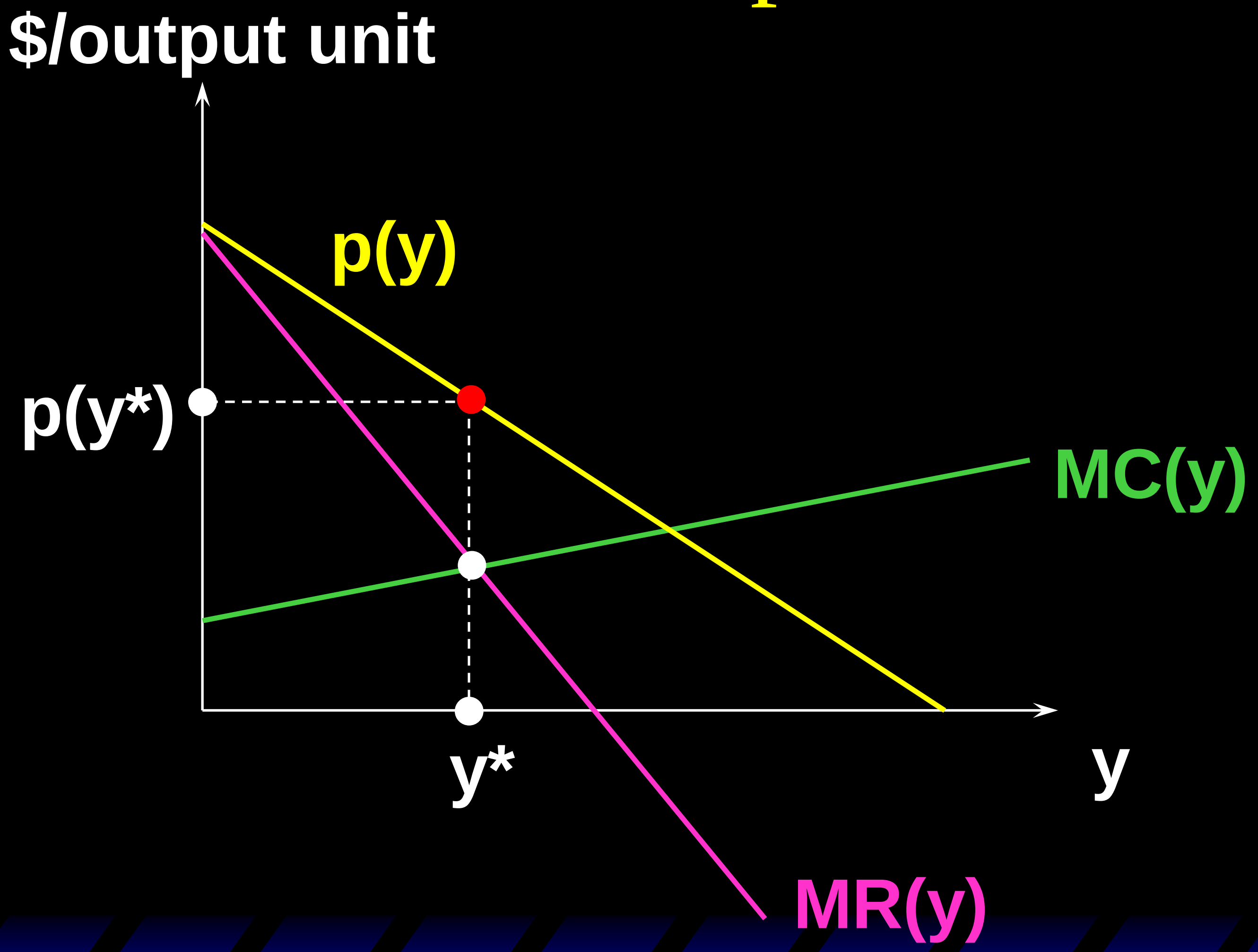
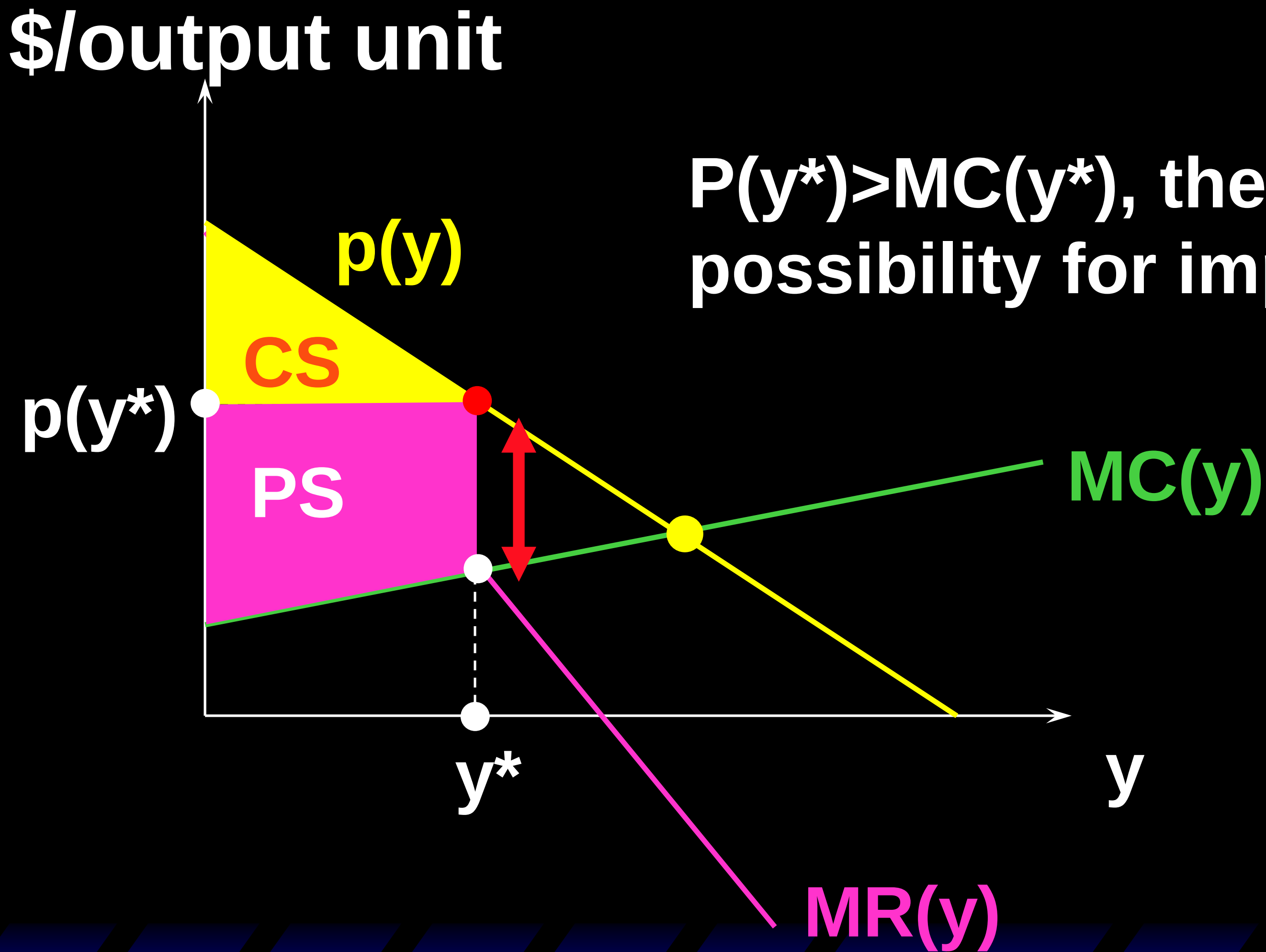
# Lecture 16

## Price Discrimination

# Review: Uniform Pricing of a Monopolist



# The Inefficiency of Monopoly




P(y*)>MC(y*), there is possibility for improvement

# How Should a Monopoly Price?

So far a monopoly has been thought of as a firm which has to sell its product at the **same** price to every customer. This is **uniform pricing**.

There are more clever pricing schemes that allow monopolies to earn higher profits

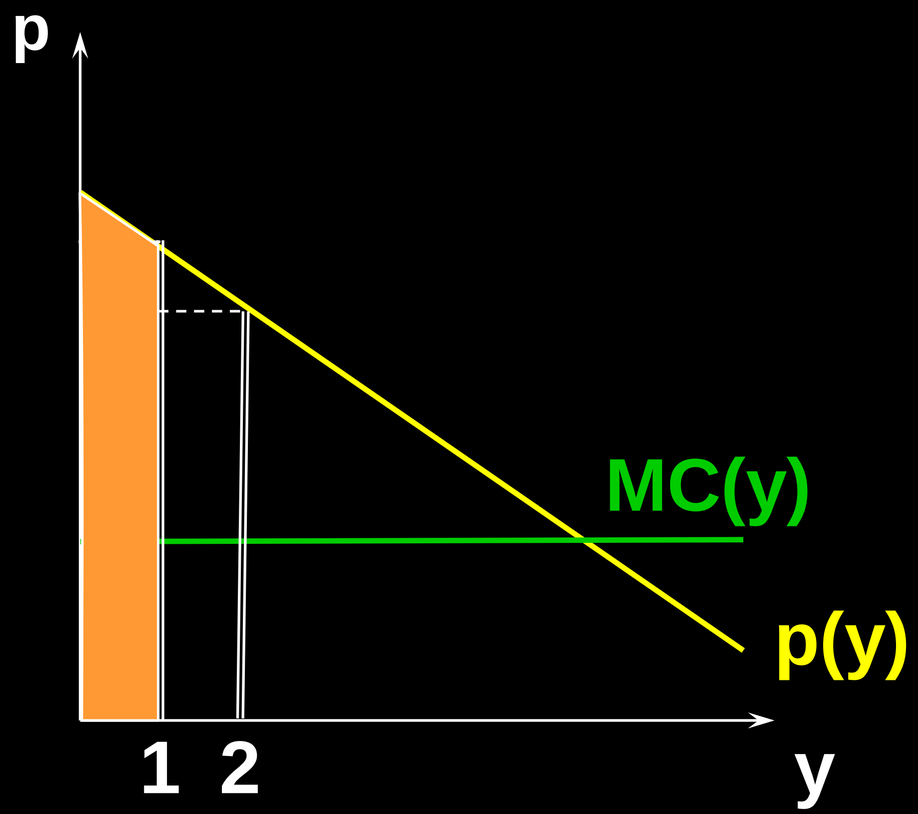
- **price discrimination (价格歧视)**
- **two-part tariff (两部分定价)**
- **bundling (捆绑销售) etc.**

# Types of Price Discrimination

**1st-degree: Each output unit is sold at a different price. Prices may differ across buyers.**

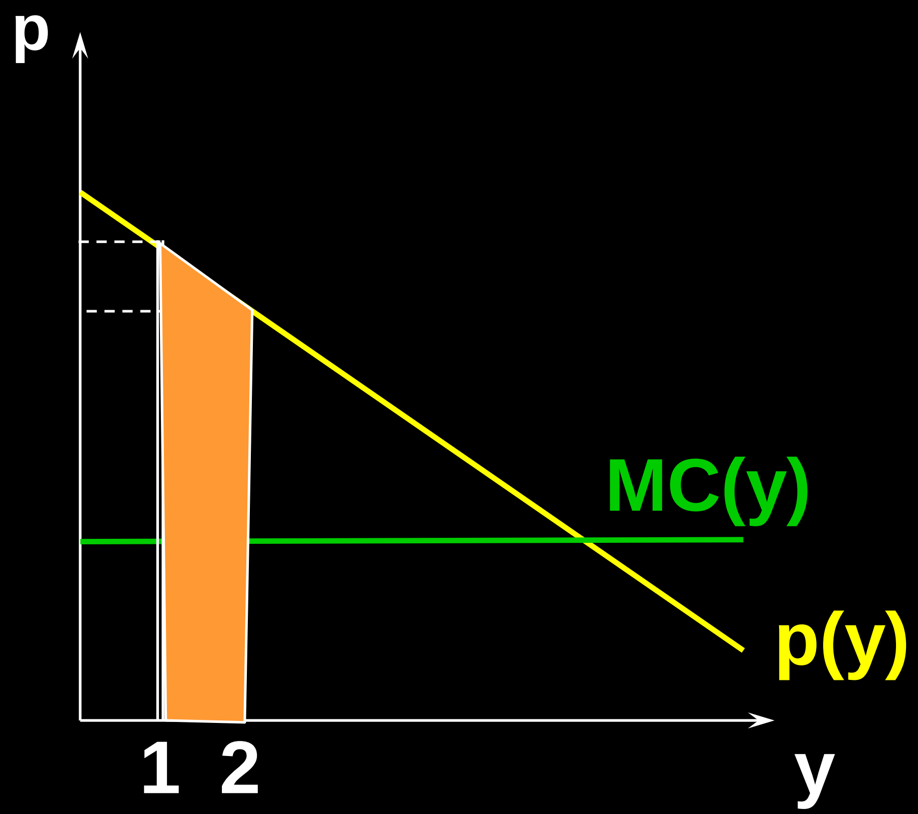
**一级价格歧视：厂商以不同的价格出售不同数量的产品，并且这些价格因人而异（完全价格歧视）。**

# First-degree Price Discrimination



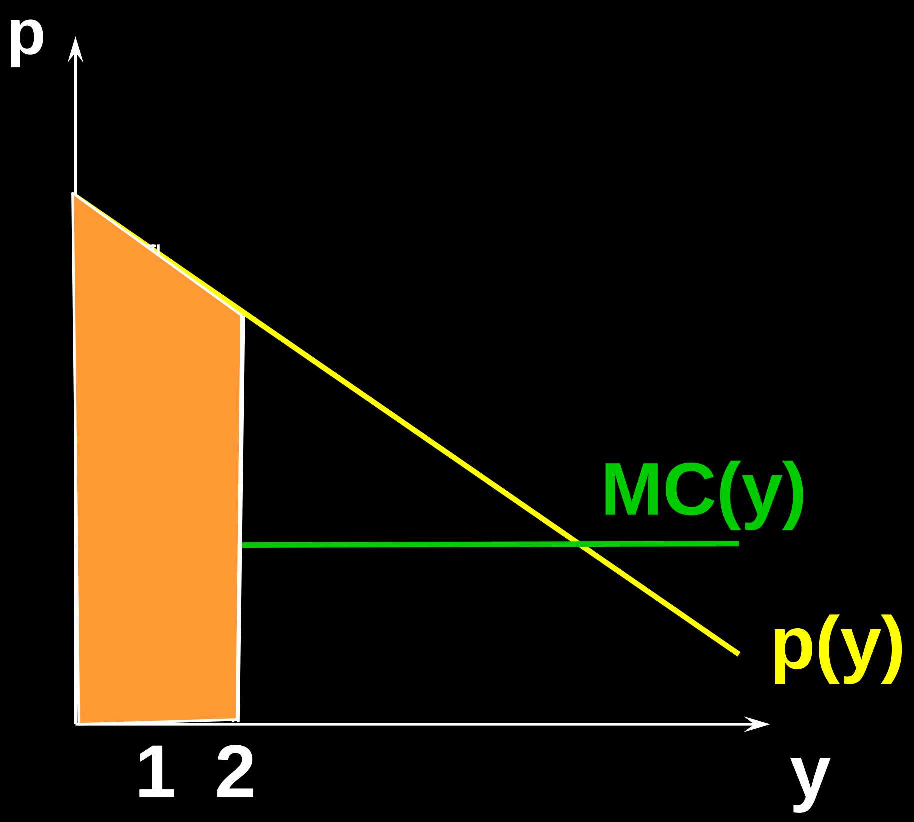

Consumer 1

# First-degree Price Discrimination



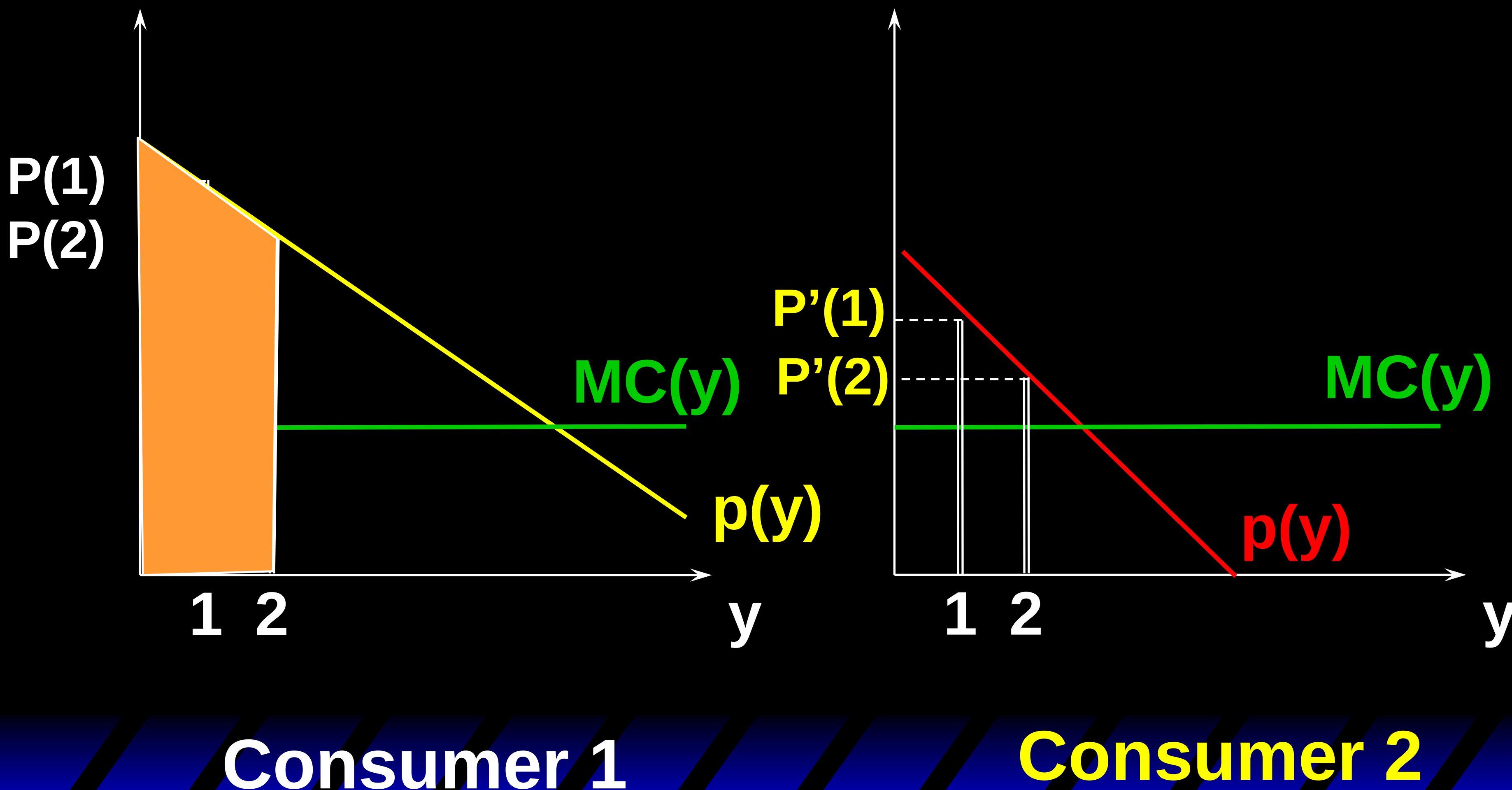

Consumer 1

# First-degree Price Discrimination

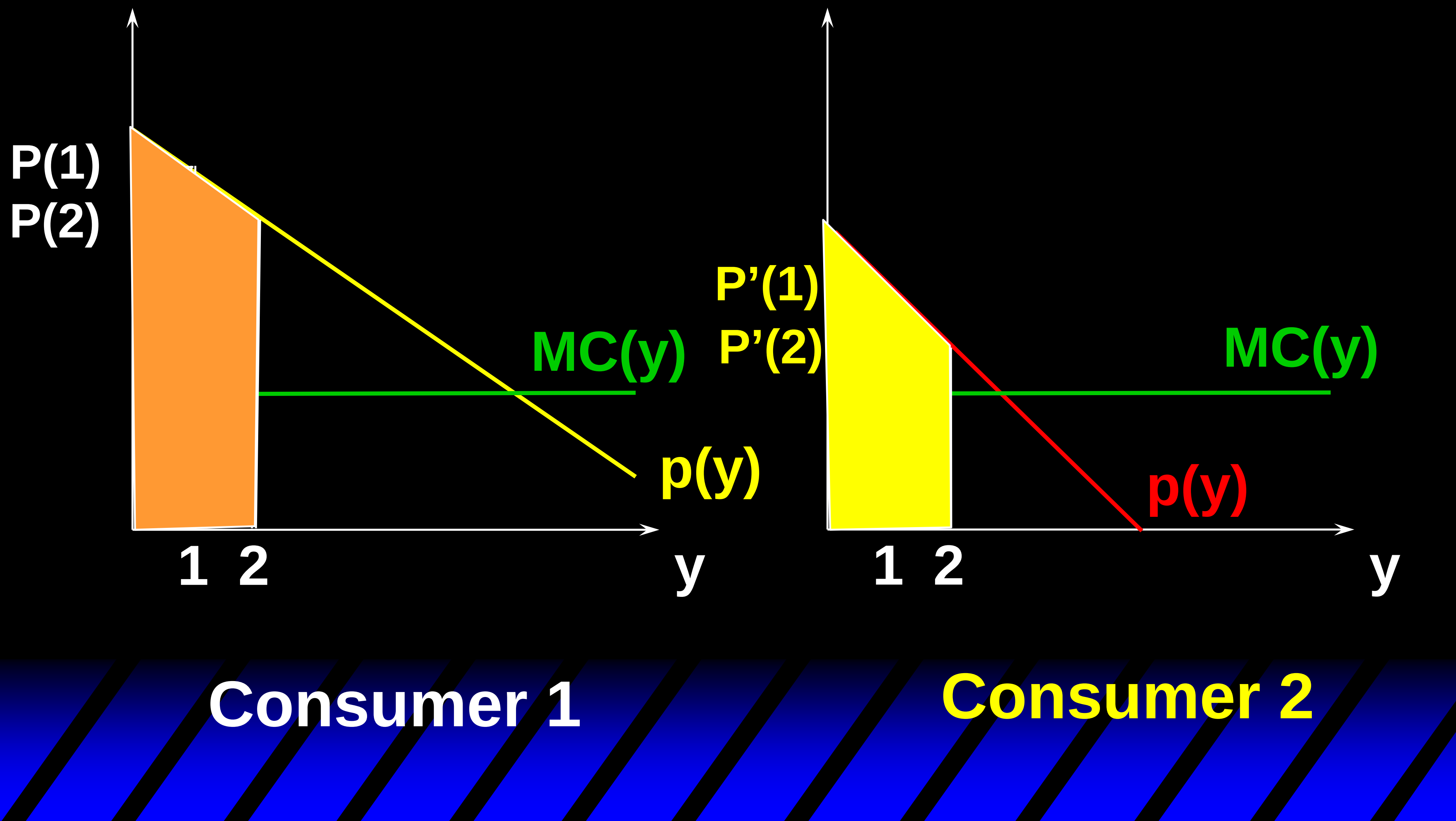



Consumer 1

# First-degree Price Discrimination



# First-degree Price Discrimination



# Types of Price Discrimination

**2nd-degree: The price paid by a buyer can vary with the quantity demanded by the buyer. But all customers face the same price schedule. E.g. bulk-buying discounts.**

二级价格歧视：厂商以不同的价格出售不同数量的产品，但购买相同数量的每个消费者都支付相同价格。

# Types of Price Discrimination

**3rd-degree: Price paid by buyers in a given group is the same for all units purchased. But price may differ across buyer groups.**
**E.g., senior citizen and student discounts vs. no discounts for middle-aged persons.**

三级价格歧视：厂商对不同的人按不同的价格出售产品，但卖给特定个人的每单位产品都以相同的价格出售（不因数量不同而不同）。

# First-degree Price Discrimination

- **Each output unit is sold at a different price. Price may differ across buyers.**
- **It requires that the monopolist can discover the buyer with the highest valuation of its product, the buyer with the next highest valuation, and so on.**

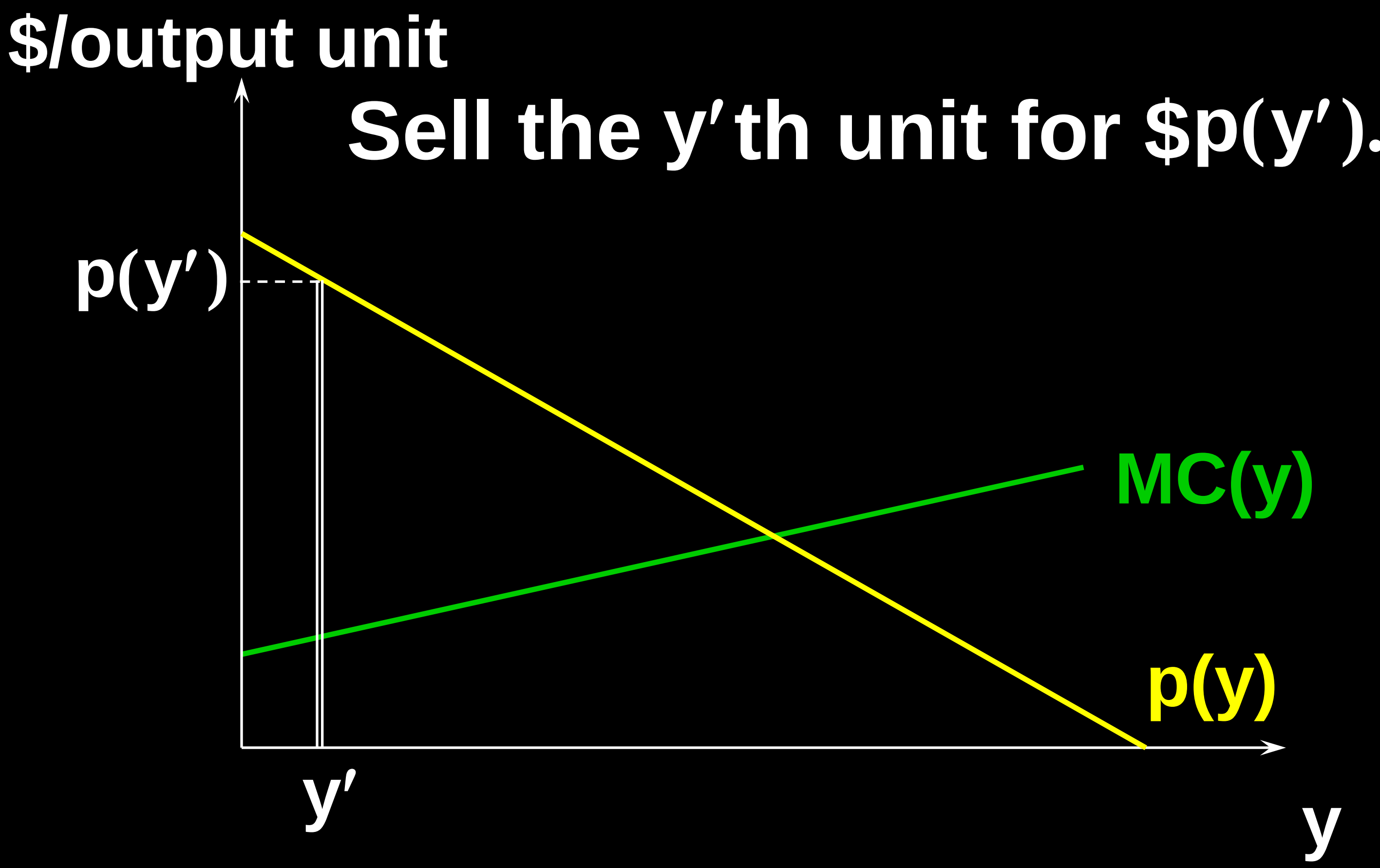
厂商拥有每一个消费者需求函数的完全信息

# Types of Price Discrimination

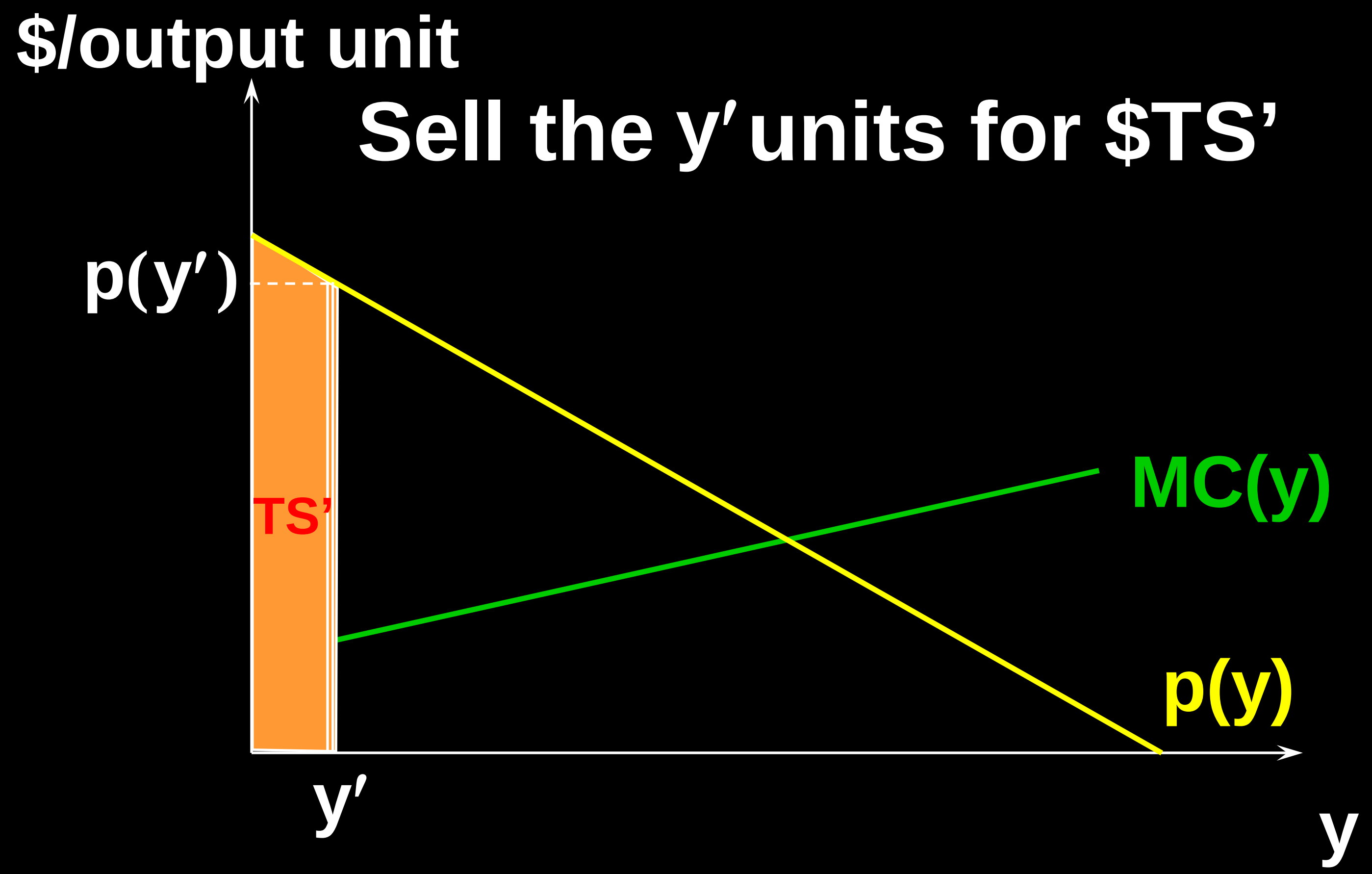
**1st-degree: Each output unit is sold at a different price. Prices may differ across buyers.**

一级价格歧视：厂商为每一个消费者所购买的每一单位产品制定不同的价格。这一价格等于该消费者对该单位产品的最高支付意愿。

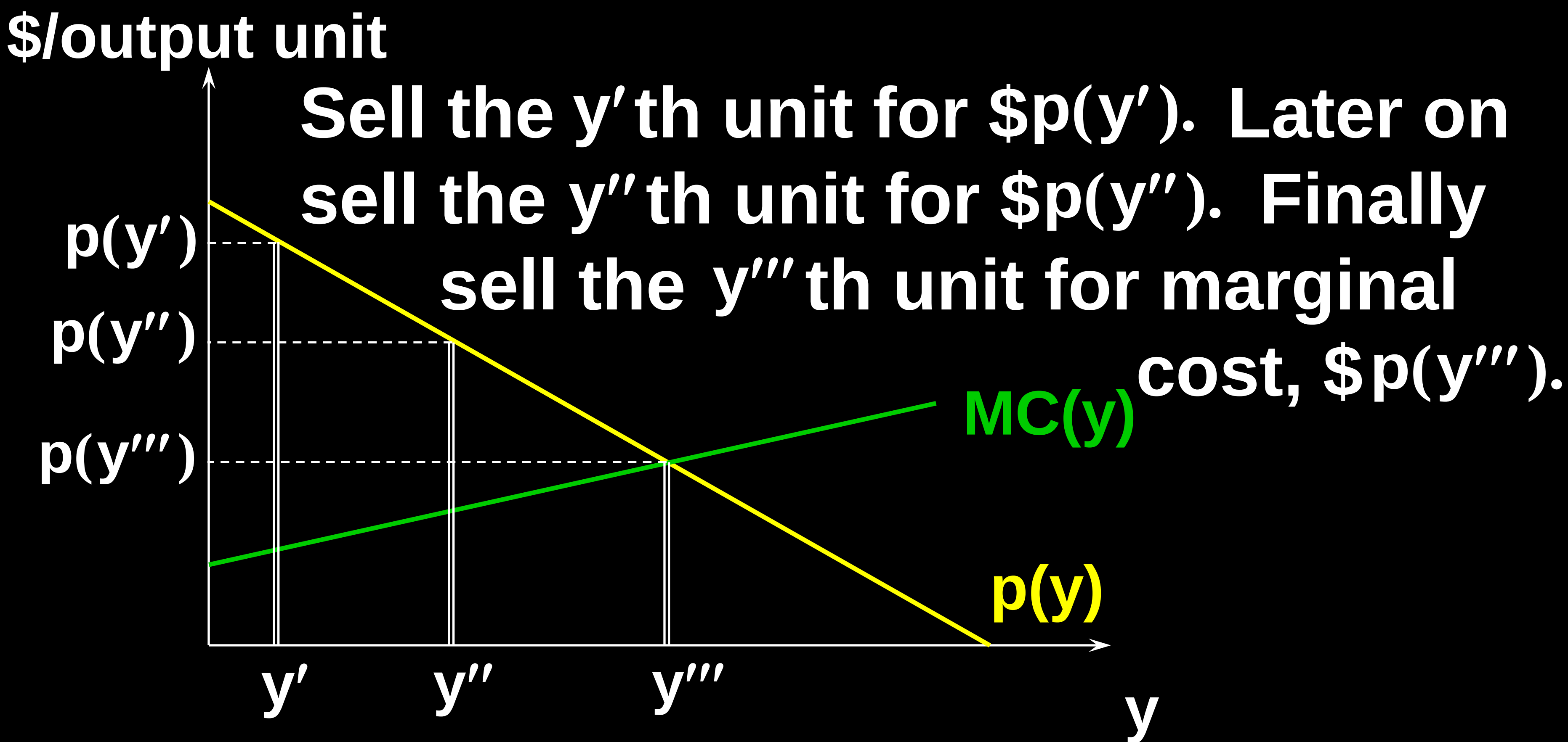
# First-degree Price Discrimination



# First-degree Price Discrimination

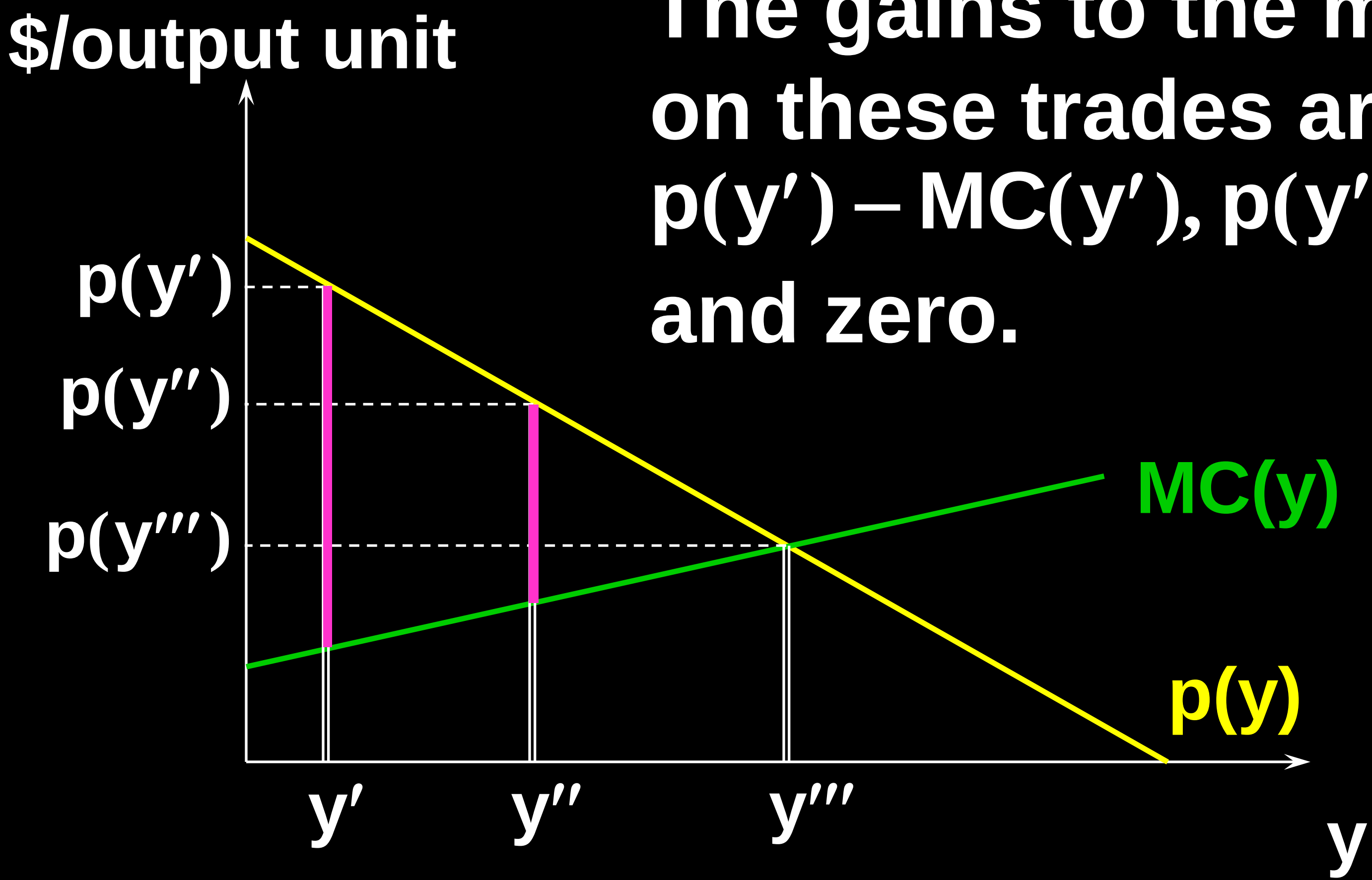


# First-degree Price Discrimination



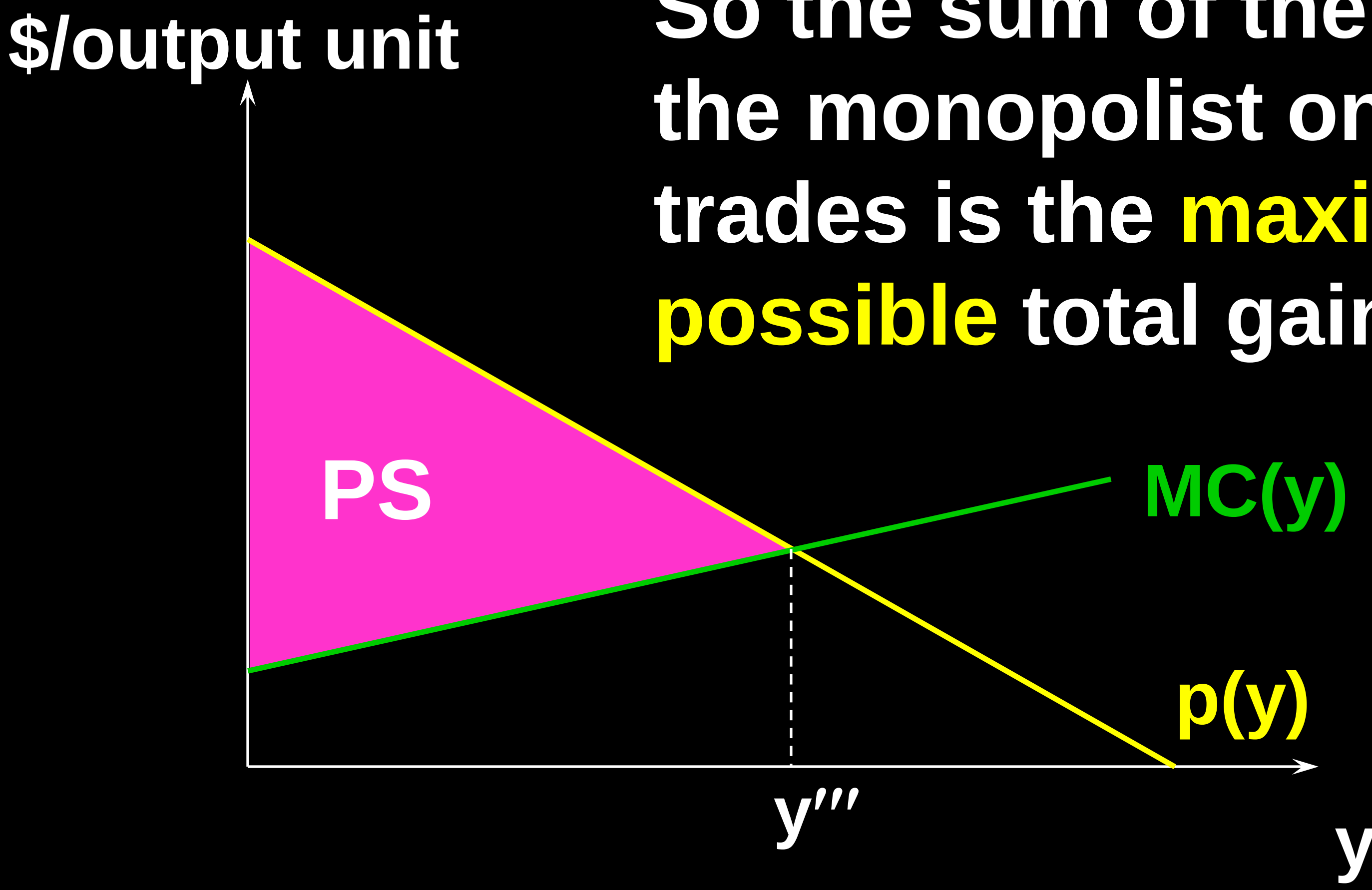
# First-degree Price Discrimination

The gains to the monopolist on these trades are: $p(y') - MC(y')$, $p(y'') - MC(y'')$ and zero.



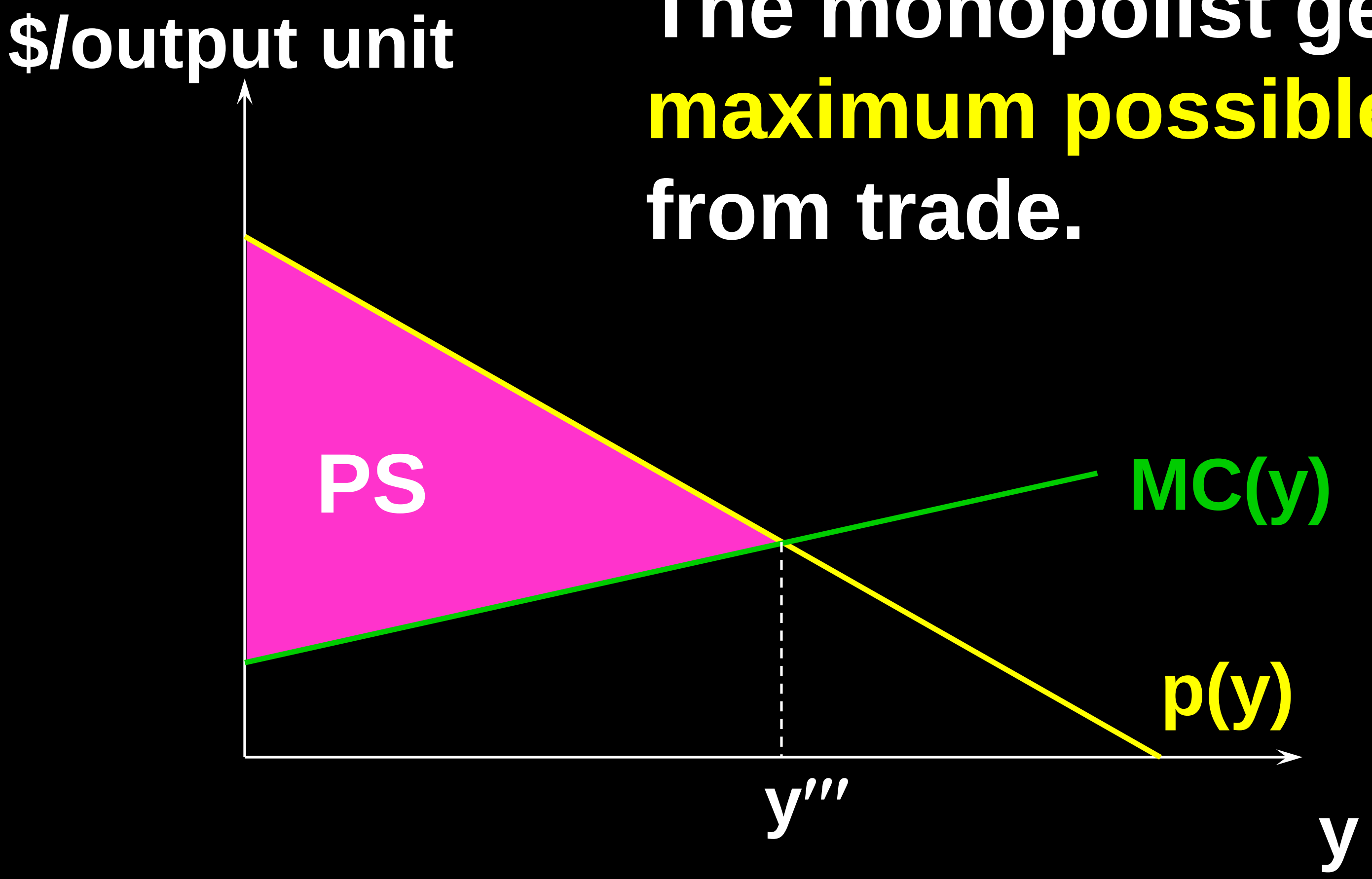
# First-degree Price Discrimination

**So the sum of the gains to the monopolist on all trades is the maximum possible total gains-to-trade.**



# First-degree Price Discrimination

The monopolist gets the **maximum possible gains** from trade.




First-degree price discrimination is **Pareto-efficient（帕累托有效率的）.**

# First-degree Price Discrimination

**First-degree price discrimination gives a monopolist all of the possible gains-to-trade, leaves the buyers with zero surplus, and supplies the efficient amount of output.**

一级价格歧视下，垄断厂商得到市场上的所有剩余，利润最大化等价于社会剩余最大化，最终产出水平是帕累托有效率的

# Second-degree Price Discrimination

- The monopolist may realize there are different types of consumers, but **it can't identify** to which type a particular consumer belongs.

  垄断厂商可能缺乏消费者需求的完全信息

- The monopolist could design **"price-quantity" packages** that appeal to different types of consumers.

  此时可通过"**价格-数量套餐**"的设计来区分和迎合不同类型的消费者

# Second-degree Price Discrimination

Different units are sold at different prices, but everyone has access to the **same price schedule**.

消费者可自由选择厂商提供的“数量-价格”套餐，这些价格不因人而异

e.g. Quantity discount; Buy 2 get 1 free

# Price Discrimination: An Example

- A monopolistic newspaper company offers news articles to readers by email.
- Two buyers: a **student** and a **manager**
  - Student: $q_s = 80 - p_s$
  - Manager: $q_m = 100 - p_m$

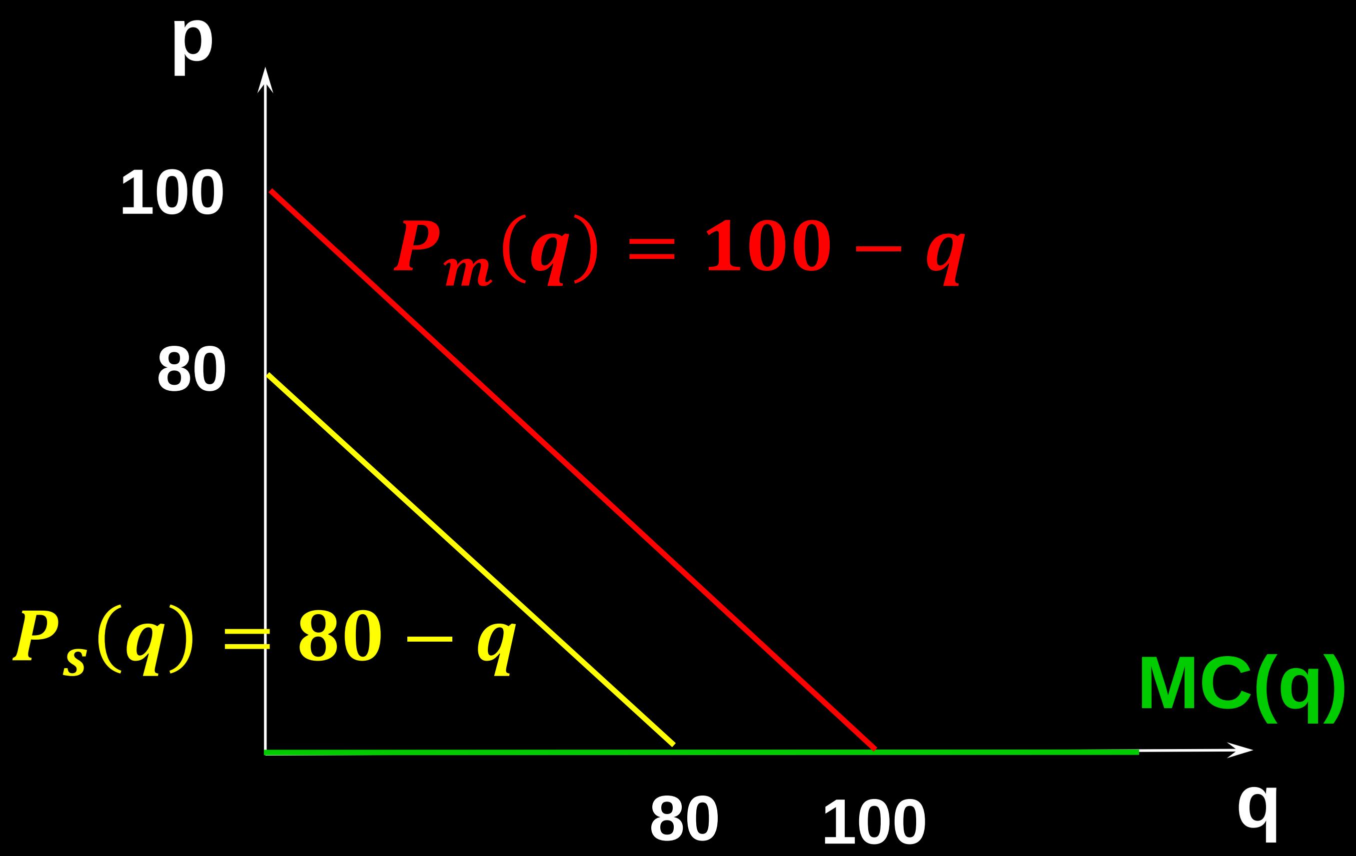
where q is the number of articles that a user requests per year

- MC = **0**

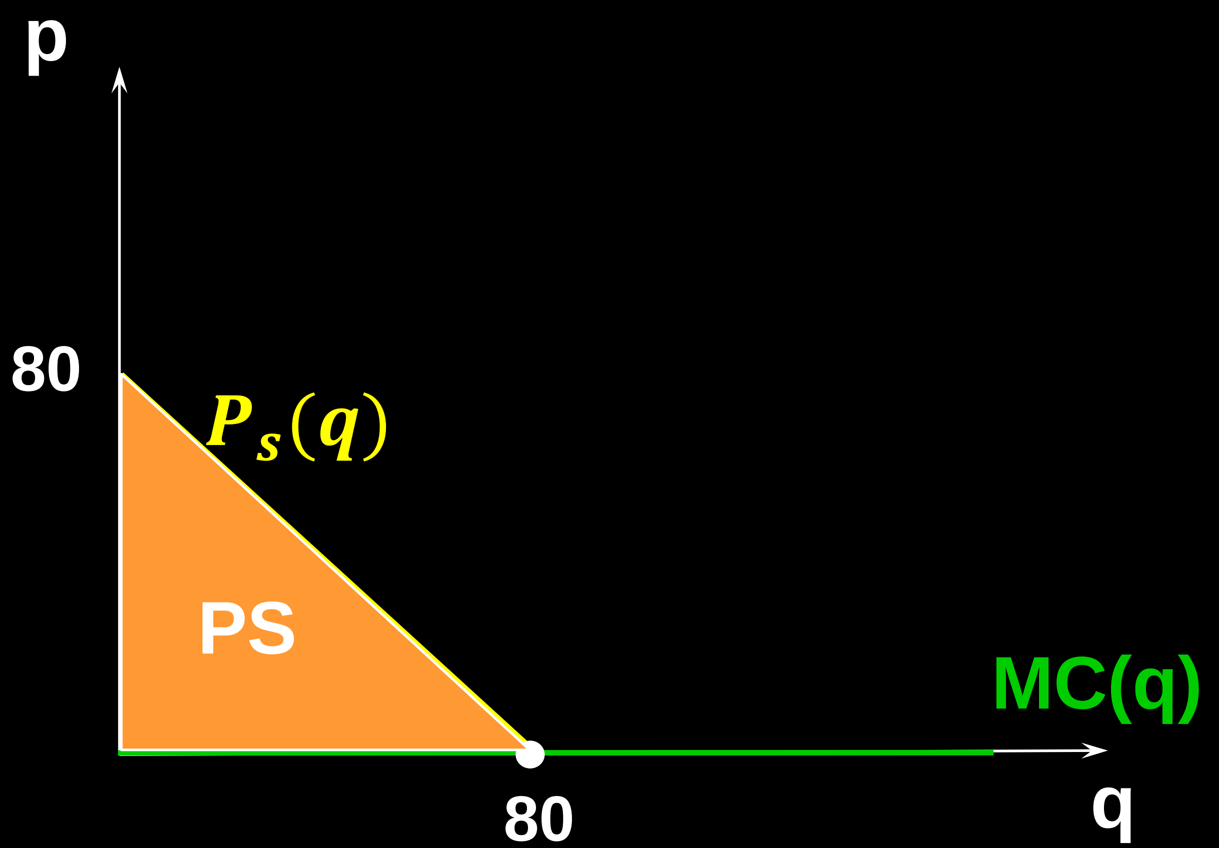
# Price Discrimination: An Example

1st degree: the company **can identify** which user is the student and which is the manager. What are the profit maximizing quantity and price for each buyer?

# Price Discrimination: An Example



# Price Discrimination: An Example

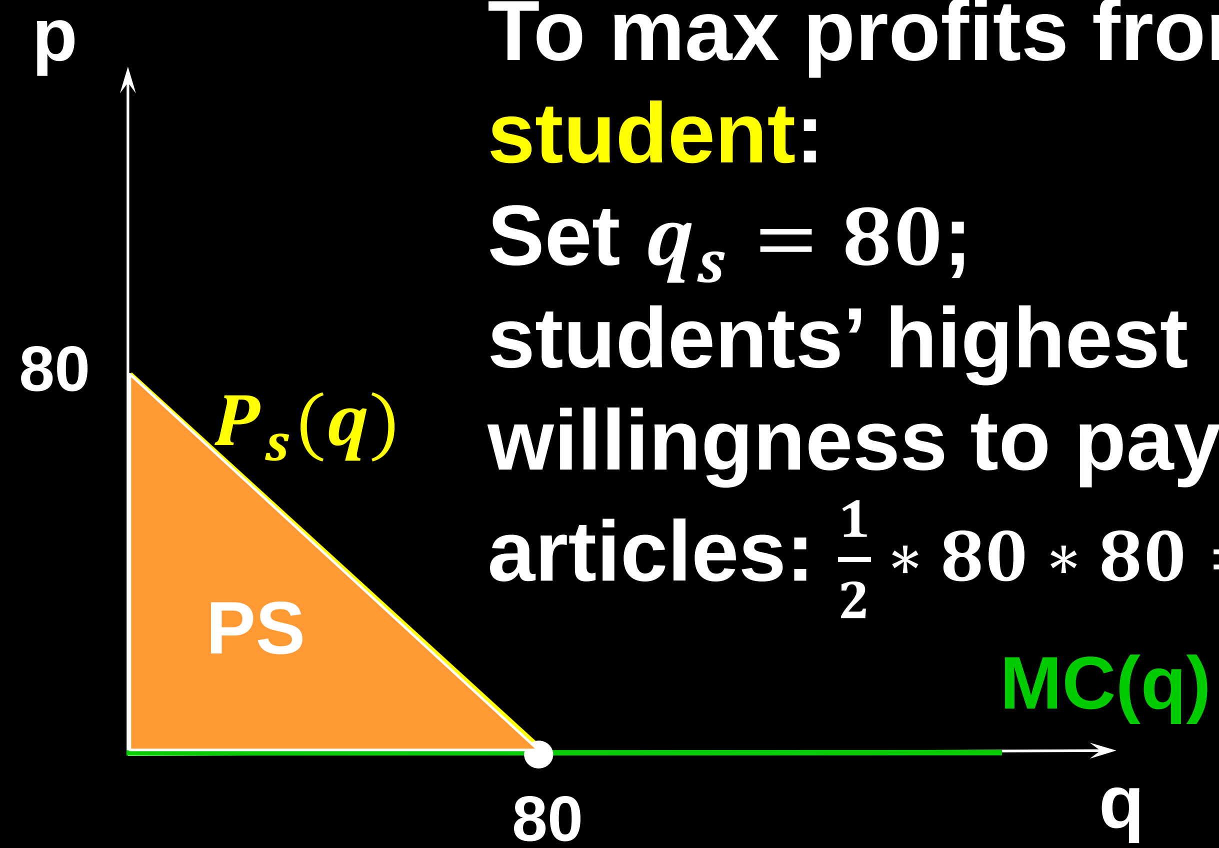


# Price Discrimination: An Example

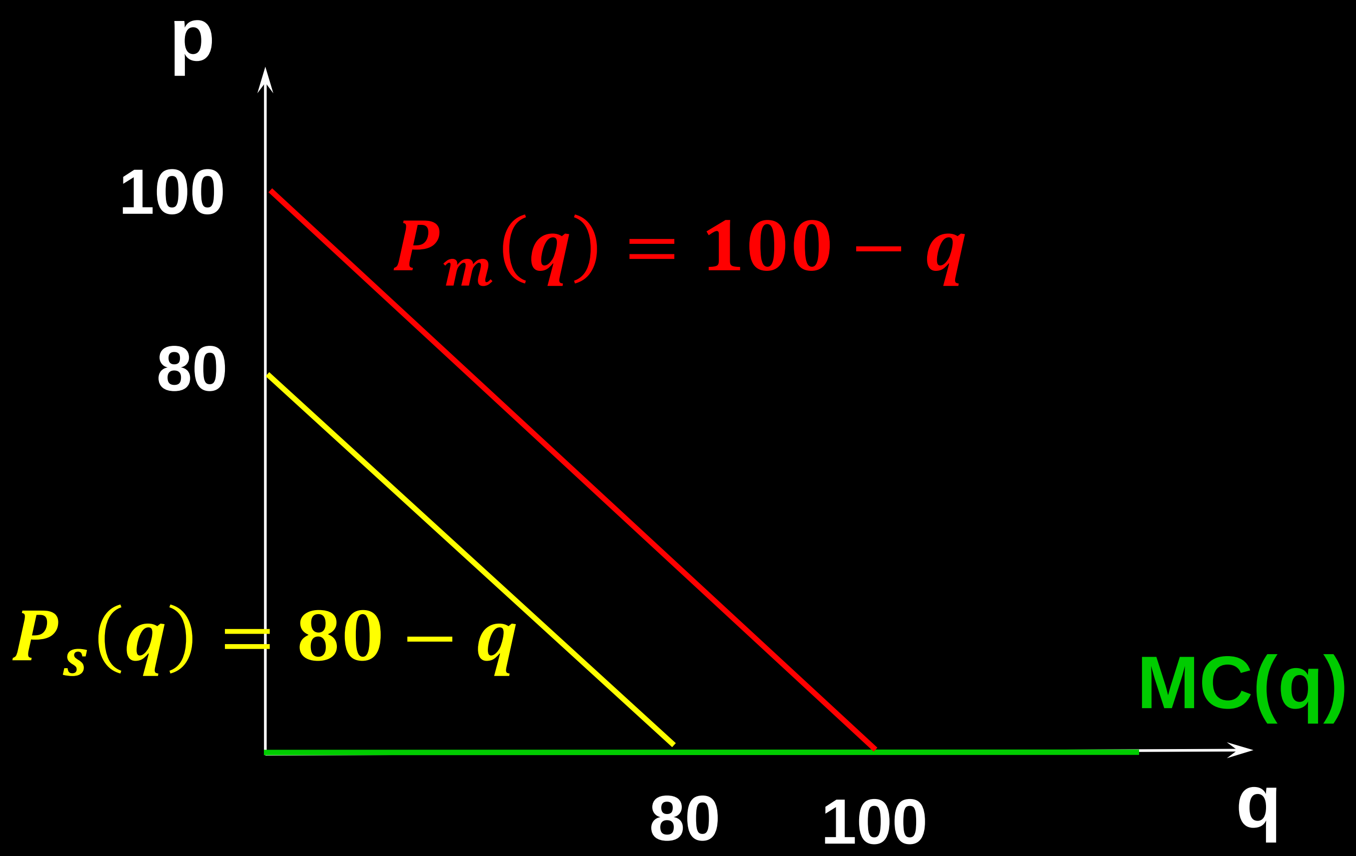
**To max profits from the student:**

**Set $q_s = 80$;**

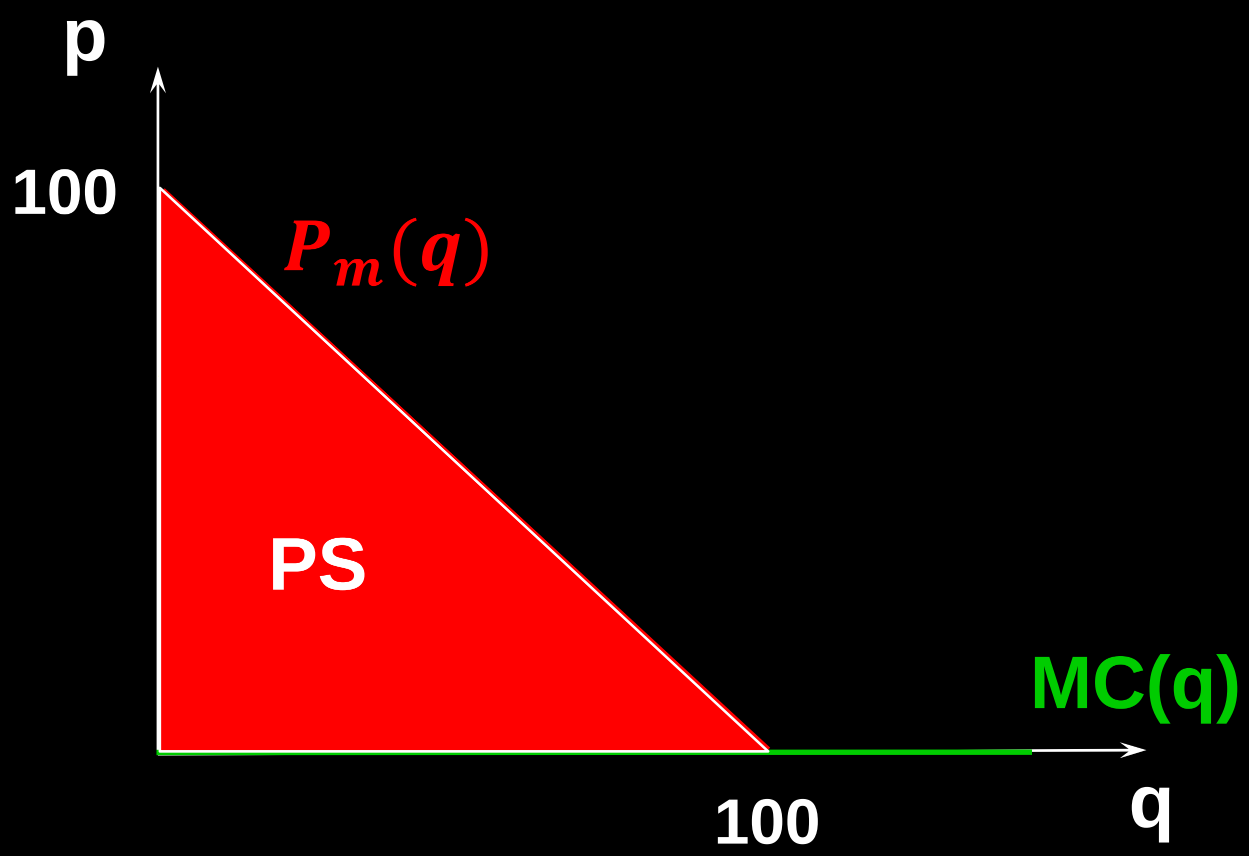
**students' highest willingness to pay for 80-articles: $\frac{1}{2} * 80 * 80 = \$32$**



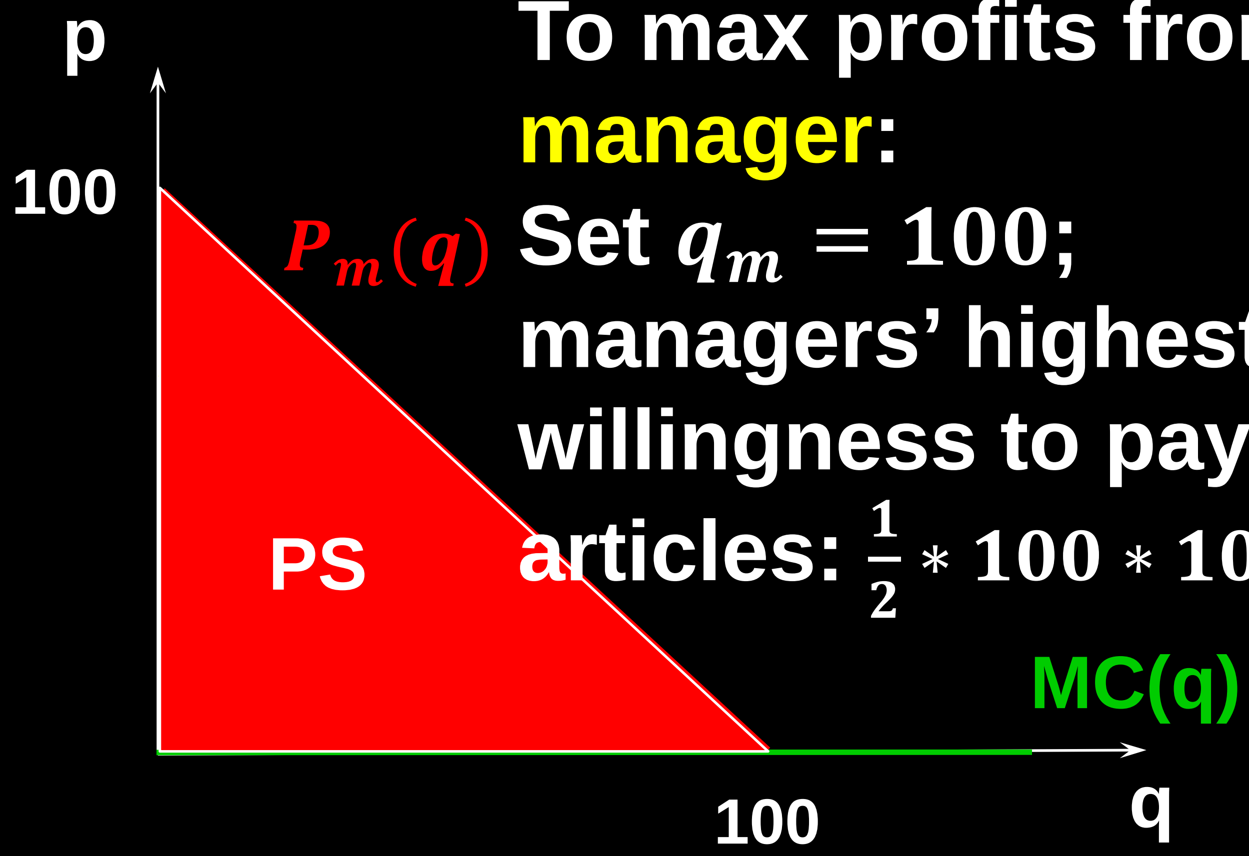
# Price Discrimination: An Example



# Price Discrimination: An Example



# Price Discrimination: An Example




**To max profits from the manager:**
**Set $q_m = 100$; managers' highest willingness to pay for 100 articles: $\frac{1}{2} * 100 * 100 = \$50$**

# Price Discrimination: An Example

1st degree: suppose the company **can identify** which user is the student and which is the manager.

The company sells 80-article to the student for $32 and 100-article to the manager for $50 (**take-it-or-leave-it offers**)

Profit = $82

# Price Discrimination: An Example

2nd degree: the company **cannot identify** which user is the student and which is the manager. It offers **quantity-price packages** to reveal each customer's type. What packages could it offer?

# Price Discrimination: An Example

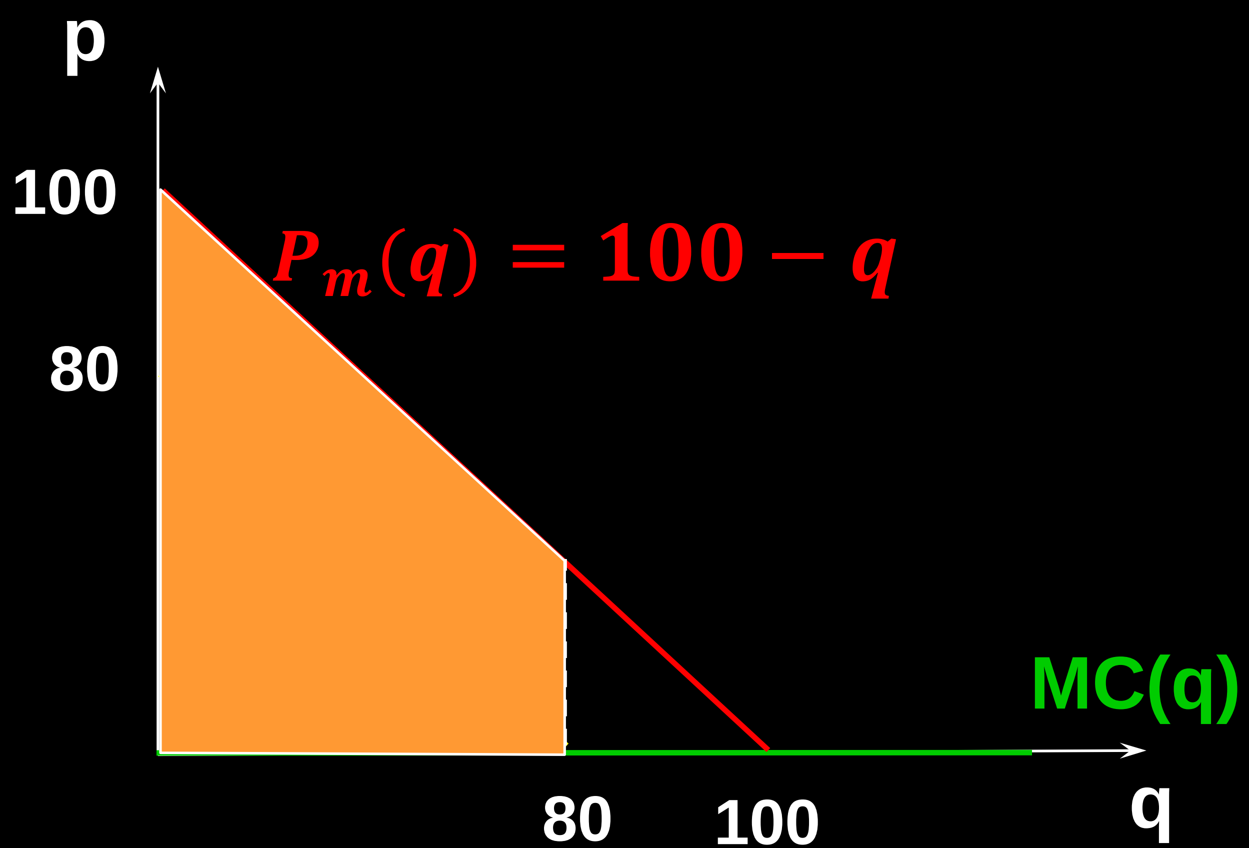
Two packages: one is ($q_1$=80, $p_1$=\$32), the other is ($q_2$=100, $p_2$=\$50).

The student will select ($q_1$=80, $p_1$=\$32) and get a net surplus = \$0

The manager:

- if ($q_2$=100, $p_2$=\$50), net surplus = \$0
- if ($q_1$=80, $p_1$=\$32), net surplus = ?

# Price Discrimination: An Example



# Price Discrimination: An Example

$$P_m = 100 - q$$

**The manager's valuation / willingness to pay for 80-article is**

$$\frac{1}{2} * (20 + 100) * 80 = \$48$$

# Price Discrimination: An Example

- Two packages: one is ($q_1$=80, $p_1$=\$32), the other is ($q_2$=100, $p_2$=\$50).
- The student will select ($q_1$=80, $p_1$=\$32) and gets a net surplus = \$0
- The manager:
  - if ($q_2$=100, $p_2$=\$50), net surplus = \$0
  - if ($q_1$=80, $p_1$=\$32), net surplus = 48 – 32 = \$16

**The manager will select ($q_1$=80, $p_1$=\$32)**

# Price Discrimination: An Example

Two packages: one is ($q_1$=80, $p_1$=\$32), the other is ($q_2$=100, $p_2$).

What is the **maximum possible** value for $p_2$?

The student will select ($q_1$=80, $p_1$=\$32) and gets a net surplus = \$0

The manager:

- if ($q_2$=100, $p_2$), net surplus = \$50 - $p_2$
- if ($q_1$=80, $p_1$=\$32), net surplus = \$16

# Price Discrimination: An Example

**The manager:**

- **if ($q_2$=100, $p_2$), net surplus = \$50 - $p_2$**
- **if ($q_1$=80, $p_1$=\$32), net surplus = \$16**

**In order for the manager to select ($q_2$=100, $p_2$),**

$$\mathbf{50 - p_2 \geq 16}$$

$$\mathbf{p_2 \leq 34}$$

**The highest price the monopolist could charge for 100-article is $\mathbf{p_2 = \$34}$**

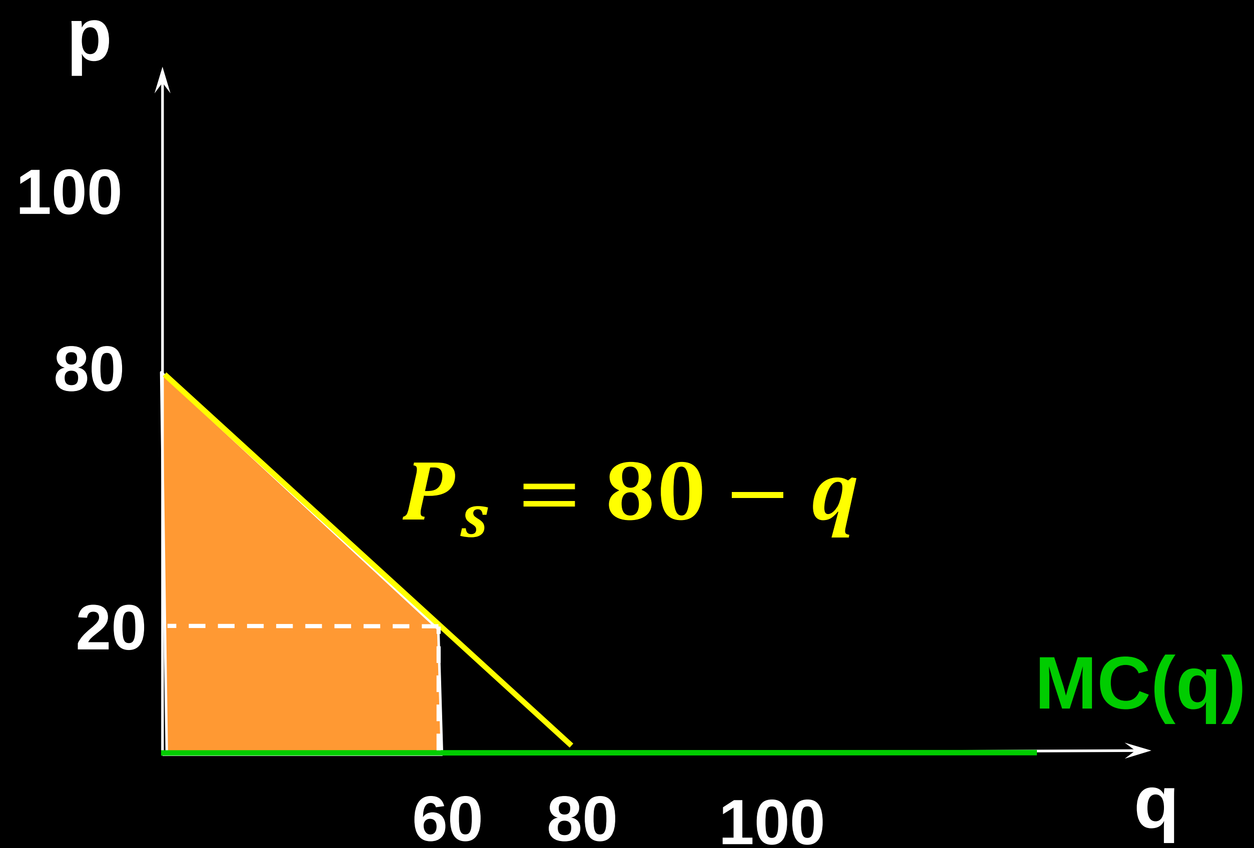
# Price Discrimination: An Example

- Two packages: one is ($q_1$=80, $p_1$=\$32), the other is ($q_2$=100, $p_2$=\$34)
- The student will select ($q_1$=80, $p_1$=\$32) and gets a net surplus = \$0
- The manager will select ($q_2$=100, $p_2$=\$34) and gets a net surplus = \$16
- The company makes a profit = 32 + 34 = \$66

# Price Discrimination: An Example

**Now suppose two packages: one is ($q_1$=60, $p_1$) – for the student, the other is ($q_2$=100, $p_2$) – for the manager. What are the maximum possible values for $p_1$ and $p_2$?**

# Price Discrimination: An Example

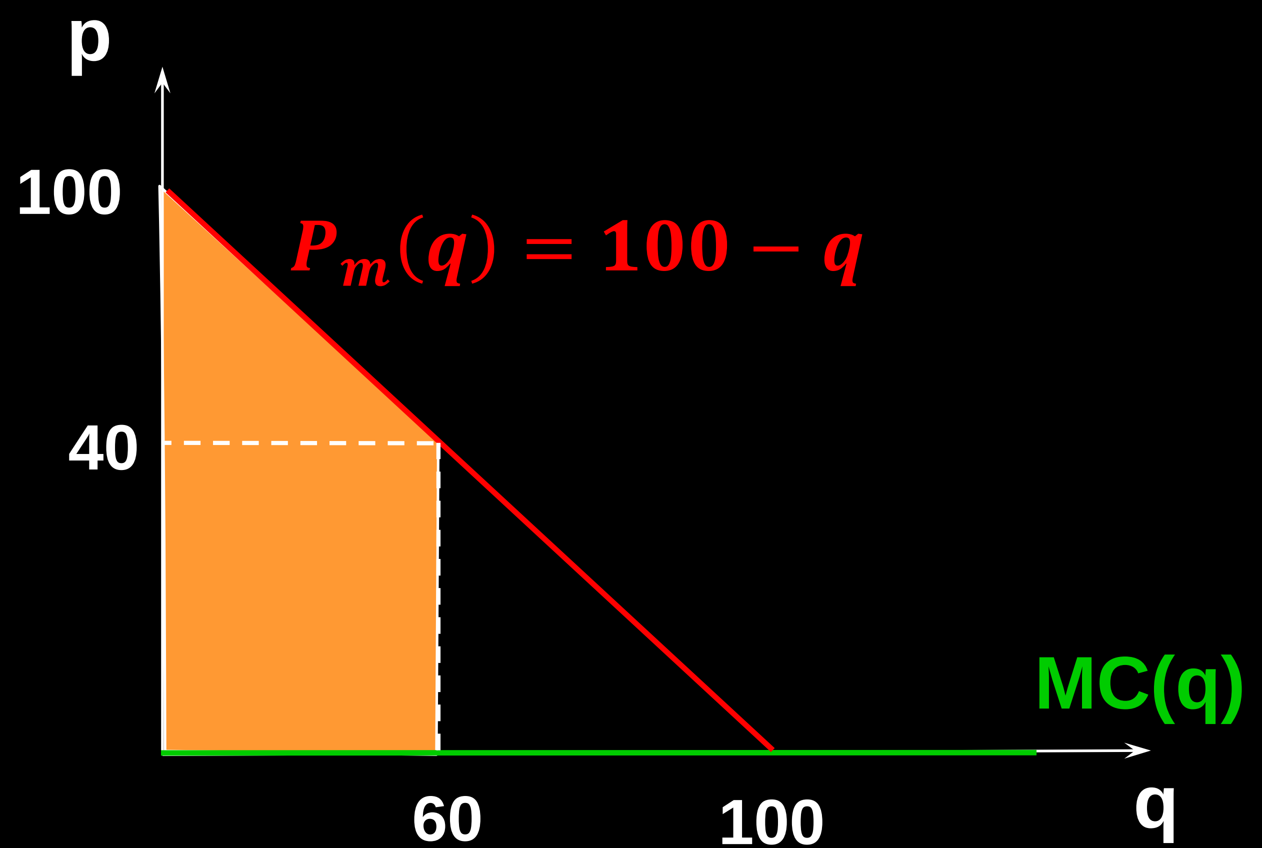



**The student's valuation for 60-article is $\frac{1}{2}(20+80) * 60 = \$30$**

# Price Discrimination: An Example

- Two packages: one is ($q_1$=60, $p_1$) – for the student, the other is ($q_2$=100, $p_2$) – for the manager.
  - the maximum $p_1 = \$30$
  - student package is ($q_1$=60, $p_1$=\$30)
- If the manager selects ($q_1$=60, $p_1$=\$30), her net surplus = ?

# Price Discrimination: An Example




**The manager's valuation for 60-article is** $\frac{1}{2}(40 + 100) * 60 = \$42$

# Price Discrimination: An Example

Two packages: one is ($q_1$=60, $p_1$=\$30), the other is ($q_2$=100, $p_2$)

- If the manager selects ($q_1$=60, $p_1$=\$30), his/her net surplus = 42-30 =\$12
- If the manager selects ($q_2$=100, $p_2$), the net surplus = 50 - $p_2$

In order for the manager to select ($q_2$=100, $p_2$), we need $50 - p_2 \geq 12$

=> $p_2 \leq 38$

# Price Discrimination: An Example

The highest price the monopolist could set for 100-article is $\mathbf{p_2 = \$38}$

# Price Discrimination: An Example

- Two packages: one is ($q_1$=60, $p_1$=\$30), the other is ($q_2$=100, $p_2$=\$38)
- The student will select ($q_1$=60, $p_1$=\$30), and gets a net surplus = \$0
- The manager will select ($q_2$=100, $p_2$=\$38) and get a net surplus = \$12
- The company makes a profit = 30 + 38 = \$68

# Price Discrimination: An Example

- Two packages: one is ($q_1$=80, $p_1$=\$32), the other is ($q_2$=100, $p_2$=\$50). The company makes a profit = \$64
- Two packages: one is ($q_1$=80, $p_1$=\$32), the other is ($q_2$=100, $p_2$=\$34). The company makes a profit = \$66
- Two packages: one is ($q_1$=60, $p_1$=\$30), the other is ($q_2$=100, $p_2$=\$38). The company makes a profit = \$68

# Price Discrimination: An Example

These exists an optimal design of price-quantity packages that maximizes the company's profits

# Third-degree Price Discrimination

**Price paid by buyers in a given group is the same for all units purchased. But price may differ across buyer groups.**

三级价格歧视：对不同的人(群)按不同的价格出售产品，但对于给定的人(群)，每单位产品都按相同价格出售

# Third-degree Price Discrimination

- A monopolist manipulates market price by altering the **quantity** of product supplied to that market.
- So the question “What discriminatory prices will the monopolist set, one for each group?” is really the question “How many units of product will the monopolist supply to each group?”

如何决定不同人群的价格？<=>如何决定卖给不同人群的产品数量？

# Third-degree Price Discrimination

- Two markets, 1 and 2. (两类人或两个细分市场)
- $y_1$ is the quantity supplied to market 1. Market 1's inverse demand function is $p_1(y_1)$.
- $y_2$ is the quantity supplied to market 2. Market 2's inverse demand function is $p_2(y_2)$.

# Third-degree Price Discrimination

- For given supply levels $y_1$ and $y_2$ the firm's profit is

$$\Pi(y_1, y_2) = p_1(y_1)y_1 + p_2(y_2)y_2 - c(y_1 + y_2).$$

- What values of $y_1$ and $y_2$ maximize profit?

# Third-degree Price Discrimination

$$\Pi(y_1, y_2) = p_1(y_1)y_1 + p_2(y_2)y_2 - c(y_1 + y_2).$$

**The profit-maximization conditions are**

$$\frac{\partial \Pi}{\partial y_1} = \frac{\partial}{\partial y_1}\left(p_1(y_1)y_1\right) - \frac{\partial c(y_1 + y_2)}{\partial (y_1 + y_2)} \times \frac{\partial (y_1 + y_2)}{\partial y_1}$$

$$= 0$$

# Third-degree Price Discrimination

$$\Pi(y_1, y_2) = p_1(y_1)y_1 + p_2(y_2)y_2 - c(y_1 + y_2).$$

**The profit-maximization conditions are**

$$\frac{\partial \Pi}{\partial y_1} = \frac{\partial}{\partial y_1}\left(p_1(y_1)y_1\right) - \frac{\partial c(y_1 + y_2)}{\partial (y_1 + y_2)} \times \frac{\partial (y_1 + y_2)}{\partial y_1}$$

$$= 0$$

$$\frac{\partial \Pi}{\partial y_2} = \frac{\partial}{\partial y_2}\left(p_2(y_2)y_2\right) - \frac{\partial c(y_1 + y_2)}{\partial (y_1 + y_2)} \times \frac{\partial (y_1 + y_2)}{\partial y_2}$$

$$= 0$$

# Third-degree Price Discrimination

$$\frac{\partial (y_1 + y_2)}{\partial y_1} = 1$$ **and** $$\frac{\partial (y_1 + y_2)}{\partial y_2} = 1$$ **so**

**the profit-maximization conditions are**

$$\frac{\partial}{\partial y_1}\left(p_1(y_1)y_1\right) = \frac{\partial c(y_1 + y_2)}{\partial (y_1 + y_2)}$$

**and** $$\frac{\partial}{\partial y_2}\left(p_2(y_2)y_2\right) = \frac{\partial c(y_1 + y_2)}{\partial (y_1 + y_2)}.$$

# Third-degree Price Discrimination

$$\frac{\partial}{\partial y_1}\left(p_1(y_1)y_1\right) = \frac{\partial}{\partial y_2}\left(p_2(y_2)y_2\right) = \frac{\partial c(y_1+y_2)}{\partial (y_1+y_2)}$$

# Third-degree Price Discrimination

$$\underbrace{\frac{\partial}{\partial y_1}(p_1(y_1)y_1) = \frac{\partial}{\partial y_2}(p_2(y_2)y_2)} = \frac{\partial\, c(y_1+y_2)}{\partial\,(y_1+y_2)}$$

$MR_1(y_1) = MR_2(y_2)$ says that the allocation $y_1$, $y_2$ **maximizes the revenue** from selling $y_1 + y_2$ output units.

E.g. if $MR_1(y_1) > MR_2(y_2)$ then an output unit should be moved from market 2 to market 1 to increase total revenue.

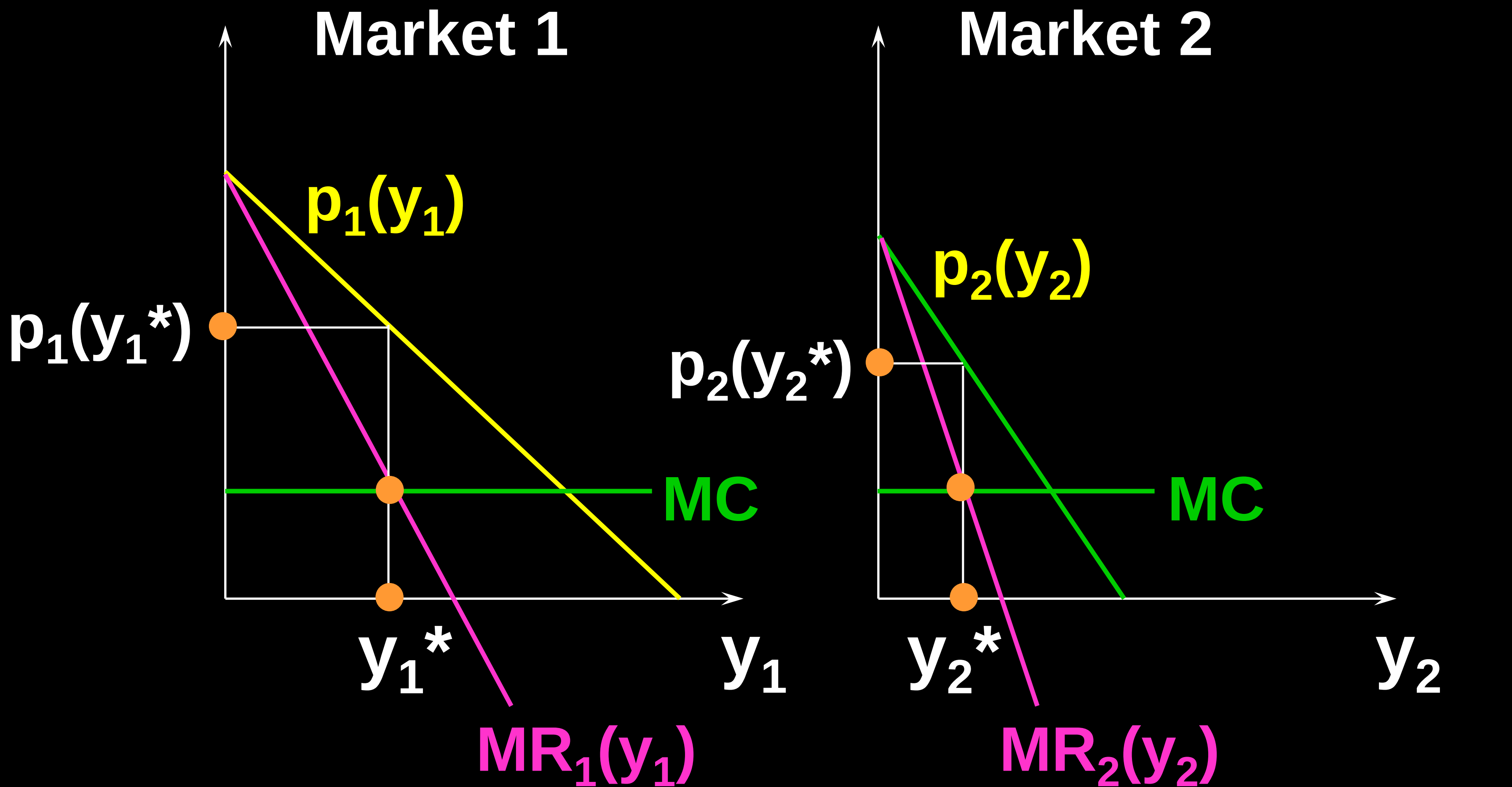
# Third-degree Price Discrimination

$$\underbrace{\frac{\partial}{\partial y_1}\left(p_1(y_1)y_1\right) = \frac{\partial}{\partial y_2}\left(p_2(y_2)y_2\right) = \frac{\partial c(y_1+y_2)}{\partial (y_1+y_2)}}$$

**The marginal revenue common to both markets equals the marginal cost if profit is to be maximized.**

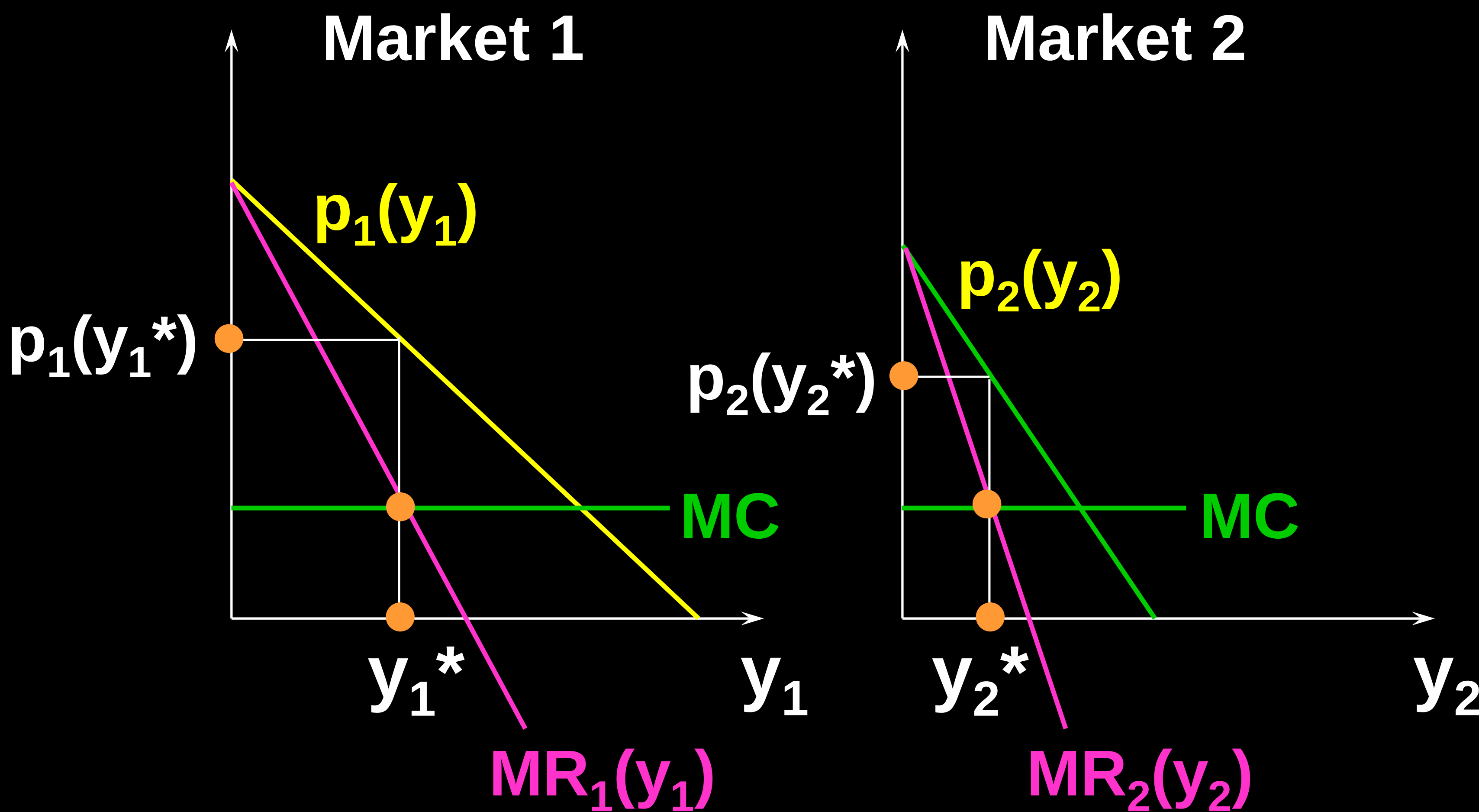
当额外生产1单位的边际成本等于每个市场上的边际收益时，垄断厂商的利润最大。

# Third-degree Price Discrimination




$MR_1(y_1^*) = MR_2(y_2^*) = MC$

# Third-degree Price Discrimination




$MR_1(y_1^*) = MR_2(y_2^*) = MC$ and $p_1(y_1^*) \neq p_2(y_2^*)$.

# Third-degree Price Discrimination

**In which market will the monopolist set the higher price?**

# Third-degree Price Discrimination

**In which market will the monopolist set the higher price?**

**Recall that**

$$\mathbf{MR_1(y_1) = \frac{d[p(y_1)y_1]}{dy_1} = p(y_1)\left[1 - \frac{1}{|\varepsilon_1|}\right]}$$

$$\mathbf{MR_2(y_2) = \frac{d[p(y_2)y_2]}{dy_2} = p(y_2)\left[1 - \frac{1}{|\varepsilon_2|}\right]}$$

# Third-degree Price Discrimination

In which market will the monopolist set the higher price?

Recall that

$$MR_1(y_1) = \frac{d[p(y_1)y_1]}{dy_1} = p(y_1)\left[1 - \frac{1}{|\varepsilon_1|}\right]$$

$$MR_2(y_2) = \frac{d[p(y_2)y_2]}{dy_2} = p(y_2)\left[1 - \frac{1}{|\varepsilon_2|}\right]$$

and

$$MR_1(y_1^*) = MR_2(y_2^*) = MC(y_1^* + y_2^*)$$

# Third-degree Price Discrimination

**So** $$p(y_1^*)\left[1-\frac{1}{|\varepsilon_1|}\right]=p(y_2^*)\left[1-\frac{1}{|\varepsilon_2|}\right]$$

**Therefore,** $p_1(y_1^*) > p_2(y_2^*)$ **only if**

$$1-\frac{1}{|\varepsilon_1|}<1-\frac{1}{|\varepsilon_2|}\Rightarrow|\varepsilon_1|<|\varepsilon_2|$$

# Third-degree Price Discrimination

So
$$p(y_1^*)\left[1-\frac{1}{|\varepsilon_1|}\right]=p(y_2^*)\left[1-\frac{1}{|\varepsilon_2|}\right]$$

Therefore, $p_1(y_1^*) > p_2(y_2^*)$ only if

$$1-\frac{1}{|\varepsilon_1|}<1-\frac{1}{|\varepsilon_2|}\Rightarrow|\varepsilon_1|<|\varepsilon_2|$$

The monopolist sets the higher price in the market where demand is less own-price elastic. 细分市场的需求越缺乏弹性，垄断厂商的定价越高

# Third-degree Price Discrimination: an Example

**A monopolist faces two markets with demand given by:**

$$D_1(p_1) = 100 - p_1$$
$$D_2(p_2) = 120 - 2p_2$$

**Total cost is given by $C(q) = q^2$. What price should it charge in each market to max profits?**

# Third-degree Price Discrimination: an Example

**Inverse demand functions**

$$\mathbf{p_1 = 100 - q_1}$$
$$\mathbf{p_2 = 60 - q_2/2}$$

**A monopolist is to max**

$$\Pi = q_1(100 - q_1) + q_2(60 - q_2/2) - (q_1 + q_2)^2$$

$$MR_1(q_1) = 100 - 2q_1 = MC = 2(q_1 + q_2)$$

$$MR_2(q_2) = 60 - q_2 = MC = 2(q_1 + q_2)$$

# Third-degree Price Discrimination: an Example

$$MR(\mathbf{q}_1) = 100 - 2q_1 = MC = 2(q_1 + q_2)$$

$$MR(q_2) = 60 - q_2 = MC = 2(q_1 + q_2)$$

=>

$$4q_1 + 2q_2 = 100$$
$$2q_1 + 3q_2 = 60$$

$$q_1^* = 22.5\,, q_2^* = 5$$
$$p_1 = 77.5\,, p_2 = 57.5$$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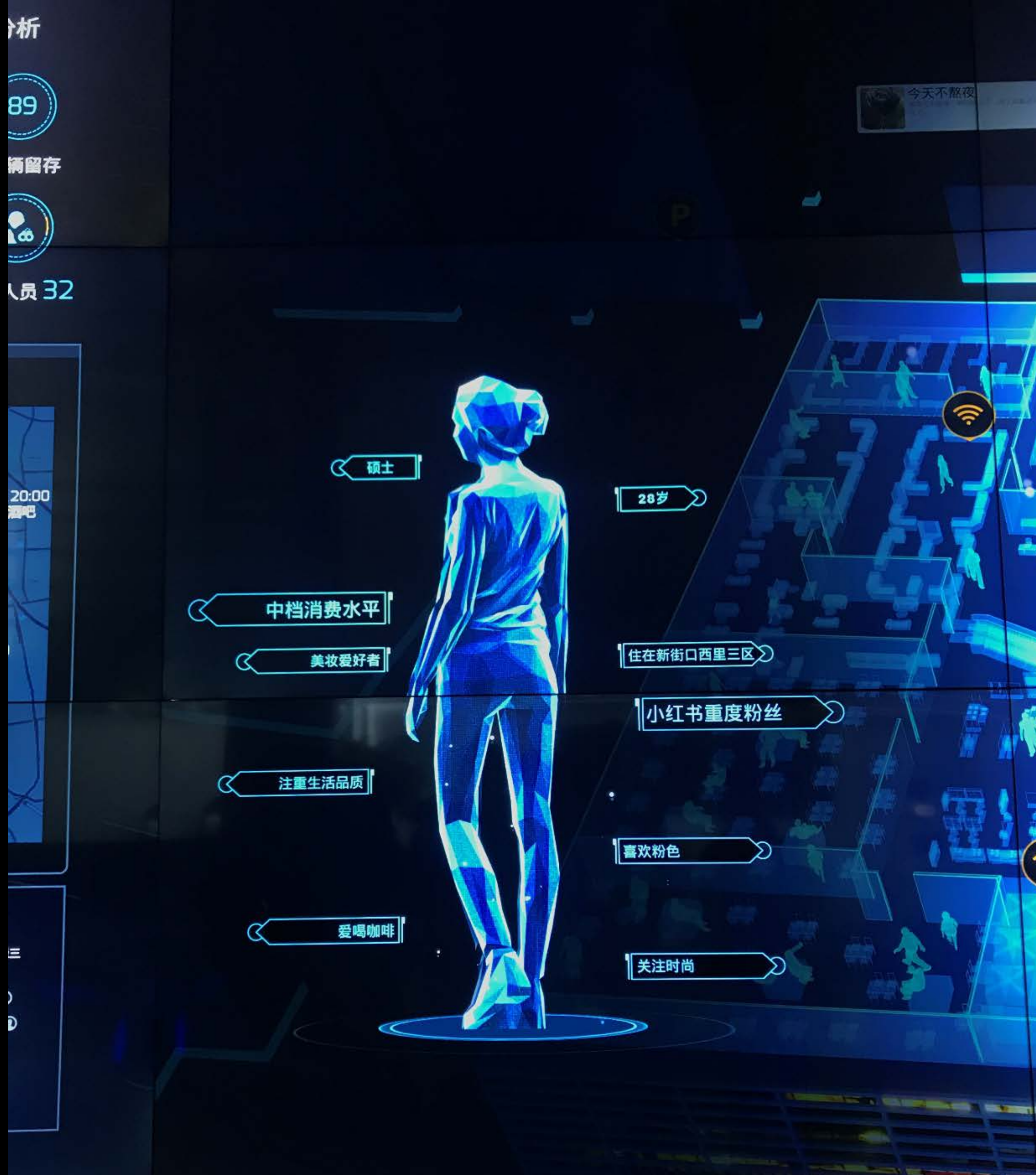

今天不熬夜
32
硕士
28岁
中档消费水平
美妆爱好者
住在新街口西里三区
小红书重度粉丝
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# Two-Part Tariffs

A two-part tariff is a lump-sum fee, $p_1$, plus a price $p_2$ for each unit of product purchased.

两部分定价: 固定费用 + 单位价格

Thus the cost of buying x units of product is

$$p_1 + p_2x.$$

# Two-Part Tariffs

Should a monopolist prefer a two-part tariff to uniform pricing, or to any of the price-discrimination schemes discussed so far?

If so, how should the monopolist design its two-part tariff?

# Two-Part Tariffs

$p_1 + p_2 x$

**Q: What is the largest that $p_1$ can be?**

# Two-Part Tariffs

$$p_1 + p_2x$$

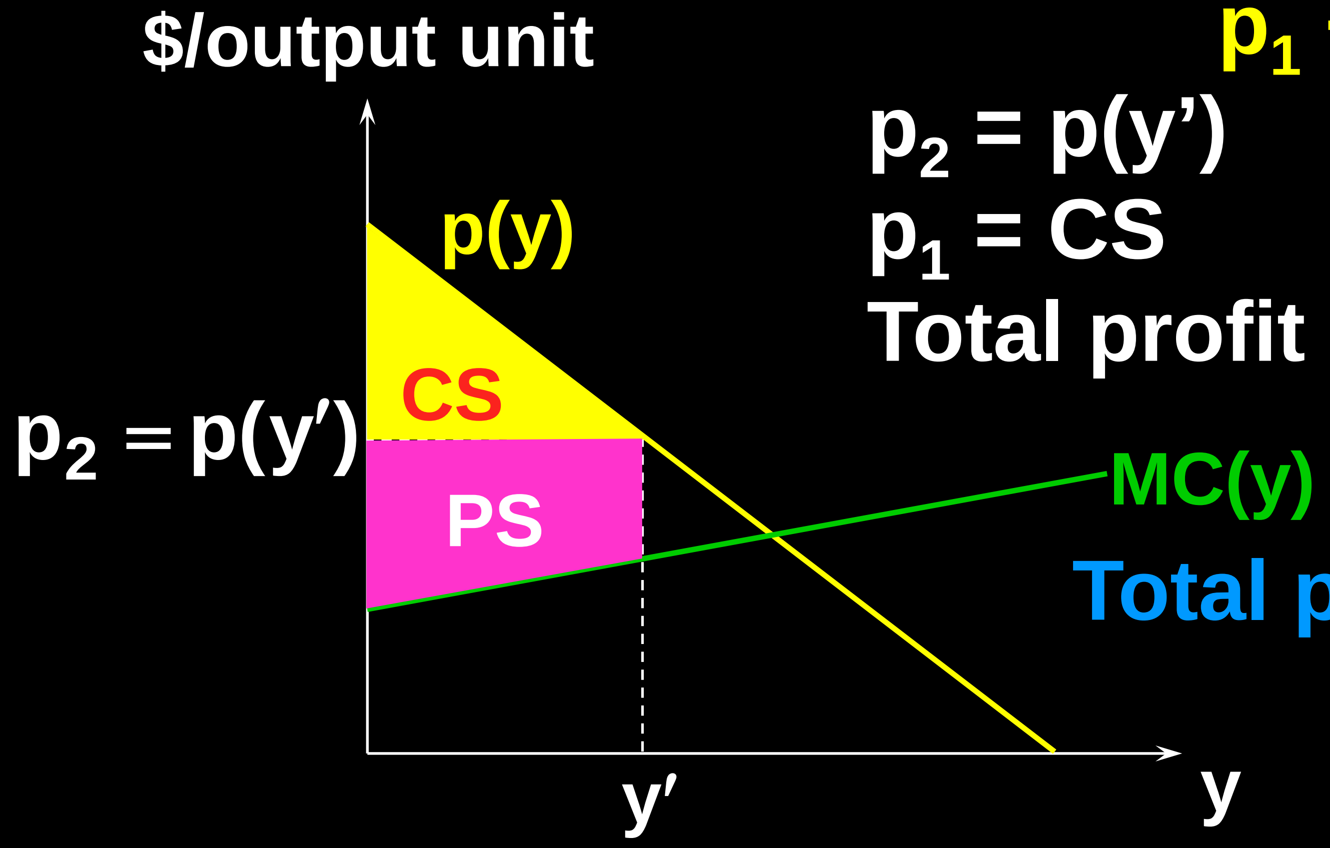
Q: What is the largest that $p_1$ can be?

A: $p_1$ is the "entrance fee" so the largest it can be is the **surplus** the buyer gains from entering the market.

Set $p_1$ = CS and now ask what should be $p_2$?

两部分定价中的固定费用是在单位价格$p_2$下消费者获得的剩余

# Two-Part Tariffs




$p_1 + p_2x$

$p_2 = p(y')$

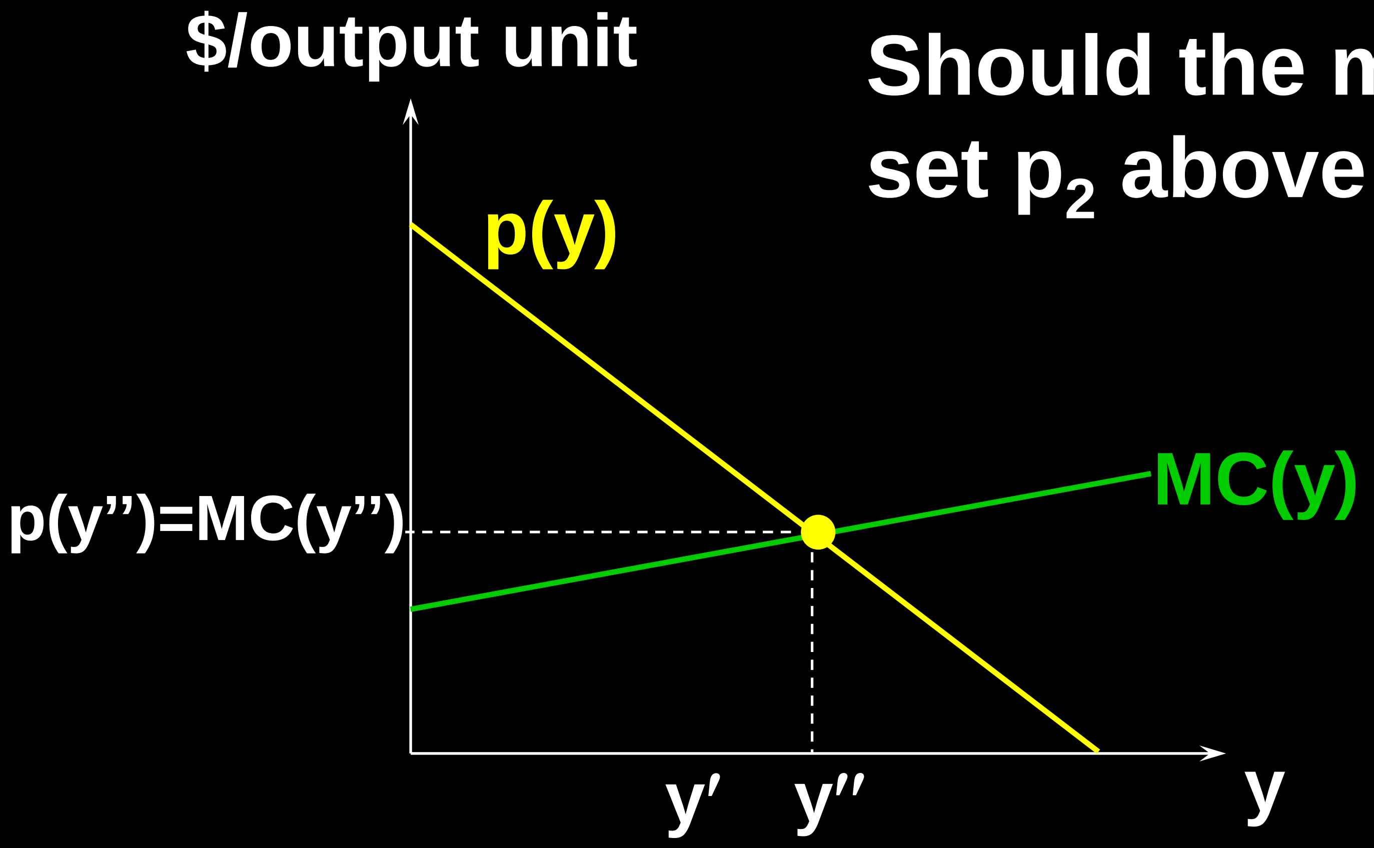
$p_1 = CS$

Total profit = CS + PS

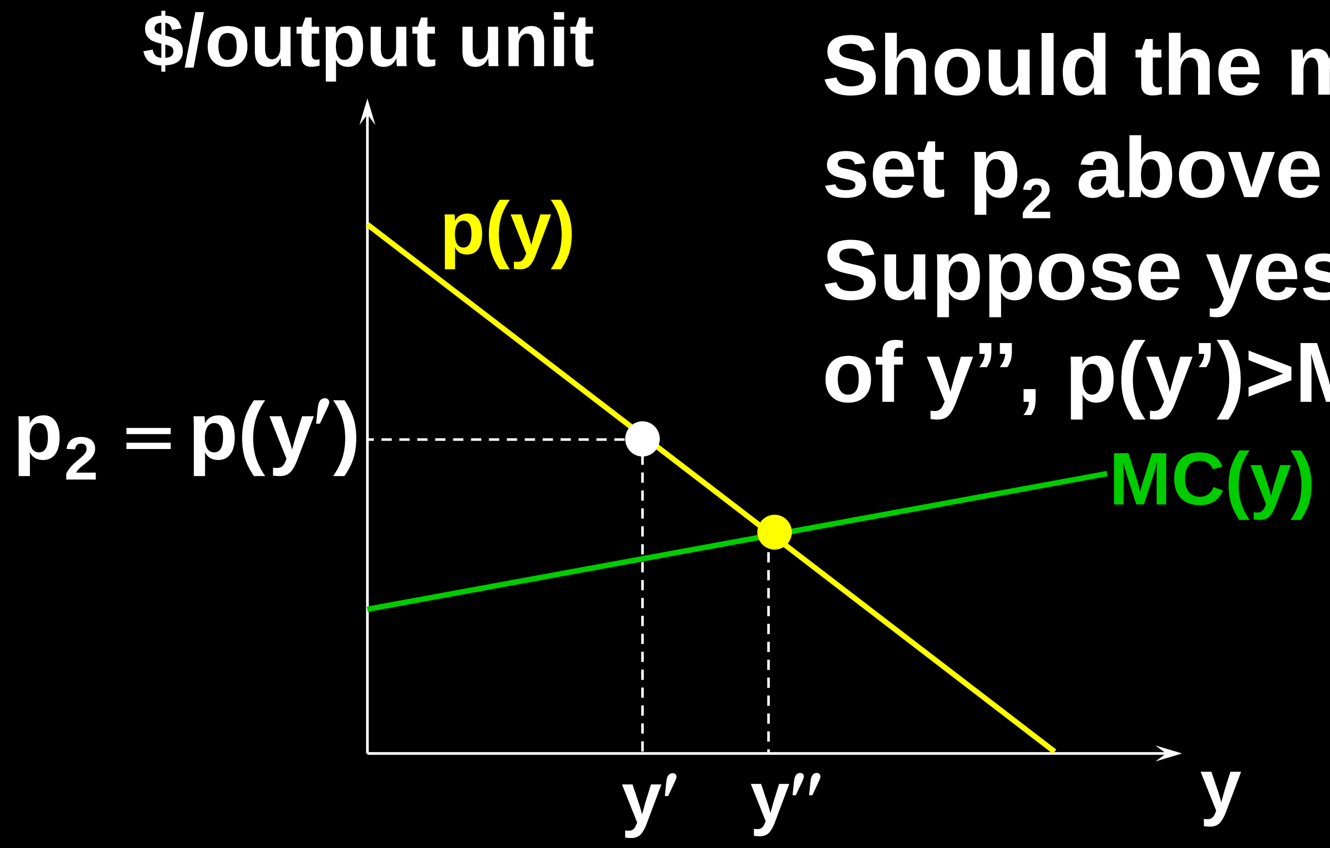
Total profit

# Two-Part Tariffs

Should the monopolist set $p_2$ above MC?

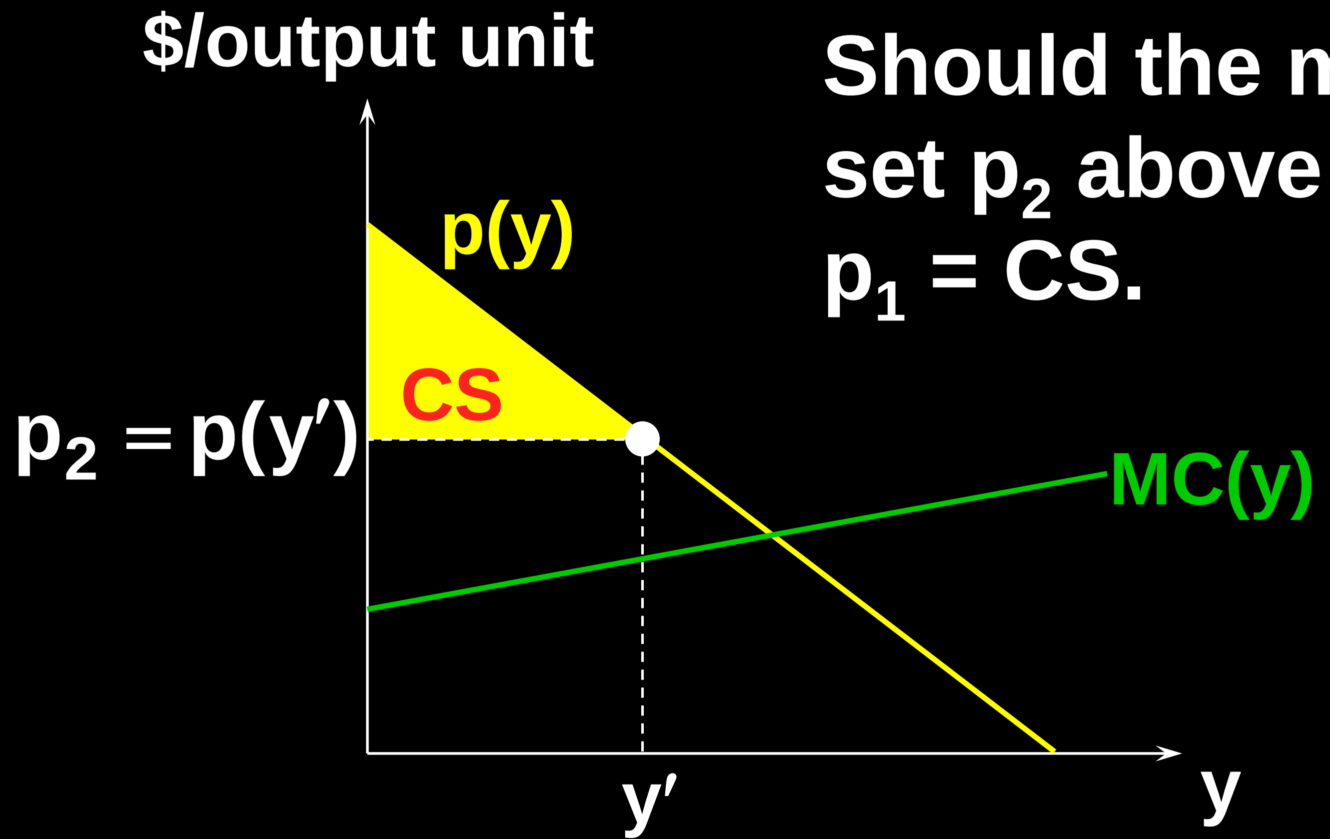


# Two-Part Tariffs




**Should the monopolist set $p_2$ above MC? Suppose yes; y' instead of y", p(y')>MC(y')**

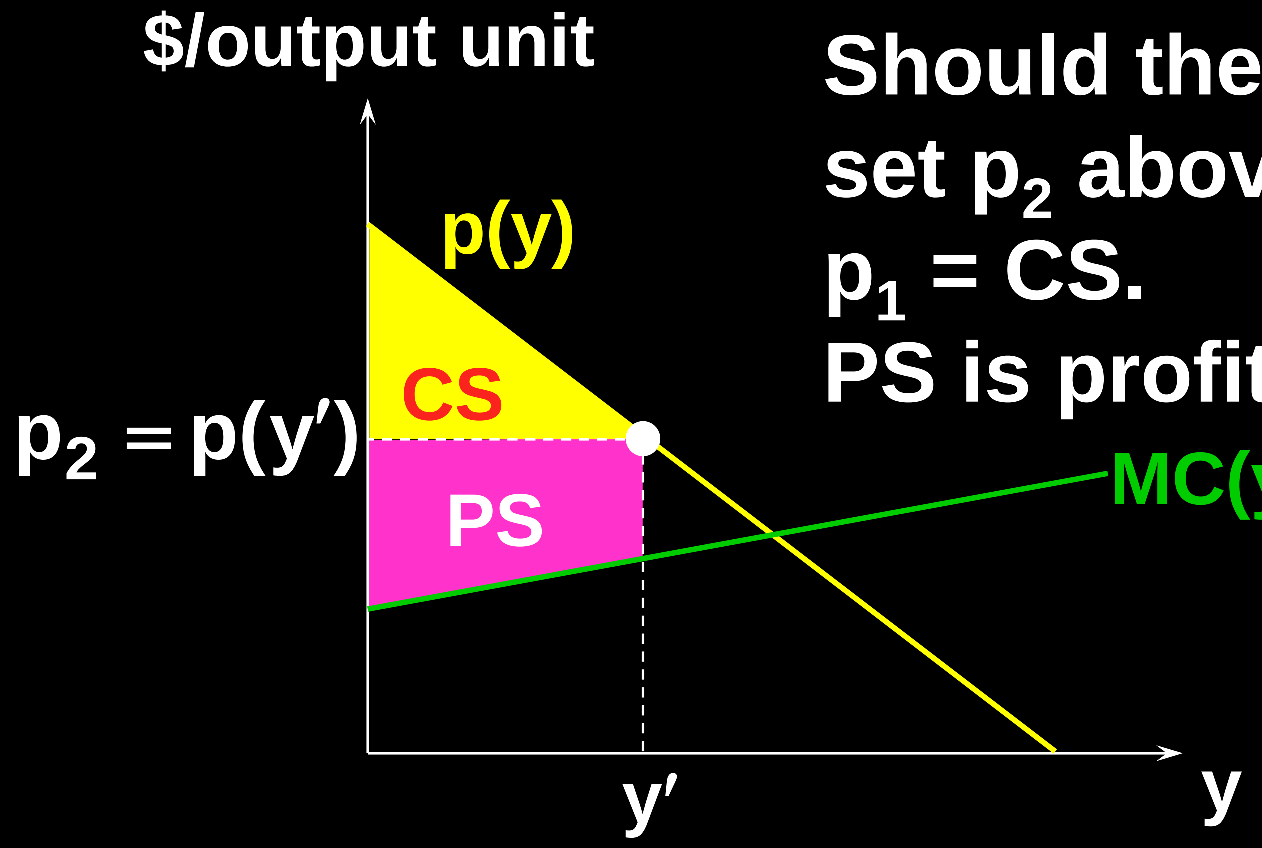
# Two-Part Tariffs




Should the monopolist set $p_2$ above MC?

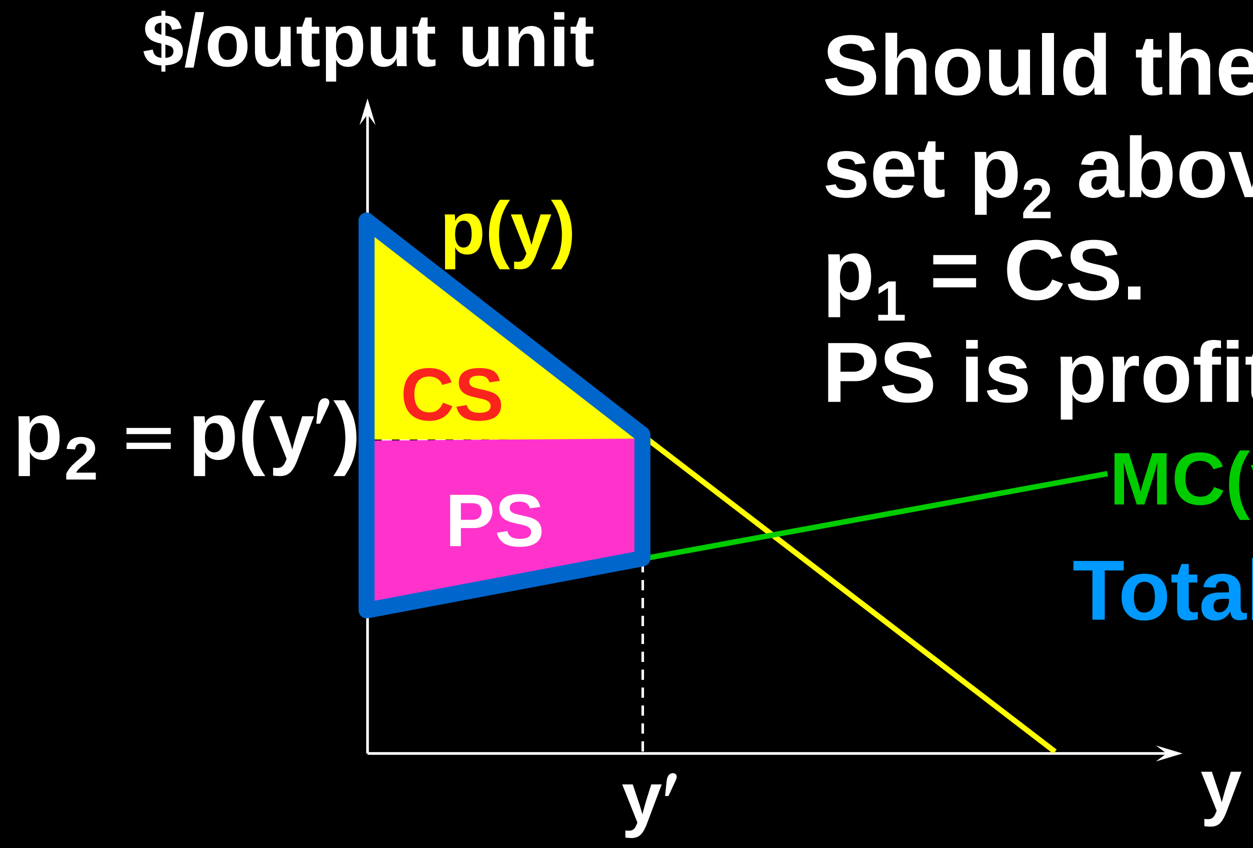
$p_1$ = CS.

# Two-Part Tariffs




Should the monopolist set $p_2$ above MC?
$p_1$ = CS.
PS is profit from sales.

# Two-Part Tariffs



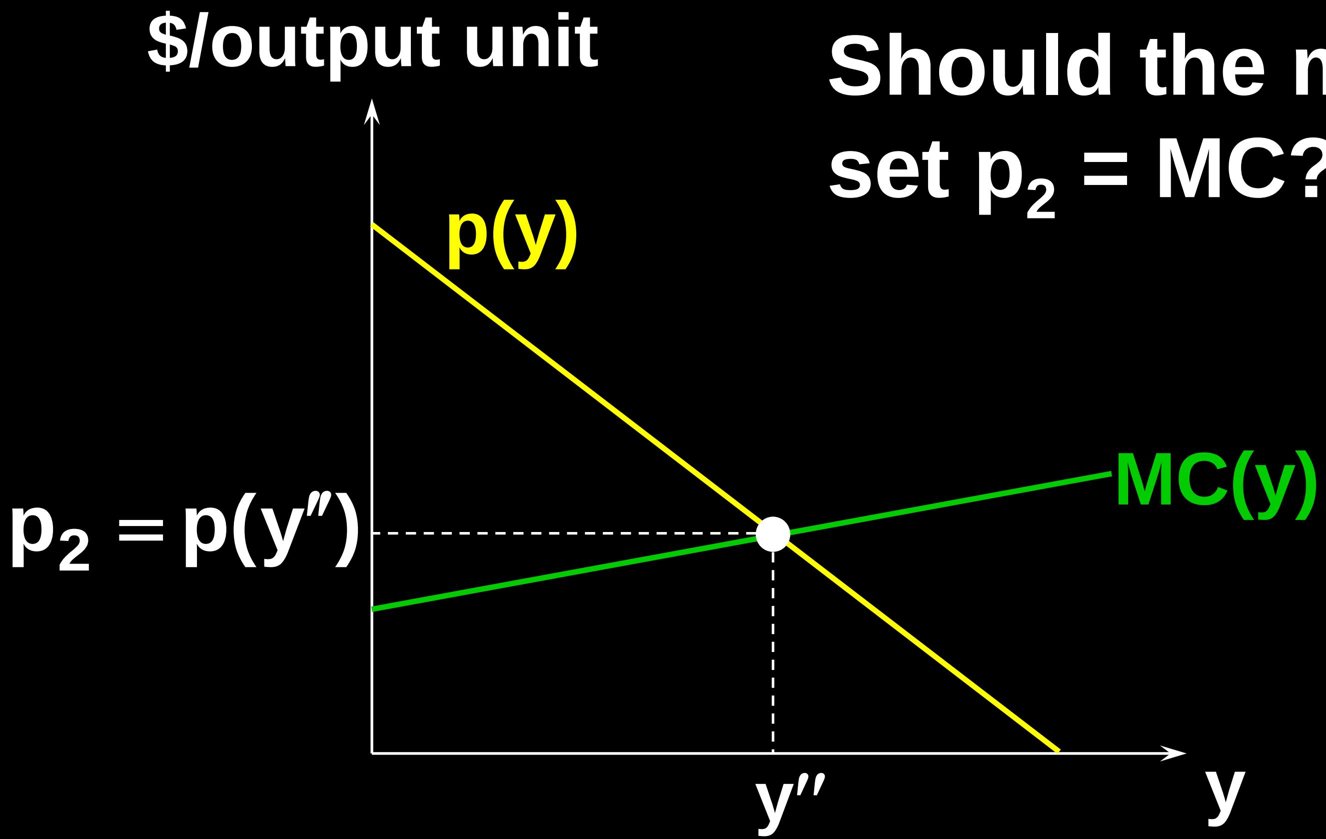

Should the monopolist set $p_2$ above MC?

$p_1$ = CS.

PS is profit from sales.

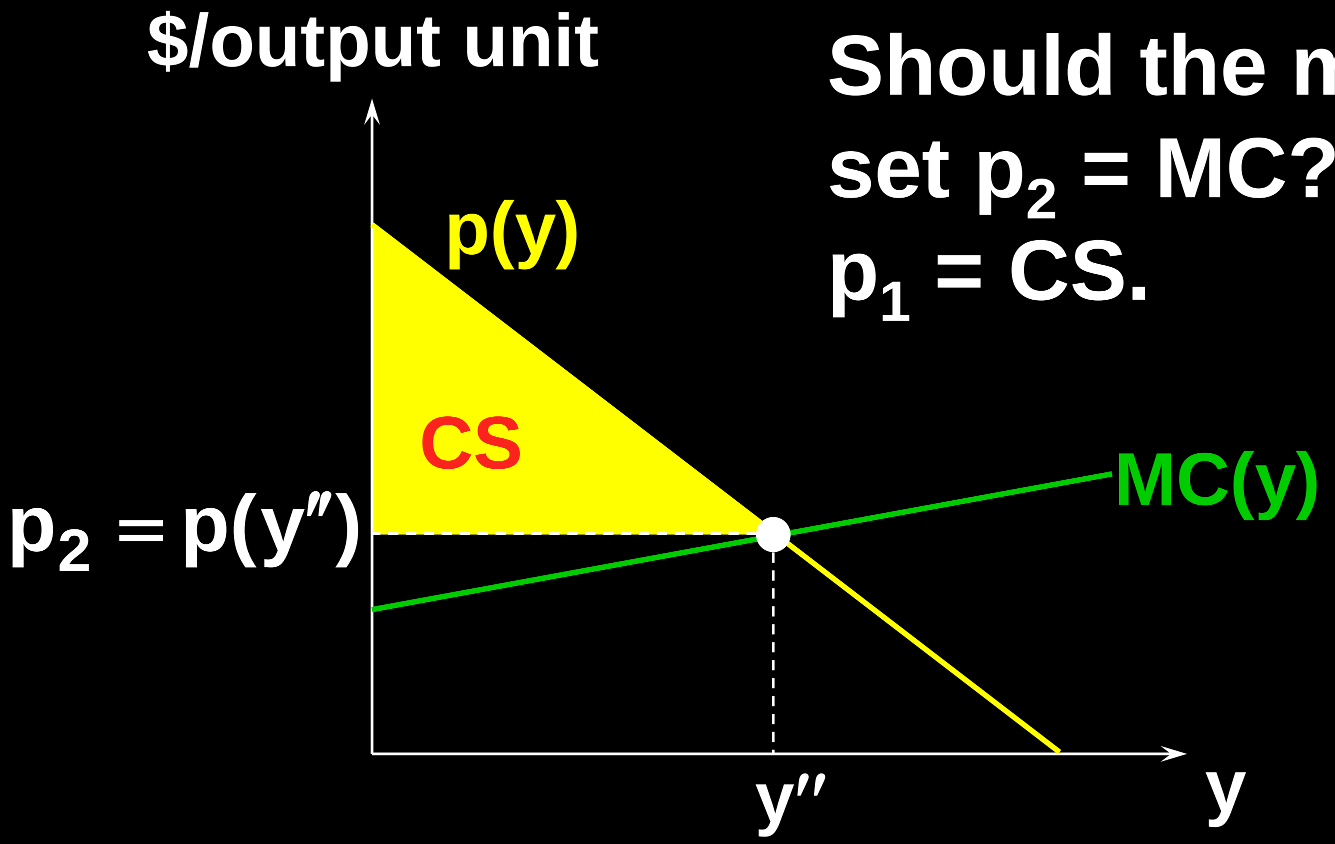
Total profit

# Two-Part Tariffs



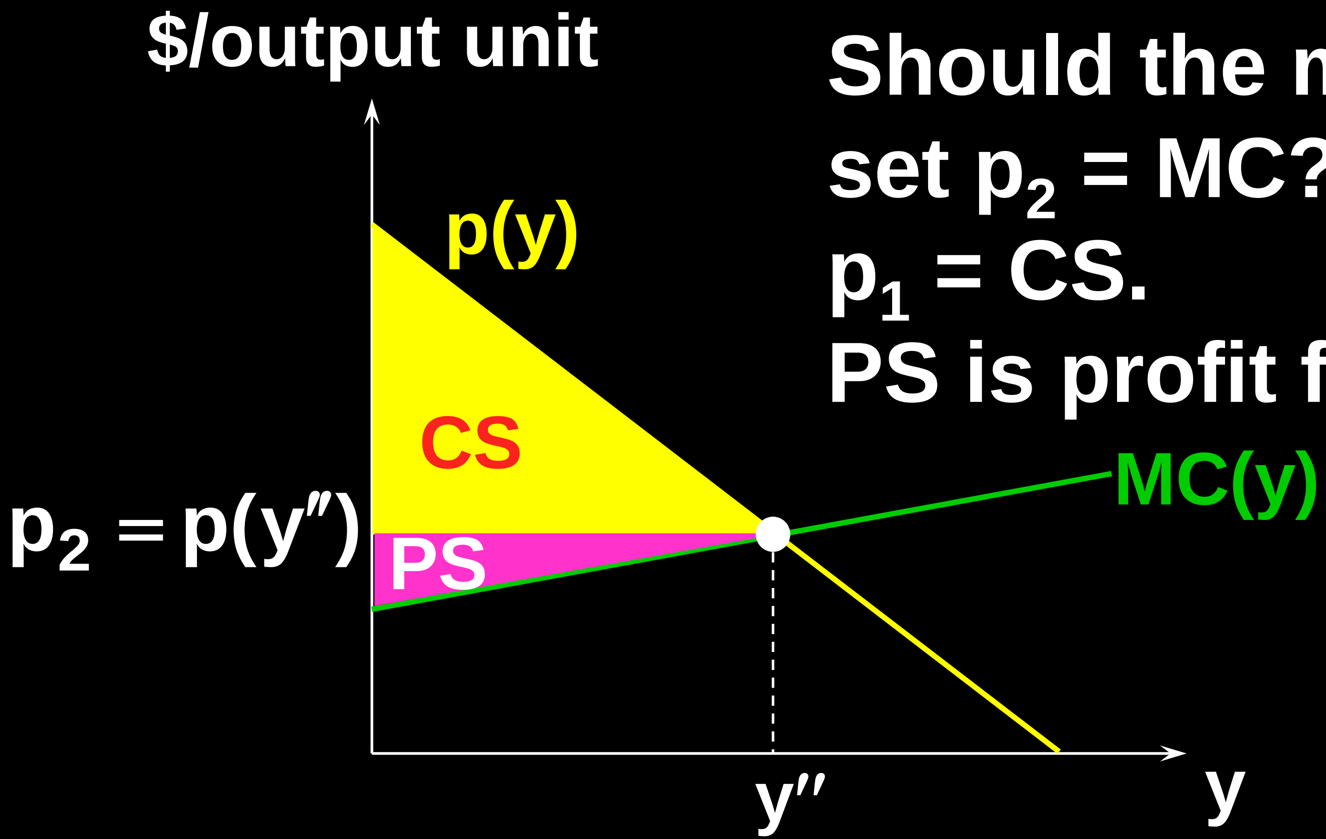

Should the monopolist set $p_2$ = MC?

# Two-Part Tariffs




Should the monopolist set $p_2$ = MC?
$p_1$ = CS.

# Two-Part Tariffs

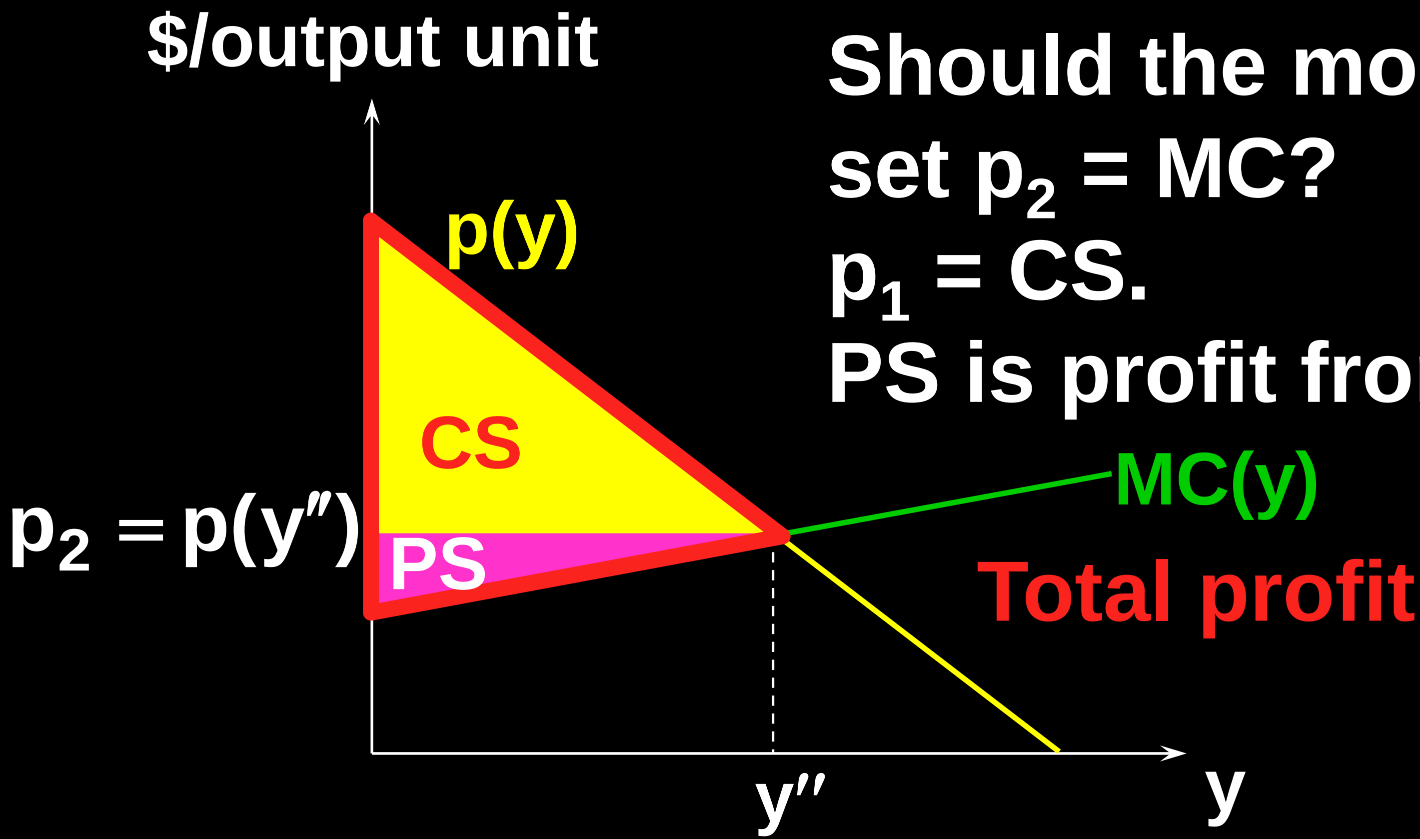



Should the monopolist set $p_2$ = MC?

$p_1$ = CS.

PS is profit from sales.

# Two-Part Tariffs

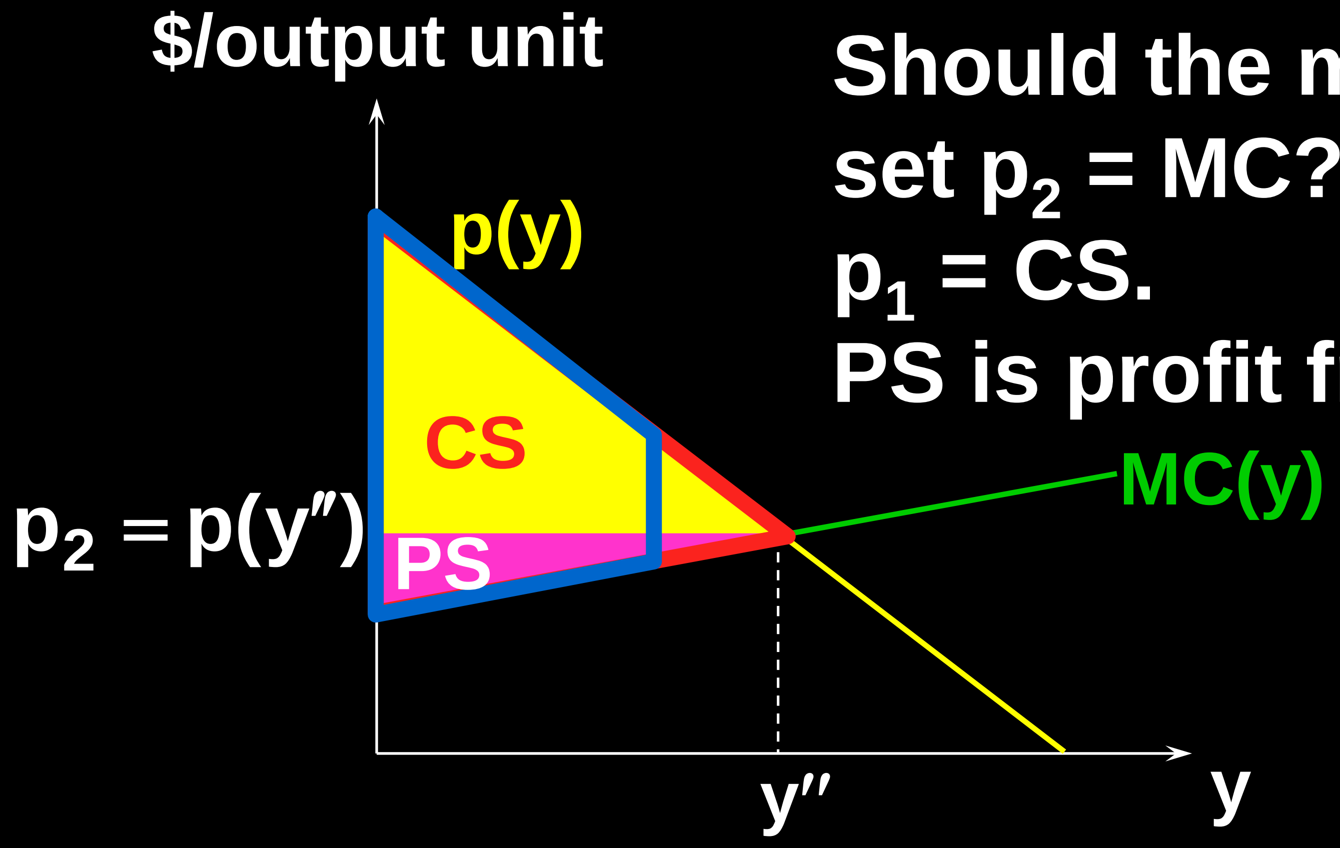



Should the monopolist set $p_2$ = MC?

$p_1$ = CS.

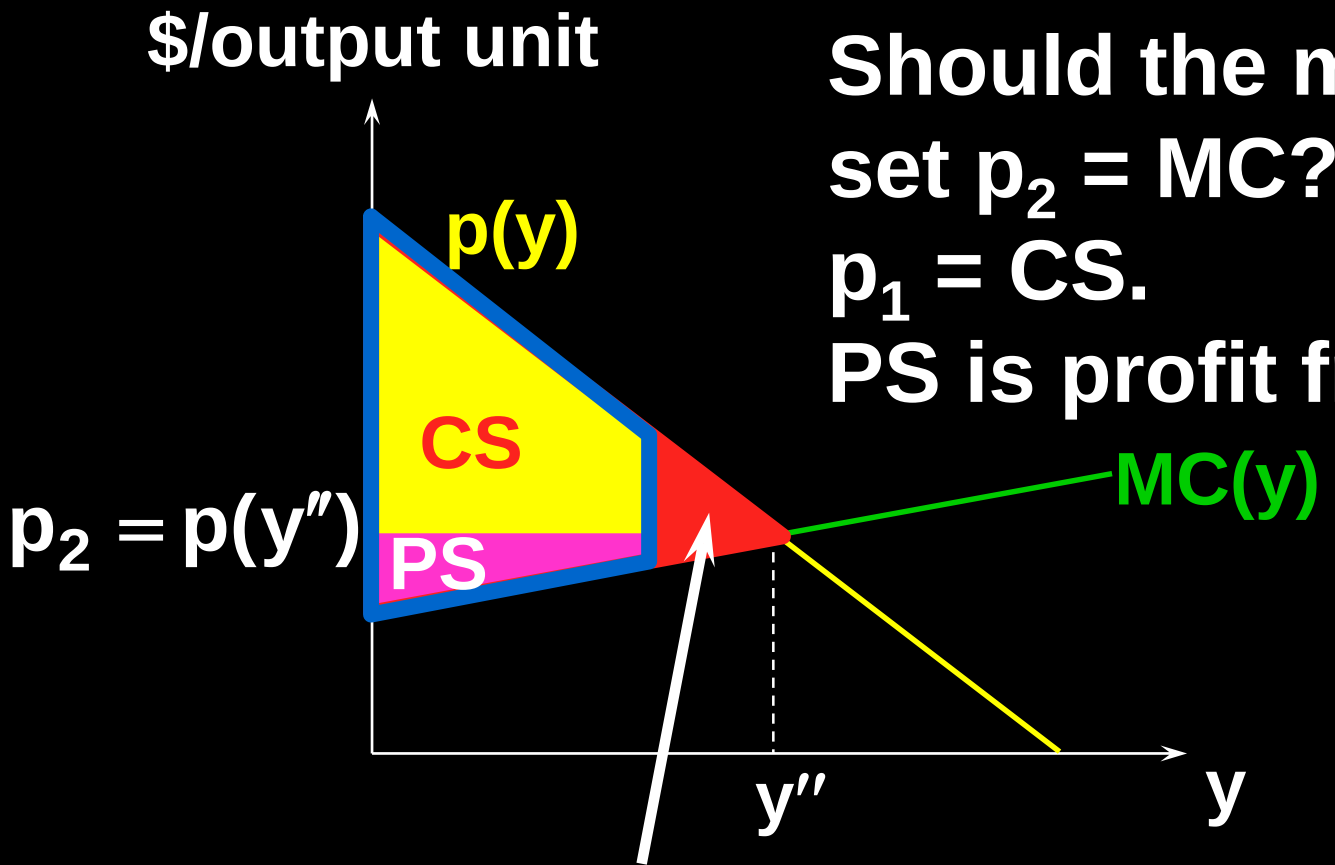
PS is profit from sales.

# Two-Part Tariffs




Should the monopolist set $p_2$ = MC?
$p_1$ = CS.
PS is profit from sales.

# Two-Part Tariffs




Should the monopolist set $p_2$ = MC?
$p_1$ = CS.
PS is profit from sales.

Additional profit from setting $p_2$ = MC.

# Two-Part Tariffs

- The monopolist maximizes its profit when using a two-part tariff by setting its per unit price $p_2$ at marginal cost and setting its lump-sum fee $p_1$ equal to Consumers' Surplus.

垄断厂商使用两部分定价，将单位价格定为边际成本、固定费用定为消费者剩余，可使利润最大化。

# Two-Part Tariffs

**A profit-maximizing two-part tariff gives an efficient market outcome in which the monopolist obtains as profit the total of all gains-to-trade.**

利润最大化时，两部分定价法下的产量满足边际成本等于单位价格的条件，这一产量使得社会总剩余最大、是有效率的产量。

# Two-Part Tariffs: An Example

**An amusement park attracts 1,000 visitors per day, and each visitor takes $\mathbf{x = 50 - 50p}$ rides, where p is the price of a ride.**

$$MC = 0$$

**Q1: If the manager sets a uniform price to max profits, how many rides will be taken per day by a typical visitor?**

$$\mathbf{p = 1 - x/50}$$

$$\mathbf{MR(x) = 1 - \frac{x}{25} = MC = 0}$$

# Two-Part Tariffs: an example

**Q1: If the manager sets a uniform price to max profits, how many rides will be taken per day by a typical visitor?**

$$p = 1 - x/50$$

$$MR(x) = 1 - \frac{x}{25} = MC = 0$$

$$x^* = 25$$

**Profit per visitor?**

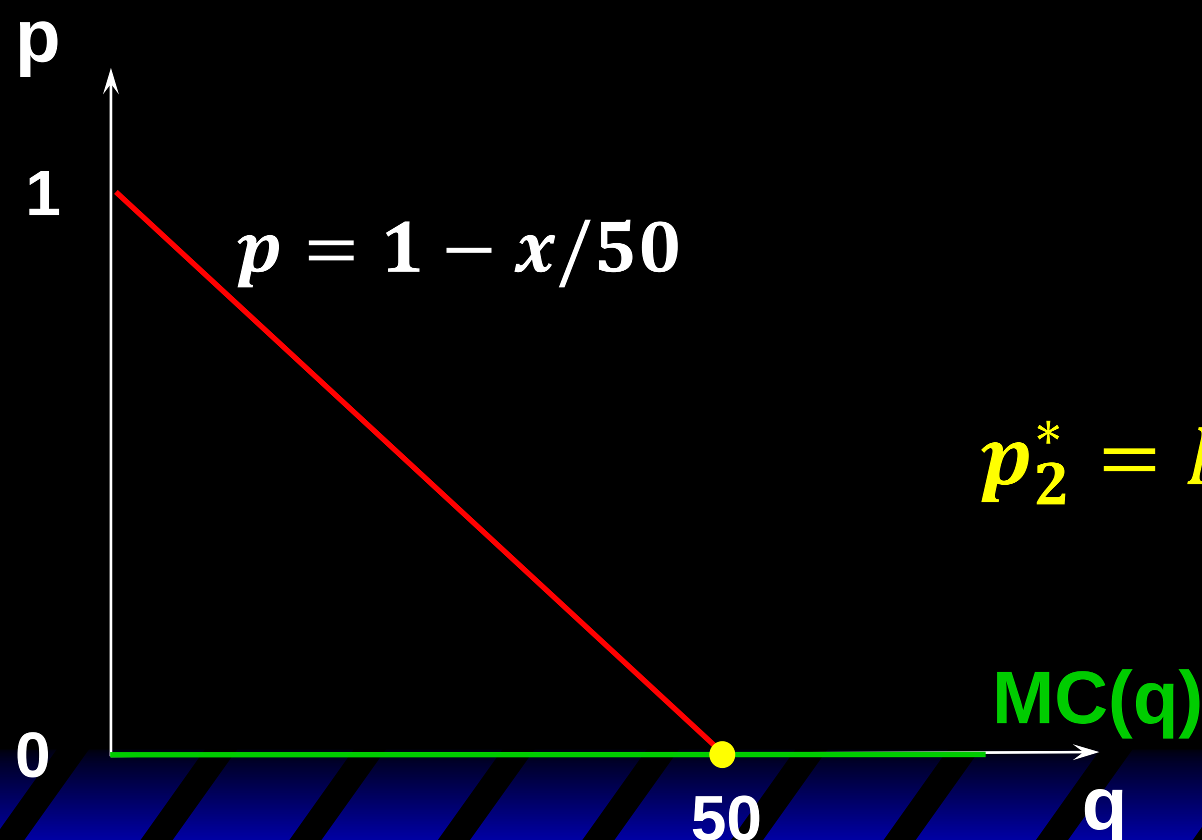
$$px^* = \left(1 - \frac{25}{50}\right) * 25 = \$12.5$$

# Two-Part Tariffs: an example

**Q2: what is the profit-maximizing two-part tariff?**

# Two-Part Tariffs: an example

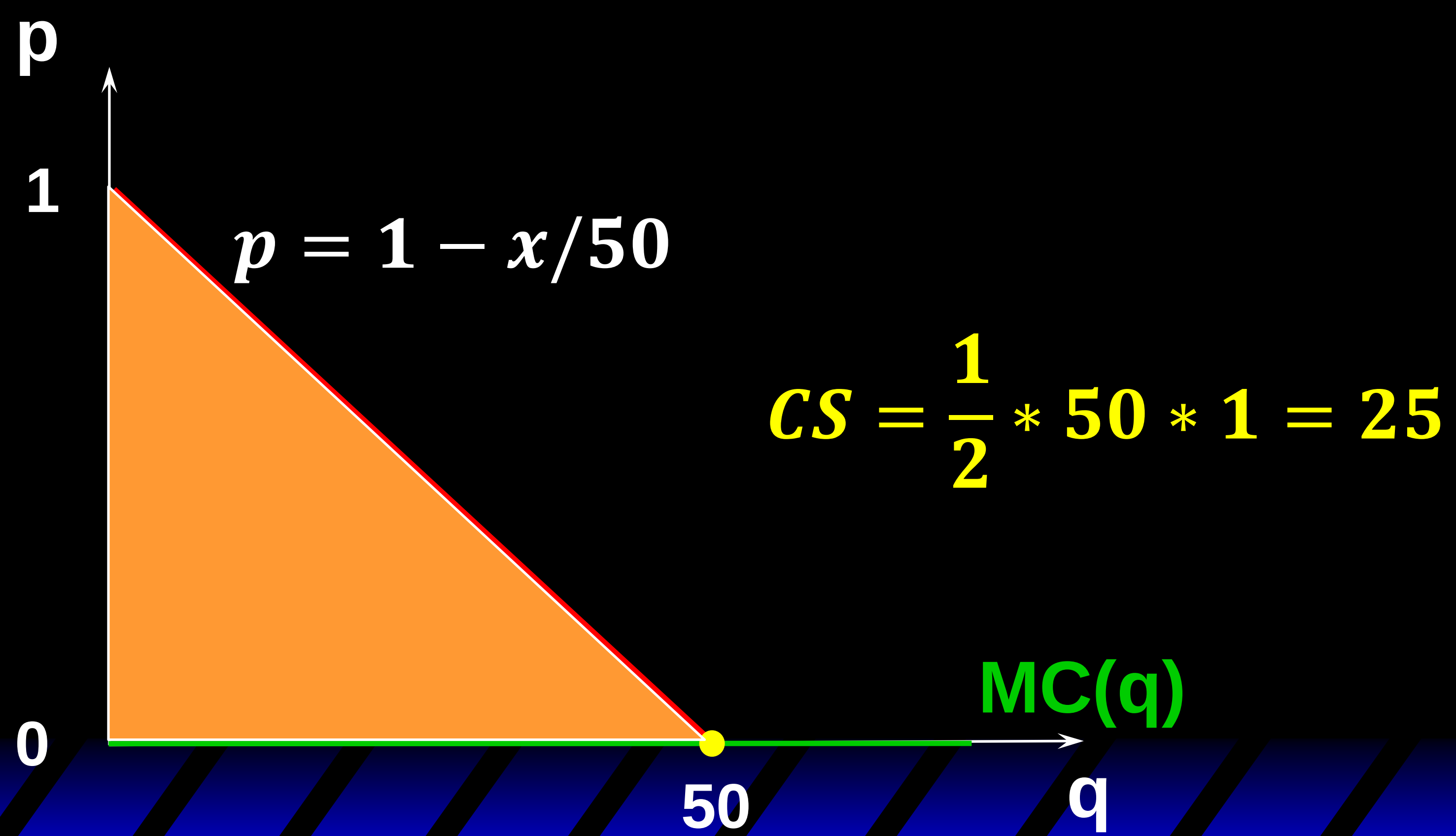
**Q2: what is the profit-maximizing two-part tariff?**




$$p_2^* = MC = 0$$

# Two-Part Tariffs: an example

**Q2: what is the profit-maximizing two-part tariff?**



# Two-Part Tariffs: an example

**Q2: what is the profit-maximizing two-part tariff?**

$$\boldsymbol{p_1 + p_2 x}$$

$$\boldsymbol{p_2^* = MC = 0}$$

$$\boldsymbol{p_1 = 25}$$