

# FORMAL LANGUAGES AND AUTOMATA THEORY

## ASSIGNMENT # 1

NAME: \_\_\_\_\_

REG#: \_\_\_\_\_

COURSE CODE: CS224

INSTRUCTOR: MUHAMMAD SAJID ALI

TOTAL MARKS: 115

---

### Instructions

- You are free to consult each other for verbal help. However, **copying or sharing the soft/hard copy with each other will not only result in the cancellation of the current assignment, but it may also impact your grade in all the future assignments and exams as well.**
- List your collaborators on the last page of your assignment. Collaborators are any people you discussed this assignment with. This is an individual assignment, so be aware of the course's collaboration policy.
- You must attach this assignment at the top of your solution.

### Tasks

Solve the following problems. The solutions may or may not be available online. If they are available, try to understand them before re-phrasing them in your own words.

**Task 1:** Consider the language  $S^*$ , where  $S = \{aa, b\}$ . How many words does this language have of length 4? of length 5? of length 6? What can be said in general (5 Marks)

**Task 2:** Consider the language  $S^*$ , where  $S = \{ab, ba\}$ . Write out all words in  $S^*$  that have seven or fewer letters. Can any word in this language contain the substrings  $aaa$  or  $bbb$ ? What is the smallest word that is not in this language? (5 Marks)

**Task 3:** Consider the language  $S^*$ , where  $S = \{a, ab, ba\}$ . Is the string  $(abbb)a$  a word in this language? Write out all words in this language with six or fewer letters. What is another way in which to describe the words in this language? Be careful, this is not simply the language of all words without  $bbb$ . (5 Marks)

**Task 4:** Consider the language  $S^*$ , where  $S = \{aa, aba, baa\}$ . Show that the words  $aabaa$ ,  $baaabaaa$ , and  $baaaaababaaaa$  are all in this language. Can any word in this language be interpreted as a string of elements from  $S$  in two different ways? Can any word in this language have an odd total number of  $a$ 's? (5 Marks)

**Task 5:** (12 Marks)

1. Let  $S = \{ab\}$  and let  $T = \{abbb\}$ . Show that  $S^* = T^*$ .
2. Let  $S = \{ab\}$  and let  $T = \{abbb\}$ . Show that  $S^* \neq T^*$ , but that  $S^* \subset T^*$
3. What principle does this illustrate?

**Task 6:** How does the situation in *Task 5* changes if we replace the operator  $*$  with the operator  $+$  as we discussed in class? Note the language  $S^+$  means the same as  $S^*$ , but does not allow the “concatenation of no words” of  $S$ . (5 Marks)

**Task 7:** Consider the language PALINDROME over the alphabet  $\{a, b\}$  (20 Marks)

1. Prove that if  $x$  is in PALINDROME, then so is  $x^n$  for any  $n$ .
2. Prove that if  $y^3$  is in PALINDROME, then so is  $y$ .
3. Prove that if  $z^n$  is in PALINDROME for some  $n$  (greater than 0) then  $z$  itself is also.
4. Prove that PALINDROME has many words of length 4 as it does of length 3.
5. Prove that PALINDROME has many words of length  $2n$  as it has of length  $2n - 1$ . How many words is that ?

**Task 8:** Prove that for all sets  $S$ . (12 Marks)

1.  $(S^+)^* = (S^*)^*$
2.  $(S^+)^+ = S^+$
3. Is  $(S^*)^+ = (S^+)^*$  for all sets  $S$ ?

**Task 09:** Construct a regular expression defining each of the following languages over the alphabet  $\Sigma = \{a, b\}$  (18 Marks)

1.  $\{w \mid w \text{ contains all strings that do not end with } aa\}$
2.  $\{w \mid w \text{ contains all strings that contain an even number of } b\text{'s}\}$
3.  $\{w \mid w \text{ contains all strings which do not contain the substring } ba\}$
4.  $\{w \mid w \text{ contains all strings that contain exactly two } b\text{'s or exactly three } b\text{'s, not more}\}$
5.  $\{w \mid w \text{ contains all strings that have exactly one double letter in them}\}$
6.  $\{w \mid w \text{ contains all strings in which the letter } b \text{ is never tripled. This means that no word contains the substring } bbb\}$

**Task 10:** Let  $r_1, r_2$  and  $r_3$  be three regular expressions. Show that the language associated with  $(r_1 + r_2)r_3$  is the same as the language associated with  $r_1r_3 + r_2r_3$ . Show that  $r_1(r_2 + r_3)$  is equivalent to  $r_1r_2 + r_1r_3$ . This will be the same as proving a “distributive law” for regular expressions. (10 Marks)

**Task 11:** Show that (9 Marks)

1. Expression  $(a + b)^* a (a + b)^* b (a + b)^*$  and  $(a + b)^* ab (a + b)^*$  are equivalent in a sense that they define same language.
2.  $(a + b)^* ab (a + b)^* + b^*a^* = (a + b)^*$
3.  $(a + b)^* ab [(a + b)^* ab (a + b)^* + b^*a^*] + b^*a^* = (a + b)^*$

**Task 12:** Show that the following pairs of regular expressions define the same language over the alphabet  $\Sigma = \{a, b\}$  (9 Marks)

1.  $(ab)^*a$  and  $a(ba)^*$
2.  $(a^* + b)^*$  and  $(a + b)^*$
3.  $(a^* + b^*)^*$  and  $(a + b)^*$