Lecture 3

Choice

Recap

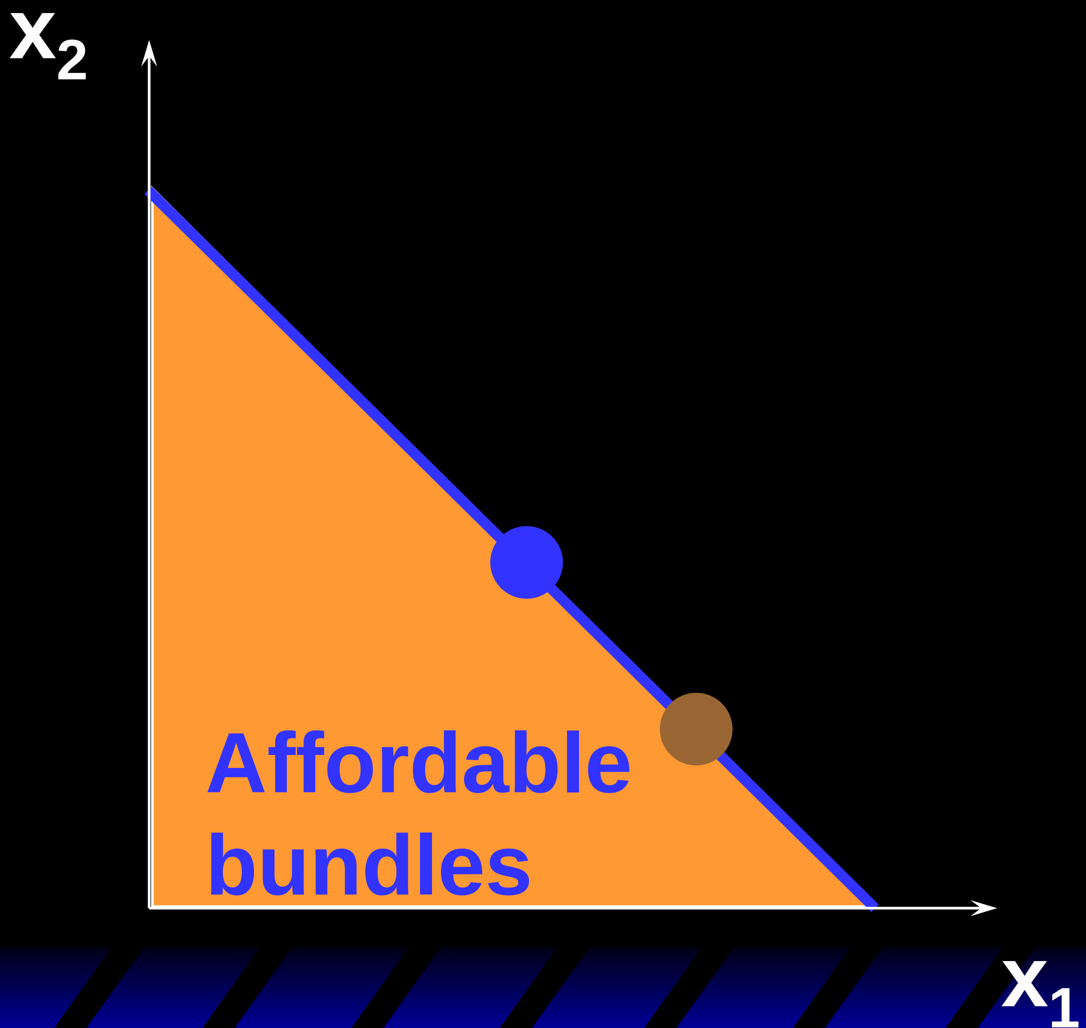
A consumer is a rational agent who always chooses the **most preferred** consumption bundle **available** to her.

To model this optimization problem, we need to model:

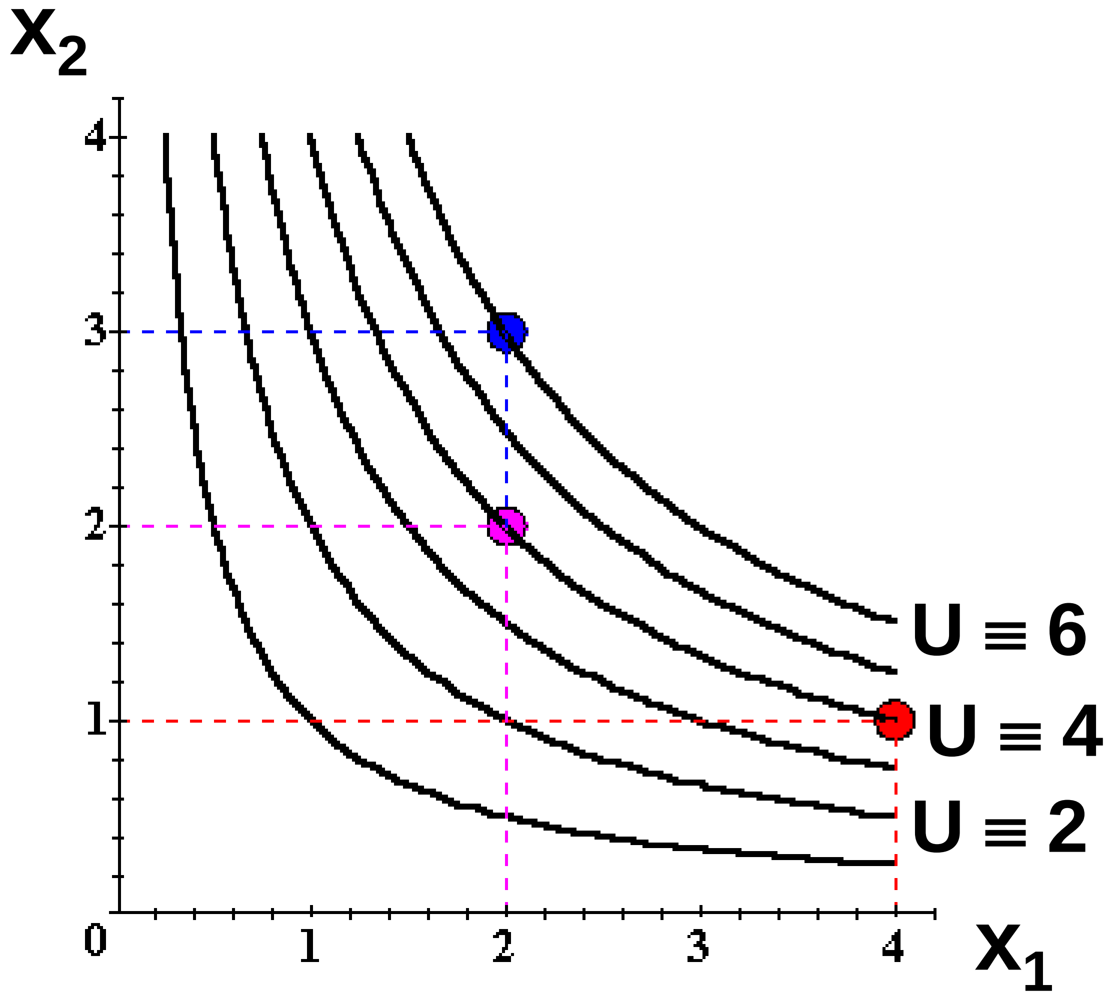
- the choice set (Lec1)
- preferences (Lec2)

How is the most preferred bundle in the choice set located? (Today)

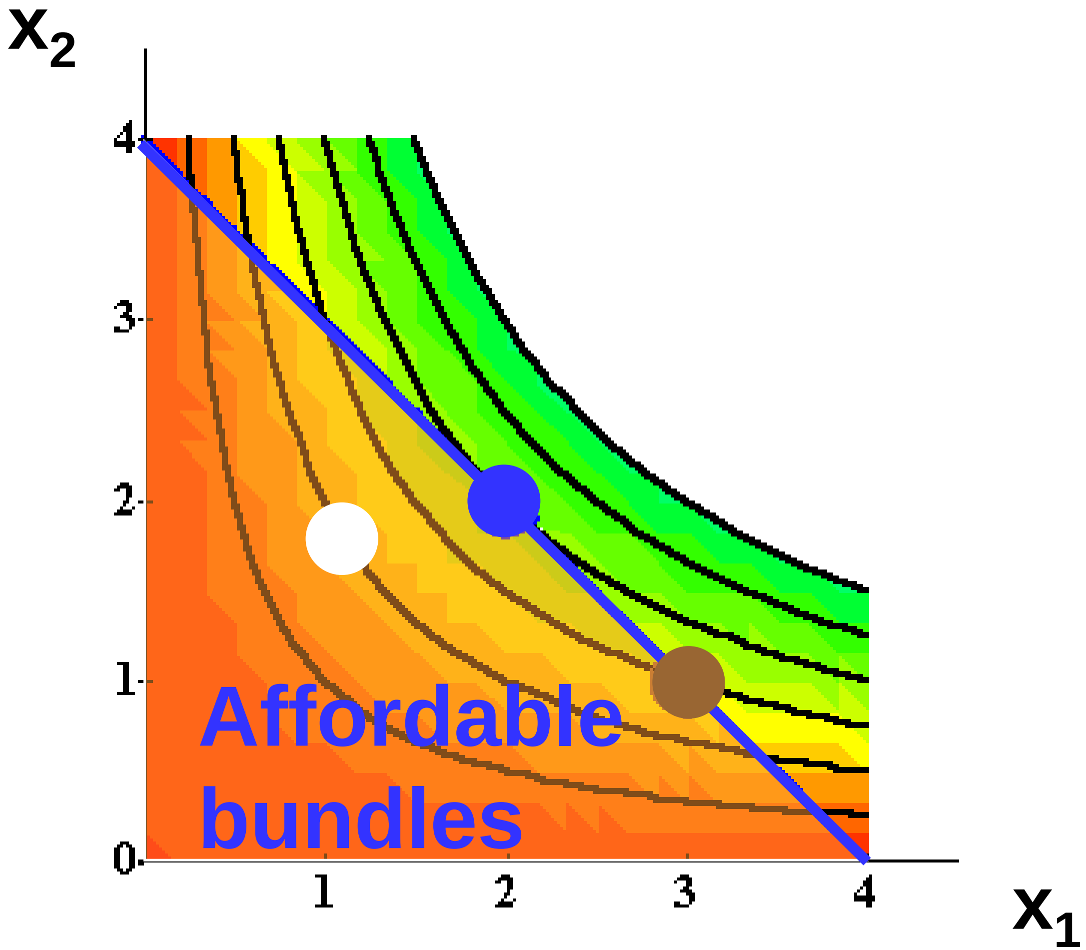
Rational Constrained Choice



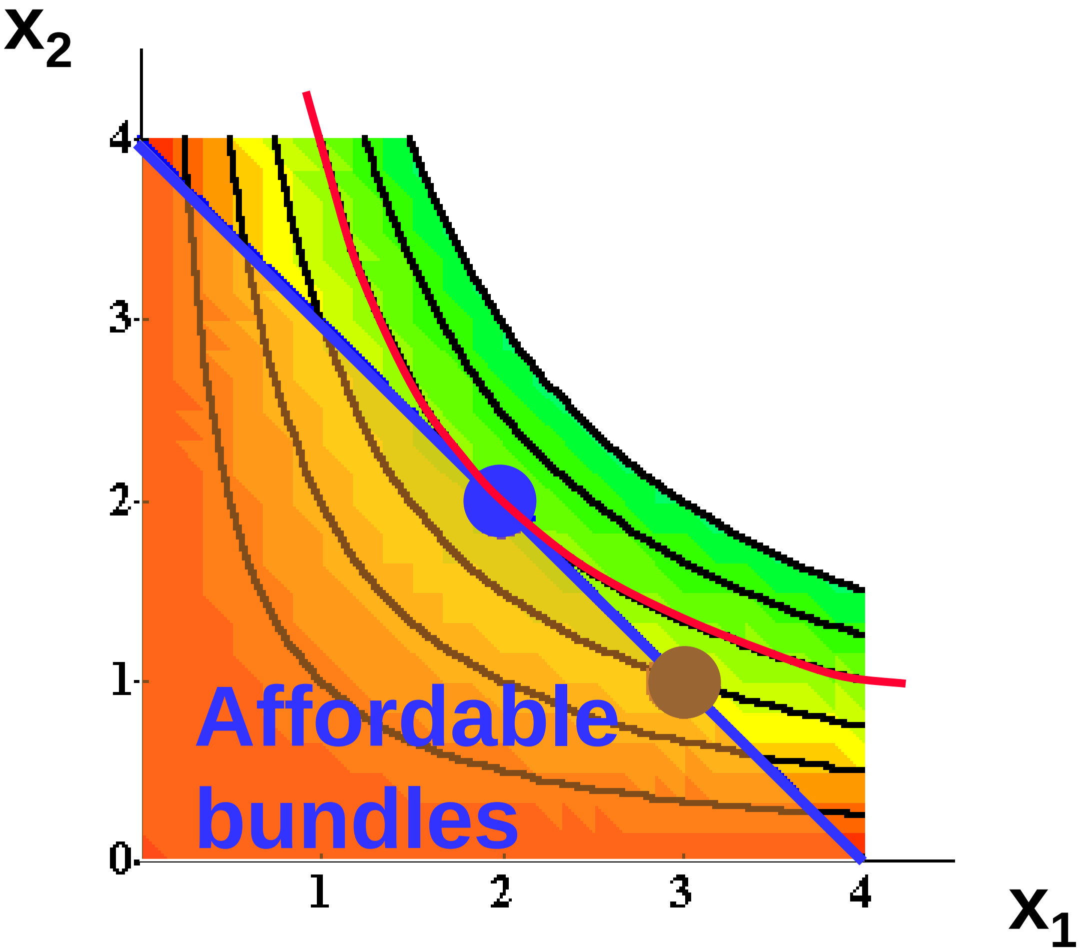
Utility Functions & Indiff. Curves



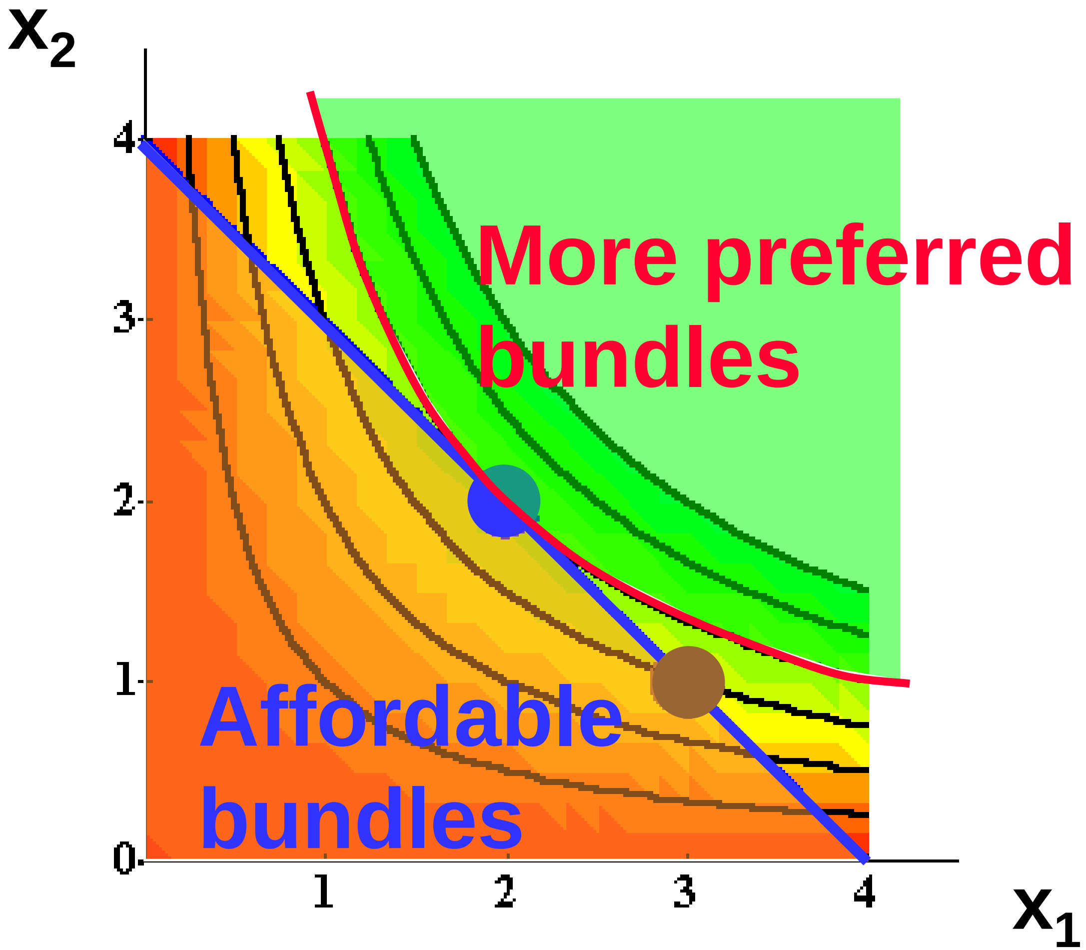
Rational Constrained Choice



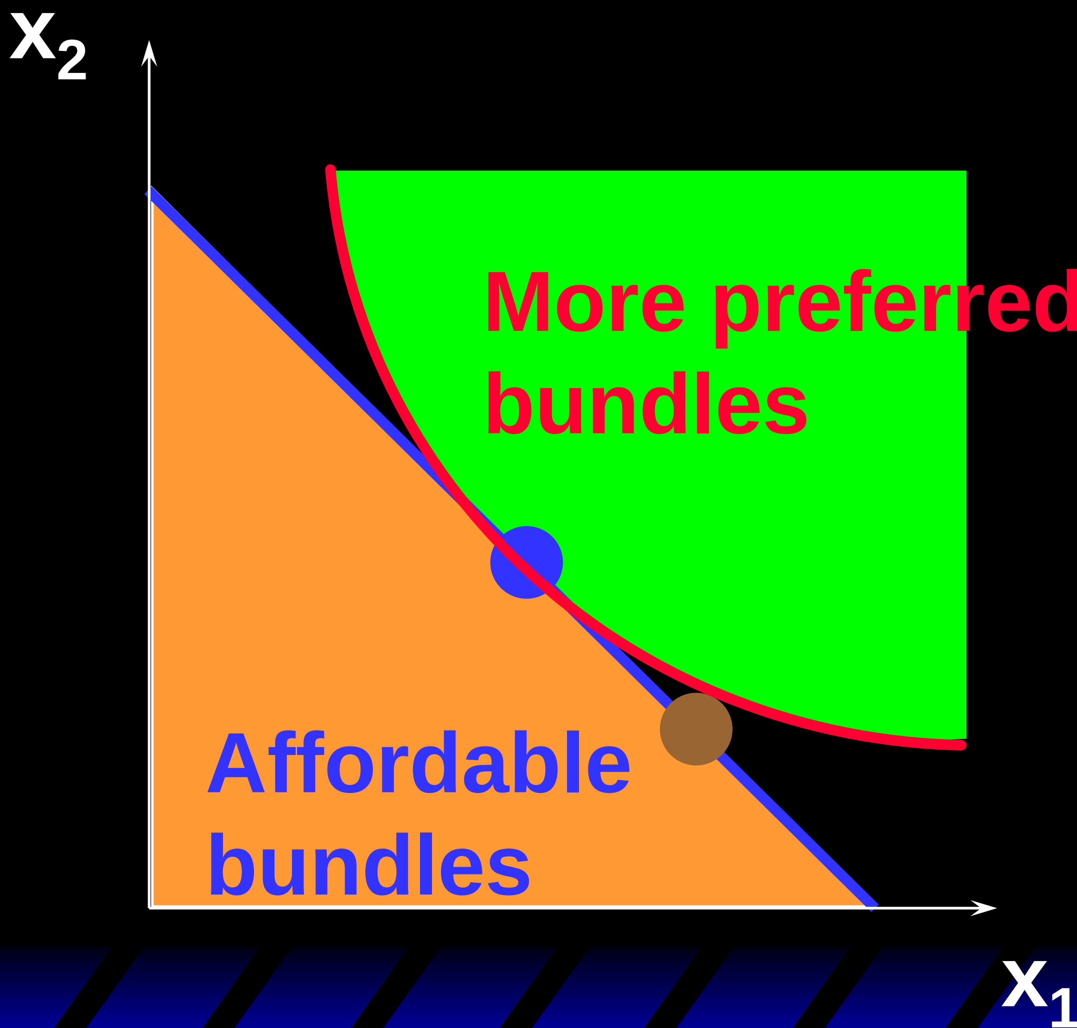
Rational Constrained Choice



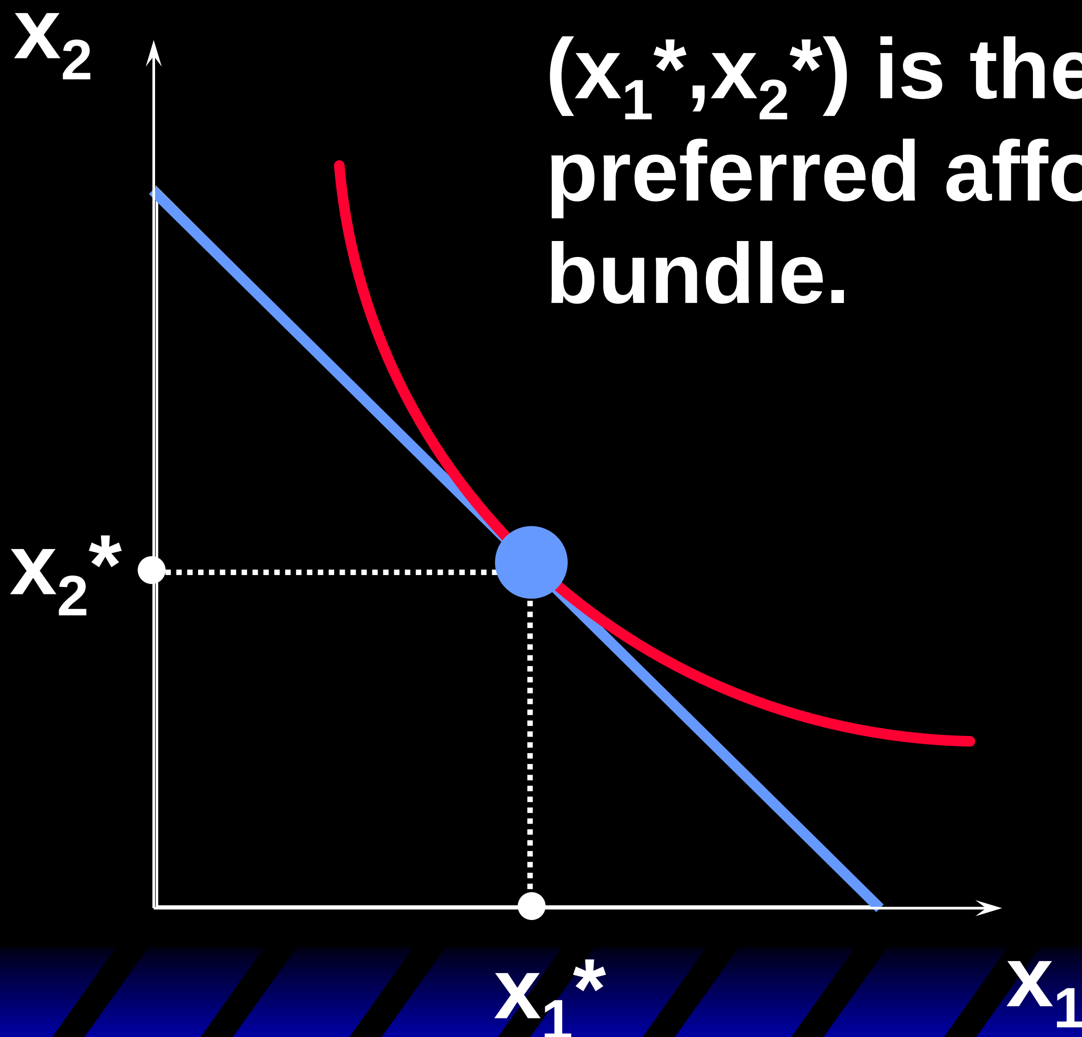
Rational Constrained Choice



Rational Constrained Choice



Rational Constrained Choice

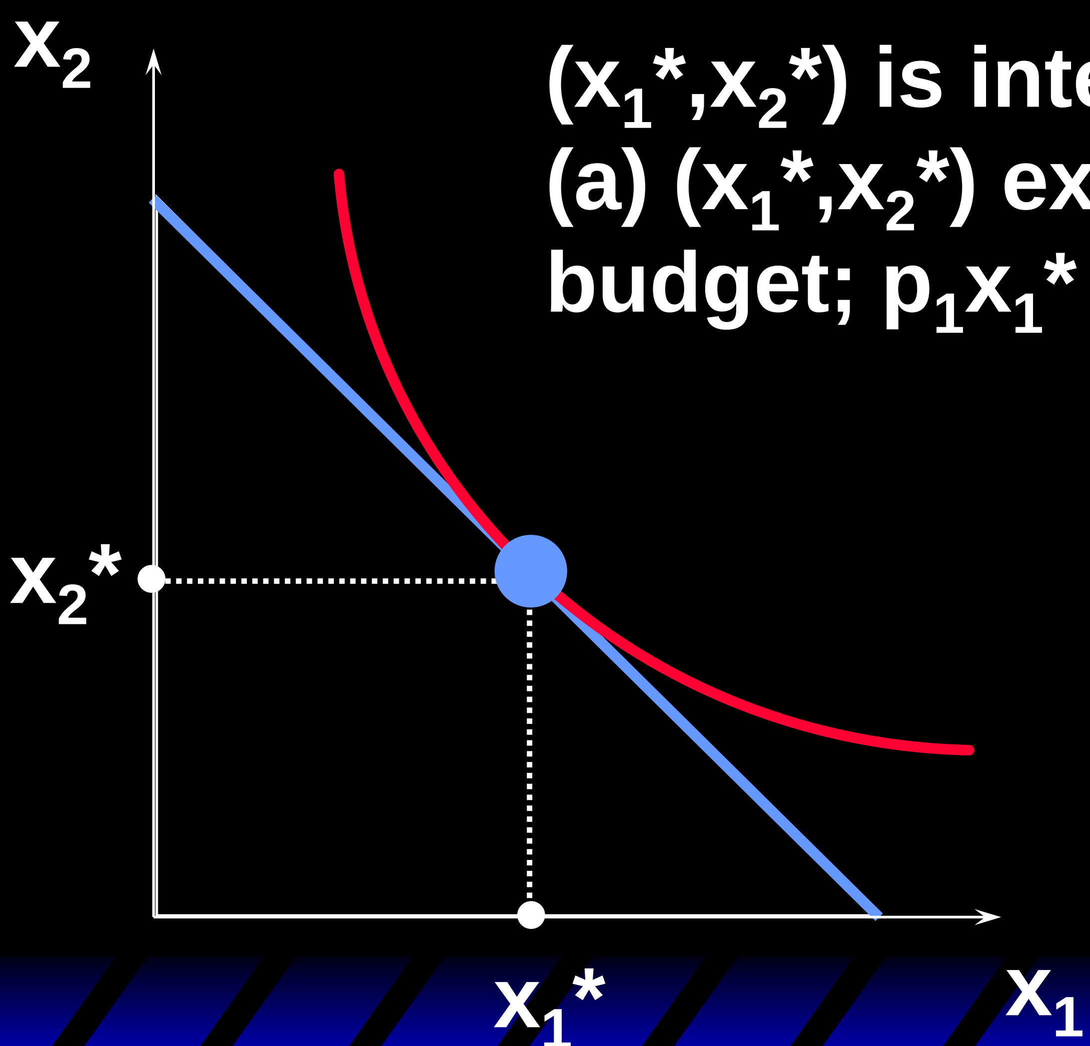


(x_1^*, x_2^*) is the most preferred affordable bundle.

Rational Constrained Choice

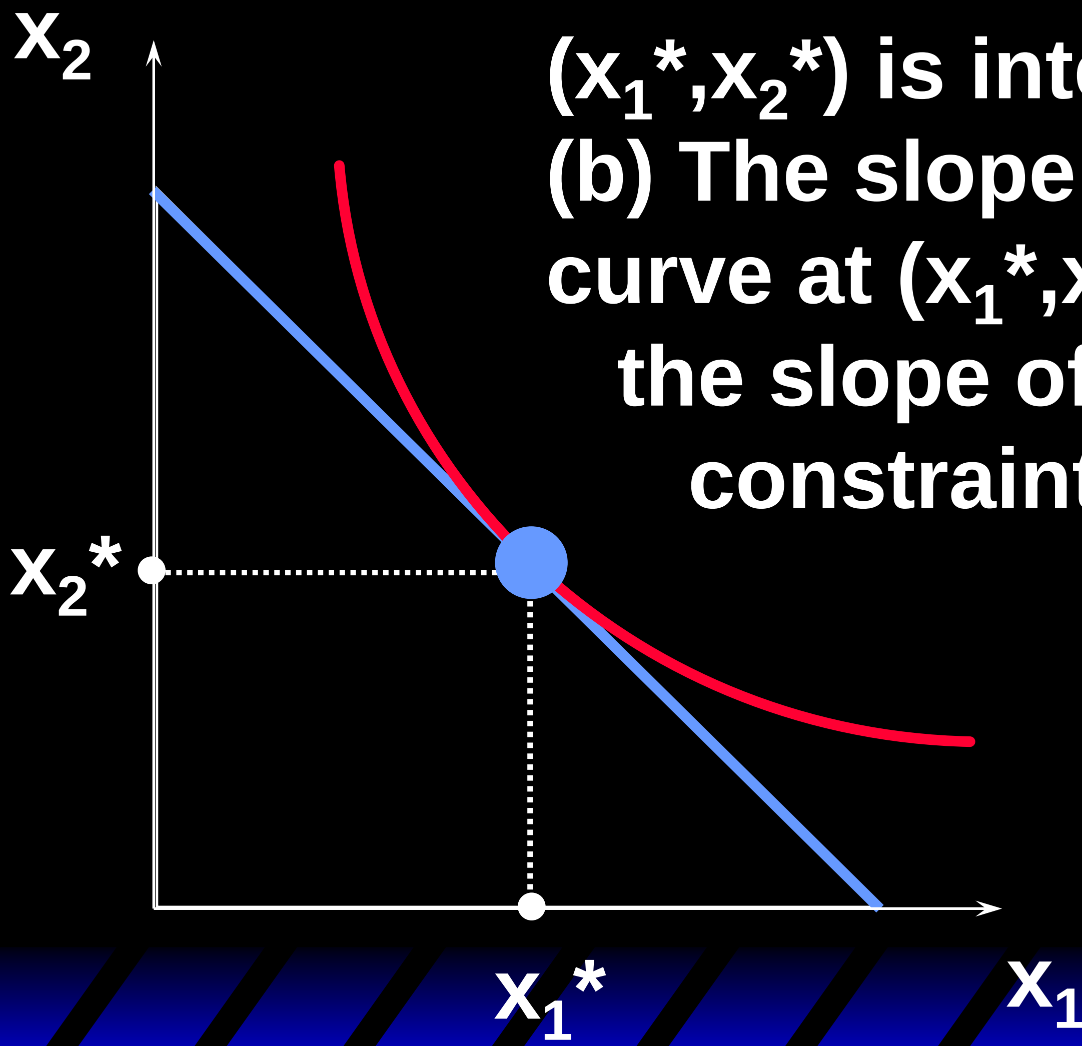
- The most preferred affordable bundle is called the consumer's **ORDINARY DEMAND** at the given prices and budget.
- Ordinary demands will be denoted by $x_1^*(p_1,p_2,m)$ and $x_2^*(p_1,p_2,m)$.

Rational Constrained Choice



(x_1^*,x_2^*) is interior.
(a) (x_1^*,x_2^*) exhausts the budget; $p_1x_1^* + p_2x_2^* = m$.

Rational Constrained Choice



(x_1^*, x_2^*) is interior .
(b) The slope of the indiff. curve at (x_1^*, x_2^*) equals the slope of the budget constraint.

Rational Constrained Choice

(x_1^*,x_2^*) satisfies two conditions:

(a) the budget is exhausted;

$$p_1x_1^* + p_2x_2^* = m$$

(b) the slope of the budget constraint, $-p_1/p_2$, and the slope of the indifference curve containing (x_1^*,x_2^*) are equal at (x_1^*,x_2^*).

$$-\frac{p_1}{p_2} = MRS = -\frac{MU_1}{MU_2}$$

Computing Ordinary Demands

- How can this information be used to locate (x_1^*,x_2^*) for given p_1, p_2 and m?

Computing Ordinary Demands - a Cobb-Douglas Example.

Suppose that the consumer has Cobb-Douglas preferences.

$$U(x_1, x_2) = x_1^a x_2^b$$

Computing Ordinary Demands - a Cobb-Douglas Example.

Suppose that the consumer has Cobb-Douglas preferences.

$$U(x_1, x_2) = x_1^a x_2^b$$

Then $$MU_1 = \frac{\partial U}{\partial x_1} = a x_1^{a-1} x_2^b$$

$$MU_2 = \frac{\partial U}{\partial x_2} = b x_1^a x_2^{b-1}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

So the MRS is

$$\mathrm{MRS} = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{ax_1^{a-1}x_2^b}{bx_1^a x_2^{b-1}} = -\frac{ax_2}{bx_1}.$$

Computing Ordinary Demands - a Cobb-Douglas Example.

So the MRS is

$$\mathrm{MRS} = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{a x_1^{a-1} x_2^{b}}{b x_1^{a} x_2^{b-1}} = -\frac{a x_2}{b x_1}.$$

At (x_1^*, x_2^*), MRS = $-p_1/p_2$ so

Computing Ordinary Demands - a Cobb-Douglas Example.

So the MRS is

$$\text{MRS} = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{ax_1^{a-1}x_2^b}{bx_1^a x_2^{b-1}} = -\frac{ax_2}{bx_1}.$$

At (x_1*,x_2*), MRS = -p_1/p_2 so

$$-\frac{ax_2^*}{bx_1^*} = -\frac{p_1}{p_2} \quad \Rightarrow \quad x_2^* = \frac{bp_1}{ap_2}x_1^*. \qquad \text{(A)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

(x_1^*, x_2^*) also exhausts the budget so

$$p_1x_1^* + p_2x_2^* = m. \qquad \text{(B)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

So now we know that

$$x_2^* = \frac{bp_1}{ap_2} x_1^* \qquad \text{(A)}$$

$$p_1 x_1^* + p_2 x_2^* = m. \qquad \text{(B)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

So now we know that

$$x_2^* = \frac{bp_1}{ap_2} x_1^* \qquad \text{(A)}$$

Substitute

$$p_1 x_1^* + p_2 x_2^* = m. \qquad \text{(B)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

So now we know that

$$x_2^* = \frac{bp_1}{ap_2} x_1^* \qquad \text{(A)}$$

Substitute

$$p_1 x_1^* + p_2 x_2^* = m. \qquad \text{(B)}$$

and get

$$p_1 x_1^* + p_2 \frac{bp_1}{ap_2} x_1^* = m.$$

This simplifies to ….

Computing Ordinary Demands - a Cobb-Douglas Example.

$$x_1^* = \frac{am}{(a+b)p_1}.$$

Computing Ordinary Demands - a Cobb-Douglas Example.

$$x_1^* = \frac{am}{(a+b)p_1}.$$

Substituting for x_1* in

$$p_1x_1^* + p_2x_2^* = m$$

then gives

$$x_2^* = \frac{bm}{(a+b)p_2}.$$

Computing Ordinary Demands - a Cobb-Douglas Example.

So we have discovered that the most preferred affordable bundle for a consumer with Cobb-Douglas preferences

$$U(x_1, x_2) = x_1^a x_2^b$$

is

$$(x_1^*, x_2^*) = \left(\frac{am}{(a+b)p_1}, \frac{bm}{(a+b)p_2} \right).$$

Computing Ordinary Demands - a Cobb-Douglas Example.

when

$$U(x_1, x_2) = x_1^a x_2^b$$

$$(x_1^*, x_2^*) = \left(\frac{am}{(a+b)p_1}, \frac{bm}{(a+b)p_2}\right).$$

$$p_1 x_1^* = p_1 \times \frac{am}{(a+b)p_1} = \frac{a}{a+b} m$$

$$p_2 x_2^* = p_2 \times \frac{bm}{(a+b)p_1} = \frac{b}{a+b} m$$

Computing Ordinary Demands - a Cobb-Douglas Example.

when

$$U(x_1, x_2) = x_1^a x_2^b$$

The consumer always spends $\frac{a}{a+b}$ of her income on x_1, and $\frac{b}{a+b}$ of her income on x_2

Rational Constrained Choice

When $x_1^* > 0$ and $x_2^* > 0$ and indifference curves have no ‘kinks’, the ordinary demands are obtained by solving:

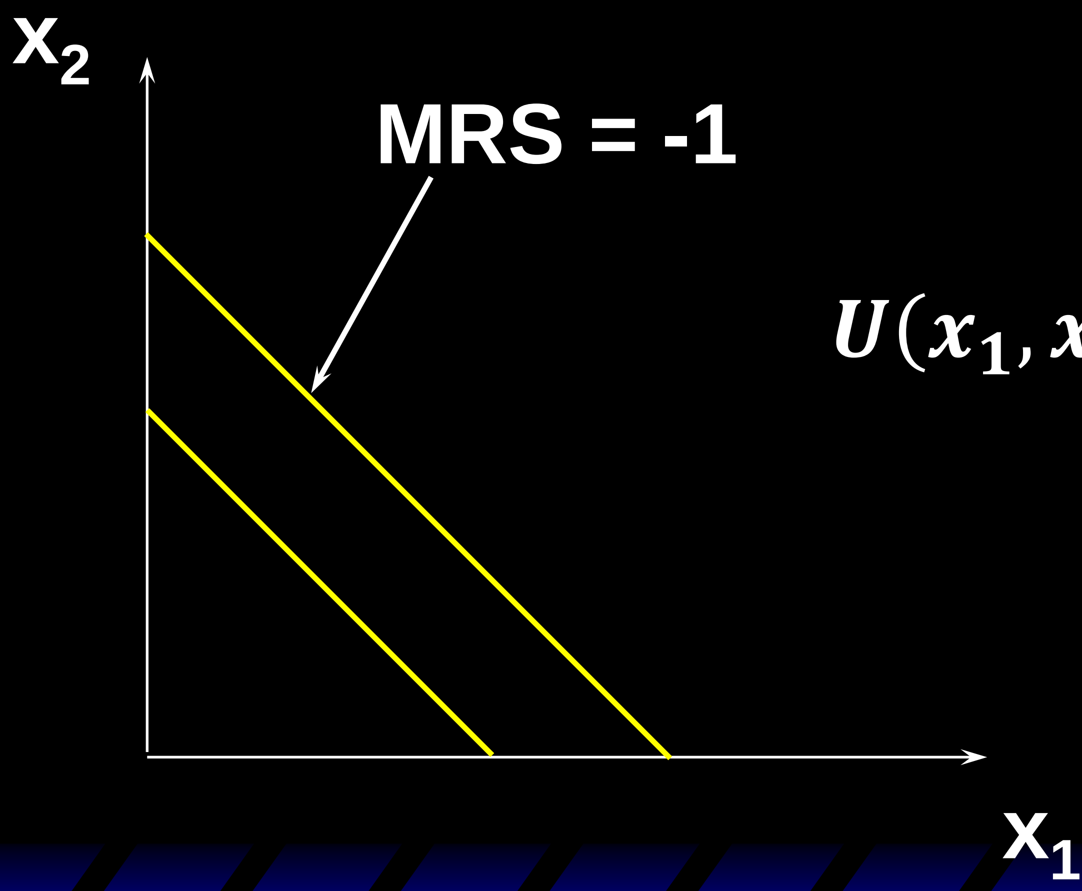
(a) $p_1x_1^* + p_2x_2^* = y$

(b) $$-\frac{p_1}{p_2} = MRS = -\frac{MU_1}{MU_2}$$

Rational Constrained Choice

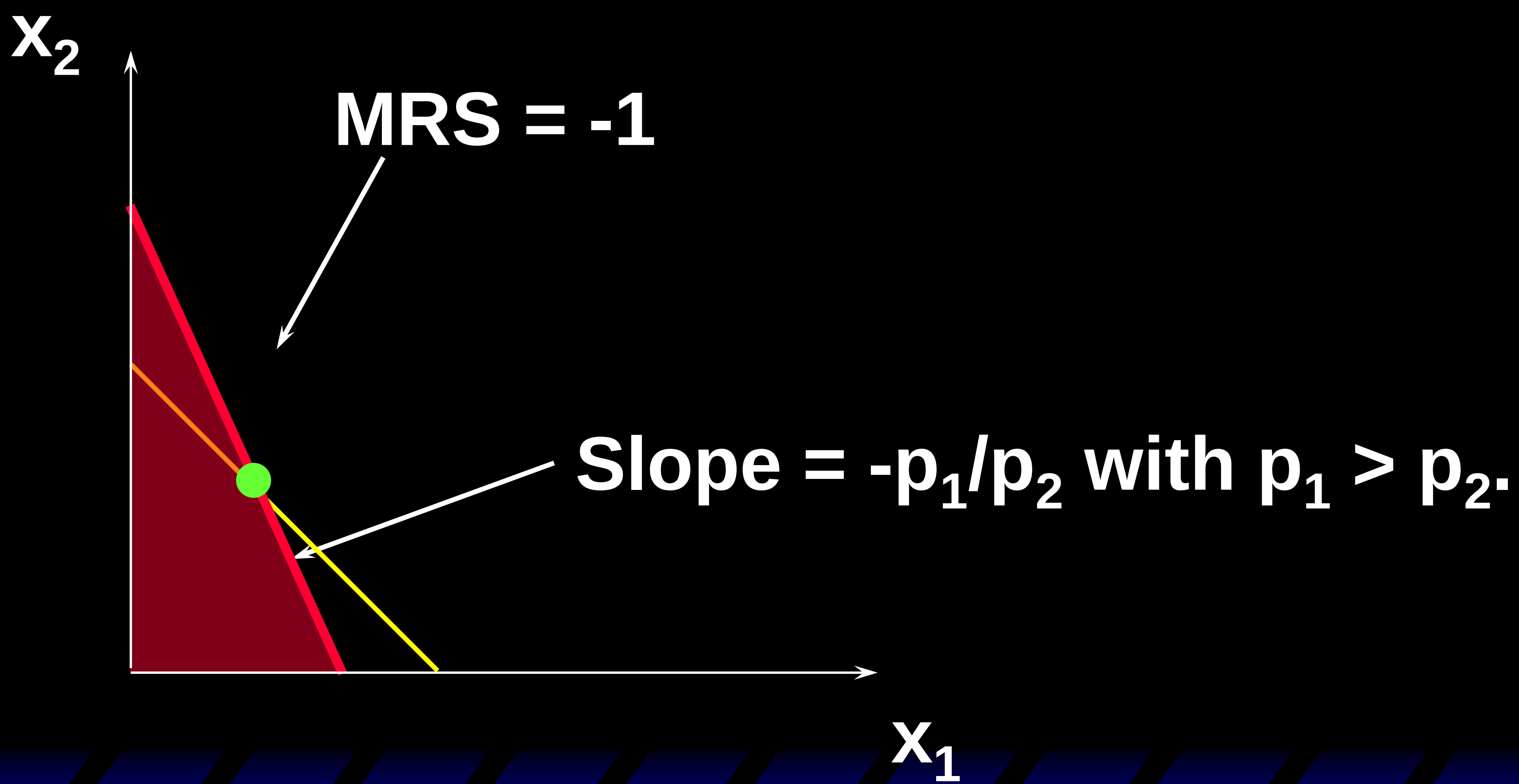
- But what if $x_1^* = 0$?
- Or if $x_2^* = 0$?
- If either $x_1^* = 0$ or $x_2^* = 0$ then the ordinary demand (x_1^*,x_2^*) is at a **corner solution** to the problem of maximizing utility subject to a budget constraint.

Examples of Corner Solutions -- the Perfect Substitutes Case

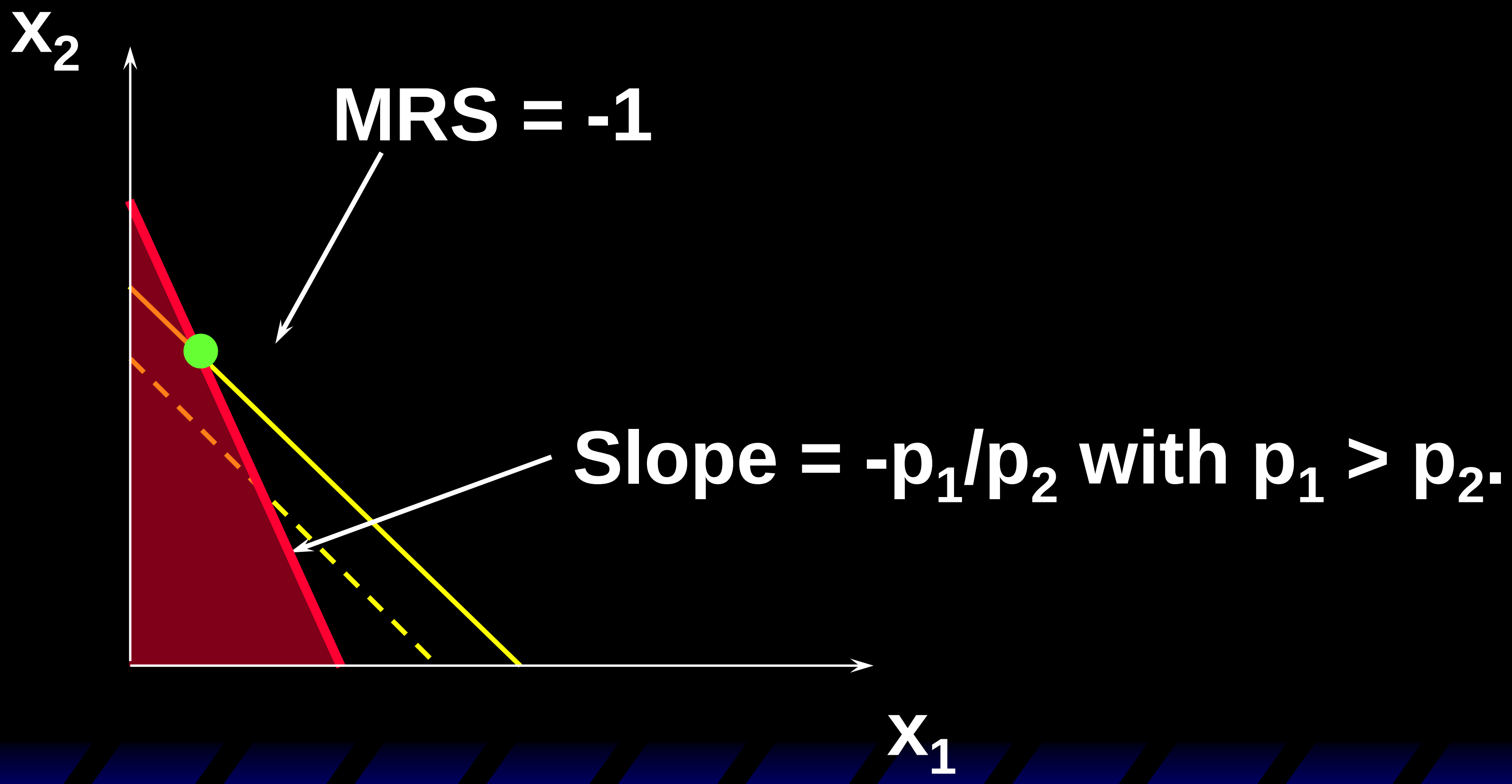


$$U(x_1, x_2) = x_1 + x_2$$

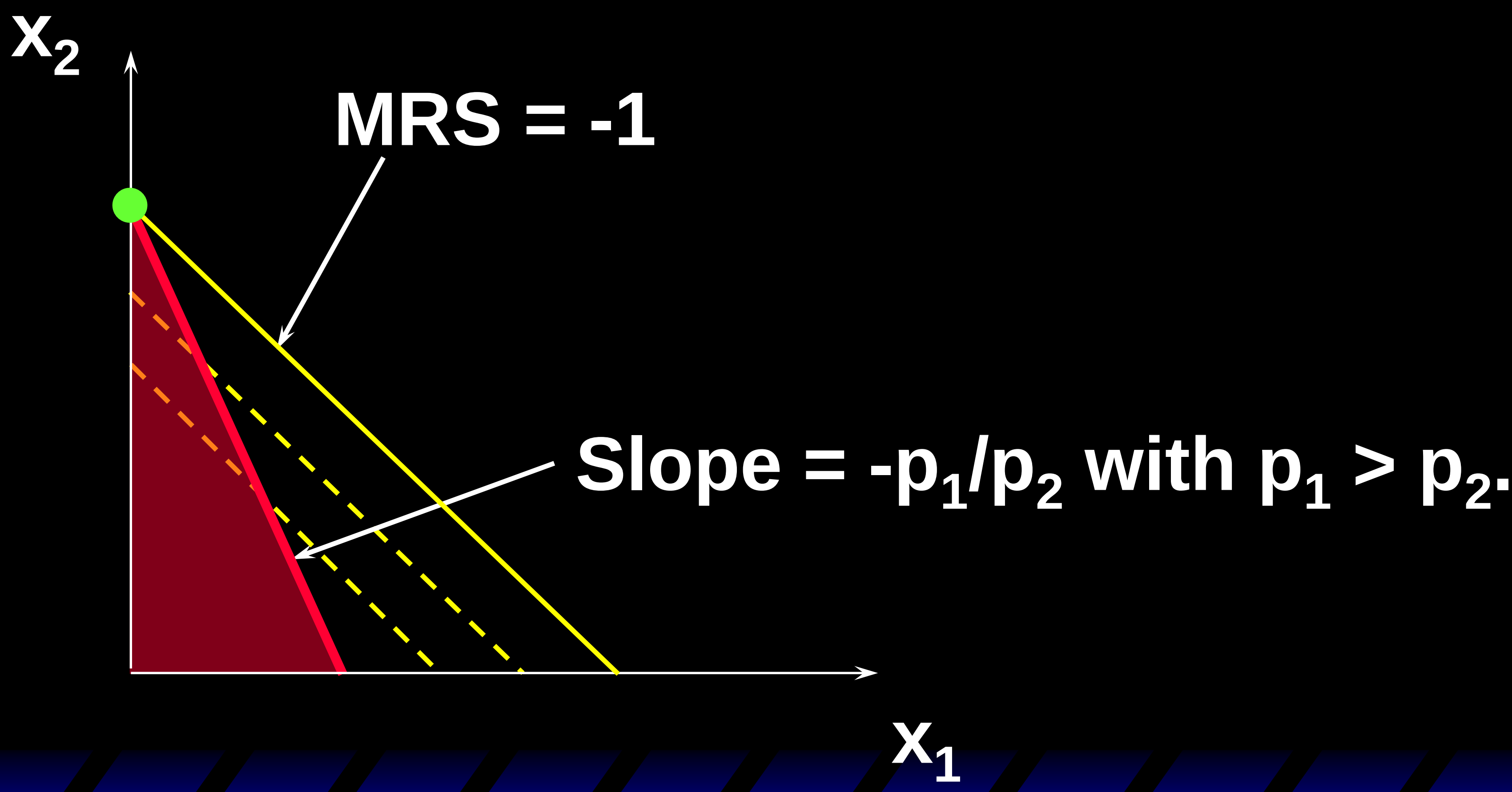
Examples of Corner Solutions -- the Perfect Substitutes Case



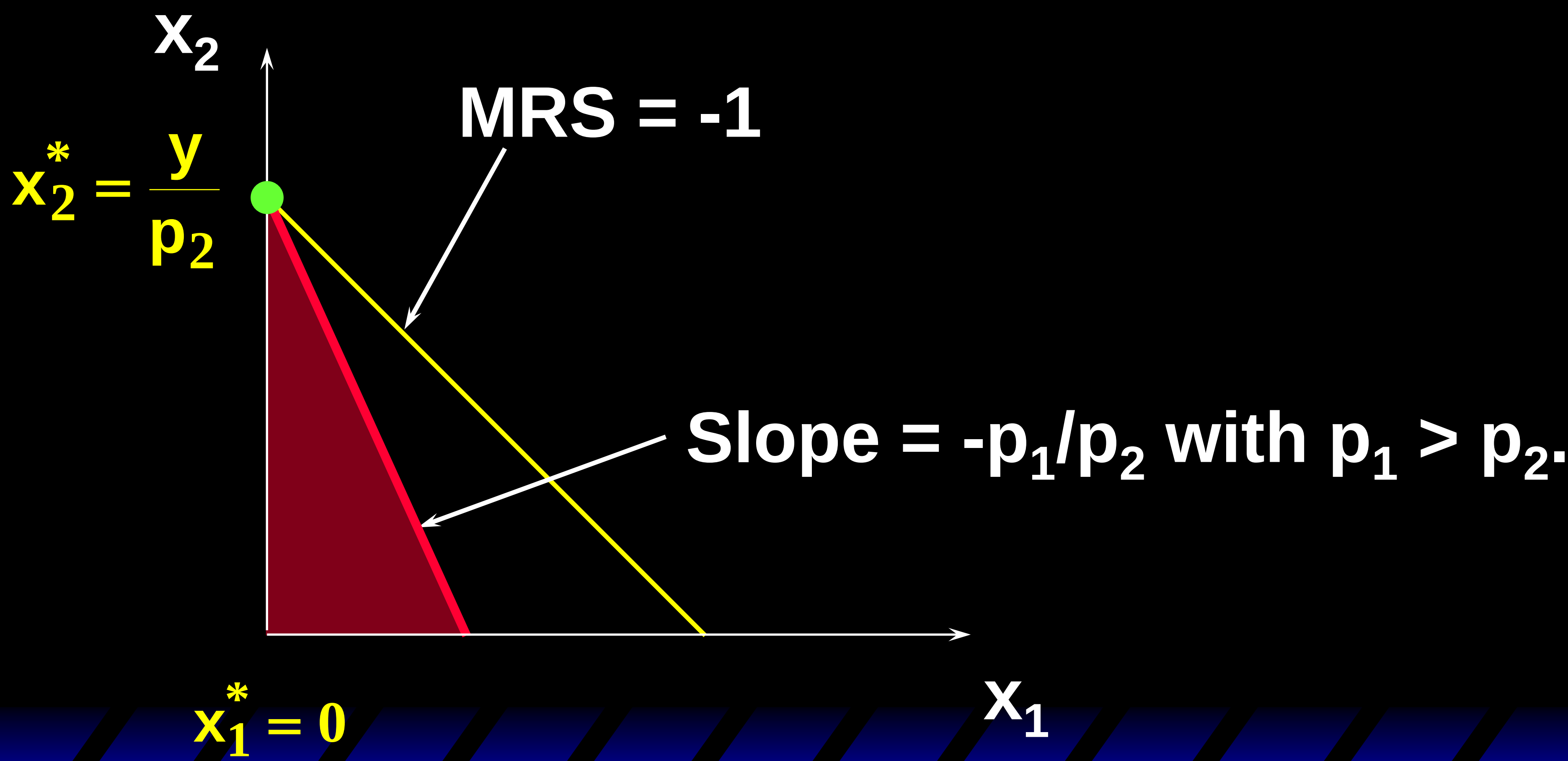
Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case

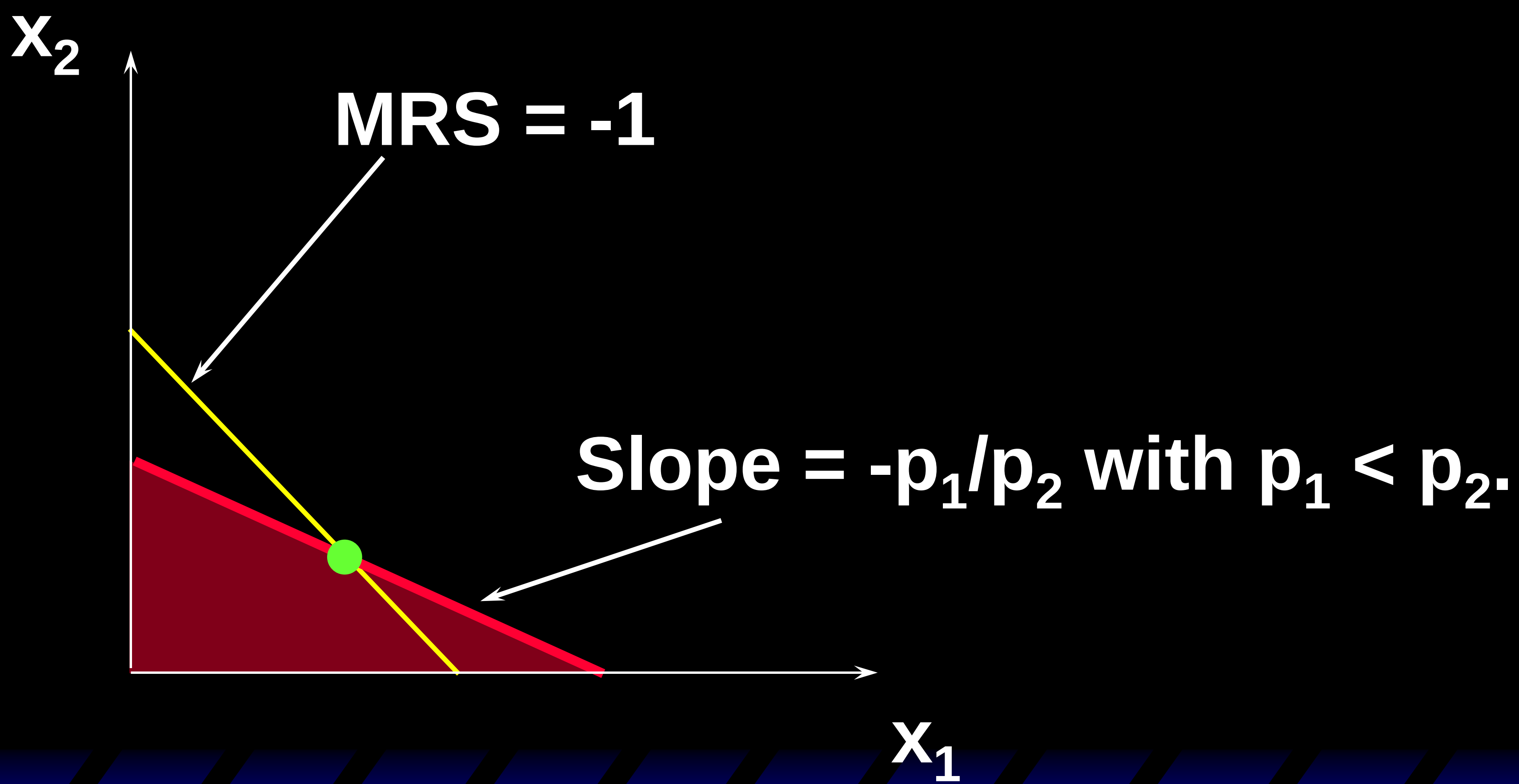


Examples of Corner Solutions -- the Perfect Substitutes Case

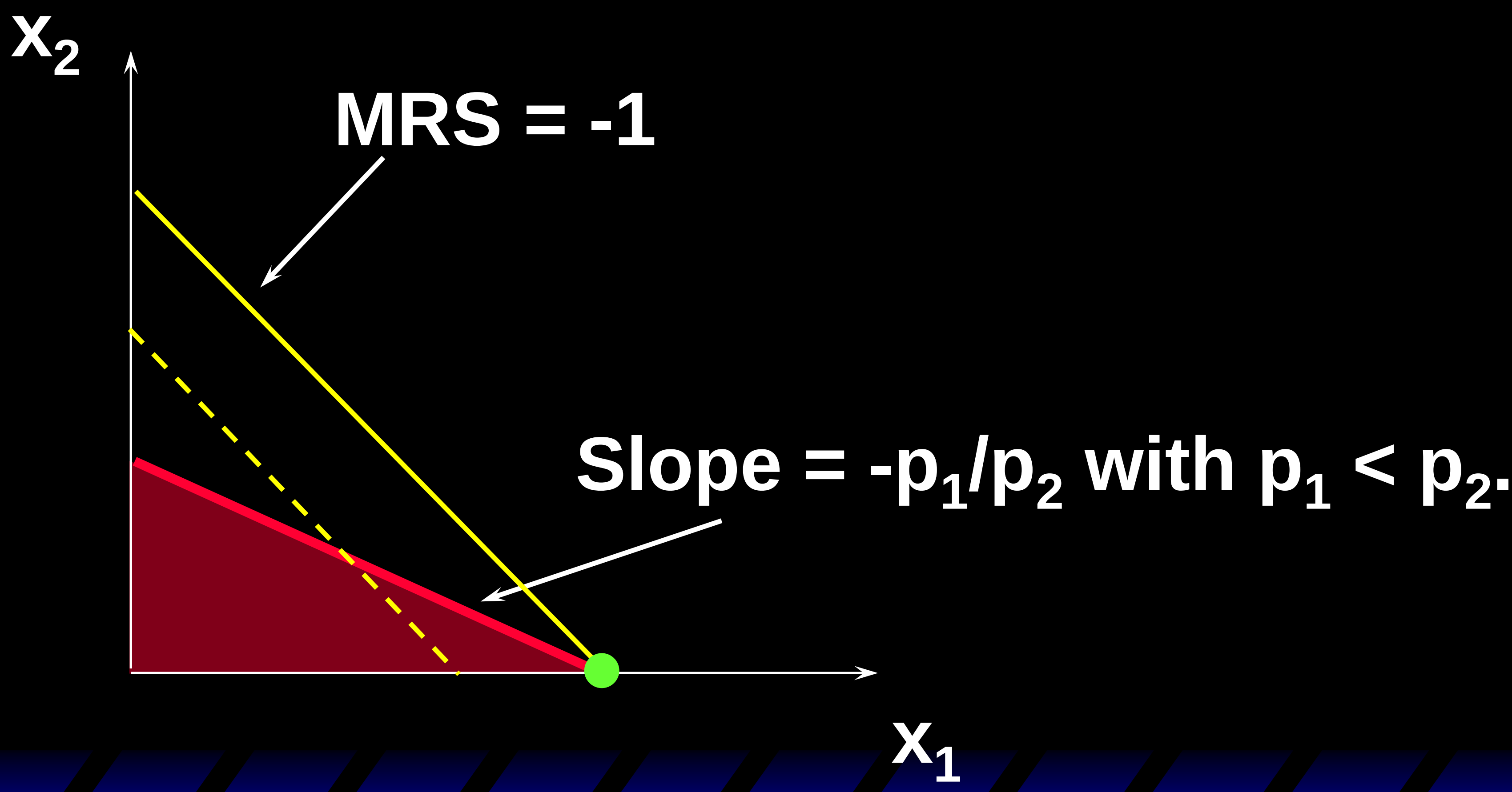
So when $U(x_1,x_2) = x_1 + x_2$, the most preferred affordable bundle is (x_1^*,x_2^*) where

$$(x_1^*, x_2^*) = \left(0, \frac{y}{p_2}\right) \quad \text{if } p_1 > p_2.$$

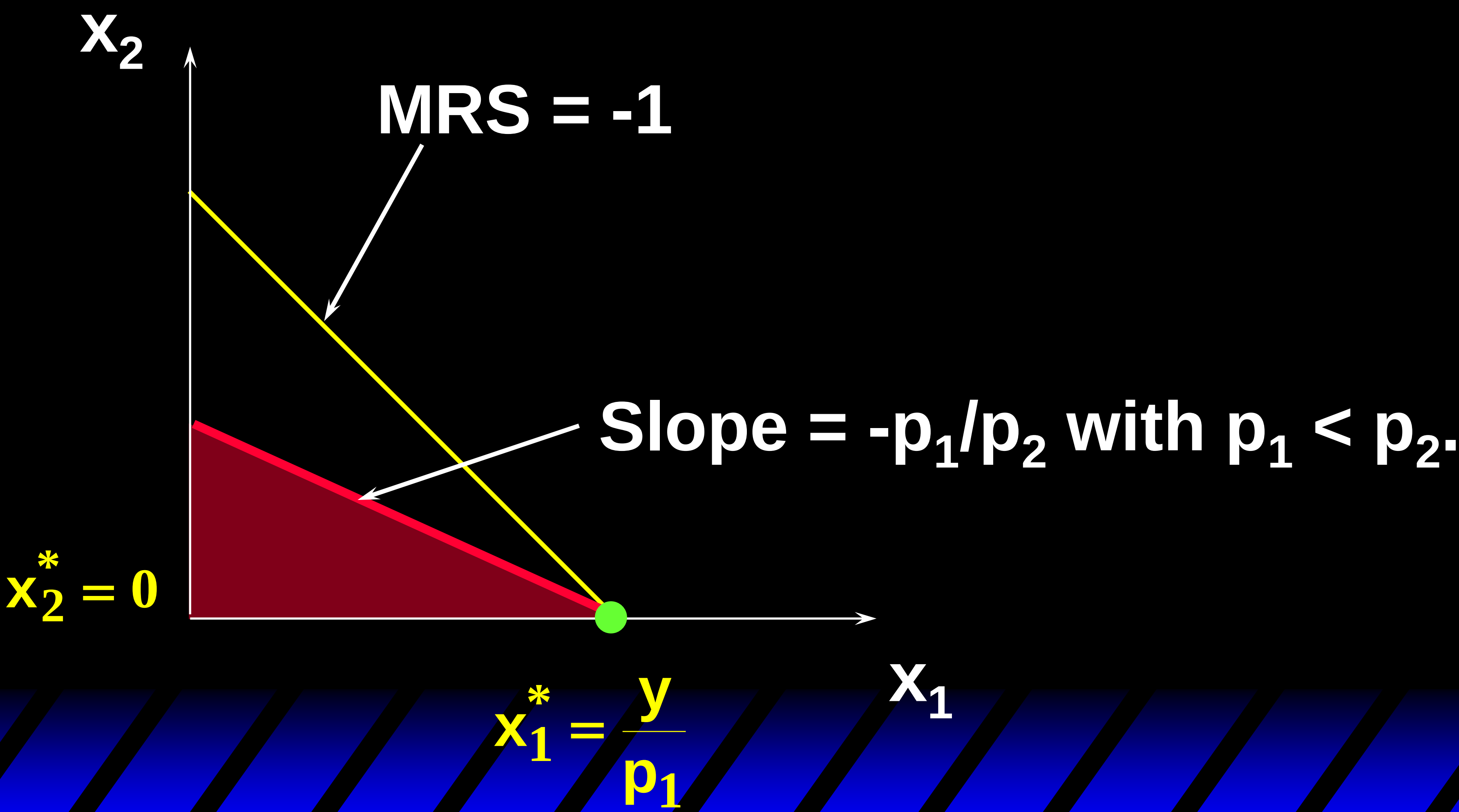
Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case

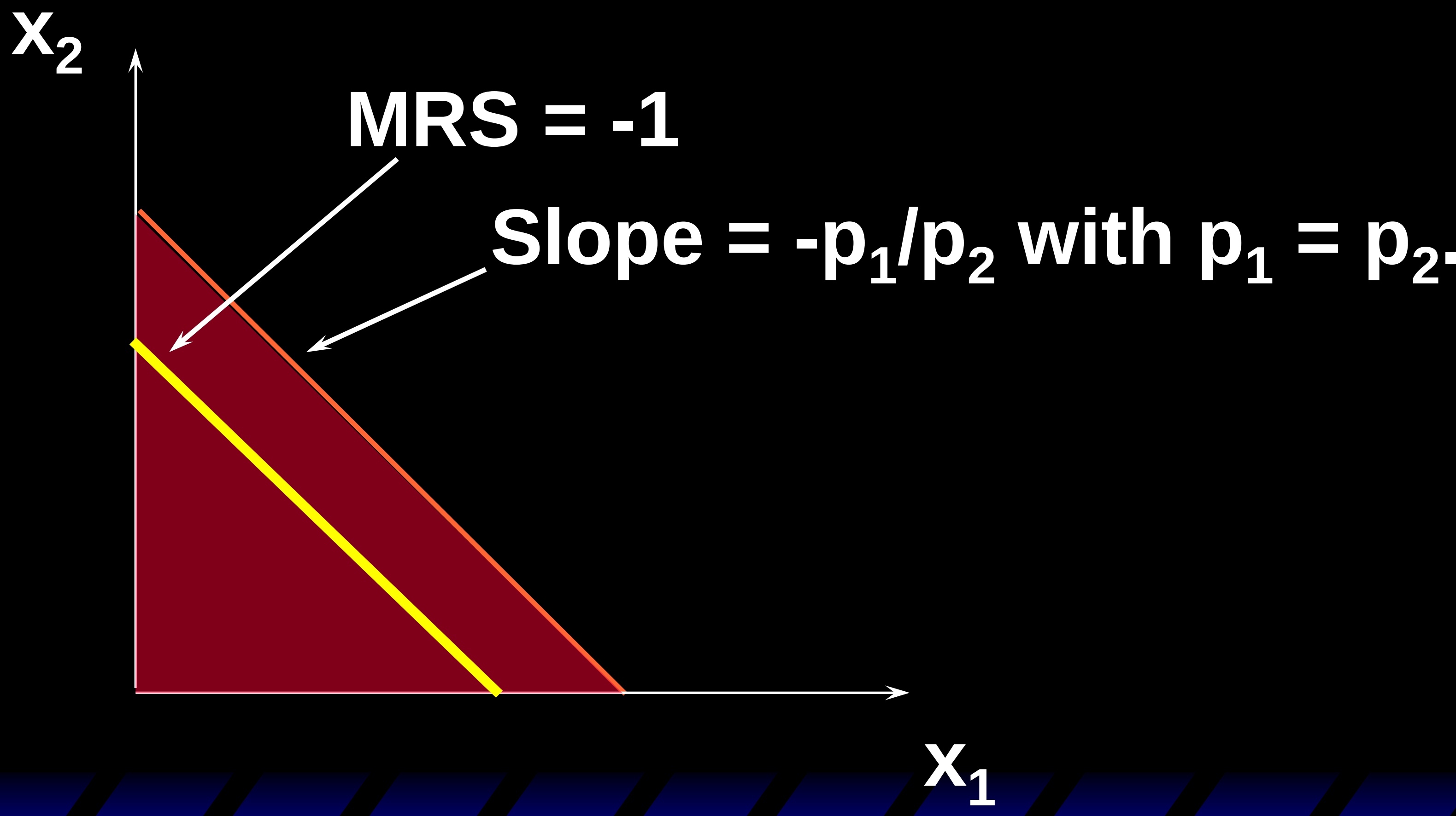


Examples of Corner Solutions -- the Perfect Substitutes Case

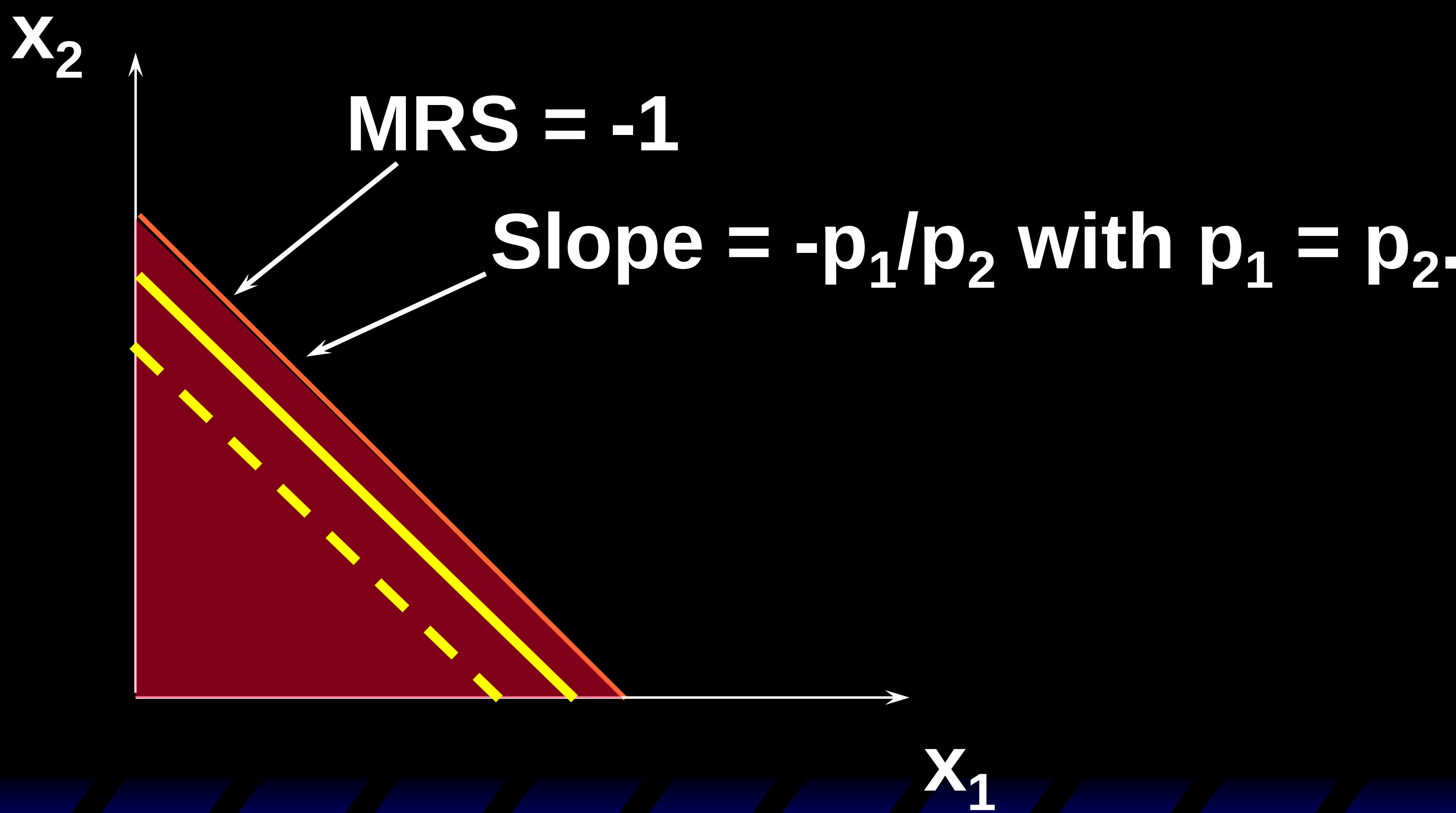
So when $U(x_1,x_2) = x_1 + x_2$, the most preferred affordable bundle is (x_1^*,x_2^*) where

$$(x_1^*, x_2^*) = \left(\frac{y}{p_1}, 0\right) \qquad \text{if } p_1 < p_2$$

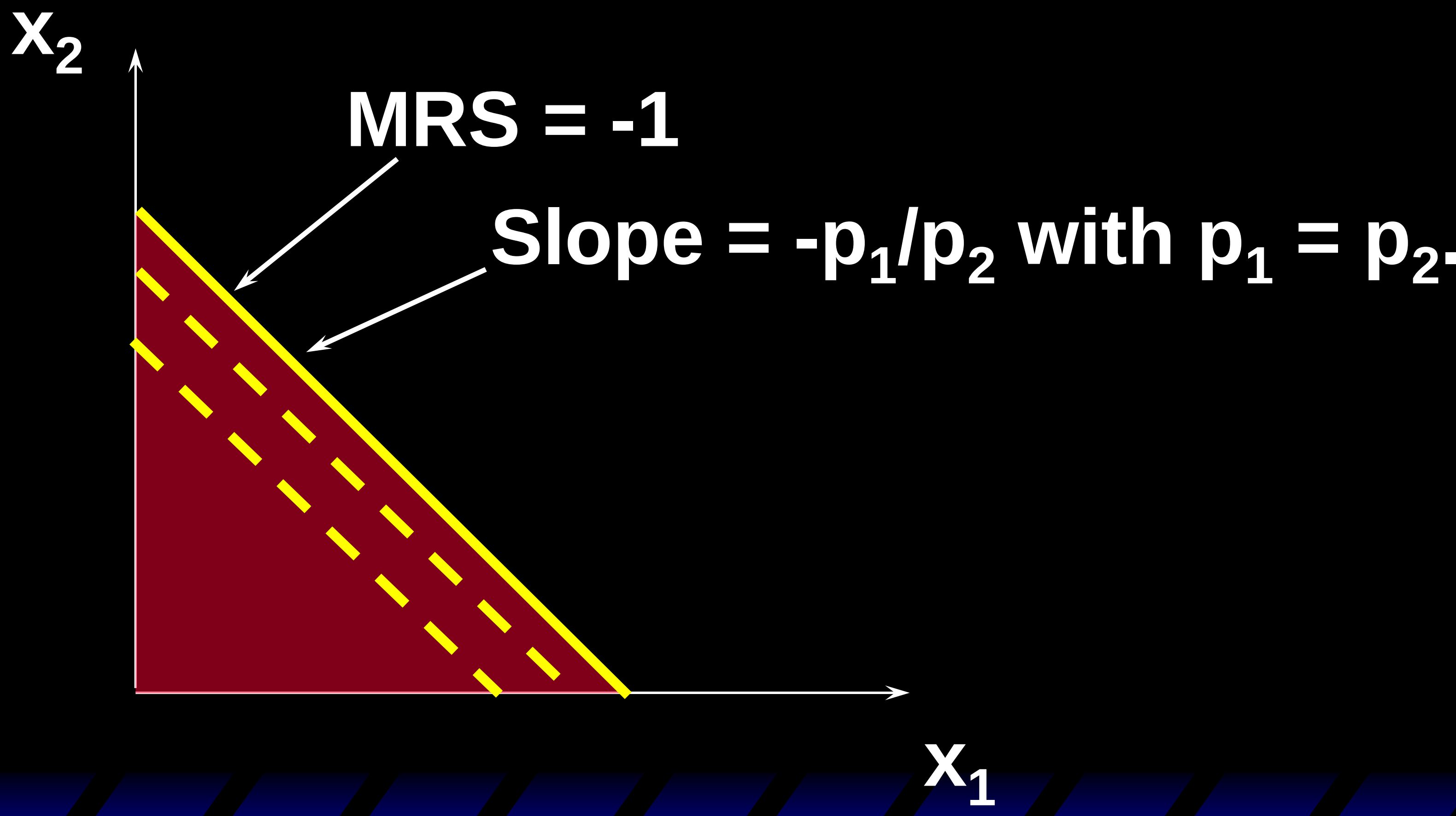
Examples of Corner Solutions -- the Perfect Substitutes Case



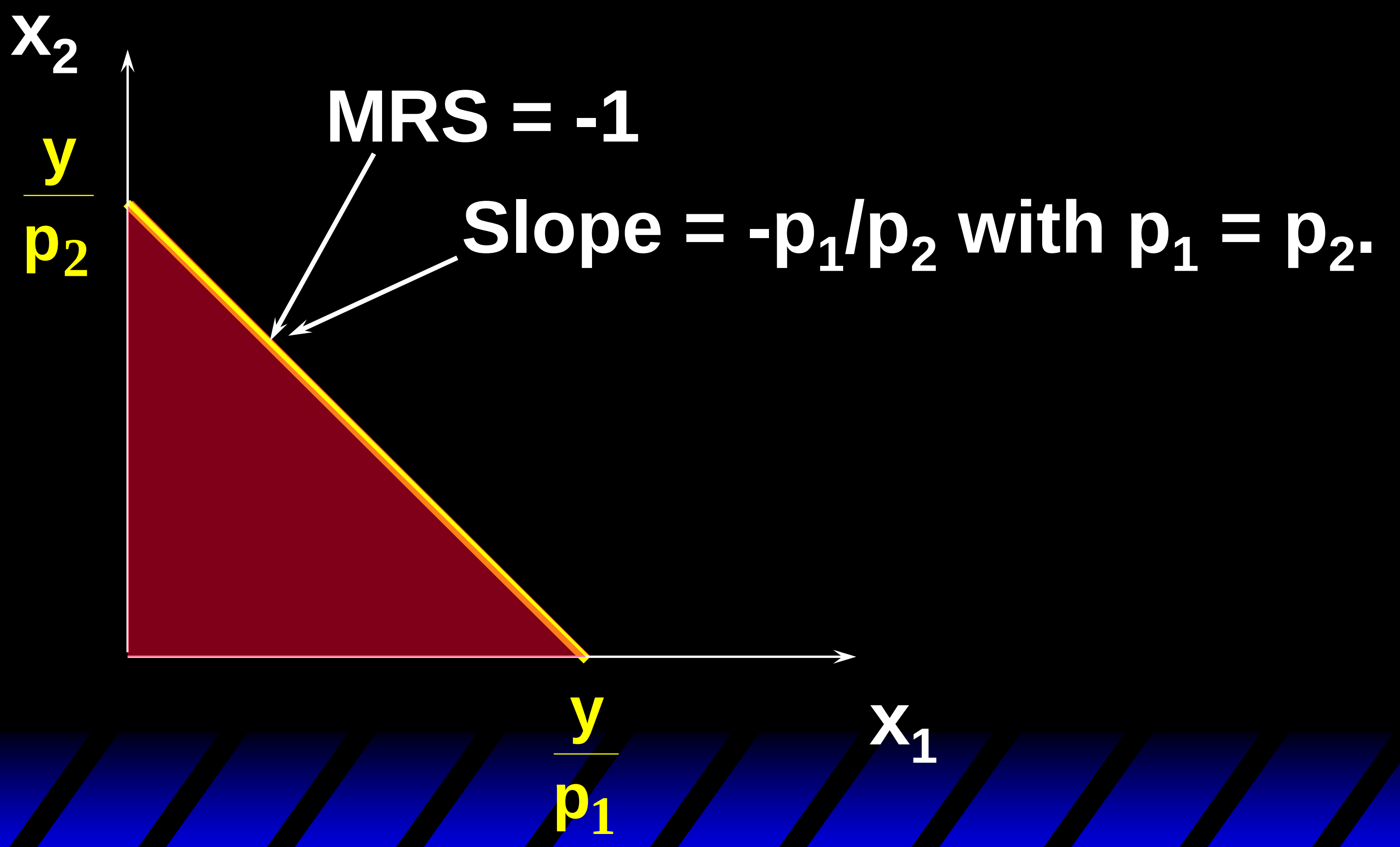
Examples of Corner Solutions -- the Perfect Substitutes Case



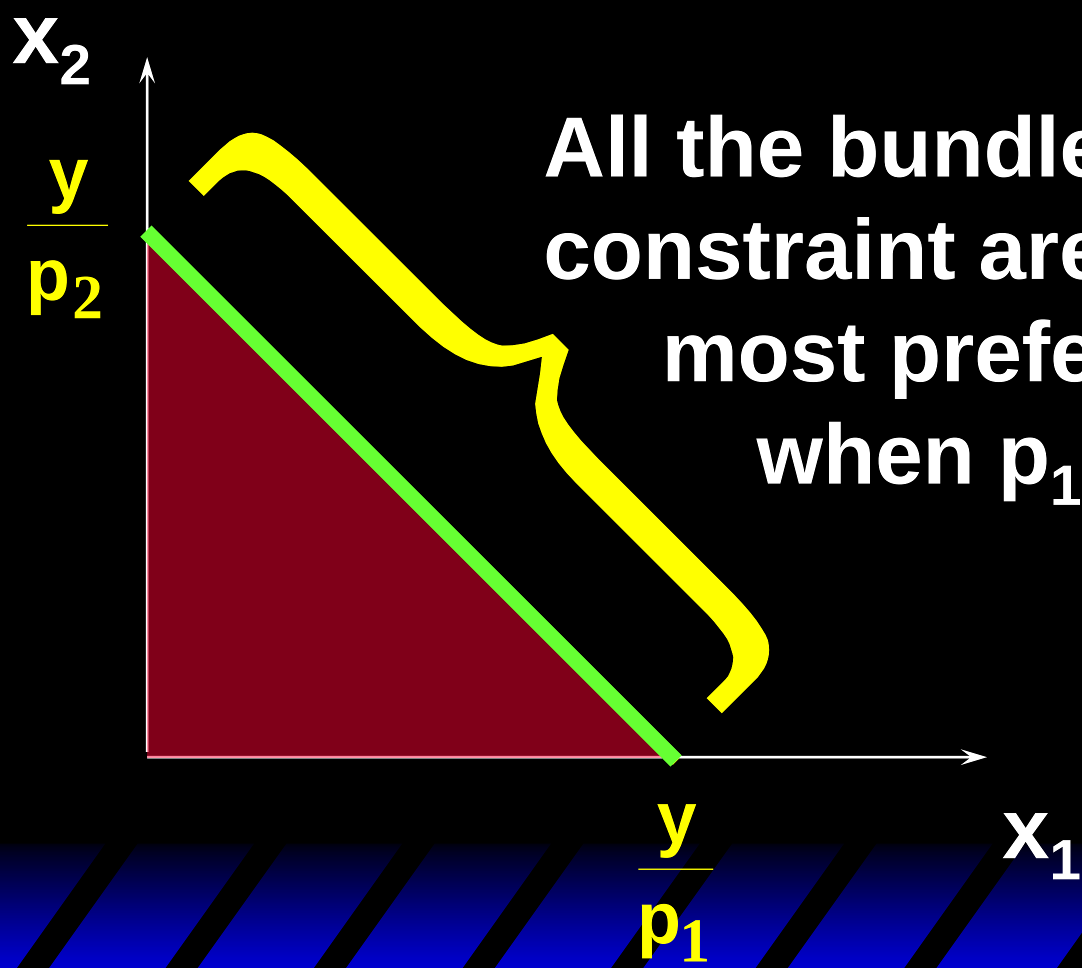
Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case

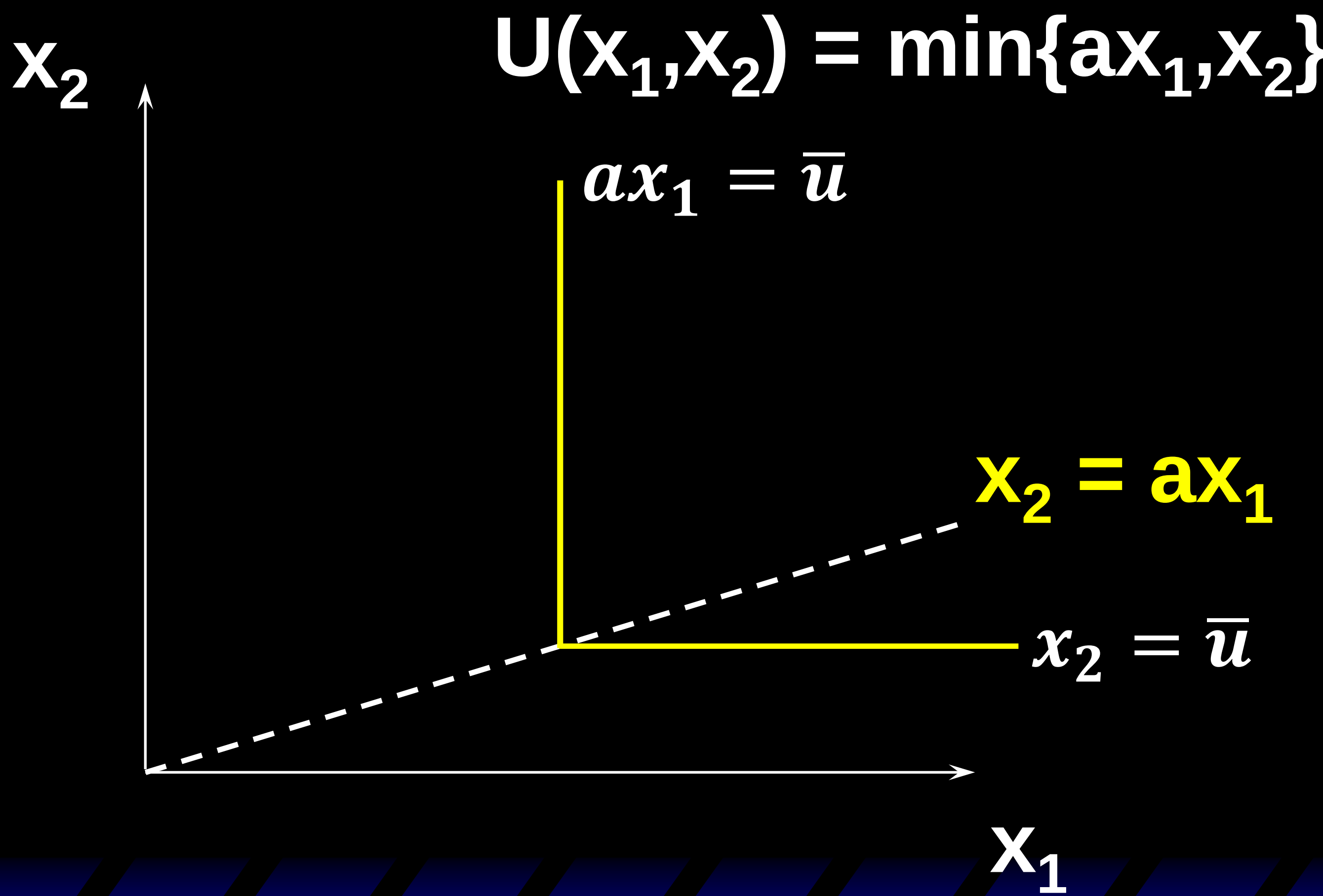


Examples of Corner Solutions -- the Perfect Substitutes Case

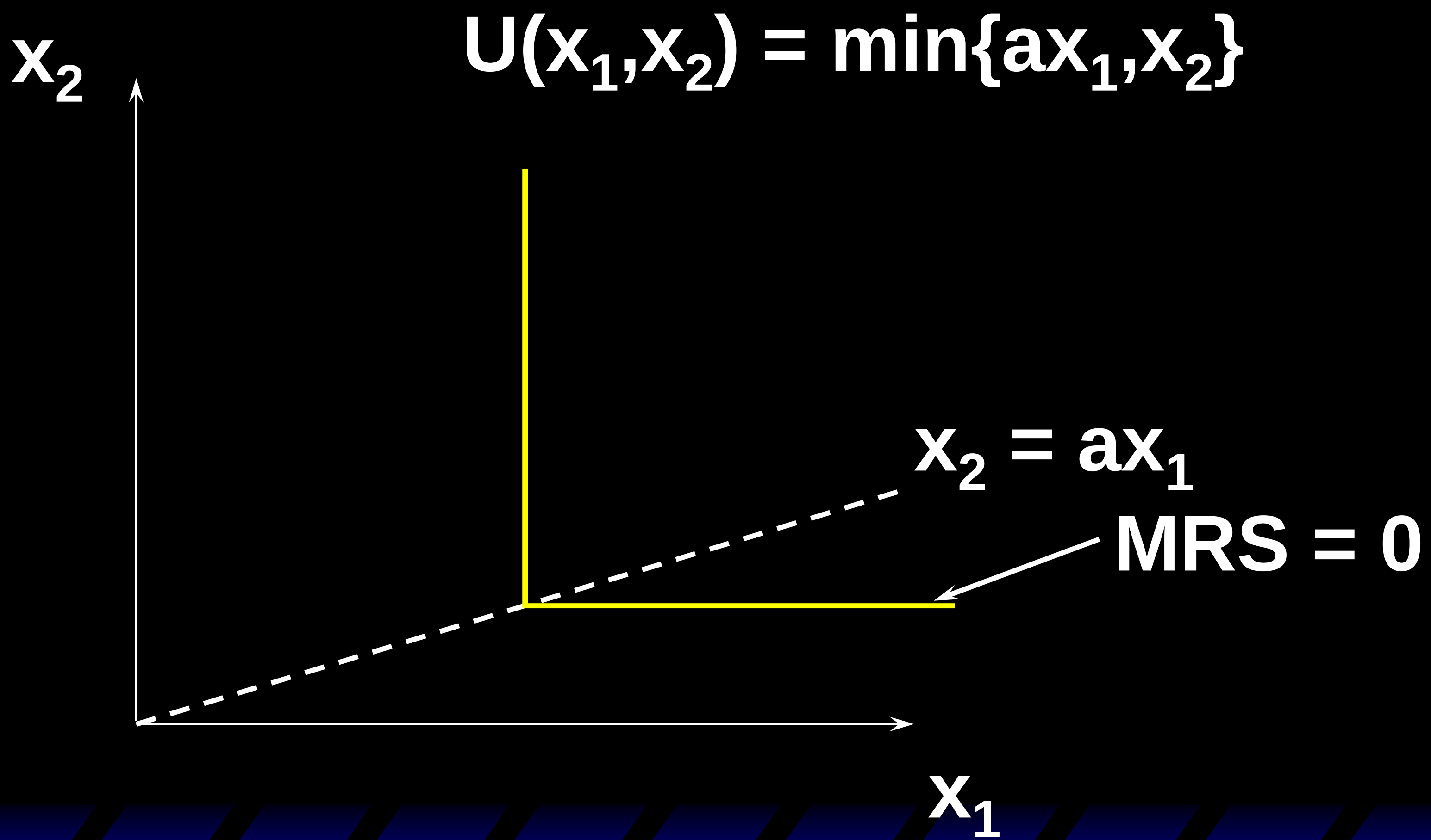


All the bundles in the constraint are equally the most preferred affordable when $p_1 = p_2$.

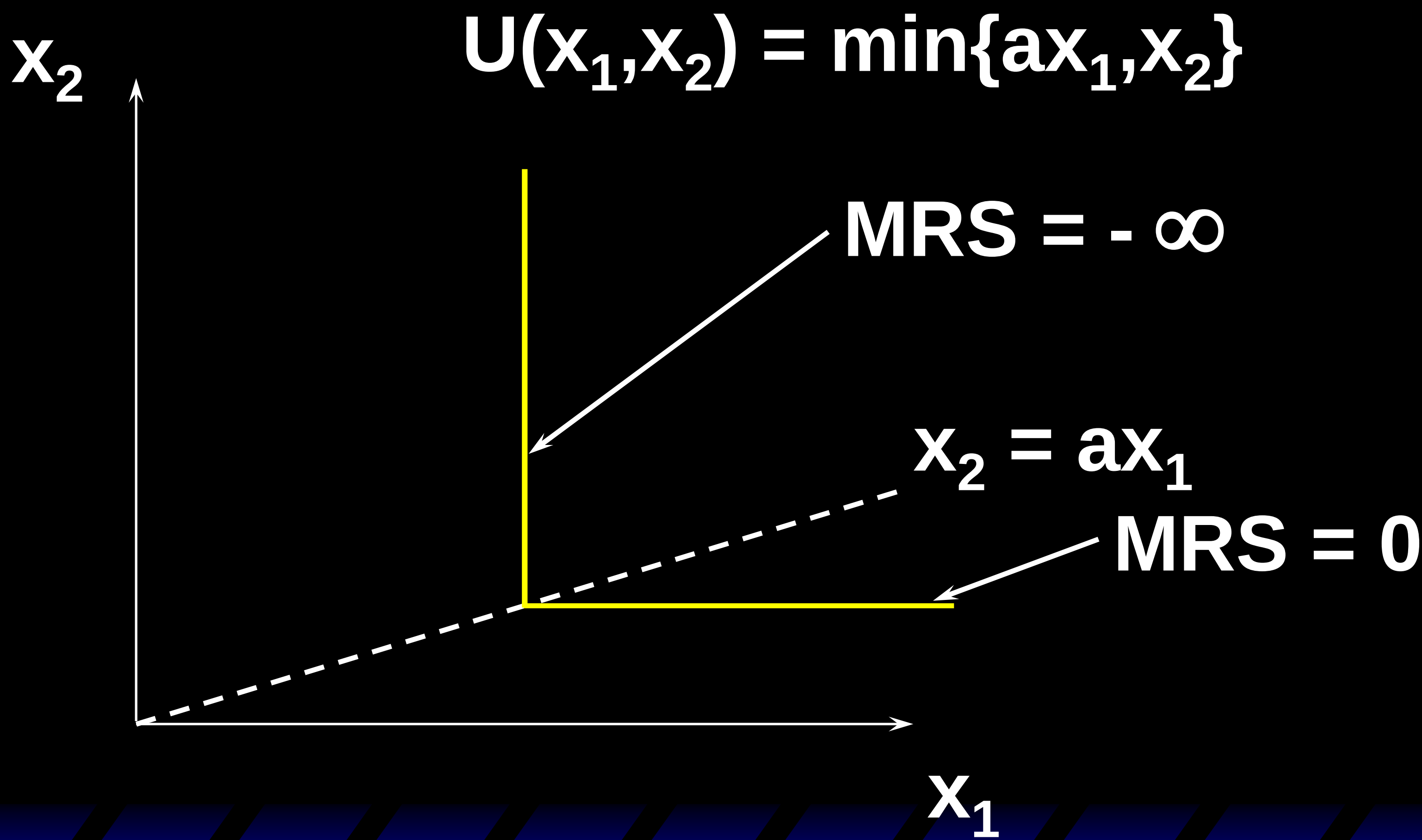
Examples of ‘Kinky’ Solutions -- the Perfect Complements Case



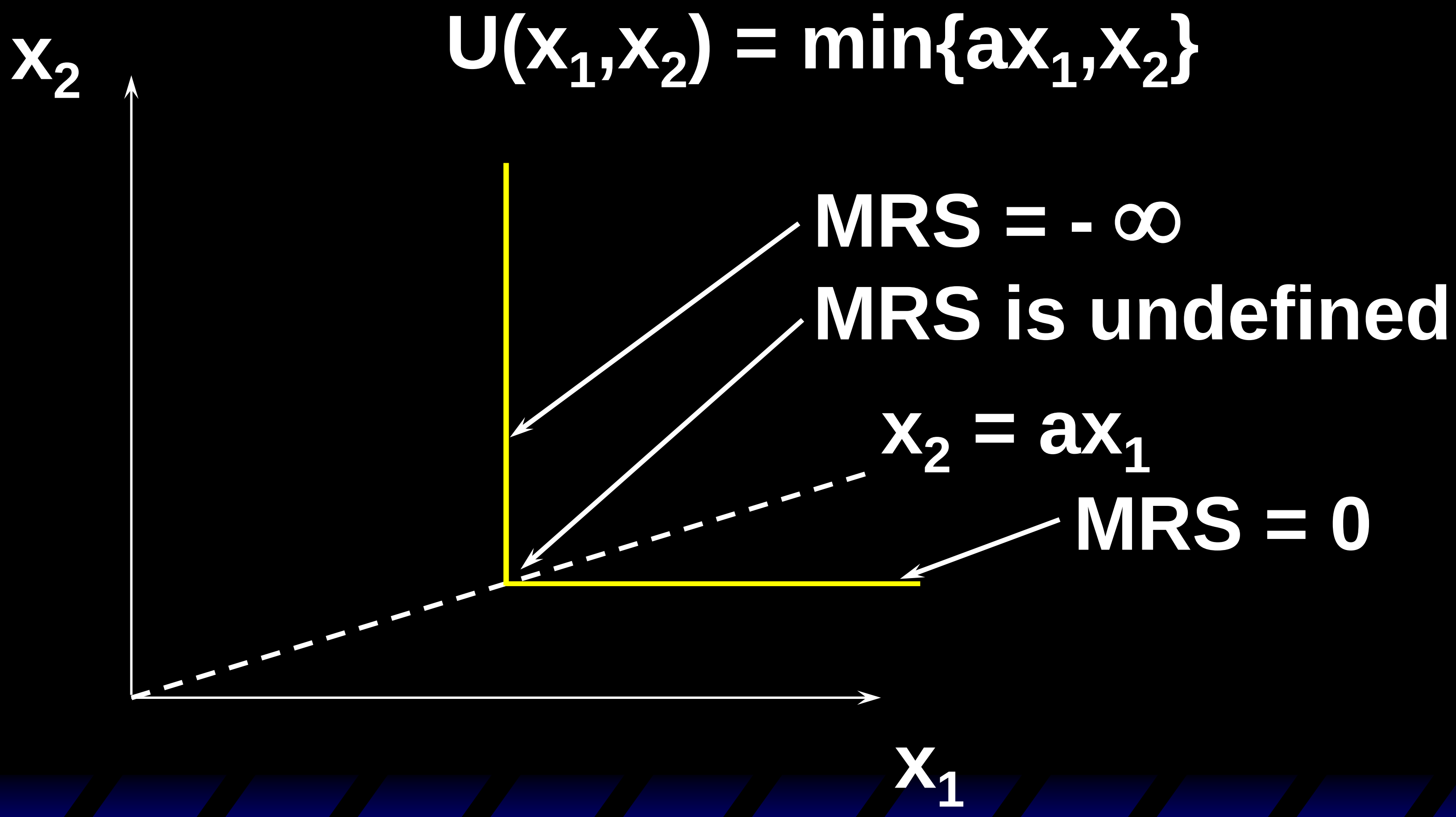
Examples of ‘Kinky’ Solutions -- the Perfect Complements Case



Examples of ‘Kinky’ Solutions -- the Perfect Complements Case

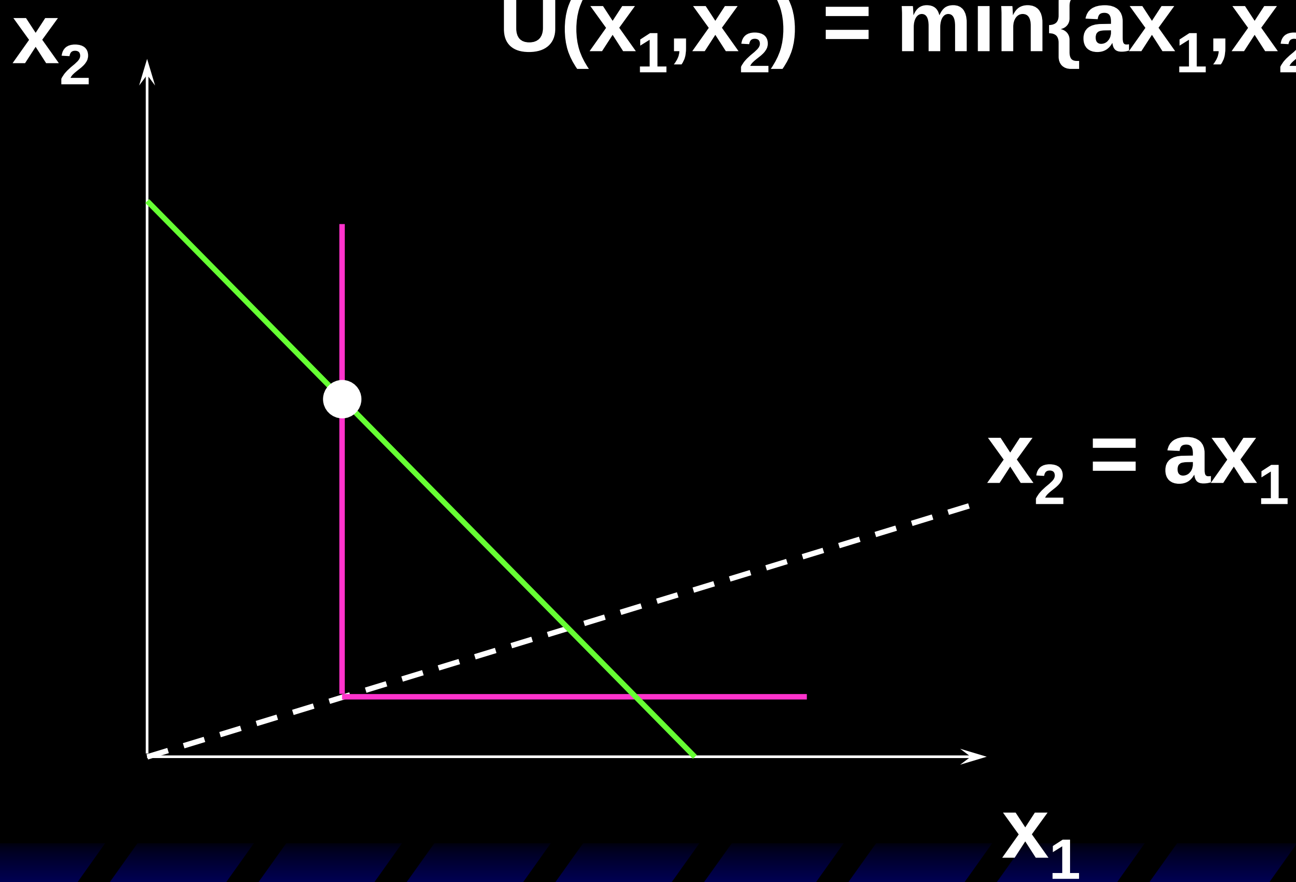


Examples of ‘Kinky’ Solutions -- the Perfect Complements Case



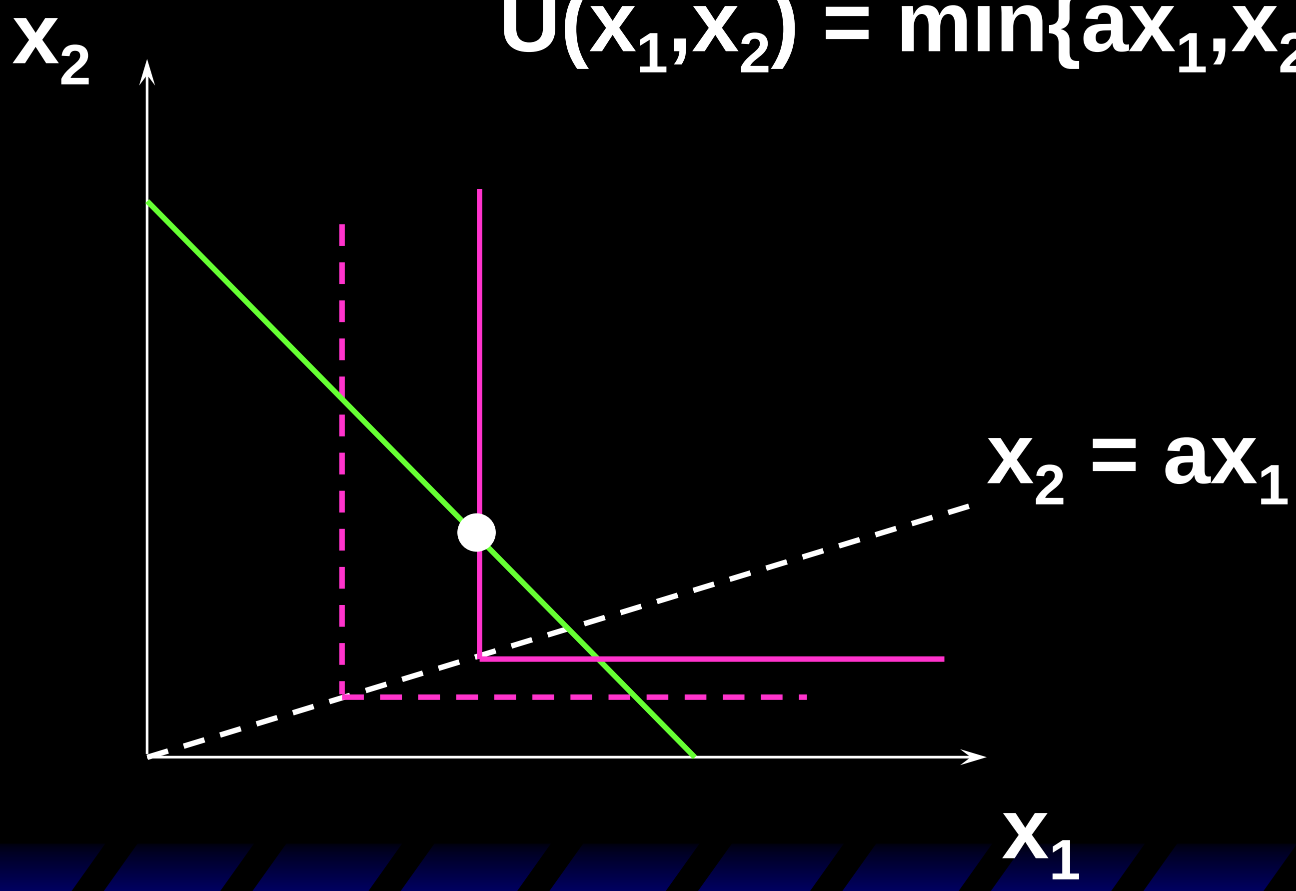
Examples of 'Kinky' Solutions -- the Perfect Complements Case

$U(x_1,x_2) = \min\{ax_1,x_2\}$



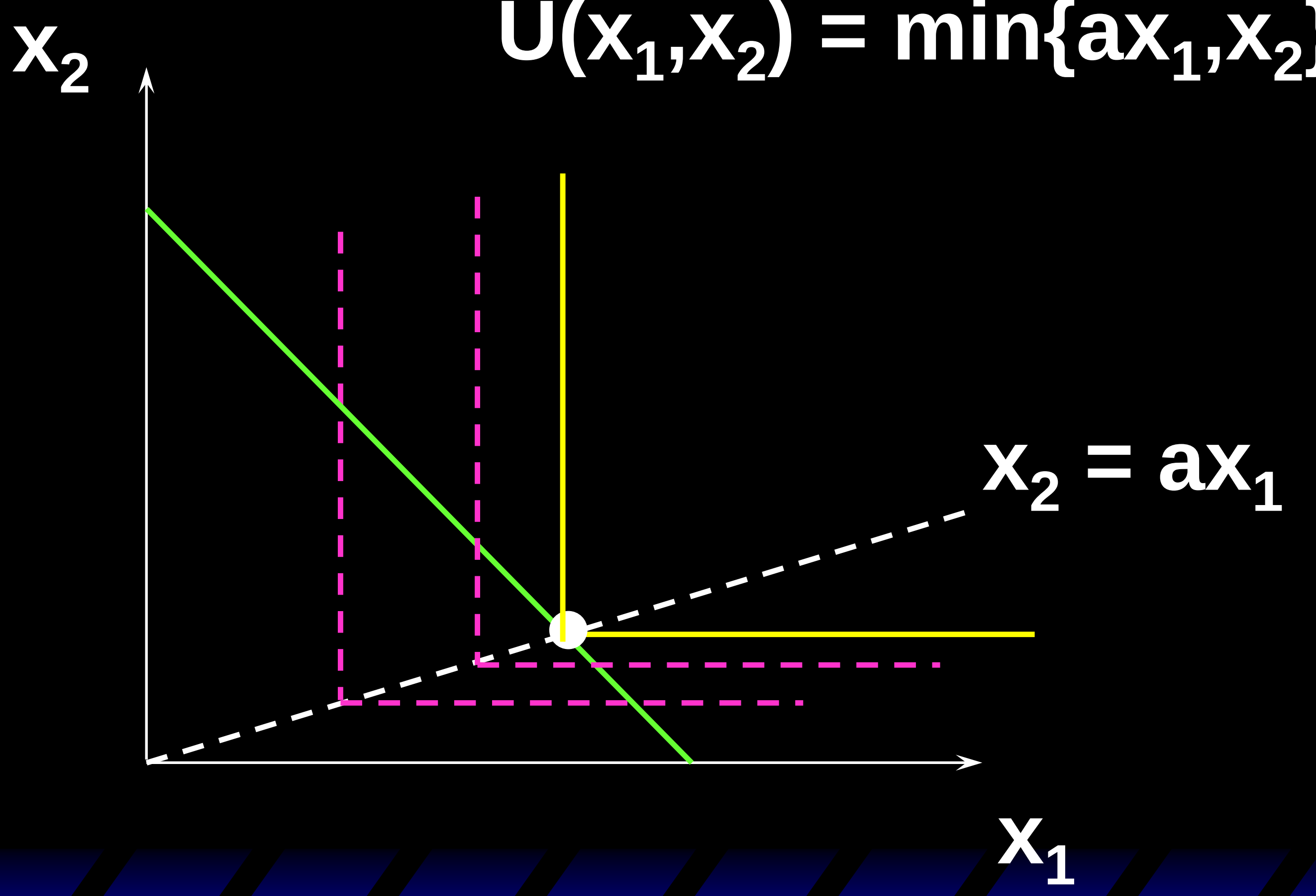
Examples of ‘Kinky’ Solutions -- the Perfect Complements Case

$U(x_1,x_2) = \min\{ax_1,x_2\}$

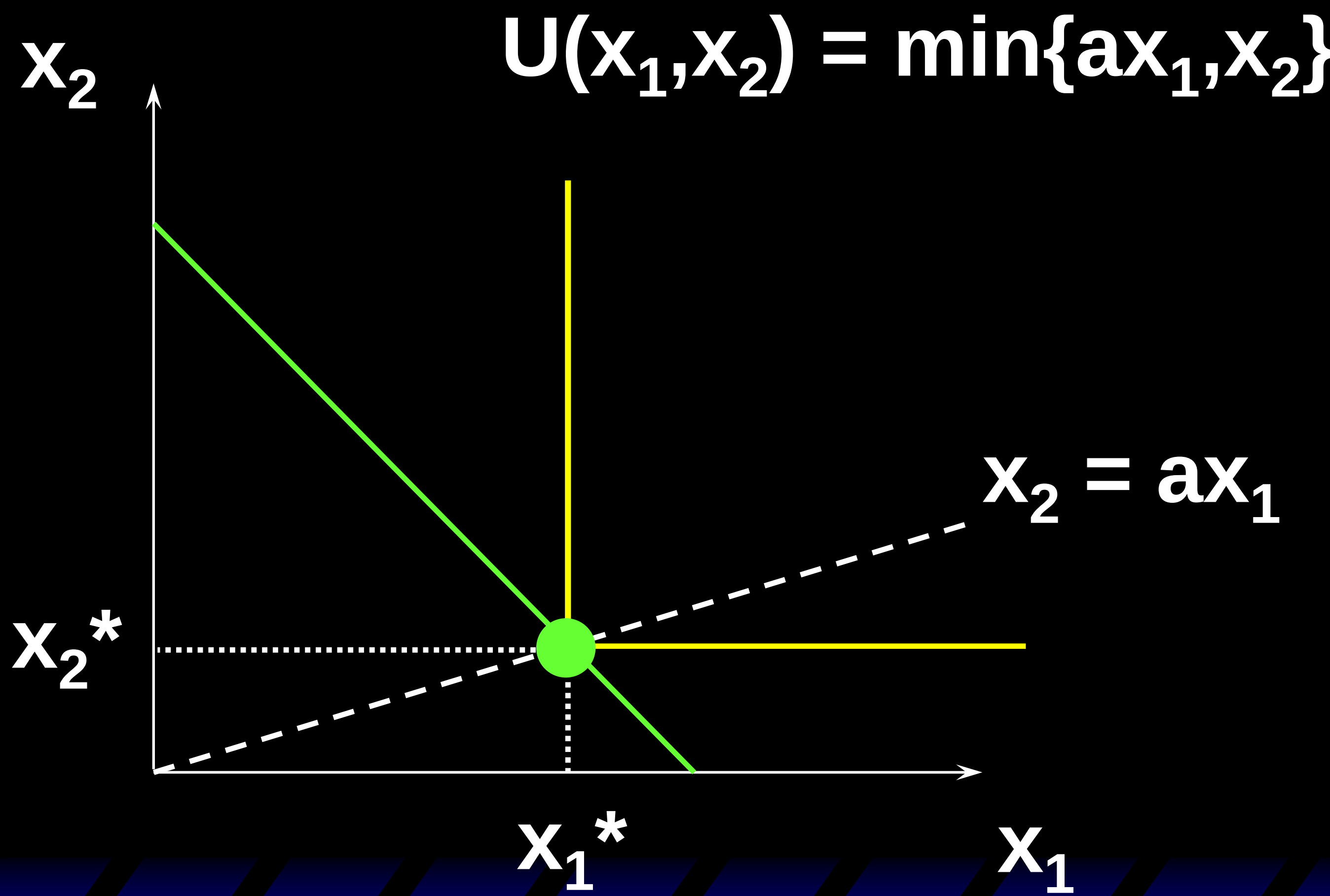


Examples of ‘Kinky’ Solutions -- the Perfect Complements Case

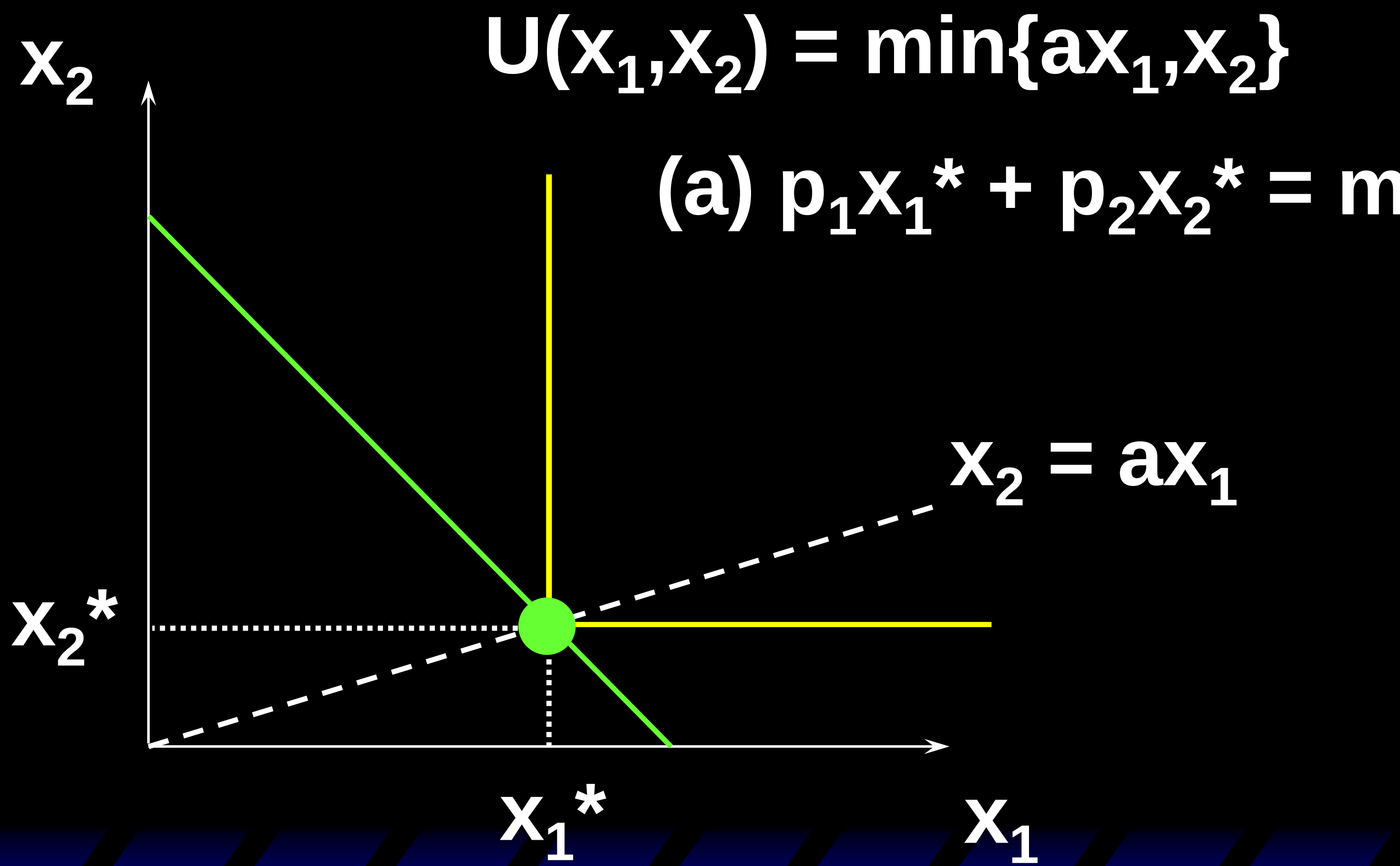
$U(x_1,x_2) = \min\{ax_1,x_2\}$



Examples of ‘Kinky’ Solutions -- the Perfect Complements Case



Examples of 'Kinky' Solutions -- the Perfect Complements Case

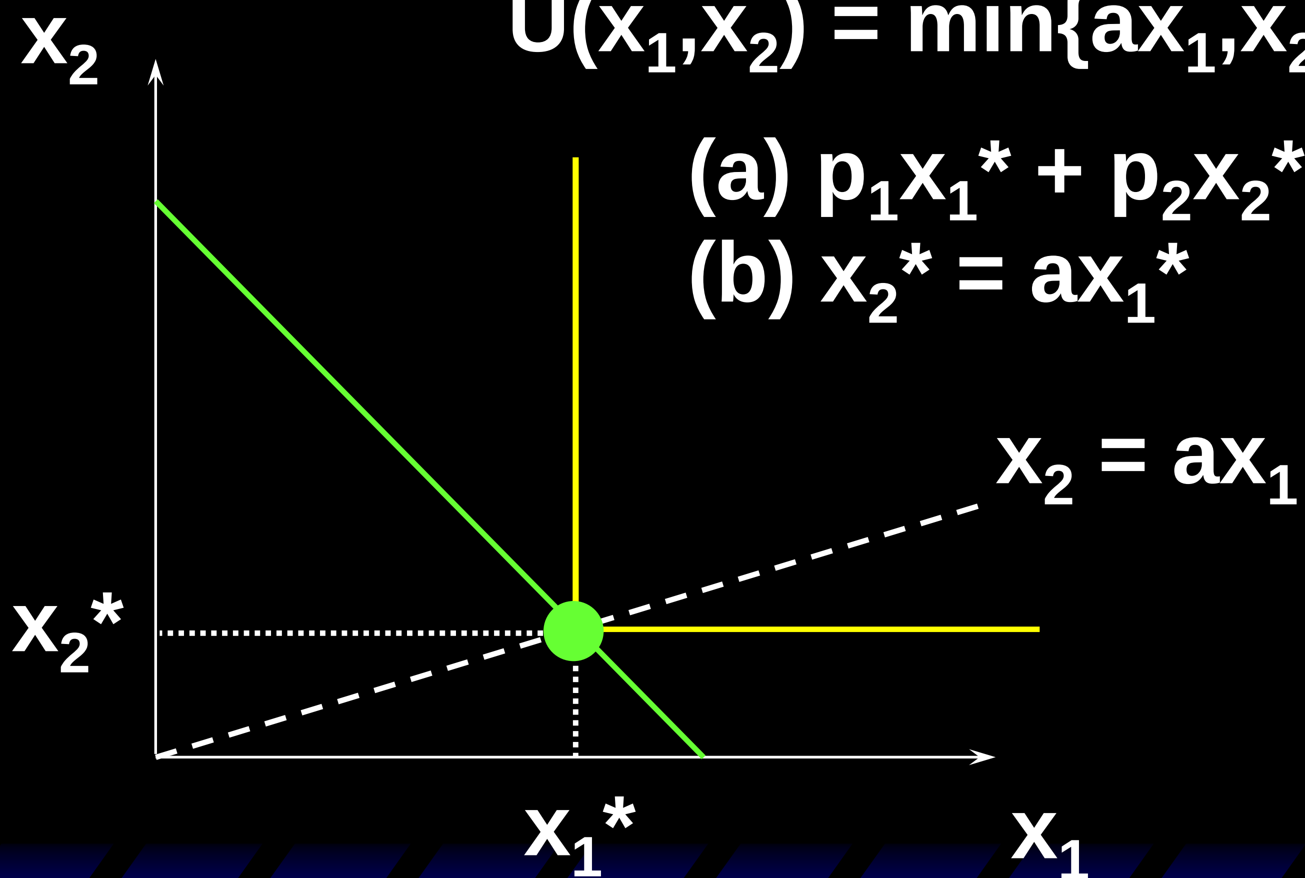


Examples of ‘Kinky’ Solutions -- the Perfect Complements Case

$U(x_1,x_2) = \min\{ax_1,x_2\}$

(a) $p_1x_1^* + p_2x_2^* = m$

(b) $x_2^* = ax_1^*$



Examples of ‘Kinky’ Solutions -- the Perfect Complements Case

(a) $p_1x_1^* + p_2x_2^* = m$; (b) $x_2^* = ax_1^*$.

Examples of 'Kinky' Solutions -- the Perfect Complements Case

(a) $p_1x_1^* + p_2x_2^* = m$; (b) $x_2^* = ax_1^*$.

Substitution from (b) for x_2^* in (a) gives $p_1x_1^* + p_2ax_1^* = m$

Examples of 'Kinky' Solutions -- the Perfect Complements Case

(a) $p_1x_1^* + p_2x_2^* = m$; (b) $x_2^* = ax_1^*$.

Substitution from (b) for x_2^* in (a) gives $p_1x_1^* + p_2ax_1^* = m$ which gives

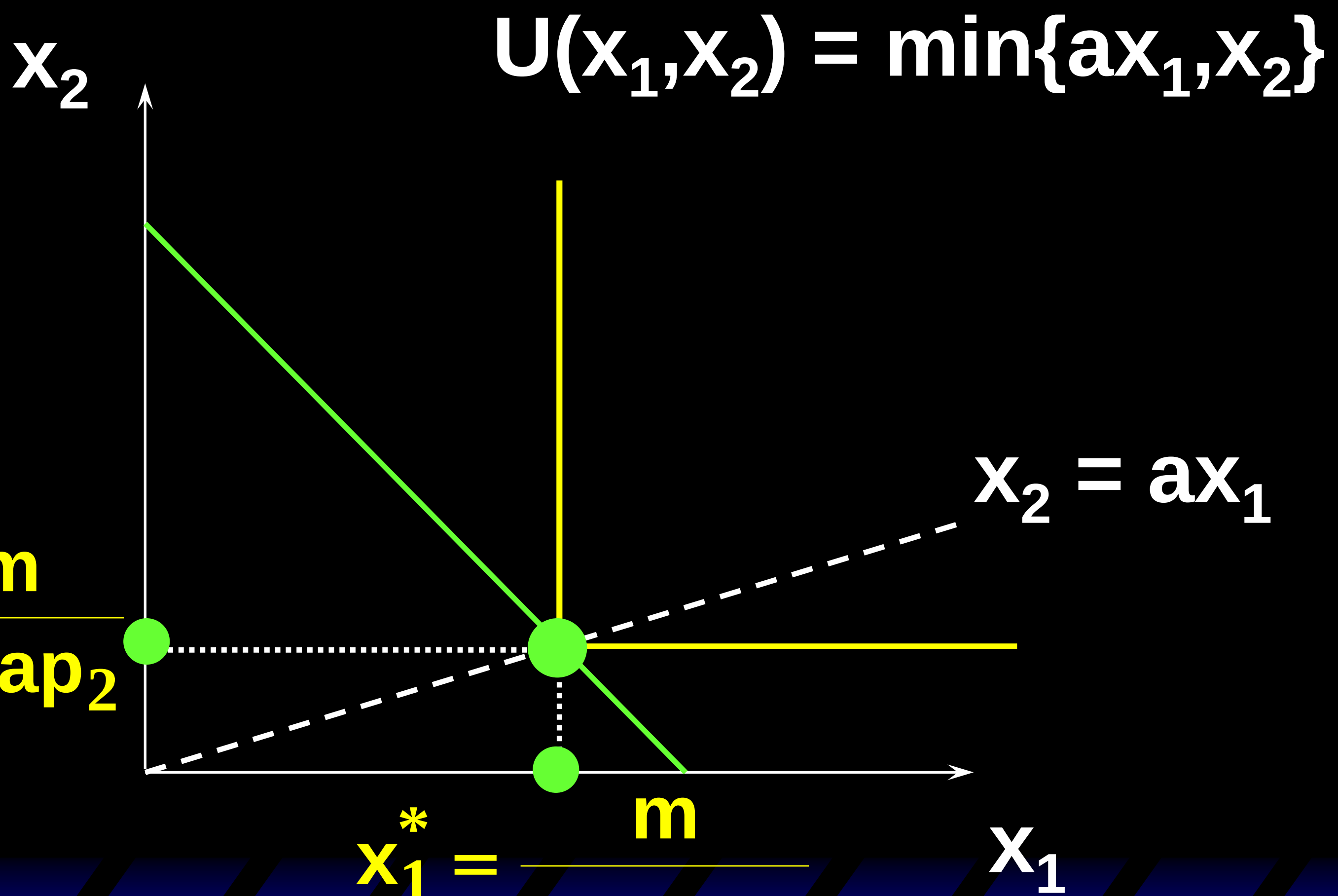
$$x_1^* = \frac{m}{p_1 + ap_2}$$

Examples of 'Kinky' Solutions -- the Perfect Complements Case

(a) $p_1x_1^* + p_2x_2^* = m$; (b) $x_2^* = ax_1^*$.

Substitution from (b) for x_2^* in (a) gives $p_1x_1^* + p_2ax_1^* = m$ which gives $$x_1^* = \frac{m}{p_1 + ap_2}; \; x_2^* = \frac{am}{p_1 + ap_2}.$$

Examples of 'Kinky' Solutions -- the Perfect Complements Case



$$x_2^* = \frac{am}{p_1 + ap_2}$$

$$x_1^* = \frac{m}{p_1 + ap_2}$$