Date 01/12/2024.

DIVISION:-

let $a, b \in \mathbb{Z}$; $a \neq 0$.

'a' divides 'b' if there is an integer 'c' such that $b = ac$.

When 'a' divides 'b' ; $\Rightarrow a|b$
∴ a is a factor/divisor of b
∴ b is a multiple of a.

Properties of Division:-

let 'a', 'b' and 'c' be integers where $a \neq 0$ then:
(i) if $a|b$ and $a|c$; then $a|(b+c)$
(ii) if $a|b$, then $a|bc$; for all integers c;
(iii) if $a|b$ and $b|c$, then $a|c$.

Division Algorithm:-

$a = dq + r$ → $a, d \in \mathbb{Z}^+$ with $0 \leq r < d$

a → dividend
d → divisor
q → quotient

also (i) $q = a \text{ div } d = \lfloor a/d \rfloor$ (div → divided by)

(ii) $r = a \bmod d = a - d\lfloor a/d \rfloor$

conditions for last equality.
$d \in \mathbb{Z}^+$
$a \in \mathbb{Z}$
$0 \leq r < d$

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EXAMPLE NO. 1:-

Determine whether $3|7$ and whether $3|12$.

$3 \nmid 7$ because $7/3$ is not an integer.
$3|12$ because $12/3$ is an integer.

EXAMPLE NO. 2:-

quotient and remainded when 101 is divided by 11?

101 ⇒ dividend = a
11 ⇒ divisor = d

By division algorithm.

$101 = 11q + r$

By the definition of quotient

$q = a \text{ div } d$

$q = 101 \text{ div } 11$
$q = 9$.

$r = a \bmod d$.
$r = 101 \bmod 11$ → outputs remainder
$r = 2$

Check

$101 = 11(9) + 2$.
$101 = 101$ proved!

EXAMPLE NO. 3:-

What are the quotient and remainder for −11 divided by 3?

$-11 = 3q + r$.

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EXAMPLE NO. 2 :-

Given 'n' and 'd' be two +ve integers. How many +ve integers not exceeding n are divisible by d.

All the positive integers divisible by d will be in the form of 'dk' where $K \in$ Positive integer.

now it is given that not exceeding n. & +ve

$$\therefore \quad 0 < dk \leq n$$

$$\Rightarrow \quad 0 < d \leq n/K \quad \times$$

or

$$0 < K \leq \lfloor n/d \rfloor \quad \checkmark$$

hence $\lfloor n/d \rfloor$ are the positive integers not exceeding n, divisible by d.

Chapter 4

4.2 Integer Representation and Algorithm.

Theorem 1 - Base b expansion of n.

let 'b' be an integer greater than 1. Then if 'n' is a positive integer, it can be represented as

$$n = a_K b^K + a_{K-1} b^{K-1} + \ldots a_1 b^1 + a_0 b^0.$$

where $K \in$ non-negative integer

$a_0, a_1 \ldots a_K$ are also non-negative integers less than b and $a_K \neq 0$.

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BINARY EXPANSION

Example no. 1 :- (Decimal expansion of an integer that has the given as its Binary expansion?)

$(10101\ 1111)_2 = 1.2^8 + 0.2^7 + 1.2^6 + 0.2^5 +$

$1.2^4 + 1.2^3 + 1.2^2 + 1.2^1 + 1.1$

$\Rightarrow 351.$

OCTAL AND HEXADECIMAL EXPANSIONS :-

Ex. 2:-

What is decimal expansion of the no. in with octal expansion $(7016)_8$?

$(7016)_8 = 7.8^3 + 0.8^2 + 1.8^1 + 6.8^0$

$= 3598.$

HEXADECIMAL NO.S / EXPANSIONS.

Require 16 different digit.

Serial No.	Binary	Hexa	Octal	Serial No.	Binary	Hexa	Octal
0	0	0	0	9	1001	9	11
1	1	1	1	10	1010	A	12
2	10	2	2	11	1011	B	13
3	11	3	3	12	1100	C	14
4	100	4	4	13	1101	D	15
5	101	5	5	14	1110	E	16
6	110	6	6	15	1111	F	17
7	111	7	7				
8	1000	8	10				

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EX: 3 :- Expand the following by 'b' expansion.

$(2AE0B)_{16} = 2.16^4 + A.16^3 + E.16^2 + 0.16^1 + B.16^0$

$\Rightarrow 2.16^4 + 10.16^3 + 14.16^2 + 0.16^1 + 11.16^0$

$\Rightarrow 175627$ (output in decimal)

A, B, C, D, E, F are replaced with corresponding decimal integers

BINARY → HEXADECIMAL.

$(1110\ 0101)_2$ as hexadecimal is 2^4

∴ make groups of 4 digits.

$(\underline{1110}\ \underline{0101})_2$

↓ ↓

$(E)_{16}$ $(5)_{16}$ $\Rightarrow (E5)_{16}$ Ans!

Imp ✱

BASE CONVERSIONS:-

Algorithm for constructing the base b expansion of an integer 'n'.

First divide n by b to obtain a quotient (q_0) and remainder (a_0).

i.e

$n = bq_0 + a_0$ $\quad 0 \le a_0 < b$ $\quad a_0$ = remainder.

⇒ rightmost digit in b expansion of n.

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Next divide q_0 by b to obtain.

$$q_0 = bq_1 + a_1 \qquad 0 \leq a_1 < b.$$

we see that a_1 is the 2nd digit from the right in the b's expansion of n.

* Continue the process by successively dividing the quotients by b.

Example 4.

Find the Octal Expansion of $(12345)_{10}$.

$$12435 = 8 . 1543 + 1.$$

$$1543 = 8 . 192 + 7$$

$$192 = 8 . 24 + 0.$$

$$24 = 8 . 3 + 0$$

$$3 = 8 . 0 + 3$$

MSB

$(30071)_8$ Ans!

Example 5:-

Find hexadecimal expansion for $(177130)_{10}$.

$$177130 = 16 . 11070 + 10.$$
$$11070 = 16 . 691 + 14$$
$$691 = 16 . 43 + 3$$

$$43 = 16 . 2 + 11$$
$$2 = 16 . 0 + 2$$

$(2B3EA)_{16}$ Ans!

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EXAMPLE 6:-

FIND BINARY EXPANSION.

$(241)_{10}$.

$241 = 2 \cdot 120 + 1$

$120 = 2 \cdot 60 + 0$

$60 = 2 \cdot 30 + 0$

$30 = 2 \cdot 15 + 0$

$15 = 2 \cdot 7 + 1$

$7 = 2 \cdot 3 + 1$

$3 = 2 \cdot 1 + 1$

$1 = 2 \cdot 0 + 1$ MSB.

$(11110001)_2$

CONVERT YOUR REG NO into HEXADECIMAL.

$2023428 = 16 \cdot 126464 + 4$

$126464 = 16 \cdot 7904 + 0$

$7904 = 16 \cdot 494 + 0$

$494 = 16 \cdot 30 + 14$

$30 = 16 \cdot 1 + 14$

$1 = 16 \cdot 0 + 1$

1, 14, 14, 0, 0, 4.

$(1EE004)_{16}$.

BINARY, OCTAL, HEXA, DECIMAL INTERCONVERSIONS.

EXAMPLE 7:-

$(1111101011\ 1100)_2$

Octal ← To

$(1111101011\ 1100)_2$

Start from LSB.

3, 7, 2, 7, 4

$\Rightarrow (37274)_8$

Similarly group to 4 for hexa.

$(765)_8 \rightarrow$ to $\rightarrow$ Binary

$7 = 111$

$6 = 110$

$5 = 101$

} give each digit its 3-Bit Binary equivalent

$\therefore (111110101)_2$

$$\begin{array}{r} 7 \\ 8 \\ \hline 15 \end{array} \Rightarrow 1 \times 10 + 5.$$

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$(A8D)_{16}$ into Binary

$A = 1010$
$8 = 1000$
$D = 1101$

$(101010001101)_2$.

ADDITION ALGORITHM.

Adding pairs of Binary digits together with carries when they occur; to compute the sum of two integers.

$a = (a_{n-1}, a_{n-2} \dots a_1, a_0)_2$

$b = (b_{n-1}, b_{n-2} \dots b_1, b_0)_2$

Now using (a) & (b) are 2 Binary No.s using logic described

$a_0 + b_0 = C_0 . 2 + S_0.$
$a_1 + b_1 + C_0 = C_1 . 2 + S_1$
$a_2 + b_2 + C_1 = C_2 . 2 + S_2.$
$\vdots$
$a_{n-1} + b_{n-1} + C_{n-2} = C_{n-1} . 2 + S_{n-1}$

→ this is carryover flow and will be considered MSB. ⇒ S_n

The summed result in Binary will be

$(S_n \; S_{n-1} \; S_{n-2} \dots S_1 \; S_0)$

—x—

$a = (1110)_2$

$b = (1011)_2$.

start from LSB.

$a_0 + b_0 \Rightarrow 0 + 1 = 0.2 + 1.$

$a_1 + b_1 \Rightarrow 1 + 1 + 0 = 1.2 + 0$

$a_2 + b_2 \Rightarrow 1 + 0 + 1 = 1.2 + 0$

$a_3 + b_3 \Rightarrow 1 + 1 + 1 \Rightarrow 1.2 + 1.$

$(11001)_2$.