"Logic is the basis of all mathematical and of all automated reasoning."

Proposition:- A declarative sentence (that declares a fact) that is either true or false but not both.

e.g : 1+3 = 2. — False

Wastington DC is the capital of America ? — True.

The propositions are denoted by propositional variables / sentinal variables (which are simple letters).

Atomic proposition cannot be expressed in simpler terms.

Compound proposition is formed by one or more existing proposition using logical operators.

Negation:-

Let 'p' be a proposition such that the negation of p is denoted as '$\neg p$' and translates as.

'It is not the case that p.'

Example

Proposition:- Haris mobile has atleast 64 GB of storaage. → p.

$\neg p$: It is not the case that Haris mobile phone has 64 GB of storage.

simpler : Haris mobile does not have atleast 64 GB of storage.

simpler : Haris mobile phone has less 64 GB of storage.

Conjuction ($\wedge$)

It is true when both are true and otherwise not.

disjuction. ($\vee$)

It is false when both 'p' & 'q' are false.

Exclusive OR. (XOR):-

let 'p' and 'q' be propositions. ($p \oplus q$

Exclusive or is denoted by XOR.

The proposition is True when exactly one of p & q is true otherwise it is false.

p	q	$p \wedge q$	$p \oplus q$)
T	T	T	F
T	F	T	T
F	T	T	T
F	F	F	F

Example:-

p = 'A student can have salad with dinner'.

q = 'A student can have soup with dinner.'

$p \oplus q$ = A student can have soup or salad, but not both with dinner.

simpler

$p \oplus q$ = A student can have soup or a salad with dinner.

Conditional:-

let 'p' and 'q' be two propositions such that.

'p' is an hypothesis and 'q' is a conclusion. Then,

The conditional '$p \rightarrow q$' is false when p is true but q is false and otherwise it is True.

p	q	$p \rightarrow q$
T	T	**T**
T	F	F → only this condition can be false
F	T	**T**
* F	F	**T**

$\boxed{\text{Terminology of Conditionals}}$

- if 'p' then 'q'
- if p, q
- p is sufficient for q
- q if p
- q when p
- a necessary conditional for p is q
- q unless ¬p
- p implies q
- ✓ p only if q
- q whenever p
- q is necessary for p
- q follow from p
- q provided that p.

a sufficient condition for 'q' is 'p'.

is the Value of the Variable x after the statement?

if $2+2=4$ then $x := x+1$.

↳ assignment of $x+1$ to x.

If $x=0$ before the statement is encountered.
Because $2+2=4$ hence x has the value $0+1=1$ after the statement (∵ true) is encountered.

Conditional:- $p \to q$
Converse:- $q \to p$ → whenever, unless, sufficient, provided
Contrapositive:- $\neg q \to \neg p$ equivalent
Inverse :- $\neg p \to \neg q$. equivalent.

When two compound propositions have the same truth values, regardless of the truth values of its propositional variables, they are called as equivalent.

Example:-
conditional ← "The home team wins whenever its raining".

Conditional: If it is raining, then the home team wins.

Contrapositive If the home team does not win, then it is not raining.

Converse If the home team wins, then it is raining.

Inverse "If it is not raining, the the home team does not win."

Bi- Conditional. (p if and only if q):

The bi-conditional statement $p \leftrightarrow q$ is true when p and q have same truth values and false otherwise.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditional are often expressed using an 'if, then' or 'only if' construction.

* conjuction operator takes precedence over disjuction operator.

$p \vee q \wedge r \Rightarrow p \vee (q \wedge r)$

* Conditional & Biconditional operaters have low precedence over conj. & disj.

$p \rightarrow q \vee r \Rightarrow p \rightarrow (q \vee r)$

deals with countable, distinct elements (e.g integers, graphs & logic).

involves continuous quantities that vary smoothly (e.g: calculus, real number & functions).

* Discrete focuses on separate values while continuous deals with uninterrupted ranges.

...ITIONAL EQUVILANCE :-

...ompound proposition which is always true, no matter what is the truth values of the propositional variable that occur in it, is called as tautology.

A compound proposition that is always false is called a contradiction.

A compound proposition that is neither true nor false is called a contigency.

LOGICALLY EQUIVILAENT:-

Compound propositions that have the same truth values.

* A compound proposition p & q is logically equivalent if $p \leftrightarrow q$ is a tautology; denoted by $p \equiv q$.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q.$$

LOGICAL EQUIVALENCES :-

$\left.\begin{array}{l} p \wedge T = p \\ p \vee F = p \end{array}\right\}$ identity law.

$\left.\begin{array}{l} p \vee p = p \\ p \wedge p = p \end{array}\right\}$ Idempotent law

$\left.\begin{array}{l} p \vee T = T \\ p \wedge F = F \end{array}\right\}$ Domination law.

$\neg(\neg p) = p$. double negation law.

$\left.\begin{array}{l} p \vee q = q \vee p \\ p \wedge q = q \wedge p \end{array}\right\}$ commutative

$\left.\begin{array}{l} (p \vee q) \vee r \equiv p \vee (q \vee r) \\ (p \wedge q) \wedge r \equiv p \wedge (q \wedge r) \end{array}\right\}$ Associative law.

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ } distributive law.

$p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$ } Absorption law.

$p \vee \neg p \equiv T$
$p \wedge \neg p \equiv F$ } Negation law.

LOGICAL EQUIVILANCES INVOLVING CONDITIONALS.

$p \to q = \neg p \vee q$ (conditional equivalence).
$p \to q \equiv \neg q \to \neg p$ and so on .... pg no. 52.

Predicates:-

Propositional function; let P(x) becomes a ~~predicate~~ proposition. once the value of x has been defined.

or a predicate.

Propositional function's may have more than one variables. e.g

$\underbrace{Q}_{\text{predicate}}(\underbrace{x, y}_{\text{variables}})$.

e.g $x = y + 3$.

* The statements that describe valid input are known as pre-conditions.

* The conditions that the output should satisfy when the program has runned is called post condition.

...mple :

temp := x

$x := y$

$y :=$ temp.

$P(x,y)$ ⇒ precondition

where $x=a,\ y=b$.

$Q(x,y)$ ⇒ post condition.

where $x=b,\ y=a$.

Quantification:-

Defining the range of values for which a propositional function may have a certain truth value as a proposition.

↙ ↘

Universal

(predicate is true for every element in consideration)

Existential.

(one or more elements under consideration for which the element is true.).

—————— x ——————

and otherwise not. are false.