Lecture 2: Part 1

Preferences

Recap

A consumer is a rational agent who always chooses the most preferred consumption bundle available to her.

To model this optimization problem, we need to model:

- **the choice set (Lec1)**
- **preferences (Today)**

Preference Relations

Preference relations are ordinal relations that compare two different consumption bundles, x and y:

- **strict preference: x is more preferred than is y; $x \succ y$**
- **indifference: x is exactly as preferred as is y; $x \sim y$**
- **weak preference: x is as at least as preferred as is y; $x \succcurlyeq y$ ($x \succ y$ or $x \sim y$)**

偏好关系是在两个商品组合间进行比较的一种次序关系。

Preference Relations

$x \succeq y$ and $y \succeq x$ imply $x \sim y$.

Preference Relations

$x \succsim y$ and $y \succsim x$ imply $x \sim y$.

$x \succsim y$ and (not $y \succsim x$) imply $x \succ y$.

Preference Relations

$x \succsim y$ and $y \succsim x$ imply $x \sim y$.

$x \succsim y$ and (not $y \succsim x$) imply $x \succ y$.

严格偏好($\succ$)和无差异($\sim$)都可以由弱偏好关系($\succsim$)等价地表示

Assumptions about Preference Relations

Completeness (完备性): For any two bundles x and y it is always possible to make the statement that either

$$x \succsim y$$

or

$$y \succsim x.$$

Assumptions about Preference Relations

Reflexivity (自反性): Any bundle x is always at least as preferred as itself; *i.e.*

$$x \succsim x.$$

Assumptions about Preference Relations

Transitivity (传递性): If
x is at least as preferred as y, and
y is at least as preferred as z, then
x is at least as preferred as z; *i.e.*

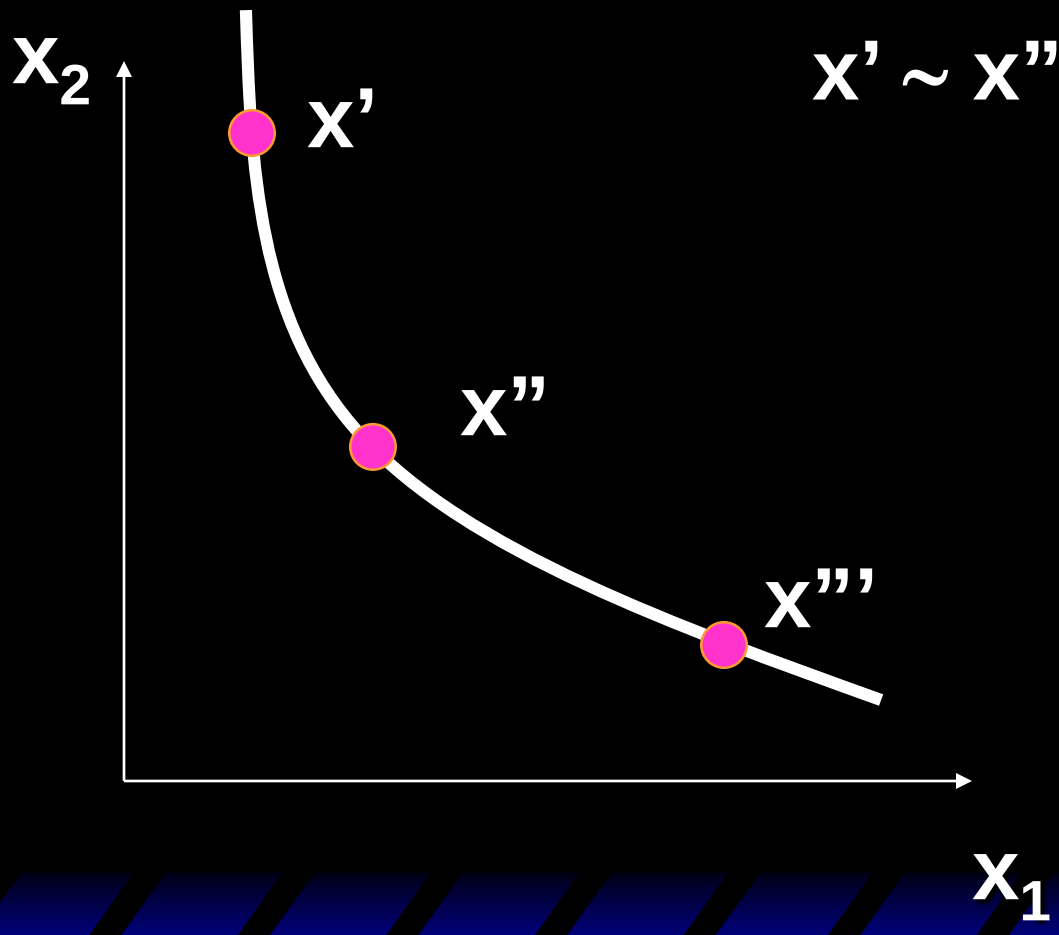
$$x \succsim y \text{ and } y \succsim z \rightarrow x \succsim z.$$

Indifference Curves

Take a reference bundle x'. The set of all bundles equally preferred to x' is the indifference curve containing x'; the set of all bundles y ~ x'.

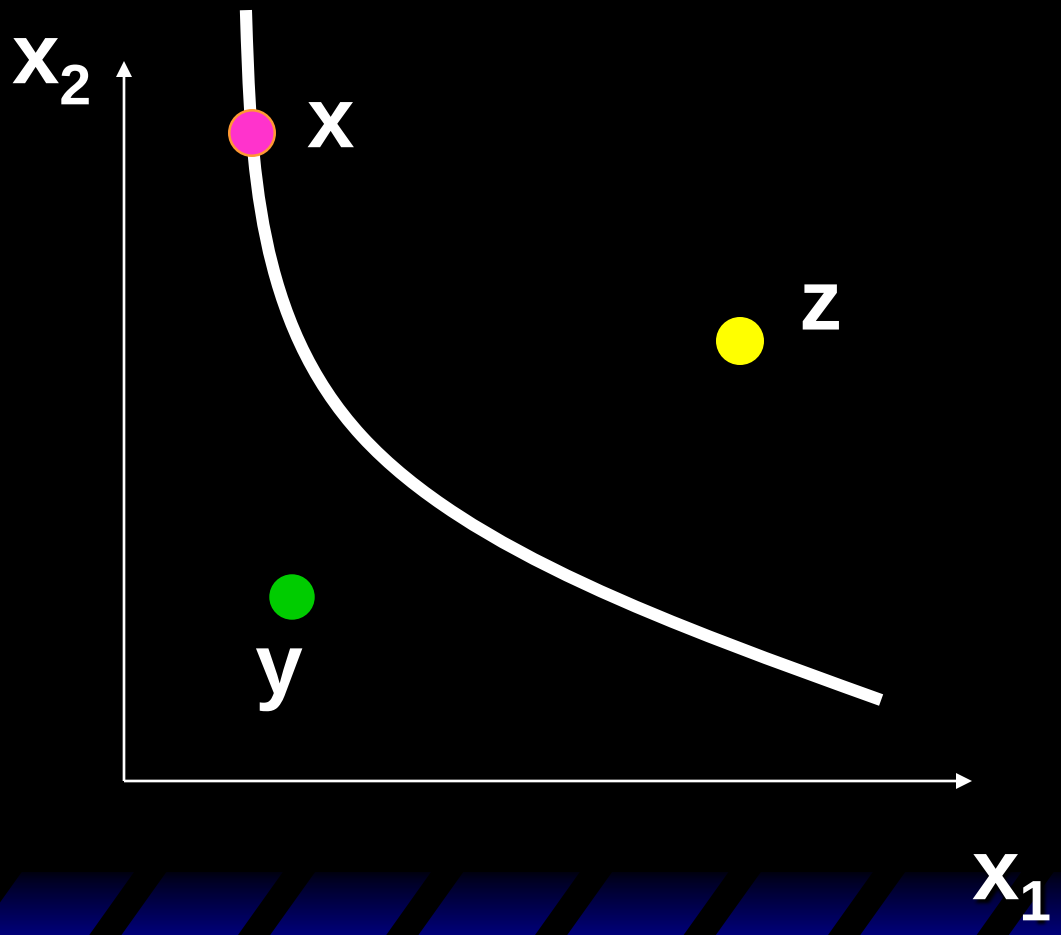
一条经过商品组合x'的无差异曲线是所有和x'受到同样偏好的商品组合的集合。

Indifference Curves

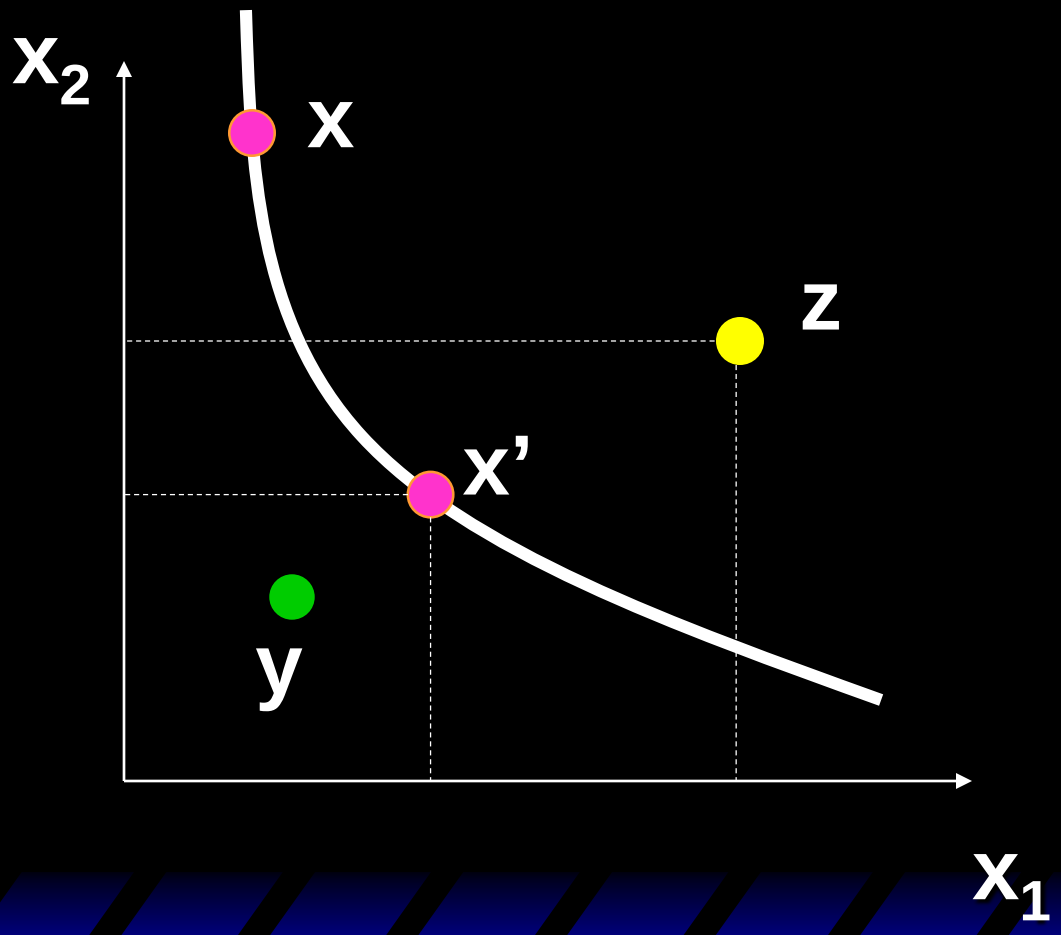


$x' \sim x'' \sim x'''$

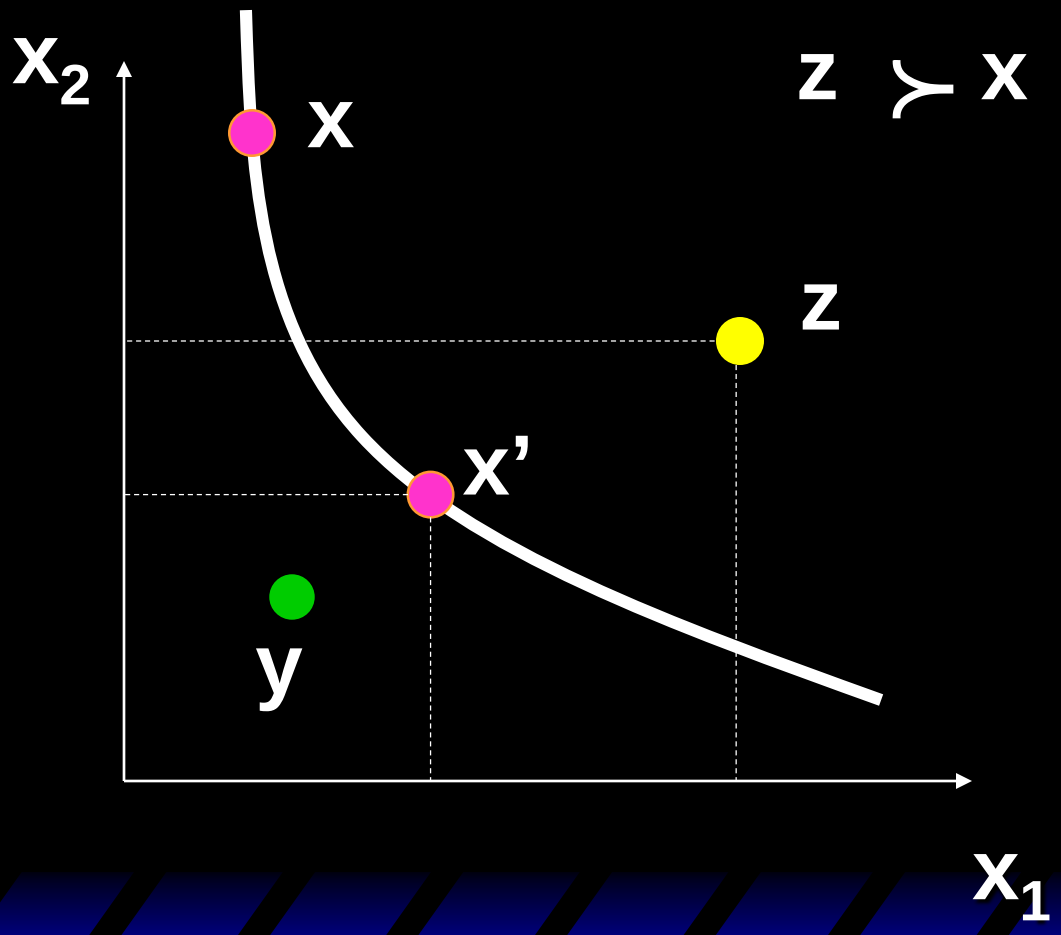
Indifference Curves



Indifference Curves

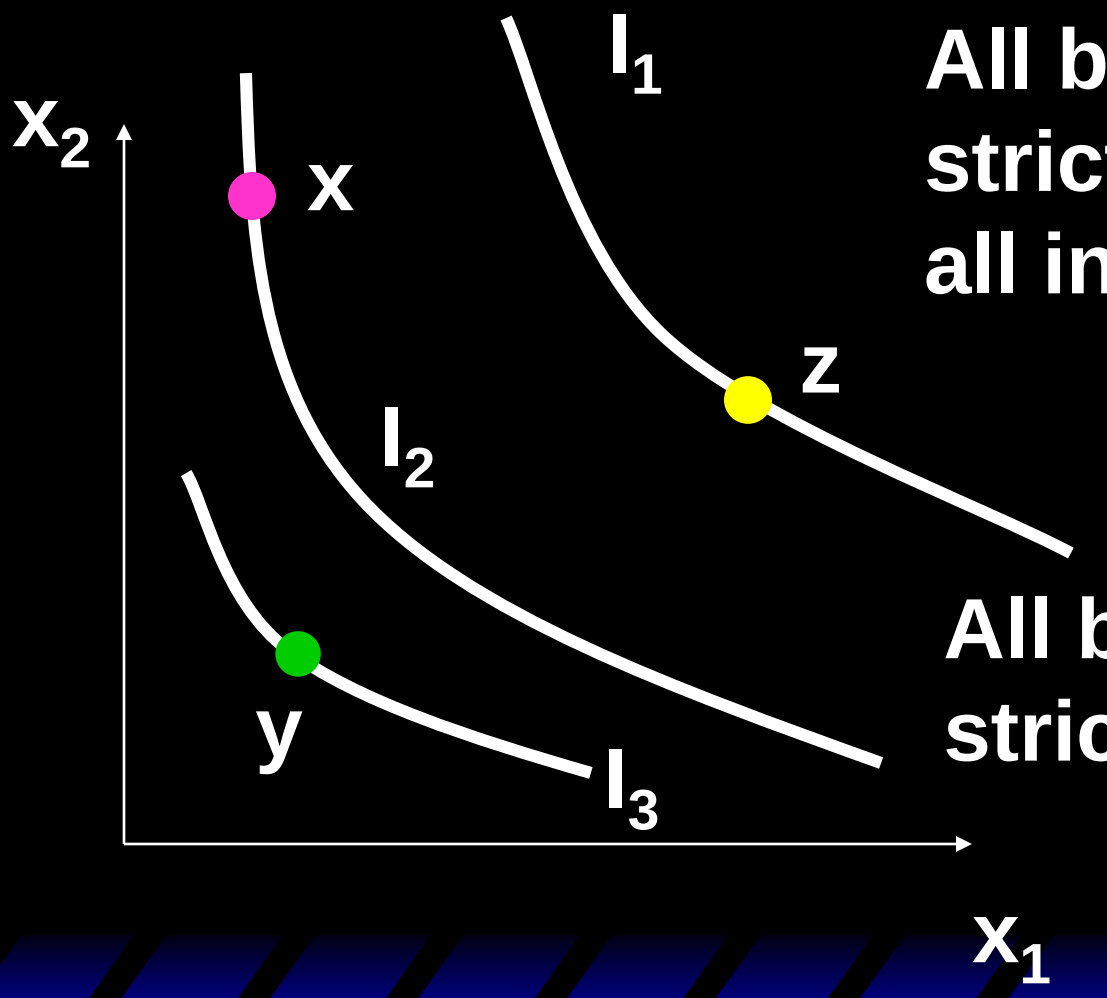


Indifference Curves



$z \succ x \succ y$

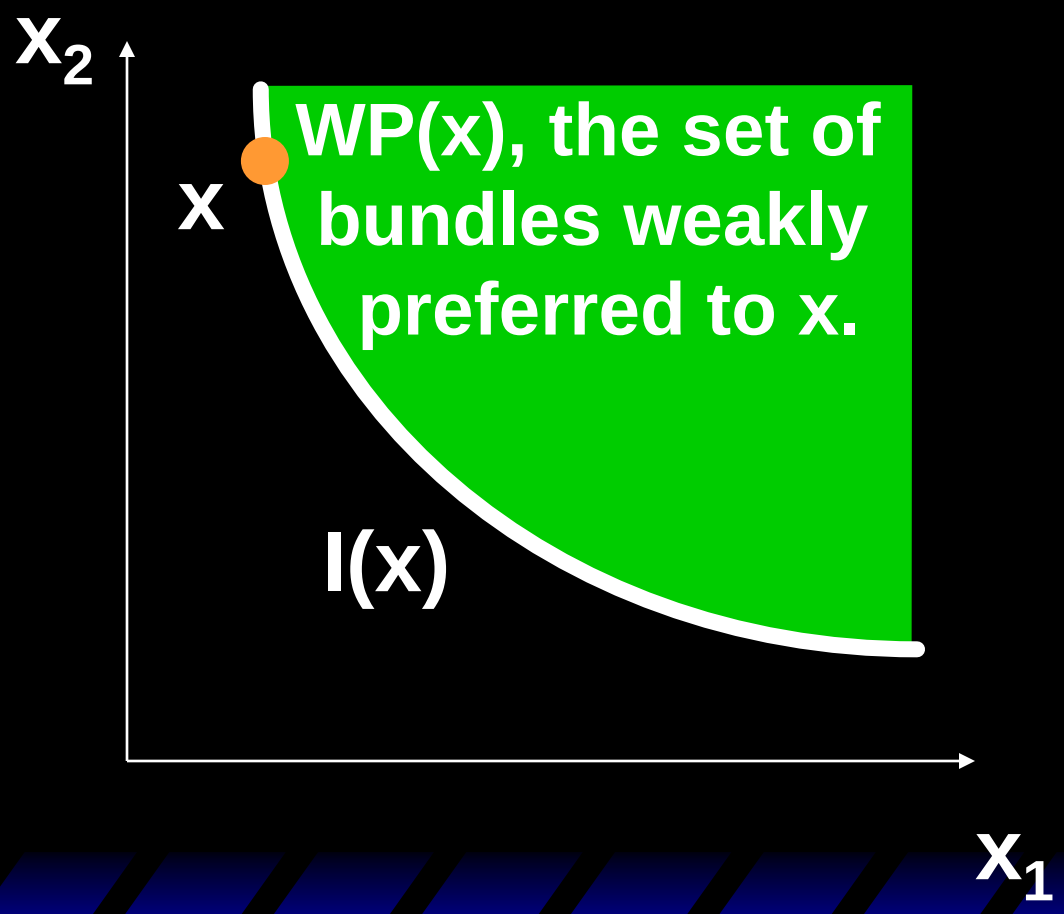
Indifference Curves



All bundles in I_1 are strictly preferred to all in I_2.

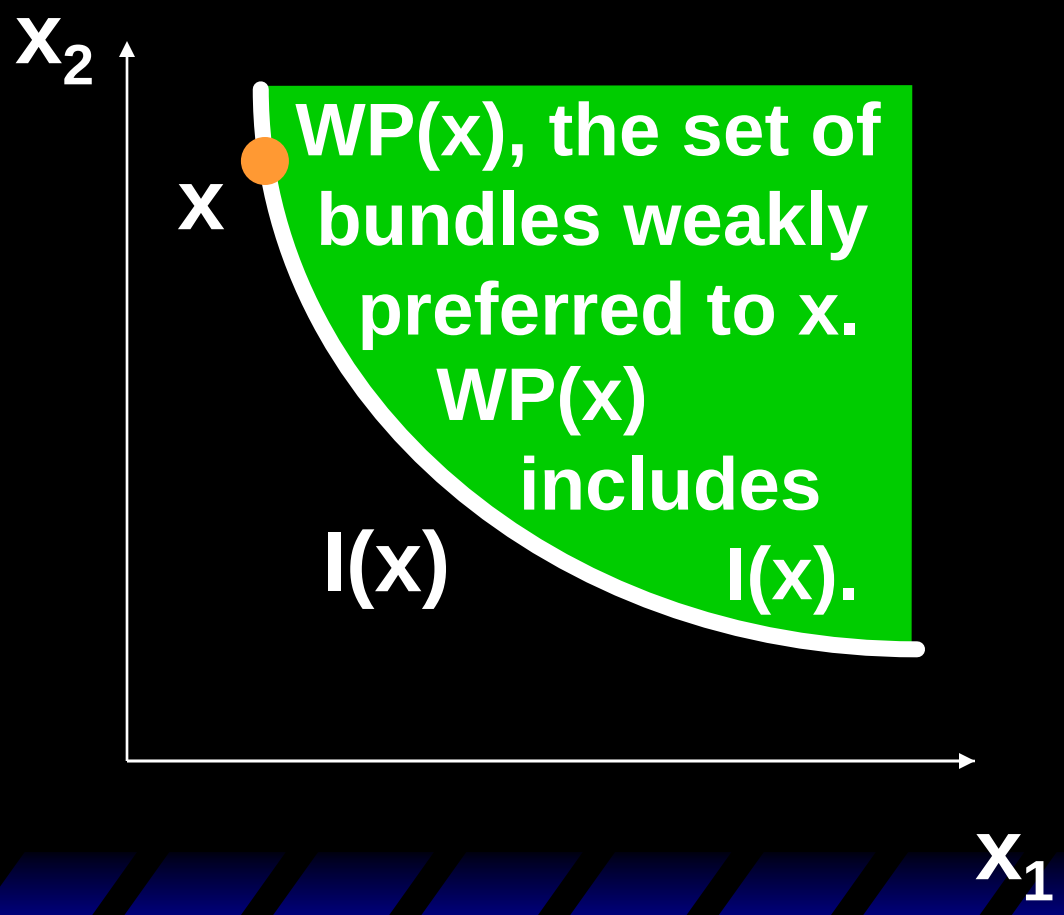
All bundles in I_2 are strictly preferred to all in I_3.

Indifference Curves



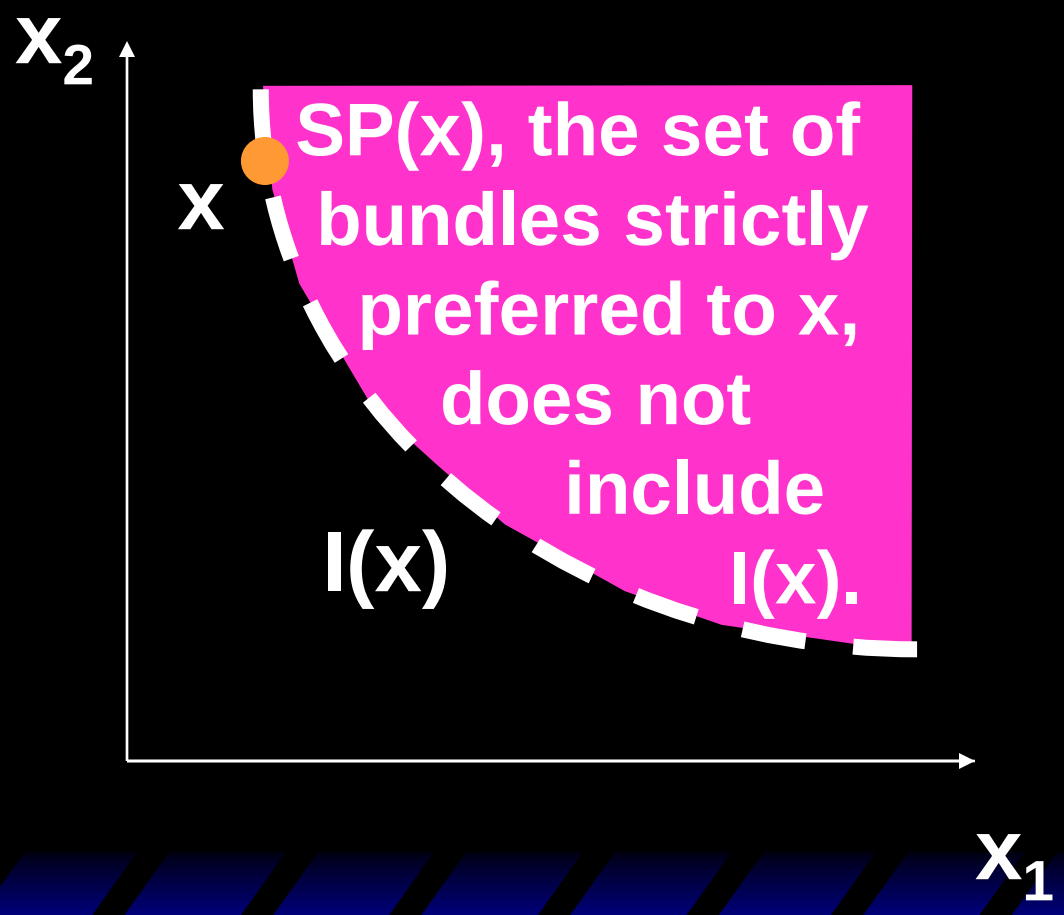
x的弱偏好集 (weakly preferred set)：所有弱偏好于x的商品组合的集合。

Indifference Curves



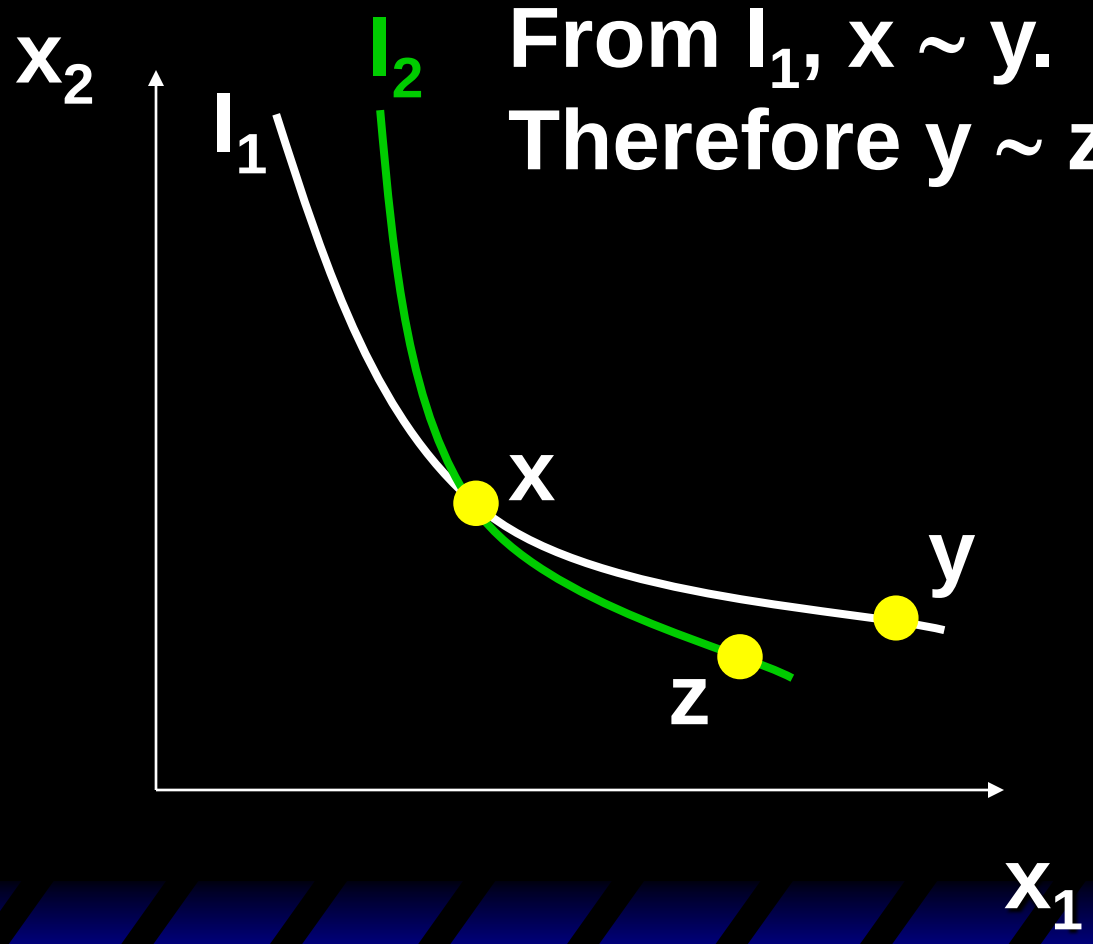
x的弱偏好集包含经过x的无差异曲线。

Indifference Curves



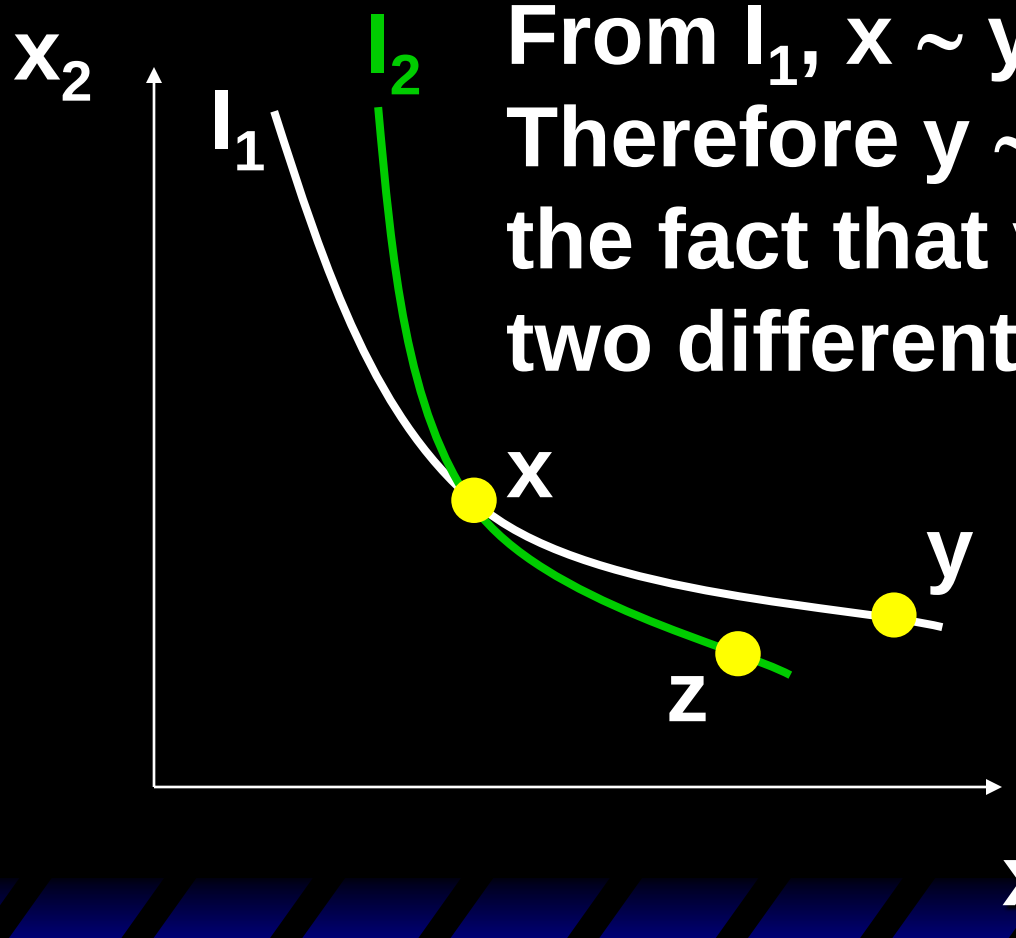
x的严格偏好集(strictly preferred set)不包含经过x的无差异曲线。

Indifference Curves Cannot Intersect



From I_1, x ~ y. From I_2, x ~ z. Therefore y ~ z.

Indifference Curves Cannot Intersect



From I_1, x ~ y. From I_2, x ~ z. Therefore y ~ z. It contradicts the fact that y and z are on two different curves.

两条不同的无差异曲线不可能相交（反证法）

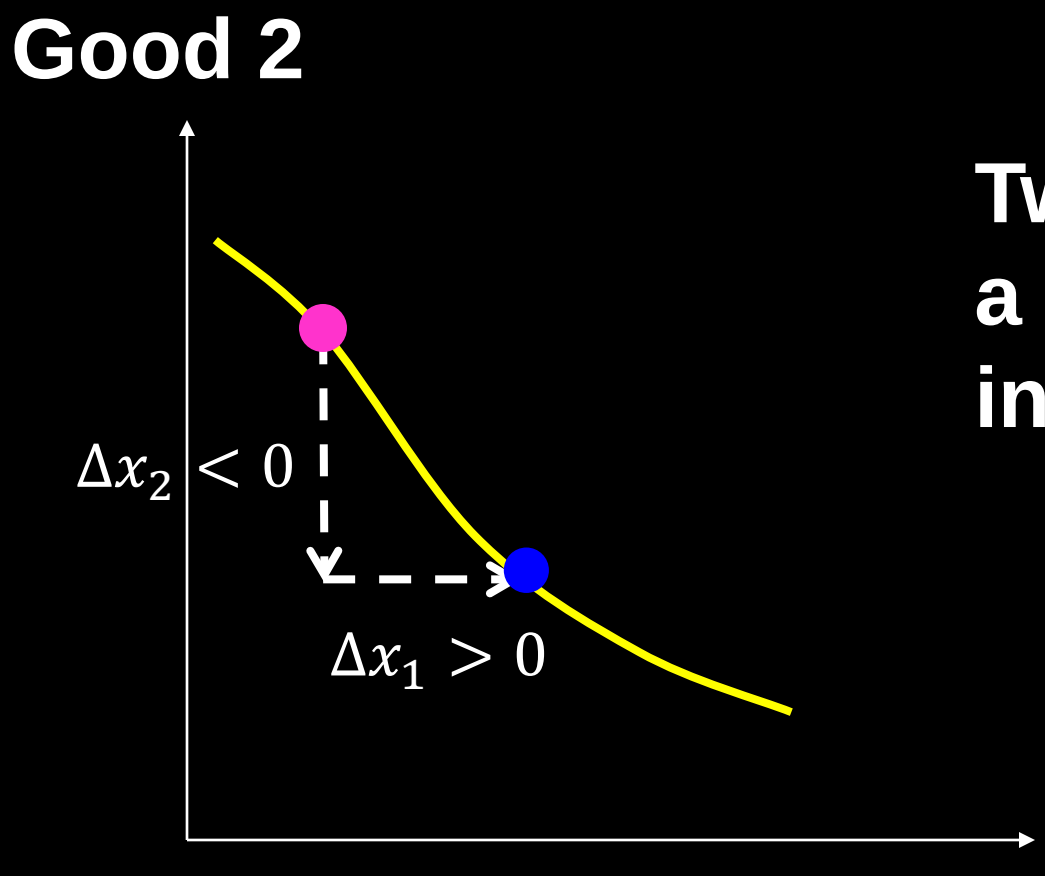
Slopes of Indifference Curves

When more of a commodity is always preferred, the commodity is a good

数量越多越受偏好的商品被称为“好商品”

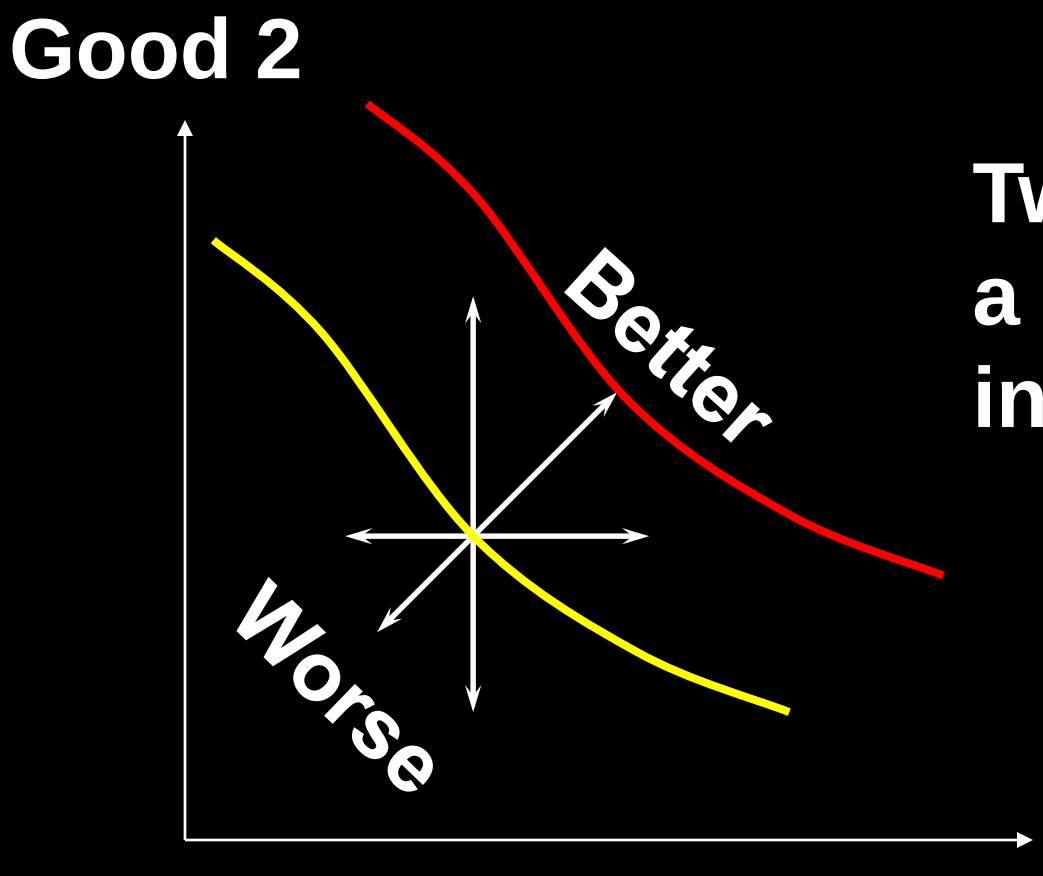
If every commodity is a good, then indifference curves are negatively sloped.

Slopes of Indifference Curves



Two goods ➡ a negatively sloped indifference curve.

Slopes of Indifference Curves



Two goods ➡ a negatively sloped indifference curve.

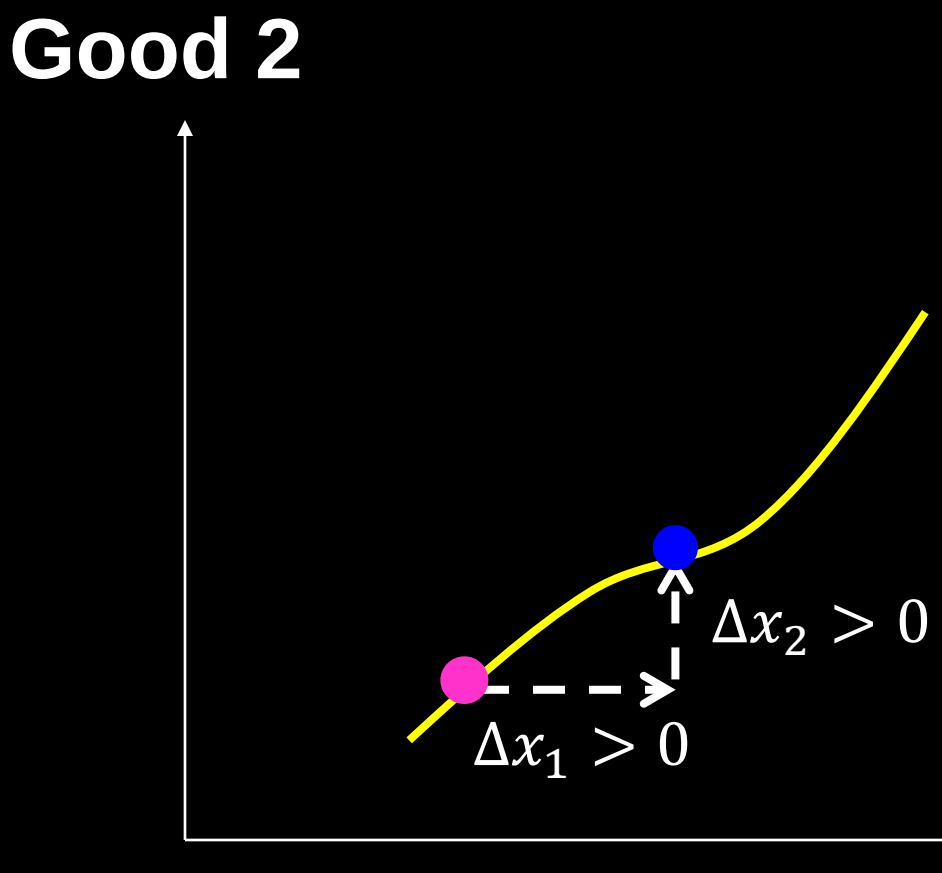
当两种商品都是好商品时，无差异曲线斜率为负；且离原点越远受到的偏好程度越高。

Slopes of Indifference Curves

If less of a commodity is always preferred, then the commodity is a bad.

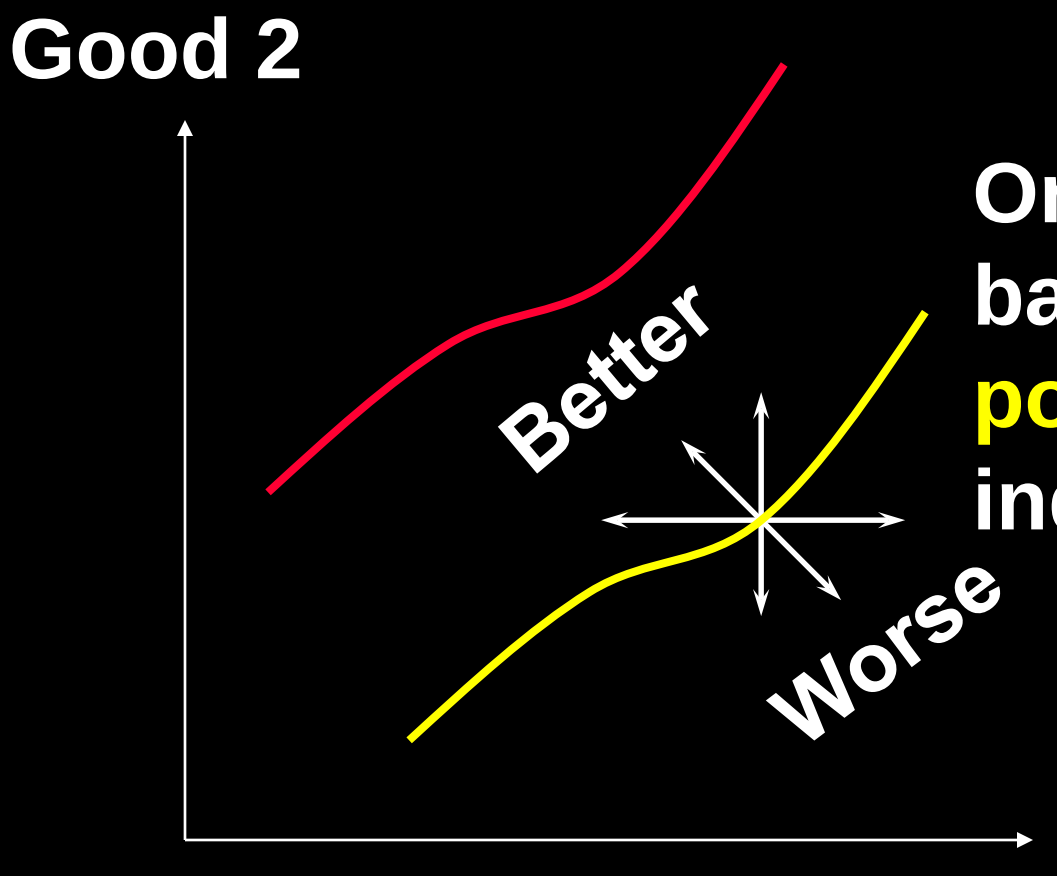
数量越少越受偏好的商品被称为“厌恶品”

Slopes of Indifference Curves



One good and one bad ⇒ a positively sloped indifference curve.

Slopes of Indifference Curves



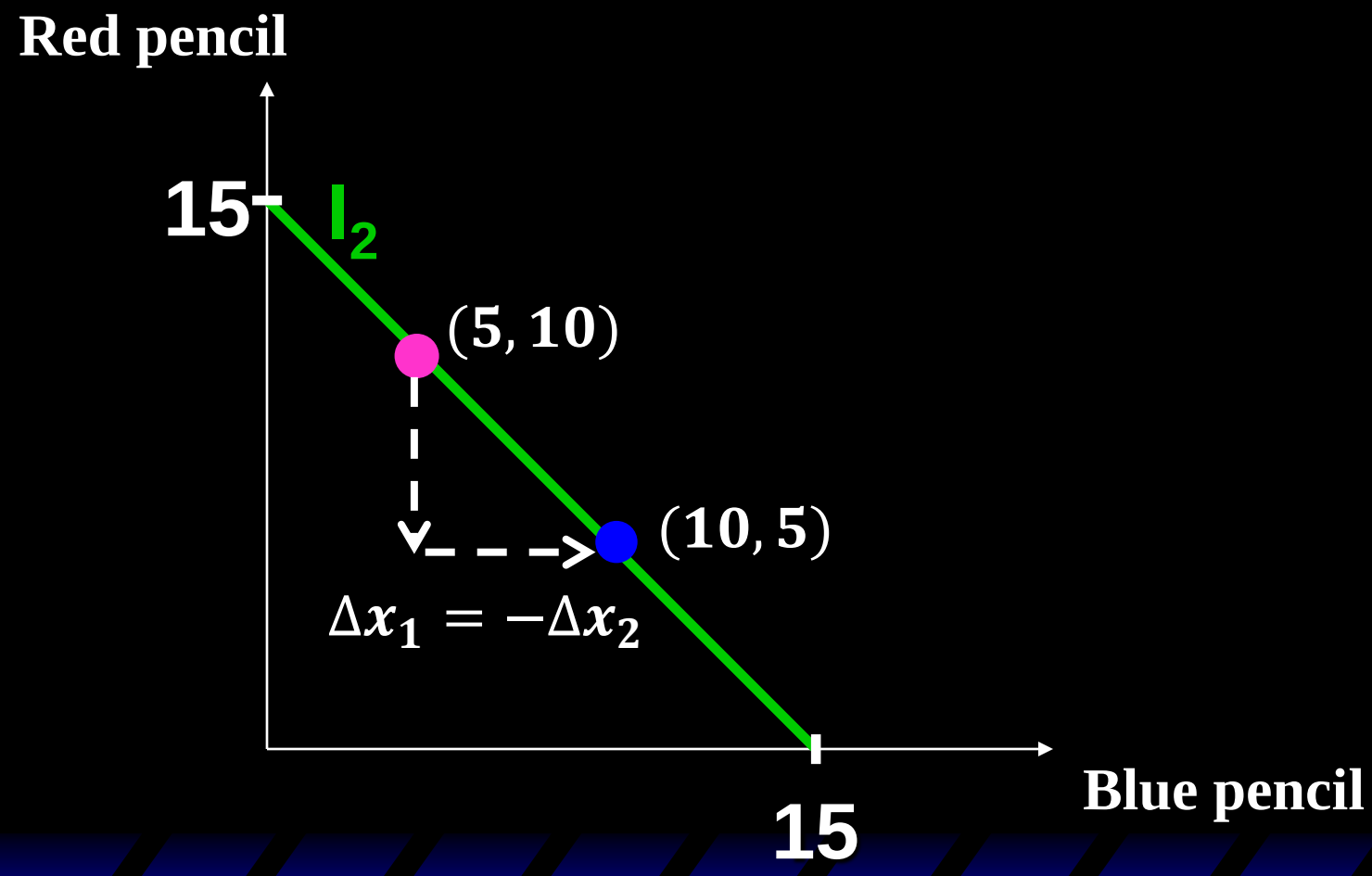
One good and one bad → a positively sloped indifference curve.

Extreme Cases of Indifference Curves; Perfect Substitutes

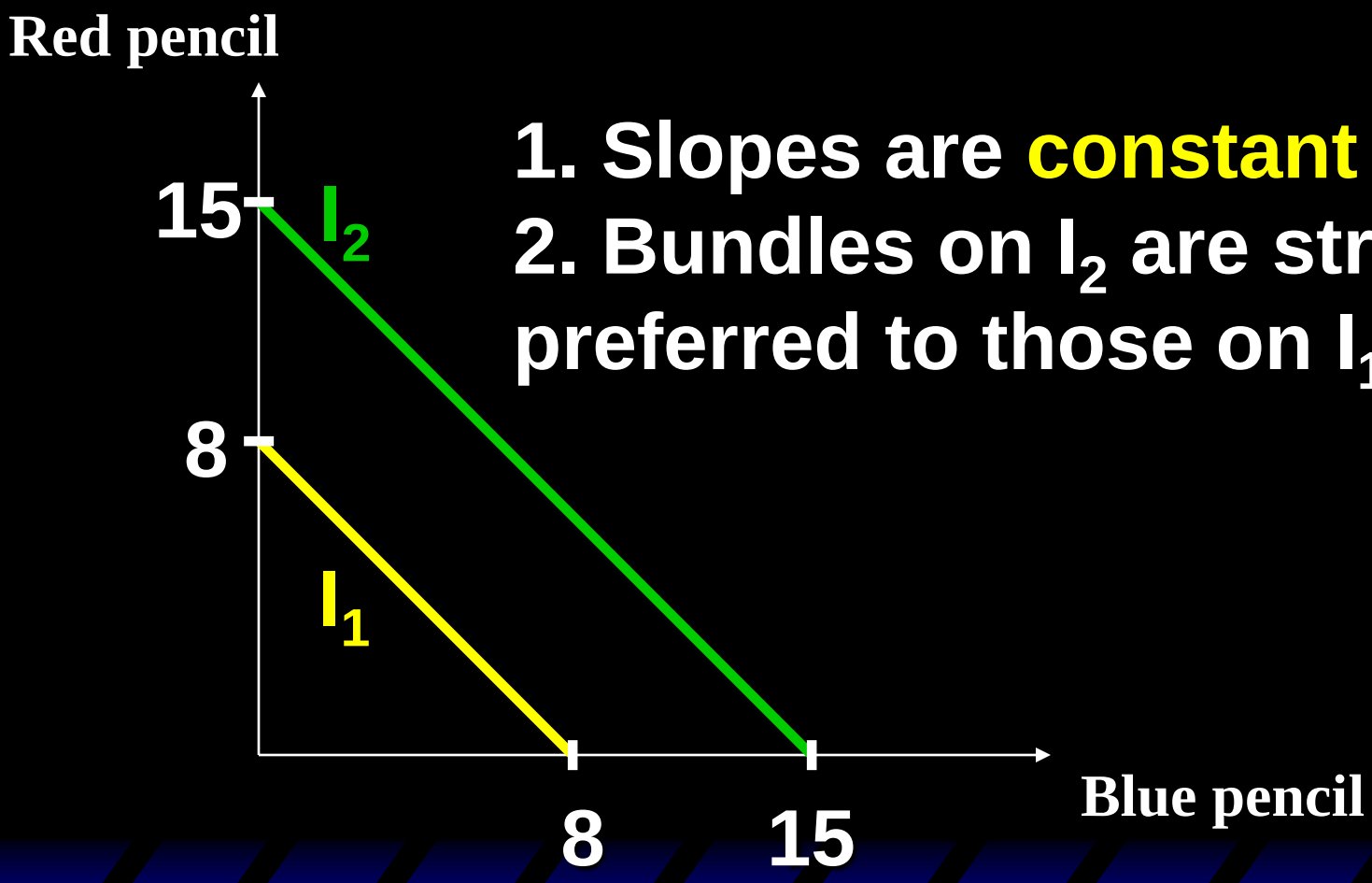
If a consumer always regards 1 unit of commodity 1 and a constant units of commodity 2 as equivalent, then the two commodities are perfect substitutes (完全替代品).

- **e.g. blue pencil and red pencil; $20 bill and $10 bill**

Extreme Cases of Indifference Curves; Perfect Substitutes



Extreme Cases of Indifference Curves; Perfect Substitutes

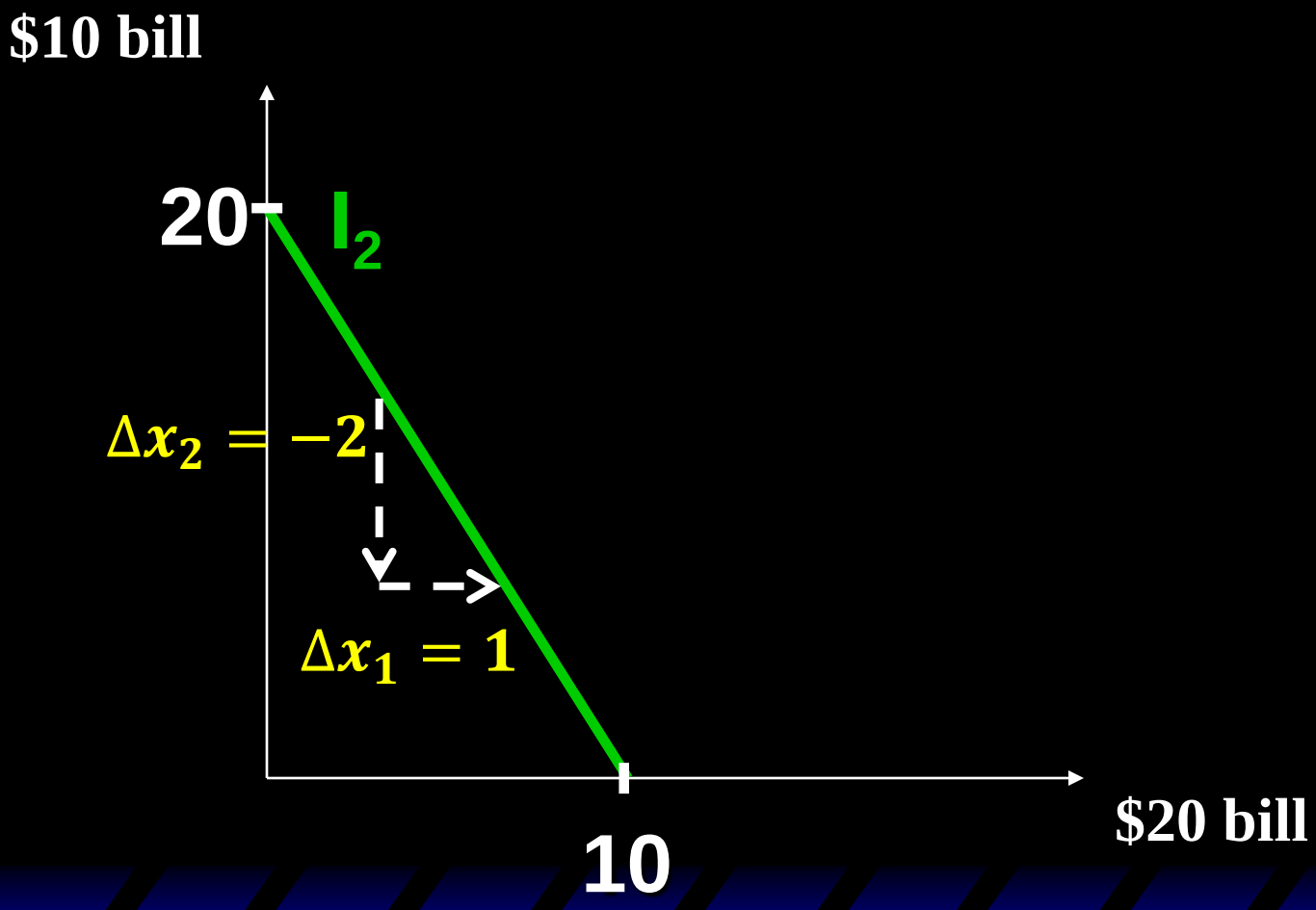


1. Slopes are **constant** at 1.
2. Bundles on I_2 are strictly preferred to those on I_1.

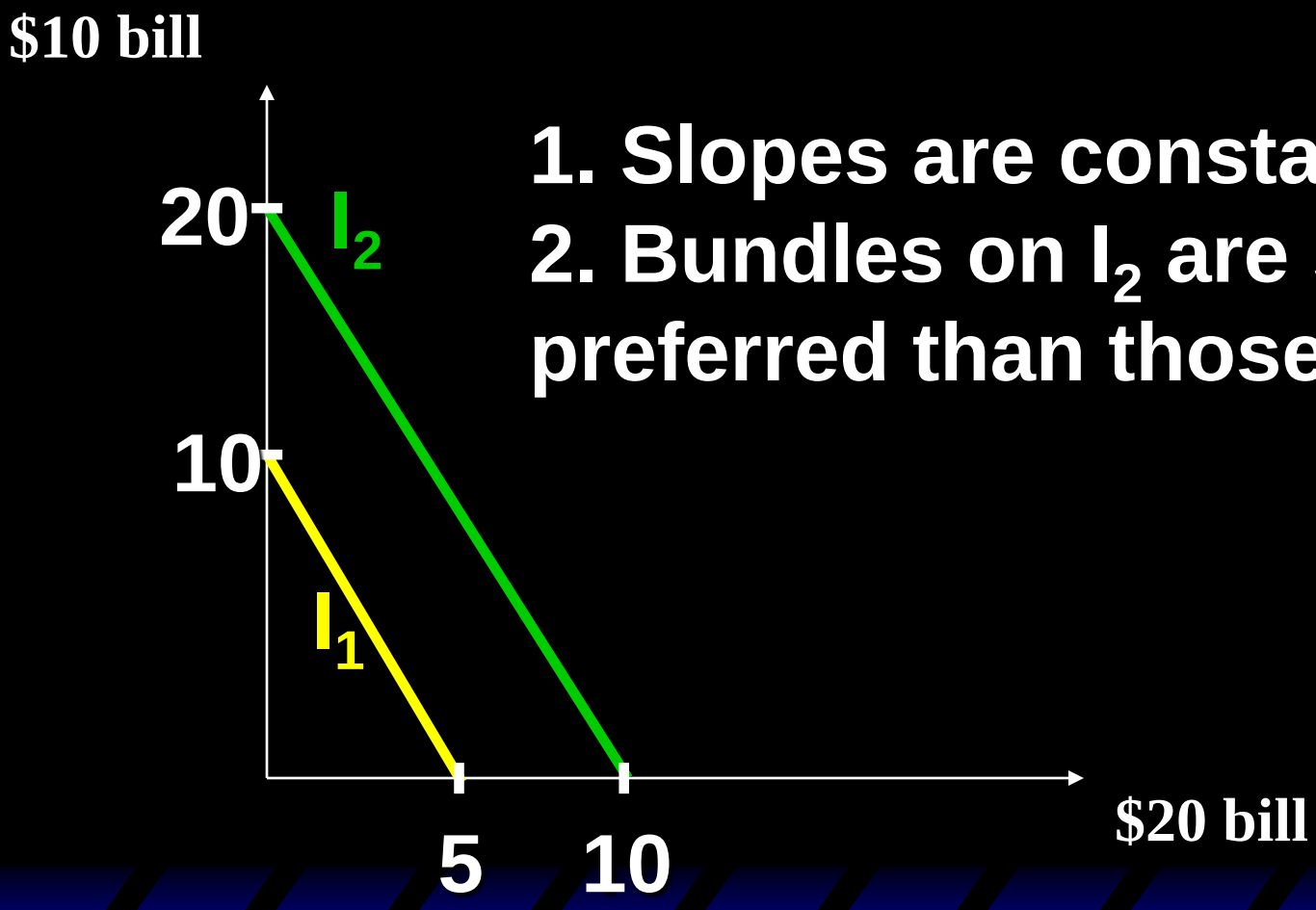
Extreme Cases of Indifference Curves; Perfect Substitutes

If x_1 is the \$20 bill and x_2 is the \$10 bill, what would the indifference curves look like?

Extreme Cases of Indifference Curves; Perfect Substitutes



Extreme Cases of Indifference Curves; Perfect Substitutes



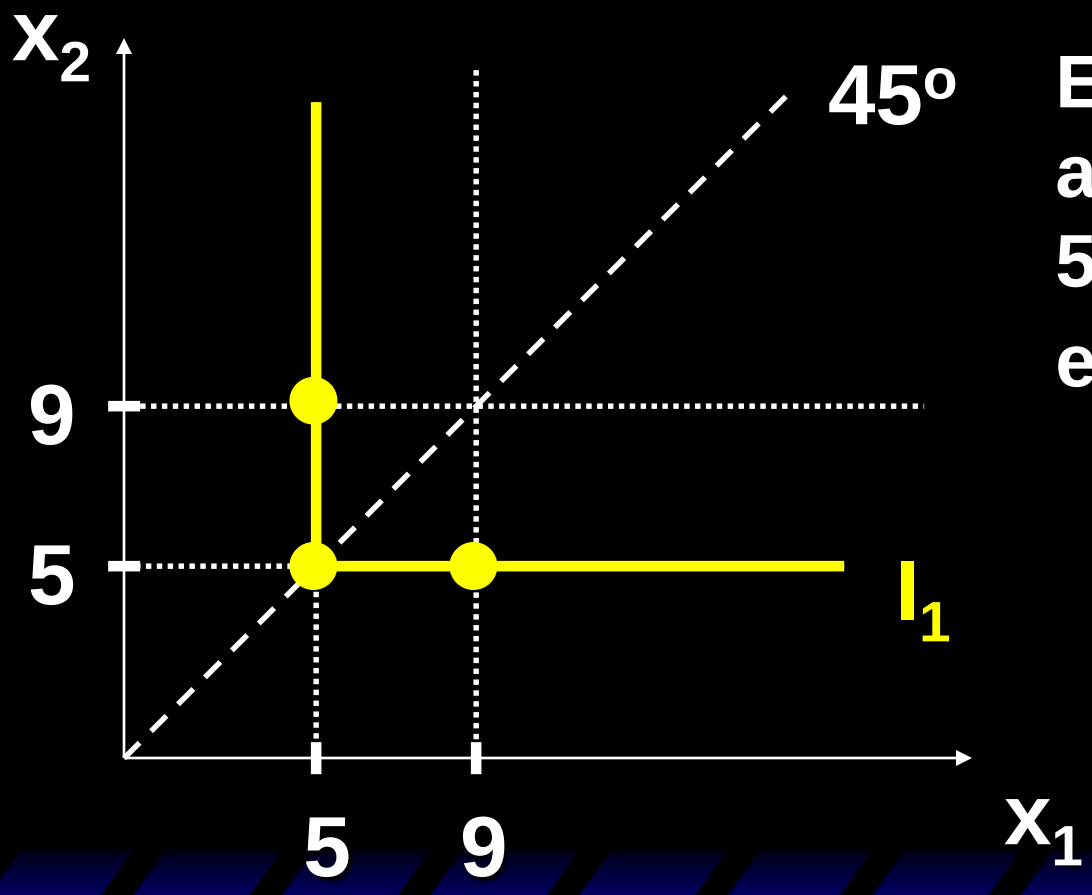
1. Slopes are constant at 2.
2. Bundles on I_2 are strictly preferred than those on I_1.

Extreme Cases of Indifference Curves; Perfect Complements

If a consumer always consumes 1 unit of commodity 1 with a constant units of commodity 2 (e.g. one-to-one), then the commodities are perfect complements.

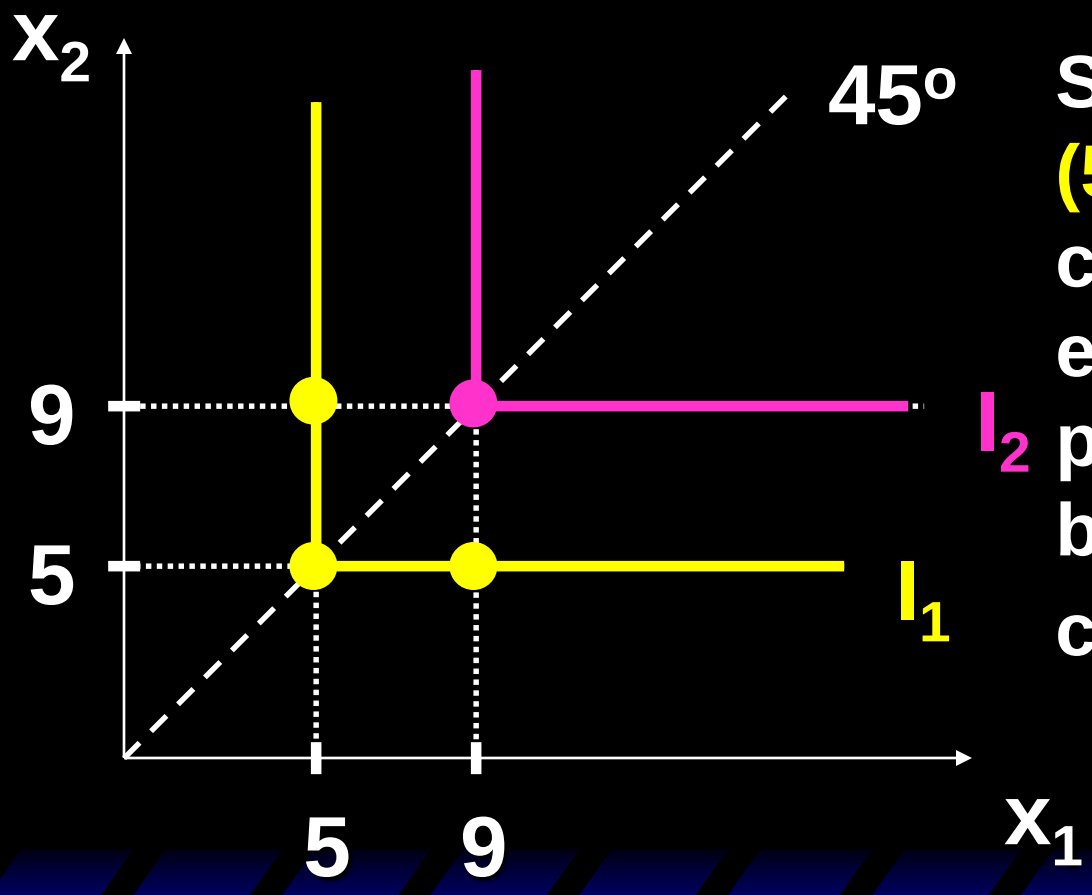
始终以固定的比例一起消费的两种商品叫做完全互补品

Extreme Cases of Indifference Curves; Perfect Complements



Each of (5,5), (5,9) and (9,5) contains 5 pairs so each is equally preferred.

Extreme Cases of Indifference Curves; Perfect Complements

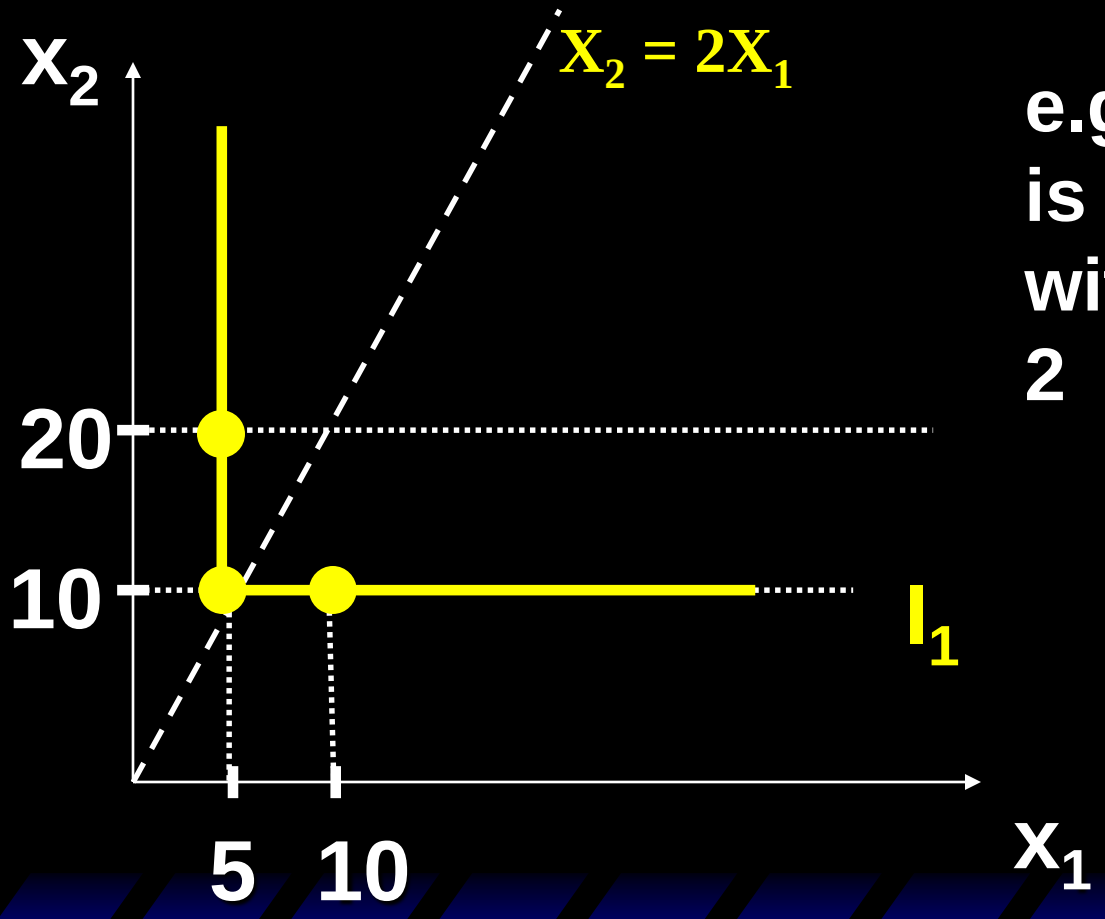


Since each of (5,5), (5,9) and (9,5) contains 5 pairs, each is less preferred than the bundle (9,9) which contains 9 pairs.

Extreme Cases of Indifference Curves; Perfect Complements

What if 1 unit of commodity 1 is always consumed with 2 units of commodity 2? What will the indifferent curves look like?

Extreme Cases of Indifference Curves; Perfect Complements



e.g. 1 unit of good 1 is always consumed with 2 units of good 2

Well-Behaved Preferences

A preference relation is "well-behaved" if it is

- **monotonic and convex.**

Monotonicity (单调性): More of any commodity is always preferred (*i.e.* no satiation and every commodity is a good).

Well-Behaved Preferences

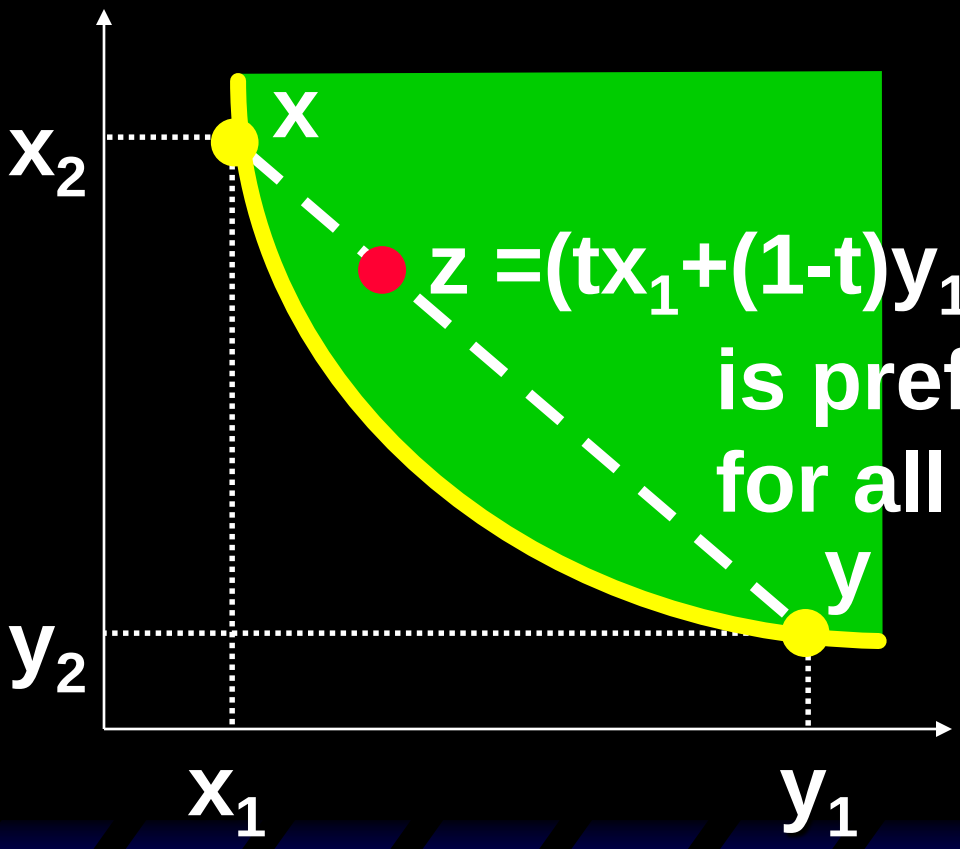
Convexity (凸偏好): Mixtures of two equally preferred bundles are (at least weakly) preferred to the bundles themselves.

If $x \sim y$ and $z = t*x + (1-t)*y$, then

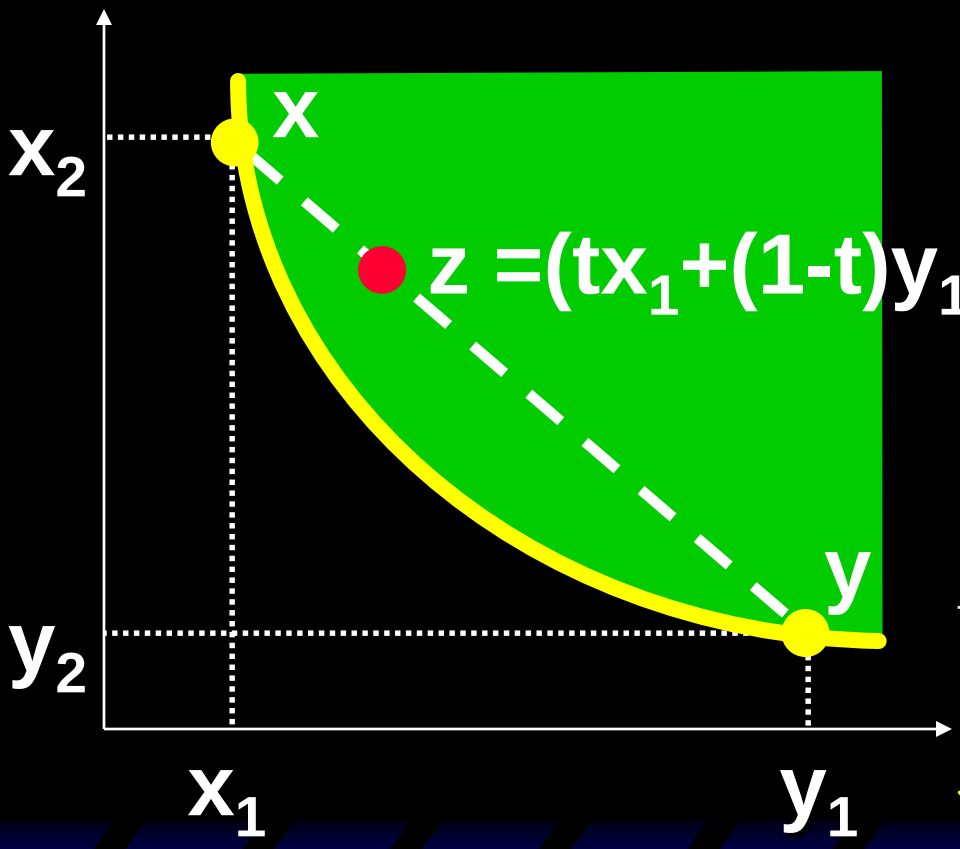
$$z \succcurlyeq x \text{ and } z \succcurlyeq y \ (\forall\ 0 \leq t \leq 1)$$

对任意两个受到同样偏好的商品组合x和y做混合（加权平均），混合后的新组合受到的偏好程度至少和初始组合受到的偏好程度一样高。

Well-Behaved Preferences -- Convexity.



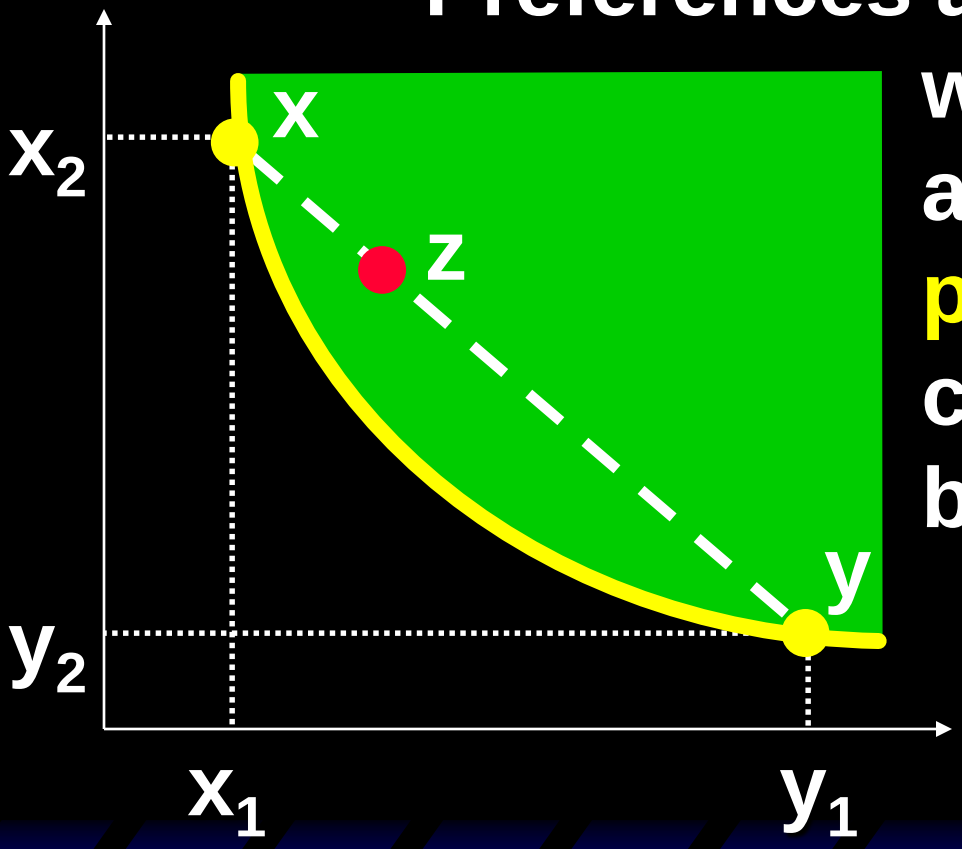
Well-Behaved Preferences -- Convexity.



Convexity means that "average is preferred to extreme"

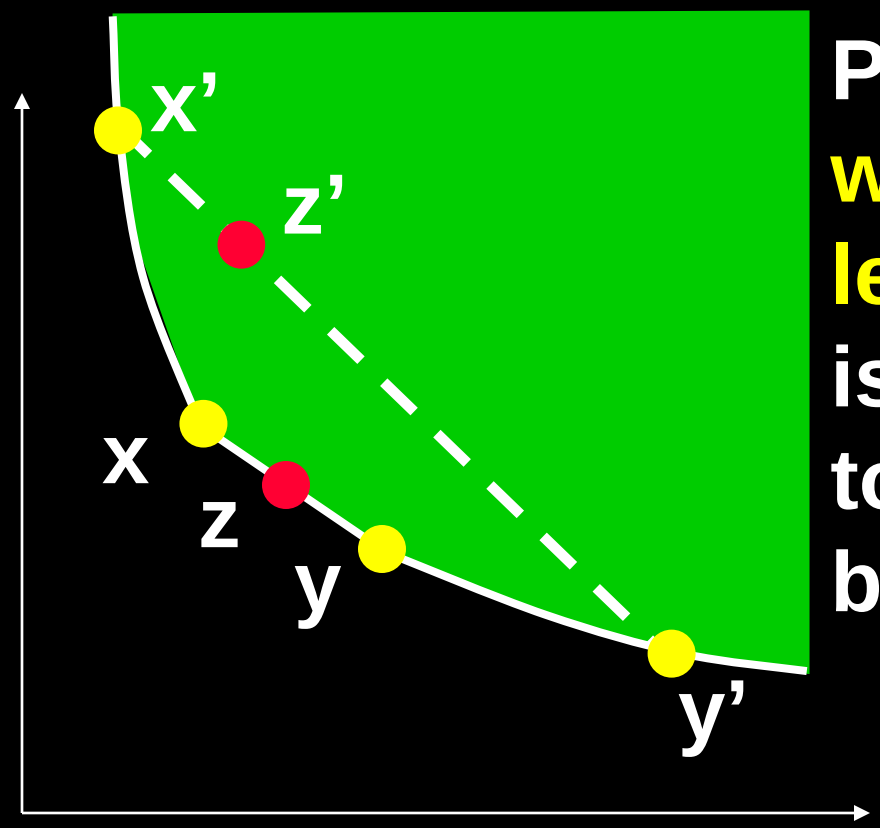
"平均比极端更受偏好"

Well-Behaved Preferences -- Convexity.



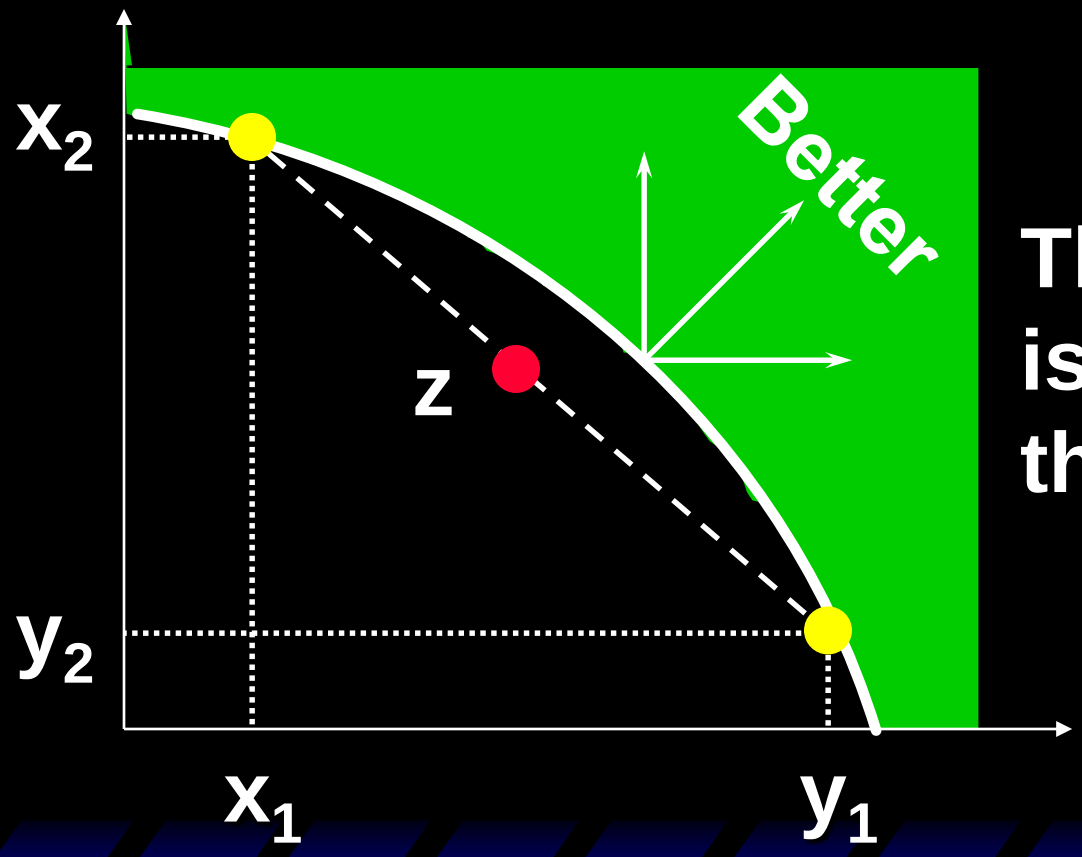
Preferences are strictly convex when all mixtures z are strictly preferred to their component bundles x and y.

Well-Behaved Preferences -- Weak Convexity.



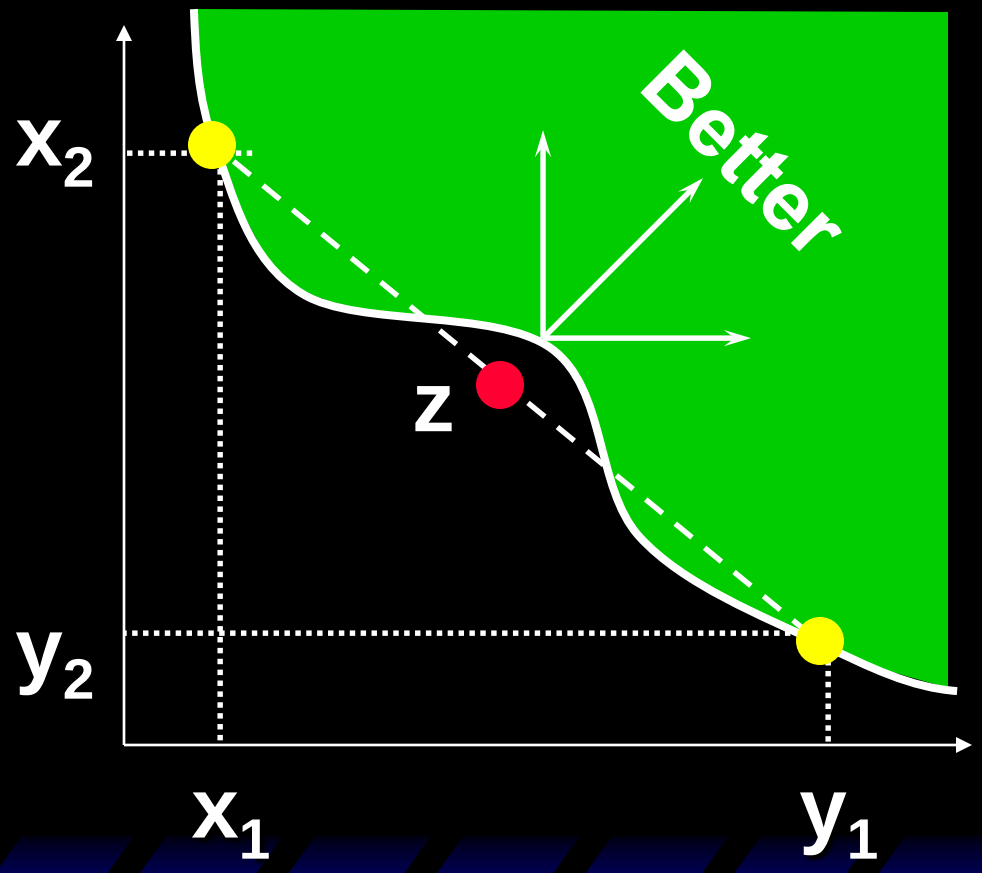
Preferences are weakly convex if at least one mixture z is equally preferred to a component bundle.

Non-Convex Preferences



The mixture z is less preferred than x or y.

More Non-Convex Preferences



The mixture z is less preferred than x or y.

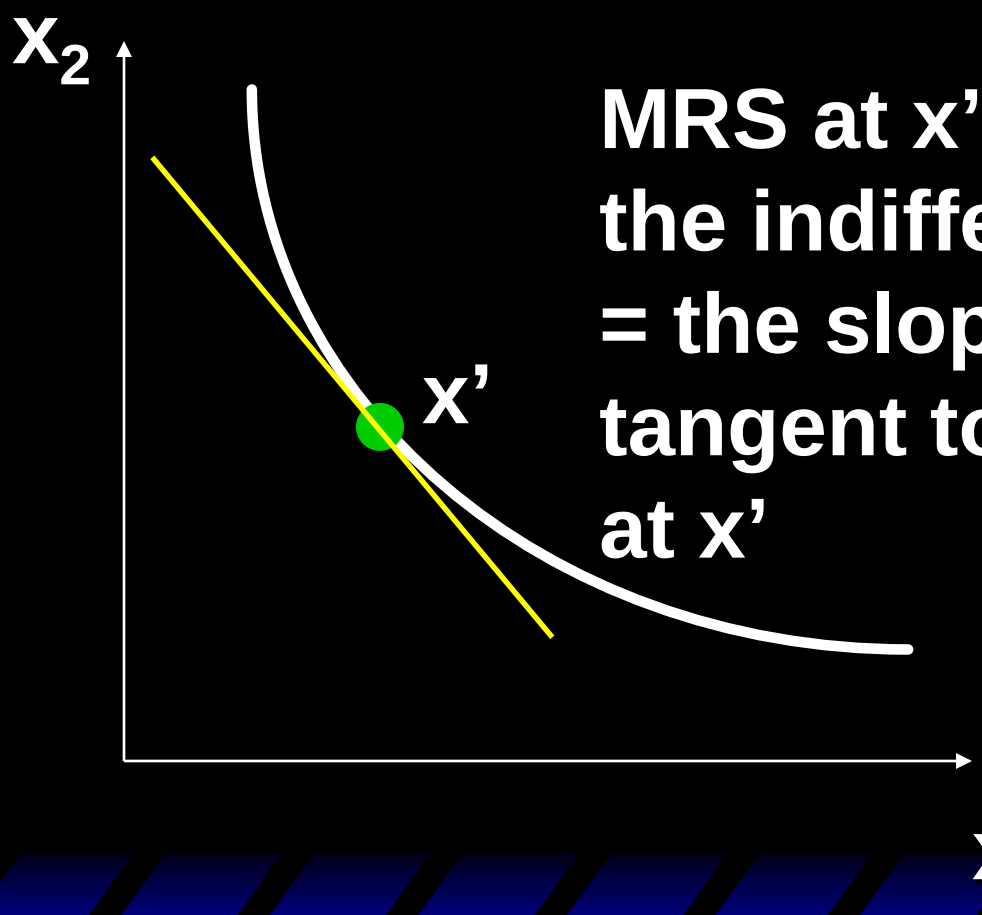
Slopes of Indifference Curves

The slope of an indifference curve is its marginal rate-of-substitution (MRS).

边际替代率

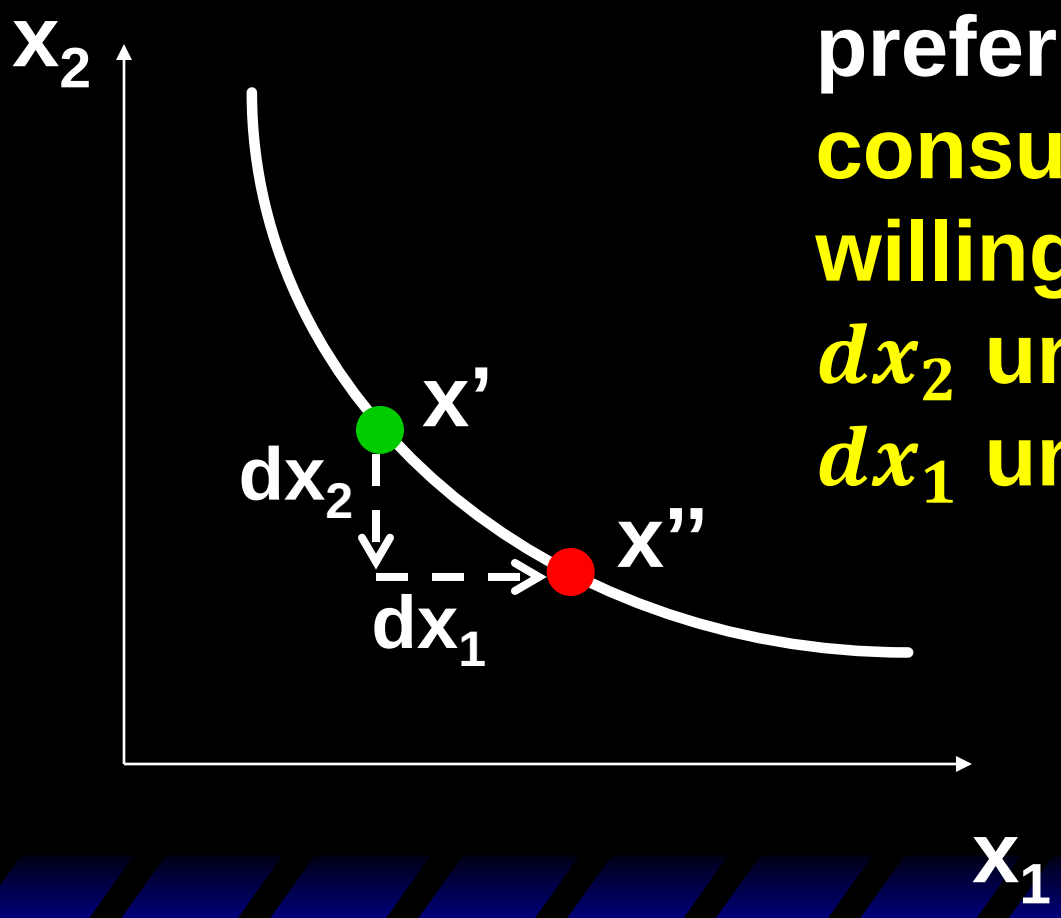
What does MRS represent?

Marginal Rate of Substitution



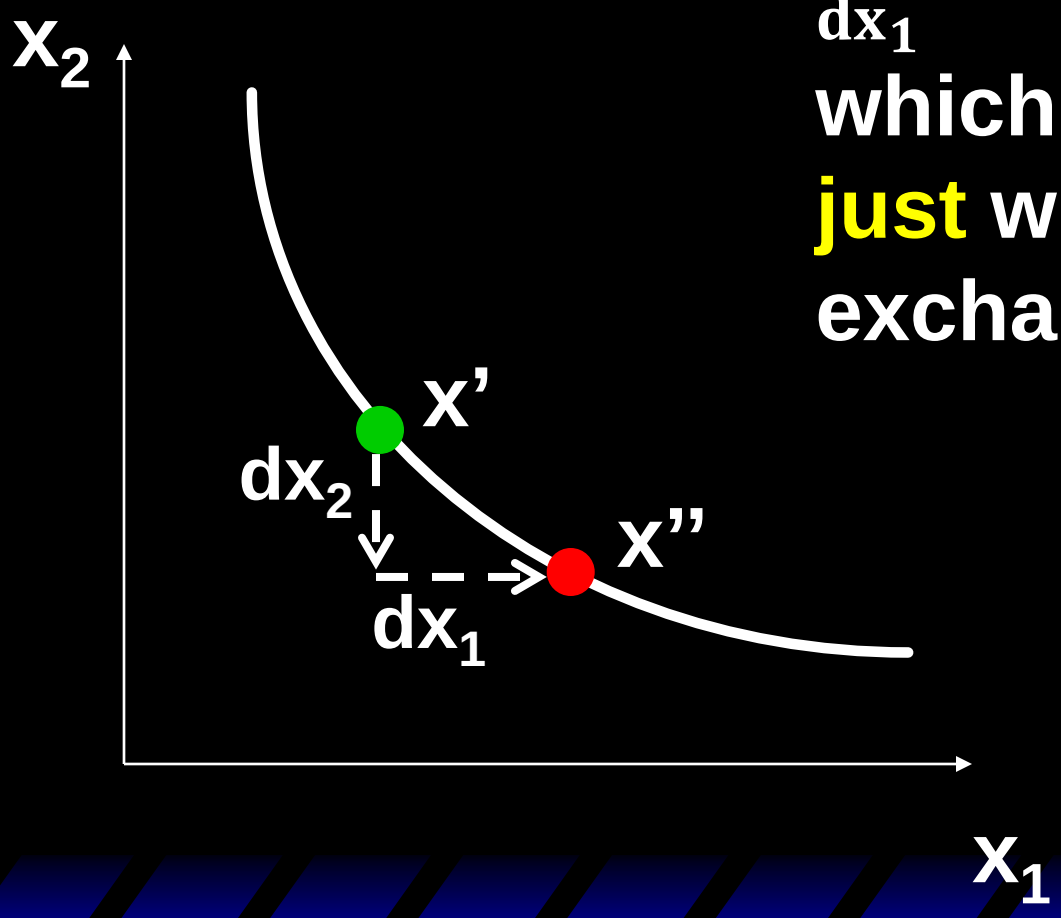
MRS at x' is the slope of the indifference curve at x' = the slope of the line tangent to the indiff. curve at x'

Marginal Rate of Substitution



x' and x'' are equally preferred. i.e. the consumer is just willing to exchange dx_2 units of good2 for dx_1 units of good 1.

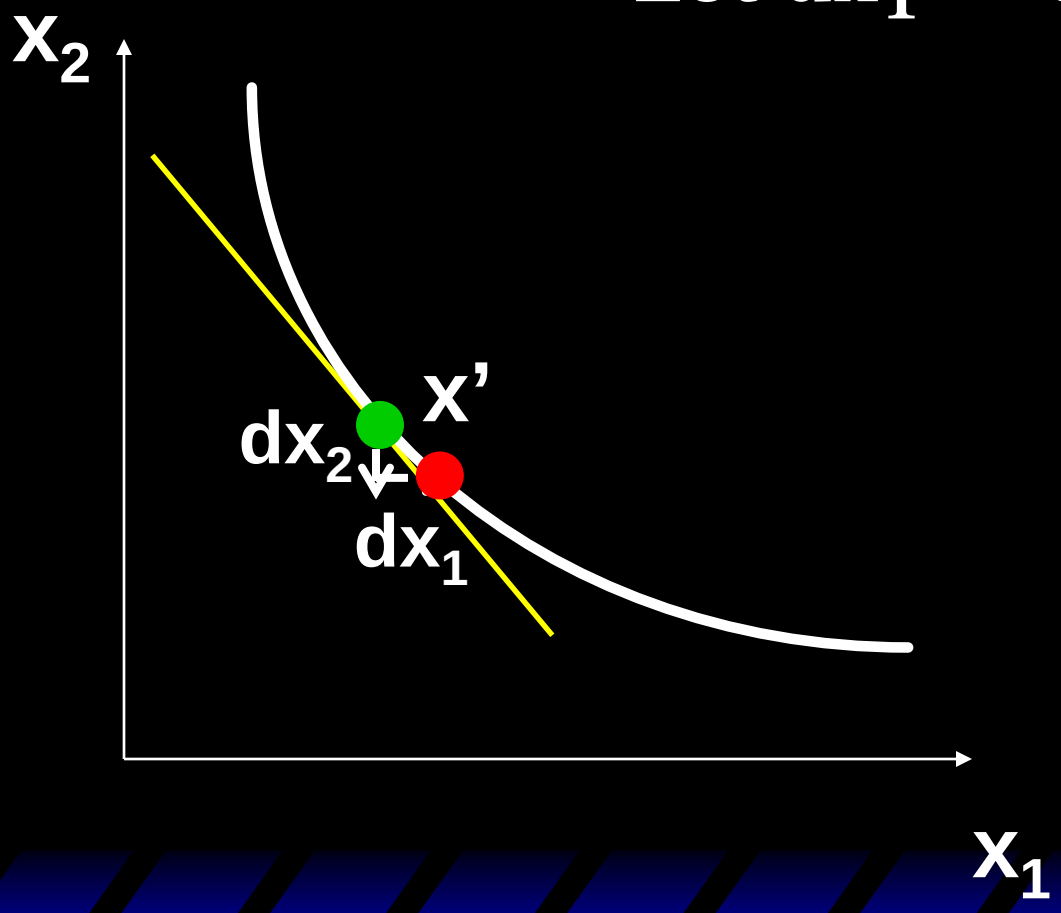
Marginal Rate of Substitution



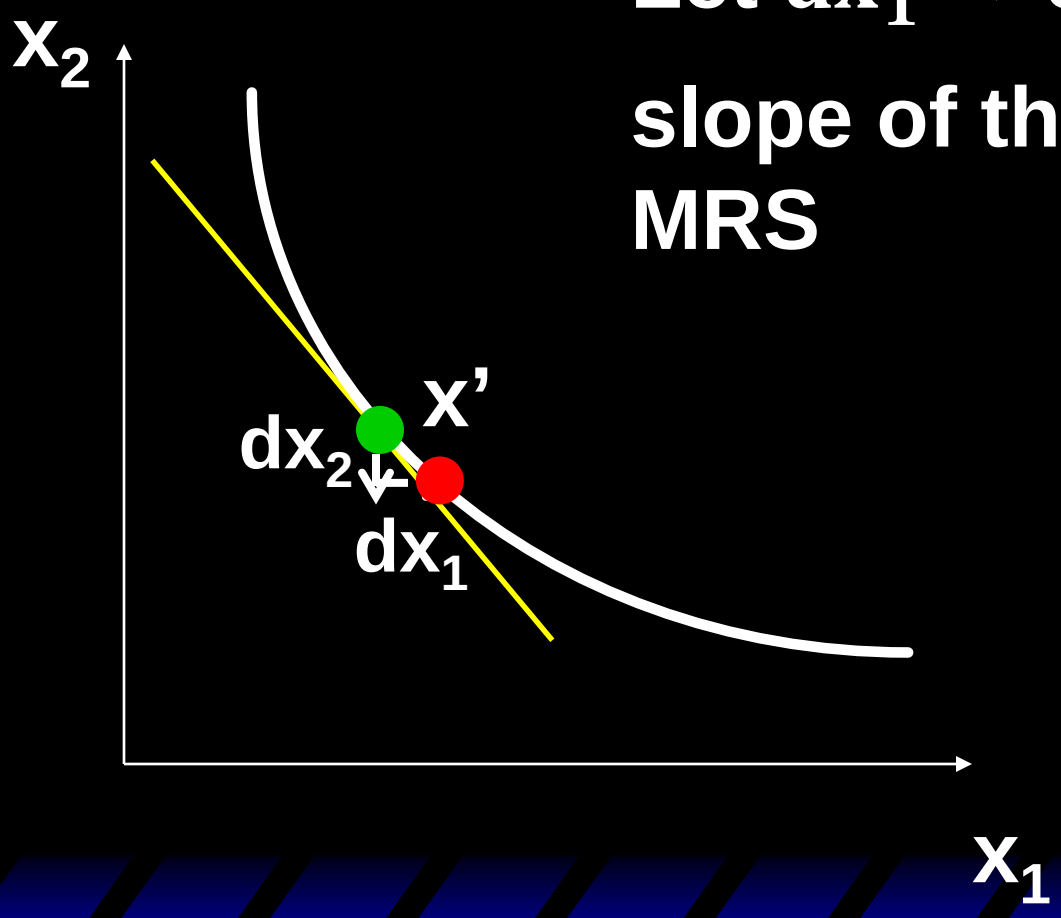
$\frac{dx_2}{dx_1}$ is the ratio at which the consumer is **just** willing to exchange x_2 for x_1

Marginal Rate of Substitution

Let $dx_1 \to 0$

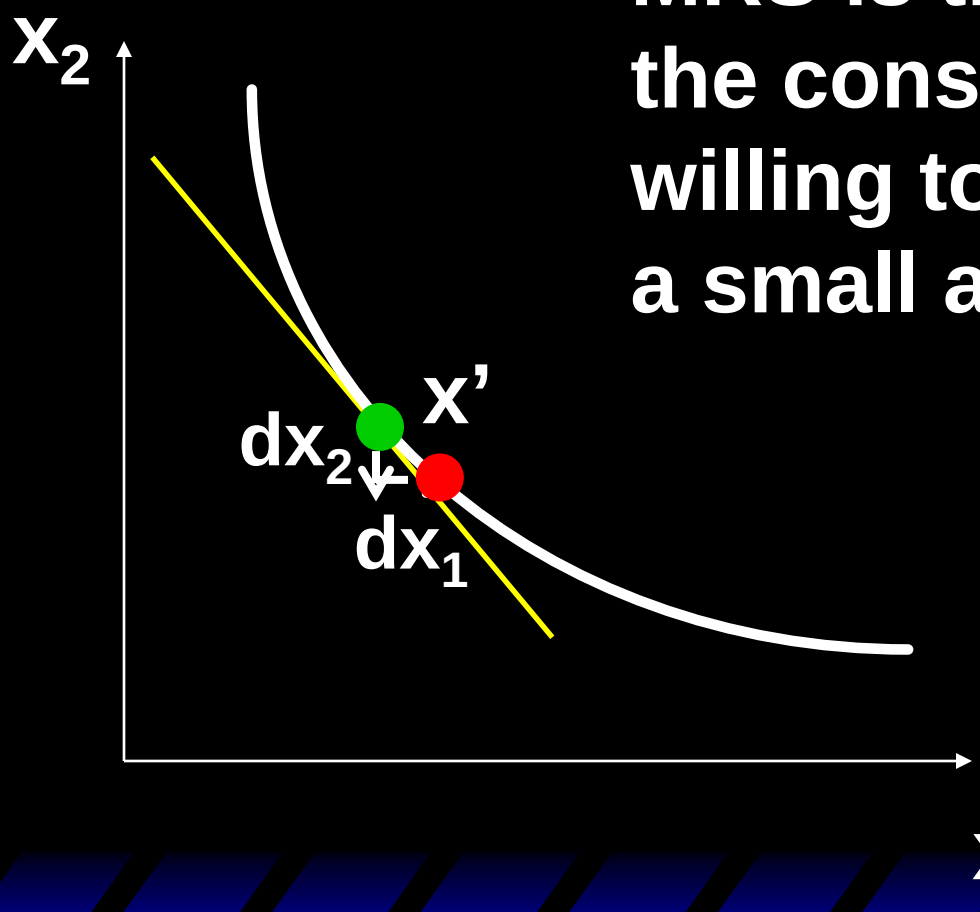


Marginal Rate of Substitution



Let $dx_1 \to 0$, $\frac{dx_2}{dx_1} \to$ the slope of the tangent line = MRS

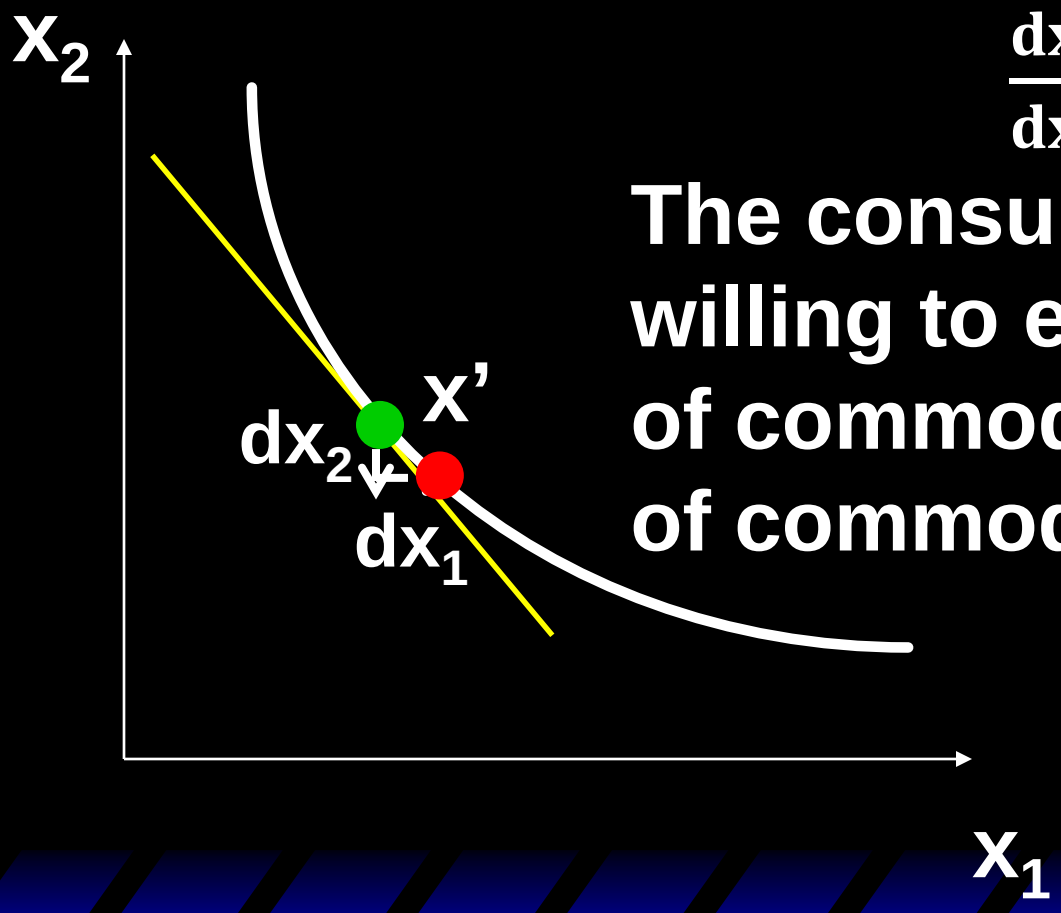
Marginal Rate of Substitution



MRS is the ratio at which the consumer is just willing to exchange x_2 for a small amount of x_1

边际替代率是消费者恰好愿意用一种商品去替代另一种商品的比率。

Marginal Rate of Substitution

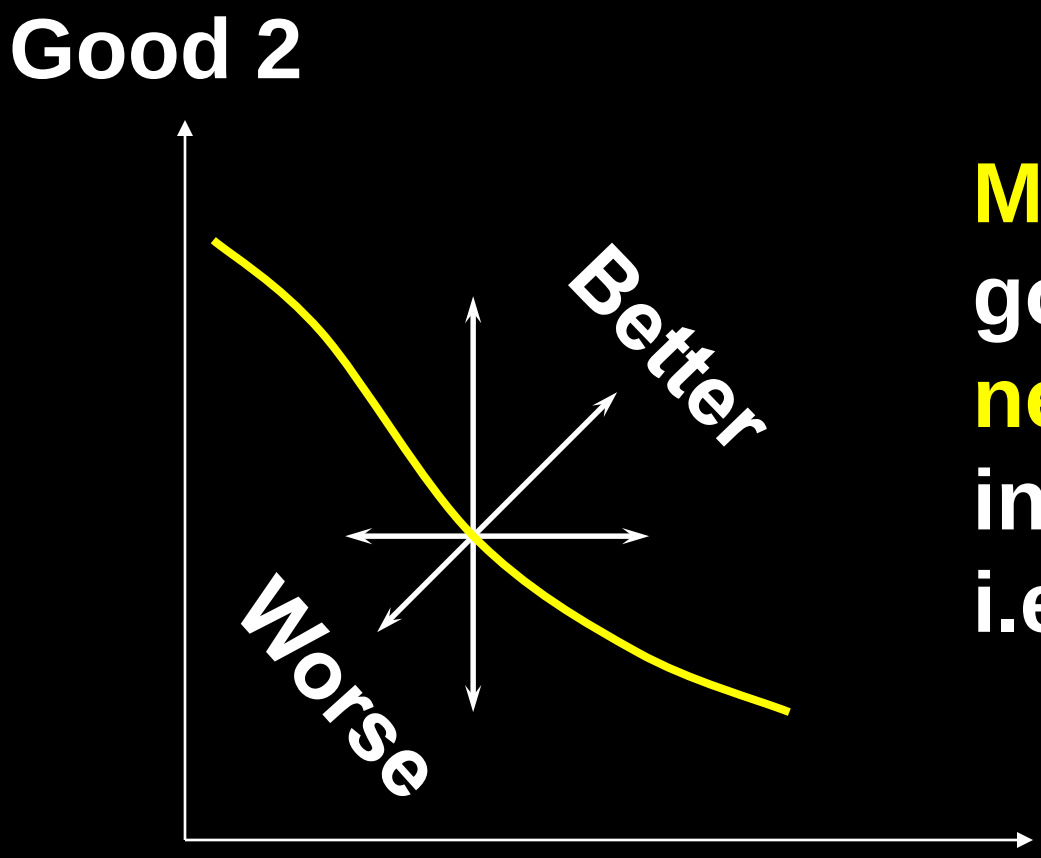


e.g. If $MRS = -2$ at x', then

$$\frac{dx_2}{dx_1} = -2$$

The consumer is just willing to exchange 2 units of commodity 2 for 1 unit of commodity 1

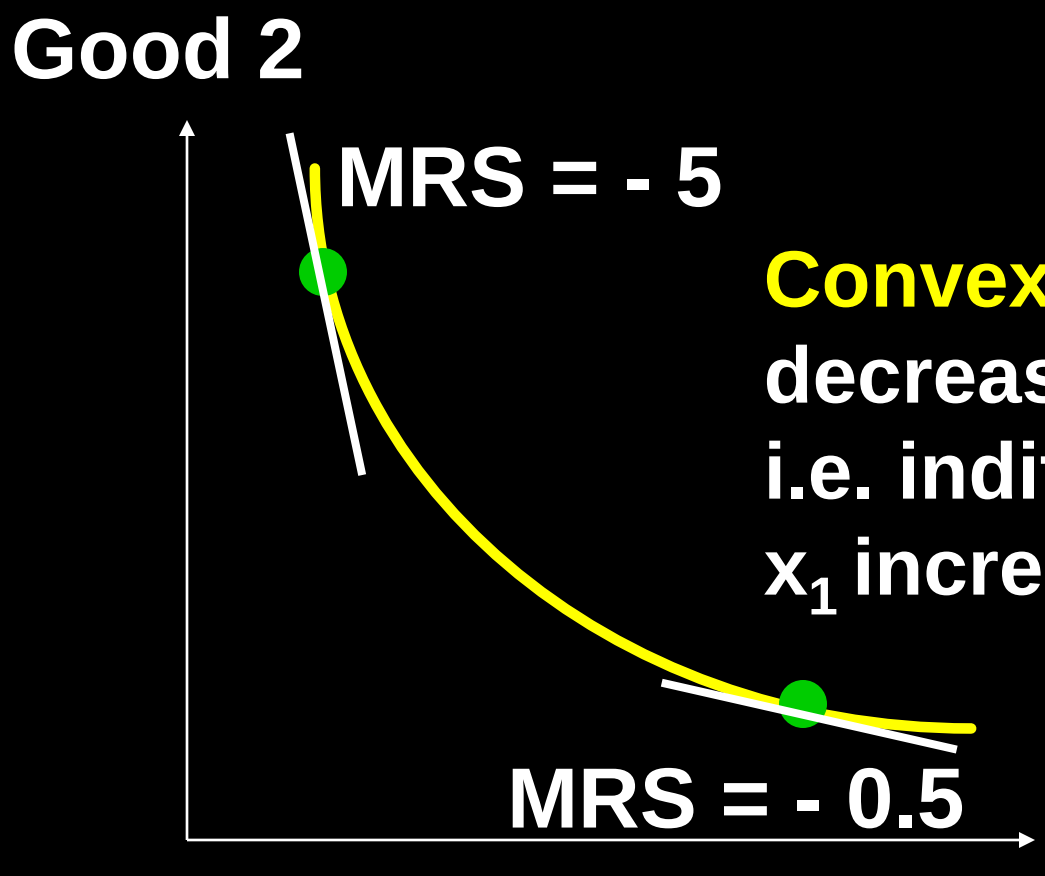
MRS & Ind. Curve Properties



Monotonicity =>Two goods => a **negatively** sloped indifference curve, i.e. MRS < 0

单调性偏好意味着无差异曲线向下倾斜

MRS & Ind. Curve Properties



Convexity => |MRS| decreases with x_1
i.e. indiff. curve flattens as x_1 increases

凸偏好性意味着无差异曲线随x_1的增加而变得平缓