Chapter no. 1

variables

$x_1 \cdots x_n$

then the linear equ

$a_1x_1 + a_2x_2 \cdots a_nx_n = b$

where 'b' & 'a₁ ⋯ aₙ' are constants / real or complex no.

one or more linear equs
=> system of linear equs

A sol. of linear equ or the sol set are all the values that make the equ satisfied at both ends.

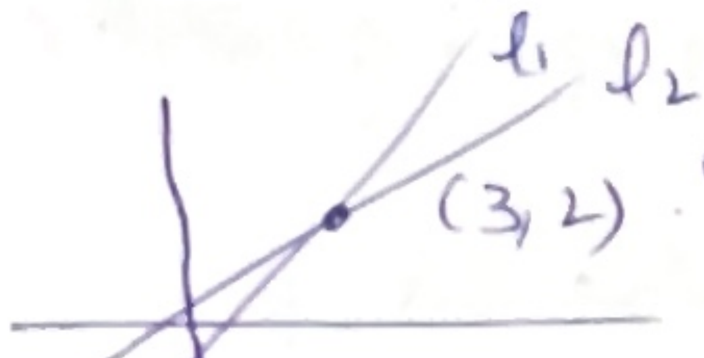
Equivalent linear equss have the same solution set.

Consider

$x_1 - 2x_2 = -1 \quad l_1$

$-x_1 + 3x_2 = 3 \quad l_2$

if sol of the system is (3, 2) it is shown graphically as



① exactly one solution.

if l_1 & l_2 coincide they have infinite ② many sol.

if they are parallel i.e no intersection ③ point then no solution

∴ there are ① ② ③ (sol) possibilities of equations.

system of linear equations

↙ consistent (1/many sols)

↘ Inconsistent (no solution).

Solving a matrix form of linear equs) is done using elementary row operations

① Replacement.
② Interchange.
③ Scaling.

*note If augmented matrices of 2 linear systems are row equivalent then the two systems have the same solution set.

remember

$$\begin{bmatrix} x & x & x & : & x \\ x & x & x & : & x \\ 0 & 0 & 0 & : & 15 \end{bmatrix}$$

↳ $0 \neq 15$ ∴ the system is inconsistent.

Row reduced echelon form obtained for any matrix is unique and single.

↳ theorem no. 1

Pivot is a no zero no. in the pivot position need to create zeros via row operations.

forward phase
row - reduction algorithm
backward phase

produce zeros above leading entries produces unique row reduced echelon form.

*interchanging rows to select your pivot correctly, computer selects the largest value → phenomena of partial pivoting

Free variables :-

No of equs < no. of variables

or

$$\begin{bmatrix} x & x & x & : & x \\ x & x & x & : & x \\ 0 & 0 & 0 & : & 0 \end{bmatrix} \quad 0 = 0 \text{ form}$$

Parametric descrip of solsets

inconsistent system has empty solution set. even if it has free variables. ∴ sol set will have no parametric representation

e.g of parametric desp.

$$\begin{cases} x_1 = -6x_2 - 3x_4 \\ x_2 \text{ is free} \\ x_3 = 5 + 4x_4 \\ x_4 = \text{free} \\ x_5 = 7 \end{cases}$$

Existence → consistent or non consistent

Uniqueness → free variables or not.

Theorem 2:
Existence & Uniqueness Theorem.

linear system is consistent iff rightmost col of augment matrix does not have a pivot col. ie iff echelon form of augemented matrix has no rows of the form

$[0\ 0\ 0 \ldots : b]$ $b \neq 0$.

once Consistent
(either)

unique sol   infinite sol.

~x~

Column Vector / Vector
matrix with one colum

$W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ $w_1, w_2 \in \mathbb{R}$ real nos.

↓ vector

↓ The set of all vectors with 2 entries is denoted by $\mathbb{R}^2$.

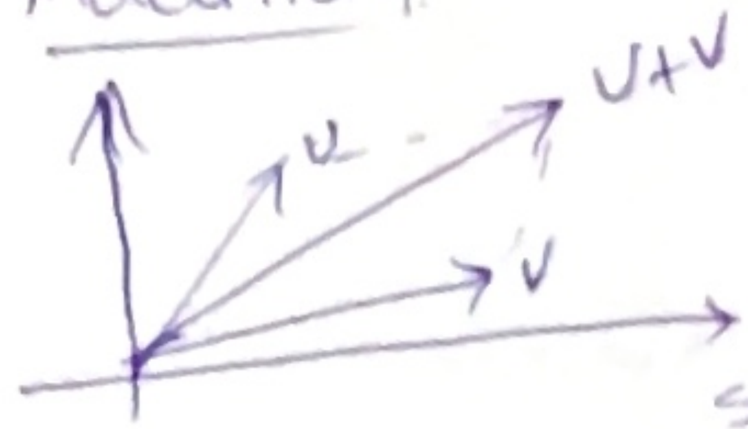
$\mathbb{R}^2$ equal
$\begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ But $\begin{bmatrix} b \\ 7 \end{bmatrix} \neq \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$\mathbb{R}^2$ sum
$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

Scalar Multiple $\mathbb{R}^2$
$cu \Rightarrow 2\begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

scalar

Parallelogram Rule of Addition.



you can thus show this geomatrically using vectors.

length of a line segment
x

$(a,b)\ (c,d)$

$\Rightarrow \sqrt{(a-c)^2 + (b-d)^2}$

Vector in $\mathbb{R}^3$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and similarly $\mathbb{R}^n$ $\begin{bmatrix} a \\ b \\ \vdots \\ z \end{bmatrix}$

* vector with all zero entries is called a zero vector.

Linear combination of vectors
consider vectors $v_1 \ldots v_n$ and scalars $c_1 \ldots c_p$

then $v_1c_1 + v_2c_2 \ldots v_nc_p = y$

this 'y' vector is called the linear combination of the vectors v_n and weights c_p.

Note
vector equation.

$v_1c_1 + v_2c_2 \ldots v_nc_p = y$

is same as (augmented matrix)

$[v_1\ c_2 \ldots c_n : y]$

A linear combi... vectors $v_1 \ldots v_k$ ... if the solution to the augmented matrix exists.

The set of all the linear combinations possible from $v_1 \ldots v_p$ is called $\text{span}\{v_1 \ldots v_p\}$

if $v_1 \ldots v_p \in \mathbb{R}^n$ then the subset of $\mathbb{R}^n$ is $\text{span}\{v_1 \ldots v_p\}$.

also called as all the vectors that can be written in the form $v_1c_1 + v_2c_2 \ldots v_pc_n$.

Note:
asking whether b lies in $\text{span}\{v_1 \ldots v_p\}$ thus means that does it lie in the subset of linear combinations of $v_1 \ldots v_p$ (i.e it is their LC)

it is same as for solving $[v_1 \ldots v_p : b]$ for a solution.

* span contains every scalar multiple of v_1 and especially zero vector must be in $\text{span}\{v_1 \ldots v_p\}$.

...ubspace of V.

NULL space

NULL space of an $m \times n$ matrix NUL A is the set of all the solutions of the homogeneous equ. $[Ax = 0]$.

* it is also defined as set of all the vectors of $\mathbb{R}^n$ mapped at zero vector of $\mathbb{R}^m$.

Theorem 2

* The NUL space of A is a subspace of $\mathbb{R}^n$, also. solution set of $Ax = 0$ is subspace of $\mathbb{R}^n$.

* ∴ NUL space should satisfy 3 properties of subspace.

—x— —x—

If I ask you to find the spanning set of NUL A you will make parametric vector descrip of $Ax = 0$ as $[A \; \vdots \; 0]$ sol.

Col space.

Set of all the linear combinations of the cols of A.

If $A = [a_1 \cdots a_n]$ then $\text{col} A = \text{span}\{a_1 \cdots a_n\}$.

Theorem 3

Note col A is also a subspace of $\mathbb{R}^m$.

also note:-

The col A is all in $\mathbb{R}^m$ iff the equ $Ax = b$ has a sol for each b in $\mathbb{R}^m$. where $A \to m \times n$ matrix.

Row space

For a matrix A

The set of linear comb of rows is called as the row space of A.

Row A is also a subspace of $\mathbb{R}^n$. where n is the no. of entries of row.

→ this Linear independant NUL A equal no of free variables in the equ $Ax = 0$

Note

$\text{Row} A = \text{Col} A^T$.

* kernel of T => NULL space (Linear T)

Range of T = Col A.

Basis

let H be a subspace of, the set of vectors of B in V is a Basis for H if.

① B is linearly independant set

② the subspace spanned by B coincides with H.

$H = \text{span}\{B\}$

Theorem 5

* it is the minimum possible set to form/span for H.

~x~

if A, B are row equ. and their row spaces are the same then all non zero rows are the Basis in echelon form.

Chapter no. 04

Vector space :-

a nonempty set of objects called vectors such that all the vectors u, v and w in V (vector space) follow the following rules of addition & multiplication.

① $u+v = v+u$

② $(u+v)+w = u+(v+w)$

* ③ There a zero vector such that $V = u+0 = u$

* ④ The sum u & v denoted by $u+v$ lies in V

⑤ $c(u+v) = cu+cv$

⑥ $(c+d)u = cu+du$

$c(du) = (cd)u$.

⑦ $1.u = u$

* ⑧ The multiplication of u with any const. lies in V

[$\mathbb{R}^n$ where $n \geq 1$ are all vector spaces]

\+ doubly infinite sequence of no.s

\+ Discrete time signals.

\+ Degree n polynomials set is a vector space if

-> contains zero polynomial.

-> sum of two polynomials should have degree $\leq$ org degree.

-> multiplication with scalar should lie within the degree

and $(p^n)+(-p^n) = 0$.

\+ function

All the rules are same and checked within for the degree.

Vector subspace

It is a subset H of a vector V such that it has the following 3 properties

(i) zero vector of V is in H.

(ii) H is closed under addition i.e u & v in H, the sum $u+v$ is in H.

(iii) H is closed under multiplication by scalars. i.e for each u in H and c as scalar the vector cu is in H.

* The set consisting of only zero vector in a vector space V is a subspace of V called as the zero subspace.

\+ Polynomials sub space

\+ Infinite set of supported signals subspace.

Note

$\mathbb{R}^2$ is not a subspace $\mathbb{R}^3$ because $\mathbb{R}^2$ is not even a subset of $\mathbb{R}^3$

$\mathbb{R}^2$ it has 2 entries.

$\mathbb{R}^3$ it has 3 entries.

Theorem no. 1

If $v_1 \dots v_p$ are in vector space V then span $\{v_1 \dots v_p\}$

is an $n \times n$ matrix D such that $AD = I$

A^T is an invertiable matrix.

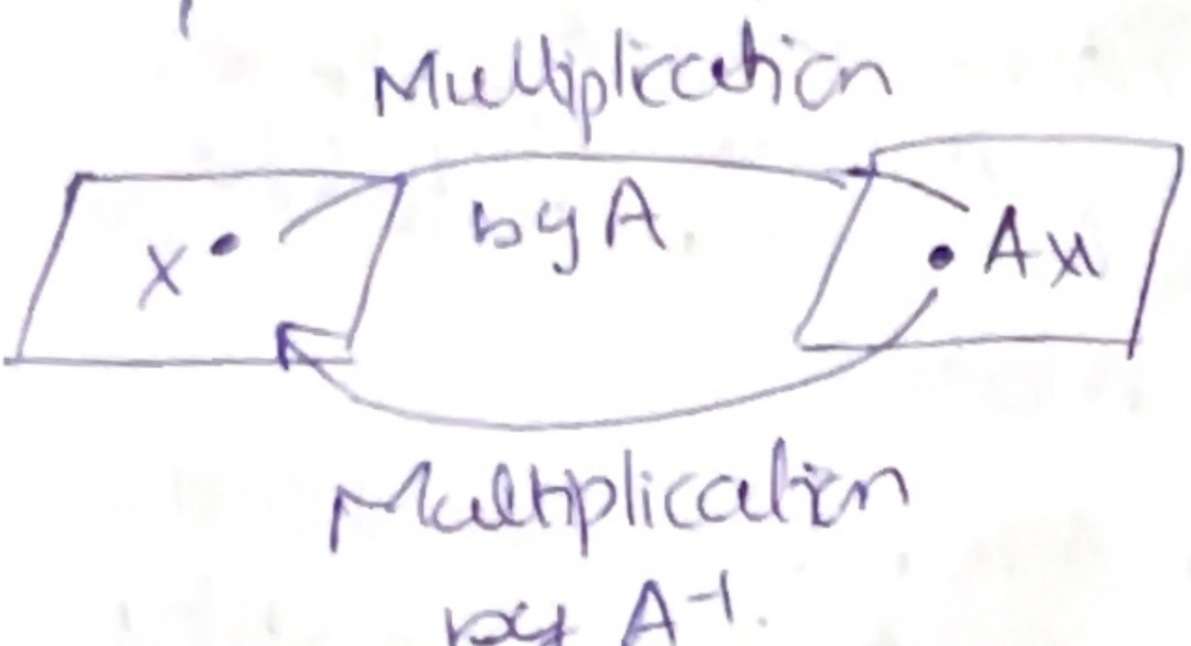
---x---

Note;

if $AB = I$

Both A & B are invertible with.

$A = B^{-1}$ & $A = B^{-1}$.



Linear Transformation T

$T: \mathbb{R}^n \to \mathbb{R}^n$ is said to be invertible $\to$ iff A is invertable

if there exists a funct $S: \mathbb{R}^n \to \mathbb{R}^n$ such that

$T(S(x)) = x \quad \forall x \in \mathbb{R}^n$ ①

$T(S(x)) = x \quad \forall x \in \mathbb{R}^2$ ②

If such as S exists it is unique and linear T given as

$S(x) = A^{-1}(x)$

where $S \Rightarrow T^{-1}$.

Theorem 10

Row-column expansion of AB.

if A is $m \times n$ & B is $n \times p$ matrix.

$$AB = \begin{bmatrix} col_1(A) & col_2 A & \cdots \end{bmatrix} \times \begin{bmatrix} row_1 B \\ row_2 B \\ \vdots \end{bmatrix}$$

$\Rightarrow col_1 A \, row_1 B + col_2 A \, row_2 B \cdots col_n A \, row_n B$

---x---

say your given

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$

then A^{-1}?

$$\begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \Rightarrow \begin{bmatrix} I_p & 0 \\ 0 & I_q \end{bmatrix}$$

~x~

LU Factorization

let A is an $m \times n$ matrix that can be row reduced to echelon form without row interchanges.

$A = LU$.

$L \to$ invertible $= m \times m$ lower triangular matrix with 1's on its diagonal.

& U is an $m \times n$ echelon form of A

$A = LU$.

$Ax = b$.

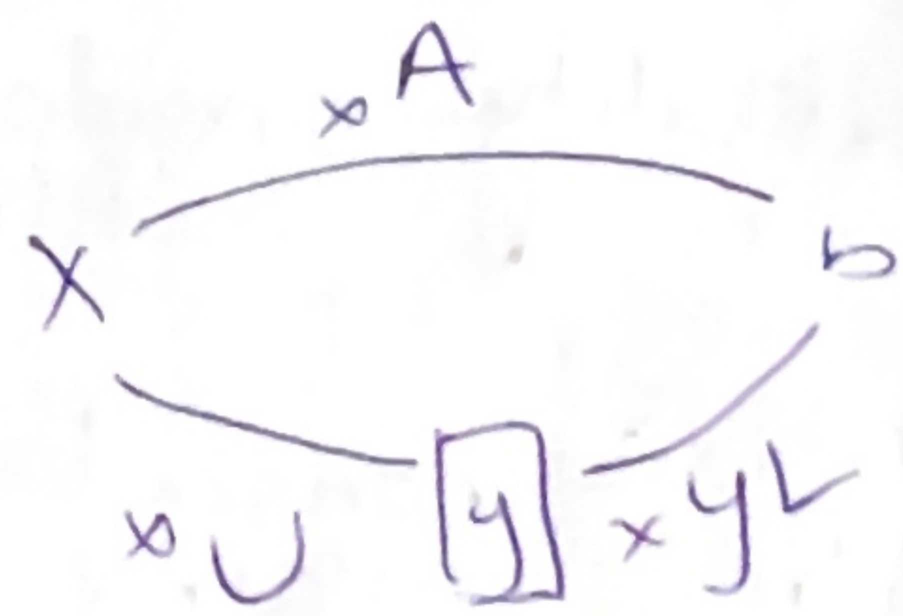
$LUx = b$.

$ULx = b$.

$LUx = b$

$\boxed{Ly = b.}$

$\boxed{Ux = y}$



$L \Rightarrow$ pick 1st pivot divide i with it & put in L matrix, the pick make zeros beneath 1's pivot then pick 2nd pivot repeat same process.

checkerboard pattern is checked through.

$x^T M x = 0$

$x^T x \neq 0$

$A = \begin{bmatrix} 0 & 01 \\ 01 & 0 \end{bmatrix}$ is a matrix that if multiplied to a vector

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ it will reverse the x_1 & x_2 coordinates as.

$Ax = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$.

~x~

A nxn matrix A is said to be <u>invertible</u> if there exist a matrix C, such that

$AC = I$

or $CA = I$

where $C \in A^{-1}$.

not invertibility → singular matrix → $\boxed{\text{Det } A = 0}$

invertible matrix → nonsingular matrix → $\boxed{\text{Det } A \neq 0}$

<u>Theorem of 4</u> if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Det $A \neq 0$ (nonsingular and Matrix is invertible

$A^{-1} = |A|\, A^{adj}$

$\text{Adj } A \Rightarrow \begin{bmatrix} d & -d \\ -c & a \end{bmatrix}$

<u>Theorem no. 5</u>

If A is an invertible nxn matrix the for each b in $\mathbb{R}^n$ the equation $Ax = b$ has the unique solution

$x = A^{-1} b$.

<u>Theorem no. 6</u>

1. if A is invertible then A^{-1} is also invertible.

2. if A & B are invertible then AB is inveritble

$(AB)^{-1} = B^{-1} A^{-1}$.

3. if A is invertible then A^T is also invertible.

$(A^T)^{-1} = (A^{-1})^T$

<u>Elementary Matrices</u>

ones obtained by performing a single elementary row operation.

on identity matrix

<u>Theorem no 7</u>

An nxn matrix A is invertiable iff A is rowequivalent to I_n, and thus any sequence of elementary rowops. that reduces A to I_n also transform I_n to A^{-1}.

Theorem no. 8:

$A \Rightarrow n \times n$ matrix.

a. A is an inveritable matrix.

b. A is rowequivalent to nxn identity matrix.

c. A has n pivots

d. A has only, $Ax = 0$ only trivial solution of equ.

e. Columns of A are linearly independant.

f. The linear transformation $x \mapsto Ax$ is 1:1

g: The equ. $Ax = b$ has atleast 1 solution of b in $\mathbb{R}^n$.

h: The columns of A span $\mathbb{R}^n$.

i. $x \mapsto Ax$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$.

j. C exisit such that $C \times A = I$

...geneous
...ution set:

...inear equs with
solution as:

$Ax = 0$.

where 0 is the zerovector of $\mathbb{R}^m$

Such a system always has one solution i.e $x = 0$ (zerovector) called the trivial solution.

Does $Ax=0$ has a non trivial solution? i.e a non zerovector x satisfying $Ax=0$

it has non trivial iff the equation has atleast 1 free variable.

~x~

Consider that you have a solution of $Ax = 0$ as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_2 + 2x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x_2 \\ x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 2x_3 \\ 0 \\ x_3 \end{bmatrix}$$

$$\Rightarrow x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} x_3 \\ 0 \\ 1 \end{bmatrix}$$

this is the general representation of our solution in parametric vector form

generally as

$x = su + tv$.

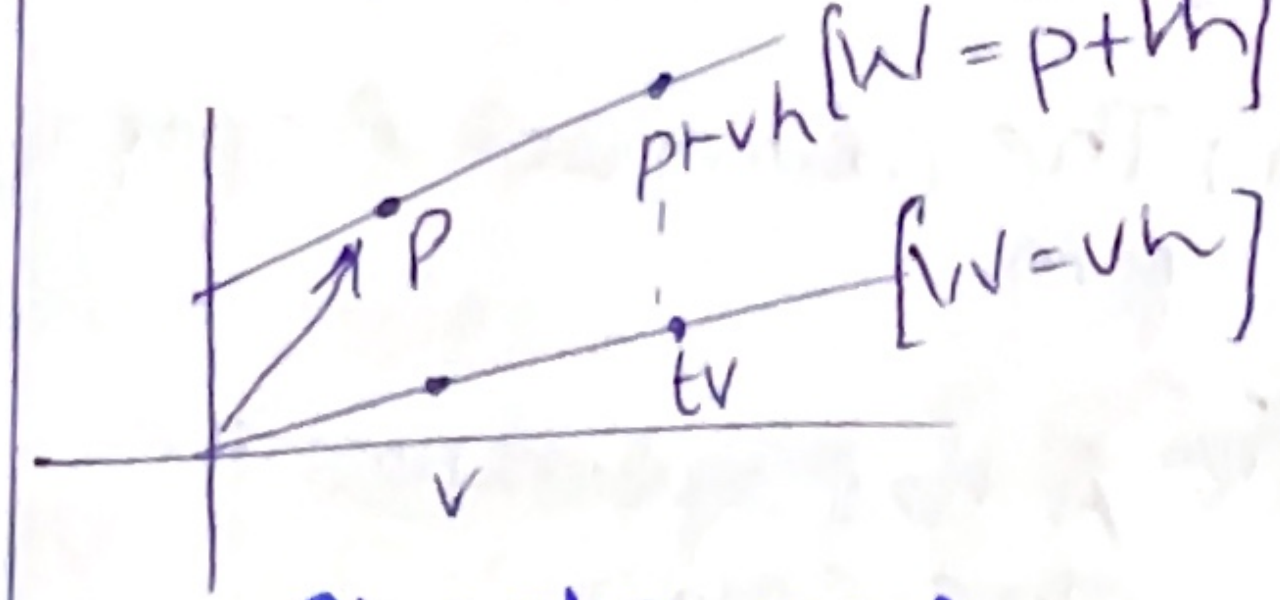
if we had only one variable of x say $x = x_3 t$.

generally $x = vt$.

Similarly
Solution of a Nonhomogeneous equation/systems i.e
$Ax = b$.

is $x = p + tv$
or $\boxed{w = p + v_h}$
where 'v_h' is the solution of $Ax=0$
and the same give b of $Ax = b$ is added through h p.

This represented as:



Chapter no.2

A diagnol matrix is an $m \times n$ matrix whose non diagonal entries are zero.

$A(Bx) = (AB)x$.

If A is an $m \times n$ matrix with columns $b_1 \dots b_p$ & B matrix of $m \times n$ with then product AB

is

$$A[b_1 \dots b_p] \Rightarrow [Ab_1 \; Ab_2 \; \dots \; Ab_p]$$

AB matrix.

And the each columns of AB is a linear combination of the columns of A using weights from the corresponding columns of B.

$$\boxed{row_i(AB) = row_i(A)B}$$

Theorem 2
→ laws on matrices

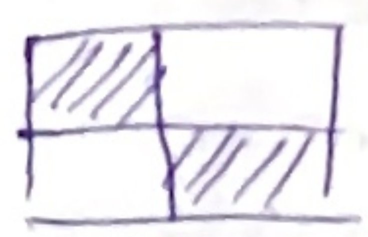
Theorem 3.

$(A^T)^T = A$

$(A+B)^T = A^T + B^T$.

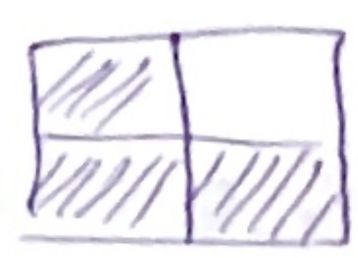
$(rA)^T$ where r is scalar
$\Rightarrow r(A)^T$.

$(AB)^T = B^T A^T$. ★



$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

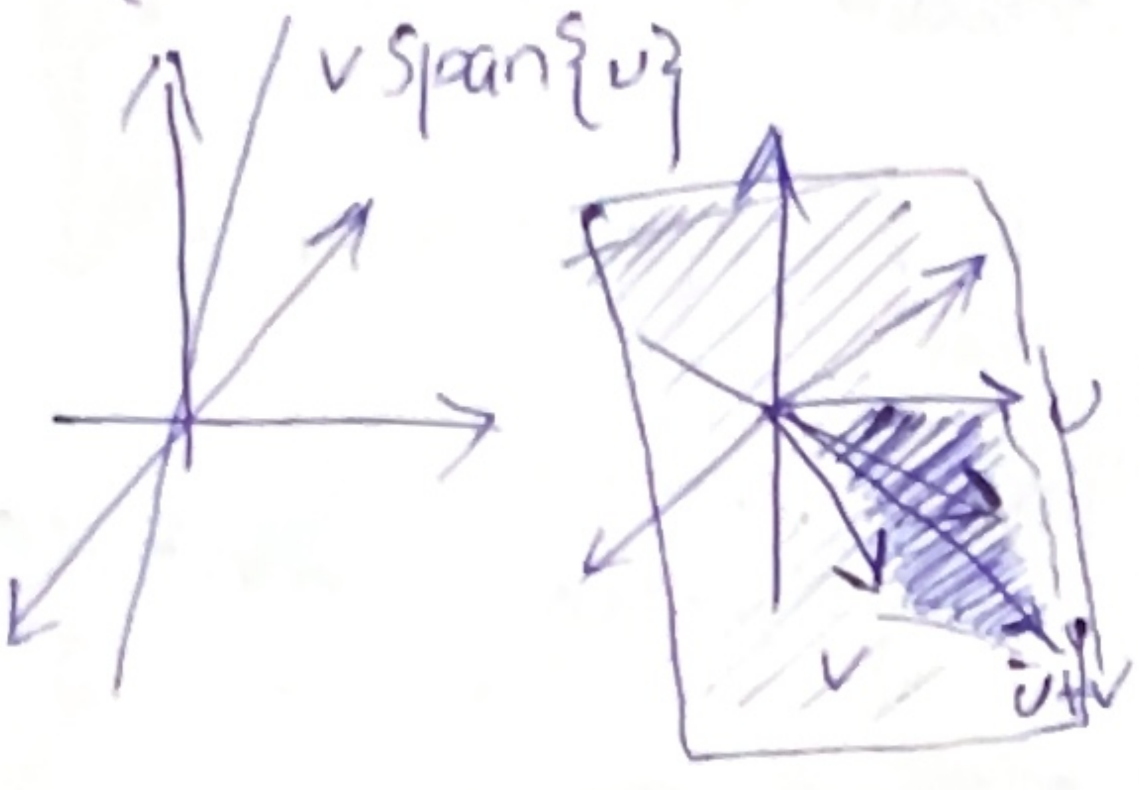


$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

let $u, v \in \mathbb{R}^3$
span{u} vs span{u,v}.
↓
means all the LC of the vector u.
And span{u,v} means all the LC of the vector u & v. where u & v are not multiples of each othe.

Span{u,v} also means a plane in $\mathbb{R}^3$ with u, v and 0.
↳ meaning it contains lie throug u & 0 and v & 0.



LC Applications.

{no. of units} {cost per unit} = {total cost}
$\underbrace{\qquad}_{x}$

Theorem 3

Ax = b where A => m×n matrix.

has the same solution as the vector equ.

$x_1 a_1 + x_2 a_2 + \dots x_n a_n = b$. and also same as linear equs.
$[a_1\ a_2 \dots a_2 : b]$.

and ...

* so we equalizied three different systems as one!

∴ Now deduce.

$Ax = b$ has a sol if b is the linear combination of cols of A (concept derived from vectors).

∴ is $Ax = b$ consistent mean does A lie in the span{cols of A}. where cols of $A \in \mathbb{R}^m$.
If yes, then b in $\mathbb{R}^m$ is a linear combination of cols of A in $\mathbb{R}^m$ / cols of A than span $\mathbb{R}^m$. or generate.

Theorem 4: [A = m×n matrix all below are true for all]

(i) for b in $\mathbb{R}^m$, the equ $Ax = b$ has a sol.

(ii) Each b in $\mathbb{R}^m$ is the linear combination of cols of A.

(iii) The columns of A span $\mathbb{R}^m$.

(iv) A has a pivot position in every row. ↓

* ∴ The Theorem 4 is about the coefficient matrix and not the augmented matrix. because an augmented row [A b] having pivot positioned every row. may or may not be consistent

row operation ...

Theorem no.

$A(u+v) = Au + Av$
$A(cv) = cAu$

where A = m×n matrix

and c is a const or scalar while u & v are the vectors.

Consider for (a)

$$[a_1\ a_2\ a_3]\begin{bmatrix} u_1 + v_1 \\ v_2 + u_2 \\ v_3 + u_3 \end{bmatrix}$$

$(u_1 + v_1) a_1 +$
$a_2 (u_2 + v_2)$
$a_3 (v_3 + u_3)$.

$\Rightarrow (u_1 a_1 + u_2 a_2 + u_3 a_3) + (v_1 a_1 + v_2 a_2 + a_3 v_3)$.

Consider for b

$$[a_1\ a_2\ a_3]\begin{bmatrix} cu_1 \\ cu_2 \\ cu_3 \end{bmatrix}$$

$c(a_1 u_1) + c(u_2 a_2) + c(u_3 a_3)$.

$\Rightarrow c(u_1 a_1 + u_2 a_2 + u_3 a_3)$

$\Rightarrow c(uA)$.
$\underbrace{\qquad}_{x}$

tion

of

n

if

...ransformation

$\mathbb{R}^2 \to \mathbb{R}^2$

...s called as the shear transformation.

Note

The transformation or mapping of T is said to be linear if

(i) $T(u+v) = T(u)+T(v)$

$\forall$ u, v is in the domain of T.

(ii) $T(cu) = cT(u)$

$\forall$ scalars c and all u in the domain of T.

* $T(0) = 0$.

Matrix of Linear Transformation

Every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is actually a matrix transformation

$x \mapsto Ax$.

* linear transformations are recognized completely by what they do on identity matrix.

thus

Note

$T(x) = x_1 T(e_1) + x_2 T(e_2)$.

Theorem 10 *

let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear Transformation then there exists a unique matrix A such that

$T(x) = Ax$

$\forall$ x is in $\mathbb{R}^n$.

A is an $m \times n$ matrix that can be written as

$A = [T(e_1) \;\dots\; T(e_n)]$

↳ this is called as the standard matrix of linear transformation.

Note

The mapping $T: \mathbb{R}^n \to \mathbb{R}^m$ is said to be onto $\mathbb{R}^m$ if each b in $\mathbb{R}^m$ is the image of atleast one x in $\mathbb{R}^n$.

It is not onto $\mathbb{R}^m$ if there is some b in $\mathbb{R}^m$ for which $T(x)=b$ has no solution.

The mapping $T: \mathbb{R}^n \to \mathbb{R}^m$ is said to be one to one if each b in $\mathbb{R}^m$ is an image of atmost 1 x in $\mathbb{R}^n$.

Theorem 11 *

If $T: \mathbb{R}^n \to \mathbb{R}^m$ is a linear Transformation then it is one to one if the equation $T(x)=0$ has only trivial solution.

Theorem 12 *.

$T: \mathbb{R}^n \to \mathbb{R}^m$ to be a linear transformation and let A be a standard matrix for T then.

a. T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ iff columns of A span $\mathbb{R}^m$.

b. T is one one iff cols of A are linearly independent

1.7 Chapter 1 CTND.

The vector equation is said to be linearly independant if

$x_1v_1 + x_2v_2 + \cdots x_pv_p = 0$.

has only trivial solution.

Similarly the set of vectors $\{v_1 \ldots v_p\}$ with weight $c_1 \ldots c_p$ are linearly independant as.

$c_1v_1 + c_2v_2 + \cdots c_pv_p = 0$.

* if asked if the set of vectors is linearly independant then it must be solved to determine if it has a non trivial solution, if yes, then it is linearly dependant.

this also applys to find if the cols of a matrix are Linearly independant.

Solve $Ax = 0$

$[A : 0]$ should have only trivial solution.

also. (Theorem 7)

set of vectors if not multiples of each other are also Linearly independant

Theorem no. 8 *

If a set of vectors contains more vectors than the no. of entries in each vector the set of vectors is linearly dependant.

$p > n$.

Theorem no. 9 *

If a set of vectors $\{v_1 \ldots v_p\}$ in $\mathbb{R}^n$ contains a zero vector, then the set is linearly dependant.

~ x ~

$Ax = B$

Say the multiplication of A with x transform A into b.

Also consider example.

$$\underset{A}{\begin{bmatrix} x & x & x & x \\ x & x & x & x \end{bmatrix}} \underset{x}{\begin{bmatrix} x \\ x \\ x \\ x \end{bmatrix}} = \underset{b}{\begin{bmatrix} x \\ x \end{bmatrix}}$$

then it means all the vectors x in $\mathbb{R}^4$ transformed into b of $\mathbb{R}^2$

* The concept of correspondence of x to Ax is a function from one set of vectors to another.

This movement from $\mathbb{R}^n$ to say $\mathbb{R}^m$ is assigned through a Transformation T where say

$$\begin{array}{cc} \mathbb{R}^n & \mathbb{R}^m \\ x & T(x) \\ \text{Domain} & \text{Codomain} \end{array}$$

image of x

Represented as

$T : \mathbb{R}^n \rightarrow \mathbb{R}^m$.

set of all $T(x)$ are called Range of x.

~ x ~

...x / 8

...is the Basis of ...the vecter space V such that

$B = \{b_1 \cdots b_n\}$.

then there exists a set of scalars to produce a vector x from V such that.

$X = c_1 b_1 + c_2 b_2 + c_3 b_3 \cdots c_n b_n$

*

These weights to produce x are especially knowas the coordinates of x relative to the Basis B.

$[X]_B = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$

* Basis will change the grid system not the position of coordinates.

let a matrix of Basis

$P_B = [B_1 \; B_2 \; .. B_n]$

then.

$\boxed{X = [P_B][X]_B}$

$[P_B]$ ↓ matrix of Basis.

$[X]_B$ ↓ coordinates

P_B is called as the change of coordinates matrix.

Dimensions of a Vector space

Theorem 10

If a vecter space V has Basis $B = \{b_1 \ldots b_n\}$ then any set in V containing more than n vecters is Linearly dependant.

Theorem 11

If a vecter space V has a basis of n vecters, then every basis of V must contain exactly n vecters.

Dim V vecter space is spanned by this finite set.

The no. of vecters in the basis of V => dim V.

* if a V is not spanned by a finite set, then V is said to be infinite dimensional.

in general

$\dim P^n \Rightarrow n+1.$

$\dim \mathbb{R}^n = n.$

~x~

Theorem 12

$\dim H \leq \dim V.$

Rank :- given by.

Dim of Col space + Nulity of A (Dim of of the Null space)

rank A + nulity of A = no. of columns in A.

rank A ↓ no. of pivot columns of A

nulity of A ↓ no. of nonpivot cols of A ↓ ie free variables.

Invertiable matrix

CNTD. $A = n \times n$ matrix.

$\text{NULL } A = \{0\}$

$\text{nullity} A = 0$

$\text{rank } A = n$.

$\text{col } A = \mathbb{R}^n$

~x~

Change of Basis.

Theorem 15.

$$\boxed{X_C = \underset{C \leftarrow B}{P} [X_B]}$$

$$\underset{C \leftarrow B}{P} = [b_1]_C [b_2]_C \cdots [b_n]_C.$$

Note.

$$(P_{B \leftarrow C})^{-1} \Rightarrow \underset{C \leftarrow B}{P}.$$

Solving

$$[c_1 \; c_2 : b_1 \; b_2]$$

to get.

$$\sim \left[I : \underset{C \leftarrow B}{P}\right]$$

dim row A = dim col A

rank A = dim of col A

X_1 - po

X_2 - mag

X_3

X_4

X_5

X_6 - hydrogen

$KMnO_4$ $\begin{bmatrix} \\ \end{bmatrix}$

H_2O $\begin{bmatrix} \\ \end{bmatrix}$

$\begin{bmatrix} \\ \end{bmatrix}$

and apply row reduc

A flow in = flow out

B