Inferential Statistics and Applied Probability

Assignment # 2

(DEADLINE: 15/12/2023)

REG#: _____ NAME: _____

COURSE CODE: DS211 INSTRUCTOR: MUHAMMAD SAJID ALI

TOTAL MARKS: 90

Instructions

- You are free to consult each other for verbal help. However, **copying or sharing the soft/hard copy with each other will not only result in the cancellation of the current assignment, but it may also impact your grade in all the future assignments and exams as well**.
- List your collaborators on the last page of your assignment. Collaborators are any people you discussed this assignment with. This is an individual assignment, so be aware of the course's collaboration policy.
- You must attach this assignment at the top of your solution.
- All questions are mapped to CLO1

Handwritten Tasks – attempt each of the following task by hand on the A4 page.

Task 1: Let X be a random variable defined by drawing a card from a standard deck of 52 playing cards. If the card is a knave (jack), queen, or king, then X=11. If the card is an ace, then X=1. For any other card (two through ten), X takes the value of the number on the card. Now define a second random variable Y by the following procedure. When you evaluate X, you look at the color of the card. If the card is red, then Y = X – 1; otherwise, Y = X + 1. **(10 marks)**

1. What is P({X <= 2})?
2. What is P({X >= 10})?
3. What is P({X >= Y})?
4. What is the probability distribution of Y – X?
1. What is P({Y >= 12})?

Task 2: Magic the gathering is a popular card game. Cards can be land cards, or other cards. We consider a game with two players. Each player has a deck of 40 cards. Each player shuffles their deck, then deals seven cards, called their hand. The rest of each player's deck is called their library. Assume that player one has 10 land cards in their deck and player two has 20. Write L_1 for the number of lands in player one's hand and L_2 for the number of lands in player two's hand. Write L_t for the number of lands in the top 10 cards of player one's library. **(15 marks)**

1. Write S = L1 + L2. What is P({S = 0})?
2. Write D = L1 – L2 . What is P({D = 0})?
3. what is the probability distribution for L1?
4. Write out the probability distribution for $P(L_1 \mid L_t = 10)$.
5. Write out the probability distribution $P(L_1 \mid L_t = 5)$.

Task 3: Suppose that among the 200 students registered in DS221, there are 187 students that have taken both a calculus and a linear algebra class in the past, and there are 2 students that have taken neither. **(10 marks)**

1. How many students have taken at least one of those two math classes in the past?
2. Now suppose furthermore that the number of students that have not taken linear algebra is 4 times the number of students that have not taken calculus. How many students have taken a linear algebra class in the past?

Task 4: A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows. **(10 marks)**

P(D|P1) = 0.01, P(D|P2) = 0.03, P(D|P3) = 0.02,

Where $P(D|P_j)$ is the probability of a defective product, given plan j. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

Task 5: A large industrial firm uses three local motels to provide overnight accommodations for its clients. From past experience it is known that 20% of the clients are assigned rooms at the Ramada Inn, 50% at the Sheraton, and 30% at the Lakeview Motor Lodge. If the plumbing is faulty in 5% of the rooms at the Ramada Inn, in 4% of the rooms at the Sheraton, and in 8% of the rooms at the Lakeview Motor Lodge, what is the probability that **(10 marks)**

1. a client will be assigned a room with faulty plumbing?
2. a person with a room having faulty plumbing was assigned accommodations at the Lakeview Motor Lodge?

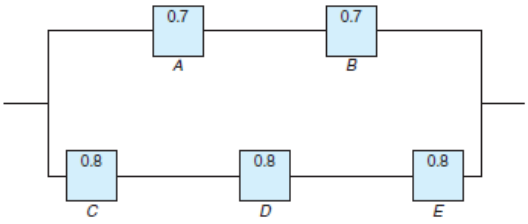
Task 6: Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as. **(10 marks)**

		x		
$f(x,y)$		1	2	3
y	1	0.05	0.05	0.10
	3	0.05	0.10	0.35
	5	0.00	0.20	0.10

1. Evaluate the marginal distribution of x.
2. Evaluate the marginal distribution of y .
3. Find P(y = 3 | x = 2).

Task 7: From a box containing 4 dimes and 2 nickels, 3 coins are selected at random without replacement. Find the probability distribution for the total T of the 3 coins. Express the probability distribution graphically as a probability histogram. **(10 marks)**

Task 8: A real estate agent has 8 master keys to open several new homes. Only 1 master key will open any given house. If 40% of these homes are usually left unlocked, what is the probability that the real estate agent can get into a specific home if the agent selects 3 master keys at random before leaving the office? A circuit system is given in figure below. Assume the components fail independently. **(10 marks)**



6. What is the probability that the entire system works?
7. Given that the system works, what is the probability that the component a is not working?

Task 9: Let A, B and C are events in a sample space while A and B are disjoint events. We know. **(05 marks)**

P(A) = 2P(B), P(C|A) = 2/7, P(C|B) = 4/7

What is *P(C | (A U B))* ?

Submission:

Submit both of your handwritten (as a document, i.e. Word or PdF) and programming tasks (as notebooks along the dataset) on *Teams*.