true.

Date:

then

due to $p \wedge q$ both 'p' & 'q' are true.

$\therefore (p \wedge q) \rightarrow p$ here p is true

so hence proved that it is a tautology.

(b) $p \rightarrow (p \vee q)$

let p is true then

$p \vee q$ = true

hence proved that $p \rightarrow (p \vee q)$ is a tautology.

(c) $\neg p \rightarrow (p \rightarrow q)$.

let $\neg p$ is true then p is false and now either q is true or false $p \rightarrow q$ is alway going to be true, hence proved tautology.

(d) $(p \wedge q) \rightarrow (p \rightarrow q)$

$\neg(p \rightarrow q) \longrightarrow p$.

let $\neg(p \rightarrow q)$ is true it means.

$p \rightarrow q$ is false thus we say that p is true and q is false

hence proved that its a tautology.

(f) $\neg(p \to q) \to \neg q$

let the same assumptions as of the previous ones.

14.

(given)

(a) $= [\neg p \wedge (p \vee q)] \to q$.

let $\neg p \wedge (p \vee q)$ is true it means both $\neg p$ and $(p \vee q)$ are true.

it means $\neg p = T$ and $p = F$ and $p \vee q = T$ $\therefore q = T$ hence proved tautology.

(b). $[(p \to q) \wedge (q \to r)] \to (p \to r)$

Now let

$(p \to q) \wedge (q \to r)$ are true.

Then $p \to q$ is true and $q \to r$ is also true.

Date:

(c) $[p \wedge (p \to q)] \to q$

let $p \wedge (p \to q)$ is true ; it means both (p) & $(p \to q)$ are true ; then for $p \to q$ to be true 'q' should be true; hence proved!

(d) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

Similar reasoning.

impl
*

15. Prove the following as tautologies using logical equivalences only.

a. $(p \wedge q) \to p$.

$(p \wedge q) \to p \Rightarrow$ apply logical equivalence.

$(p \Rightarrow p) \vee (q \to p)$ opening by logical equivalence.

$(\neg p \vee p) \vee (q \to p)$

By commutative law $(p \vee \neg p) \vee (q \to p)$

By negation law $(T) \vee (q \to p)$

By commutative law.

$(q \to p) \vee (T)$

$\Rightarrow T.$

hence by negation law again proved!

Imp to write detail of every law that is used.

Date:

(b) $p \to (p \vee q)$

By logical equivalence.

$(p \to p) \vee (p \to q)$

By logical equivalance.

$(\neg p \vee p) \vee (\neg p \vee q)$

By commutative law.

$(p \vee \neg p) \vee (\neg p \vee q)$

By negation law.

$(T) \vee (\neg p \vee q) \Rightarrow T.$

(c) $\neg p \to (p \to q)$

By logical equivalence.

$p \vee (p \to q)$

$p \vee (\neg p \vee q)$

By associative law

$(p \vee \neg p) \vee q$

By negation law.

$(T \vee q) \Rightarrow T$ hence proved.

(d)

$(p \wedge q) \to (p \to q)$

$[p \to (p \to q)] \vee [q \to (p \to q)]$

$\neg p \vee (p \to q) \vee \neg q \vee p \to q$

$(\neg p \vee \neg p \vee q) \vee (q \vee \neg q \vee \neg p)$

Date:

$\neg p \vee (\neg p \vee q) \vee (T \vee \neg p)$

$\Rightarrow T$ hence proved!

16. Similar to question 15.

(a) $[\neg p \wedge (p \vee q)] \rightarrow q$

$\neg(\neg p \wedge (p \vee q)) \vee q.$

$[\neg(\neg p) \vee \neg(p \vee q)] \vee q$

$p \vee (\neg(\neg p) \wedge \neg q)] \vee q$

$[p \vee (\bar{p} \wedge \neg q)] \vee q.$

$(T \wedge \neg q) \vee q$

$T \vee q \Rightarrow T$ hence proved!

(b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

$[(\neg p \vee q) \wedge (\neg q \vee r)] \rightarrow (p \rightarrow r)$

$\neg[(\neg p \vee q) \wedge (\neg q \vee r)] \wedge (\neg p \vee r)$

$[\neg(\neg p \vee q) \vee \neg(\neg q \vee r)] \wedge (\neg p \vee r)$

$$(\neg(\neg p) \vee \neg q \quad \vee \quad \neg(\neg q) \vee \neg r) \wedge (\neg p \vee r)$$

$$(p \vee \neg q \vee q \vee \neg r) \wedge \neg p \vee r$$

$$((p \vee \neg r) \vee T)) \wedge \neg p \vee r$$

T heneeproved!

imp! (c) $[p \wedge (p \to q)] \to q$.

$$[p \wedge (\neg p \vee q)] \to q$$

$$\neg [p \wedge (\neg p \vee q)] \vee q.$$

$$\neg [(p \wedge \neg p) \vee (p \wedge q)] \vee q$$

$$[F \vee (p \wedge q)] \to q$$ should not had opened logical equivalence

$$(p \wedge q) \to q.$$

now open logical equivalence

$$(p \to q) \vee (q \to q)$$

$$(\neg p \vee q) \vee (\neg q \vee q)$$

T hence proved!

$\wedge (q \to r)$

Date:

d. $[(p \vee q) \wedge (p \to r)] \to r$

imp *

$[(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)] \to r$

using distributive law to make more compact

$[(p \vee q) \wedge ((\neg p \wedge \neg q) \vee r))] \to r.$

Now by logical equivalence.

$\neg [(p \vee q) \wedge ((\neg p \wedge \neg q) \vee r))] \vee r.$

By Demorgan's law

$[\neg (p \vee q) \vee \neg ((\neg p \wedge \neg q) \vee r)] \vee r.$

$(\neg p \wedge \neg q) \vee (\neg (\neg p \wedge \neg q) \wedge \neg r)] \vee r$

$(\neg p \wedge \neg q) \vee ([(p \vee q) \wedge \neg r] \vee r$

$[(\neg p \wedge \neg q) \vee ((p \vee q) \wedge \neg r))] \vee r$

distributive.

Now by negation law again.

$[(\neg p \wedge \neg q) \vee (p \vee q) \wedge ((\neg p \wedge \neg q) \vee \neg r)] \vee r.$

$[(\neg p \vee (p \vee q)) \wedge (\neg q \vee (p \vee q)) \wedge (\neg p \wedge \neg q) \vee \neg r]$

$(\neg p \vee p) \vee q) \quad (p \vee (\neg q \vee q)) \wedge \quad \vee r$

$T \wedge T \quad \wedge (\neg p \wedge \neg q) \vee \neg r] \vee r$

Date:

$[(\neg p \wedge \neg q) \vee \neg r] \vee r$

$(\neg p \wedge \neg q) \vee (\neg r \vee r)$

$(\neg p \wedge \neg q) \vee T \Rightarrow T$ hence proved!

18. TT
19. TT

20. $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \vee \neg q)$ are logically equivalent.

By logical equivalence

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$(p \rightarrow q) \wedge (q \rightarrow p)$

$(\neg p \vee q) \wedge (\neg q \vee p)$

Now by distributive property.

$((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)$

$(\neg p \overset{\wedge \neg q}{\vee} (q \wedge \neg q)) \vee ((\neg p \wedge p) \vee (q \wedge p))$

identity law

$((\neg p \wedge \neg q) \vee F) \vee (F \vee (q \wedge p))$

commutative law.

$(p \wedge q) \vee (\neg p \wedge \neg q)$

proved!

Date: ______

21. $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.

very imp *

$\neg(p \leftrightarrow q)$

$\neg((p \to q) \wedge (q \to p))$

$\neg(p \to q) \vee \neg(q \to p)$

$\neg(\neg p \vee q) \vee \neg(\neg q \vee p)$

$(p \wedge \neg q) \vee (q \wedge \neg p)$

$\vee q$

$((p \wedge \neg q) \vee q) \wedge ((p \wedge \neg q) \vee \neg p)$

$(p \vee q) \wedge (q \vee \neg q) \wedge (p \vee \neg p) \wedge$

$((p \vee q) \wedge T) \wedge (T \wedge (\neg q \vee \neg p))$

$(p \vee q) \wedge (\neg q \vee \neg p)$

$(p \vee q) \wedge (\neg p \vee \neg q)$

$\Rightarrow (\neg p \vee \neg q)\ (q \vee p)$

Now by logical equivalent

$(p \to \neg q) \wedge (\neg q \to p)$

$(p \to \neg q)$ hence proved.

$p \leftrightarrow \neg q$

$(p \to \neg q) \wedge (\neg q \to p)$

$(\neg p \vee \neg q) \wedge (q \vee p)$

$((\neg p \vee \neg q) \wedge q) \vee ((\neg p \vee \neg q) \wedge p)$

$((\neg p \wedge q) \vee (\neg q \wedge q)) \vee$

$((\neg p \wedge p) \vee (\neg q \wedge p))$

$(\neg q \vee \neg p)$ $((\neg p \wedge q) \vee (F)) \vee$

$(\neg q \wedge p) \vee (F)$

$(\neg p \wedge q) \vee (\neg q \wedge p)$

$(\neg p \vee \neg q) \wedge$ and so on.

So need to solve both ends.

22. $p \to q$ and $\neg q \to \neg p$.

$(\neg p \vee q)$ $\quad$ $\neg(\neg q) \vee \neg p$.

$\neg p \vee q$ $\quad$ $q \vee \neg p$

$\neg p \vee q \equiv \neg p \vee q$ proved!

23. $\neg p \leftrightarrow q$ & $p \leftrightarrow \neg q$.

imp! * $(\neg p \to q) \wedge (q \to \neg p)$ ✓

$(\neg(\neg p) \vee q) \wedge (\neg q \vee \neg p)$ ✓

$(p \vee q) \wedge (\neg q \vee \neg p)$. ✓

commutative law.

$(\neg p \vee \neg q) \wedge (q \vee p)$.

$\neg p \vee \neg q \wedge (\neg(\neg q) \vee p)$ ✓

$(p \to \neg q) \wedge (\neg q \to p)$.

$(p \leftrightarrow \neg q)$ hence proved!

24. $\neg(p \oplus q)$ and $p \leftrightarrow q$.

$\neg((p \vee q) \wedge (\neg p \vee \neg q))$

$\neg(p \vee q) \vee \neg(p \vee \neg q)$.

OR. apply distributive law!

Date:

$\neg[(p \wedge (\neg p \vee \neg q)) \vee (q \wedge (\neg p \vee \neg q))]$

$\neg[(p \wedge \neg p) \vee (p \wedge \neg q) \vee (q \wedge \neg p) \vee (q \wedge \neg q)]$

$\neg[(p \wedge \neg q) \vee (q \wedge \neg p)]$

$\neg(p \wedge \neg q) \wedge \neg(q \wedge \neg p)$

$(\neg p \vee \neg(\neg q)) \wedge (\neg q \vee \neg(\neg p))$

$(\neg p \vee q) \wedge (\neg q \vee p)$

$(p \rightarrow q) \wedge (q \rightarrow p)$

$(p \leftrightarrow q)$ hence proved!

25. $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$.

$\neg((p \rightarrow q) \wedge (q \rightarrow p))$

$[\neg(p \rightarrow q) \vee \neg(q \rightarrow p)]$

$(\neg(\neg p \vee q) \vee \neg(q \vee p))$

$(p \wedge \neg q) \vee (q \wedge \neg p)$

$(p \wedge \neg q \vee q) \wedge ((p \wedge \neg q) \vee \neg p)$

$(p \wedge \neg q) \vee q \wedge ((p \wedge \neg q) \vee \neg p)$

$((p \vee q) \wedge (\neg q \vee q)) \wedge ((p \vee \neg p) \wedge (\neg q \vee \neg p))$

$((p \vee q) \wedge T) \wedge (T \wedge (\neg q \vee \neg p))$

$(p \vee q) \wedge (\neg q \vee \neg p)$

$(\neg p \vee \neg q) \wedge (p \vee q)$.

$(\neg q \vee \neg p)$ $(\neg p \rightarrow q)$

$(q \rightarrow \neg p)(\neg p \rightarrow q)$.

$(\neg p \leftrightarrow q)$ hence proved!

26.

$(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$.

$(\neg p \vee q) \wedge (\neg p \vee r)$

or

$(p \rightarrow q) \wedge (p \rightarrow r)$.

Rather use: $p \rightarrow (q \wedge r)$.

$\neg p \vee (q \wedge r)$

$(\neg p \vee q) \wedge (\neg p \vee r)$

$(p \rightarrow q) \wedge (p \rightarrow r)$ hence proved!

Date:

27. $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$.

$\neg(p \vee q) \vee r$

$(\neg p \wedge \neg q) \vee r$

$(\neg p \vee r) \wedge (\neg q \vee r)$

$(p \to r)\ (q \to r)$ hence proved!

28. $(p \to q) \vee (p \to r)$ & $p \to (q \vee r)$

$\neg p \vee (q \vee r)$

$(\neg p \vee q) \vee (\neg p \vee r)$

$(p \to q) \vee (p \to r)$ hence proved!

29. $(p \to r) \vee (q \to r)$ & $p \wedge q \to r$

$(p \wedge q) \to r$

$\neg(p \wedge q) \vee r$

$(\neg p \vee \neg q) \vee r$

$(\neg p \vee r) \vee (\neg q \vee r)$

$(p \to r) \vee (q \to r)$ proved!