

dot controls shape as it \uparrow $\sqrt{6}$
it comes close to normal.

T-DISTRIBUTION:-

has a symmetric and bell shaped curve like normal dist.

heavier tails (more prop in ext values)

let say we have a population that is normally distributed; with a sample drawn to be too small in size.

then we would follow the t-distribution. with $N-1$ degrees of freedom.

$$T = \frac{\text{mean}\{x\} - \text{pop mean}\{x\}}{\text{std err}\{x\}}$$

the amount of probability mass at tails of t -dist is controlled by parameter v (df).

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$v \rightarrow \infty$ it becomes standard normal dist.
(least heavy tail)

Degree of Freedom: (df)

defined how many values are free to vary after we estimate something like mean.

e.g

Sample: $x_1 = 27, x_2 = 5, x_3 = 8$.

Sample mean $\Rightarrow 6.67$.

degree of freedom $\Rightarrow 3 - 1 :-$

(once we know two values the third is determined).

* df: helps to determine shape of t -dist; which is used for confidence intervals and hypothesis testing.

Question:- Why cannot we use Z -dist for smaller sample size?

(i) Z -dist assumes that the population std. deviation is known.

however in the smaller samples, the pop. std. deviation is not measured from unbiased estimator rather the sample leading to more uncertainty. (\rightarrow from)

\rightarrow because small sample are more variable and less reliable for such estimates.

* CLT works better with $n > 30$

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Confidence Interval for the mean;

$$\bar{X} + t_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right)$$

→ std deviation of sample.

↓
t-critical values.

Example:-

$$n = 8.$$

$$\text{sample mean} \Rightarrow 0.26 + 0.03 \Rightarrow 0.29.$$

$$\text{std deviation of sample} \Rightarrow 0.074.$$

$$CI = 99\%.$$

Now using formula.

$$1 - \alpha \Rightarrow 1 - 0.99$$

$$\Rightarrow 0.01$$

for two tails.

↓ ↓

$$0.005$$

$$0.005 + 0.99$$

$$0.005$$

$$0.995$$

$$0.29 \pm (2.898) \frac{0.074}{\sqrt{8}}$$

$$\Rightarrow 0.29 \pm 0.05.$$

$$t_{\alpha/2} \Rightarrow t_{0.005} \Rightarrow -2.898.$$

$$t_{0.995} \Rightarrow +2.898.$$