

# Inferential Statistics and Applied Probability

## Assignment # 2

(DEADLINE: 15/12/2023)

REG#: 2023428

COURSE CODE: DS211

CS - Section C

TOTAL MARKS: 90

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### Instructions

- You are free to consult each other for verbal help. However, **copying or sharing the soft/hard copy with each other will not only result in the cancellation of the current assignment, but it may also impact your grade in all the future assignments and exams as well.**
- List your collaborators on the last page of your assignment. Collaborators are any people you discussed this assignment with. This is an individual assignment, so be aware of the course's collaboration policy.
- You must attach this assignment at the top of your solution.
- All questions are mapped to CLO1

**Handwritten Tasks** – attempt each of the following task by hand on the A4 page.

**Task 1:** Let  $X$  be a random variable defined by drawing a card from a standard deck of 52 playing cards. If the card is a knave (jack), queen, or king, then  $X=11$ . If the card is an ace, then  $X=1$ . For any other card (two through ten),  $X$  takes the value of the number on the card. Now define a second random variable  $Y$  by the following procedure. When you evaluate  $X$ , you look at the color of the card. If the card is red, then  $Y = X - 1$ ; otherwise,  $Y = X + 1$ . **(10 marks)**

1. What is  $P(\{X \leq 2\})$ ?
2. What is  $P(\{X \geq 10\})$ ?
3. What is  $P(\{X \geq Y\})$ ?
4. What is the probability distribution of  $Y - X$ ?
5. What is  $P(\{Y \geq 12\})$ ?

**Task 2:** Magic the gathering is a popular card game. Cards can be land cards, or other cards. We consider a game with two players. Each player has a deck of 40 cards. Each player shuffles their deck, then deals seven cards, called their hand. The rest of each player's deck is called their library. Assume that player one has 10 land cards in their deck and player two has 20. Write  $L_1$  for the number of lands in player one's hand and  $L_2$  for the number of lands in player two's hand. Write  $L_t$  for the number of lands in the top 10 cards of player one's library. **(15 marks)**

1. Write  $S = L_1 + L_2$ . What is  $P(\{S = 0\})$ ?
2. Write  $D = L_1 - L_2$ . What is  $P(\{D = 0\})$ ?
3. what is the probability distribution for  $L_1$ ?
4. Write out the probability distribution for  $P(L_1 | L_t = 10)$ .
5. Write out the probability distribution  $P(L_1 | L_t = 5)$ .

**Task 3:** Suppose that among the 200 students registered in DS221, there are 187 students that have taken both a calculus and a linear algebra class in the past, and there are 2 students that have taken neither. **(10 marks)**

1. How many students have taken at least one of those two math classes in the past?
2. Now suppose furthermore that the number of students that have not taken linear algebra is 4 times the number of students that have not taken calculus. How many students have taken a linear algebra class in the past?

**Task 4:** A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows. (10 marks)

$$P(D|P1) = 0.01, P(D|P2) = 0.03, P(D|P3) = 0.02,$$

Where  $P(D|P_j)$  is the probability of a defective product, given plan  $j$ . If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

**Task 5:** A large industrial firm uses three local motels to provide overnight accommodations for its clients. From past experience it is known that 20% of the clients are assigned rooms at the Ramada Inn, 50% at the Sheraton, and 30% at the Lakeview Motor Lodge. If the plumbing is faulty in 5% of the rooms at the Ramada Inn, in 4% of the rooms at the Sheraton, and in 8% of the rooms at the Lakeview Motor Lodge, what is the probability that (10 marks)

1. a client will be assigned a room with faulty plumbing?
2. a person with a room having faulty plumbing was assigned accommodations at the Lakeview Motor Lodge?

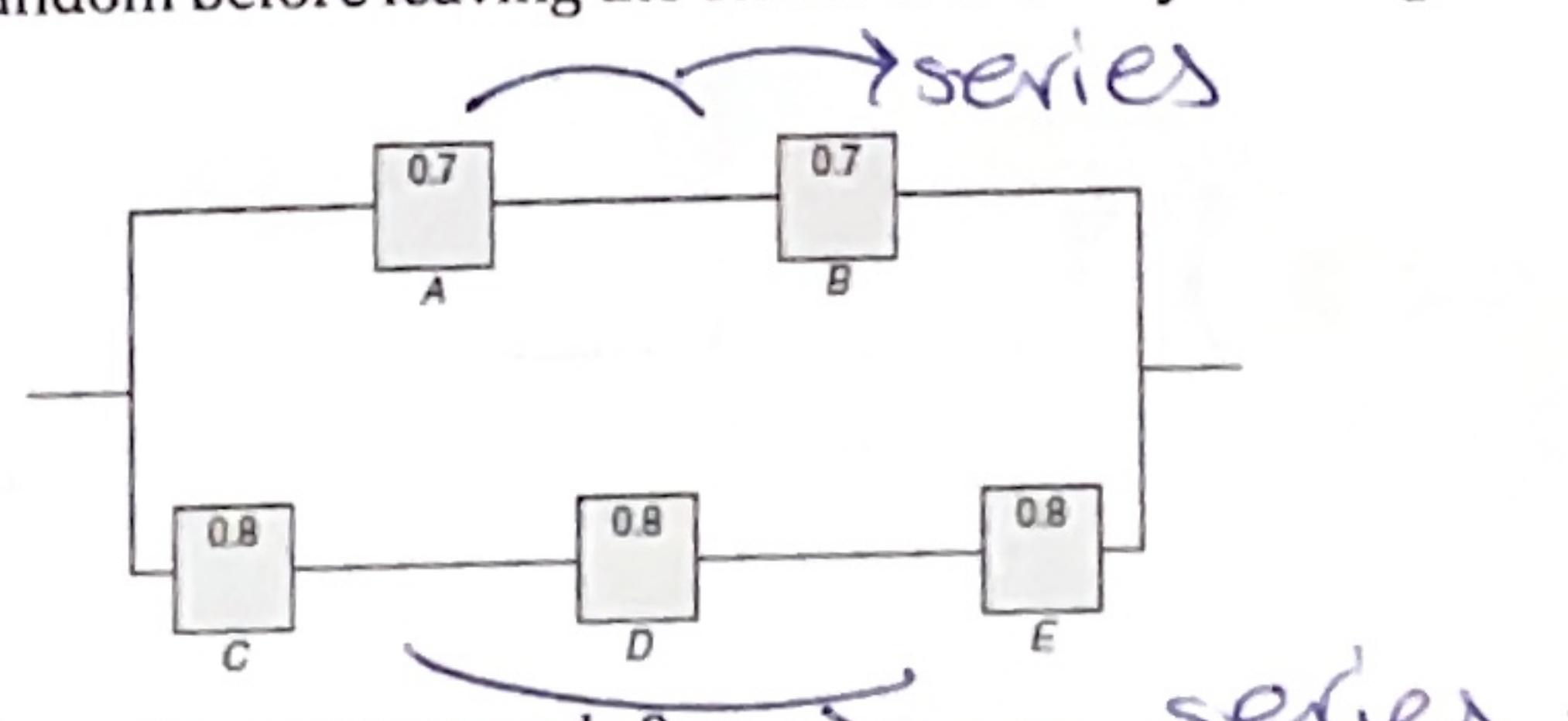
**Task 6:** Let  $X$  denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let  $Y$  denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as. (10 marks)

		$x$		
		1	2	3
$y$	1	0.05	0.05	0.10
	3	0.05	0.10	0.35
	5	0.00	0.20	0.10

1. Evaluate the marginal distribution of  $x$ .
2. Evaluate the marginal distribution of  $y$ .
3. Find  $P(y = 3 | x = 2)$ .

**Task 7:** From a box containing 4 dimes and 2 nickels, 3 coins are selected at random without replacement. Find the probability distribution for the total  $T$  of the 3 coins. Express the probability distribution graphically as a probability histogram. (10 marks)

**Task 8:** A real estate agent has 8 master keys to open several new homes. Only 1 master key will open any given house. If 40% of these homes are usually left unlocked, what is the probability that the real estate agent can get into a specific home if the agent selects 3 master keys at random before leaving the office? A circuit system is given in figure below. Assume the components fail independently. (10 marks)



6. What is the probability that the entire system works?
7. Given that the system works, what is the probability that the component  $a$  is not working?

**Task 9:** Let  $A$ ,  $B$  and  $C$  are events in a sample space while  $A$  and  $B$  are disjoint events. We know. (05 marks)

$$P(A) = 2P(B), P(C/A) = 2/7, P(C/B) = 4/7$$

$$\text{What is } P(C / (A \cup B)) ?$$

#### Submission:

Submit both of your handwritten (as a document, i.e. Word or PDF) and programming tasks (as notebooks along the dataset) on Teams.

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21/01/2023

## ASSIGNMENT NO.2

### Task no. 1

$X = \text{drawing card from a deck of 52 cards}$

$X = 11$  (for Jack, queen or king).

$X = 1$  (Ace)

$X = \text{value of the card (for any card b/w 2 to 10)}$

$Y = \text{Random Variable representing color}$

$Y = X - 1$  (Red cards)

$Y = X + 1$  (non-red Cards)

$$\textcircled{1} \quad P(X \leq 02)$$

Now  $X$  can be;

$X = 1$  (4 aces in the deck).

$X = 2$  (4 cards with 2 in the deck).

hence for the given condition

$X \leq 2$  there are 8 possible outcomes.

$$\therefore P(X \leq 02) = \frac{8}{52} \Rightarrow \frac{2}{13}$$

$$\textcircled{2} \quad P(X \geq 10)$$

$X = 10$  (4 possible cards)

Jack (4 possible cards)  
Queen " "  
King " "  
 $\int = 12$

$4+12 \Rightarrow$  Total possible outcomes of the given condition.

$$\therefore \frac{16}{52} = \frac{4}{13}.$$

③  $P(X \geq Y)$

$Y$  depends upon  $X$  for its value as;

$$Y = X-1 \text{ (Red)}$$

$$Y = X+1 \text{ (Non Red).}$$

$X \geq Y$  means.

①  $X \geq X-1$  always True.

$X \geq X+1$  always false

It means that

$P(X \geq Y)$  occurs as an outcome only if red cards and thus 26 red cards in a deck.

$$\frac{26}{52} = \frac{1}{2}.$$

④ Probability distribution of  $Y-X$

given that  $Y$  depends upon the card's color.  $Y=X-1$  (Red)  
 $Y=X+1$  (non red).

$$P(Y-X=-1) = P(\text{red card}) \Rightarrow \frac{26}{52}$$

$$P(Y-X=+1) = P(\text{black card}) \Rightarrow \frac{26}{52}.$$

$$P(Y-X) = \begin{cases} \frac{1}{2} & \text{if } Y-X=1 \\ \frac{1}{2} & \text{if } Y-X=-1. \end{cases}$$

⑤  $P(X \geq 12)$

$Y \geq 12$ .

means

$$X-1 \geq 12$$

$X \geq 13$ . this is impossible because at max  $X=11$

$$+1 \geq 12$$

$$x \geq 11$$

This thus represents all jacks, queens and kings.

But this was derived from

$$Y = X + 1 \text{ (i.e. black cards)}$$

$\therefore$  Black Jack, queen and kings. i.e. 6 in no.

$$P(Y \geq 12) \frac{6}{52} = \frac{3}{26}$$

Task 2:-  
 $\sim x \sim$

Player 1 & Player 2 with a deck of 40 cards each.

Player 1 has 10 land cards out of 40 cards.

Player 2 has 20 land cards out of 40 cards.

$L_1$  = No. of land cards in Player 1's hand of 7 cards.

$L_2$  = no. of land cards in Player 2's hand of 7 cards.

$L_t$  = No. of land card in top 10 cards of Player 1's library.

$$\textcircled{1} \quad P(S=0) \text{ where } S = L_1 + L_2$$

This asks for the probability that the sum of the land cards in both player's hands are 0.

$$L_1 = 0 \text{ & } L_2 = 0$$

$$P(L_1 = 0) = \frac{\binom{30}{7}}{\binom{40}{7}}$$

no. of non-land cards

$$P(L_2 = 0) = \frac{\binom{20}{7}}{\binom{40}{7}}$$

$$P(S=0) = P(L_1=0) \cdot P(L_2=0)$$

$$P(S=0) = P(L_1=0) \cdot P(L_2=0)$$

because the draws are independent

$$\binom{30}{7} \Rightarrow 2593055$$

$$\binom{20}{7} \Rightarrow 11,520$$

$$\binom{40}{7} \Rightarrow 18643560$$

$$= \frac{(2593055)}{18643560} \times \frac{11520}{18643560}$$

$$\Rightarrow \frac{0.139 \times 0.00416}{0.000578}$$

$$\textcircled{2} \cdot P(D=0) \Rightarrow \frac{0.000578}{D = L_1 - L_2}$$

$$k=0.$$

$$\frac{\binom{10}{0} \binom{30}{7-0}}{\binom{40}{7}} \cdot \frac{\binom{20}{0} \binom{20}{7}}{\binom{40}{7}} \Rightarrow 0.00058$$

$$k=1$$

$$\frac{\binom{10}{1} \binom{30}{7-1}}{\binom{40}{7}} \cdot \frac{\binom{20}{1} \binom{20}{7-1}}{\binom{40}{7}}$$

$$\Rightarrow 0.00614$$

$$k=2$$

$$\frac{\binom{10}{2} \binom{30}{7-2}}{\binom{40}{7}} \cdot \frac{\binom{20}{2} \binom{20}{7-2}}{\binom{40}{7}} \Rightarrow 0.0205$$

$$k=3$$

$$\frac{\binom{10}{3} \binom{30}{7-3}}{\binom{40}{7}} \cdot \frac{\binom{20}{3} \binom{20}{7-3}}{\binom{40}{7}}$$

$$\frac{\binom{10}{6} \binom{30}{7-6}}{\binom{40}{7}} \cdot \frac{\binom{20}{1} \binom{20}{7-1}}{\binom{40}{7}} \Rightarrow 0.0182$$

$$P(D=0) = \sum_{k=0}^7 P(L_1=k) \cdot P(L_2=k)$$

$$\frac{\binom{10}{4} \binom{20}{7-4}}{\binom{40}{7}} \Rightarrow 0.00623$$

$$\frac{\binom{10}{5} \binom{30}{7-5}}{\binom{40}{7}} \Rightarrow 0.00077$$

$$\frac{\binom{10}{6} \binom{30}{7-6}}{\binom{40}{7}} \cdot \frac{\binom{10}{1} \binom{10}{7-1}}{\binom{40}{7}} \Rightarrow 0.000019$$

Probability dist of  $L_1$ .

$L_1$  represents the land cards drawn from the player 1's hand.

They can be upto 7.

Represented as:

$$P(L_1 = k) \frac{\binom{10}{k} \binom{30}{7-k}}{\binom{40}{7}}$$

ways to choose.

$\binom{10}{k}$  represents  $k$  lands from 10 lands

$\binom{30}{7-k}$  ways to choose  $7-k$  non lands from 30 non land cards.

Now the value of  $k$  ranges from 0 - 7 depending on how many land cards are drawn.

thus:

$k=0$

$$\frac{\binom{10}{0} \binom{30}{7}}{\binom{40}{7}} \Rightarrow 0.139.$$

$k=1$

$$\frac{\binom{10}{1} \binom{30}{6}}{\binom{40}{7}} \Rightarrow 0.287$$

$k=2$

$$\frac{\binom{10}{2} \binom{30}{5}}{\binom{40}{7}} \Rightarrow 0.302$$

$k=3$

$$\frac{\binom{10}{3} \binom{30}{4}}{\binom{40}{7}} \Rightarrow 0.180$$

$k=4$

$$\frac{\binom{10}{4} \binom{30}{3}}{\binom{40}{7}} \Rightarrow 0.072$$

$k=5$

$$\frac{\binom{10}{5} \binom{30}{2}}{\binom{40}{7}} \Rightarrow 0.017$$

$k=6$

$$\frac{\binom{10}{6} \binom{30}{1}}{\binom{40}{7}} \Rightarrow 0.002$$

$k=7$

$$\frac{\binom{10}{7} \binom{30}{0}}{\binom{40}{7}} \Rightarrow 0.0001$$

$$④ P(L_1 | L_t = 10)$$

Now it asks for the probability that the top 10 cards in the deck are land cards for player 1.

$$k = 0, 1, 2, 3, \dots$$

$$P(L_t = k) = \frac{\binom{10}{k} \binom{30}{70-k}}{\binom{40}{7}}$$



Basically we want to find player 1 drawing a land card in hand if all top 10 are land; then it means that the player will have all his hand cards equal to land cards.

$$\therefore 7 - k = 0 \text{ & } k = 7$$

$$\Rightarrow P(L_1 = k | L_t = 10)$$

$$\Rightarrow \frac{\binom{10}{k}}{\binom{40}{7}}$$

Now we can calculate for each value of k

(i.e.  $k = 0, \dots, 7$ ) no. of cards drawn given that they were all chosen from land cards.

$$k=0$$

$$P(L_1 = k | L_t = 10) \Rightarrow \frac{\binom{10}{0}}{120}$$

$$\Rightarrow \frac{1}{120}$$

$$k=1 \quad \frac{\binom{10}{1}}{120} \Rightarrow \frac{10}{120} = \frac{1}{12}$$

$$k=2 \quad \frac{\binom{10}{2}}{120} \Rightarrow \frac{45}{120} = \frac{3}{8}$$

$$k=3 \quad \frac{\binom{10}{3}}{120} \Rightarrow \frac{120}{120} = 1$$

4

$$\frac{\binom{10}{4}}{120} \Rightarrow \frac{210}{120} \Rightarrow \frac{7}{4}$$

k=5

$$\frac{\binom{10}{5}}{120} \Rightarrow \frac{252}{120} \Rightarrow \frac{21}{10}$$

k=6

$$\frac{\binom{10}{6}}{120} \Rightarrow \frac{210}{120} \Rightarrow \frac{7}{4}$$

k=7

$$\frac{\binom{10}{7}}{120} \Rightarrow \frac{120}{120} \Rightarrow 1.$$

 $\sim \times \sim$ 

$$\textcircled{5} \quad P(L_1 | L_t=5)$$

Probability that the player draws k land cards into his hand given that no. of land cards at the top 10 cards of the deck are  $L_t=5$ .

$$\Rightarrow P(L_1=k | L_t=5) \Rightarrow \frac{\binom{5}{k} \binom{35}{7-k}}{\binom{40}{7}}$$

k=0

$$\frac{\binom{5}{0} \binom{35}{7-0}}{\binom{40}{7}} \Rightarrow 0.3607.$$

k=1

$$\frac{\binom{5}{1} \binom{35}{6}}{\binom{40}{7}} \Rightarrow 0.7303.$$

k=2

$$\frac{\binom{5}{2} \binom{35}{5}}{\binom{40}{7}} \Rightarrow 0.6385.$$

k=3

$$\frac{\binom{5}{3} \binom{35}{4}}{\binom{40}{7}} \Rightarrow 0.2806,$$

k=4

$$\frac{\binom{5}{4} \binom{35}{3}}{\binom{40}{7}} \Rightarrow 0.0591.$$

$$K=5 \quad \left(\begin{array}{c} 5 \\ 5 \end{array}\right) \quad \left(\begin{array}{c} 35 \\ 7-5 \end{array}\right)$$

$$\frac{\phantom{11111}}{\phantom{11111}} \quad \Rightarrow \quad \underline{0.0049}.$$

### TASK NO. 3

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Total no. of students  $\Rightarrow 200$

No. of students who have taken both  $\Rightarrow 187$  calculus and LA.

No. of students who have not taken both  $\Rightarrow 2$ .

No. of students that have not taken linear algebra are  $\frac{Y}{4}$  times than those who have taken.

① The no. of students who have take atleast one of the two classes.

$$200 - 2 \Rightarrow 198.$$

②

$x$  = students who have not taken calculus in the past.

$y$  = students who have not taken linear algebra in the past.

$$y = 4x.$$

A = who have taken calculus class

B = who have take linear algebra class

- all the elements that belong to A, B or Both.

section:- have taken both.

or L.

$$|A \cup B| = 198$$

(all those have atleast taken one).

$$|A \cap B| = 187$$

(all those who have taken both classes).

Now  
—x—

$$|A \cup B| = (|A \cap B|) + |A| + |B|$$

$$|A| + |B| = |A \cup B| + |A \cap B|$$

$$\Rightarrow 198 + 187$$

$$\Rightarrow 385$$

Students who did not take calculus.

$$\Rightarrow 200 - |A| = x$$

Students who did not take L.A.

$$\Rightarrow 200 - |B| = y$$

$$|A| = 200 - x$$

$$|B| = 200 - y \\ = 200 - 4x$$

$$|A| + |B| = 385$$

$$200 - x + 200 - 4x = 385$$

$$400 - 5x = 385$$

$$5x = 15$$

$$\boxed{5x = 15}$$

$$y = 4(3)$$

$$\boxed{y = 12}$$

Ans!

#### TASK NO.4

—x—

$$P(D|P_1) = 0.01$$

$$P(D|P_2) = 0.03$$

$$P(D|P_3) = 0.02$$

$$P(D|P_j)$$

$$P(P_1) = 0.3$$

$$P(P_2) = 0.2$$

$$P(P_3) = 0.5$$

$$P(D) = P(D|P_1)(P_1) +$$

$$P(D|P_2)(P_2) +$$

$$P(D|P_3)(P_3)$$

$$\Rightarrow (0.01)(0.3) + \Rightarrow 0.019$$

$$(0.03)(0.2) +$$

$$(0.02)(0.5)$$

$$P(P_2|D) = \frac{(P)(D|P_2)P(P_2)}{P(D)}$$

$$\frac{(0.03)(0.2)}{0.019} \Rightarrow 0.3158$$

$$\Rightarrow \frac{(0.02)(0.5)}{0.019} \Rightarrow 0.5263$$

$\therefore P_3$  has the highest probability of being the executed plan given the product was defective.

$$P(P_j|D) = \frac{P(D|P_j) \cdot P(P_j)}{P(D)}$$

$$P(P_1|D) = \frac{P(D|P_1) \cdot P(P_1)}{P(D)}$$

$$\Rightarrow \frac{(0.01)(0.3)}{0.019}$$

$$\Rightarrow 0.1579$$

## TASK NO. 5

$\rightarrow R$

$$P(\text{Ramada}) \Rightarrow 0.2$$

$$P(\text{Sheraton}) \Rightarrow 0.5$$

$$P(\text{Lakeview}) \Rightarrow 0.3$$

①  $P(\text{Plumbing} | \text{Ramada}) \Rightarrow 0.5$

$$P(\text{,,} | \text{Sheraton}) \Rightarrow 0.03$$

$$+ 0.01 = 0.04$$

$$P(\text{,,} | \text{LakeView}) \Rightarrow 0.08$$

$$P(P) = P(P|R) \xrightarrow{P(R)} + P(P|S) \xrightarrow{P(S)}$$

$$+ P(P|L) P(L)$$

$$\Rightarrow 0.05 \times 0.2 + 0.04 \times 0.5$$

$$+ 0.08 \times 0.3$$

$$\boxed{\Rightarrow 0.059}$$

$$P(L|P) \Rightarrow \frac{P(P|L) P(L)}{P(L)}$$

$$\Rightarrow \frac{0.08 \times 0.3}{0.054}$$

$$\boxed{\Rightarrow 0.444}$$

— X —

## TASK 6 :-

① Marginal dist of X.

X=1

$$P(X=1) \Rightarrow P(X=1, Y=1) +$$

$$P(X=1, Y=2) + P(X=1, Y=3)$$

$$\Rightarrow 0.05 + 0.05 + 0.00$$

$$\underline{0.05 + 0.05} = \boxed{0.10}$$

X=2

$$P(X=2) \Rightarrow P(X=2, Y=1) +$$

$$P(X=2, Y=2) + P(X=2, Y=3)$$

$$\Rightarrow 0.05 + 0.10 + 0.20$$

$$\boxed{\underline{0.35}}$$

$X=3$

$$P(X=3) \Rightarrow P(X=3, Y=1) + P(X=3, Y=2)$$

$\uparrow P(X=3, Y=3)$

$$0.10 + 0.35 + 0.10 \Rightarrow \boxed{0.55}$$

### ③ Marginal Dist of Y

$Y=1$

$$P(Y=1) \Rightarrow P(Y=1, X=1) +$$

$$P(X=2, Y=1) + P(X=3, Y=1)$$

$$\Rightarrow 0.5 + 0.05 + 0.10 \Rightarrow \boxed{0.20}$$

Similarly

$Y=2$

$$\Rightarrow 0.05 + 0.10 + 0.35 \Rightarrow 0.45$$

$Y=5$

$$\Rightarrow 0.00 + 0.0 - 0.10 \Rightarrow \boxed{0.50}$$

$$\Rightarrow \boxed{0.30}$$

$$③ P(Y=3 | X=2)$$

$$P(Y=3 | X=2) \Rightarrow$$

Using Bayes's theorem

$$\frac{P(X=x | Y=y) \cdot P(Y=y)}{P(X=x)}$$

$$\frac{0.50 \times 0.20}{0.35} \Rightarrow \frac{0.10}{0.35}$$

$$\Rightarrow 0.286.$$

### TASK NO. 7

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Sample Set = 4 dimes &  
Total . 2 nickels.

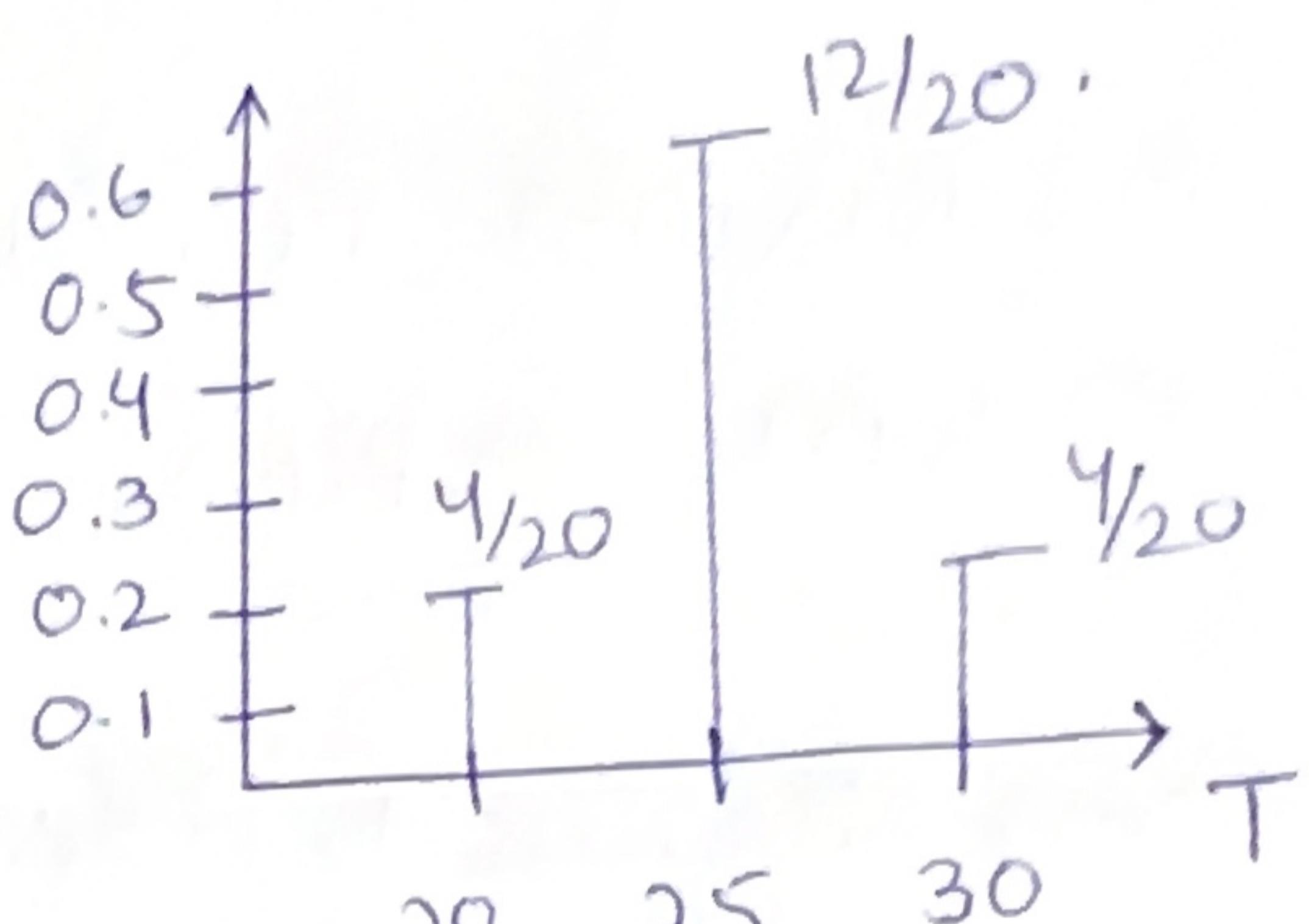


Selection makes the following sets without replacement.

$$\{(DNN), (DDN), (NDN), (DDD)\}$$

6  
 & draw sample  
 have multiple combinations  
 whose probability dist is given  
 below as;

$$\frac{^4C_1 \cdot ^2C_2}{^6C_3} \Rightarrow \frac{4}{20}, T \Rightarrow 10+5+5 \\ \Rightarrow 20$$



$$\frac{^4C_2 \cdot ^2C_1}{^6C_3} \Rightarrow \frac{12}{20}, T = 10+10+5 \\ \Rightarrow 25$$

$$\frac{^4C_3 \cdot ^2C_0}{^6C_3} \Rightarrow \frac{4}{20}, T = 10+10+10 \\ \Rightarrow 30.$$

### Task no. 9:-

$$P(A) = 2P(B)$$

A & B are disjoint sets.

$$P(C|A) = 2/7$$

$$P(E|B) = 4/7$$

$$P(C|A \cup B) \Rightarrow ?$$

as disjoint

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 2P(B) + P(B)$$

$$P(A \cup B) \Rightarrow 3P(B)$$

$$P(C|A \cup B)$$

$$P((A \cup B) \cap C)$$

$$P(A \cup B).$$

$$\frac{P(B \cap C) + P(A \cap C)}{P(A \cup B)}$$

also.

$$P(C|A) = \frac{P(C \cap A)}{P(A)}$$

$$P(A \cap C) = P(C|A)P(A).$$

$$\Rightarrow 2P(B) \times 2/7 \Rightarrow \frac{4}{5} P(B)$$

again.

$$P(C|B) = \frac{P(C \cap B)}{P(B)}$$

$$P(C|B) \times P(B) = P(C \cap B)$$

$$P(C \cap B) = \frac{4}{7} P(B).$$

Using all value to determine.

$$\begin{aligned} P(C|A \cup B) &\Rightarrow \frac{\frac{4}{7} P(B) + \frac{4}{7} P(B)}{3P(B)} \\ &\Rightarrow \frac{1.14P(B)}{3P(B)} \Rightarrow \frac{1.14}{3} \\ &\Rightarrow 0.38 \end{aligned}$$

### Task no. 8

As the working of complete system depends upon the working of each components.

The components A & B will work when both of them are working.

$$P(A \cap B) = (0.7)^2$$

Similarly C & D & E are in series.

$$P(C \cap D \cap E) = (0.8)^3$$

if either one of the series system works

Thus the probability that the system works

$$\begin{aligned} ① P_{\text{system}} &\Rightarrow P((A \cap B) \cup (C \cap D \cap E)) \\ &\Rightarrow P(A \cap B) + P(C \cap D \cap E) - P(A \cap B \cap C \cap D \cap E) \\ &\Rightarrow (0.7)^2 \cdot (0.8)^3 - 0.25 \end{aligned}$$

$$\boxed{\Rightarrow 0.75.}$$

$$② P(A' | \text{System is working})$$

$$\Rightarrow \frac{P(\bar{A} \cap \text{System is working})}{P(\text{System works})}$$

$$\Rightarrow \frac{(0.3)(0.7)(0.8)^3}{0.75}$$

$$\Rightarrow \frac{0.2}{(0.3)^2 (0.8)^3}$$

$$(0.4) + \frac{(0.6) \times (0.375)}{0.625}$$

P(gets into the house)

$\Rightarrow P(\text{the house is open}) + P(\text{house locked opened with a key})$ .

$$\begin{aligned} P(\text{key works}) &\Rightarrow \frac{7}{2} \binom{2}{1} \binom{1}{1} \\ &\Rightarrow \frac{3}{8} \binom{2}{3} \\ &\Rightarrow 0.375 \end{aligned}$$