Lecture 19

Exchange

Overview

So far, we have been ignoring the effects **across** markets, i.e. how events in one market affect conditions in other markets

Our modeling has been in one market at a time – this is called the **Partial Equilibrium** (局部均衡)

Overview

This lecture: the study of General Equilibrium (一般均衡)

- **how demand and supply conditions interact in several markets to determine the prices of many markets**
- **A simple exchange economy with 2 agents and 2 goods (纯交换经济)**

Exchange

Two consumers, A and B.

Their endowments of goods 1 and 2 are $\omega^A = (\omega_1^A, \omega_2^A)$ and $\omega^B = (\omega_1^B, \omega_2^B)$.

E.g. $\omega^A = (6,4)$ and $\omega^B = (2,2)$.

The total quantities available are $\omega_1^A + \omega_1^B = 6 + 2 = 8$ units of good 1 and $\omega_2^A + \omega_2^B = 4 + 2 = 6$ units of good 2.

Exchange

Consumer A's utility is $\mathrm{U}^A(x_1^A, x_2^A)$

Consumer B's utility is $\mathrm{U}^B(x_1^B, x_2^B)$

We place A and B in a **competitive** market, and would like to know

- whether they will trade
- if they trade, what the prices (p_1, p_2) will be
- whether the post-trade allocation is efficient

Exchange

Edgeworth and Bowley devised a diagram, called an Edgeworth box, to show all possible allocations of the available quantities of goods 1 and 2 between the two consumers.

埃奇沃思方框图

Starting an Edgeworth Box

方框的宽代表经济体中商品2的总量

Height =

$\omega_2^A + \omega_2^B$

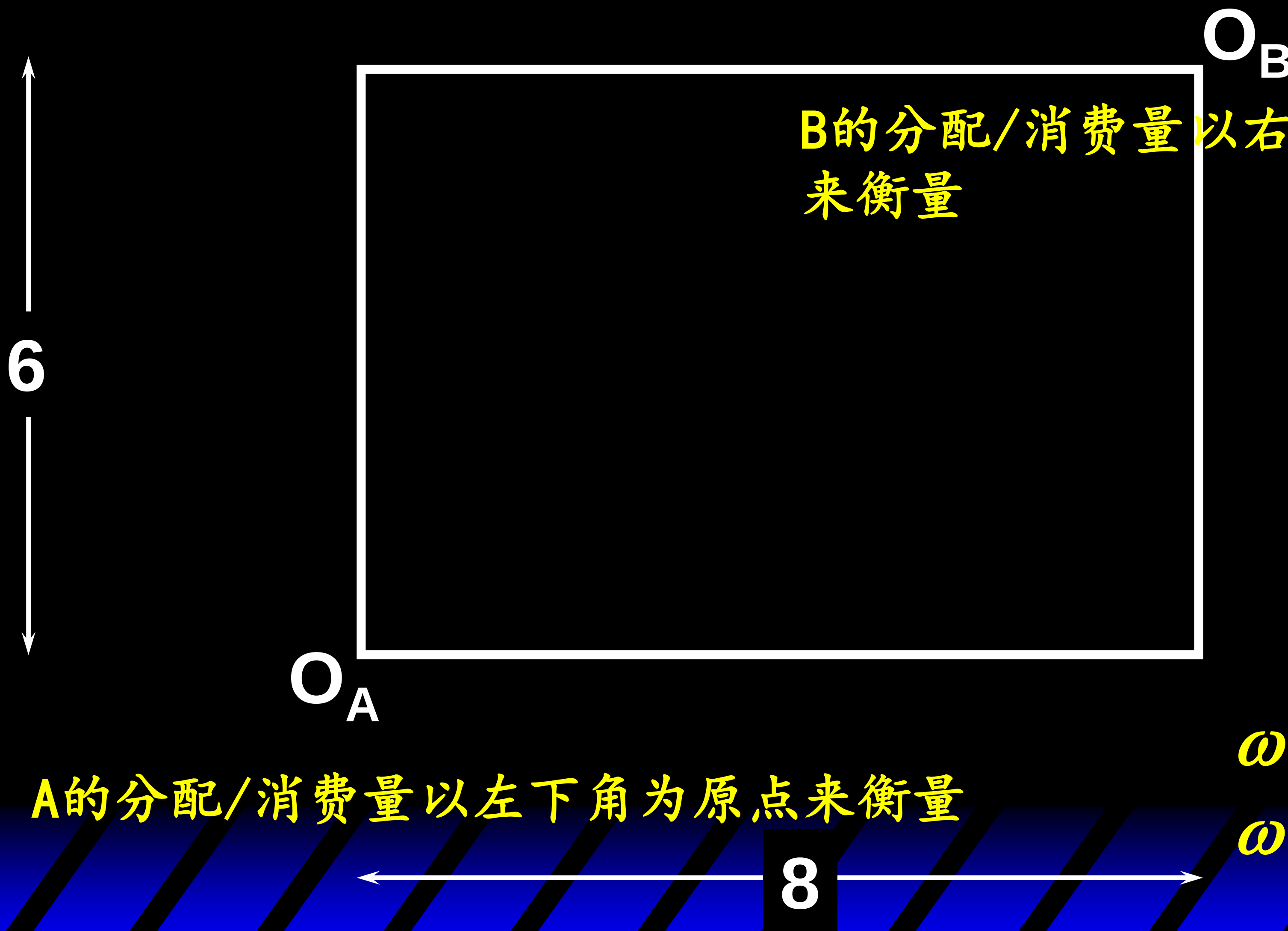
$= 4 + 2$

$= 6$

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

方框的长代表经济体中商品1的总量

The Endowment Allocation



B的分配/消费量以右上角为原点来衡量

A的分配/消费量以左下角为原点来衡量

$$\omega^A = (6,4)$$
$$\omega^B = (2,2)$$

The Endowment Allocation

Height = $\omega_2^A + \omega_2^B$

$= 4 + 2$

$= 6$

The endowment allocation is

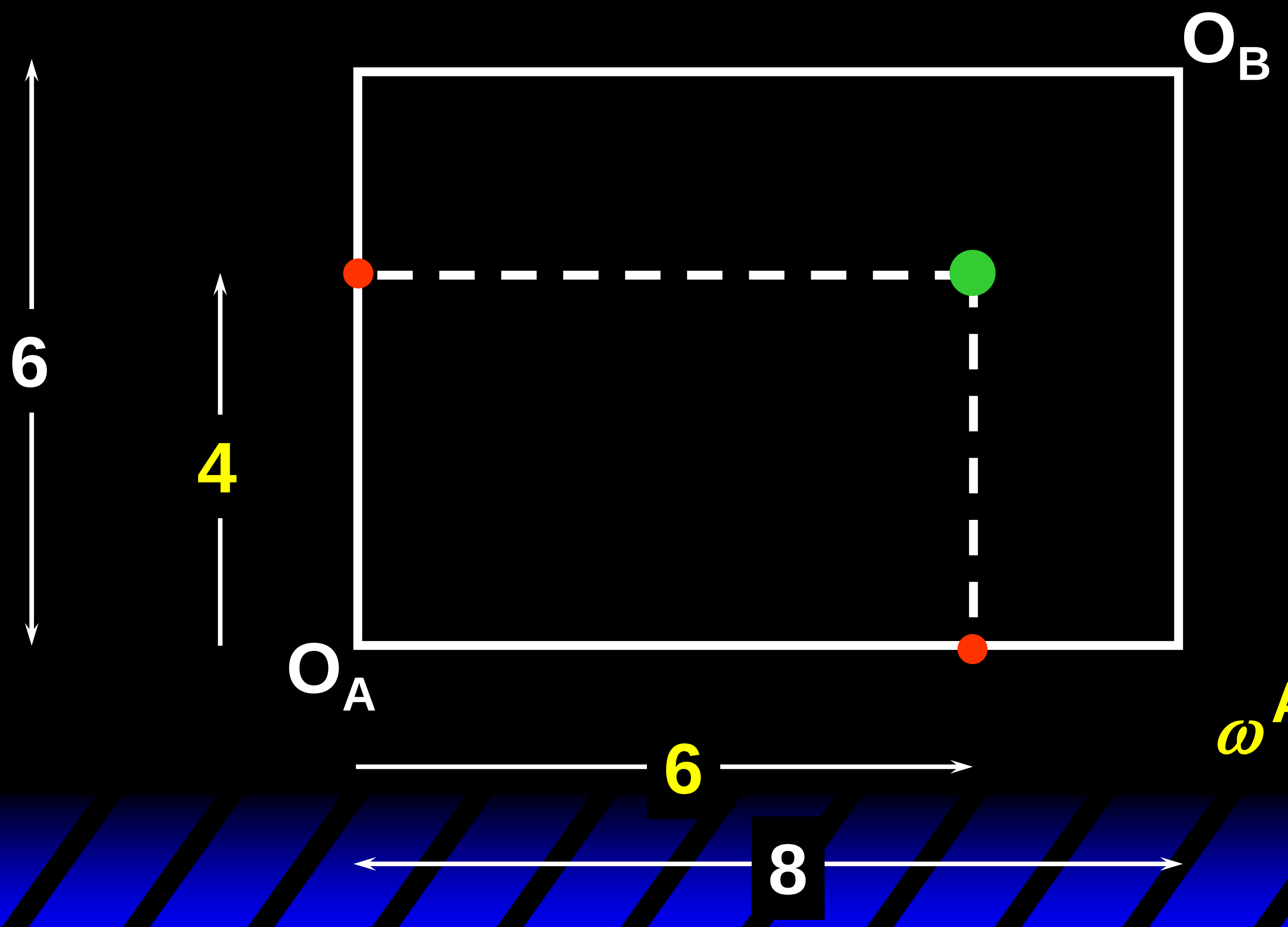
$\omega^A = (6,4)$

and

$\omega^B = (2,2)$.

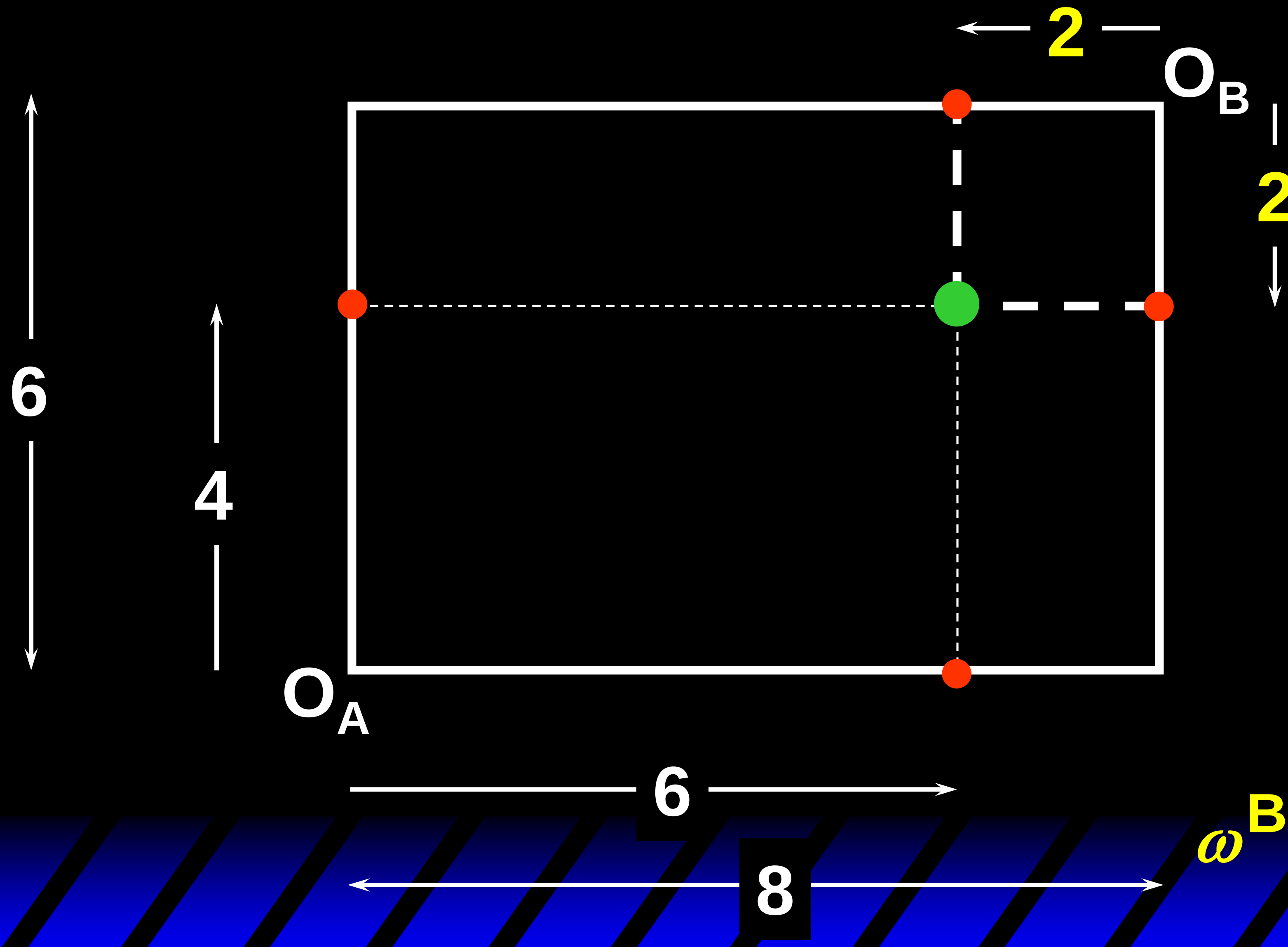
Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

The Endowment Allocation



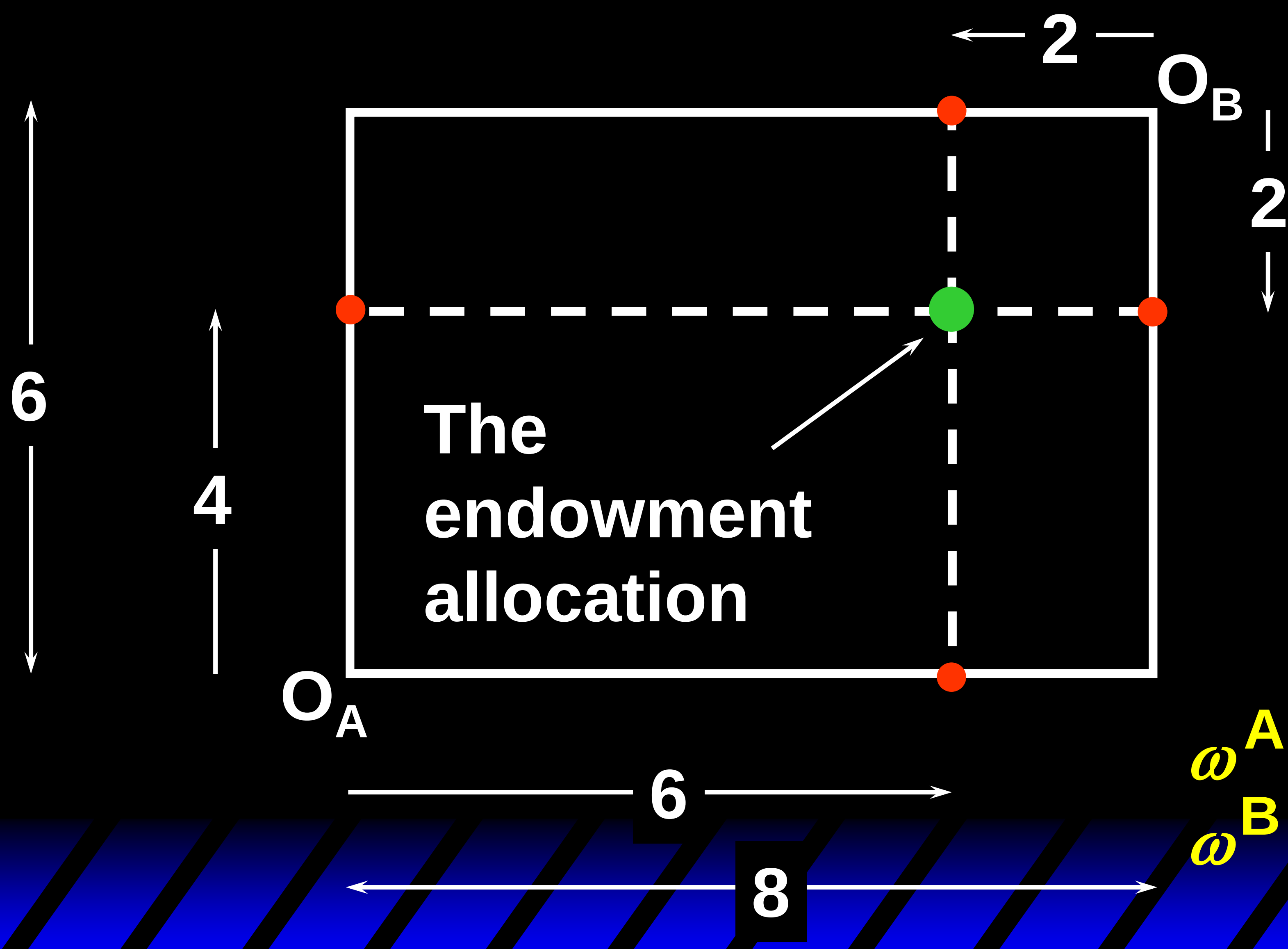
$\omega^A = (6,4)$

The Endowment Allocation



$\omega^B = (2,2)$

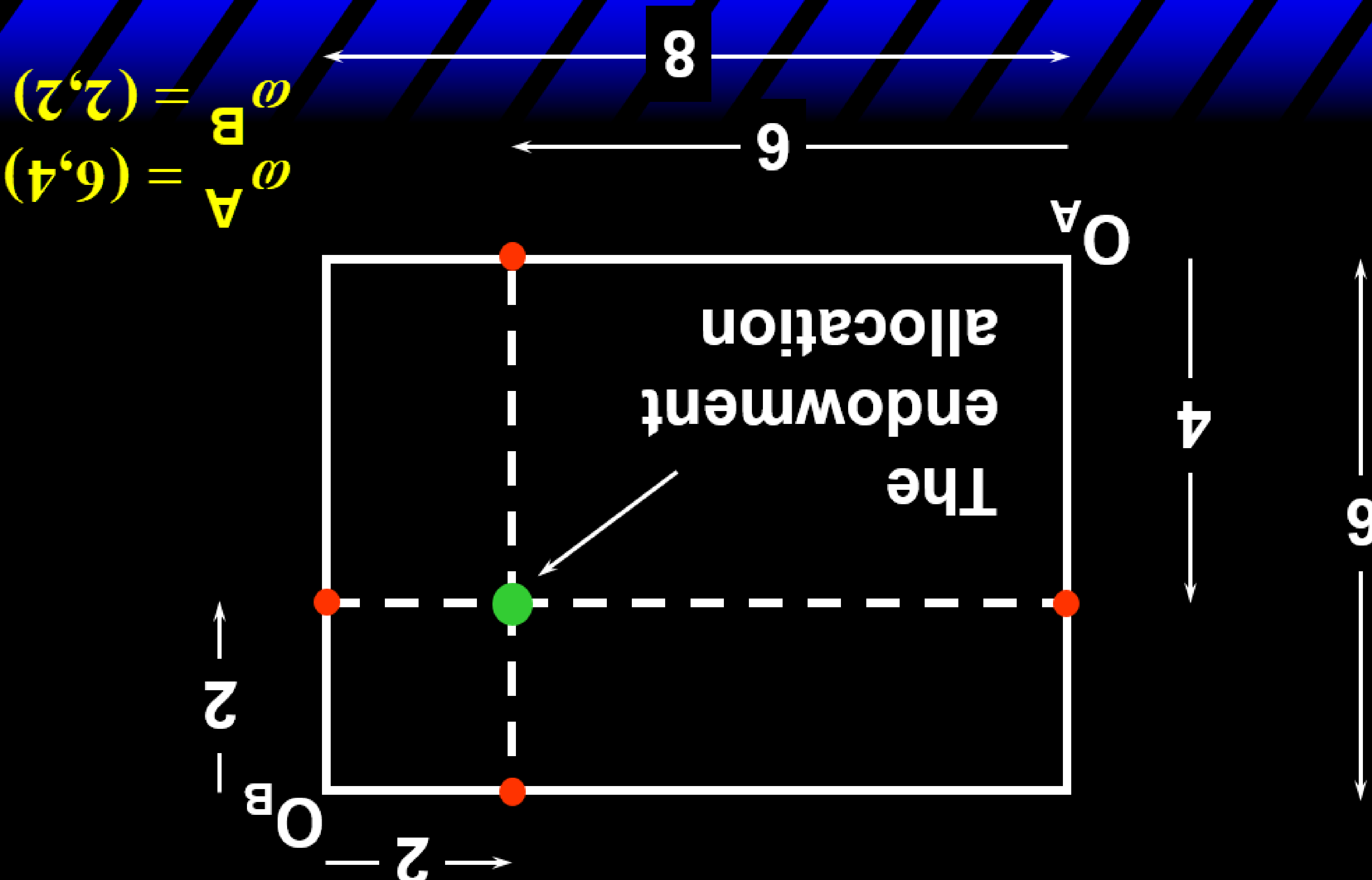
The Endowment Allocation



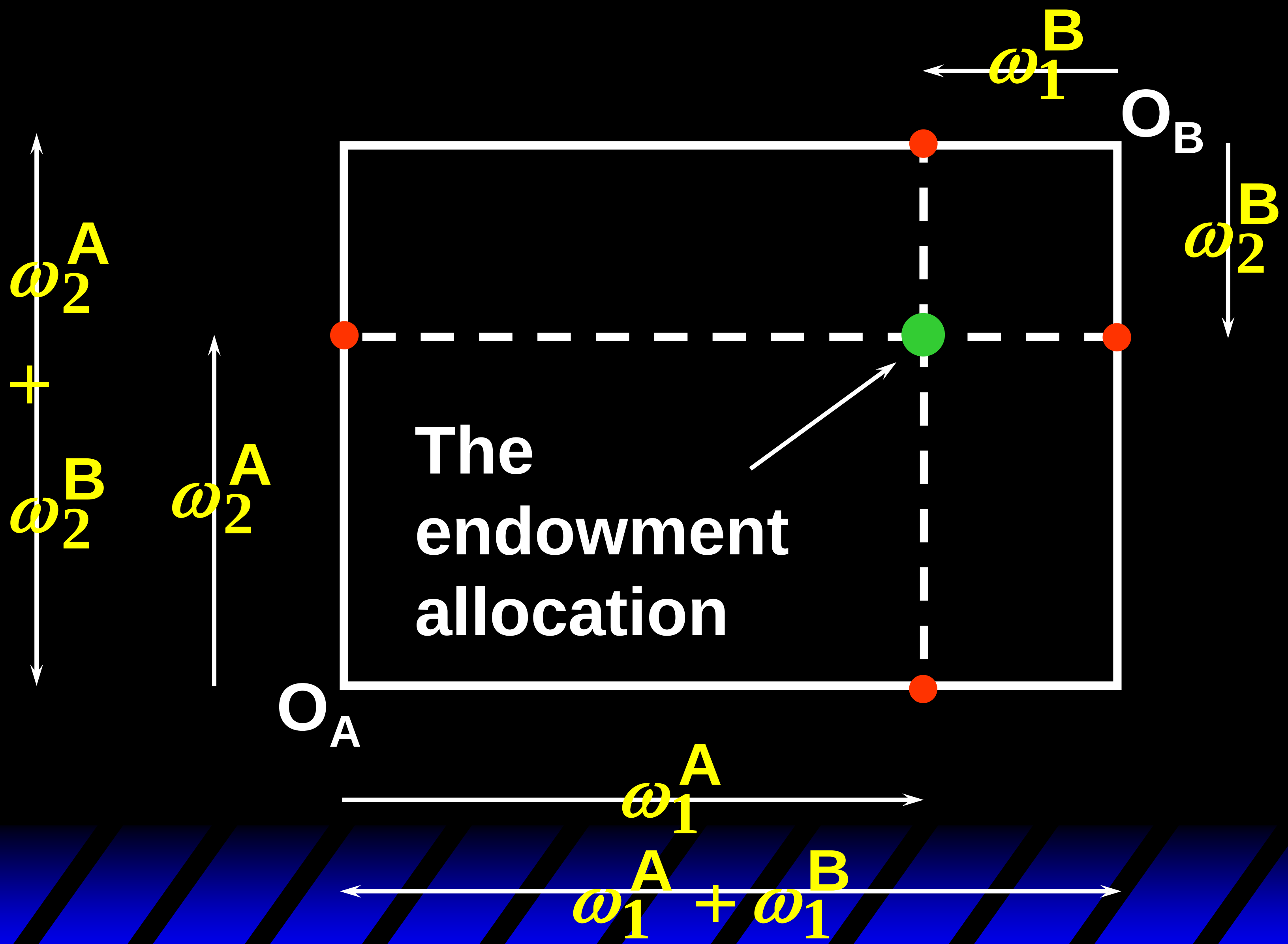
$\omega^A = (6,4)$

$\omega^B = (2,2)$

180 Degree Rotation



The Endowment Allocation



Feasible Allocations

What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?

How can all of the **feasible allocations (可行的分配)** be depicted by the Edgeworth box diagram?

Feasible Allocations

(x_1^A, x_2^A) denotes an allocation to consumer A.

(x_1^B, x_2^B) denotes an allocation to consumer B.

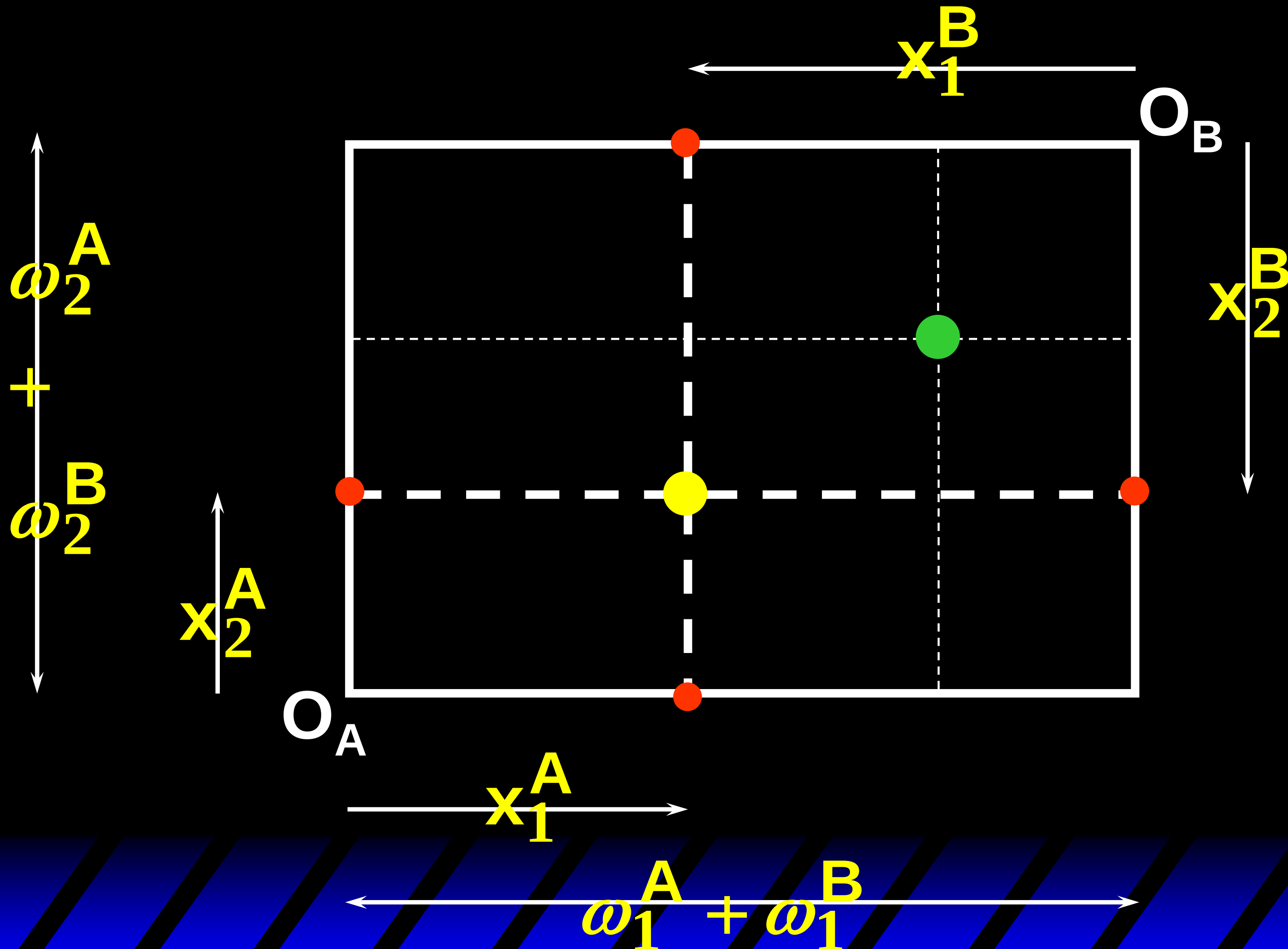
An allocation is feasible if and only if

$$x_1^A + x_1^B \le \omega_1^A + \omega_1^B$$

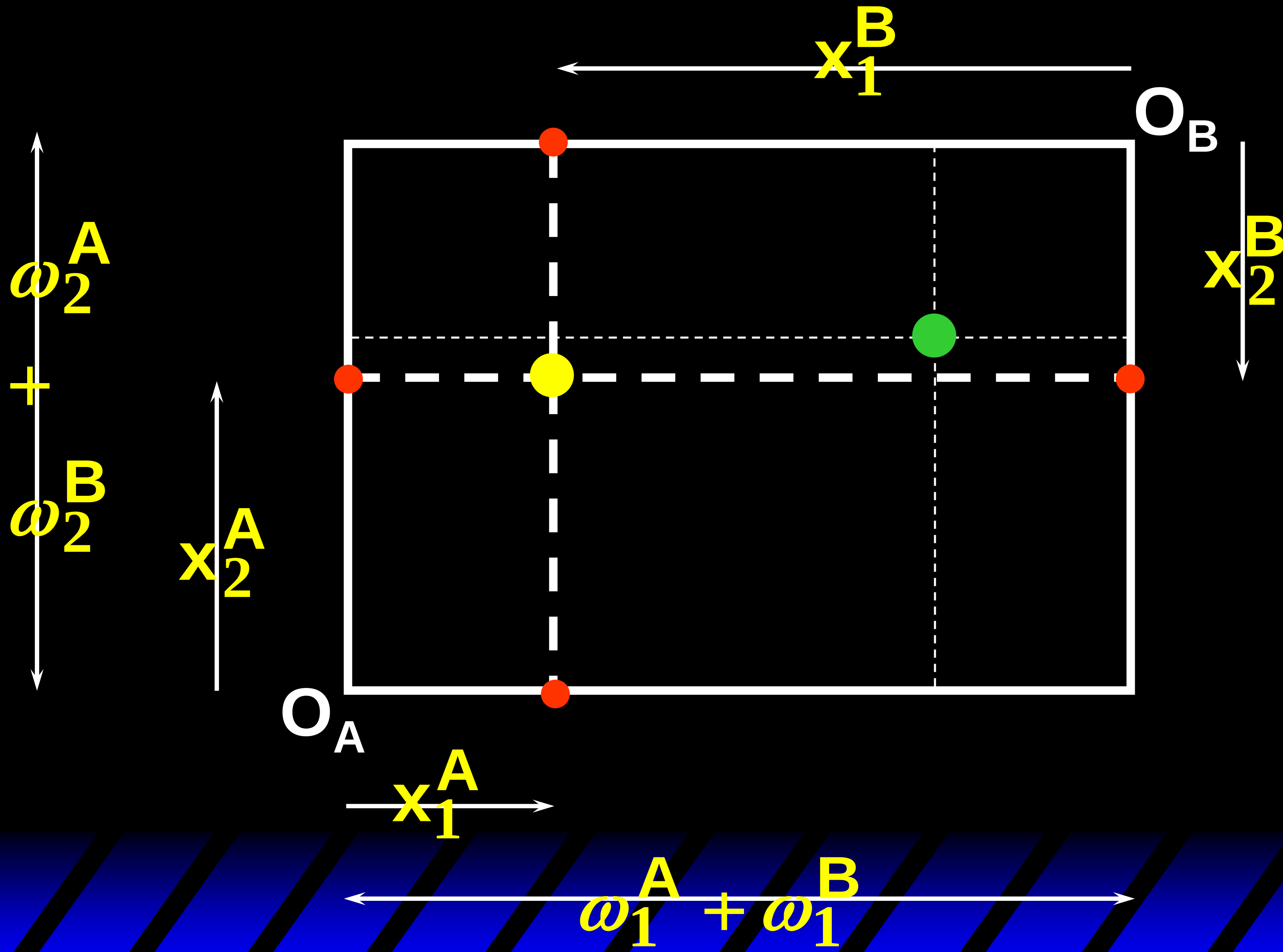
and $x_2^A + x_2^B \le \omega_2^A + \omega_2^B$.

每一商品的总消费量都不超过该商品在经济中的总禀赋量

Feasible Allocations



Feasible Allocations



Feasible Allocations

All points in the box, including the boundary, represent feasible allocations of the combined endowments.

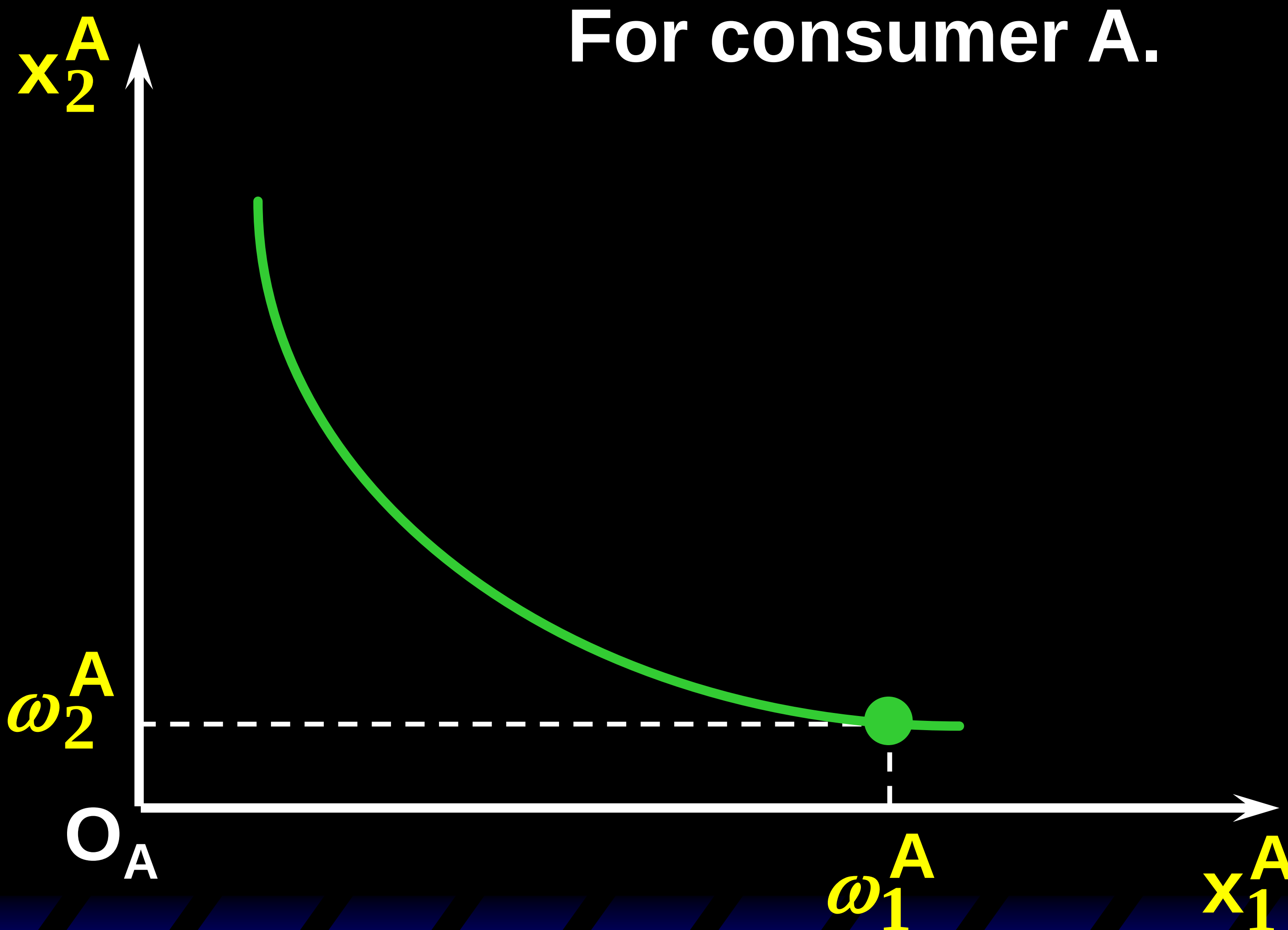
埃奇沃思方框中的任意一点，包括边框上的点，都代表一种可行的分配。在该点处，每一种商品的总消费恰好等于总禀赋量。

Feasible Allocations

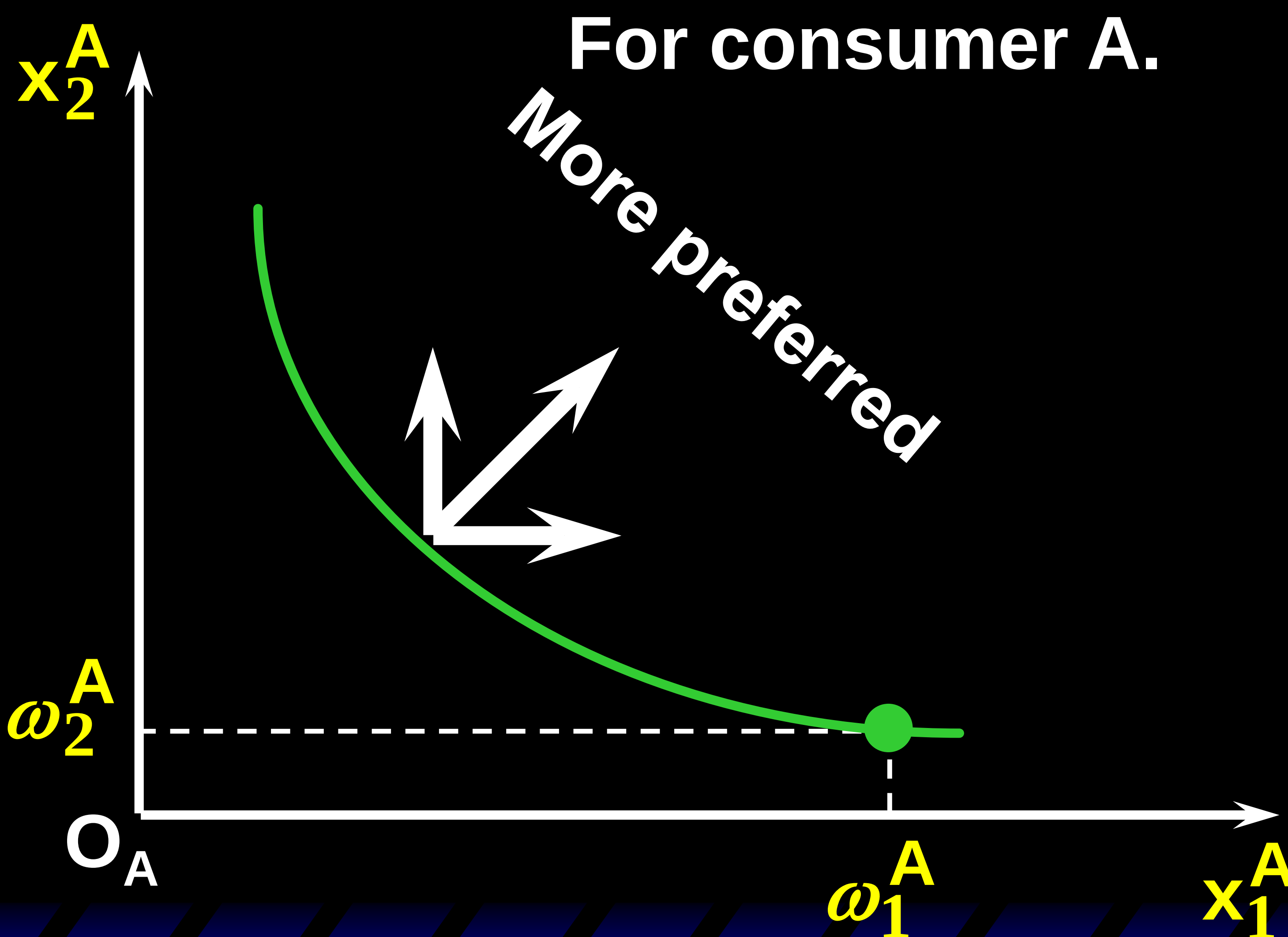
Which allocations make both consumers better off?

相比于禀赋点，哪些新的分配方案使得双方的境况都变好？

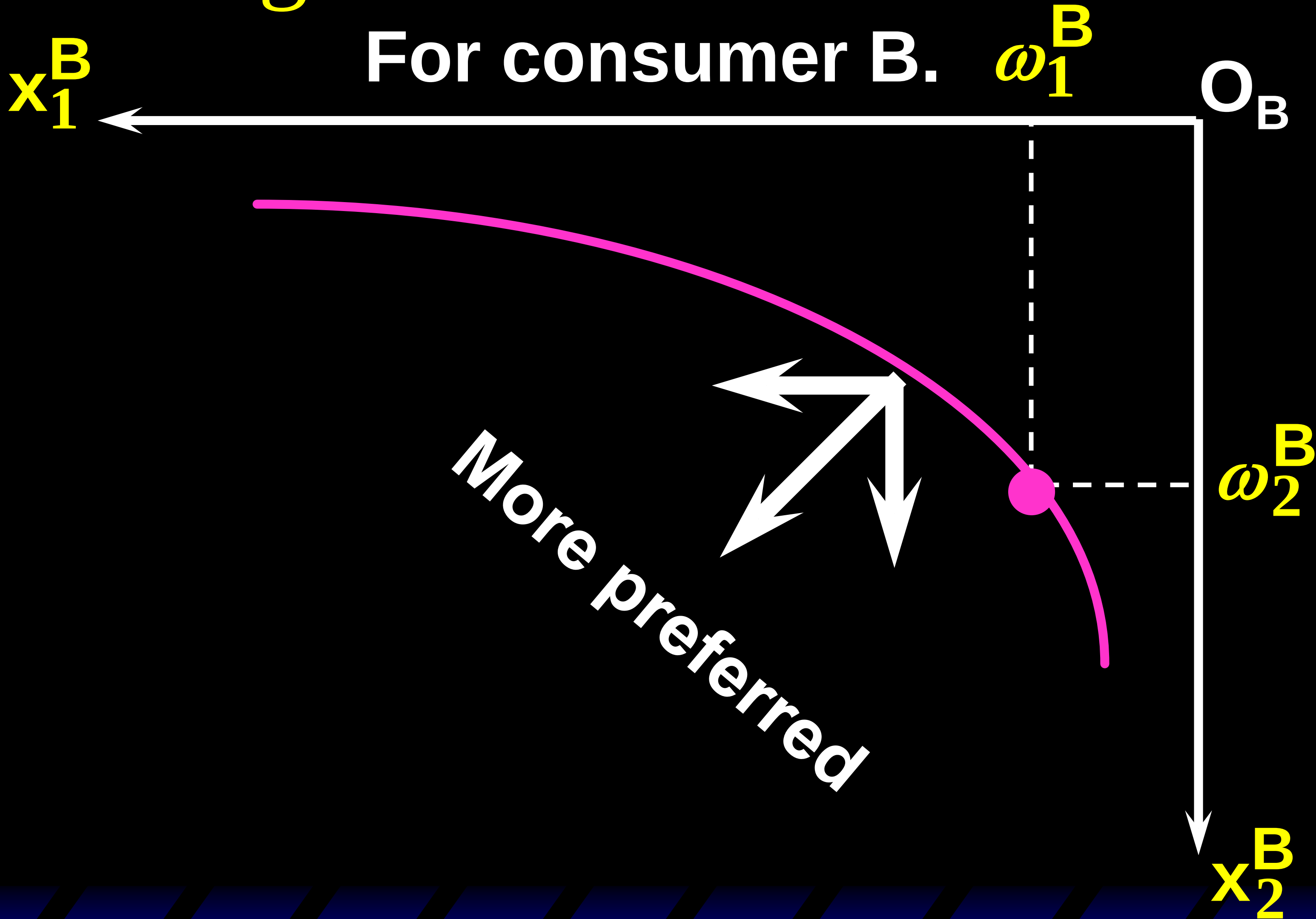
Adding Preferences to the Box



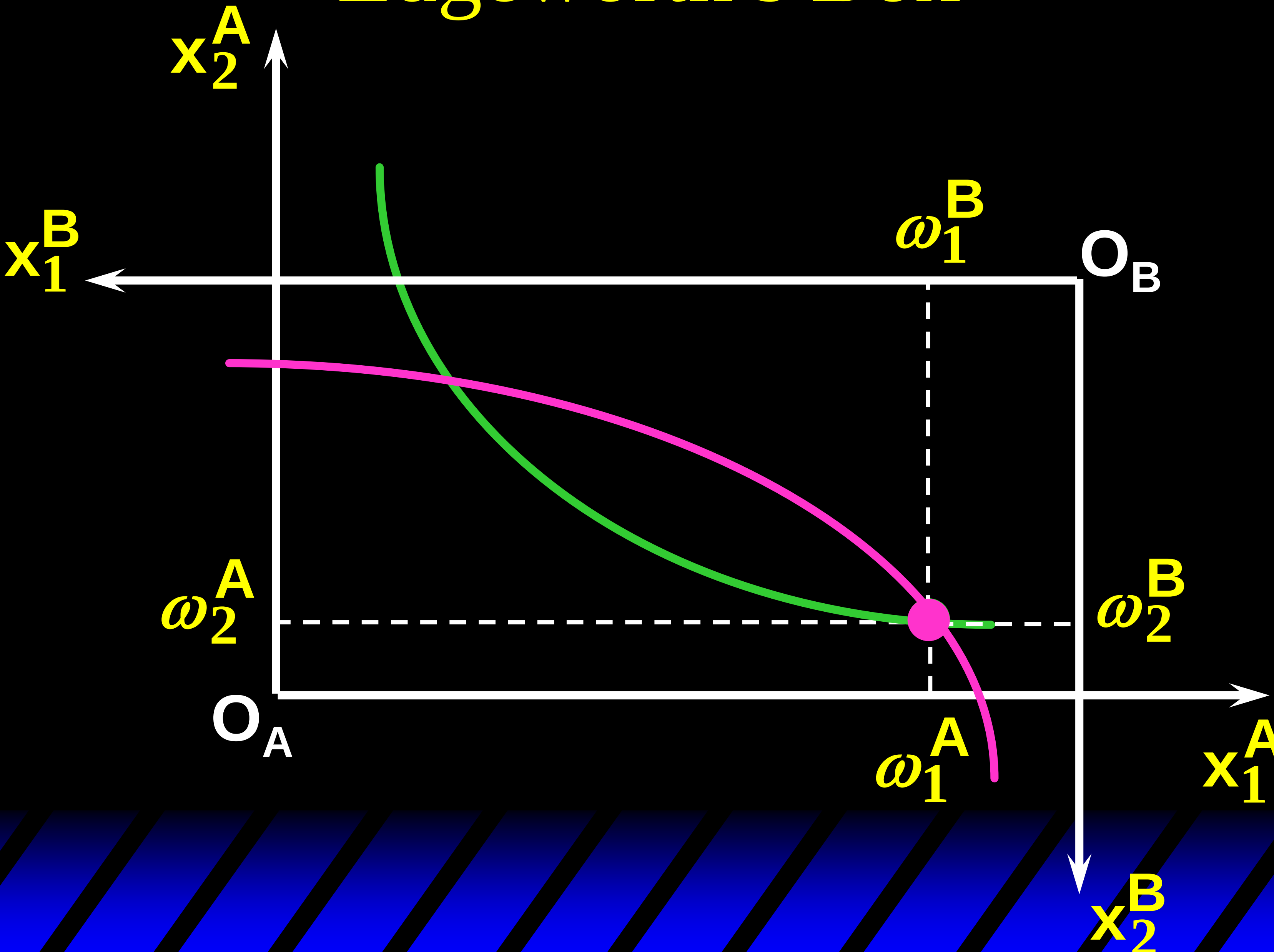
Adding Preferences to the Box



Adding Preferences to the Box



Edgeworth's Box



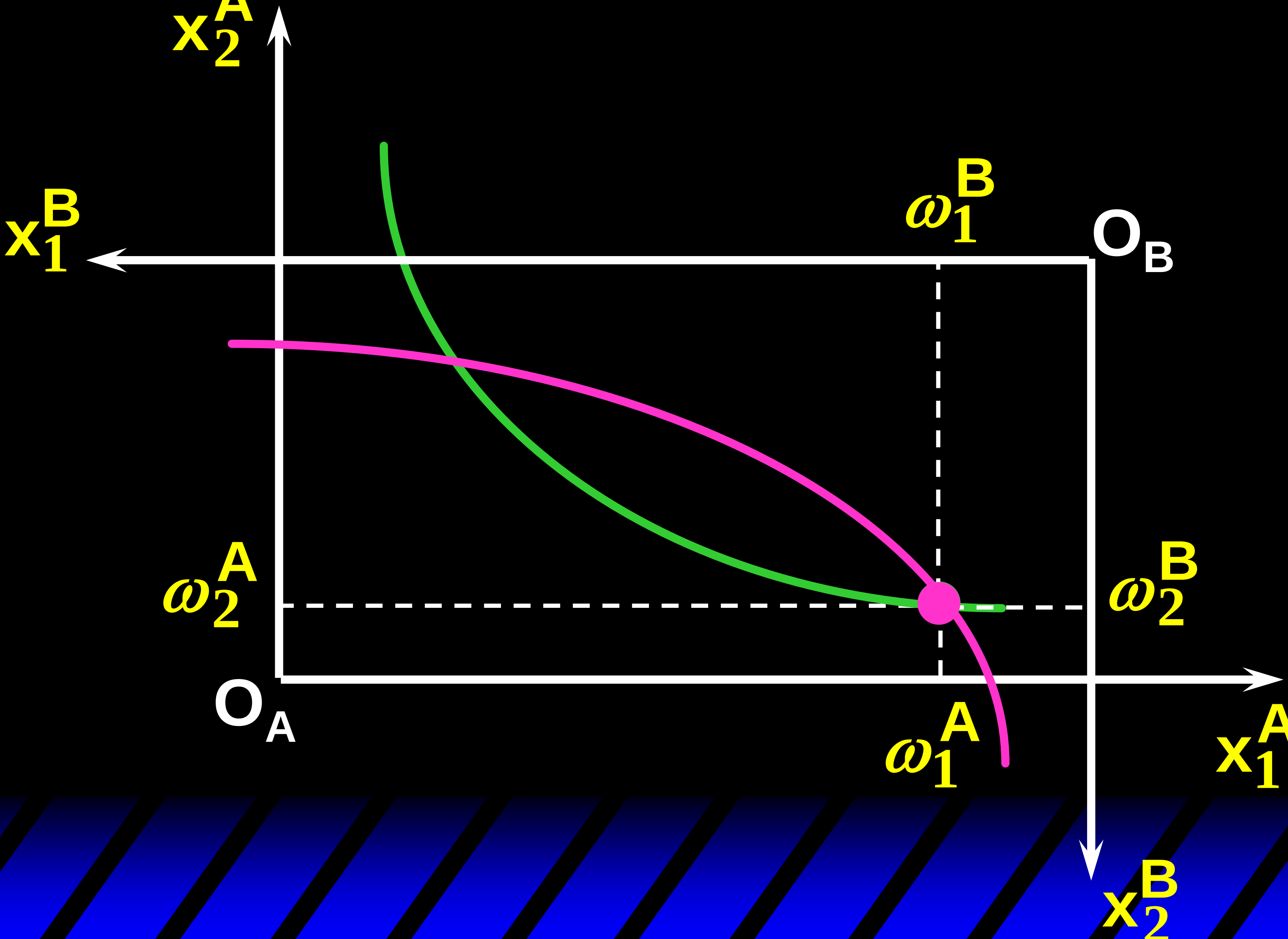
Pareto-Improvement

An allocation of the endowment that improves the welfare of a consumer without reducing the welfare of another is a Pareto-improving allocation.

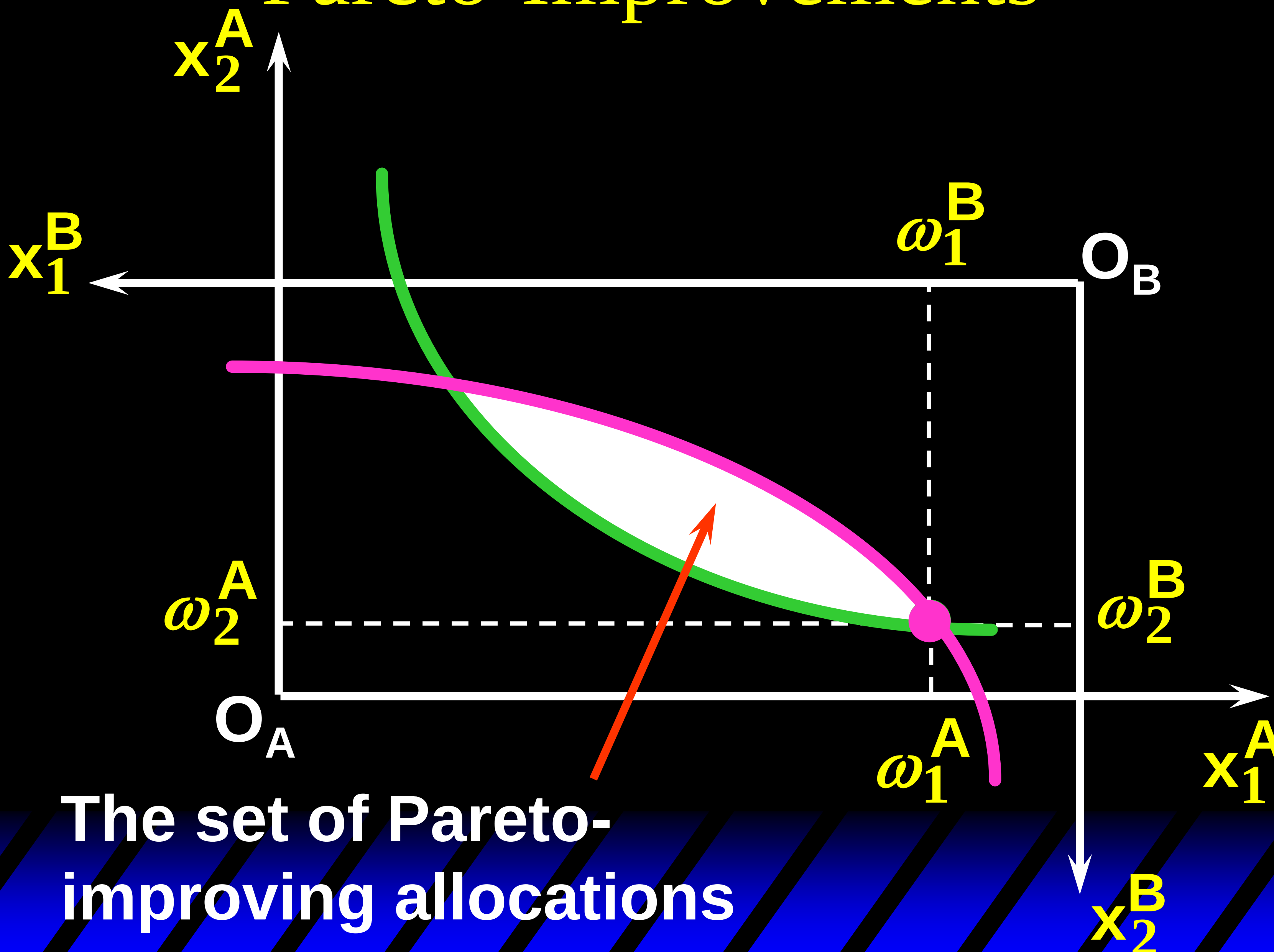
帕累托改进：一种分配，与禀赋分配相比能在不使任何人境况变差的情况下，使得一部分人的境况变好。

Where are the Pareto-improving allocations?

Edgeworth's Box

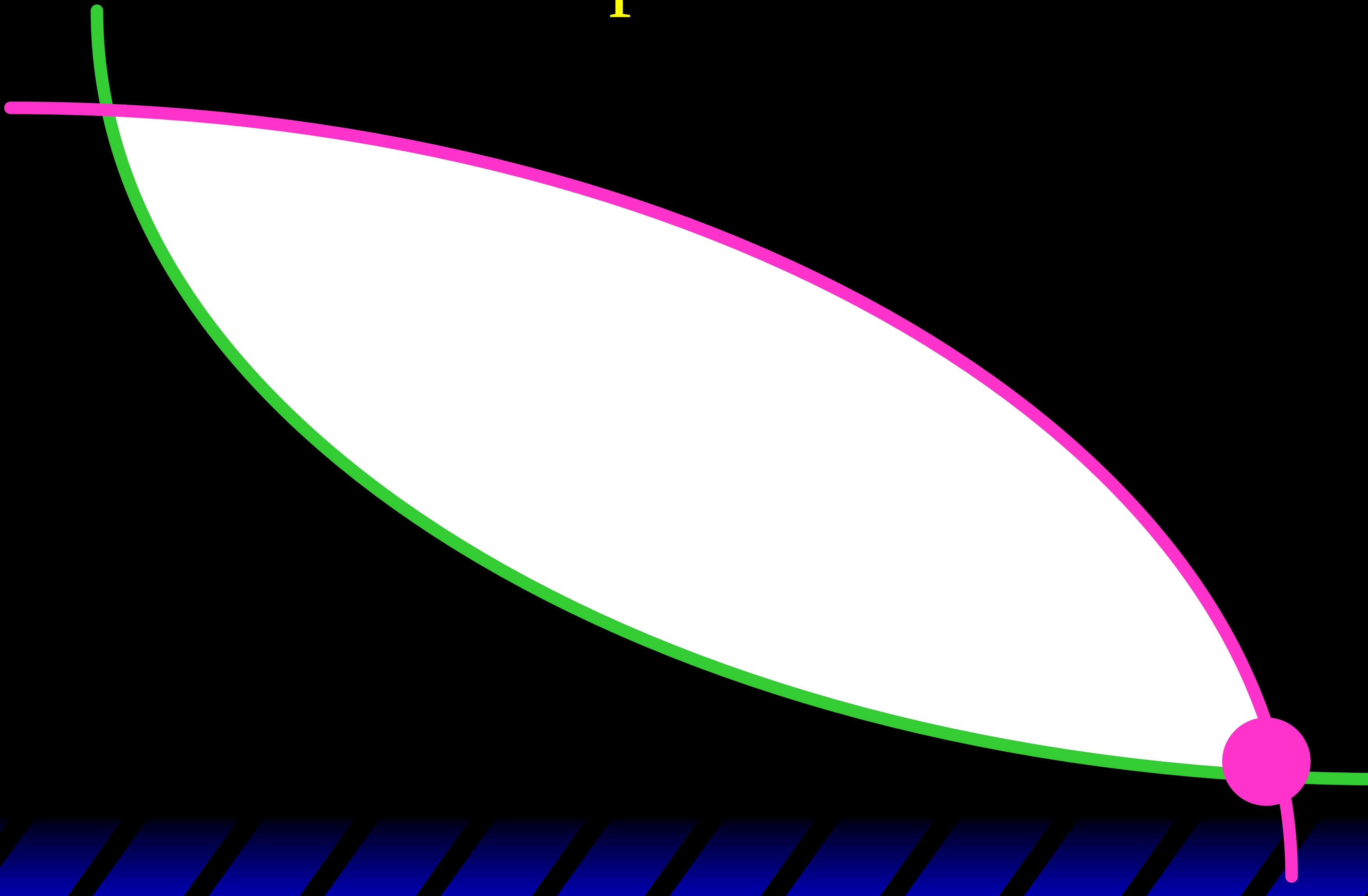


Pareto-Improvements

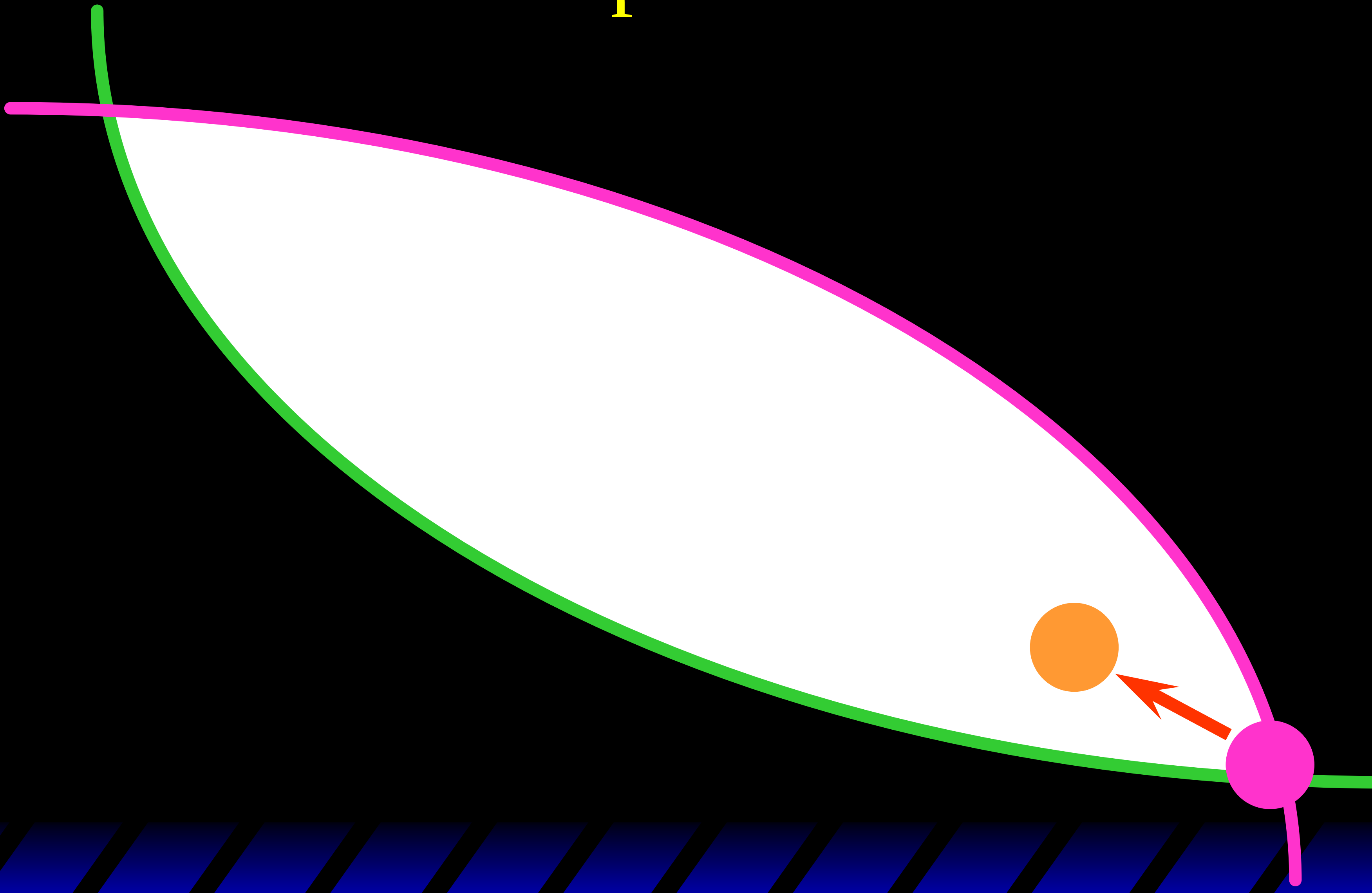


The set of Pareto-improving allocations

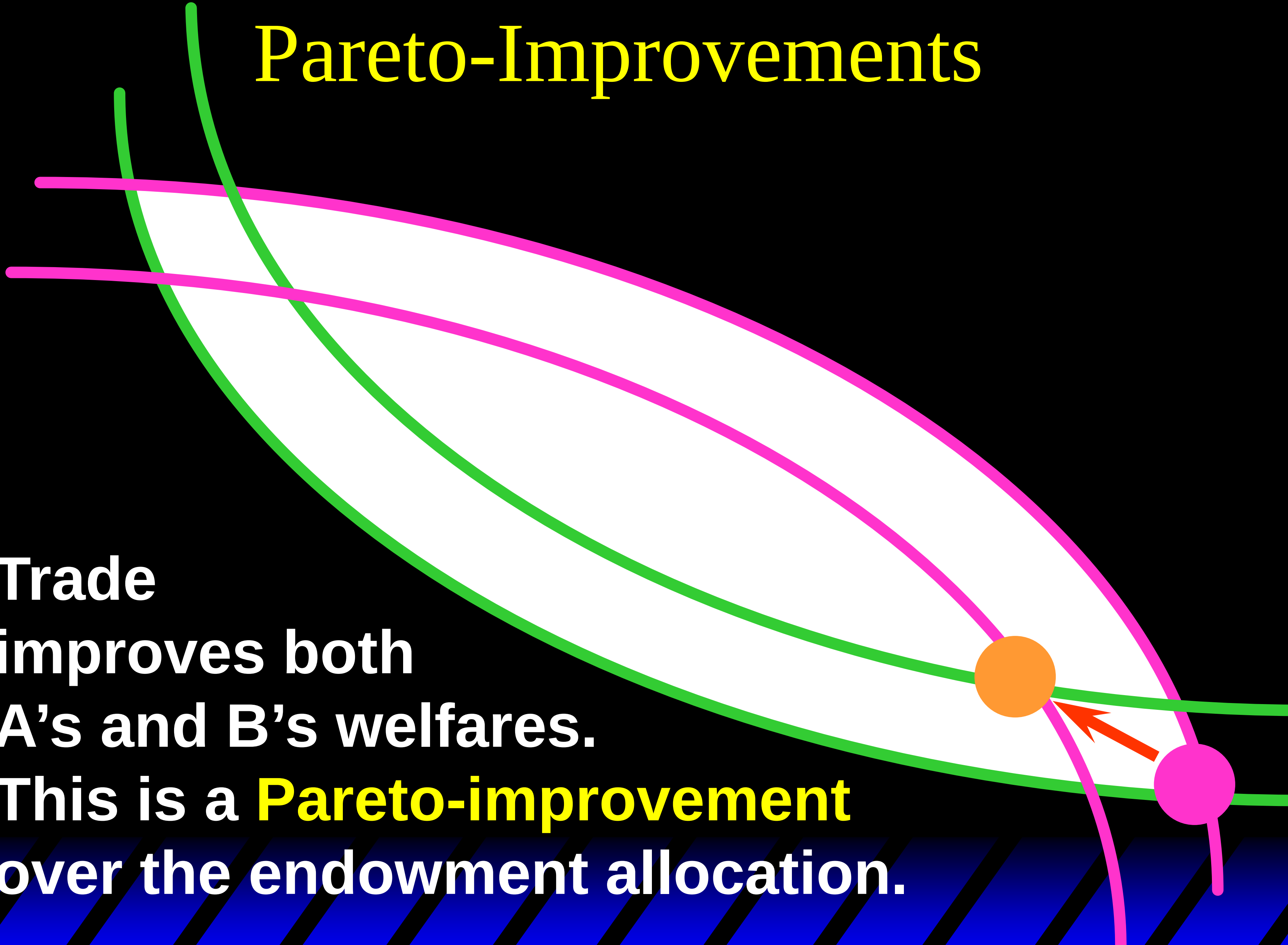
Pareto-Improvements



Pareto-Improvements



Pareto-Improvements



Trade improves both A's and B's welfares.
This is a Pareto-improvement over the endowment allocation.

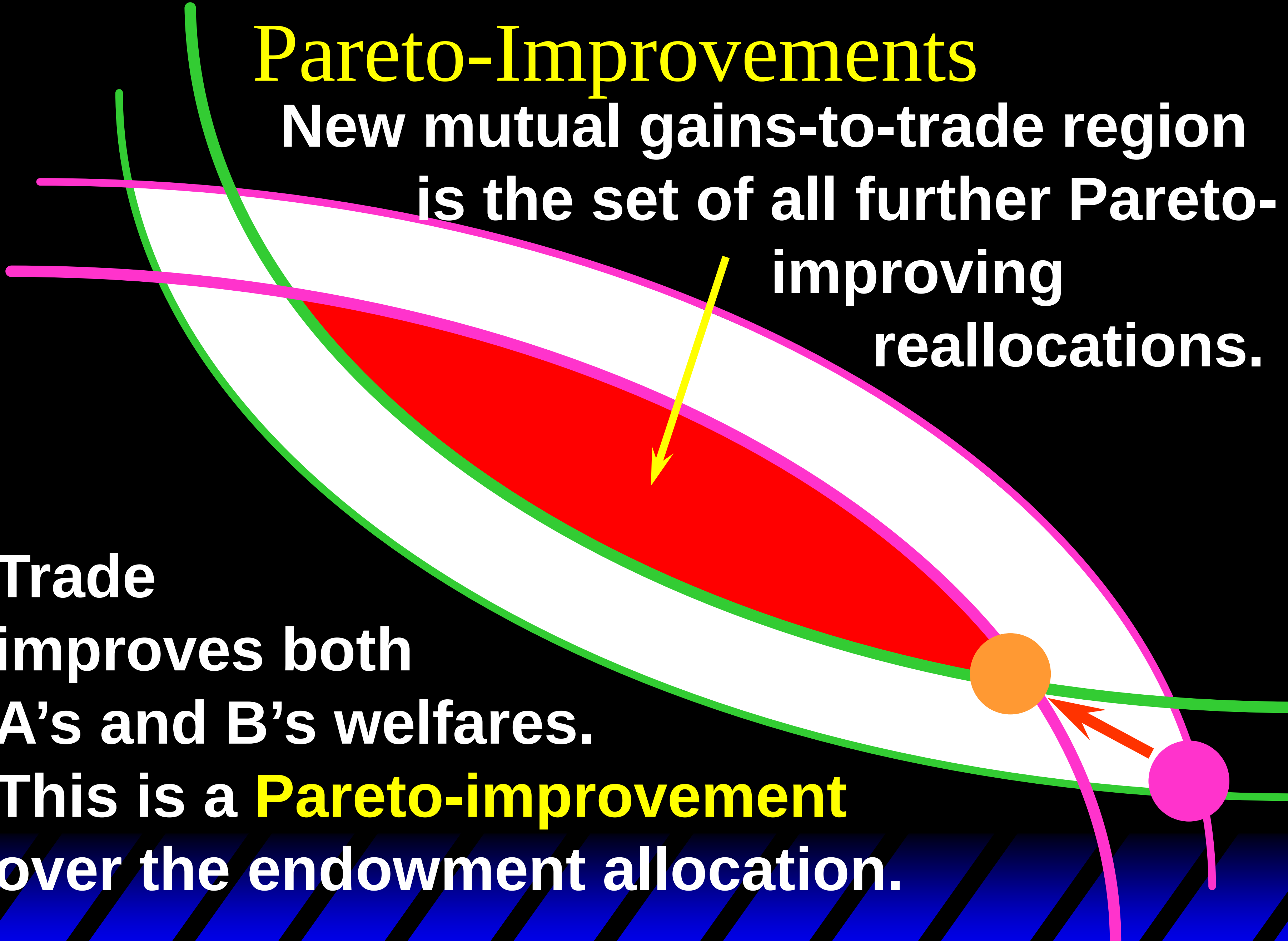
Pareto-Improvements

- **Since each consumer can refuse to trade, the only possible outcomes from exchange are Pareto-improving allocations.**

  若存在帕累托改进的可能性，就存在交易空间，双方都不会接受当前的分配方案。

- **But which particular Pareto-improving allocation will be the outcome of trade?**

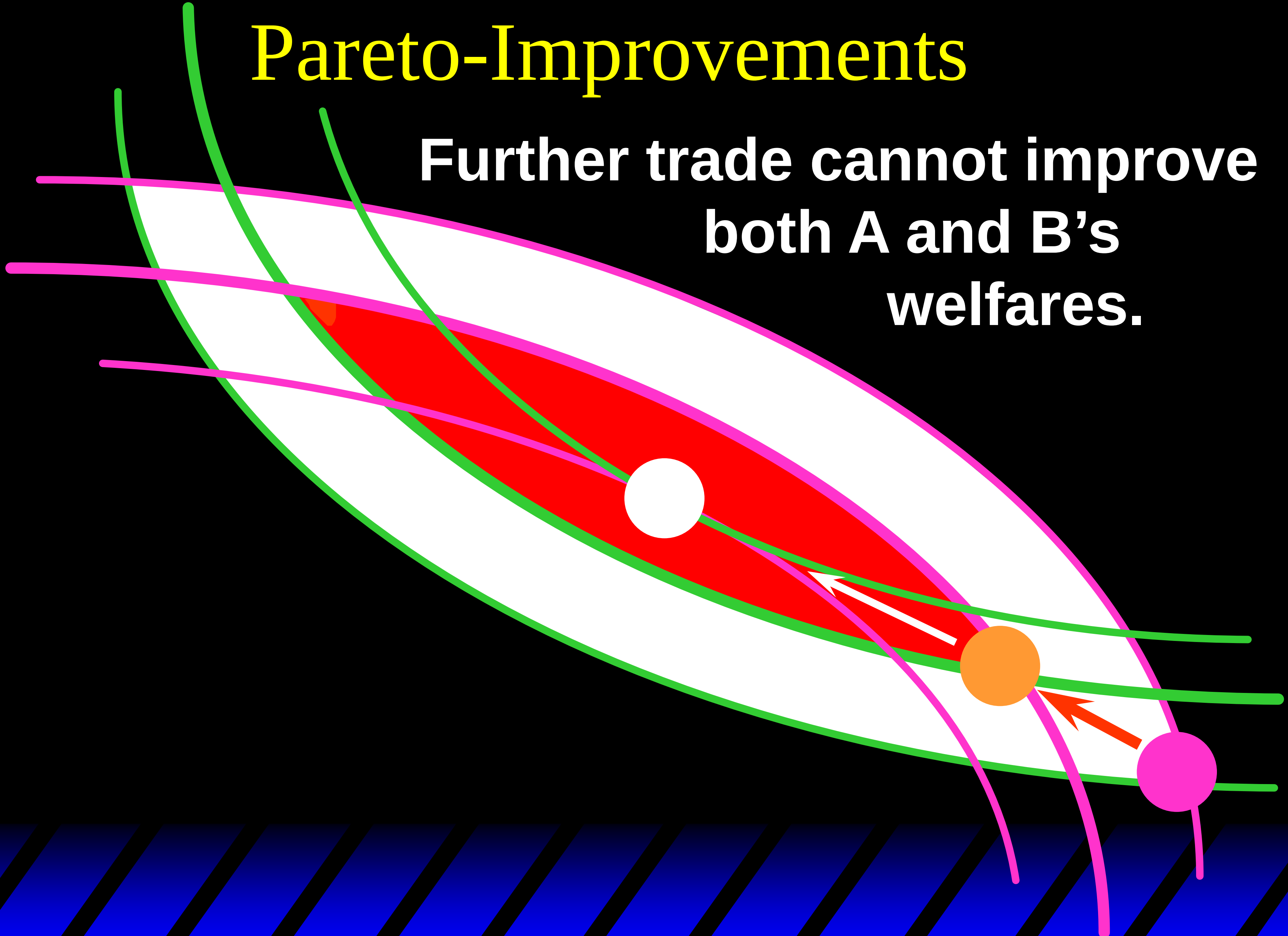
Pareto-Improvements



New mutual gains-to-trade region is the set of all further Pareto-improving reallocations.

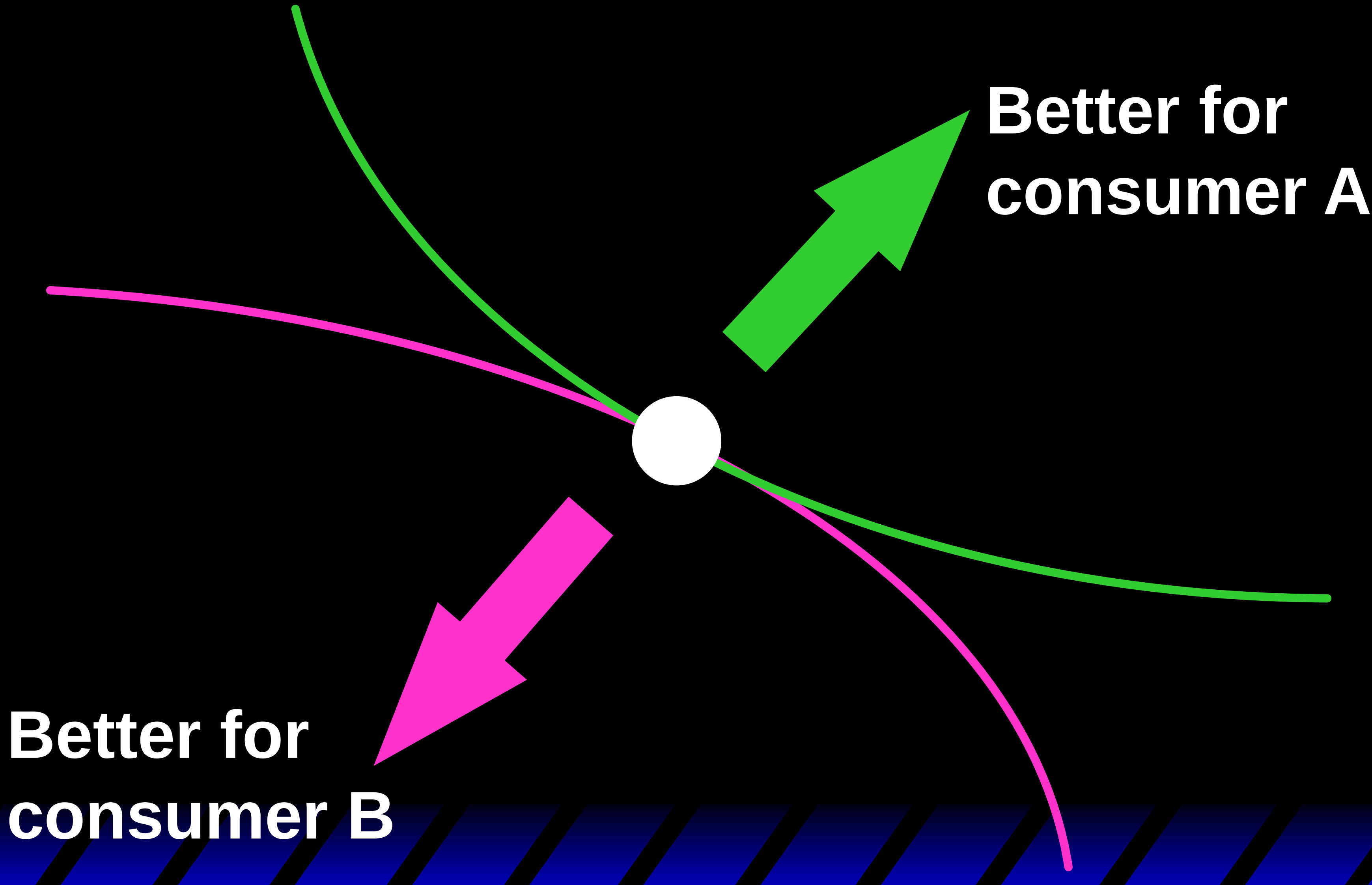
Trade improves both A's and B's welfares. This is a Pareto-improvement over the endowment allocation.

Pareto-Improvements

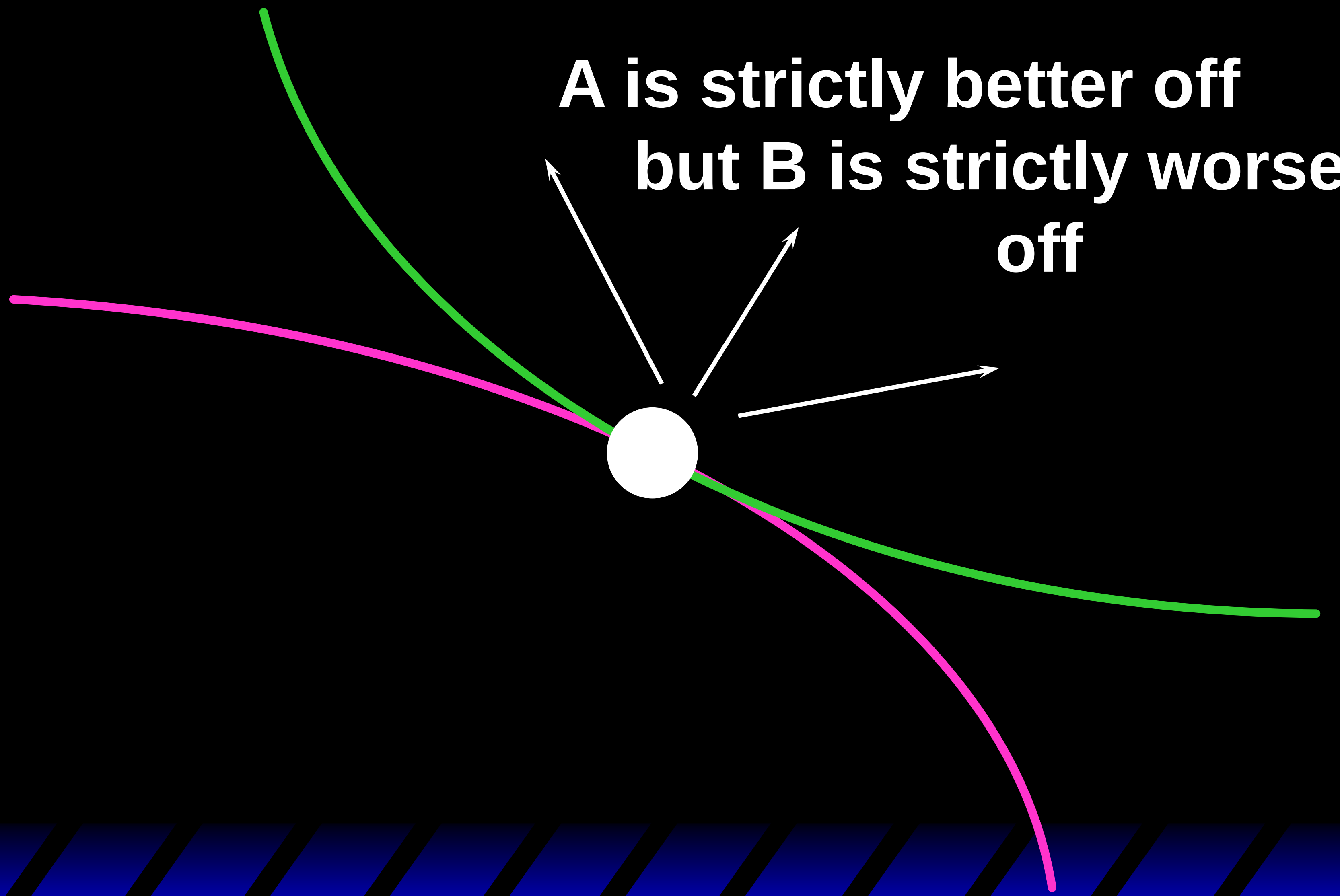


Further trade cannot improve both A and B's welfares.

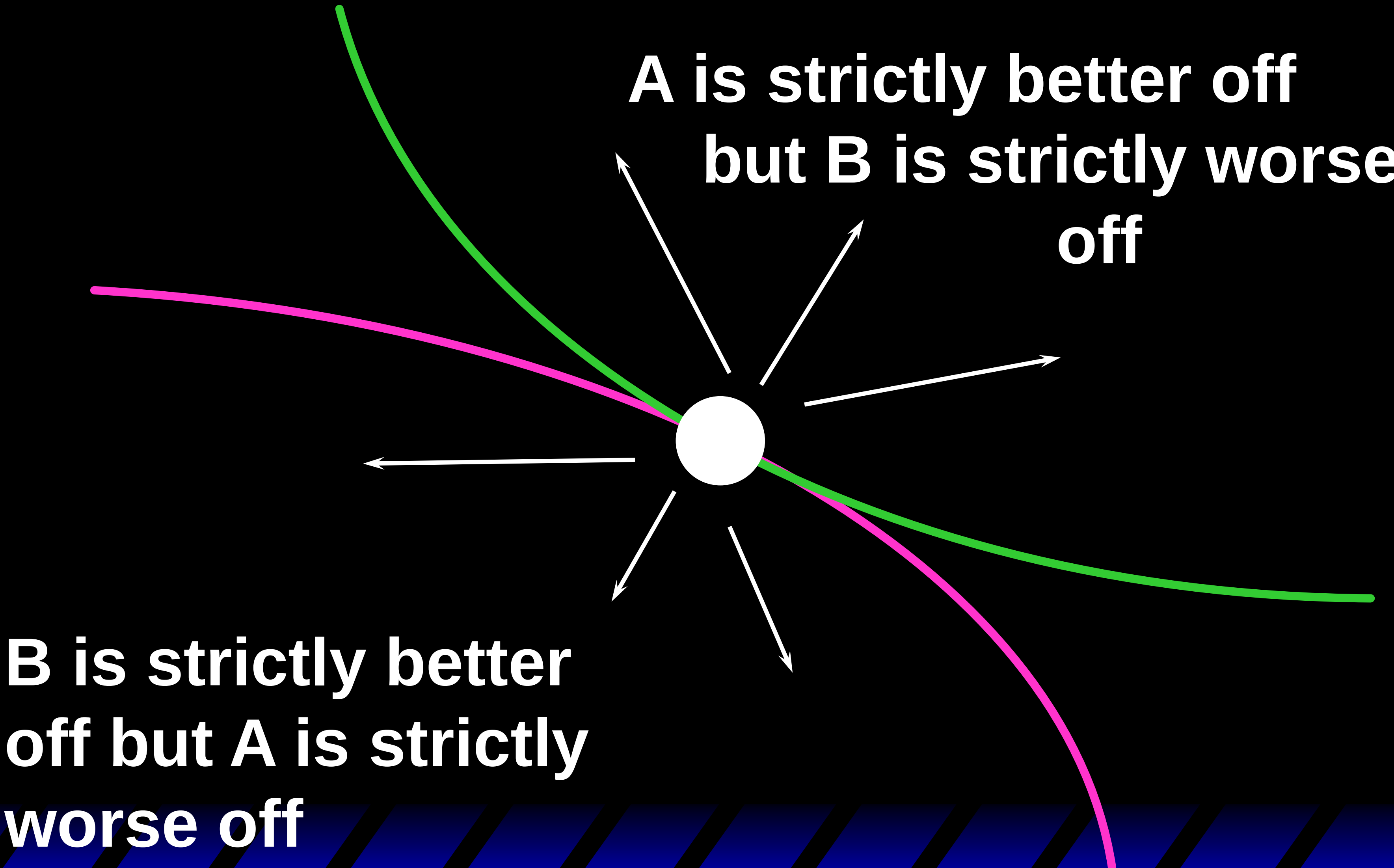
Pareto-Optimality



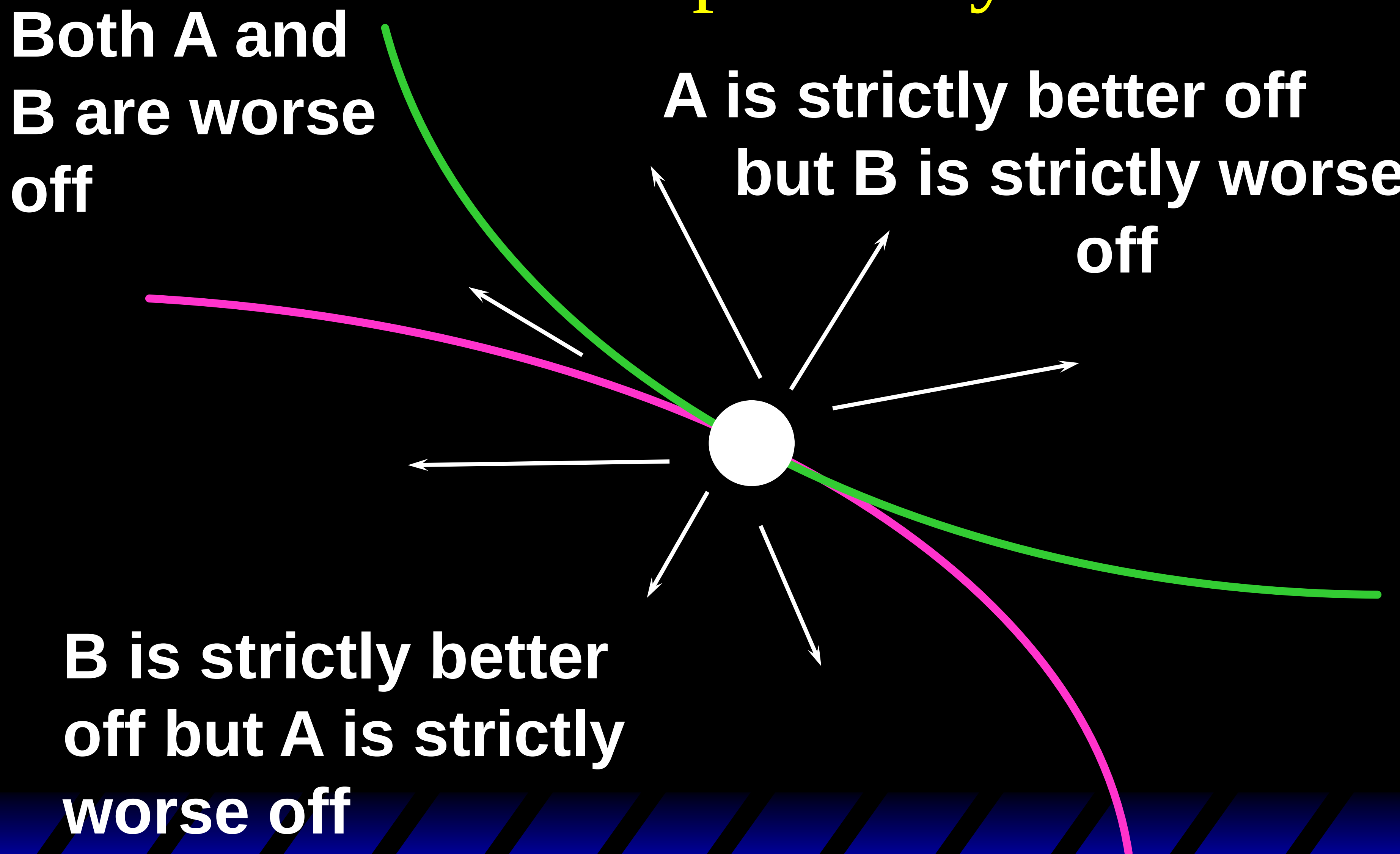
Pareto-Optimality



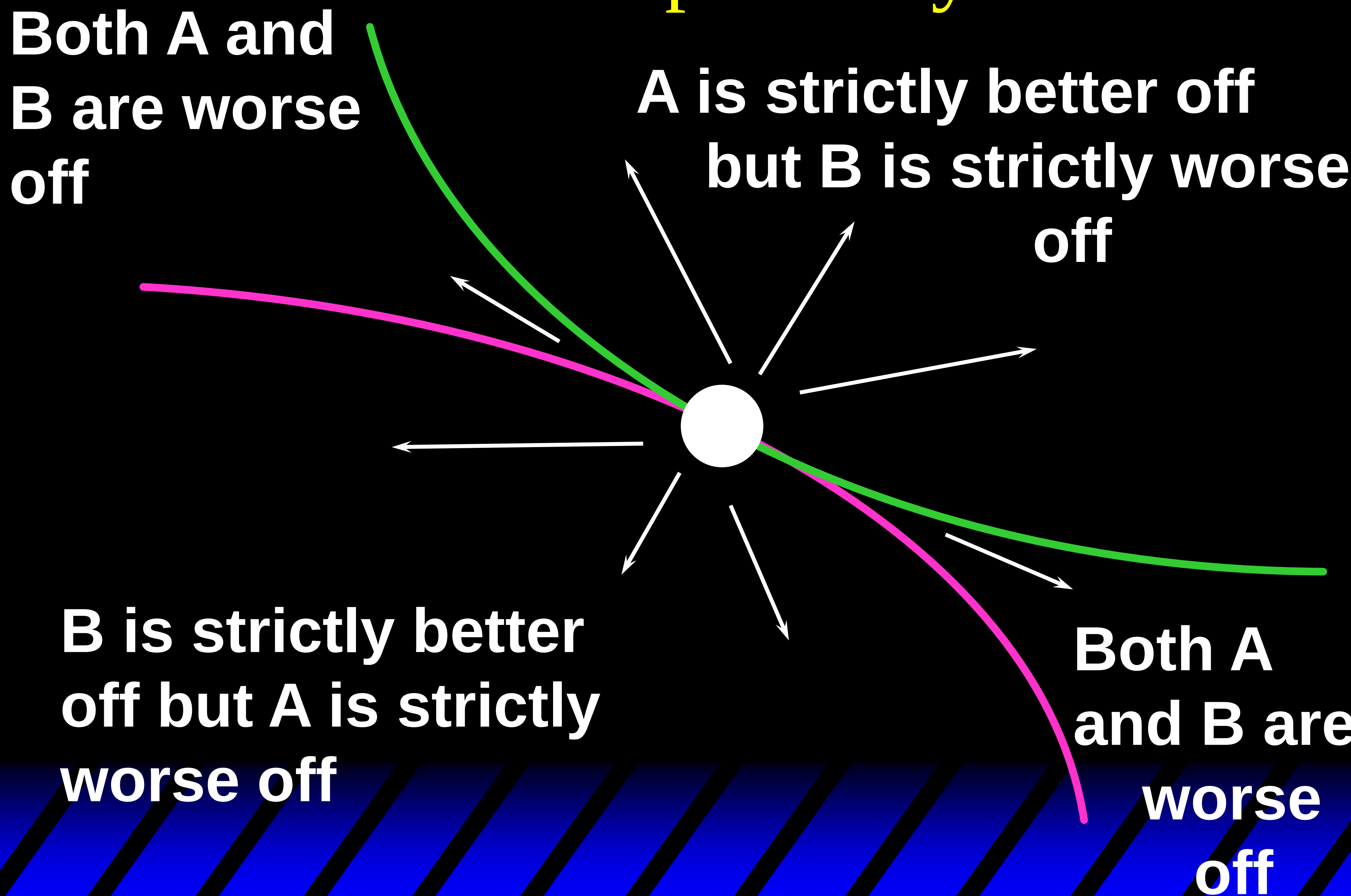
Pareto-Optimality



Pareto-Optimality



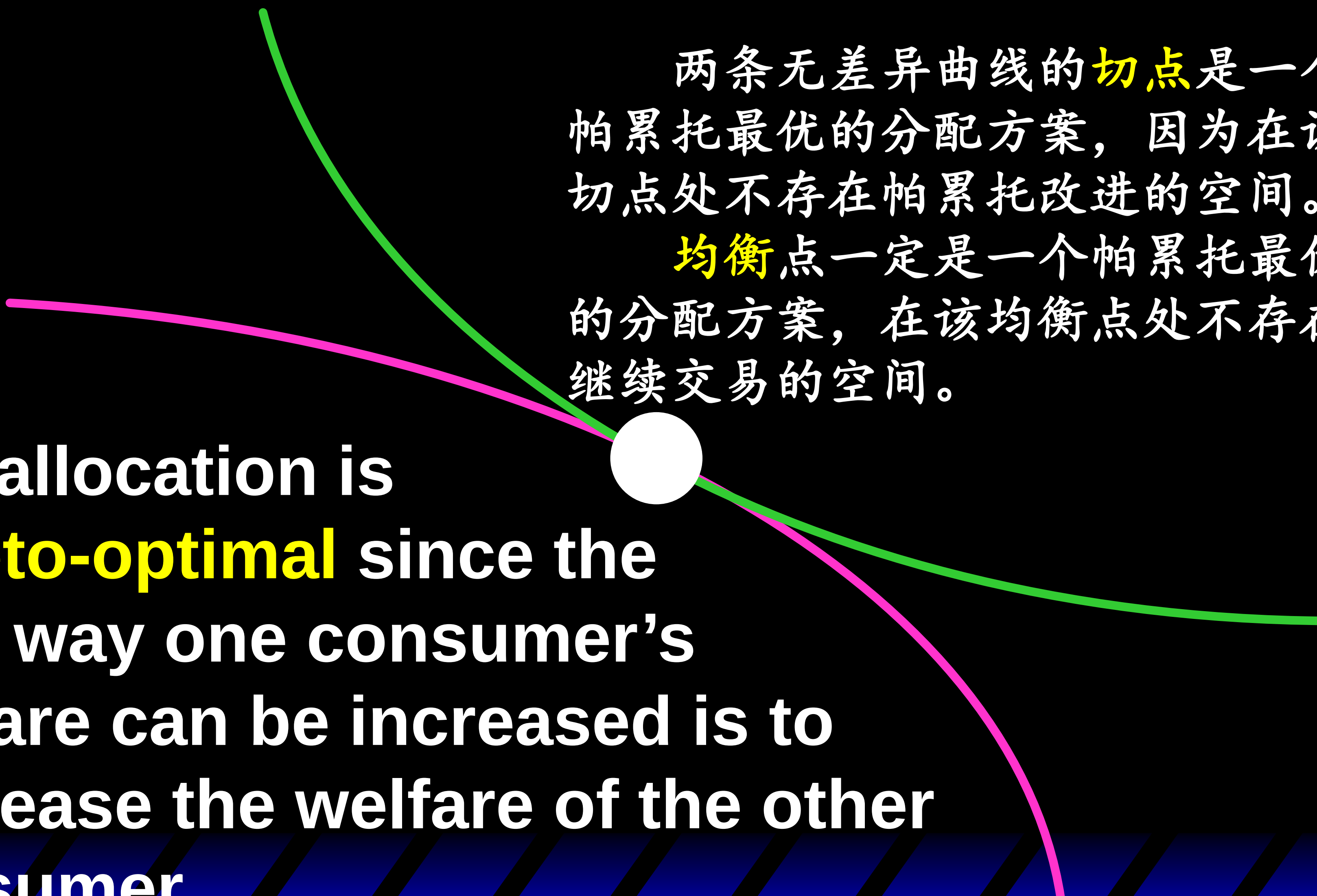
Pareto-Optimality



Pareto-Optimality

两条无差异曲线的**切点**是一个帕累托最优的分配方案，因为在该切点处不存在帕累托改进的空间。

均衡点一定是一个帕累托最优的分配方案，在该均衡点处不存在继续交易的空间。

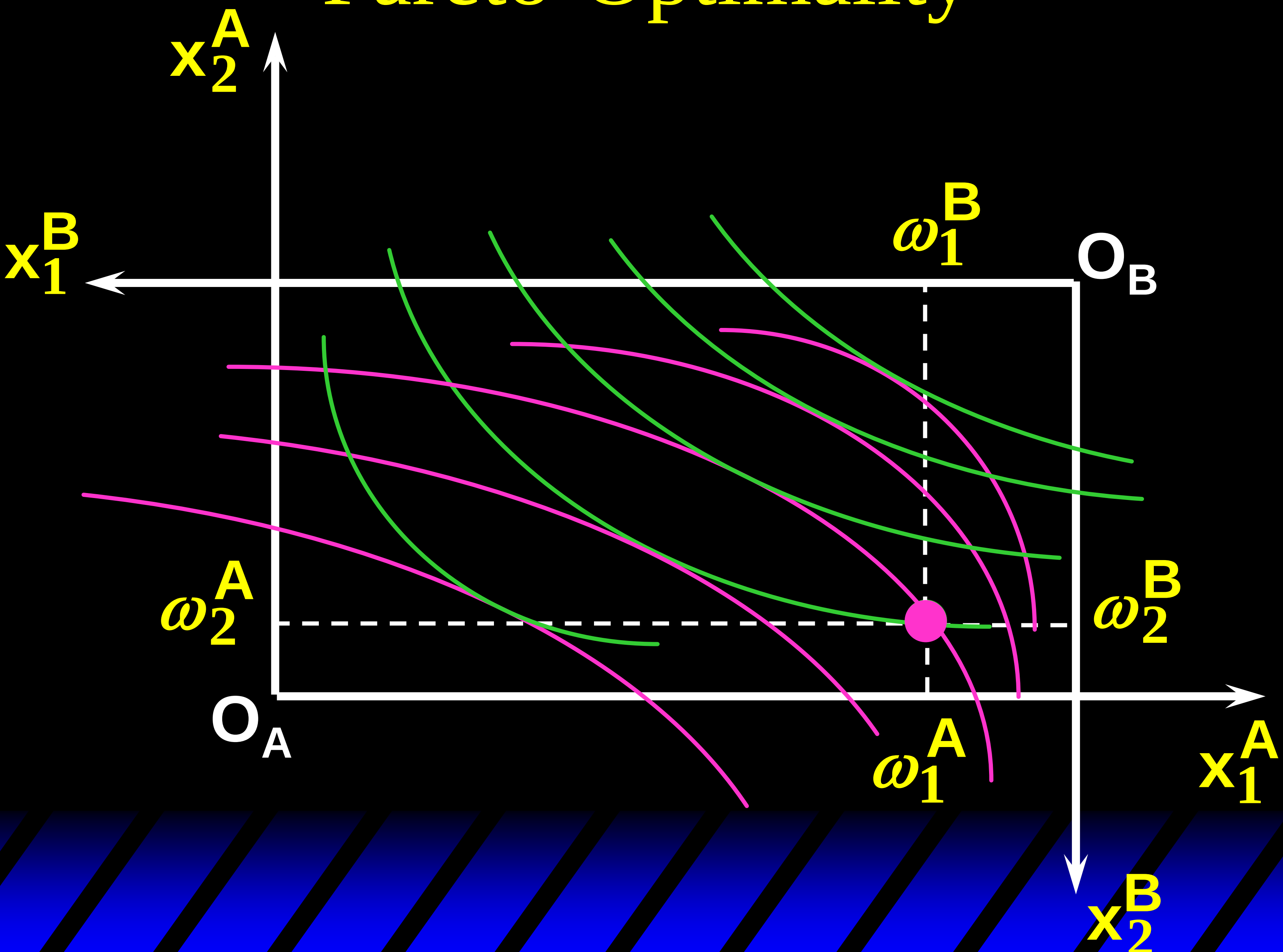


The allocation is **Pareto-optimal** since the only way one consumer’s welfare can be increased is to decrease the welfare of the other consumer.

Pareto-Optimality

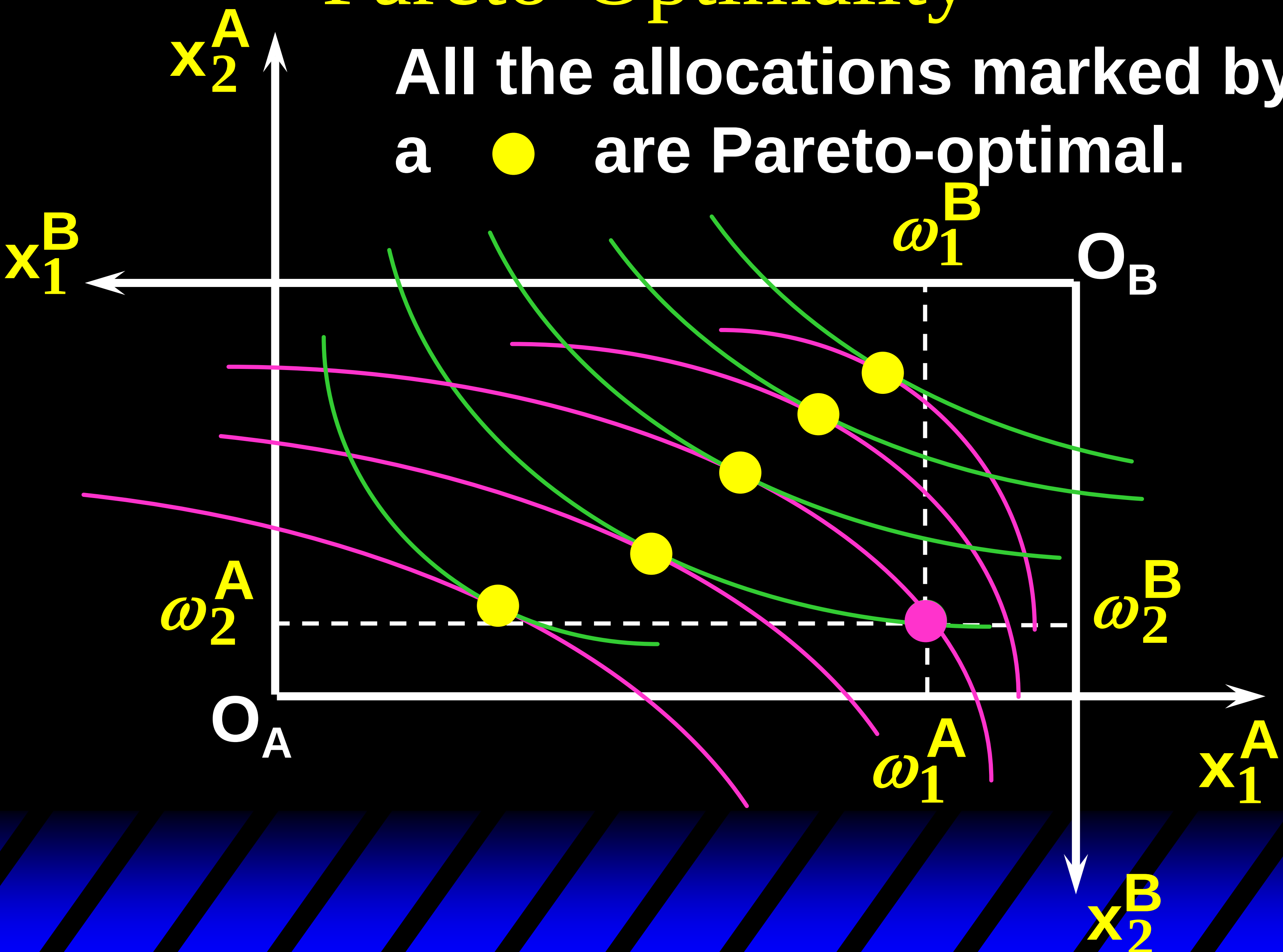
Where are **all** of the Pareto-optimal allocations of the endowment?

Pareto-Optimality



Pareto-Optimality

All the allocations marked by a ● are Pareto-optimal.



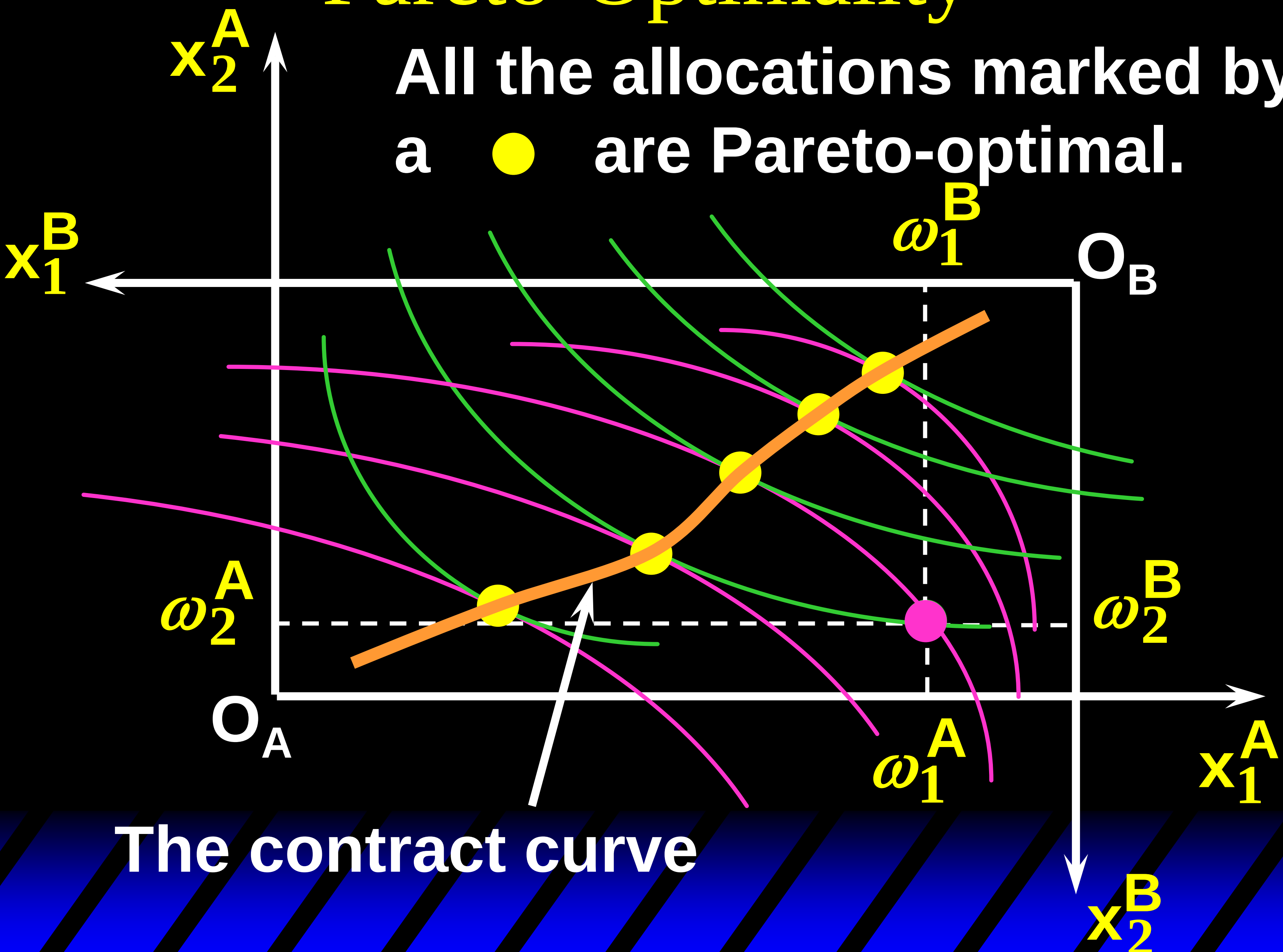
Pareto-Optimality

The contract curve is the set of all Pareto-optimal allocations.

帕累托最优分配的集合叫做契约曲线。

Pareto-Optimality

All the allocations marked by a ● are Pareto-optimal.



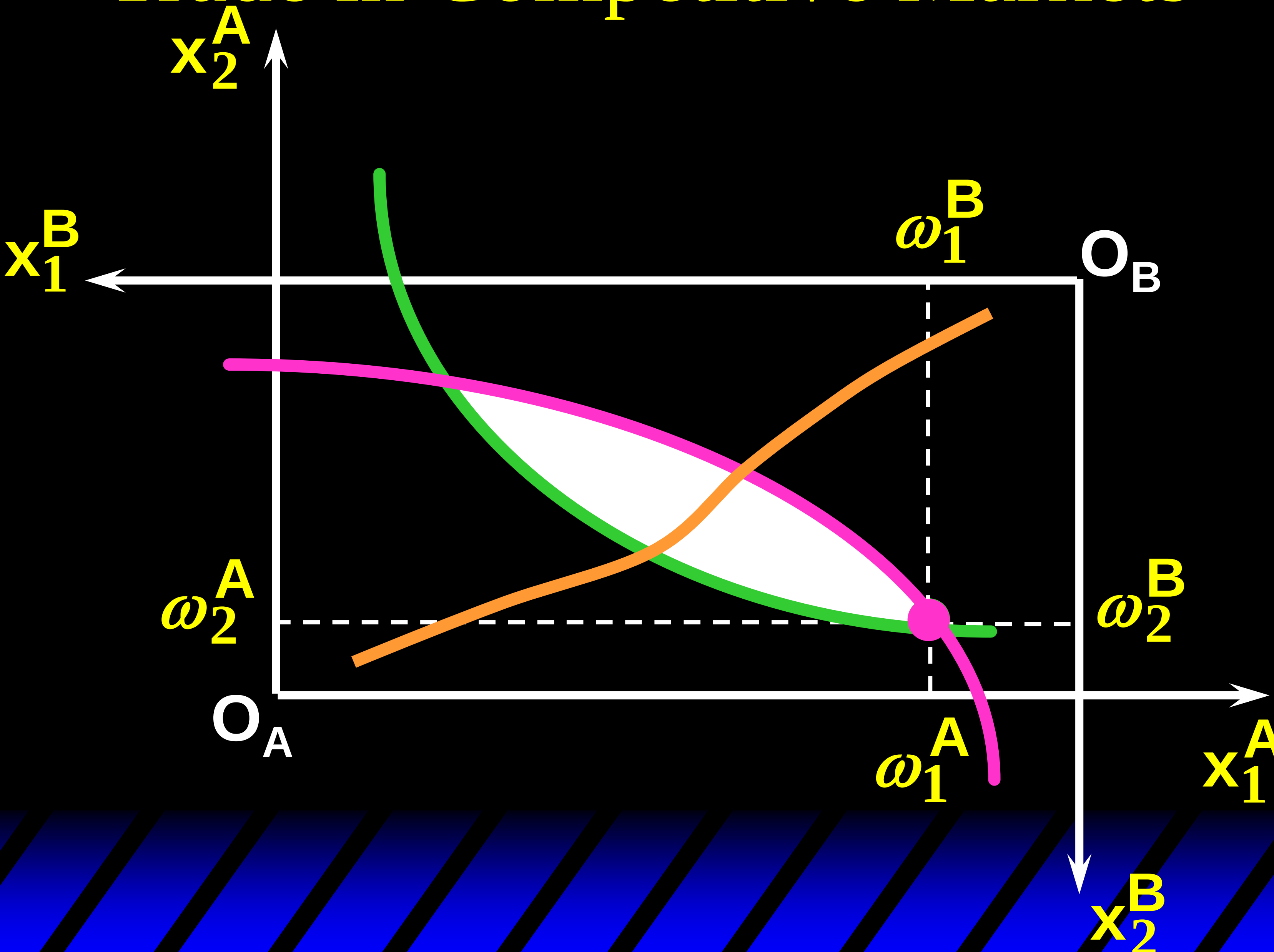
Pareto-Optimality

In a perfectly competitive market, to which of the many allocations on the contract curve will consumers trade?

若存在帕累托改进空间，双方都不接受当前分配 => 均衡处不存在帕累托改进的空间 => 均衡分配在契约曲线上

均衡分配在契约曲线上的哪一点？

Trade in Competitive Markets



Trade in Competitive Markets

- Consider trade in perfectly **competitive** markets.
- Each consumer is a **price-taker** trying to maximize her own utility given p_1, p_2 and her own endowment.

Trade in Competitive Markets

We are looking for the equilibrium prices (p_1^*, p_2^*) such that:

1) A chooses (x_1^{*A}, x_2^{*A}) to max $\mathrm{U}^A(x_1^A, x_2^A)$
s.t. $p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A$

2) B chooses (x_1^{*B}, x_2^{*B}) to max $\mathrm{U}^B(x_1^B, x_2^B)$
s.t. $p_1 x_1^B + p_2 x_2^B = p_1 \omega_1^B + p_2 \omega_2^B$

Trade in Competitive Markets

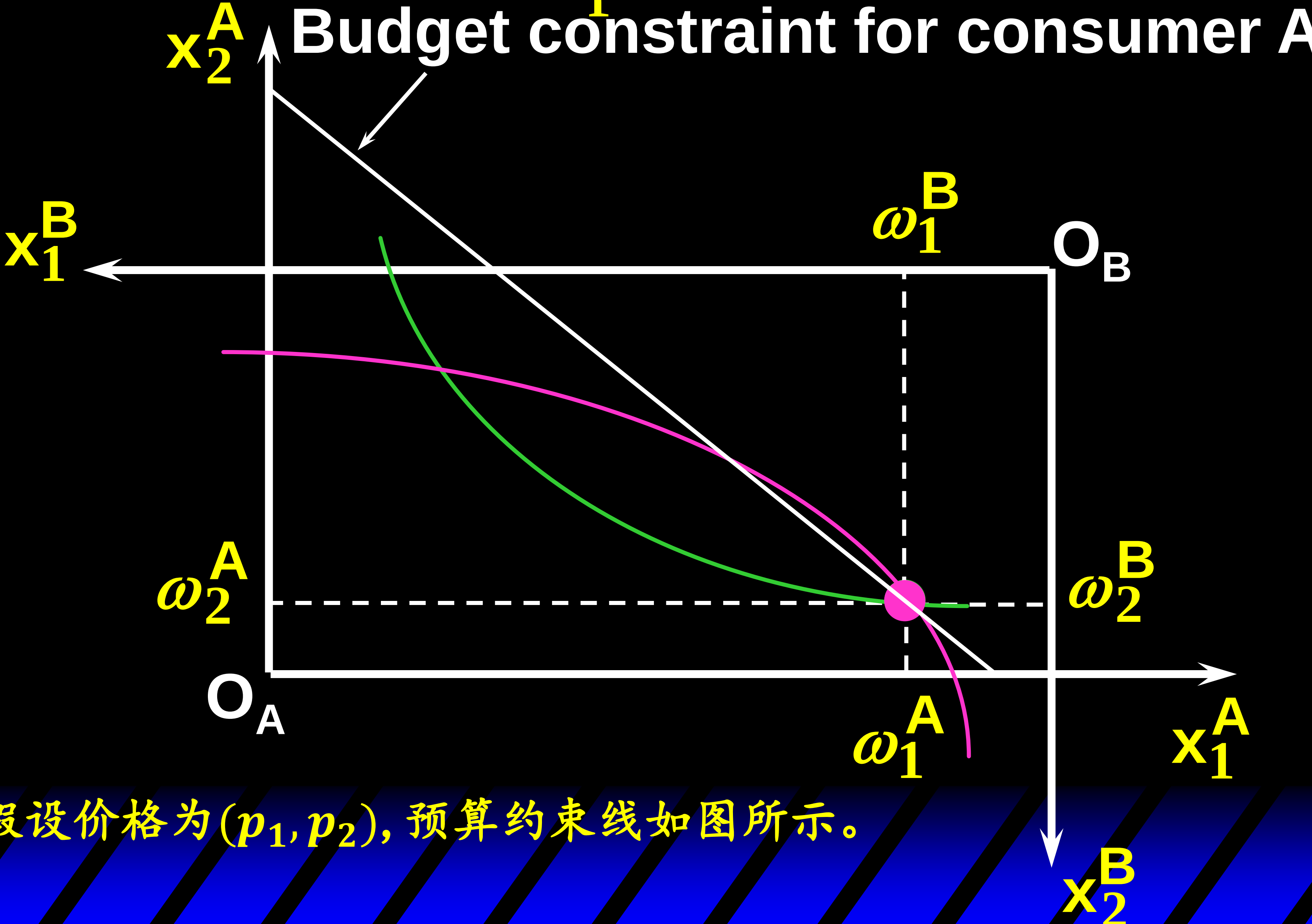
We are looking for the equilibrium prices (p_1^*, p_2^*) such that:

3) Markets clear, i.e. total consumption of x_1 (x_2) = total endowment of x_1 (x_2)

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_2^B$$

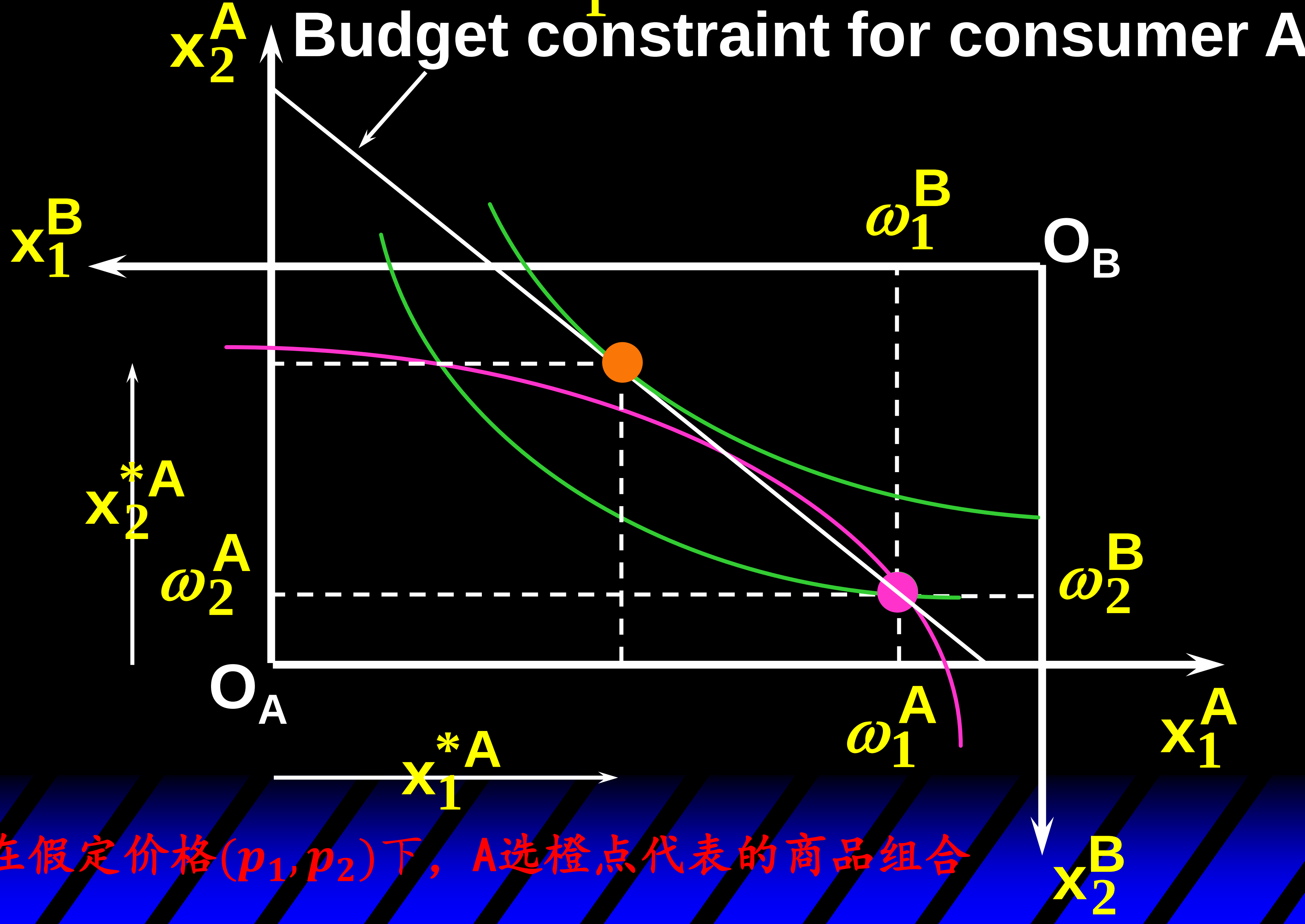
$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$$

Trade in Competitive Markets



假设价格为(p_1, p_2)，预算约束线如图所示。

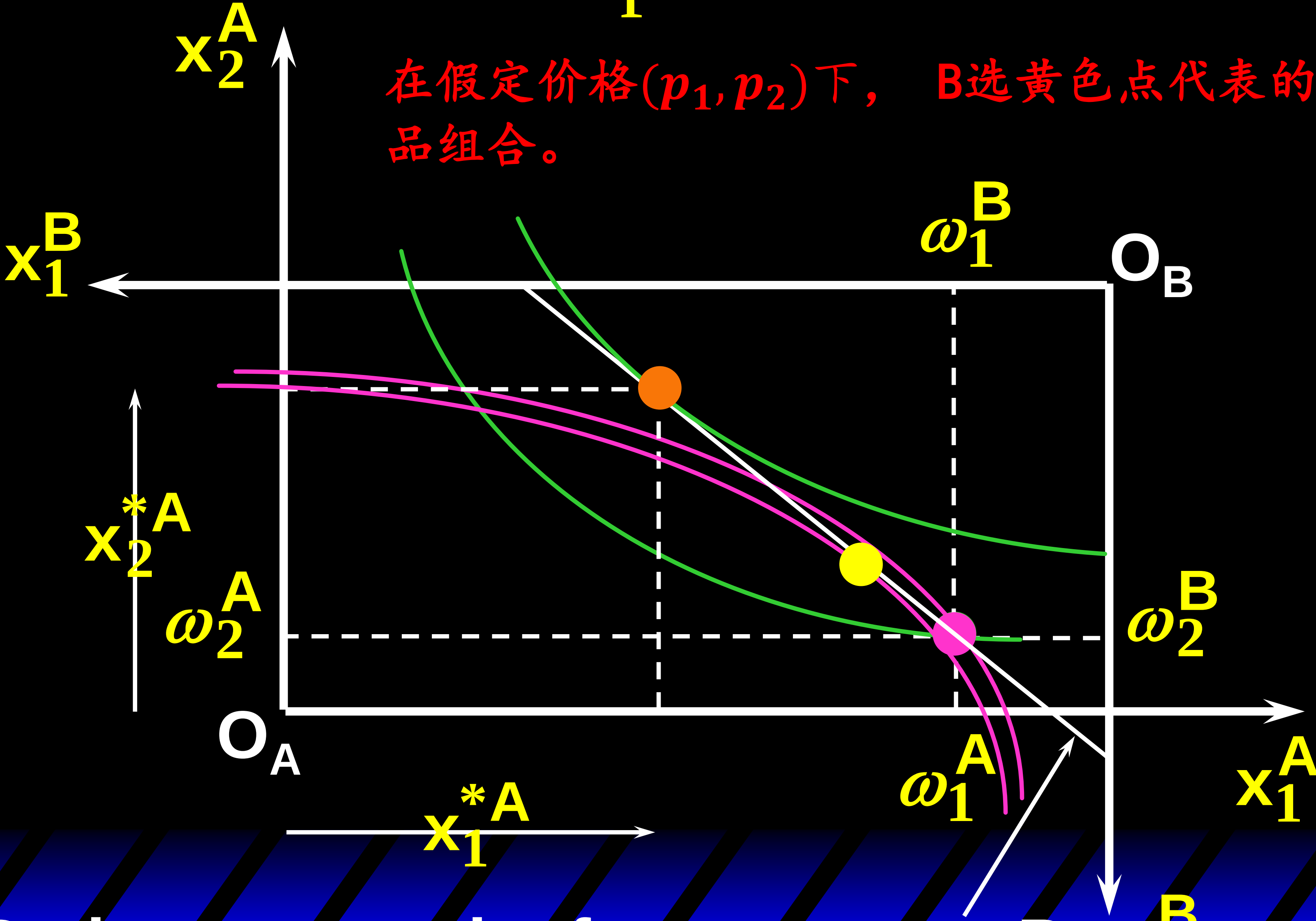
Trade in Competitive Markets



在假定价格(p_1,p_2)下，A选橙点代表的商品组合

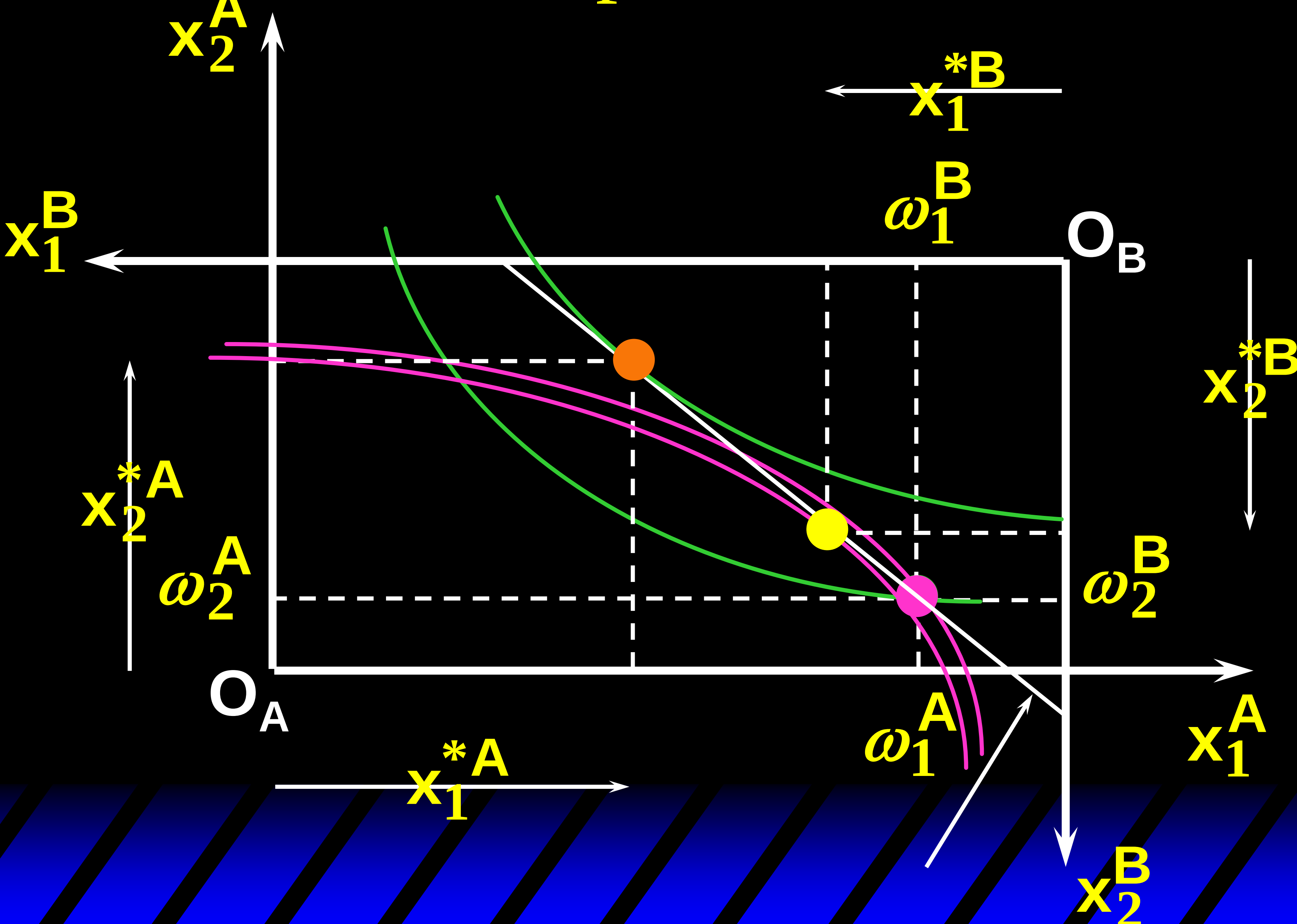
Trade in Competitive Markets

在假定价格(p_1, p_2)下， B选黄色点代表的商品组合。



Budget constraint for consumer B

Trade in Competitive Markets



Trade in Competitive Markets

在假定的价格下，

$$x_1^{*A} + x_1^{*B} < \omega_1^A + \omega_1^B$$

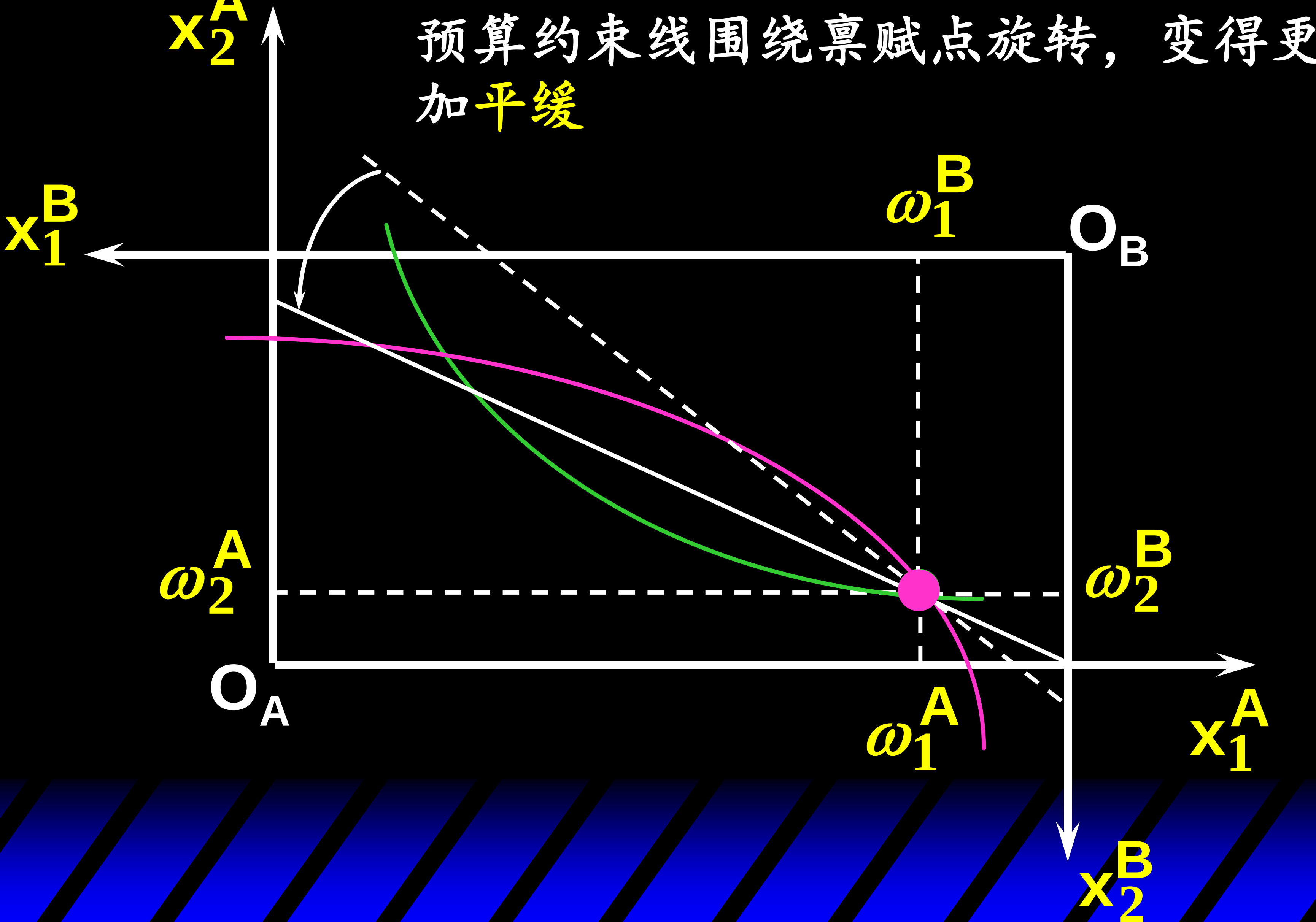
$$x_2^{*A} + x_2^{*B} > \omega_2^A + \omega_2^B$$

市场不出清；x_1的总需求小于总供给 (excess supply)、x_2的总需求大于总供给 (excess demand); 因此该假定价格不是均衡价格

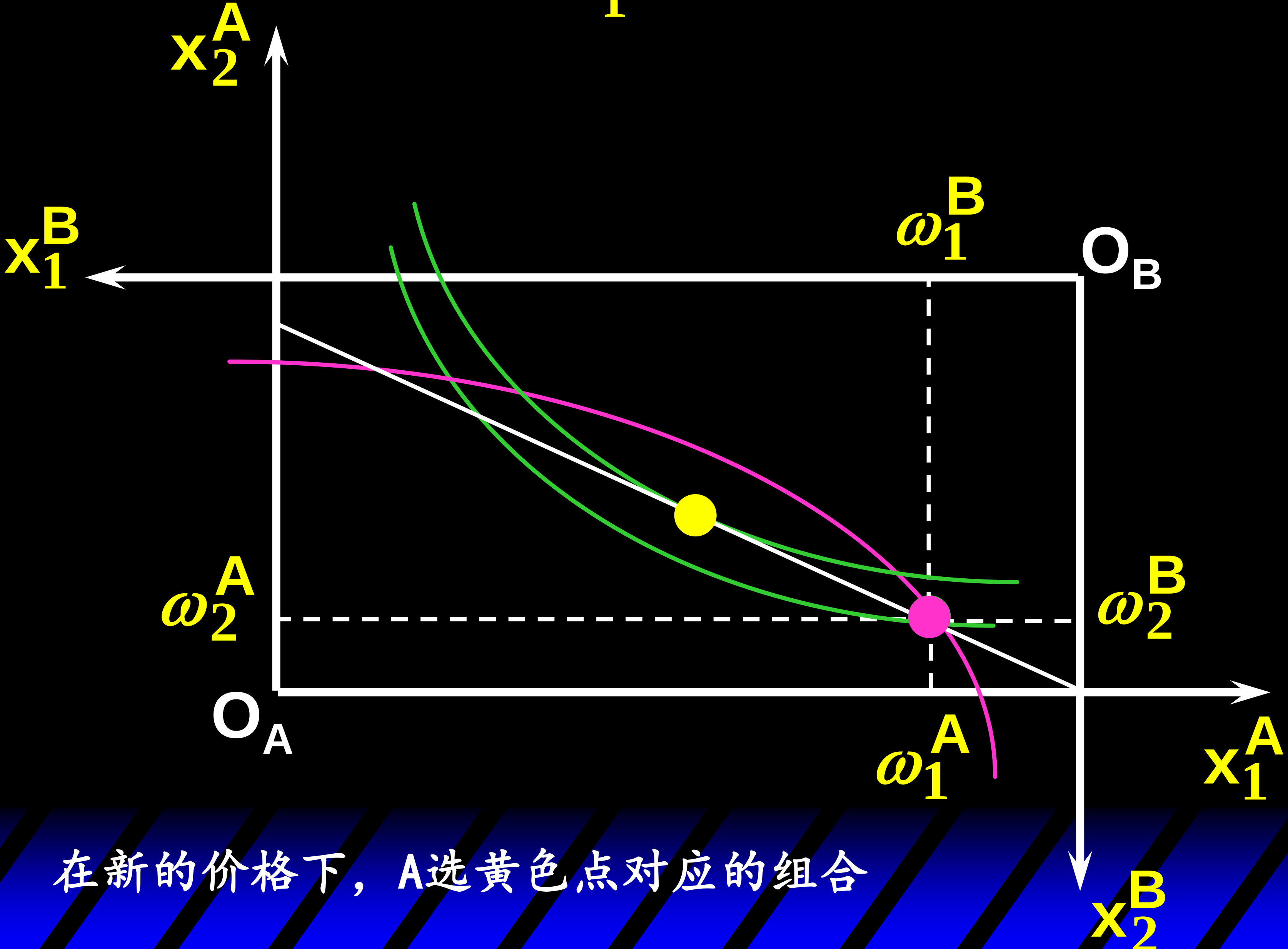
Trade in Competitive Markets

- Since there is an excess demand for commodity 2, p_2 will **rise**.
- Since there is an excess supply of commodity 1, p_1 will **fall**.
- The slope of the budget constraints is $- p_1/p_2$ so the budget constraints will pivot about the endowment point and become **less steep**.

Trade in Competitive Markets

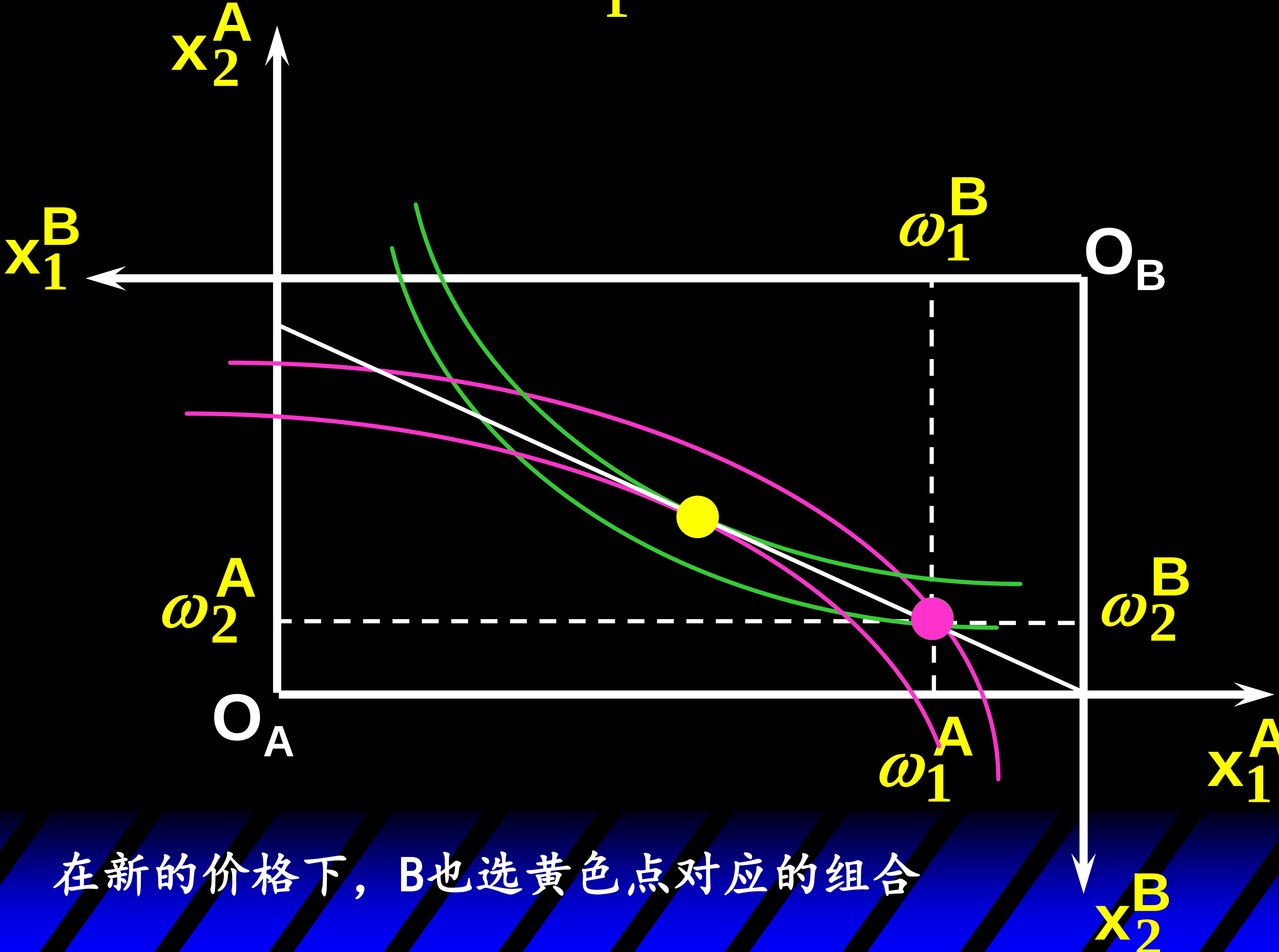


Trade in Competitive Markets



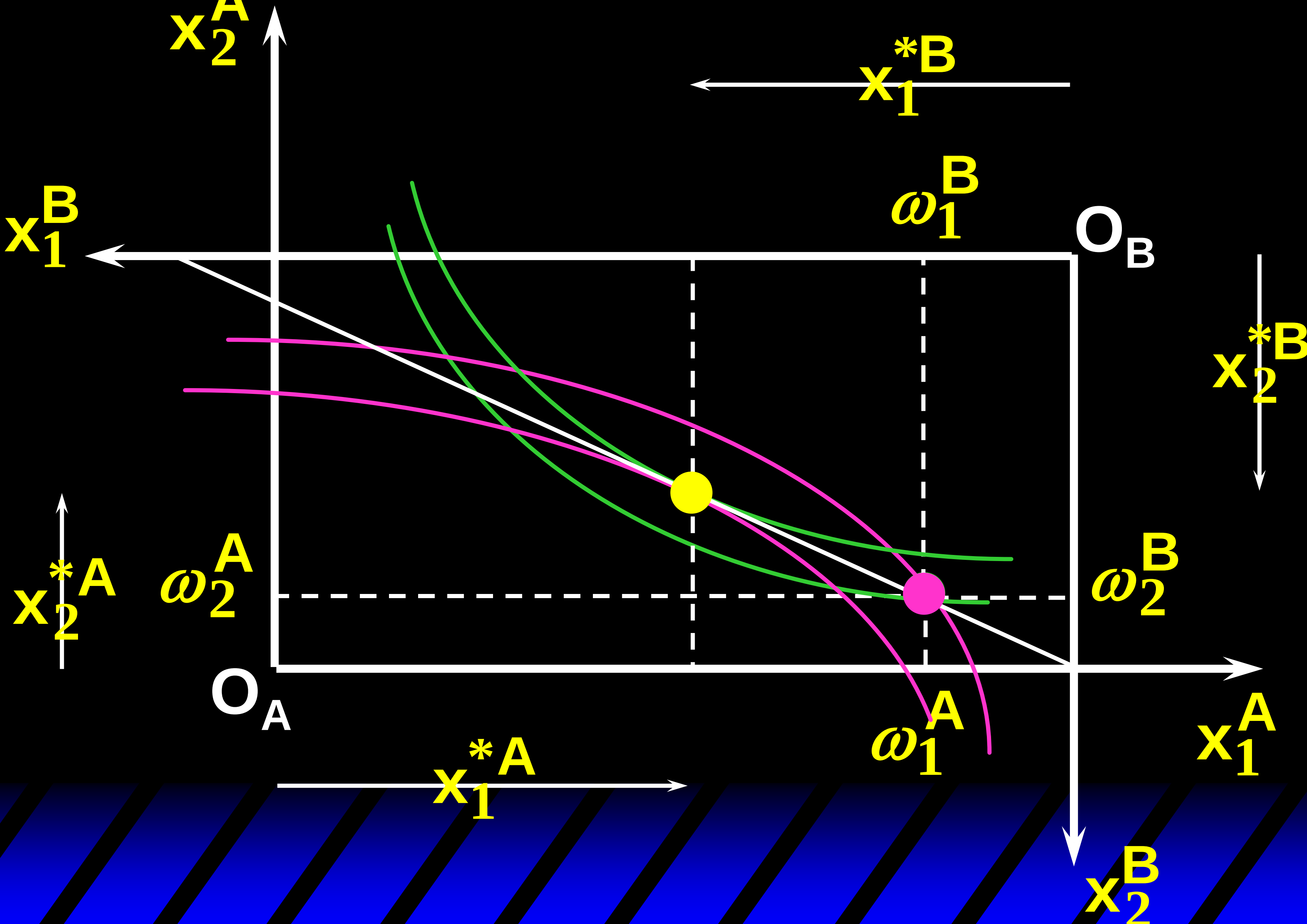
在新的价格下，A选黄色点对应的组合

Trade in Competitive Markets



在新的价格下，B也选黄色点对应的组合

Trade in Competitive Markets



Trade in Competitive Markets

A **general equilibrium** occurs when prices p_1 and p_2 cause both the markets for commodities 1 and 2 to clear; i.e.

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

and $$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B.$$

在新价格下，1) 消费者效用最大化，2) 市场出清 => 该价格为竞争性均衡价格

Trade in Competitive Markets

- At the new prices p_1 and p_2 both markets clear; there is a general equilibrium.
- Trading in competitive markets achieves a particular Pareto-optimal allocation of the endowments.
  - 无差异曲线在均衡点相切，竞争性均衡是一个帕累托最优分配
- This is an example of the **First Fundamental Theorem of Welfare Economics** 福利经济学第一定理

First Fundamental Theorem of Welfare Economics

Given that consumers' preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy's endowment.

在一个完全竞争的纯交换经济中，若消费具有良性偏好，那么竞争性均衡是帕累托最优的。

Trade in Competitive Markets: An Example

Two consumers: A and B;

Two commodities, x_1 and x_2;

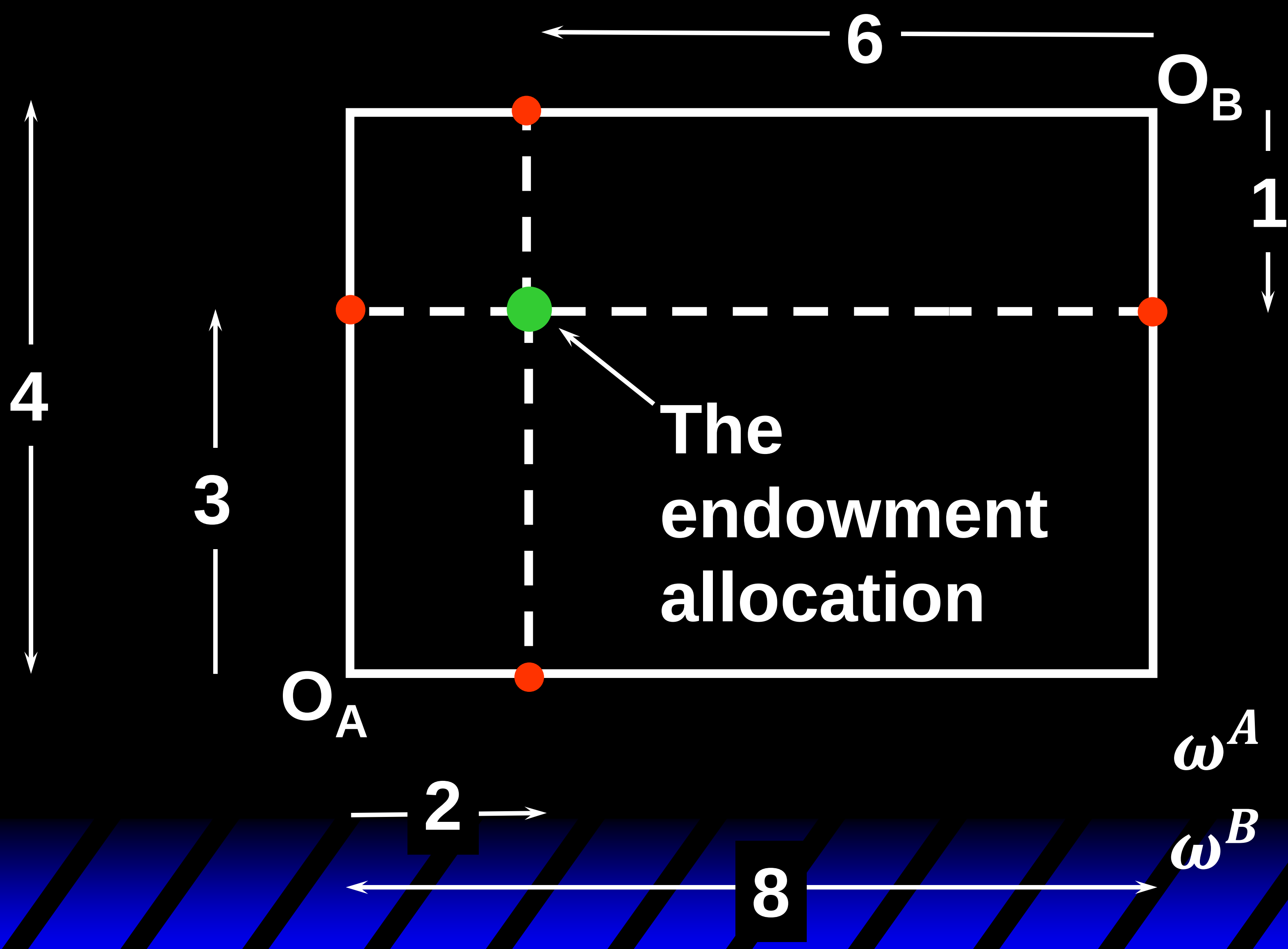
$U^A(x_1^A, x_2^A) = x_1^A x_2^A$

$U^B(x_1^B, x_2^B) = x_1^B x_2^B$

Endowment $(\omega_1^A, \omega_2^A) = (2, 3)$, $(\omega_1^B, \omega_2^B) = (6, 1)$

Q1: Draw the Edgeworth box. Locate the endowment point in the box.

An Example



$$\omega^A = (2,3)$$
$$\omega^B = (6,1)$$

Trade in Competitive Markets: An Example

Q2: Find the contract curve.

The contract curve is the set of all Pareto efficient allocations.

At a given Pareto efficient allocation, indifference curves are tangent to each other => same slope

$$U^A(x_1^A, x_2^A) = x_1^A x_2^A$$

$$slope_A = MRS = -x_2^A / x_1^A$$

$$U^B(x_1^B, x_2^B) = x_1^B x_2^B$$

$$slope_B = MRS = -x_2^B / x_1^B$$

Trade in Competitive Markets: An Example

$$slope_A = MRS = -x_2^A / x_1^A$$
$$slope_B = MRS = -x_2^B / x_1^B$$

At Pareto efficient allocations,

$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B}$$

Trade in Competitive Markets: An Example

$$slope_A = MRS = -x_2^A/x_1^A$$
$$slope_B = MRS = -x_2^B/x_1^B$$

At Pareto efficient allocations,

$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B}$$

Both goods are exhausted:

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B = 8$$
$$x_2^A + x_2^B = \omega_2^A + \omega_2^B = 4$$
$$x_1^B = 8 - x_1^A$$
$$x_2^B = 4 - x_2^A$$

Trade in Competitive Markets: An Example

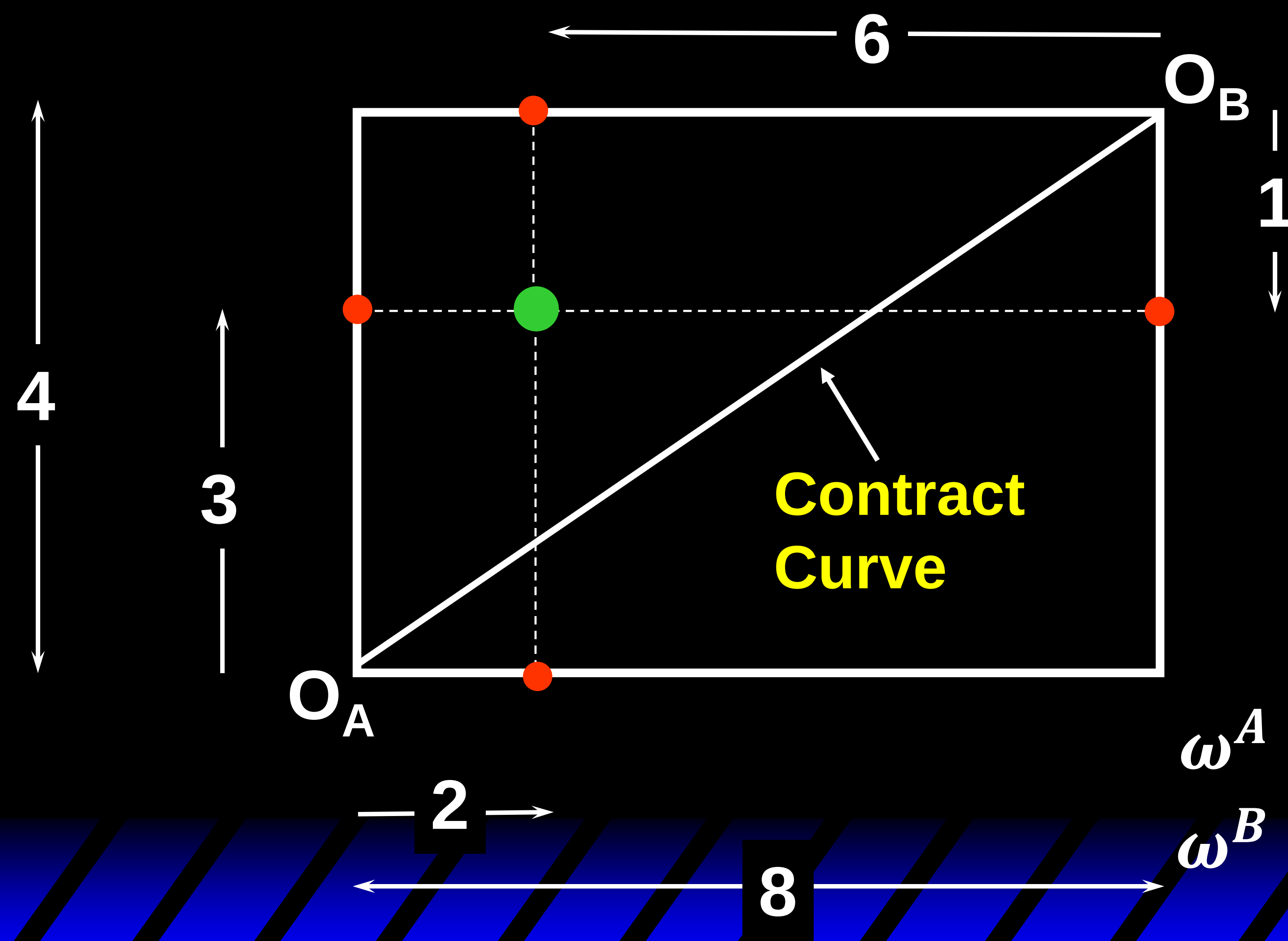
$$x_1^B = 8 - x_1^A$$
$$x_2^B = 4 - x_2^A$$
$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B} = \frac{4 - x_2^A}{8 - x_1^A}$$

Cross multiplying gives:

$$x_2^A\left(8 - x_1^A\right) = x_1^A(4 - x_2^A)$$
$$8x_2^A - x_1^A x_2^A = 4x_1^A - x_1^A x_2^A$$
$$x_2^A = \frac{1}{2}x_1^A$$

(斜率为1/2的、位于方框图内部的线段)

An Example



$$\omega^A = (2,3)$$
$$\omega^B = (6,1)$$

Trade in Competitive Markets: An Example

Q3: Find the competitive equilibrium prices.

- Recall that, in a competitive equilibrium, 1) consumers max utility 2) markets clear

- Denote the equilibrium prices as (p_1, p_2)

- We only care about the relative price so we normalize $p_2 = 1$ (treat x_2 as a numeraire)

Trade in Competitive Markets: An Example

$$\left(\omega_1^A, \omega_2^A\right) = (2, 3), \left(\omega_1^B, \omega_2^B\right) = (6, 1)$$

$$m^A = 2p_1 + 3p_2 = 2p_1 + 3$$

Consumer A maximizes

$$U^A\left(x_1^A, x_2^A\right) = x_1^A x_2^A$$

subject to

$$p_1 x_1^A + x_2^A = m^A = 2p_1 + 3$$

$$x_1^{*A} = \frac{2p_1 + 3}{2p_1}, x_2^{*A} = \frac{2p_1 + 3}{2}$$

Trade in Competitive Markets: An Example

$$\left(\omega_1^A, \omega_2^A\right) = (2,3), \left(\omega_1^B, \omega_2^B\right) = (6,1)$$

$$m^B = 6p_1 + p_2 = 6p_1 + 1$$

Consumer B maximizes

$$U^B\left(x_1^B, x_2^B\right) = x_1^B x_2^B$$

subject to

$$p_1 x_1^B + x_2^B = m^B = 6p_1 + 1$$

$$x_1^{*B} = \frac{6p_1 + 1}{2p_1}, x_2^{*B} = \frac{6p_1 + 1}{2}$$

Trade in Competitive Markets: An Example

$$\left(\omega_1^A, \omega_2^A\right) = (2, 3), \left(\omega_1^B, \omega_2^B\right) = (6, 1)$$

$$x_1^{*A} = \frac{2p_1 + 3}{2p_1}, x_2^{*A} = \frac{2p_1 + 3}{2}$$

$$x_1^{*B} = \frac{6p_1 + 1}{2p_1}, x_2^{*B} = \frac{6p_1 + 1}{2}$$

In order for markets to clear,

$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$$

$$\frac{2p_1 + 3}{2} + \frac{6p_1 + 1}{2} = 3 + 1$$

$$p_1^* = \frac{1}{2}, \mathrm{p}_2 = 1$$

Second Fundamental Theorem of Welfare Economics

Any Pareto-optimal allocation (i.e. any point on the contract curve) can be achieved by trading in competitive markets provided that endowments are first appropriately rearranged amongst the consumers.

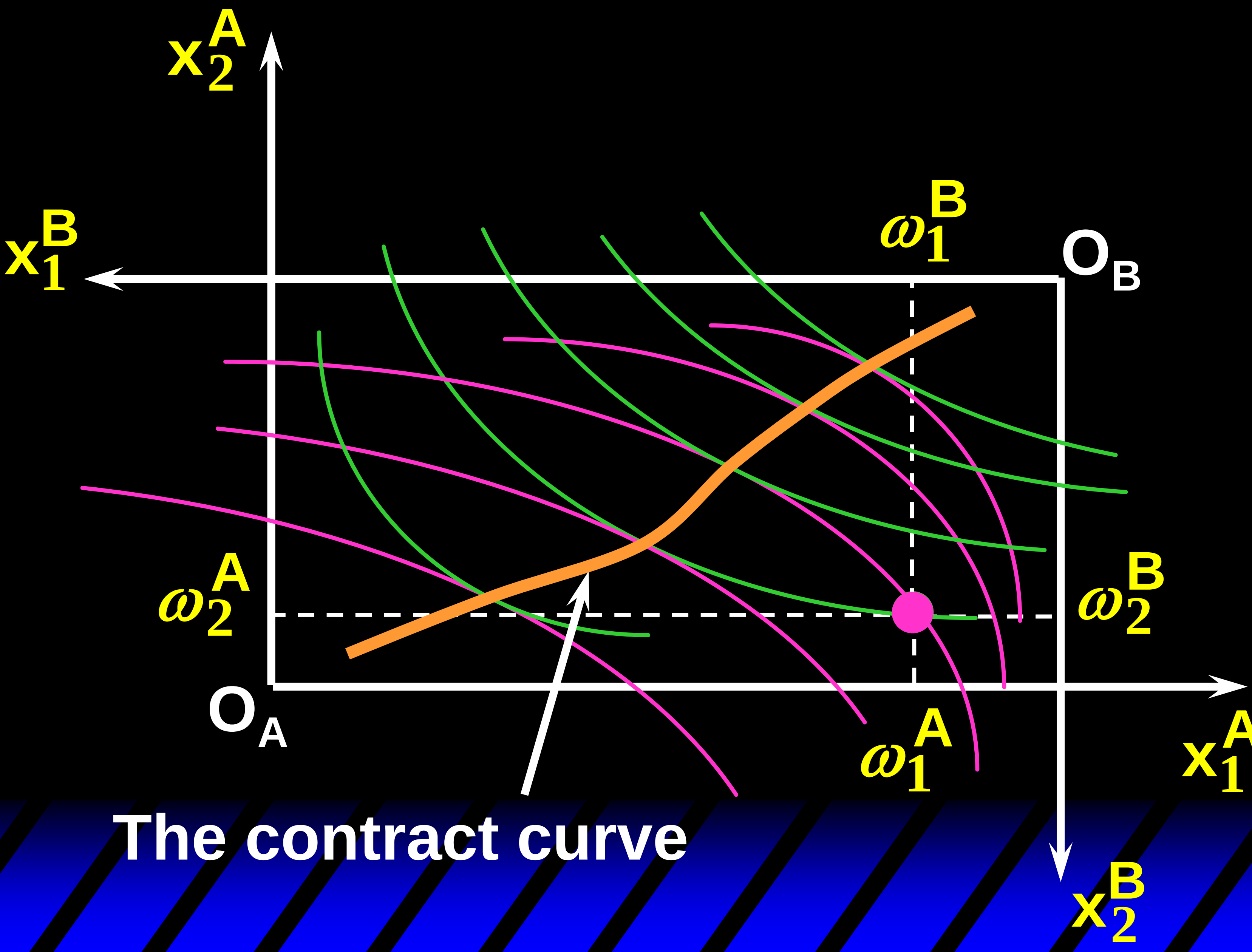
福利经济学第二定理：如果允许对初始禀赋进行一次再分配，那么任何一个帕累托最优分配都可以由竞争性市场均衡来实现。

Second Fundamental Theorem of Welfare Economics

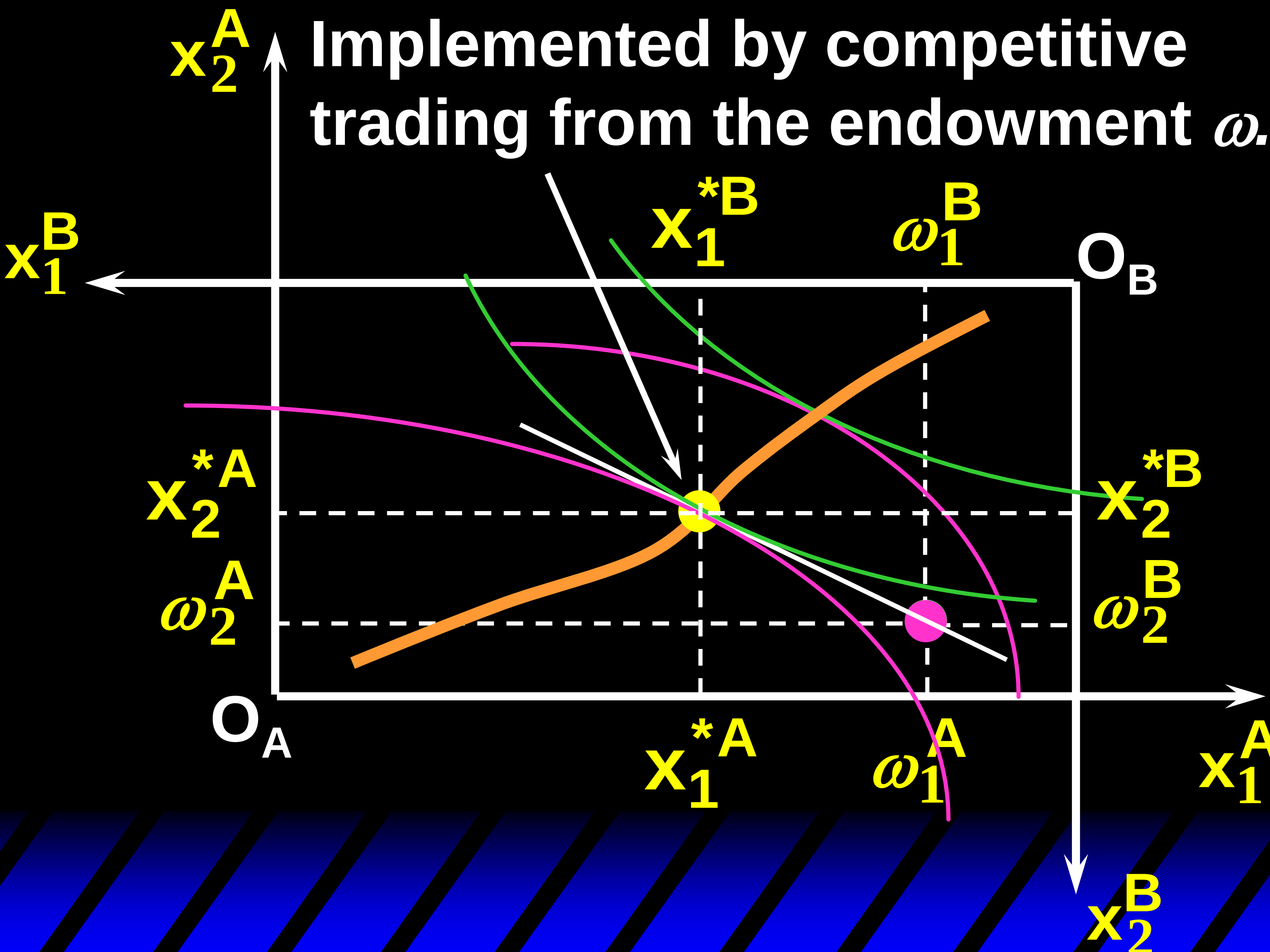
Given that consumers' preferences are well-behaved, for any Pareto-optimal allocation there are prices and an allocation of the total endowment that makes the Pareto-optimal allocation implementable by trading in competitive markets.

换言之，如果允许对初始禀赋进行一次再分配，那么存在一组价格，在该组价格下的竞争性均衡可以实现任何一个给定的帕累托最优分配。

Second Fundamental Theorem

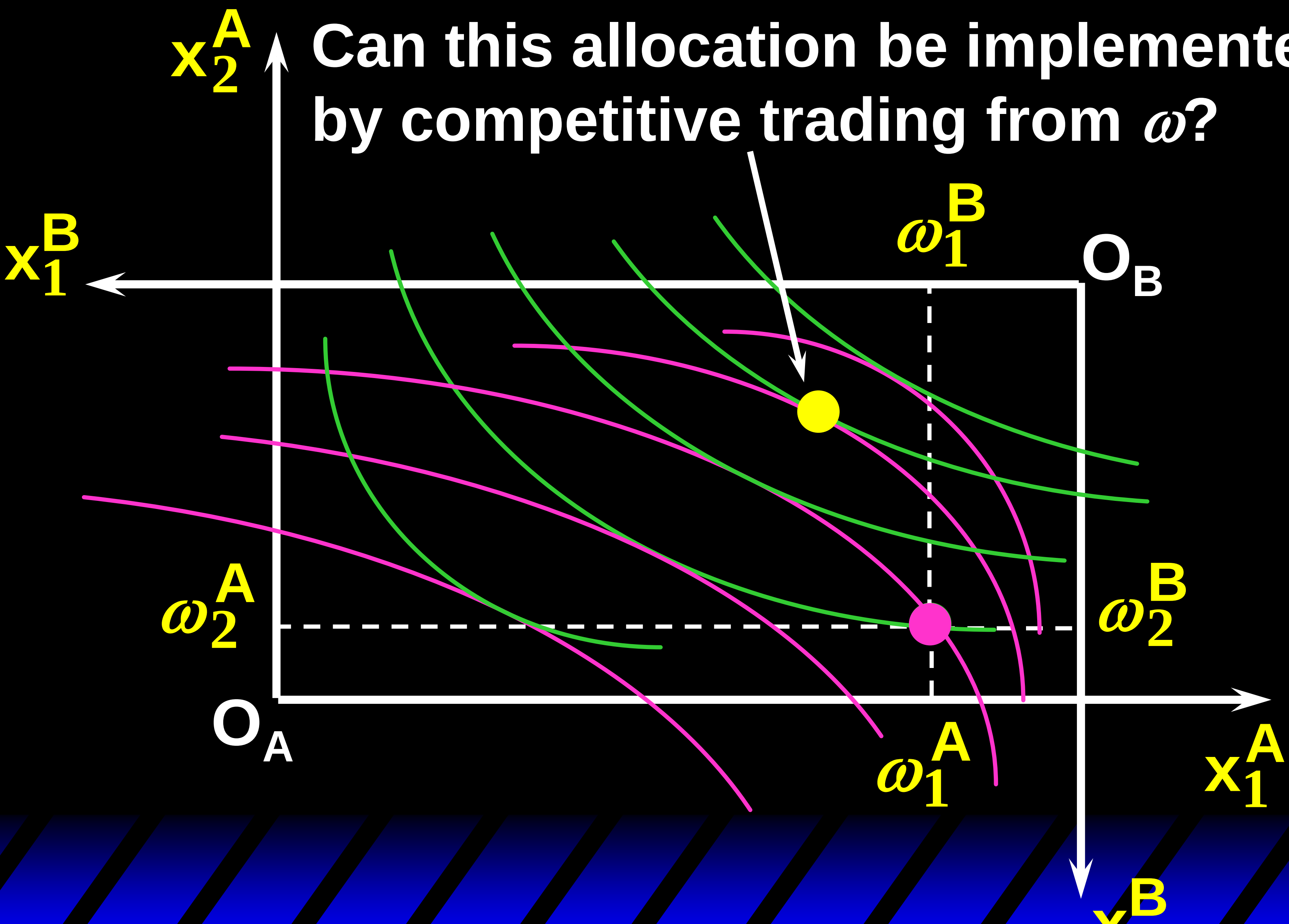


Second Fundamental Theorem



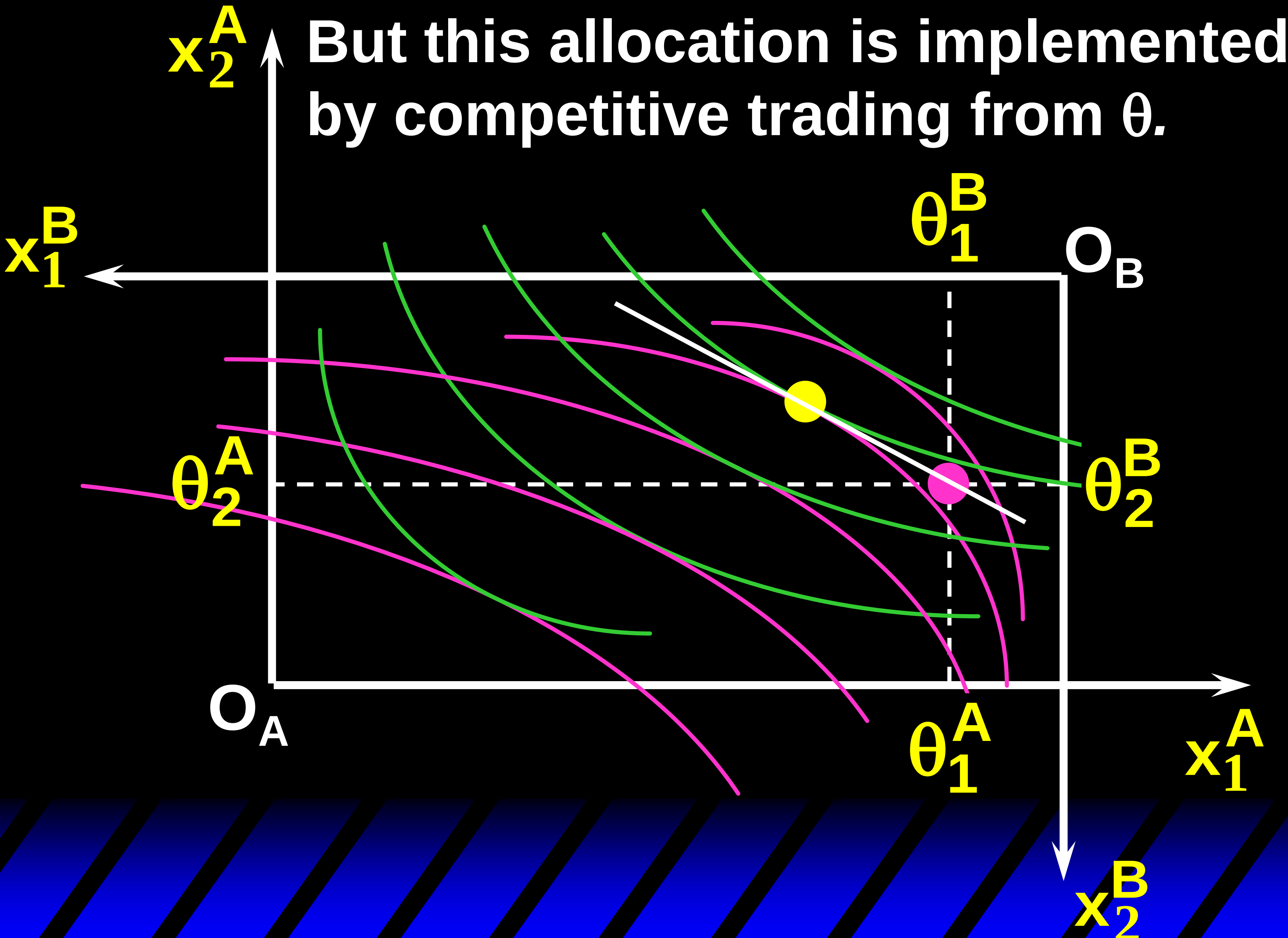
Second Fundamental Theorem

Can this allocation be implemented by competitive trading from ω?



Second Fundamental Theorem

But this allocation is implemented by competitive trading from θ.



Walras' Law

Walras' Law: If consumers' preferences are well-behaved, then the aggregate value of all excess demand in an economy is identically zero.

在纯交换经济体中，各市场超额需求/净需求的商品价值之和等于0

Walras' Law

Every consumer's preferences are well-behaved so, for any positive prices (p_1,p_2), each consumer spends all of his budget.

For consumer A:

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

For consumer B:

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Walras' Law

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Summing gives

$$p_1 (x_1^{*A} + x_1^{*B}) + p_2 (x_2^{*A} + x_2^{*B})$$

$$= p_1 (\omega_1^A + \omega_1^B) + p_2 (\omega_2^B + \omega_2^B).$$

Walras' Law

Rearranged,

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0.$$

That is, ...

某商品的消费总量与禀赋总量之差为超额需求量(excess demand)

Walras' Law

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$
$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B)$$
$$= 0.$$

This says that the summed market value of excess demands is zero for any positive prices p_1 and p_2 -- this is Walras' Law.

Implications of Walras' Law

Suppose the market for commodity A is in equilibrium; that is,

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B = 0.$$

Then

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B = 0.$$

Implications of Walras’ Law

So one implication of Walras’ Law for a two-commodity exchange economy is that if one market is in equilibrium then the other market must also be in equilibrium.

推论1：在两种商品的纯交换经济中，如果一个市场处于均衡状态，另一个市场一定也处于均衡状态

Implications of Walras' Law

What if, for some positive prices p_1 and p_2, there is an excess quantity supplied of commodity 1? That is,

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B < 0.$$

Then

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B > 0.$$

Implications of Walras' Law

So a second implication of Walras' Law for a two-commodity exchange economy is that an excess supply in one market implies an excess demand in the other market.

推论2：在两种商品的纯交换经济中，如果一个市场处于超额供给状态，另一个市场一定处于超额需求状态