Lecture 17

Game Theory

Game Theory

Game theory models strategic behavior by agents who understand that their actions affect the payoffs and therefore actions of other agents.

个体在决策时考虑其他个体会采取什么行动、以及他们的行动会对自己产生怎样影响的行为叫做策略性行为。

What is a Game?

A **game** consists of

- a set of **players** (参与者)
- a set of **strategies** (策略) for each player
- the **payoffs** (收益) to each player for every possible combination of strategy choices by the players.

A Two-Player Game in Class

- Two student volunteers, A and B
- They **simultaneously** determine how to allocate money between them
- Each student has two actions: split (平分) and steal (独吞)

A Two-Player Game in Class

If both students choose to **split** (平分), each receives ￥5

If both students choose to **steal** (独吞), each receives ￥0

A Two-Player Game in Class

- If both students choose to **split** (平分), each receives ￥5
- If both students choose to **steal** (独吞), each receives ￥0
- If student A chooses to **steal** (独吞) and B chooses to **split** (平分), A receives ￥10 and B receives ￥-5
- If student A chooses to **split** (平分) and B chooses to **steal** (独吞), A receives ￥-5 and B receives ￥10

A Two-Player Game in Class

- a set of **players**: A and B (non-cooperative)
- a set of **strategies** for each player: 平分、独吞
- the **payoffs** to each player for every possible combination of strategy choices by the players

A Two-Player Game in Class

4 possible outcomes / strategy profiles：

(平分, 平分)
(平分, 独吞)
(独吞, 平分)
(独吞, 独吞)

A strategy profile (策略组合) is a pair of strategies (s_A, s_B) where the 1st element is the strategy chosen by Player A and the 2nd is the strategy chosen by Player B.

A Two-Player Game in Class

Each outcome corresponds to a pair of payoffs.
Player A's payoff is shown first.
Player B's payoff is shown second.

Π(平分, 平分) = (5, 5)
Π(平分, 独吞) = (-5, 10)
Π(独吞, 平分) = (10, -5)
Π(独吞, 独吞) = (0, 0)

The Pay-off Matrix

		Player B	
		平分	独吞
Player A	平分	(5,5)	(-5,10)
	独吞	(10,-5)	(0,0)

收益矩阵

4 possible **outcomes**. Each outcome corresponds to a pair of **payoffs**. Player A's payoff is shown **first**. Player B's payoff is shown **second**.

The Pay-off Matrix

		Player B	
		平分	独吞
Player A	平分	(5,5)	(-5,10)
	独吞	(10,-5)	(0,0)

E.g. if A plays "平分" and B plays "独吞" then A's payoff is -5 and B's payoff is 10.

The Pay-off Matrix

		Player B	
		平分	独吞
Player A	平分	(5,5)	(-5,10)
	独吞	(10,-5)	(0,0)

What outcomes are we likely to see for this game?

The Pay-off Matrix

		Player B	
		平分	独吞
Player A	平分	(5,5)	(-5,10)
	$\boxed{\text{独吞}}$	(10,-5)	(0,0)

If B plays 平分，A's **best response** is to play 独吞

The Pay-off Matrix

		Player B	
		平分	独吞
Player A	平分	(5,5)	(-5,10)
	独吞	(10,-5)	(0,0)

If B plays 独吞，A's best response is still to play 独吞

The Pay-off Matrix

		Player B	
		平分	独吞
Player A	平分	(5,5)	(-5,10)
	独吞	(10,-5)	(0,0)

No matter what B plays, "独吞" yields a higher payoff for player A

"独吞" is a **dominant strategy (占优策略)** for player A

The Pay-off Matrix

		Player B	
		平分	独吞
Player A	平分	(5,5)	(-5,10)
	独吞	(10,-5)	(0,0)

If A plays 平分，B's **best response** is to play 独吞

The Pay-off Matrix

Player B

		平分	$\boxed{\text{独吞}}$
Player A	平分	**(5,5)**	**(-5,10)**
	独吞	**(10,-5)**	**(0,0)**

If A plays独吞，B's best response is still to play 独吞

The Pay-off Matrix

		Player B	
		平分	独吞
Player A	平分	(5,5)	(-5,10)
	独吞	(10,-5)	(0,0)

No matter what A plays, "独吞" yields a higher payoff for player B

"独吞" is a **dominant strategy** for player B

The Pay-off Matrix

		Player B	
		平分	独吞
Player A	平分	(5,5)	(-5,10)
	独吞	(10,-5)	(0,0)

In this example, we would expect (独吞, 独吞) to be an **equilibrium** outcome. In equilibrium, both players receive a payoff of 0.

Dominant Strategy Equilibrium

If all players have a dominant strategy, then there is a dominant strategy equilibrium.

A dominant strategy equilibrium is an outcome in which all players play their dominant strategies (占优策略均衡)

Dominant Strategy Equilibrium

- Not every game has a **dominant strategy equilibrium**.
- We need a more general **solution concept**.

An Example of a Two-Player Game

- The players are called A and B.
- Player A has two strategies, called "Up" and "Down".
- Player B has two strategies, called "Left" and "Right".
- The table showing the payoffs to both players for each of the four possible strategy combinations is the game's **payoff matrix**.

An Example of a Two-Player Game

Player B

		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

When B plays L, A's best response (最优应对策略) is to play U;
When B plays R, A's best response is to play D.

An Example of a Two-Player Game

Player B

		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

No dominant strategy for Player A.

When B plays L, A's best response is to play U;
When B plays R, A's best response is to play D.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

When A plays U, B's best response is to play L;

When A plays D, B's best response is to play R.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

No dominant strategy for Player B.

When A plays U, B's best response is to play L;
When A plays D, B's best response is to play R.

An Example of a Two-Player Game

		Player B: L	Player B: R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

No dominant strategy equilibrium.
What could be an equilibrium outcome for this game?

Nash Equilibrium

A strategy profile (s_A, s_B) is a Nash Equilibrium if s_A and s_B are **mutual best responses** to each other.

i.e. given player B's strategy s_B, s_A is optimal for player A; and given player A's strategy s_A, s_B is optimal for player B.

互为最优的策略组合被称作纳什均衡。当A选择s_A时，B的最优策略是s_B；当B选择s_B时，A的最优策略是s_A；那么(s_A, s_B)被称作博弈的一个纳什均衡。

Nash Equilibrium

The Nash Equilibrium is a state where no player has an incentive to deviate from his chosen strategy after knowing the strategies of the other players.

在纳什均衡处，没有任何一个参与者有单方面偏离均衡策略的动机。

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,3)	(2,1)

Is (U,R) a N.E.?

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,3)	(2,1)

Is (U,R) a N.E.?

If B plays Right then A's best reply is Down since this improves A's payoff from 1 to 2.
So (U,R) is not a Nash Equilibrium.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (D,R) a N.E.?

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (D,R) a N.E.?

If B plays Right then A’s best reply is Down.
If A plays Down then B’s best reply is Right.
So (D,R) is a Nash Equilibrium.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (D,L) a N.E.?

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (D,L) a N.E.?

If A plays Down then B's best reply is Right, so (D,L) is not a Nash Equilibrium.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (U,L) a N.E.?

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Is (U,L) a N.E.?

If A plays Up then B's best reply is Left.
If B plays Left then A's best reply is Up.
So **(U,L) is a Nash Equilibrium**.

An Example of a Two-Player Game

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

(U,L) and (D,R) are both Nash equilibria for the game.

一个博弈可能存在多个纳什均衡

Nash equilibrium does not say anything about which one should / will be played

Finding Nash Equilibria

		Player B	
		L	R
Player A	U	(<u>3</u>,9)	(1,8)
	D	(0,0)	(2,1)

When B plays L, A's best reply is U. Underline the payoff for A when the strategy profile is (U,L)

Finding Nash Equilibria

		Player B	
		L	R
Player A	U	(<u>3</u>,9)	(1,8)
	D	(0,0)	(<u>2</u>,1)

When B plays R, A's best reply is D. Underline the payoff for A when the strategy profile is (D,R)

Finding Nash Equilibria

		Player B	
		L	R
Player A	U	(<u>3</u>,<u>9</u>)	(1,8)
	D	(0,0)	(<u>2</u>,1)

When A plays U, B's best reply is L. Underline the payoff for A when the strategy profile is (U,L)

Finding Nash Equilibria

		Player B	
		L	R
Player A	U	(<u>3</u>,<u>9</u>)	(1,8)
	D	(0,0)	(<u>2</u>,<u>1</u>)

When A plays D, B's best reply is R. Underline the payoff for A when the strategy profile is (D,R)

Finding Nash Equilibria

		Player B	
		L	R
Player A	U	(<u>3</u>,<u>9</u>)	(1,8)
	D	(0,0)	(<u>2</u>,<u>1</u>)

Nash Equilibria are strategy profiles where payoffs are both underlined.

Nash Equilibrium

A **dominant strategy equilibrium** must be a Nash Equilibrium

占优策略均衡一定是纳什均衡

A Nash Equilibrium is not necessarily a dominant strategy equilibrium

纳什均衡不一定是占优策略均衡

Dominant Strategy Equilibrium Must Be a Nash Equilibrium

- **Suppose (s_A, s_B) is a dominant strategy equilibrium**
- **s_A is A's best strategy regardless of B's strategies => s_A is A's best reply when B plays s_A**
- **Similarly, s_B is B's best strategy regardless of A's strategies => s_B is B's best reply when A plays s_A**
- **(s_A, s_B) are mutual best response to each other. By definition, it is a NE.**

Dominant Strategy Equilibrium Must Be a Nash Equilibrium

Player B

Player A		平分	独吞
	平分	**(5,5)**	**(-5,10)**
	独吞	**(10,-5)**	**(0,0)**

Dominant Strategy Equilibrium Must Be a Nash Equilibrium

Player B

		平分	独吞
Player A	平分	(5,5)	(-5,<u>10</u>)
	独吞	(10,-5)	(0,<u>0</u>)

Dominant Strategy Equilibrium Must Be a Nash Equilibrium

Player B

		平分	独吞
Player A	平分	(5,5)	(-5,10)
	独吞	(10,-5)	(0,0)

Dominant Strategy Equilibrium Must Be a Nash Equilibrium

Player B

Player A

	平分	独吞
平分	(5,5)	(-5,10)
独吞	(10,-5)	(0,0)

Dominant Strategy Equilibrium Must Be a Nash Equilibrium

		Player B	
		平分	独吞
Player A	平分	(5,5)	(-5,10)
	独吞	(10,-5)	(0,0)

Nash Equilibrium

The Prisoner's Dilemma

- **Nash Equilibria are not necessarily Pareto efficient.**
- 纳什均衡不一定是帕累托有效率的
- **To see an example, consider a famous game called the Prisoner's Dilemma (囚徒困境).**

The Prisoner's Dilemma

		赵四	
		S	C
刘能	S	(-5,-5)	(-30,-1)
	C	(-1,-30)	(-10,-10)

What outcomes are we likely to see for this game?

The Prisoner's Dilemma

		赵四	
		S	C
刘能	S	(-5,-5)	(-30,-1)
	C	(-1,-30)	(-10,-10)

If 刘能 plays Silence (沉默) then 赵四's best reply is Confess (坦白).
If 刘能 plays Confess then 赵四's best reply is Confess.

The Prisoner's Dilemma

		赵四	
		S	C
刘能	S	(-5,-5)	(-30,-1)
	C	(-1,-30)	(-10,-10)

So no matter what 刘能 plays,赵四's best reply is always Confess.
Confess is a dominant strategy for赵四.

The Prisoner's Dilemma

		赵四	
		S	C
刘能	S	(-5,-5)	(-30,-1)
	C	(-1,-30)	(-10,-10)

Similarly, no matter what 赵四 plays, 刘能's best reply is always Confess. Confess is a dominant strategy for 刘能 also.

The Prisoner's Dilemma

		赵四	
		S	C
刘能	S	(-5,-5)	(-30,-1)
	C	(-1,-30)	(-10,-10)

So the only Nash equilibrium for this game is (C,C), even though (S,S) gives both 刘能 and 赵四 better payoffs.
The only Nash equilibrium is inefficient.

Who Plays When?

In all above examples the players chose their strategies simultaneously.

Such games are **simultaneous play games** (同时行动博弈)

Who Plays When?

- But there are games in which one player plays before another player.
- Such games are **sequential play games** (序贯博弈)
- The player who plays first is the **leader**. The player who plays second is the **follower**.

A Sequential Game Example

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

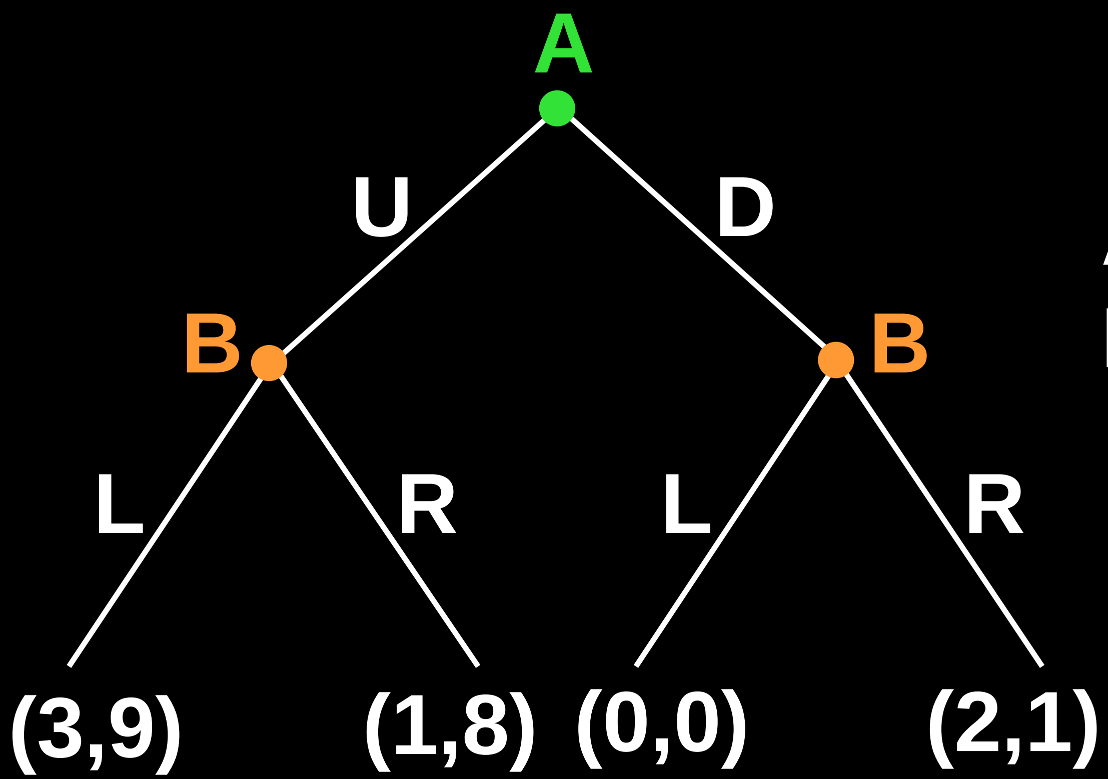
(U,L) and (D,R) are both Nash equilibria when this game is played simultaneously.

A Sequential Game Example

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Suppose instead that the game is played sequentially, with A leading and B following. We can rewrite the game in its extensive form (扩展形式).

A Sequential Game Example



A plays first.
B plays second.

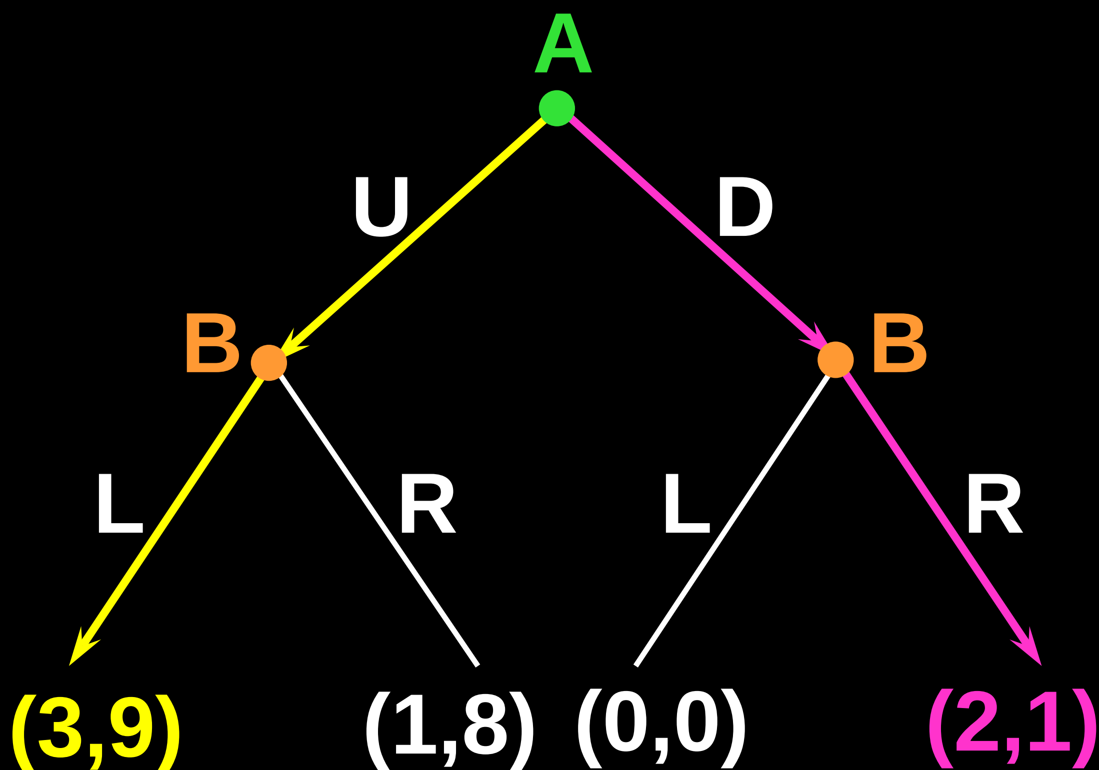
Solving Sequential Games

We use **backward induction** (逆向归纳法) to solve sequential games.

We start at the “end” of the game tree, and work “back” up the tree by solving for optimal behavior at each node.

从扩展式的最终节点出发，找出参与者在每个节点的最优策略，然后向上递推，找出参与者在上层节点处的最优策略，直到扩展式的初始点

A Sequential Game Example



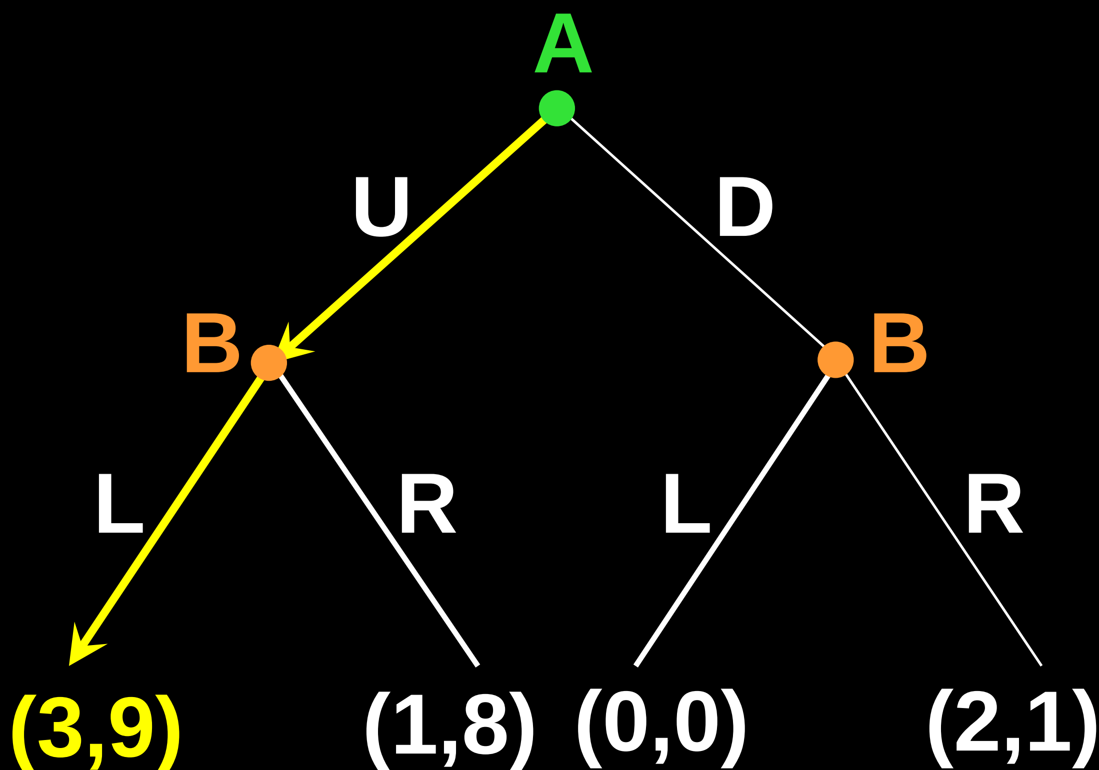
A plays first.
B plays second.

First, figure out B's optimal strategy when it is his/her time to play

If A plays U, B will play L;

If A plays D, B will play R;

A Sequential Game Example



(U,L) is the likely Nash equilibrium.

Backward induction: figure out A's optimal strategy at the original node.

If A plays U then B plays L; A gets 3.

If A plays D then B plays R; A gets 2.

Pure Strategies

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

This is our original example once more. Suppose again that play is **simultaneous**. We discovered that the game has two Nash equilibria; (U,L) and (D,R).

Pure Strategies

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Player A's has been thought of as choosing to play either U or D, but no combination of both; that is, as playing **purely** U or D.

U and D are Player A's **pure strategies (纯策略)**.

Pure Strategies

		Player B L	Player B R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Similarly, L and R are Player B's pure strategies.

Pure Strategies

		Player B	
		L	R
Player A	U	(3,9)	(1,8)
	D	(0,0)	(2,1)

Consequently, (U,L) and (D,R) are **pure strategy Nash equilibria** (纯策略纳什均衡). Must every game have at least one pure strategy Nash equilibrium?

Pure Strategies

		Player B	
		L	R
Player A	U	(1,2)	(0,4)
	D	(0,5)	(3,2)

Here is a new game. Are there any pure strategy Nash equilibria?

Pure Strategies

		Player B	
		L	R
Player A	U	(1,2)	(0,4)
	D	(0,5)	(3,2)

Pure Strategies

Player B

		L	R
Player A	U	$(\underline{1},2)$	(0,4)
	D	(0,5)	$(\underline{3},2)$

Pure Strategies

		Player B	
		L	R
Player A	U	(<u>1</u>,2)	(0,<u>4</u>)
	D	(0,5)	(<u>3</u>,2)

Pure Strategies

		Player B	
		L	R
Player A	U	(<u>1</u>,2)	(0,<u>4</u>)
	D	(0,<u>5</u>)	(<u>3</u>,2)

Pure Strategies

		Player B	
		L	R
Player A	U	(1,2)	(0,4)
	D	(0,5)	(3,2)

So the game has no Nash equilibria in pure strategies. Even so, the game does have a Nash equilibrium, but in mixed strategies.

该博弈不存在纯策略纳什均衡，但是存在混合策略那时均衡。

Mixed Strategies

- Instead of playing purely Up or Down, Player A selects a probability distribution (π_U,1-π_U), meaning that with probability π_U Player A will play Up and with probability 1-π_U will play Down.
- Player A is **mixing** over the pure strategies Up and Down.
- **The probability distribution (π_U,1-π_U) is a mixed strategy** for Player A.

A以π_U的概率选择U，1-π_U的概率选择D

Mixed Strategies

- Similarly, Player B selects a probability distribution $(\pi_L,1-\pi_L)$, meaning that with probability π_L Player B will play Left and with probability $1-\pi_L$ will play Right.
- Player B is **mixing** over the pure strategies Left and Right.
- **The probability distribution $(\pi_L,1-\pi_L)$ is a mixed strategy** for Player B.

B以π_L的概率选择L，$1-\pi_L$的概率选择R

Mixed Strategies

		Player B	
		L	R
Player A	U	(1,2)	(0,4)
	D	(0,5)	(3,2)

This game has no pure strategy Nash equilibria but it does have a Nash equilibrium in mixed strategies. How is it computed?

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, π_U	(1,2)	(0,4)
	D, $1-\pi_U$	(0,5)	(3,2)

Mixed Strategies

Player B

		L,π_L	$R,1-\pi_L$
Player A	U,π_U	(1,2)	(0,4)
	$D,1-\pi_U$	(0,5)	(3,2)

If B plays Left her expected payoff is

$$2\pi_U + 5(1-\pi_U)$$

Mixed Strategies

Player B

Player A		L, π_L	R, $1-\pi_L$
	U, π_U	(1,2)	(0,4)
	D, $1-\pi_U$	(0,5)	(3,2)

If B plays Left her expected payoff is

$$2\pi_U + 5(1-\pi_U).$$

If B plays Right her expected payoff is

$$4\pi_U + 2(1-\pi_U).$$

Mixed Strategies

Player B

		L, π_L	$R, 1-\pi_L$
Player A	U, π_U	(1,2)	(0,4)
	$D, 1-\pi_U$	(0,5)	(3,2)

If $2\pi_U + 5(1-\pi_U) > 4\pi_U + 2(1-\pi_U)$ **then B would play only Left. But there are no Nash equilibria in which B plays only Left.**

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, π_U	(1,2)	(0,4)
	D, $1-\pi_U$	(0,5)	(3,2)

If $2\pi_U + 5(1-\pi_U) < 4\pi_U + 2(1-\pi_U)$ then B would play only **Right**. But there are no Nash equilibria in which B plays only Right.

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, π_U	(1,2)	(0,4)
	D, $1-\pi_U$	(0,5)	(3,2)

So for there to exist a Nash equilibrium, B must be indifferent between playing Left or Right; i.e. $2\pi_U + 5(1-\pi_U) = 4\pi_U + 2(1-\pi_U)$

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, π_U	(1,2)	(0,4)
	D, $1-\pi_U$	(0,5)	(3,2)

So for there to exist a Nash equilibrium, B must be indifferent between playing Left or Right; i.e. $2\pi_U + 5(1-\pi_U) = 4\pi_U + 2(1-\pi_U)$

$$\Rightarrow \quad \pi_U = 3/5.$$

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

So for there to exist a Nash equilibrium, B must be indifferent between playing Left or Right; i.e. $2\pi_U + 5(1-\pi_U) = 4\pi_U + 2(1-\pi_U)$

$$\Rightarrow \quad \pi_U = 3/5.$$

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

Mixed Strategies

Player B

		L,π_L	$R,1-\pi_L$
Player A	$U, \frac{3}{5}$	(1,2)	(0,4)
	$D, \frac{2}{5}$	(0,5)	(3,2)

If A plays Up his expected payoff is

$$1 \times \pi_L + 0 \times (1 - \pi_L) = \pi_L.$$

Mixed Strategies

		Player B	
		L, π_L	R, 1-π_L
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

If A plays Up his expected payoff is

$$1 \times \pi_L + 0 \times (1 - \pi_L) = \pi_L .$$

If A plays Down his expected payoff is

$$0 \times \pi_L + 3 \times (1 - \pi_L) = 3(1 - \pi_L).$$

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

If $\pi_L > 3(1-\pi_L)$ then A would play only Up.

But there are no Nash equilibria in which A plays only Up.

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

If $\pi_L < 3(1-\pi_L)$ then A would play only **Down**. But there are no Nash equilibria in which A plays only Down.

Mixed Strategies

		Player B	
		L, π_L	R, 1-π_L
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

So for there to exist a Nash equilibrium, A must be indifferent between playing Up or Down; i.e. $\pi_L = 3(1 - \pi_L)$

Mixed Strategies

		Player B	
		L, π_L	R, $1-\pi_L$
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

So for there to exist a Nash equilibrium, A must be indifferent between playing Up or Down; i.e. $\pi_L = 3(1-\pi_L) \quad \Rightarrow \quad \pi_L = 3/4.$

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

So for there to exist a Nash equilibrium, A must be indifferent between playing Up or Down; i.e. $\pi_L = 3(1 - \pi_L) \quad \Rightarrow \quad \pi_L = 3/4.$

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2)	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

So the game's only Nash equilibrium has A playing the mixed strategy (3/5, 2/5) and has B playing the mixed strategy (3/4, 1/4).

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4)
	D, $\frac{2}{5}$	(0,5)	(3,2)

The payoffs will be (1,2) with probability

$$\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$$

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4) 3/20
	D, $\frac{2}{5}$	(0,5)	(3,2)

The payoffs will be (0,4) with probability

$$\frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$$

Mixed Strategies

		Player B L, $\frac{3}{4}$	Player B R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4) 3/20
	D, $\frac{2}{5}$	(0,5) 6/20	(3,2)

The payoffs will be (0,5) with probability

$$\frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$$

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4) 3/20
	D, $\frac{2}{5}$	(0,5) 6/20	(3,2) 2/20

The payoffs will be (3,2) with probability

$$\frac{2}{5} \times \frac{1}{4} = \frac{2}{20}$$

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4) 3/20
	D, $\frac{2}{5}$	(0,5) 6/20	(3,2) 2/20

A's expected Nash equilibrium payoff is

$$1 \times \frac{9}{20} + 0 \times \frac{3}{20} + 0 \times \frac{6}{20} + 3 \times \frac{2}{20} = \frac{3}{4}.$$

Mixed Strategies

		Player B	
		L, $\frac{3}{4}$	R, $\frac{1}{4}$
Player A	U, $\frac{3}{5}$	(1,2) 9/20	(0,4) 3/20
	D, $\frac{2}{5}$	(0,5) 6/20	(3,2) 2/20

A's expected Nash equilibrium payoff is

$$1 \times \frac{9}{20} + 0 \times \frac{3}{20} + 0 \times \frac{6}{20} + 3 \times \frac{2}{20} = \frac{3}{4}.$$

B's expected Nash equilibrium payoff is

$$2 \times \frac{9}{20} + 4 \times \frac{3}{20} + 5 \times \frac{6}{20} + 2 \times \frac{2}{20} = \frac{16}{5}.$$

How Many Nash Equilibria?

A game with a finite number of players, each with a finite number of pure strategies, has at least one Nash equilibrium.

如果一个博弈的参与者有限，且每个参与者的纯策略数量有限，那么这个博弈至少存在一个纳什均衡。

How Many Nash Equilibria?

So if the game has no pure strategy Nash equilibrium then it must have at least one mixed strategy Nash equilibrium.

如果有限参与者、有限纯策略的博弈不存在纯策略纳什均衡，那么它至少存在一个混合策略纳什均衡。

Some Applications of Game Theory

- The study of oligopolies (industries containing only a few firms)
- The study of bidding and auctions
- The study of externalities; e.g. using a common resource such as a fishery.
- The study of social interaction and human behavior