# Lecture 7

## Optimal Consumption with Endowments

# Overview

**Previously: the optimal consumption problem for a rational agent whose income is given as fixed**

**Today: the same problem for a rational agent whose income is generated from initial endowments**

拥有禀赋时的最优消费问题

# Endowments

- The list of resource units with which a consumer starts is his **endowment**.
- A consumer's endowment will be denoted by the vector $\omega$ (omega).

# Endowments

- E.g. $\omega = (\omega_1, \omega_2) = (10,2)$ states that the consumer is endowed with 10 units of good 1 and 2 units of good 2.

# Endowments

E.g. $\omega = (\omega_1, \omega_2) = (10,2)$
states that the consumer is endowed with 10 units of good 1 and 2 units of good 2.

What is the endowment's value?

For which consumption bundles may it be exchanged?

# Endowments

$p_1$=2 and $p_2$=3 so the value of the endowment $(\omega_1, \omega_2) = (10,2)$ is

$$p_1\omega_1 + p_2\omega_2 = 2 \times 10 + 3 \times 2 = 26$$

Q: For which consumption bundles may the endowment be exchanged?

A: For any bundle costing no more than the endowment's value.

# Budget Constraints Revisited

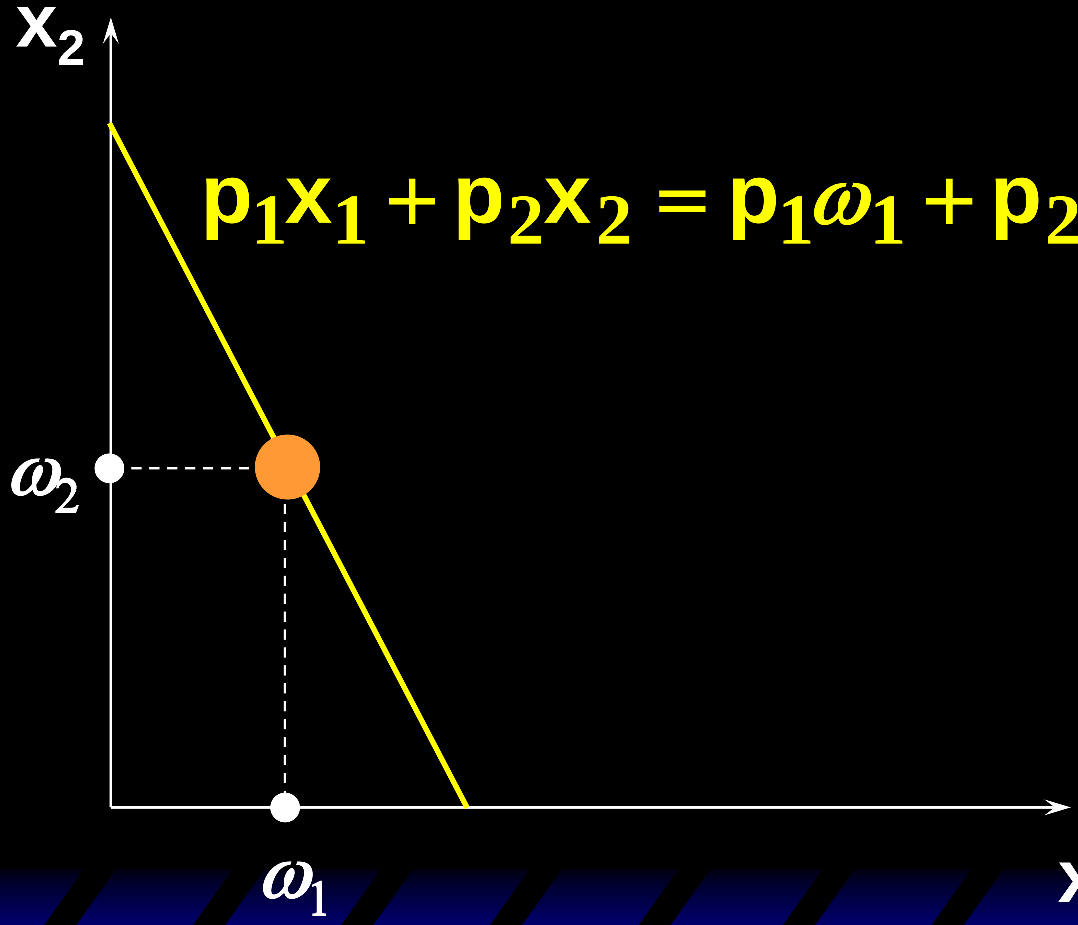
So, given $p_1$ and $p_2$, the budget constraint for a consumer with an endowment $(\omega_1, \omega_2)$ is

$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2.$$

The budget set is

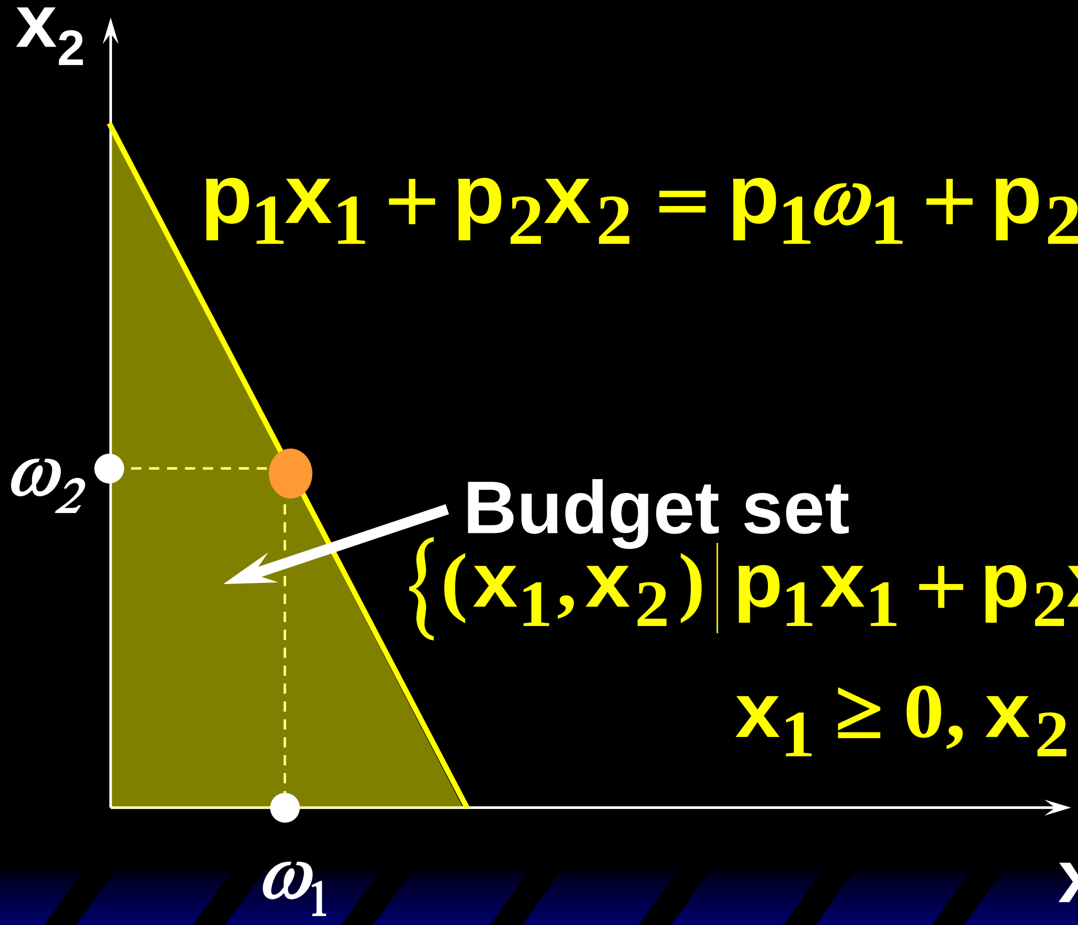
$$\left\{(x_1, x_2) \middle| p_1x_1 + p_2x_2 \le p_1\omega_1 + p_2\omega_2, \right.$$
$$\left. x_1 \ge 0, x_2 \ge 0 \right\}.$$

# Budget Constraints Revisited




$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2$$

# Budget Constraints Revisited

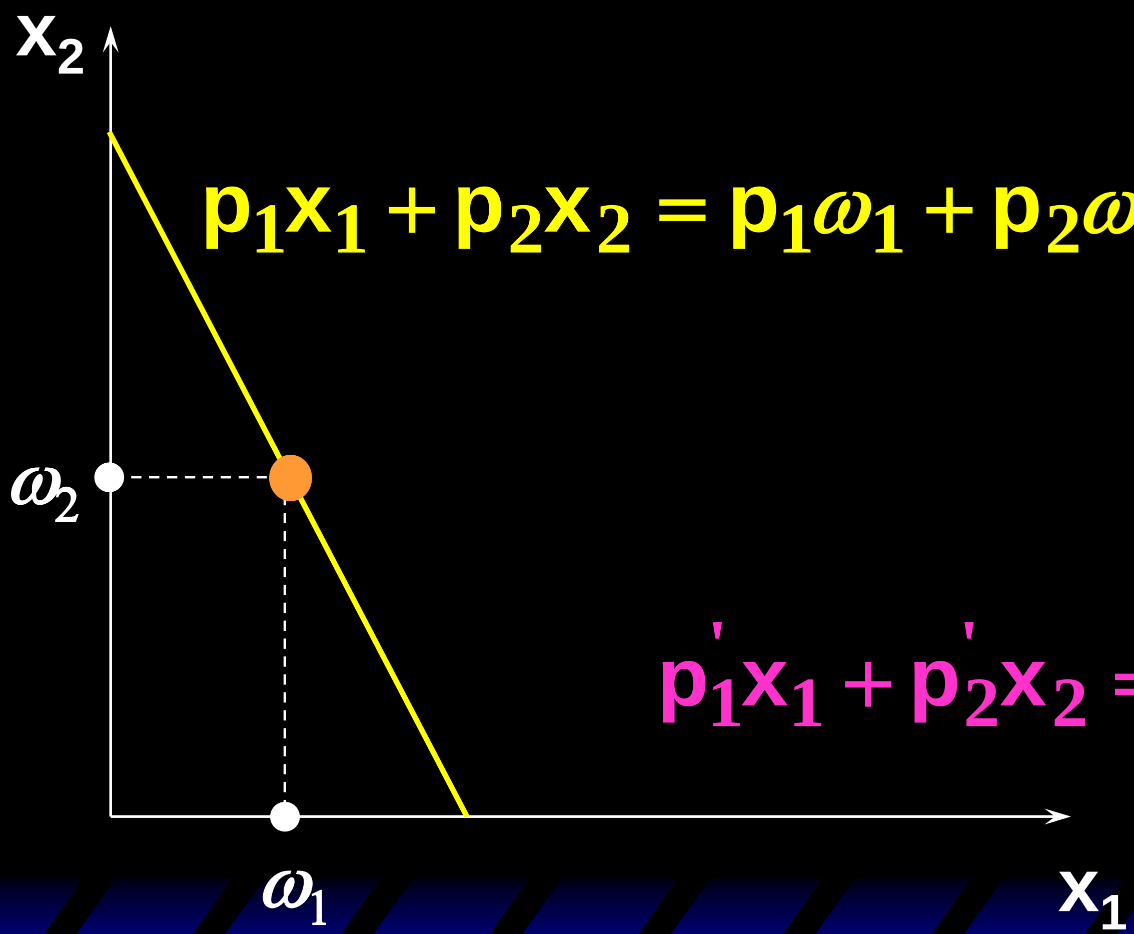



$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2$$

Budget set

$$\left\{(x_1, x_2) \,\middle|\, p_1x_1 + p_2x_2 \le p_1\omega_1 + p_2\omega_2,\ x_1 \ge 0,\ x_2 \ge 0\right\}$$

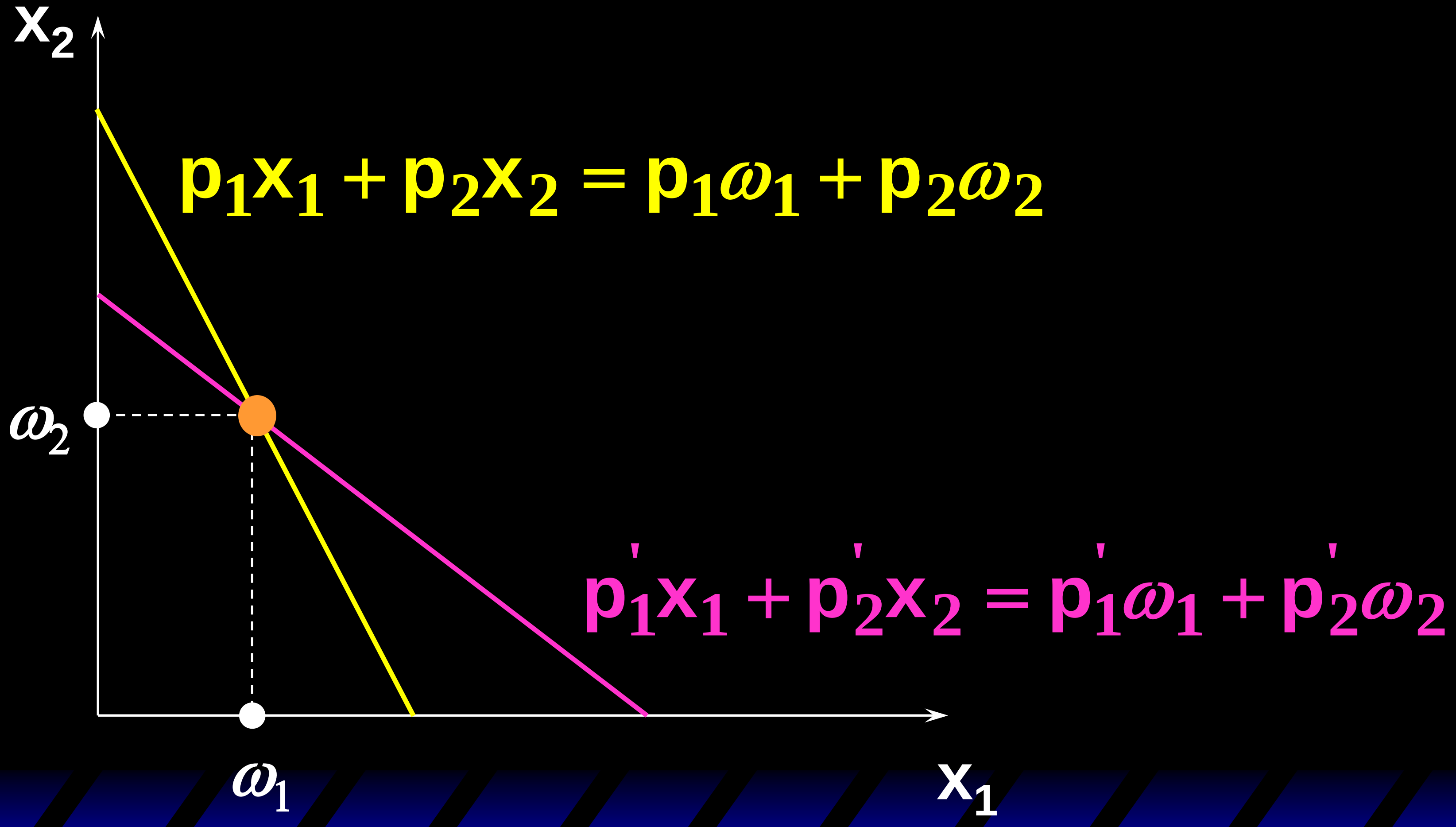
# Budget Constraints Revisited




$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2$$

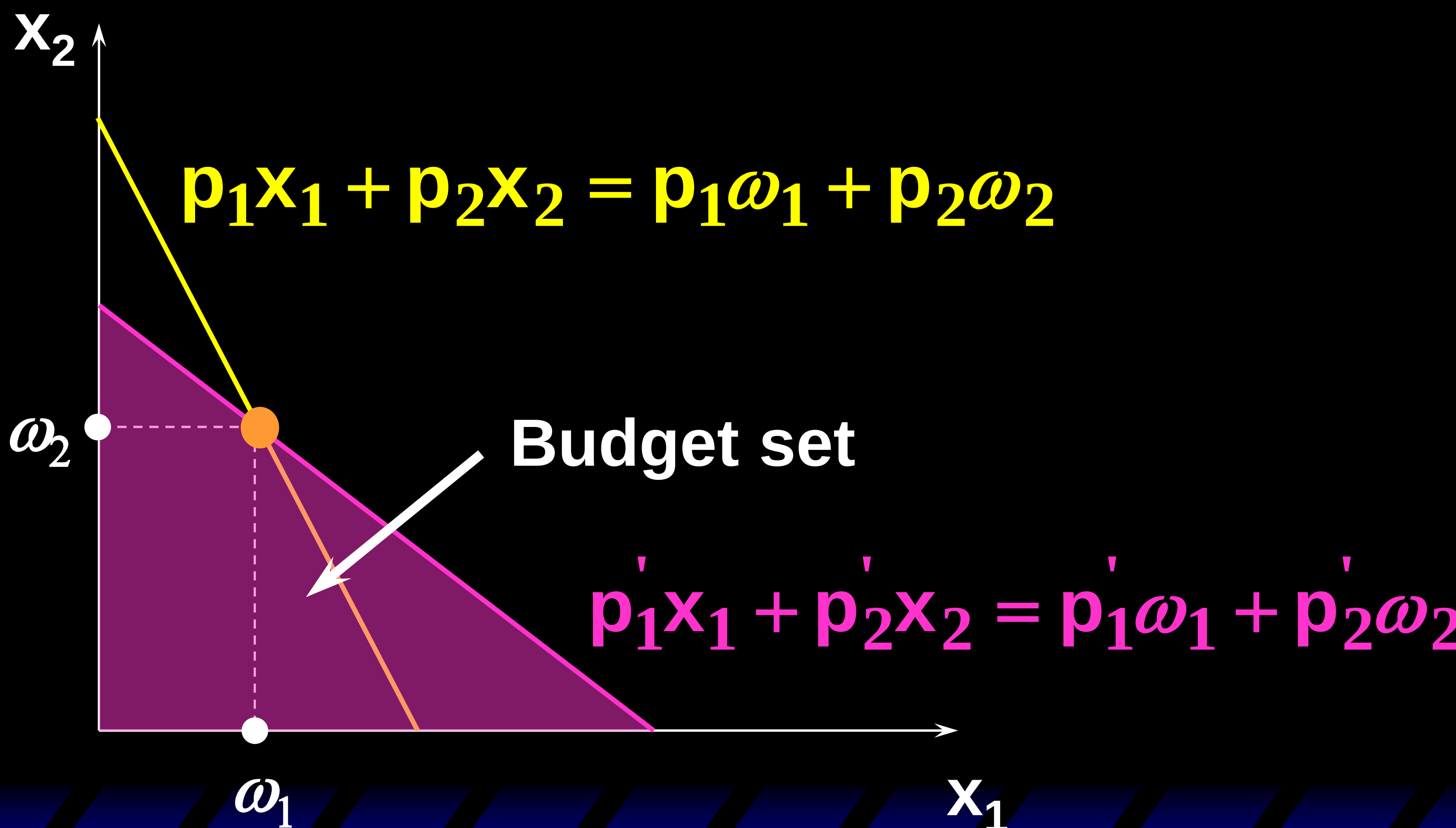
$$p_1'x_1 + p_2'x_2 = p_1'\omega_1 + p_2'\omega_2$$

# Budget Constraints Revisited




禀赋点总是位于预算约束线上

# Budget Constraints Revisited




禀赋点总是位于预算约束线上

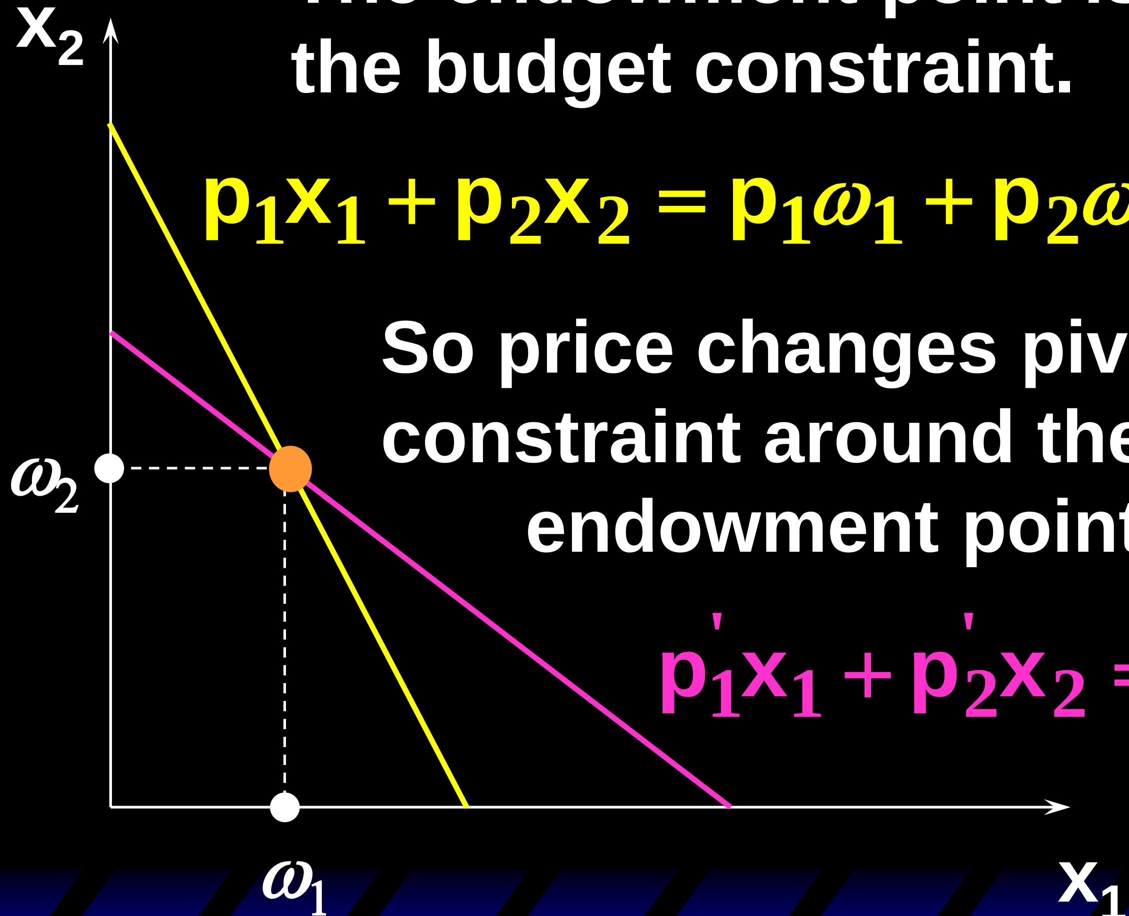
# Budget Constraints Revisited

The endowment point is always on the budget constraint.

$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2$$

So price changes pivot the constraint around the endowment point.

$$p_1'x_1 + p_2'x_2 = p_1'\omega_1 + p_2'\omega_2$$




价格的变化使得预算线围绕禀赋点旋转

# Budget Constraints Revisited

The constraint

$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2$$

is

$$p_1\underbrace{(x_1 - \omega_1)}_{\text{Net demand for x1}} + p_2\underbrace{(x_2 - \omega_2)}_{\text{Net demand for x2}} = 0.$$

The net demands for x1 and x2 can not be both positive or negative.

# Net Demands

Suppose $(\omega_1, \omega_2) = (10,2)$ and $p_1$=2, $p_2$=3. Then the constraint is

$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2 = 26.$$

If the consumer demands $(x_1^*, x_2^*)$ = (7,4), then 3 good 1 units exchange for 2 good 2 units. Net demands are

$x_1^* - \omega_1 = 7-10 = -3$ and

$x_2^* - \omega_2 = 4 - 2 = +2$.

# Net Demands

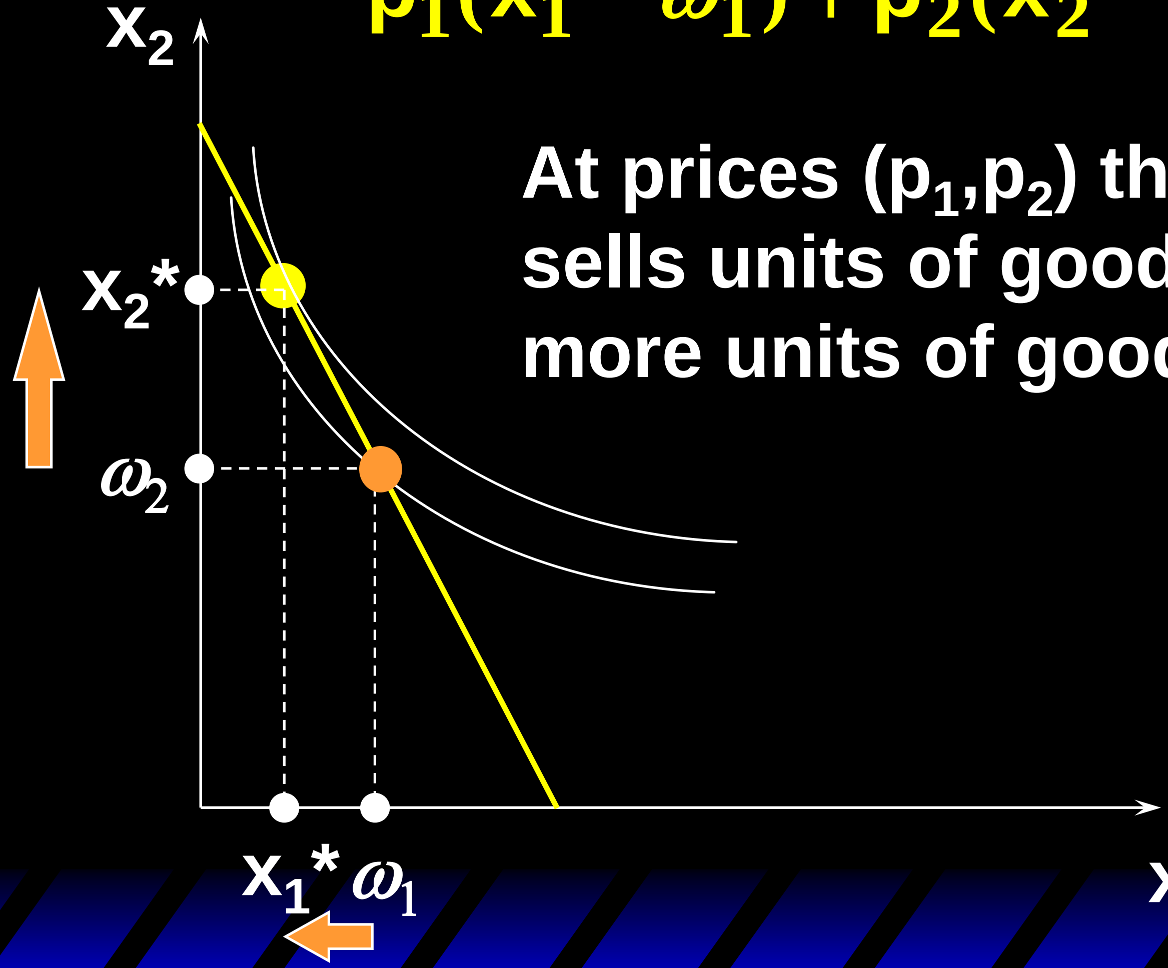
$p_1=2$, $p_2=3$, $x_1^*-\omega_1 = -3$ and $x_2^*-\omega_2 = +2$ so

$$p_1(x_1-\omega_1)+p_2(x_2-\omega_2)=$$
$$2\times(-3) \quad + \quad 3\times 2 \quad = 0.$$

The consumer is a net supplier of x1 and a net demander of x2.

# Net Demands

$$p_1(x_1 - \omega_1) + p_2(x_2 - \omega_2) = 0$$



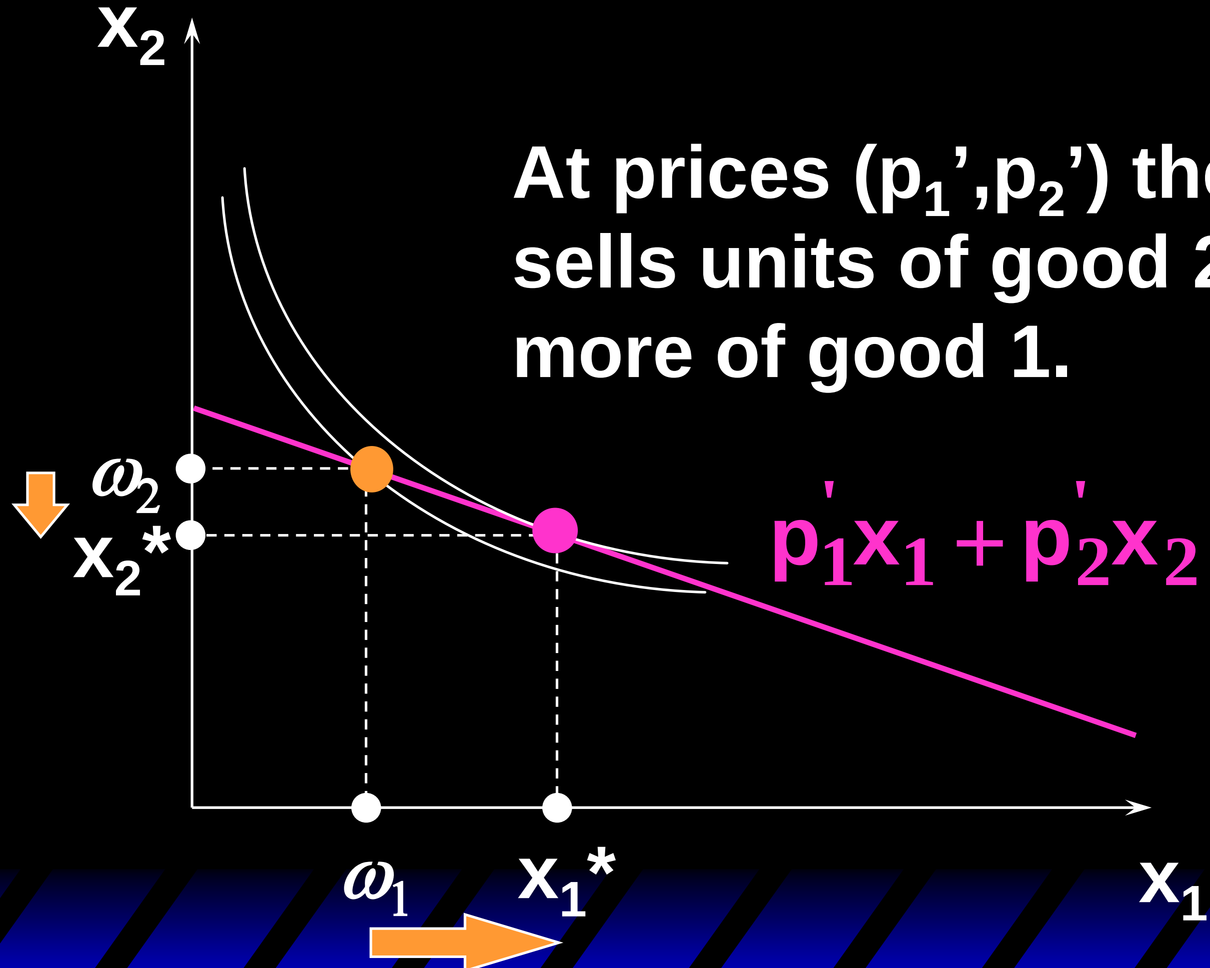

**At prices ($p_1$,$p_2$) the consumer sells units of good 1 to acquire more units of good 2.**

商品1的净供给者

# Net Demands

At prices ($p_1'$,$p_2'$) the consumer sells units of good 2 to acquire more of good 1.

$$p_1'x_1 + p_2'x_2 = p_1'\omega_1 + p_2'\omega_2$$

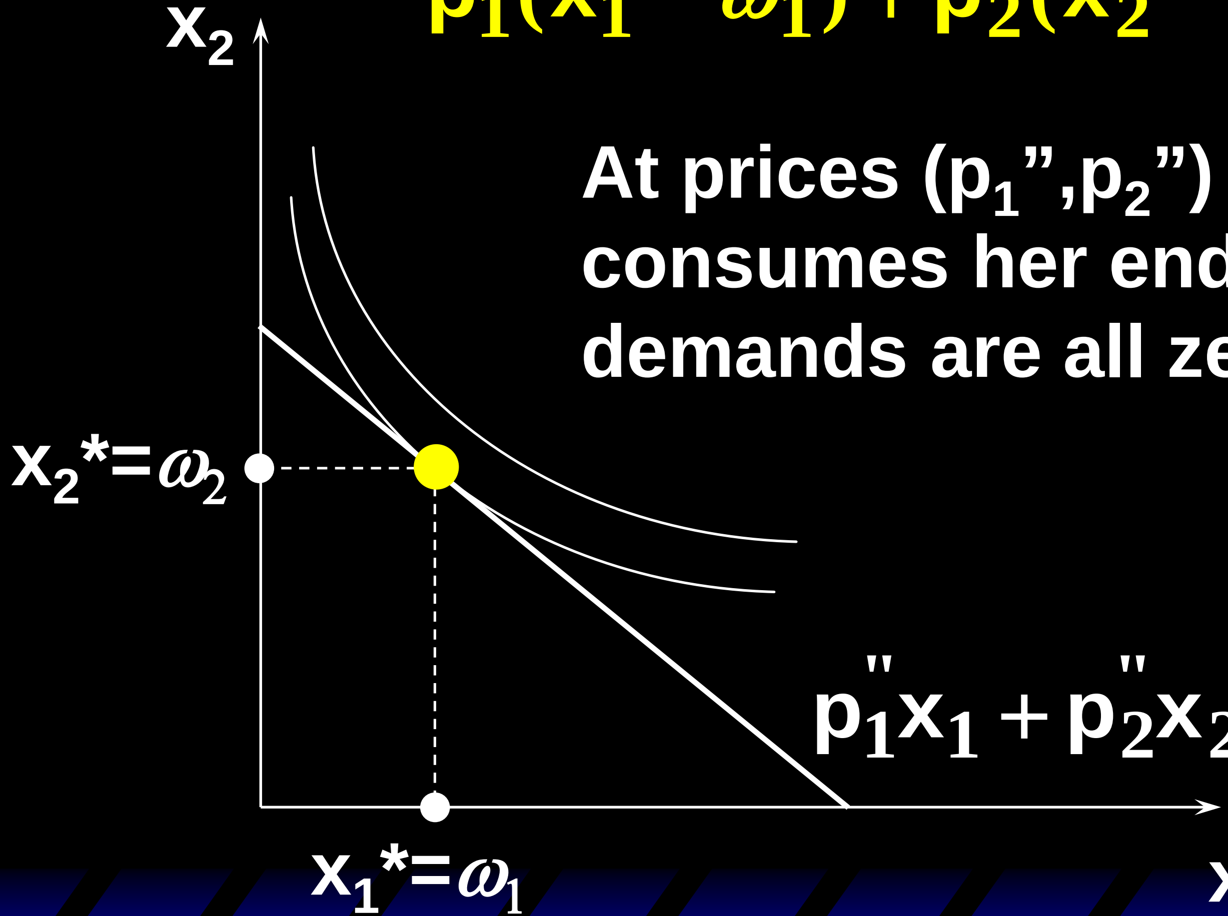



商品1的净需求者

# Net Demands

$$p_1(x_1 - \omega_1) + p_2(x_2 - \omega_2) = 0$$

At prices ($p_1$",$p_2$") the consumer consumes her endowment; net demands are all zero.




$$p_1'' x_1 + p_2'' x_2 = p_1'' \omega_1 + p_2'' \omega_2$$

禀赋点和最优消费点重合

# Labor Supply

**A worker is endowed with $m of nonlabor income and $\bar{R}$ hours of time which can be used for labor or leisure.** $\omega = (\bar{R}, m)$**.**

**Consumption good's price is $p_c$.**

**w is the wage rate.**

# Labor Supply

**The consumer's utility depends on both consumption (C) and leisure (R)**

$$U = U(C, R)$$

效用由消费数量和闲暇时间共同决定

# Labor Supply

**The consumer's utility depends on both consumption (C) and leisure (R)**

$$U = U(C, R)$$

**Q: How much time will be devoted to work in order to max utility?**

# Labor Supply

The worker's budget constraint is

$$p_c C = w(\bar{R} - R) + m$$

where C, R denote gross demands for the consumption good and for leisure. That is

$$\underbrace{p_c C + wR}_{\text{expenditure}} = \underbrace{w\bar{R} + m}_{\text{endowment value}}$$

# Labor Supply

$$\underbrace{p_c C + wR}_{\text{expenditure}} = \underbrace{w\overline{R} + m}_{\text{endowment value}}$$

# Labor Supply

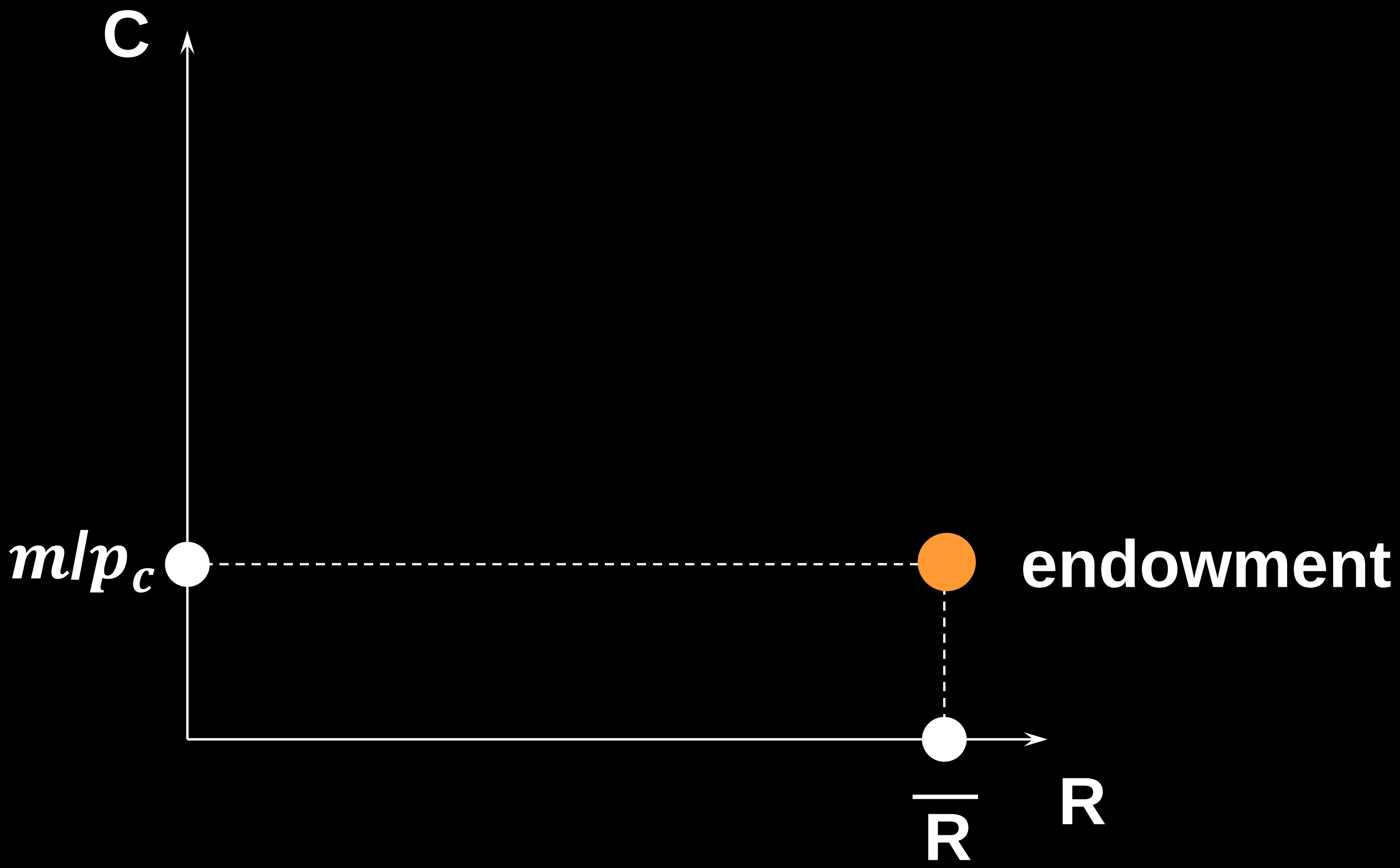
$$\underbrace{p_c C + wR}_{\text{expenditure}} = \underbrace{w\bar{R} + m}_{\text{endowment value}}$$

This is equivalent to

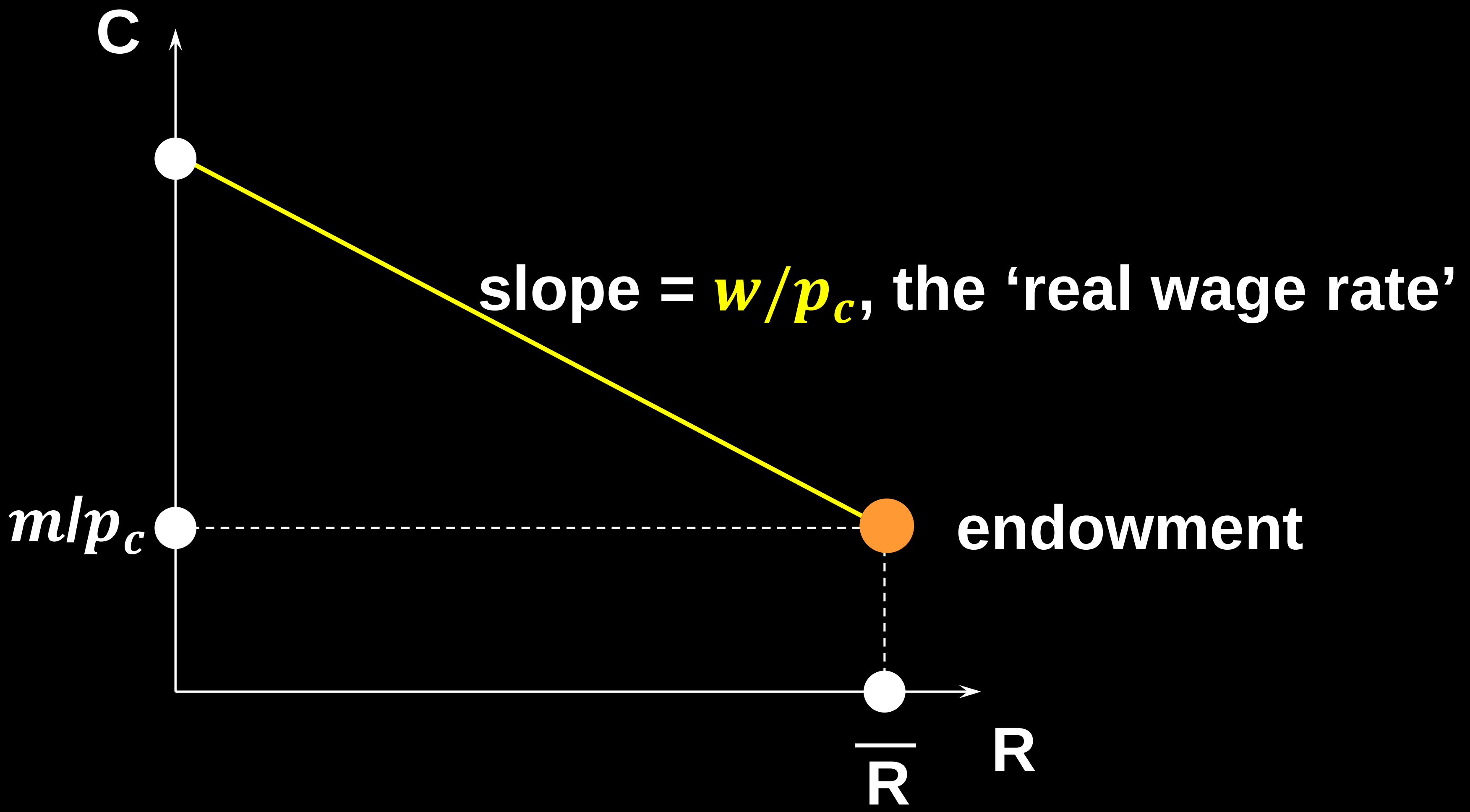
$$wR + p_c C = w\bar{R} + p_c \bar{C}$$

where $\bar{C} = \dfrac{m}{p_c}$

# Labor Supply



# Labor Supply



# Labor Supply

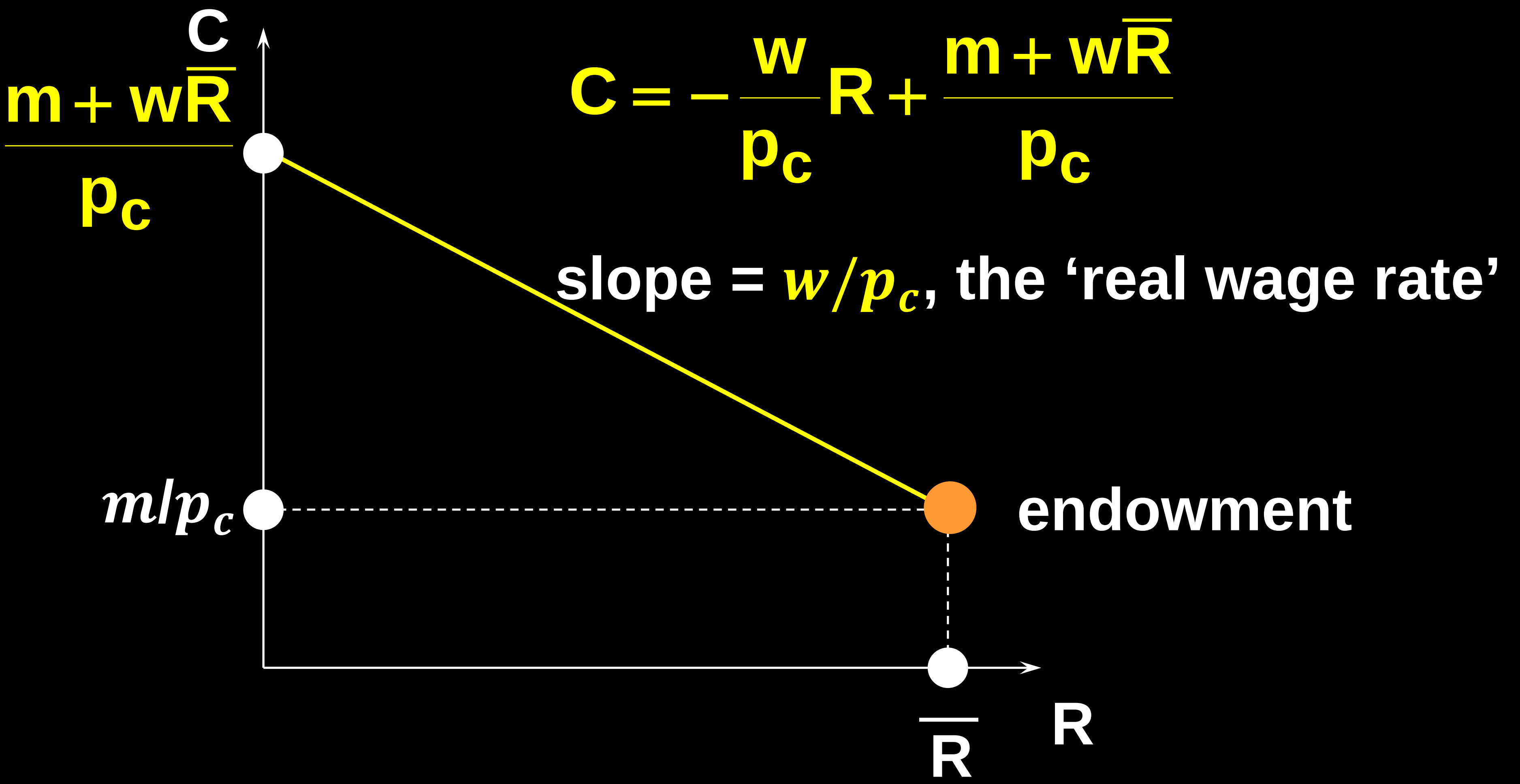
$$p_c C = w(\overline{R} - R) + m$$

rearranges to

$$C = -\frac{w}{p_c} R + \frac{m + w\overline{R}}{p_c}.$$

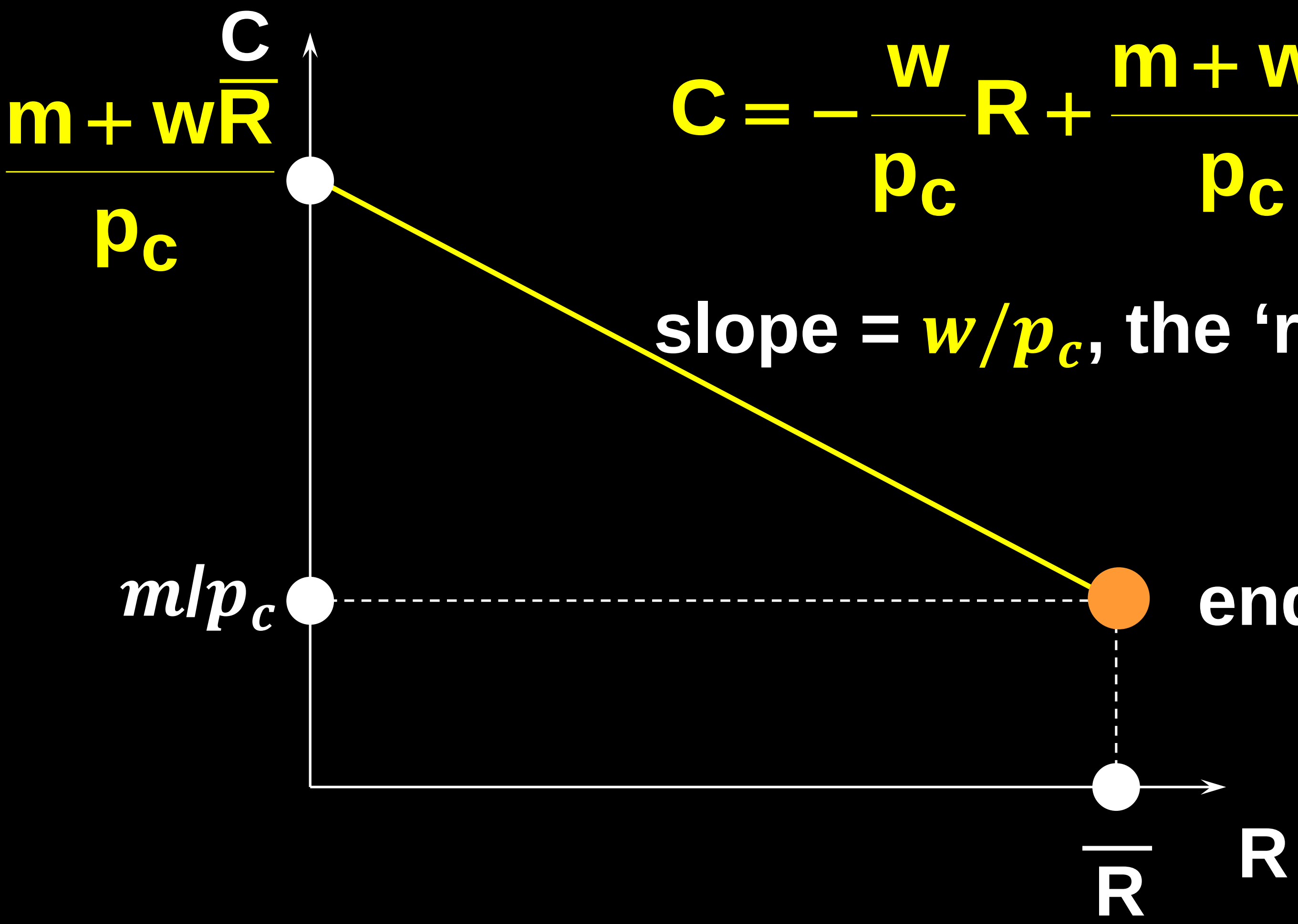
# Labor Supply

$$C = -\frac{w}{p_c}R + \frac{m + w\overline{R}}{p_c}$$



# Labor Supply

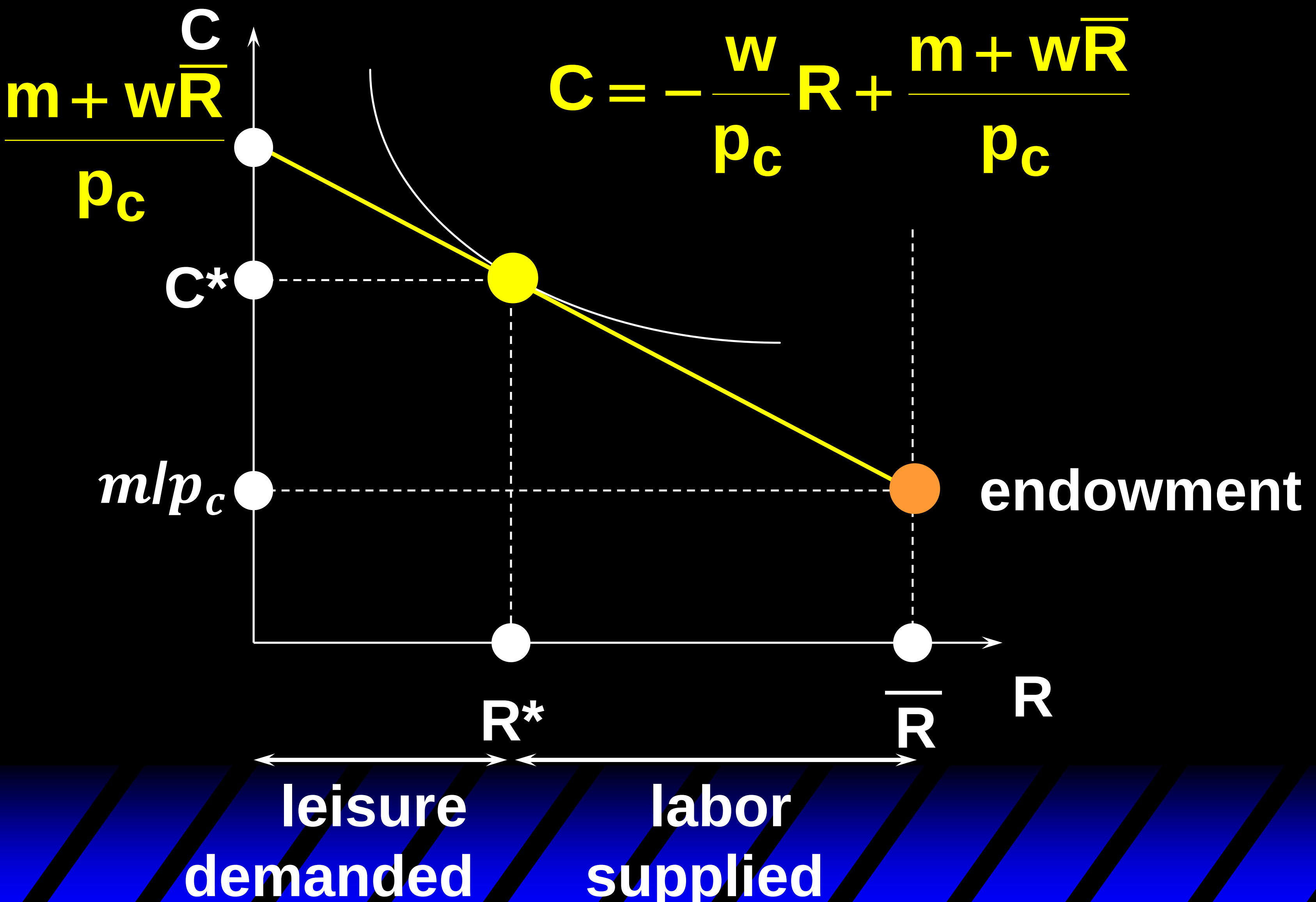
$$C = -\frac{w}{p_c}R + \frac{m + w\bar{R}}{p_c}$$



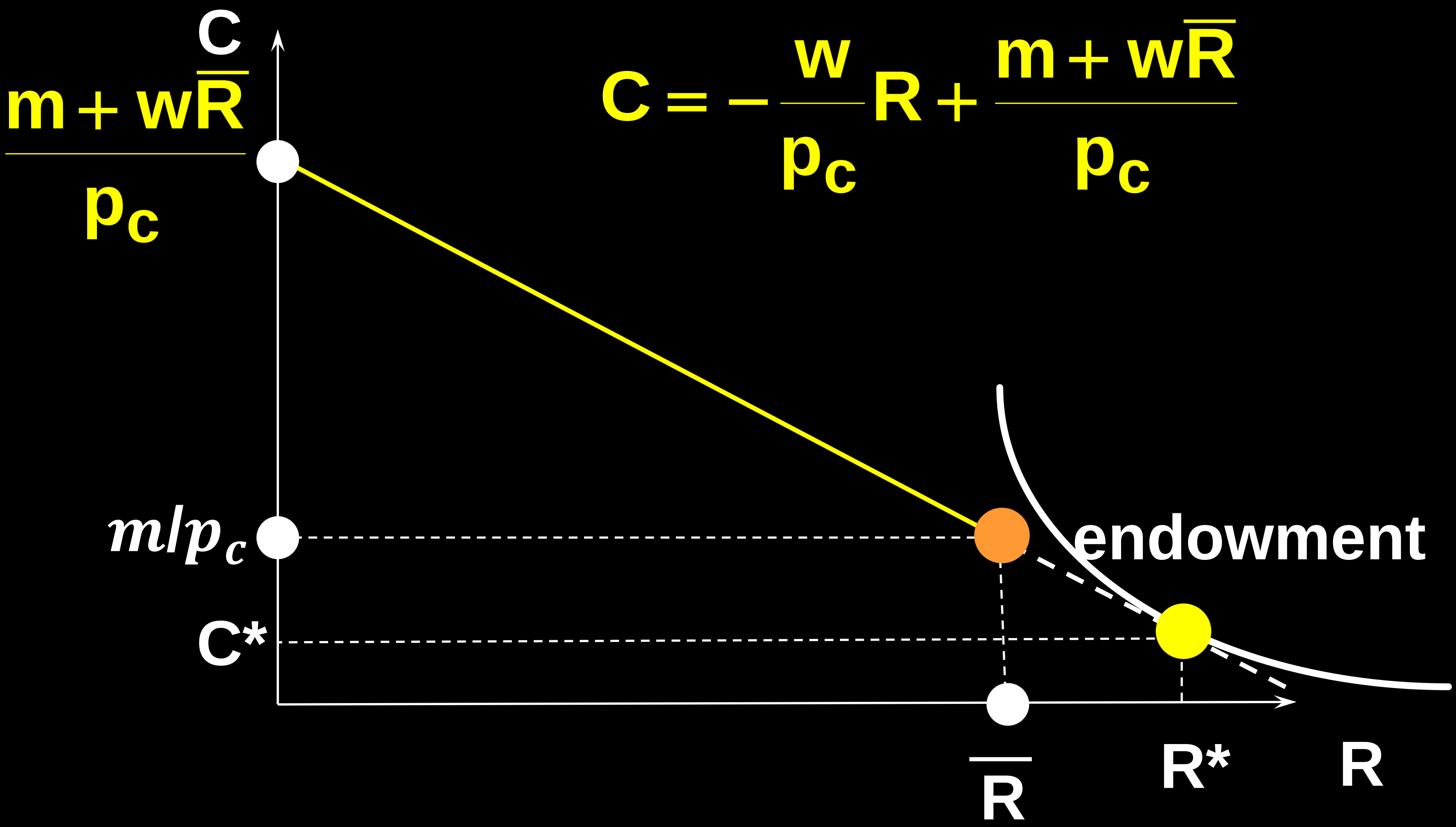

slope = $w/p_c$, the ‘real wage rate’

Note: $R \le \bar{R}$. 预算线是禀赋点以左的那部分线段。

# Labor Supply

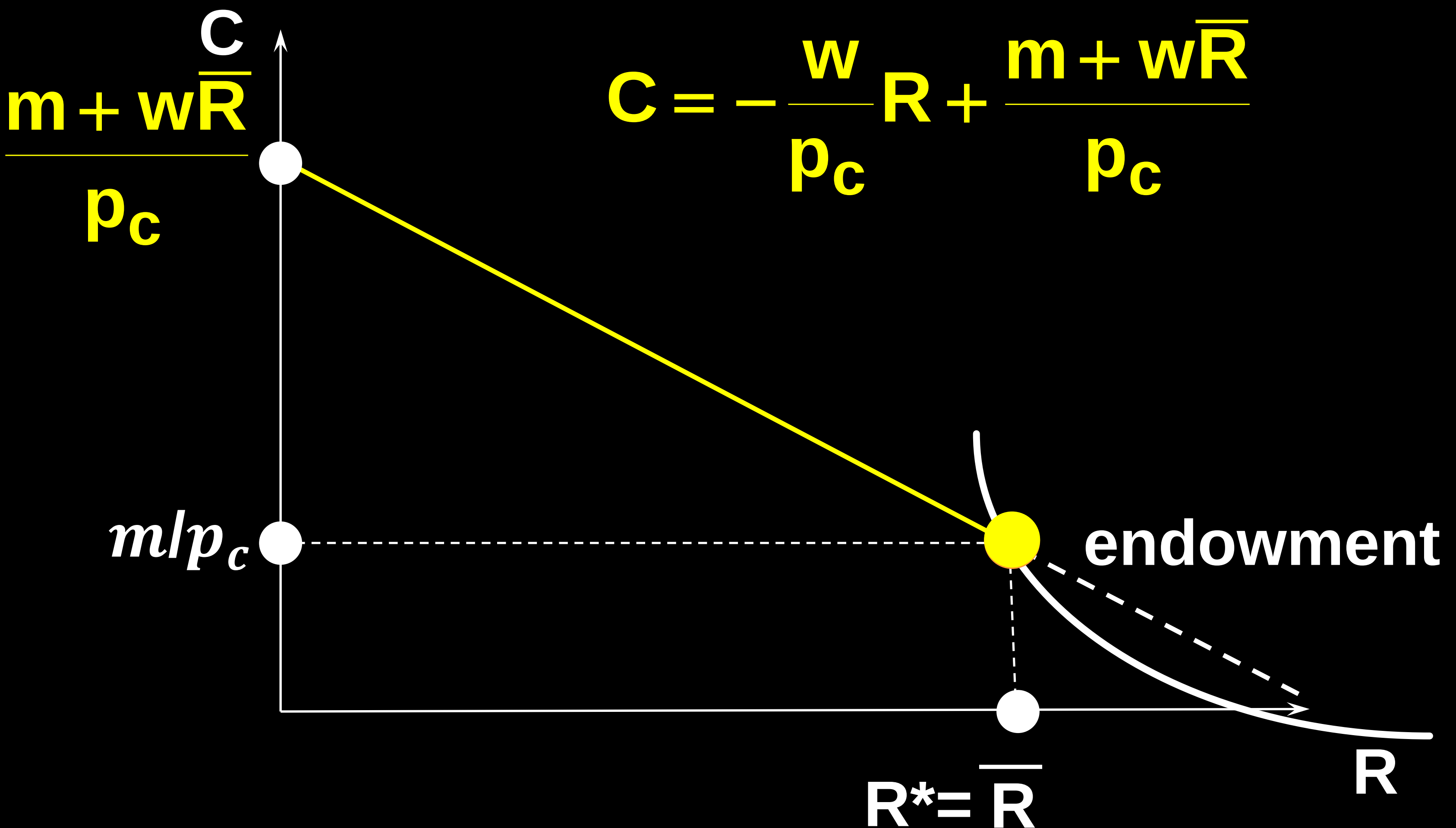


# Labor Supply




What if $R^* > \bar{R}$?

# Labor Supply




**What if $R^* > \overline{R}$?**

**- Set $R^* = \overline{R}$**

# Slutsky's Equation Revisited

Slutsky: changes to demands caused by a price change are the sum of

- a pure substitution effect, and
- an income effect.

This assumed that income y did not change as prices changed. But

$$y = p_1\omega_1 + p_2\omega_2$$

does change with price. How does this modify Slutsky's equation?

# Slutsky's Equation Revisited

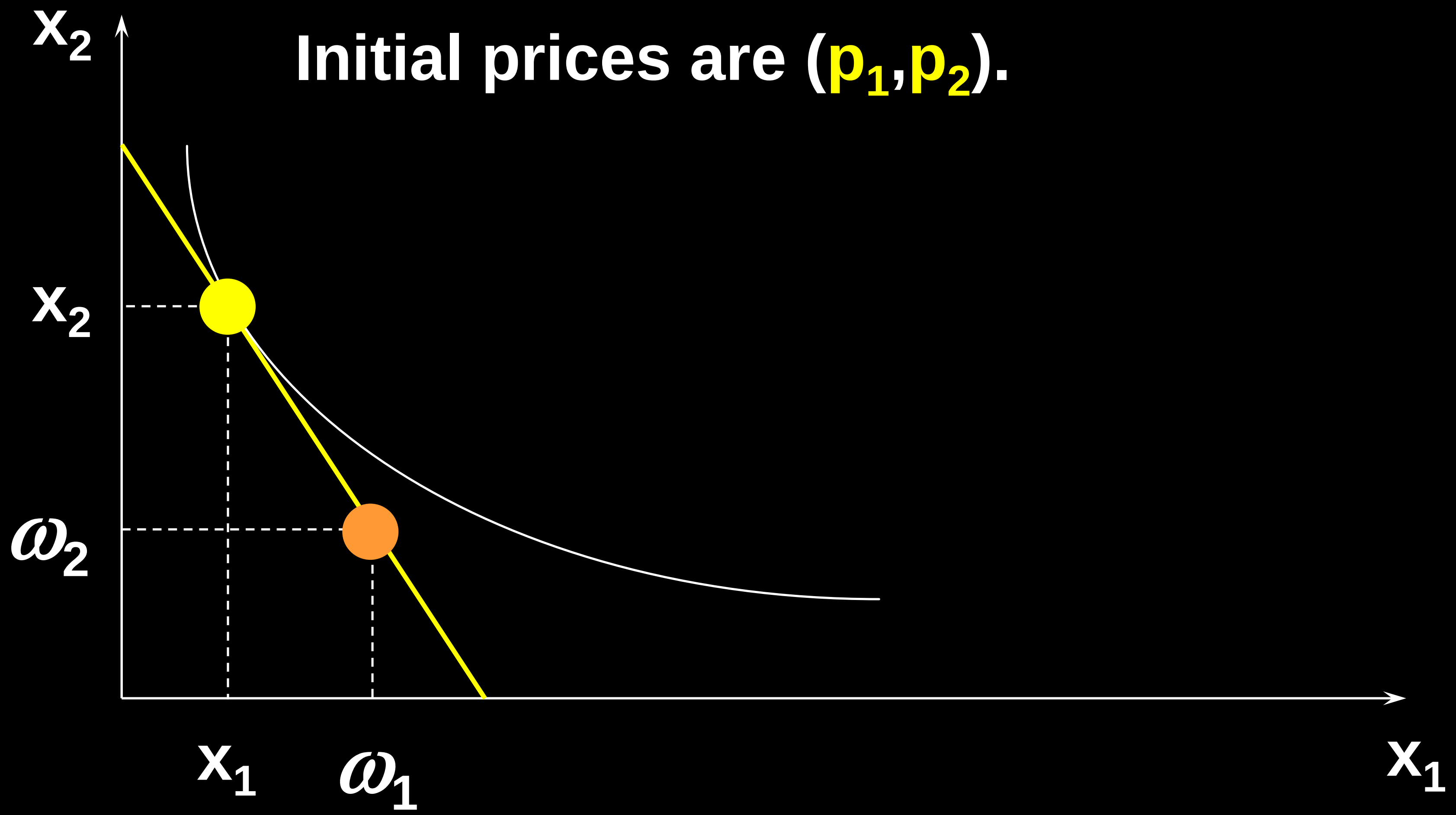
A change in $p_1$ or $p_2$ changes $y = p_1\omega_1 + p_2\omega_2$ so there will be an additional income effect, called the **endowment income effect**.

Slutsky's decomposition will thus have three components

- a pure substitution effect
- an (ordinary) income effect, and
- an endowment income effect.

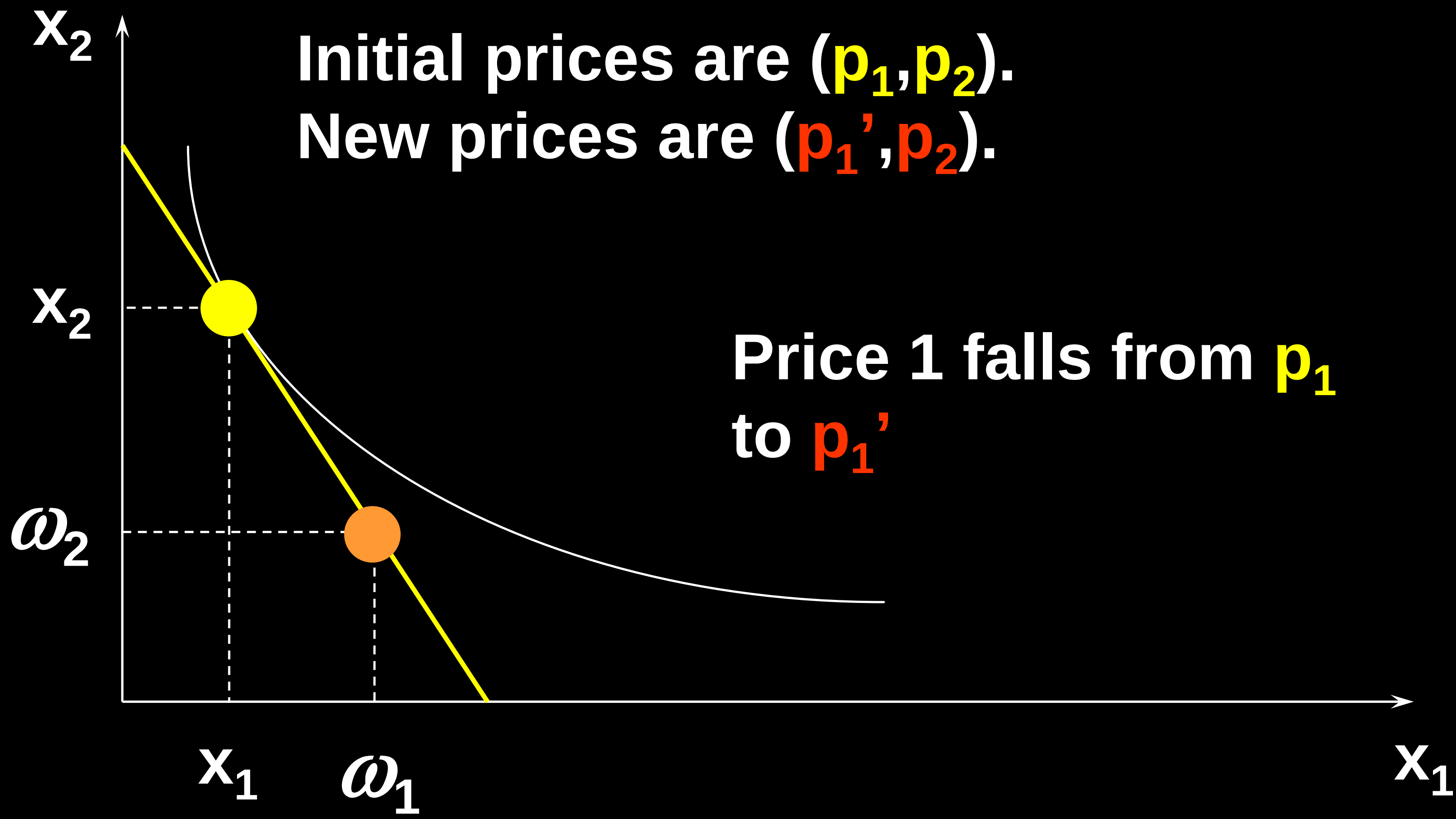
价格变化会造成禀赋收入的变化，进而造成需求的改变；这一效应被称为**禀赋收入效应**

# Slutsky's Equation Revisited



# Slutsky's Equation Revisited

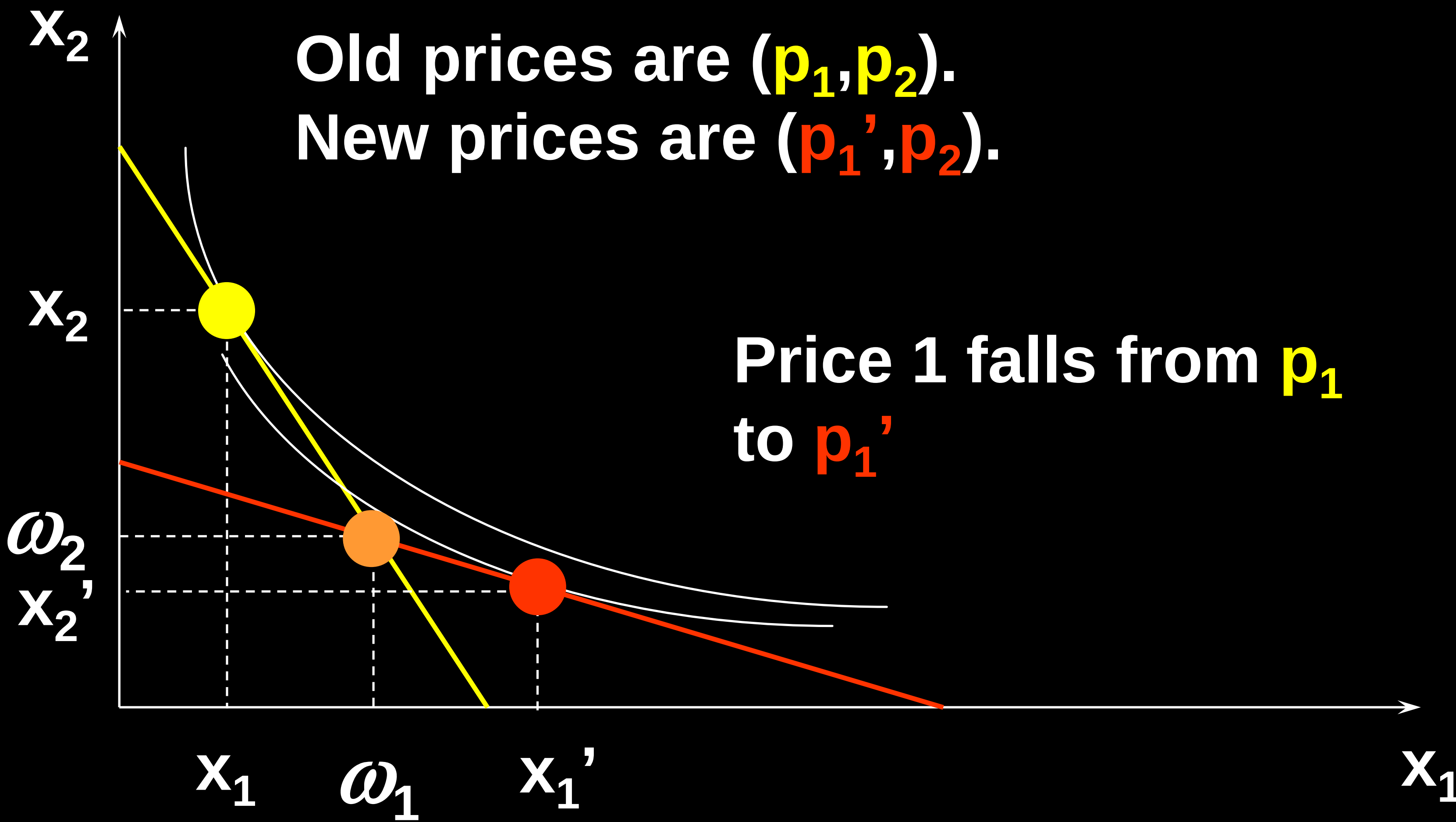
Initial prices are $(p_1,p_2)$.
New prices are $(p_1',p_2)$.




Price 1 falls from $p_1$ to $p_1'$

# Slutsky's Equation Revisited

**Old prices are ($p_1$,$p_2$).**
**New prices are ($p_1$',$p_2$).**

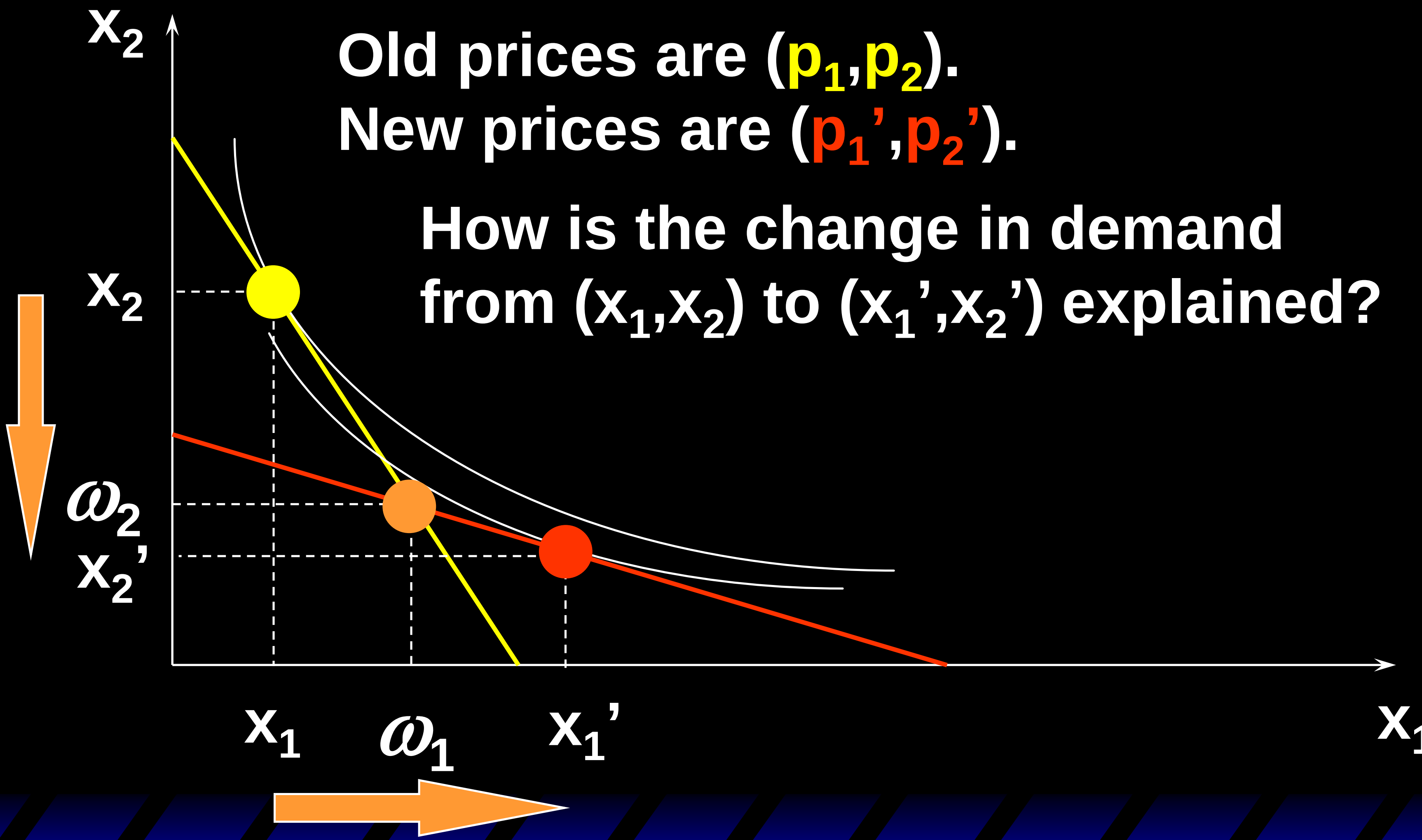



**Price 1 falls from $p_1$ to $p_1$'**

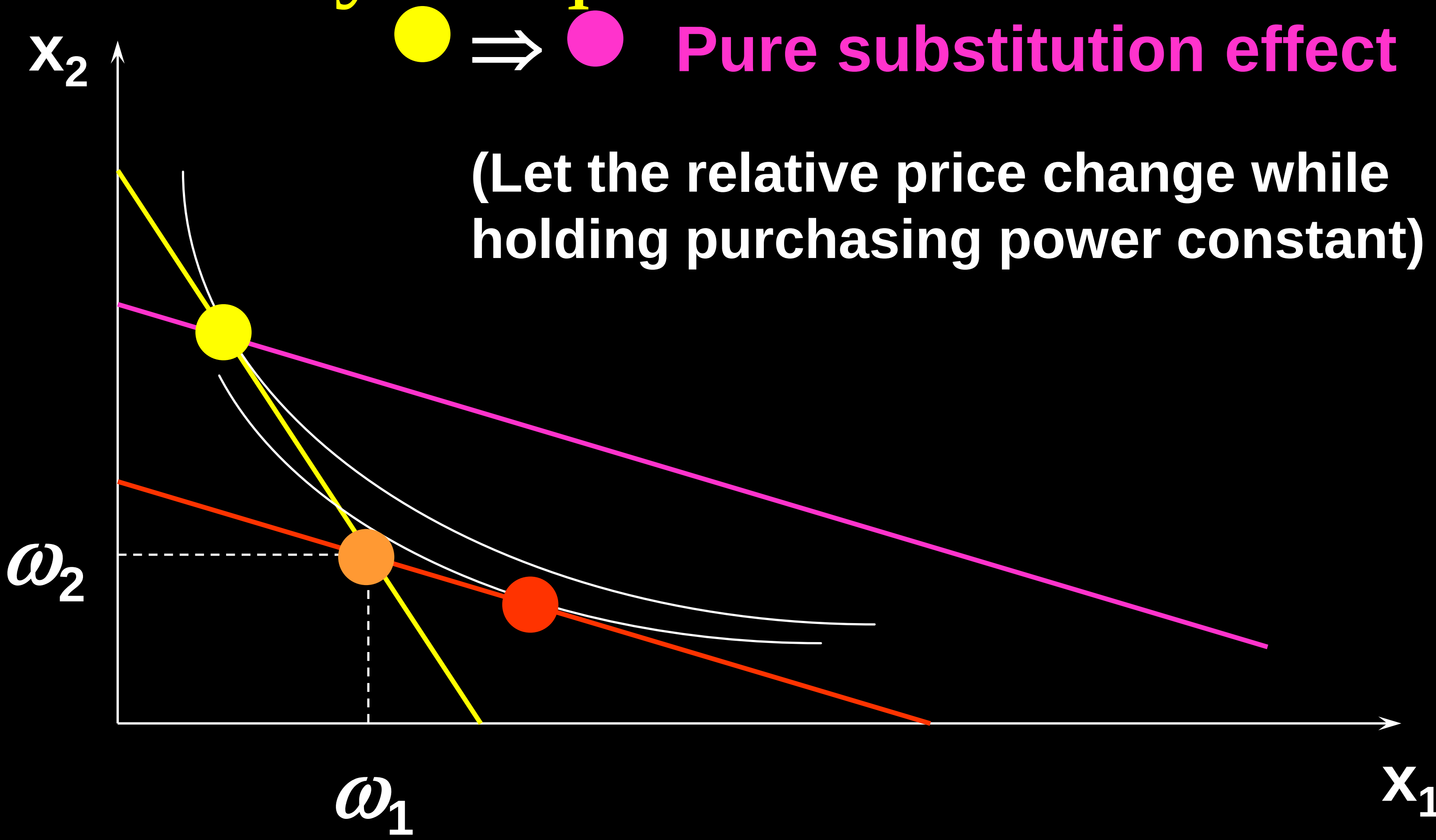
# Slutsky's Equation Revisited

Old prices are $(p_1,p_2)$.
New prices are $(p_1',p_2')$.

How is the change in demand from $(x_1,x_2)$ to $(x_1',x_2')$ explained?

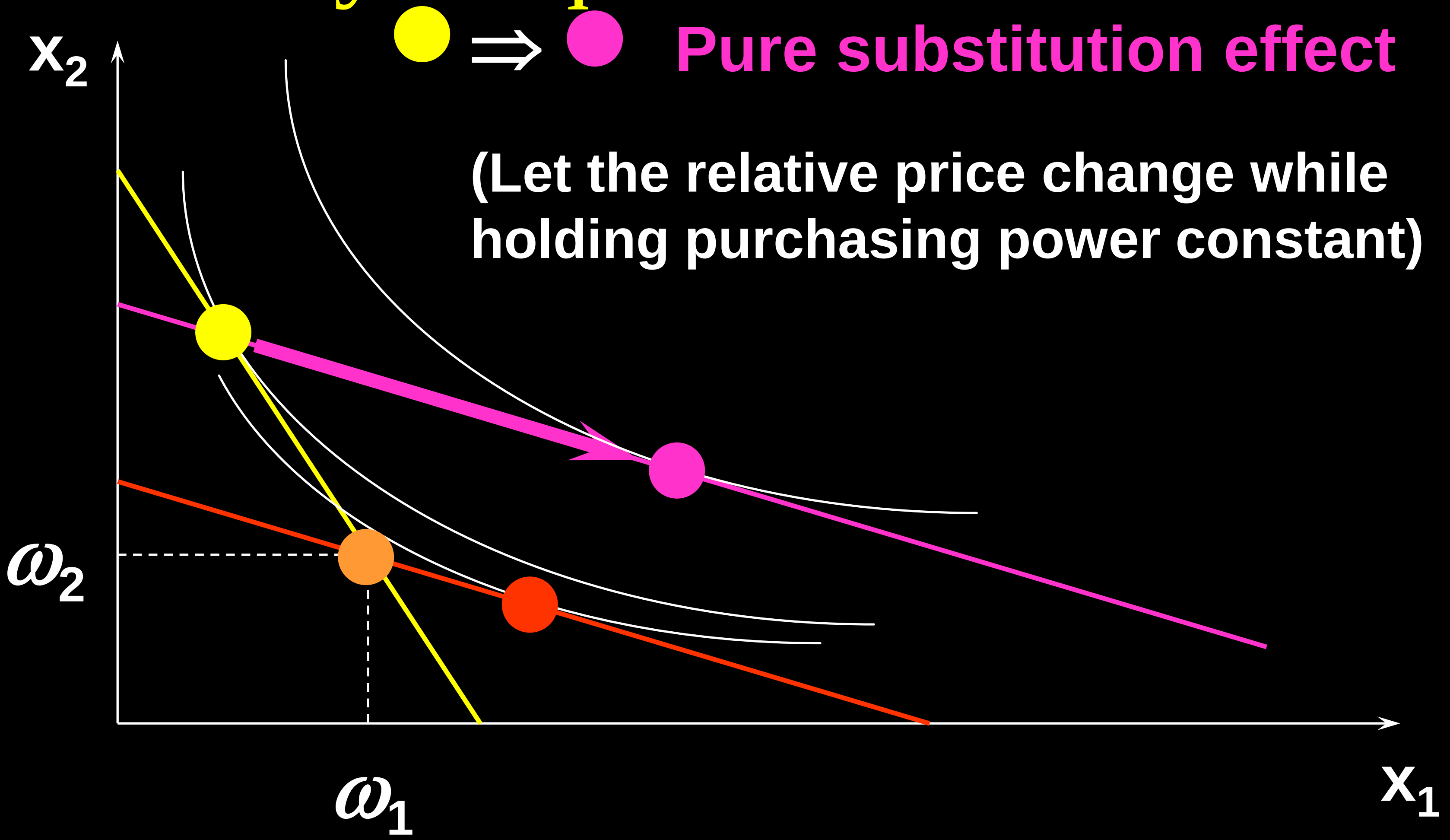


# Slutsky's Equation Revisited

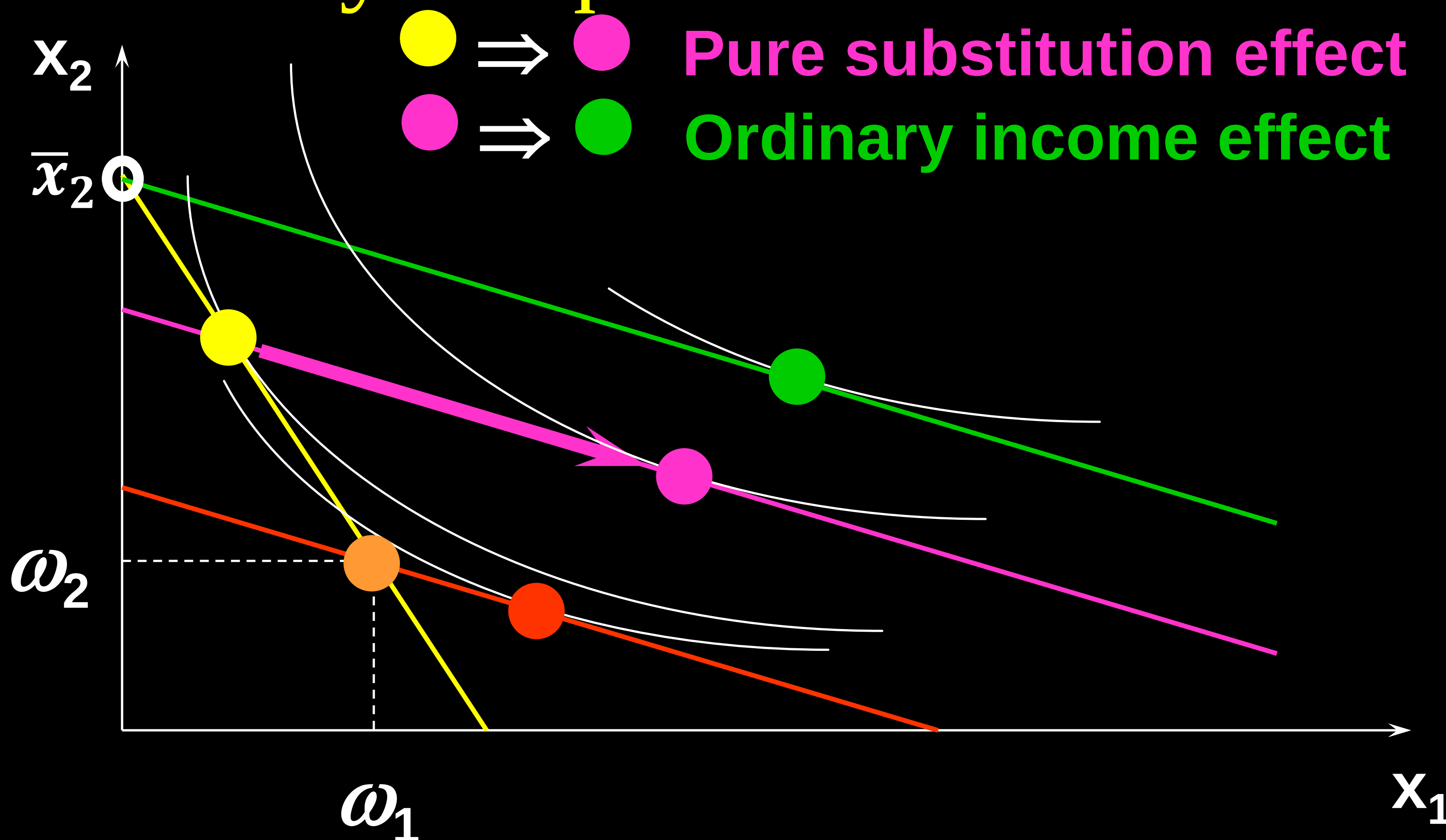



**(Let the relative price change while holding purchasing power constant)**

# Slutsky's Equation Revisited

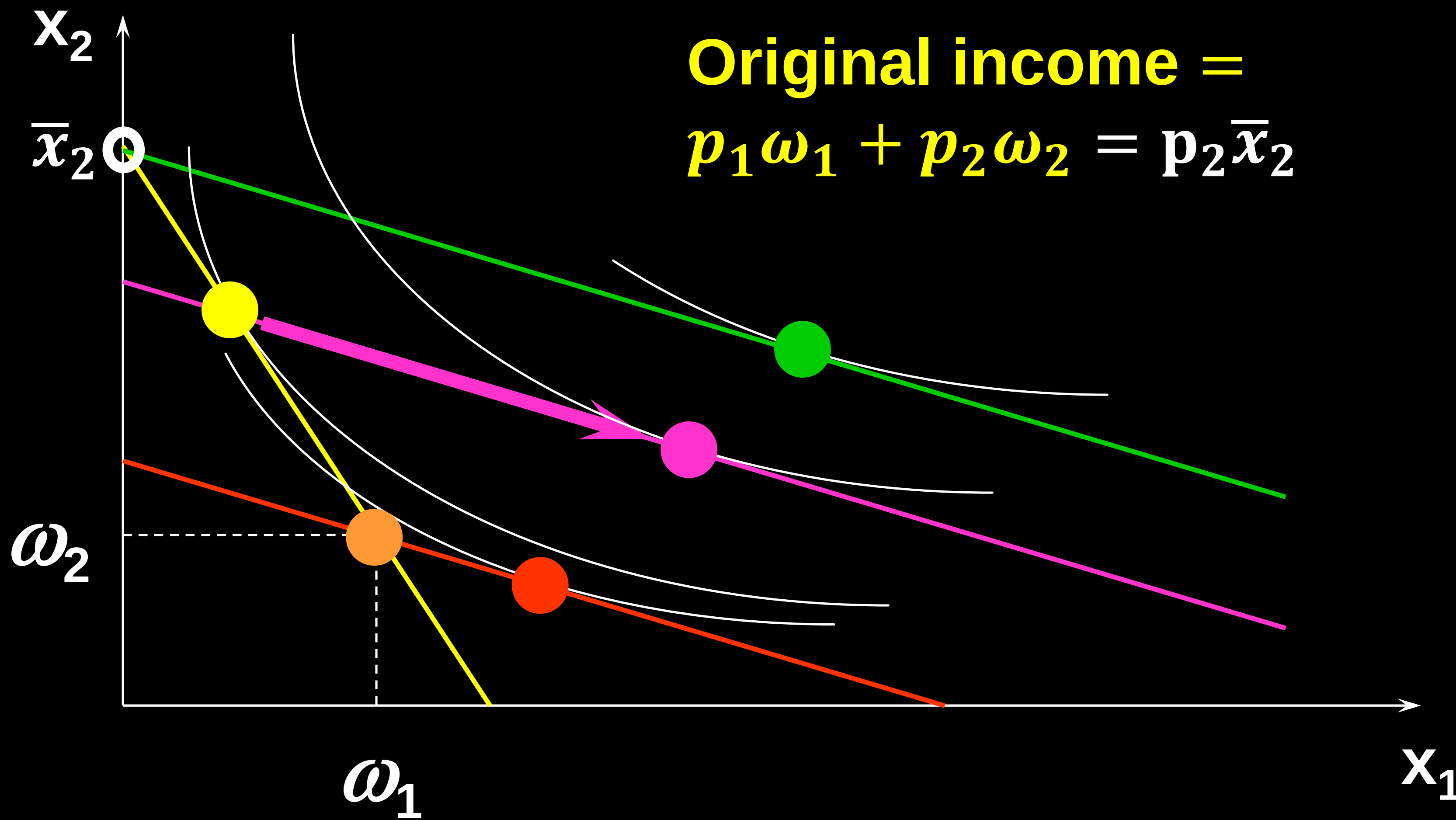


# Slutsky's Equation Revisited




**(change the income to the original income while holding prices constant)**

# Slutsky's Equation Revisited

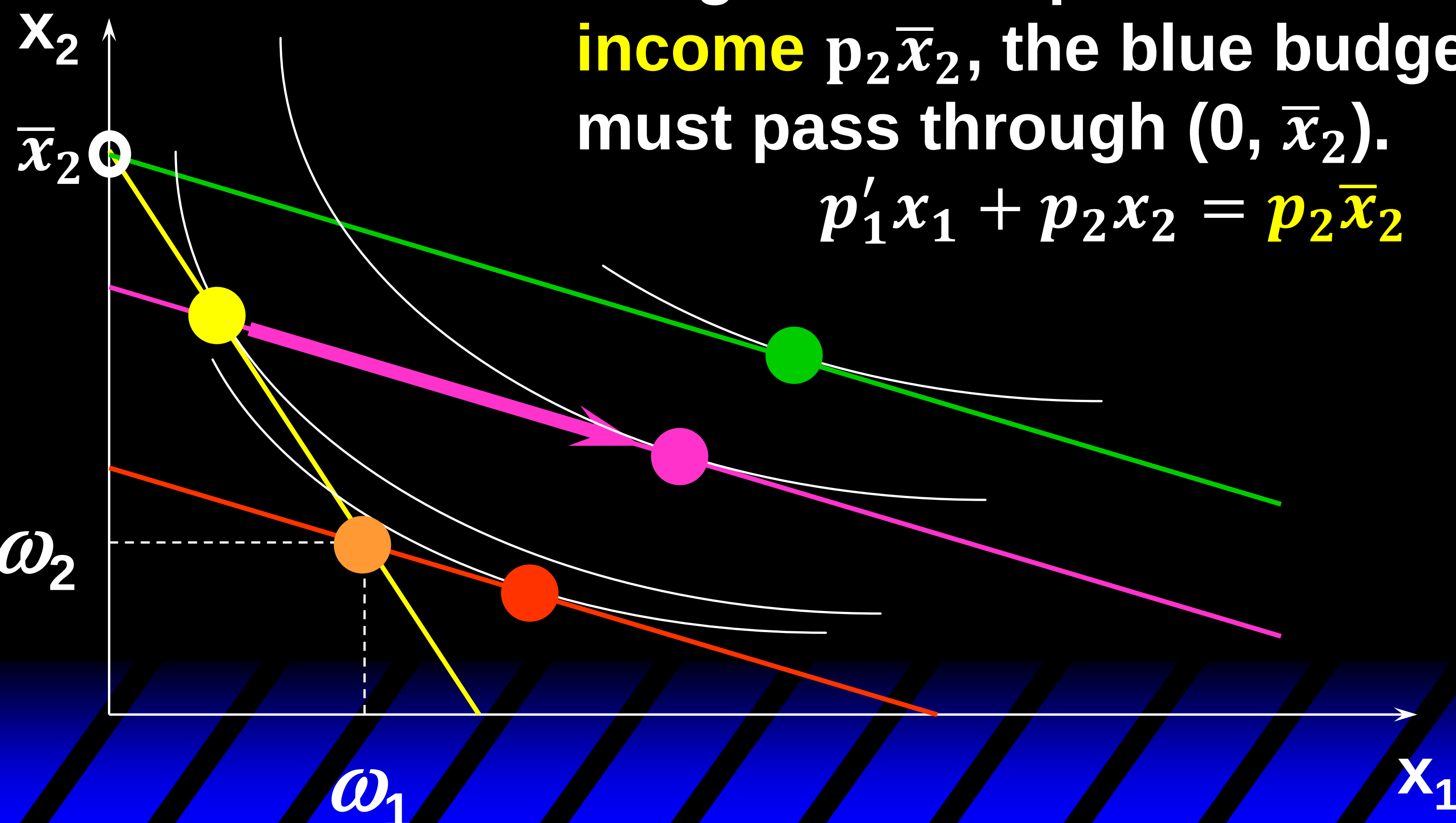



**Original income =**
$p_1\omega_1 + p_2\omega_2 = \mathbf{p}_2\overline{x}_2$

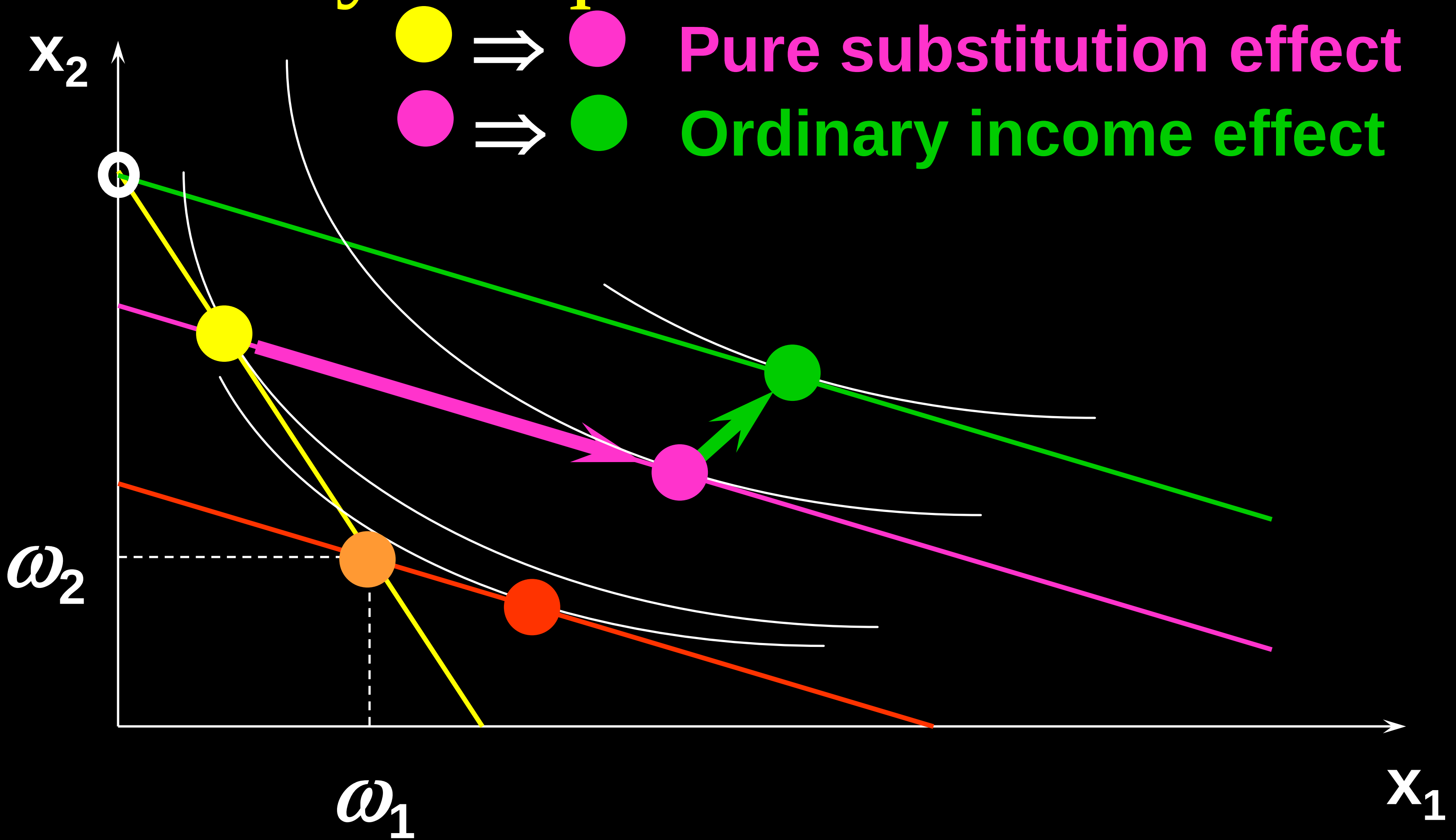
# Slutsky's Equation Revisited

**In order for the income at the blue budget to be equal to the original income $p_2\bar{x}_2$, the blue budget line must pass through $(0, \bar{x}_2)$.**

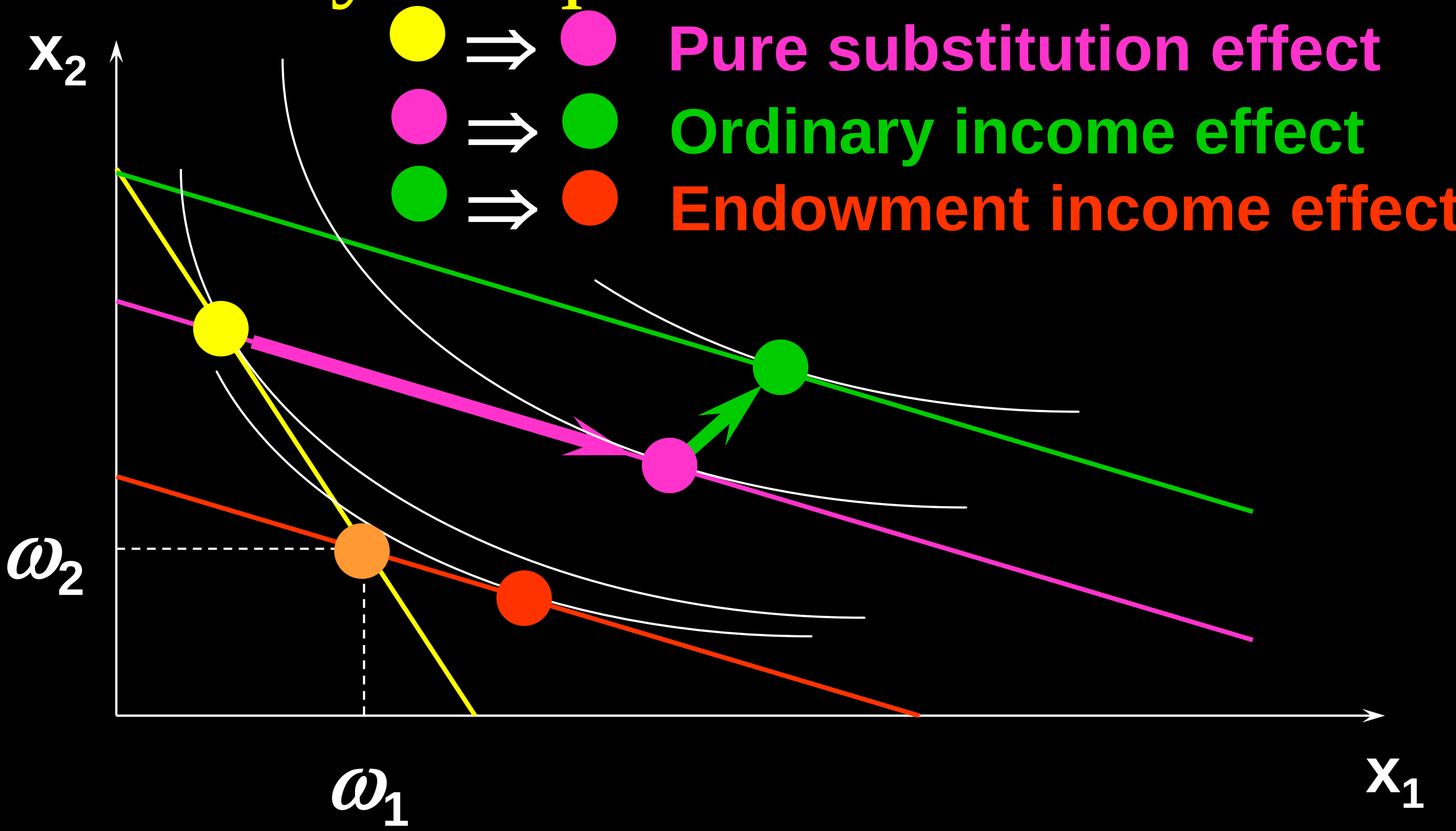
$$p_1' x_1 + p_2 x_2 = p_2 \bar{x}_2$$



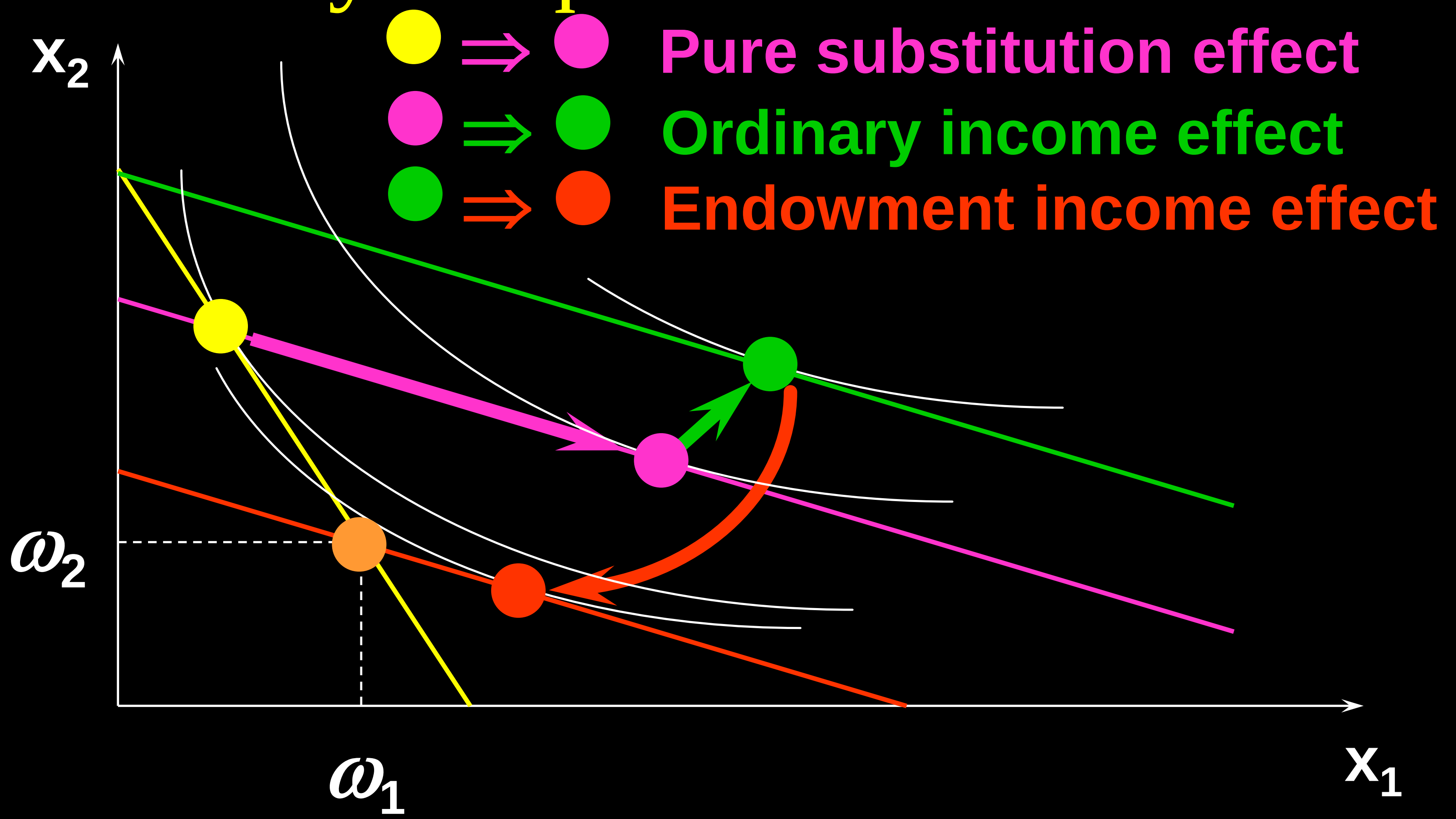
# Slutsky's Equation Revisited



# Slutsky's Equation Revisited



# Slutsky's Equation Revisited




(change the income to the **new income** while holding prices constant)

# Slutsky’s Equation Revisited

**Overall change in demand caused by a change in price is the sum of:**

**(i) a pure substitution effect**

**(ii) an ordinary income effect**

**(iii) an endowment income effect**

# Slutsky's Equation Revisited

**Step 0: derive the demand function** $x(p_1, p_2, m)$

# Slutsky's Equation Revisited

Step 0: derive the demand function $x(p_1, p_2, m)$

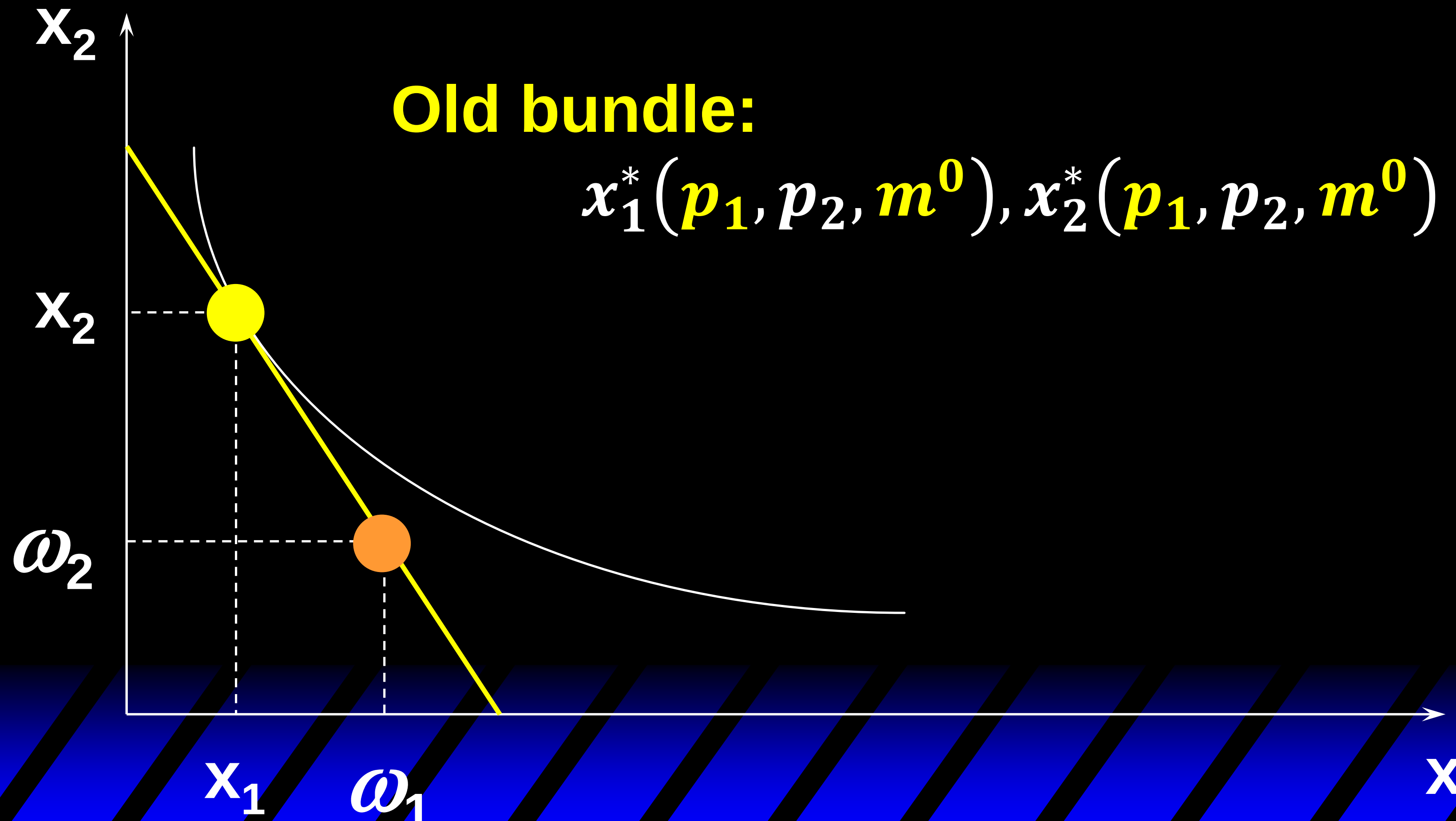
Step 1: find the old bundle, the new bundle, the intermediate bundle 1, and the intermediate bundle 2.

# The old

Old prices:($p_1$,$p_2$), old income: $m^0 = p_1\omega_1 + p_2\omega_2$

Old bundle:

$$x_1^*(p_1, p_2, m^0), x_2^*(p_1, p_2, m^0)$$

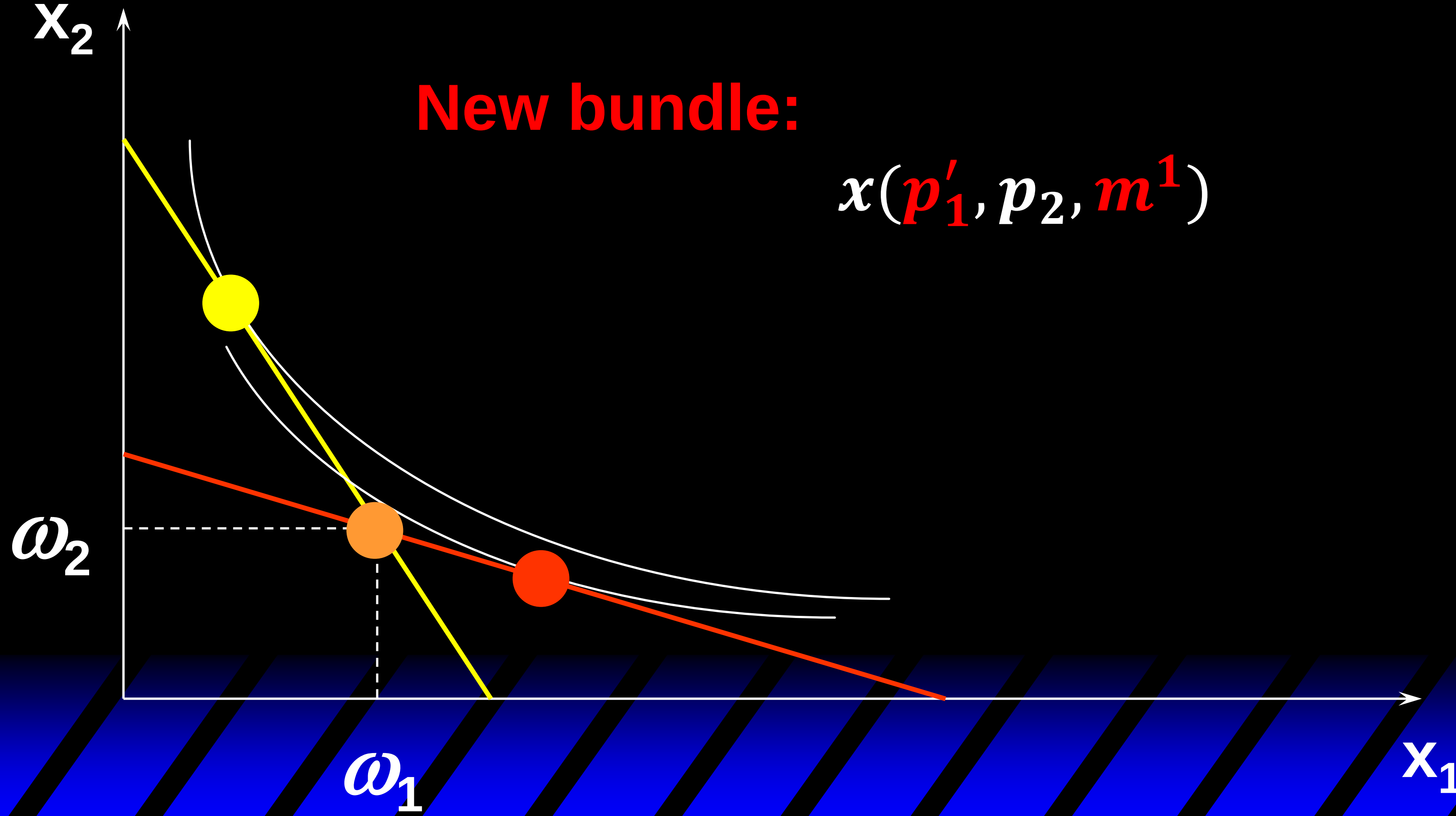


# The new

new prices: $(p_1', p_2)$, new income: $m^1 = p_1'\omega_1 + p_2\omega_2$

New bundle:

$$x(p_1', p_2, m^1)$$

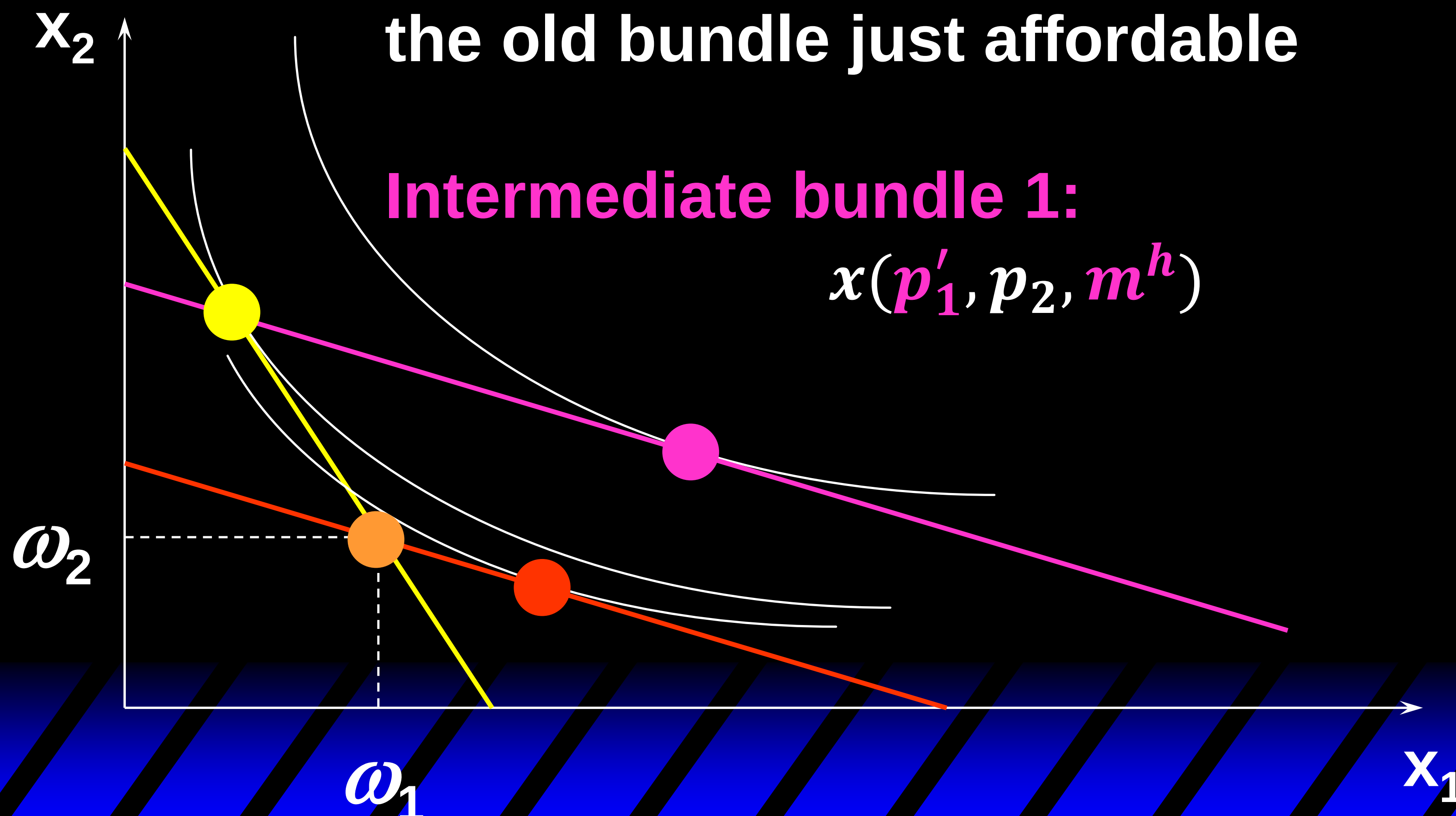


# Intermediate 1

**new prices:** $(p_1', p_2)$, **hypothetical income:** $m^h = p_1' x_1^* + p_2 x_2^*$, **which makes the old bundle just affordable**

**Intermediate bundle 1:**

$$x(p_1', p_2, m^h)$$

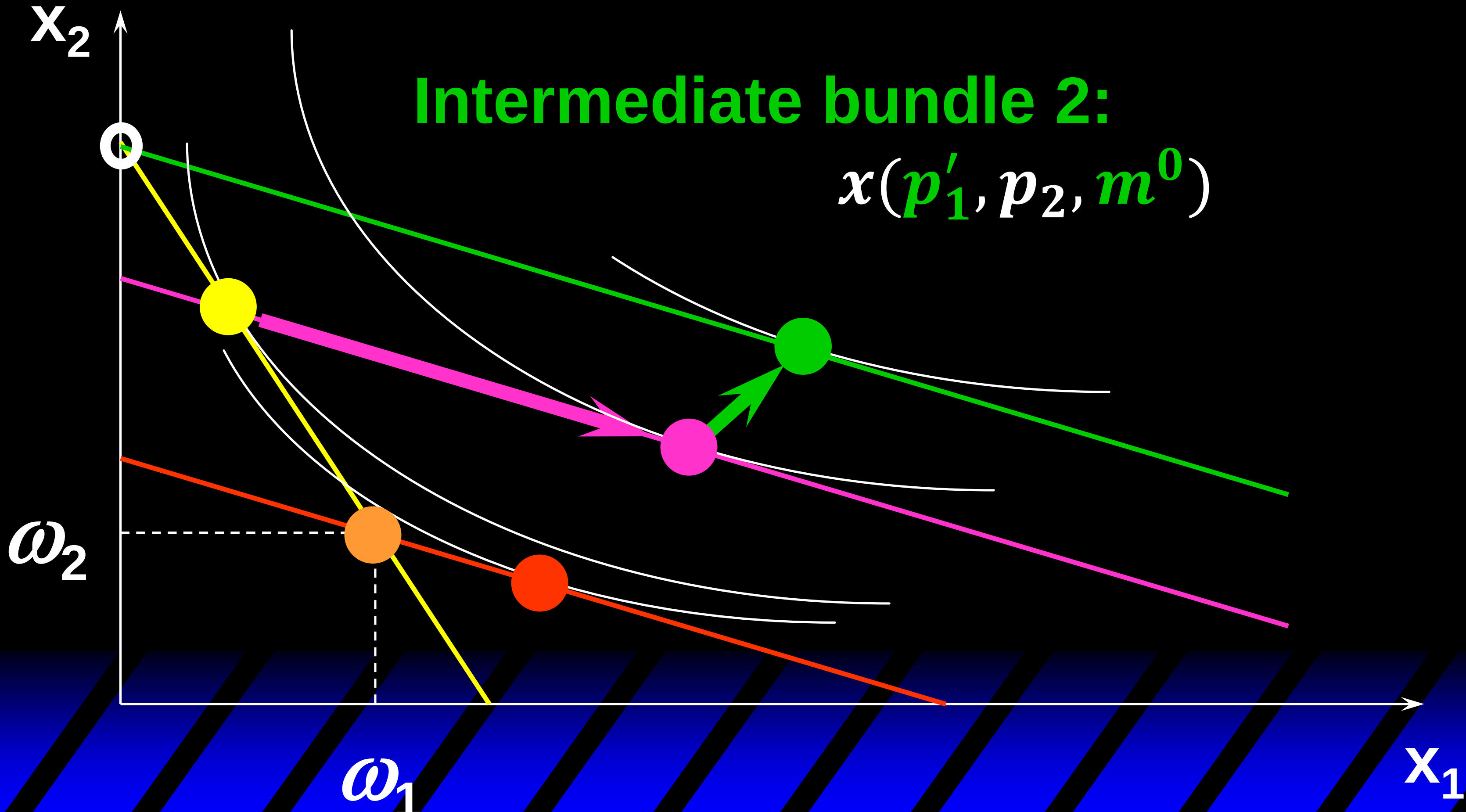


# Intermediate 2

**new prices:** $(p_1', p_2)$, **old income:** $m^0 = p_1\omega_1 + p_2\omega_2$

**Intermediate bundle 2:**

$x(p_1', p_2, m^0)$



# Slutsky's Equation Revisited

**Step 3: calculate the substitution and income effects:**

**(i) a pure substitution effect**

**(ii) an ordinary income effect**

**(iii) an endowment income effect**

# Slutsky's Equation Revisited

**Step 3: calculate the substitution and income effects:**

**(i) a pure substitution effect**

**Intermediate 1 – old**

**(ii) an ordinary income effect**

**Intermediate 2 – Intermediate 1**

**(iii) an endowment income effect**

**new – Intermediate 2**

# Rate of change version

**When income is given as fixed, the Slutsky equation is the following:**

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1^n}{\Delta p_1}$$

$$= \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{\text{due to changes in relative price}} - \underbrace{\frac{\Delta x_1(p_1, p_2, m\ )}{\Delta m} x_1}_{\text{due to changes in real income}}$$

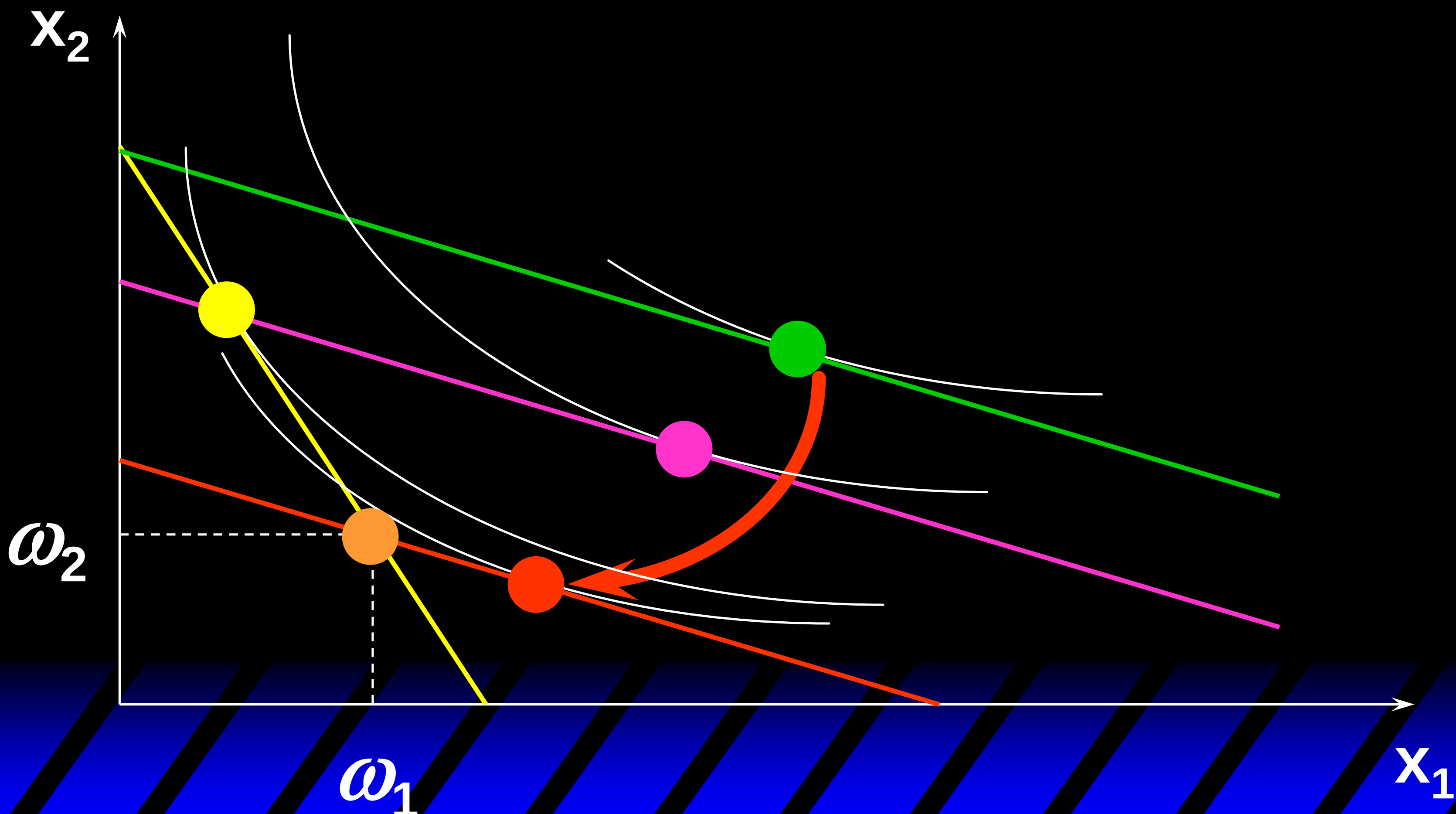
# Rate of change version

**With endowments, the price change also causes change in nominal income**

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1^n}{\Delta p_1} + \frac{\textbf{endowment effect}}{\Delta p_1}$$

$$= \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1(p_1, p_2, m)}{\Delta m} x_1 + \frac{\textbf{endowment effect}}{\Delta p_1}$$

# The Endowment Income Effect



# Rate of change version

**endowment effect**
$= \boldsymbol{x_1(p_1', p_2, m^1) - x_1(p_1', p_2, m^0)}$

# Rate of change version

**endowment effect**

$$= x_1(p_1', p_2, m^1) - x_1(p_1', p_2, m^0)$$

$$= x_1(p_1', p_2, m^0 + (m^1 - m^0)) - x_1(p_1', p_2, m^0)$$

# Rate of change version

**endowment effect**

$$= x_1(p_1', p_2, m^1) - x_1(p_1', p_2, m^0)$$

$$= x_1(p_1', p_2, m^0 + (m^1 - m^0)) - x_1(p_1', p_2, m^0)$$

**Remember that**

$$f(x + \Delta x) - f(x) \to f'(x)\Delta x$$

# Rate of change version

**endowment effect**

$$= x_1(p_1', p_2, m^1) - x_1(p_1', p_2, m^0)$$

$$= x_1(p_1', p_2, m^0 + (m^1 - m^0)) - x_1(p_1', p_2, m^0)$$

**Remember that**

$$f(x + \Delta x) - f(x) \to f'(x)\Delta x$$

**endowment effect**

$$= \frac{\Delta x_1(p_1, p_2, m)}{\Delta m}(m^1 - m^0)$$

# Rate of change version

$$\textbf{endowment effect} = \frac{\Delta x_1(p_1, p_2, m)}{\Delta m}(m^1 - m^0)$$

# Rate of change version

$$\textbf{endowment effect} = \frac{\Delta x_1(p_1, p_2, m)}{\Delta m}(m^1 - m^0)$$

$$m^1 - m^0 = p_1'\omega_1 + p_2\omega_2 - (p_1\omega_1 + p_2\omega_2)$$
$$= (p_1' - p_1)\omega_1 = \Delta \mathrm{p}_1\omega_1$$

# Rate of change version

$$\textbf{endowment effect} = \frac{\Delta x_1(p_1, p_2, m)}{\Delta m}(m^1 - m^0)$$

$$m^1 - m^0 = p_1'\omega_1 + p_2\omega_2 - (p_1\omega_1 + p_2\omega_2)$$
$$= (p_1' - p_1)\omega_1 = \Delta \mathrm{p}_1\omega_1$$

$$\textbf{endowment effect} = \frac{\Delta x_1(p_1, p_2, m)}{\Delta m}\Delta \mathrm{p}_1\omega_1$$

# Rate of change version

$$\textbf{endowment effect} = \frac{\Delta x_1(p_1, p_2, m)}{\Delta m} \Delta \mathbf{p}_1 \omega_1$$

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1(p_1, p_2, m)}{\Delta m} x_1 + \frac{\textbf{endowment effect}}{\Delta p_1}$$

$$= \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1(p_1, p_2, m)}{\Delta m} x_1 + \frac{\Delta x_1(p_1, p_2, m)}{\Delta m} \omega_1$$

$$= \frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1(p_1, p_2, m)}{\Delta m} (\omega_1 - x_1)$$

# Rate of change version

**Without endowments:**

$$\frac{\Delta x_1}{\Delta p_1} = \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{(-)} - \underbrace{\frac{\Delta x_1(p_1, p_2, m)}{\Delta m}}_{(+)\ \text{if normal}} x_1$$

# Rate of change version

**Without endowments:**

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1(p_1, p_2, m)}{\Delta m} x_1$$

**With endowments:**

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1(p_1, p_2, m)}{\Delta m} (\omega_1 - x_1)$$

# Rate of change version

**With endowments:**

$$\frac{\Delta x_1}{\Delta p_1} = \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{(-)} + \underbrace{\frac{\Delta x_1(p_1, p_2, m\ )}{\Delta m}}_{(+)\ \text{if normal}} \underbrace{(\omega_1 - x_1)}_{(?\ )}$$

# Rate of change version

**With endowments:**

$$\frac{\Delta x_1}{\Delta p_1} = \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{(-)} + \underbrace{\frac{\Delta x_1(p_1, p_2, m)}{\Delta m}}_{(+)\text{ if normal}} \underbrace{(\omega_1 - x_1)}_{(?)}$$

**If $\omega_1 - x_1 < 0$ (net demander of $x_1$),**

$$\frac{\Delta x_1(p_1, p_2, m)}{\Delta m}(\omega_1 - x_1) < 0$$

$$\frac{\Delta x_1}{\Delta p_1} < 0$$

# Rate of change version

**With endowments:**

$$\frac{\Delta x_1}{\Delta p_1} = \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{(-)} + \underbrace{\frac{\Delta x_1(p_1, p_2, m)}{\Delta m}}_{(+)\text{ if normal}} \underbrace{(\omega_1 - x_1)}_{(?)}$$

**If $\omega_1 - x_1 > 0$ (net supplier of x1),**

$$\frac{\Delta x_1(p_1, p_2, m)}{\Delta m}(\omega_1 - x_1) \lessgtr 0$$

$$\frac{\Delta x_1}{\Delta p_1} \gtrless 0$$

# An Application to the Intertemporal Choice Problem

- Let $m_1$ and $m_2$ be incomes received in periods 1 and 2.
- Let $c_1$ and $c_2$ be consumptions in periods 1 and 2.
- Let $p_1$ and $p_2$ be the prices of consumption in periods 1 and 2.

跨期消费选择

# The Intertemporal Choice Problem

- The intertemporal choice problem: Given incomes $m_1$ and $m_2$, and given consumption prices $p_1$ and $p_2$, what is the most preferred intertemporal consumption bundle $(c_1,c_2)$?
- For an answer we need to know:
  – the intertemporal budget constraint
  – intertemporal consumption preferences.

# The Intertemporal Budget Constraint

**To start, let's ignore price effects by supposing that**

$$p_1 = p_2 = \$1.$$

# The Intertemporal Budget Constraint

**To start, let's ignore price effects by supposing that**

$$p_1 = p_2 = \$1.$$

**The interest rate is denoted by r.**

- **i.e. \$1 saving today will become \$(1+r) tomorrow.**
- **\$1 tomorrow is $\frac{1}{1+r}$ in today's dollar**

# The Intertemporal Budget Constraint

**Suppose that $c_1$ units are consumed in period 1. This costs \$$c_1$ and leaves $m_1 - c_1$ saved. Period 2 consumption will then be**

$$c_2 = m_2 + (1 + r)(m_1 - c_1)$$

# The Intertemporal Budget Constraint

Suppose that $c_1$ units are consumed in period 1. This costs $\$c_1$ and leaves $m_1 - c_1$ saved. Period 2 consumption will then be

$$c_2 = m_2 + (1+r)(m_1 - c_1)$$

which is

$$c_2 = \underbrace{-(1+r)}_{\text{slope}}c_1 + \underbrace{m_2 + (1+r)m_1}_{\text{intercept}}.$$

# The Intertemporal Budget Constraint

$$c_2 = -(1+r)c_1 + m_2 + (1+r)m_1.$$

$$\underbrace{(1+r)}_{\text{“}p_1\text{”}}c_1 + \underbrace{1}_{\text{“}p_2\text{”}} \times c_2 = \underbrace{(1+r)m_1 + m_2}_{\text{“}m\text{”}}$$

**Analogous to:**

$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2$$

# The Intertemporal Budget Constraint

$$c_2 = -(1+r)c_1 + m_2 + (1+r)m_1.$$

$$\underbrace{(1+r)}_{\text{“}p_1\text{”}}c_1 + \underbrace{1}_{\text{“}p_2\text{”}} \times c_2 = \underbrace{(1+r)m_1 + m_2}_{\text{“}m\text{”}}$$

“**future-valued**” form of the budget constraint

**Analogous to:**

$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2$$

# The Intertemporal Budget Constraint

$$\mathbf{c_2 = -(1+r)c_1 + m_2 + (1+r)m_1.}$$

$$\underbrace{1 \times}_{\text{“}p_1\text{”}} c_1 + \underbrace{\frac{1}{1+r}}_{\text{“}p_2\text{”}} c_2 = \underbrace{m_1 + \frac{1}{1+r} m_2}_{\text{“}m\text{”}}$$

**Analogous to:**

$$\mathbf{p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2}$$

# The Intertemporal Budget Constraint

$$c_2 = -(1+r)c_1 + m_2 + (1+r)m_1.$$

$$\underbrace{1 \times}_{\text{"}p_1\text{"}} c_1 + \underbrace{\frac{1}{1+r}}_{\text{"}p_2\text{"}} c_2 = \underbrace{m_1 + \frac{1}{1+r} m_2}_{\text{"}m\text{"}}$$

"**present-valued**" **form of the budget constraint**

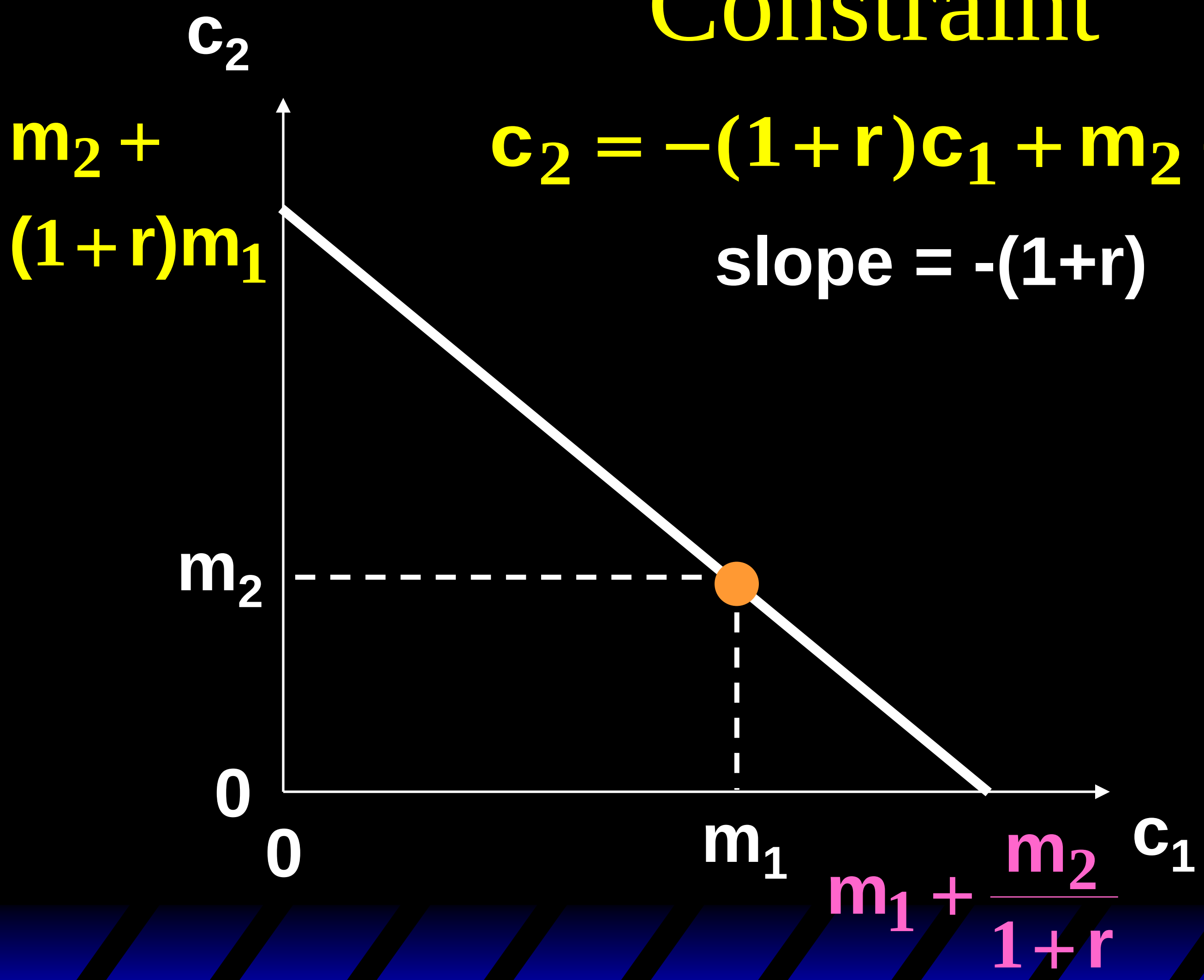
**Analogous to:**

$$p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2$$

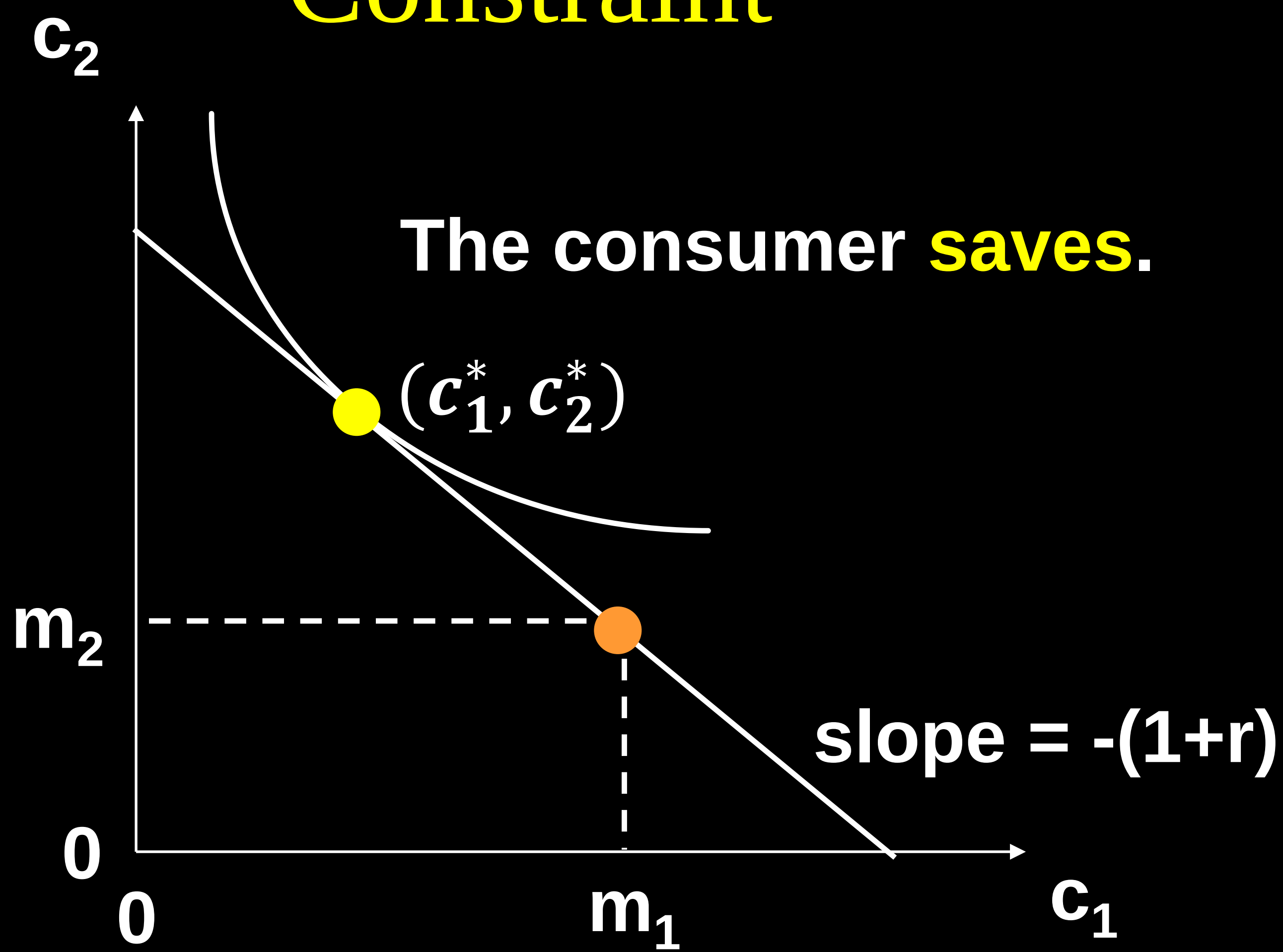
# The Intertemporal Budget Constraint

$$c_2 = -(1+r)c_1 + m_2 + (1+r)m_1.$$

slope = -(1+r)

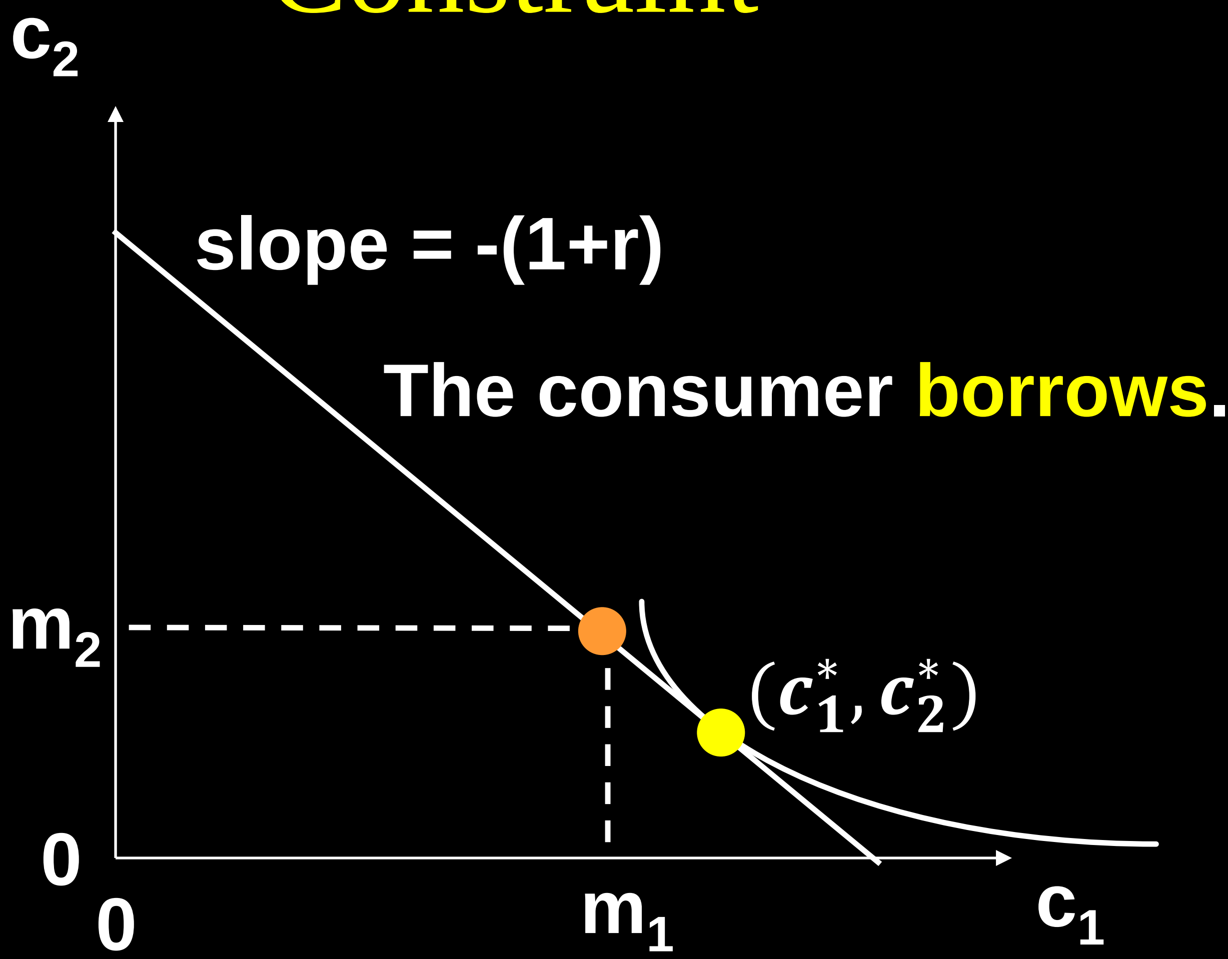


# The Intertemporal Budget Constraint



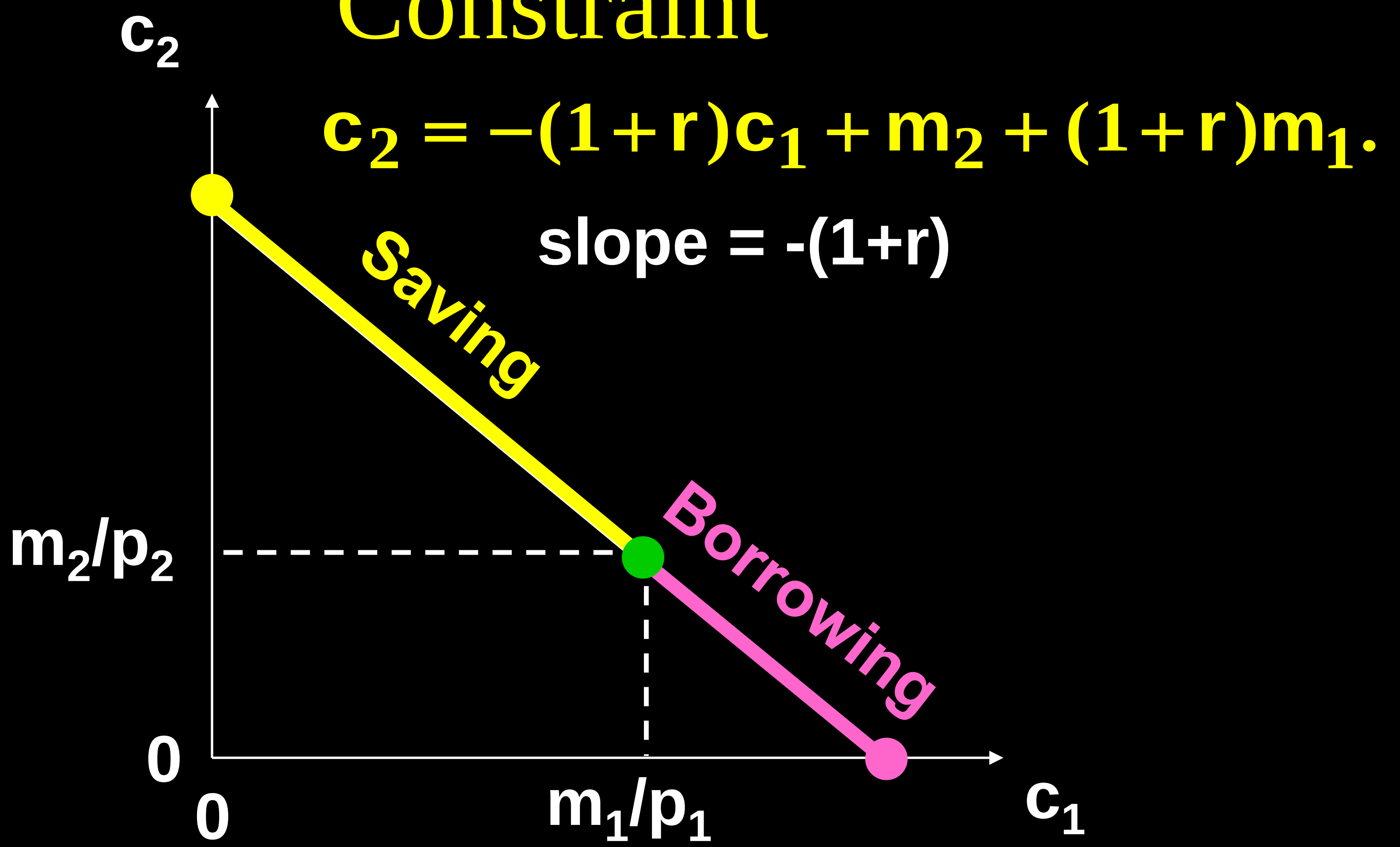

$$c_1^* < m_1$$

# The Intertemporal Budget Constraint




$c_1^* > m_1$

# The Intertemporal Budget Constraint



# The Intertemporal Budget Constraint

**Q: If the interest rate r falls, how would the budget constraint change?**

# The Intertemporal Budget Constraint

$$\mathbf{c_2 = \underbrace{-(1+r)}_{\text{slope}}c_1 + \underbrace{m_2 + (1+r)m_1}_{\text{intercept}}.}$$

$$\underbrace{(1+r)}_{\text{“}p_1\text{”}}c_1 + \underbrace{1}_{\text{“}p_2\text{”}} \times c_2 = \underbrace{(1+r)m_1 + m_2}_{\text{“}m\text{”}}$$

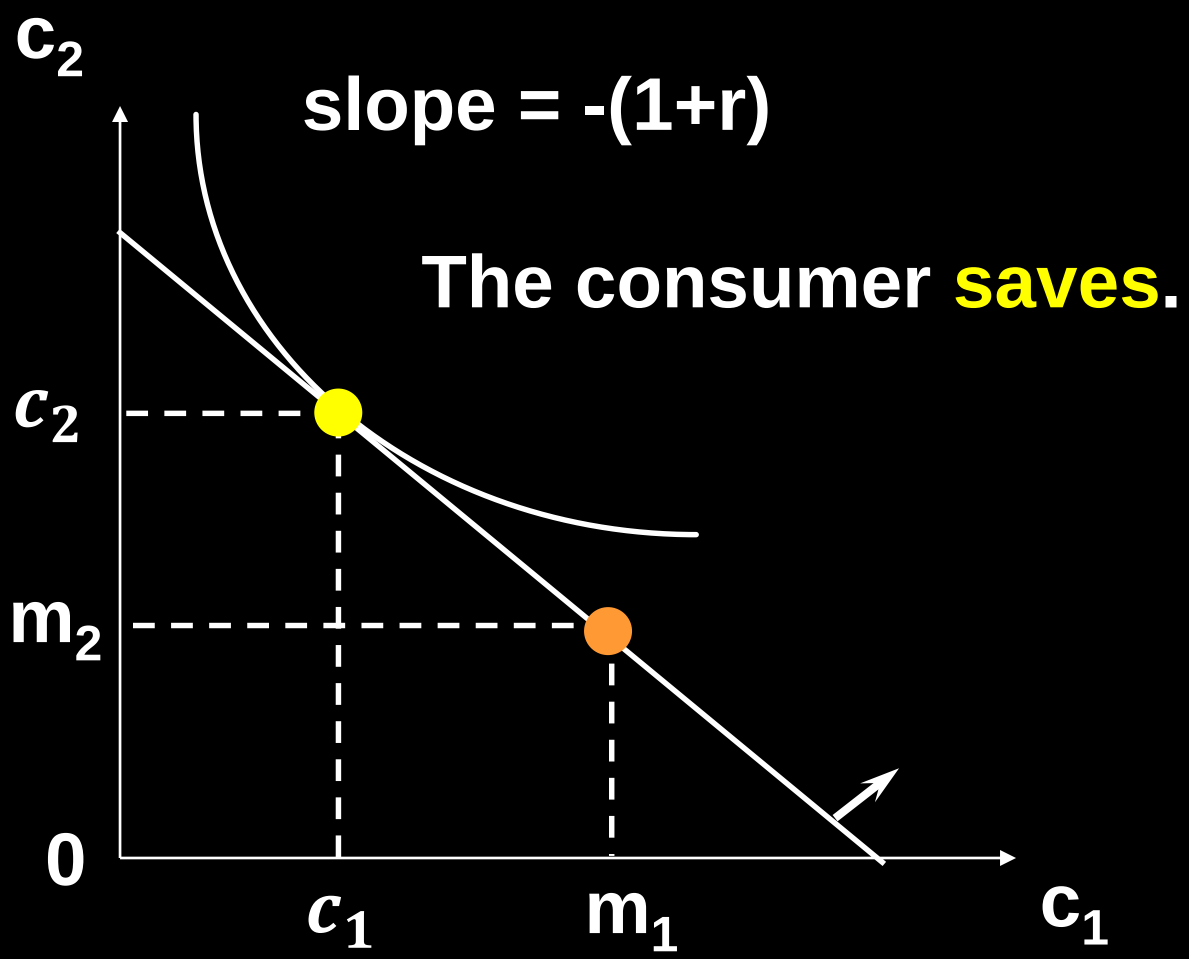
# The Intertemporal Budget Constraint

$$\underbrace{(1+r)}_{\text{“}p_1\text{”}}c_1 + \underbrace{1}_{\text{“}p_2\text{”}} \times c_2 = \underbrace{(1+r)m_1 + m_2}_{\text{“}m\text{”}}$$

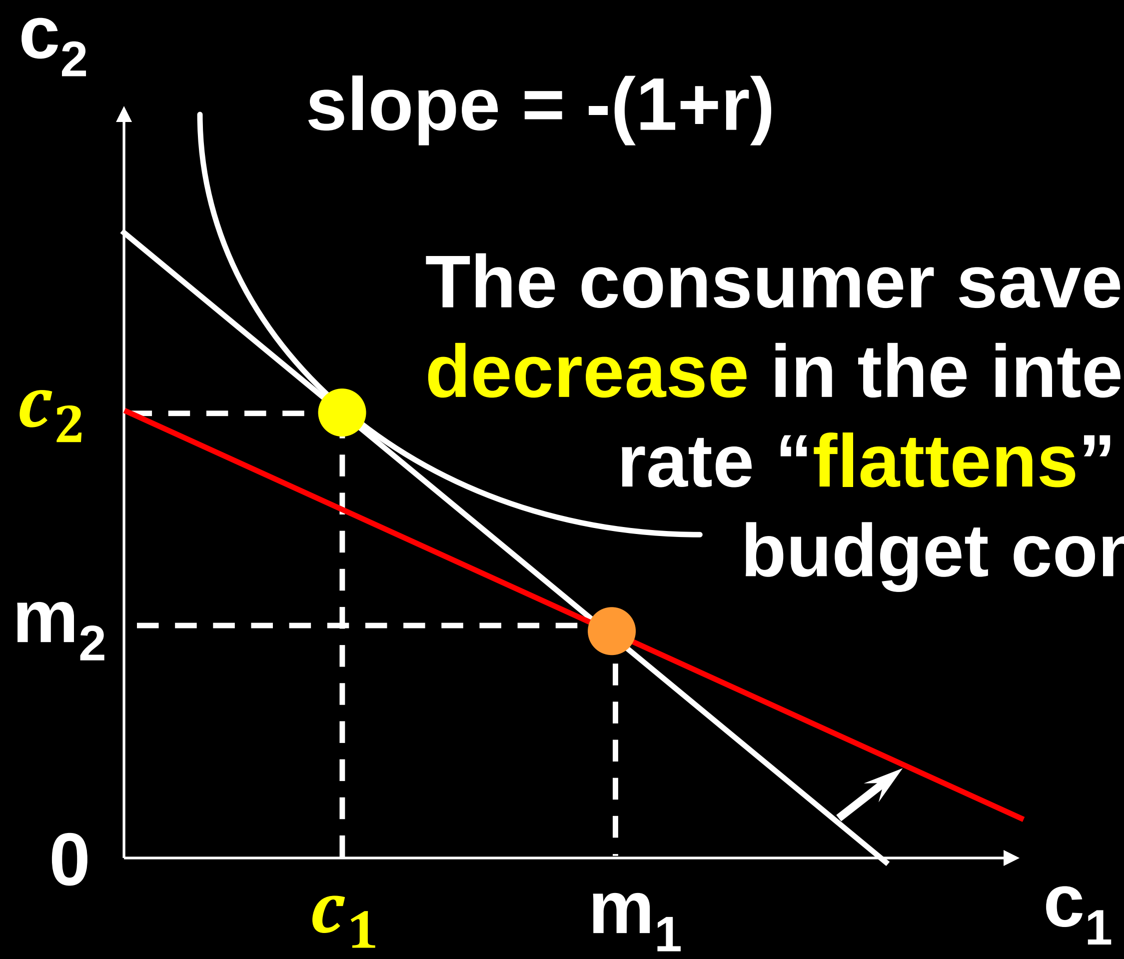
**If r falls, today's consumption $c_1$ becomes relatively cheaper.**

**The budget line pivots around $(m_1, m_2)$ and becomes flatter.**

# Comparative Statics



# Comparative Statics




The consumer saves. A **decrease** in the interest rate "**flattens**" the budget constraint.

# Comparative Statics

**Q: How would $C_1$ change?**

# Comparative Statics

**Q: How would $C_1$ change?**

**According to Slutsky equation:**

$$\frac{\Delta x_1}{\Delta p_1} = \underbrace{\frac{\Delta x_1^s}{\Delta p_1}}_{(-)} + \underbrace{\frac{\Delta x_1(p_1, p_2, m)}{\Delta m}}_{(+)\text{ if normal}} \underbrace{(\omega_1 - x_1)}_{(+)\text{ for savers}}$$

# Comparative Statics

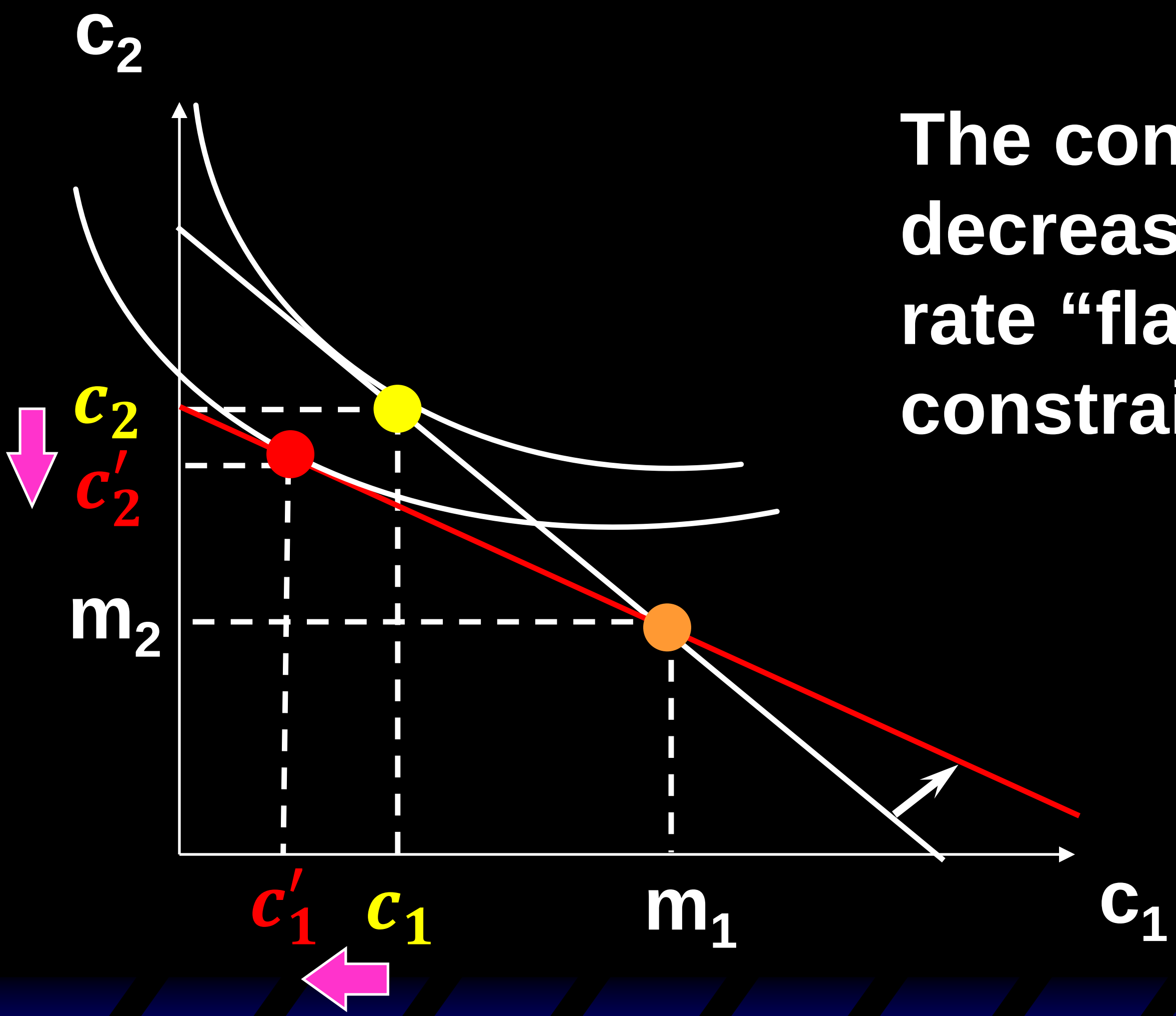
**Q: How would $C_1$ change?**

**According to Slutsky equation:**

$$\frac{\Delta c_1}{\Delta p_1} = \underbrace{\frac{\Delta c_1^s}{\Delta p_1}}_{(-)} + \underbrace{\frac{\Delta x_1(p_1, p_2, m\,)}{\Delta m}}_{(+)\text{ if normal}} \underbrace{(m_1 - c_1)}_{(+)\text{ for savers}}$$

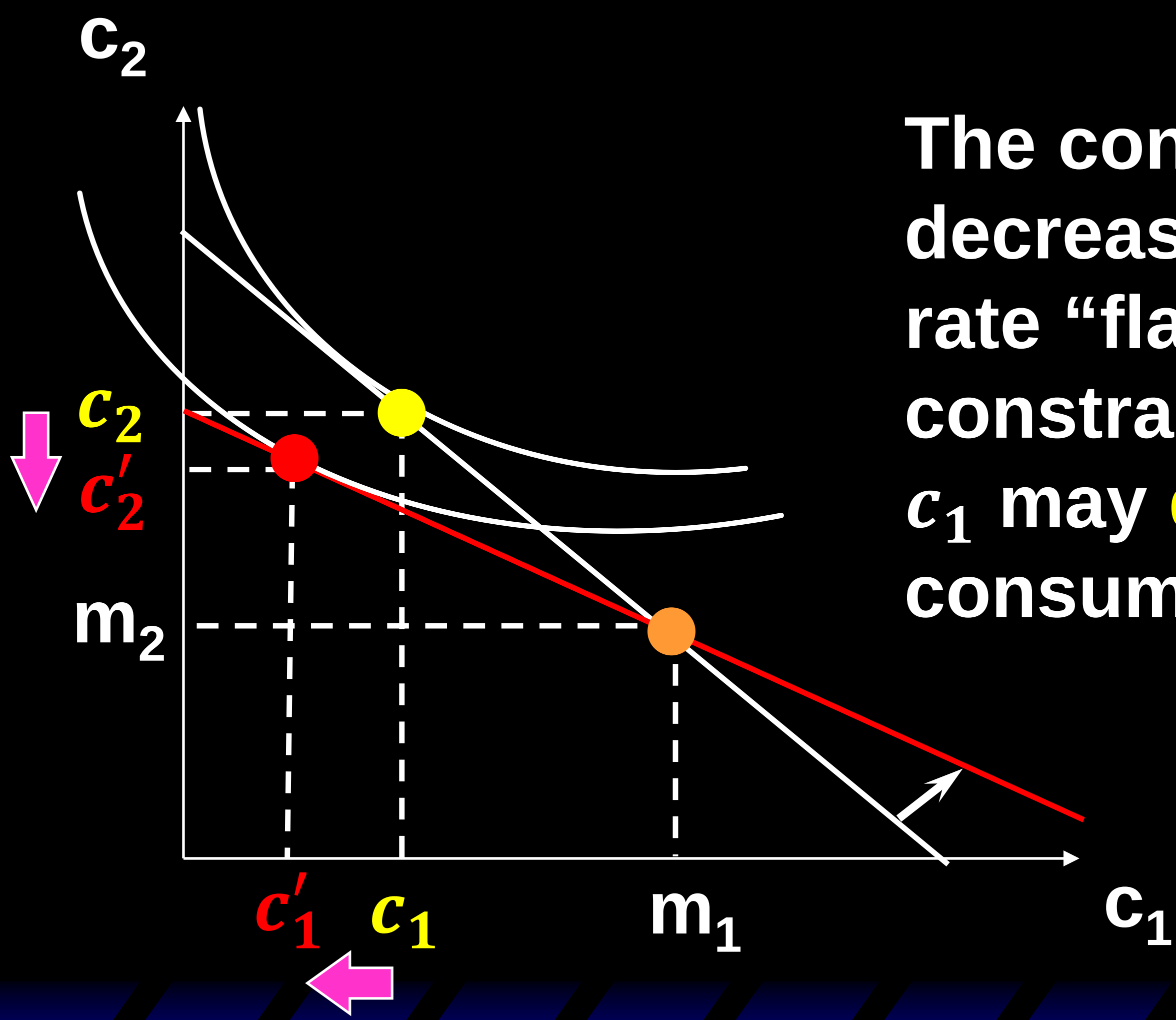
**$\frac{\Delta c_1}{\Delta p_1} \gtrless 0$ (positive or negative). $C_1$ could increase or decrease as r ($p_1$) falls.**

# Comparative Statics



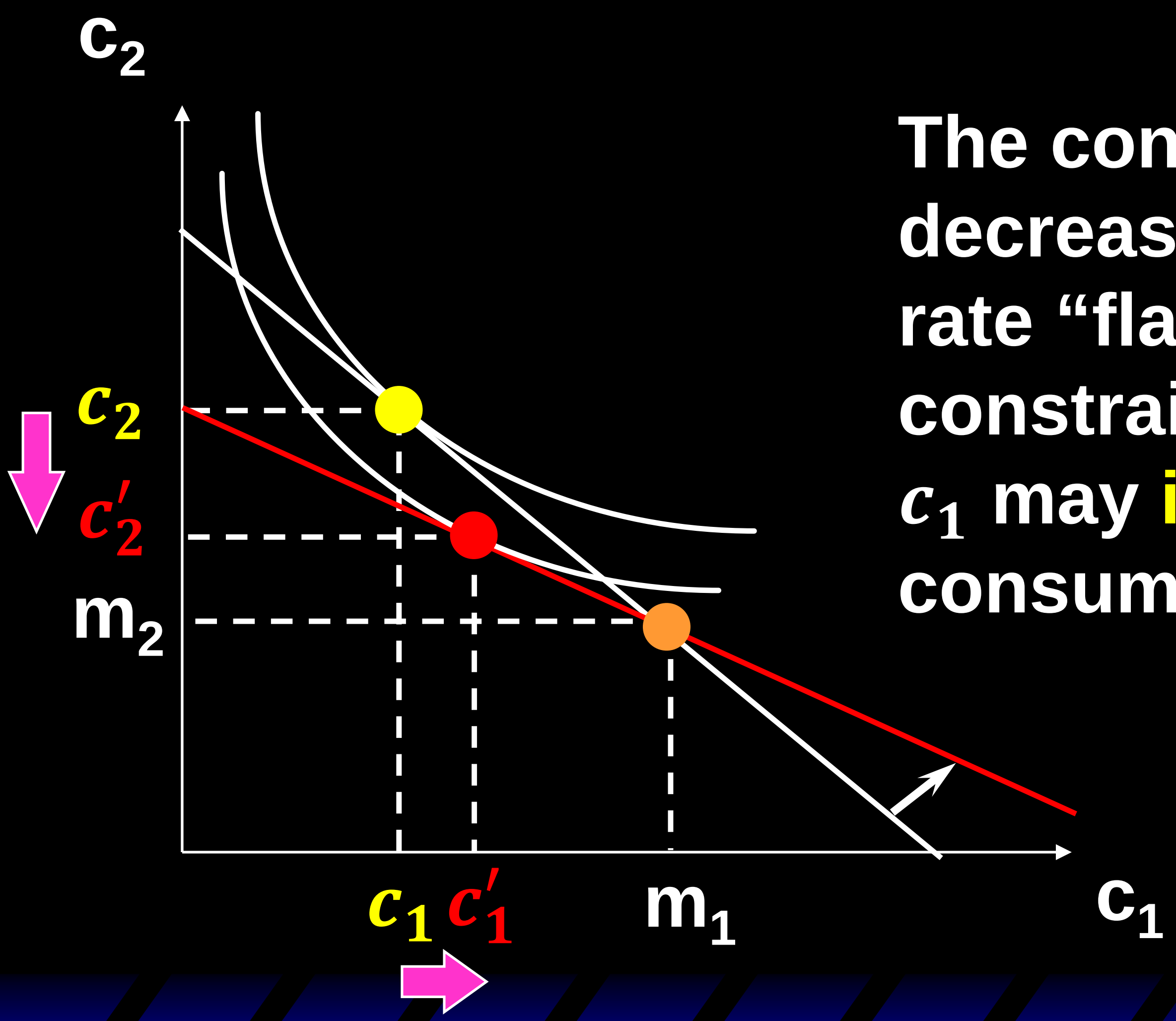

**The consumer saves. A decrease in the interest rate "flattens" the budget constraint.**

# Comparative Statics



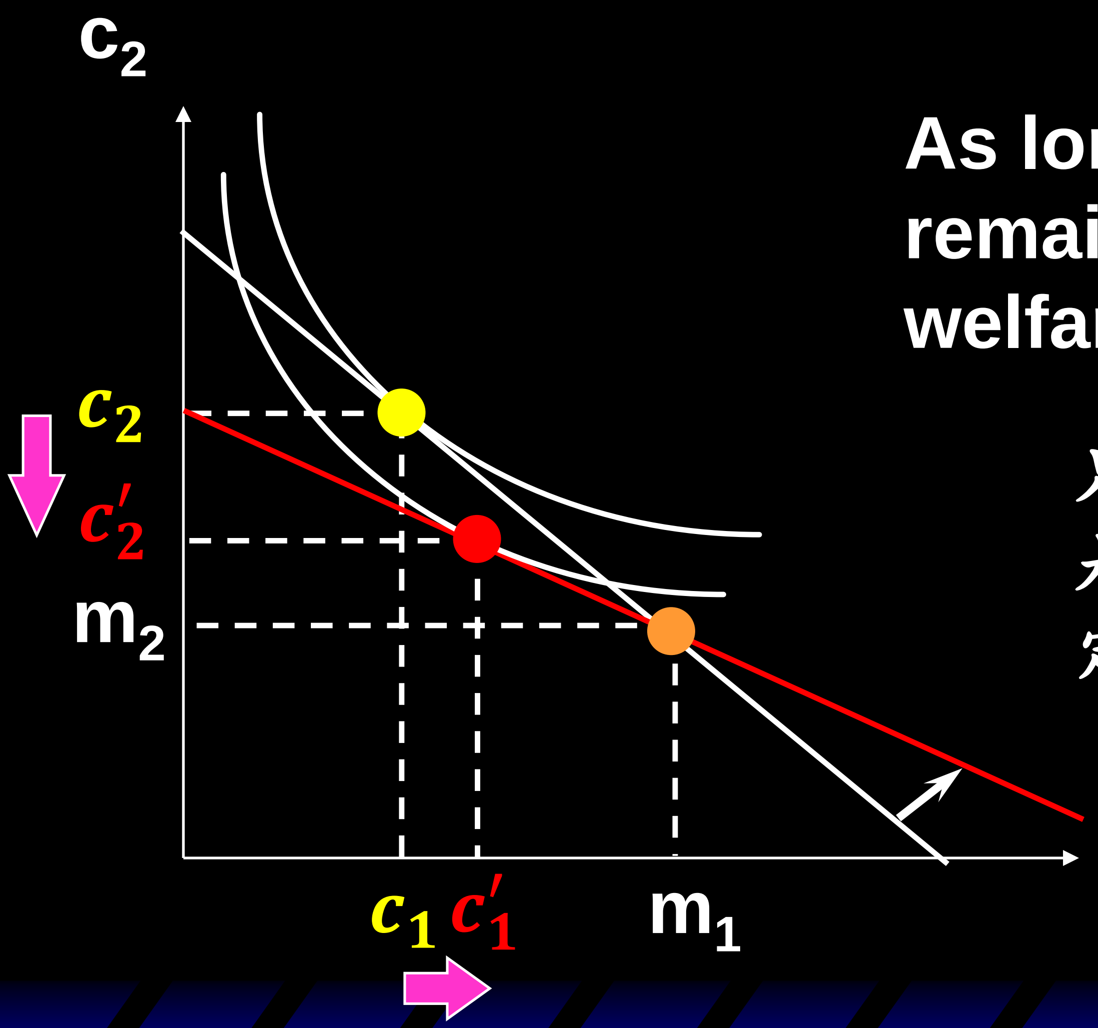

**The consumer saves. A decrease in the interest rate "flattens" the budget constraint.**
**$c_1$ may decrease, and the consumer may save more.**

# Comparative Statics




**The consumer saves. A decrease in the interest rate "flattens" the budget constraint.**
**$c_1$ may increase, and the consumer may save less.**

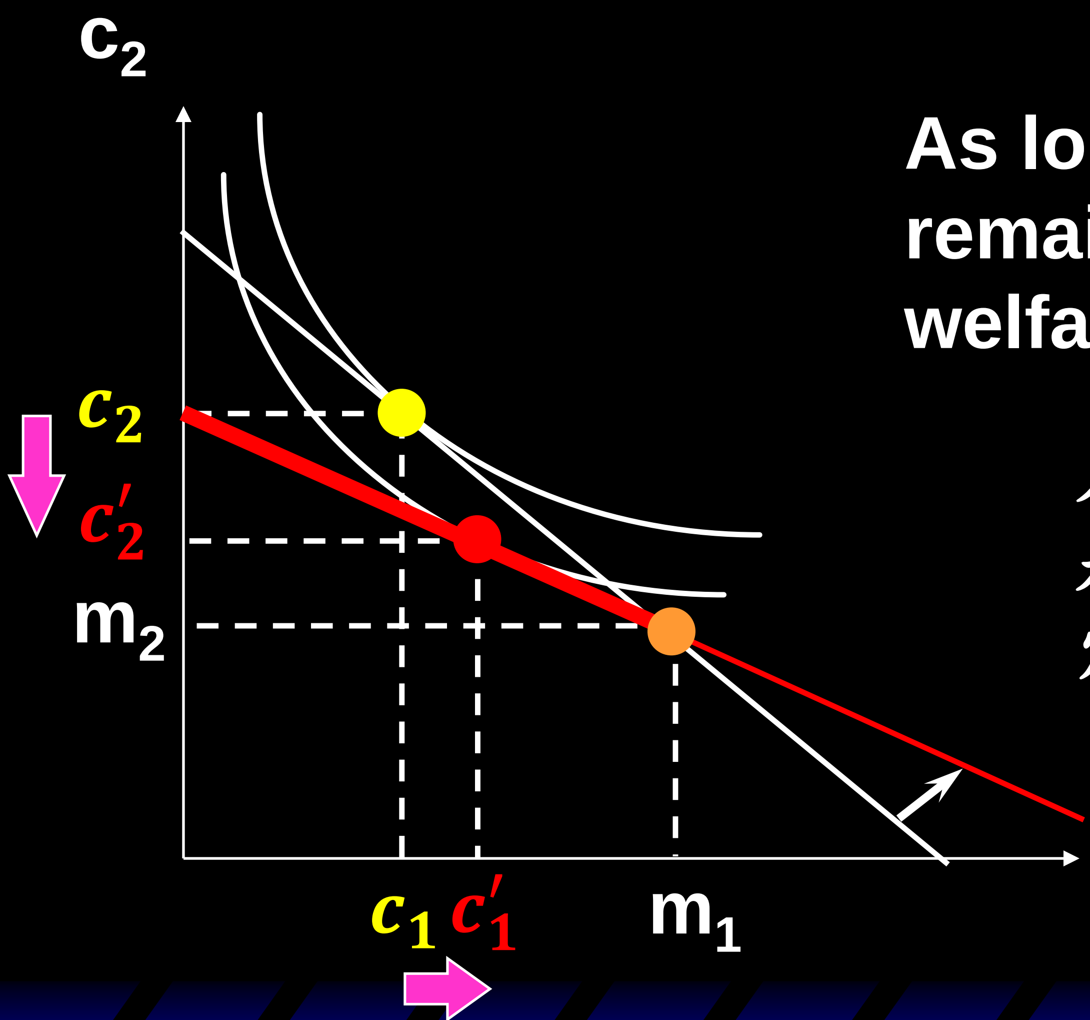
# Comparative Statics




**As long as the consumer remains a saver, the welfare will be reduced.**

只要新均衡下消费者仍是储蓄者，她的福利一定比之前下降了。

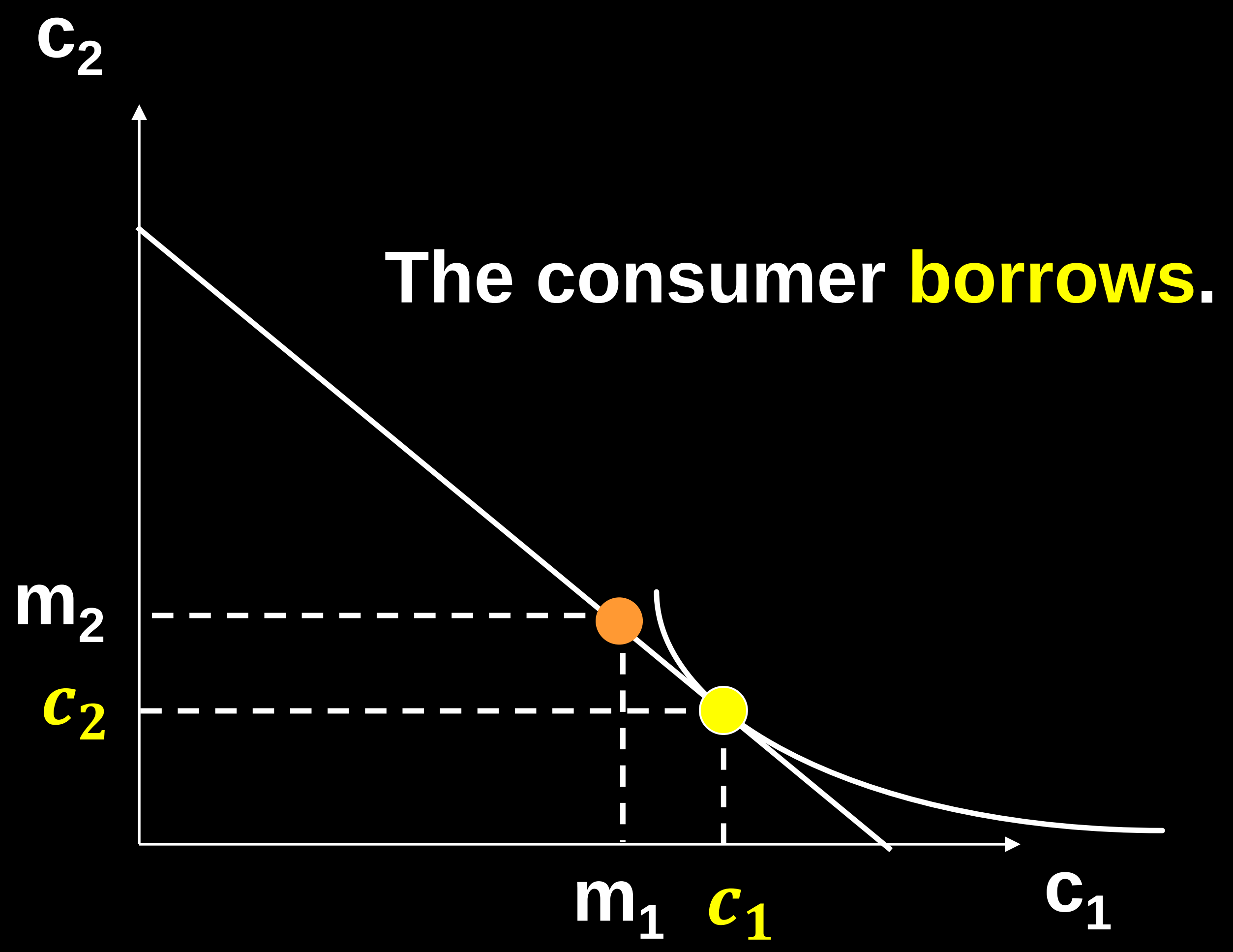
# Comparative Statics



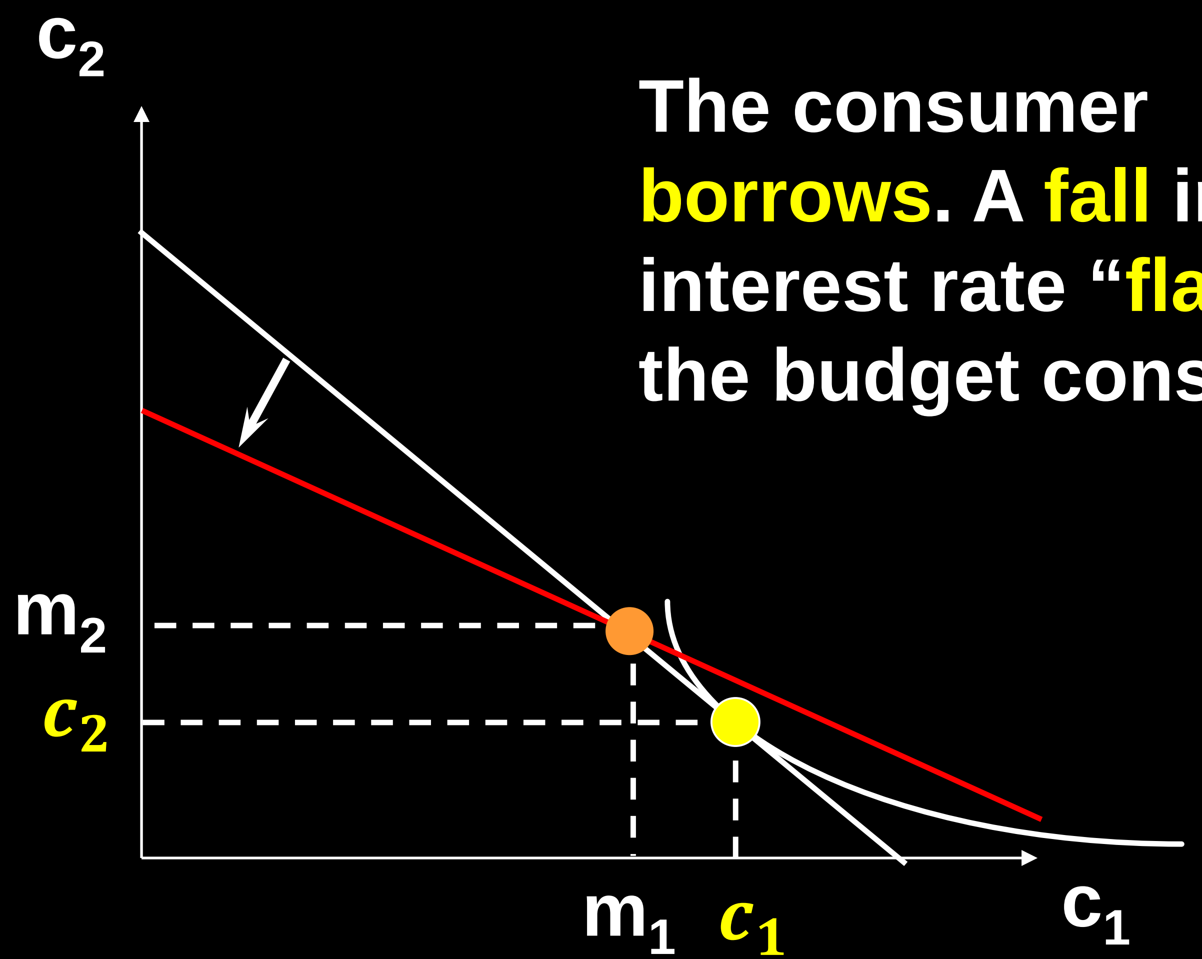

**As long as the consumer remains a saver, the welfare will be reduced.**

只要新均衡下消费者仍是储蓄者，她的福利一定比之前下降了。

# Comparative Statics



# Comparative Statics




**The consumer borrows. A fall in the interest rate "flattens" the budget constraint.**

# Comparative Statics

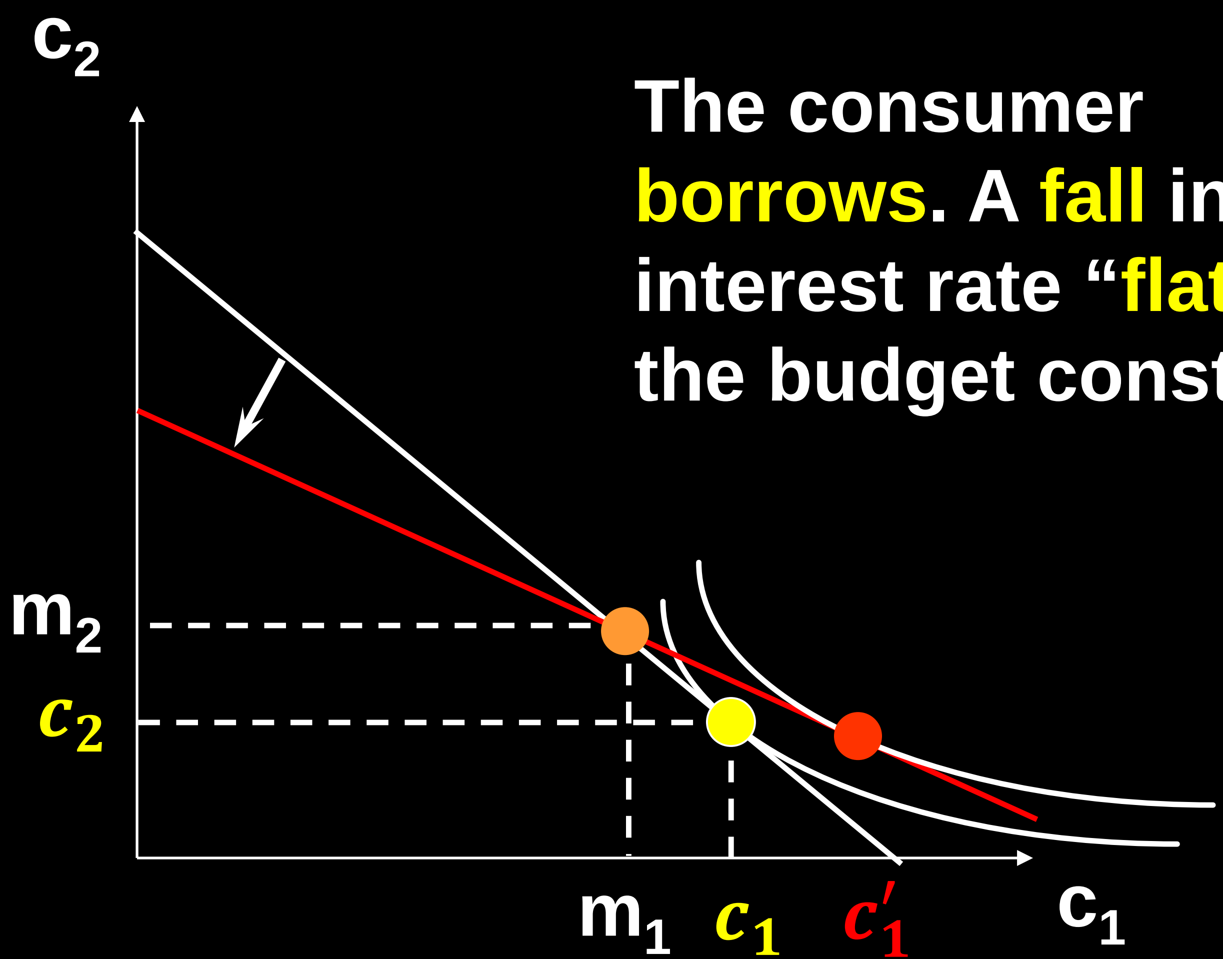
**Q: How would $C_1$ change?**

**According to Slutsky equation:**

$$\frac{\Delta c_1}{\Delta p_1} = \underbrace{\frac{\Delta c_1^s}{\Delta p_1}}_{(-)} + \underbrace{\frac{\Delta x_1(p_1, p_2, m)}{\Delta m}}_{(+)\text{ if normal}} \underbrace{(m_1 - c_1)}_{(-)\text{ for borrowers}}$$

**It must be that $\frac{\Delta c_1}{\Delta p_1} < 0$. $C_1$ increases as r ($p_1$) falls.**

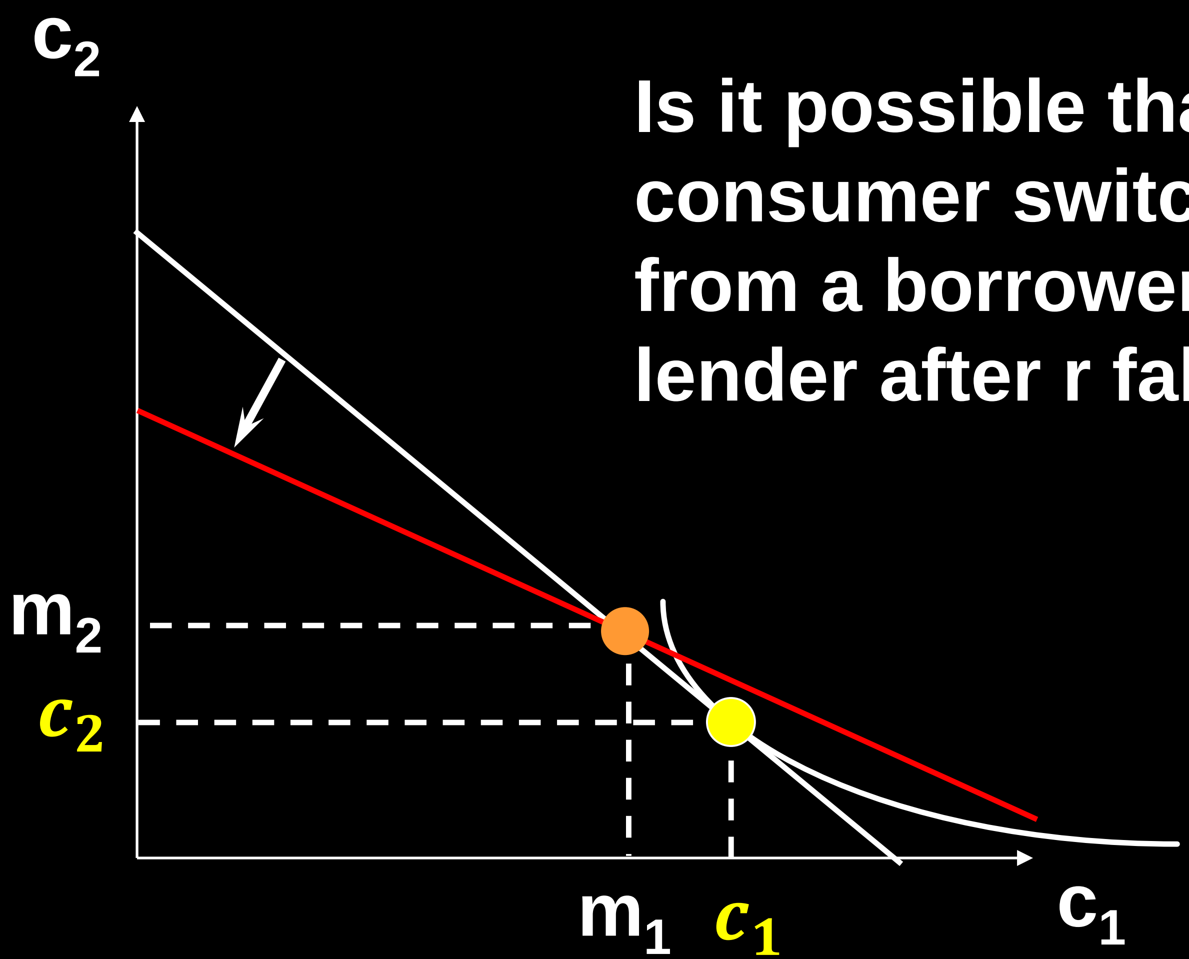
# Comparative Statics




**The consumer borrows. A fall in the interest rate “flattens” the budget constraint.**

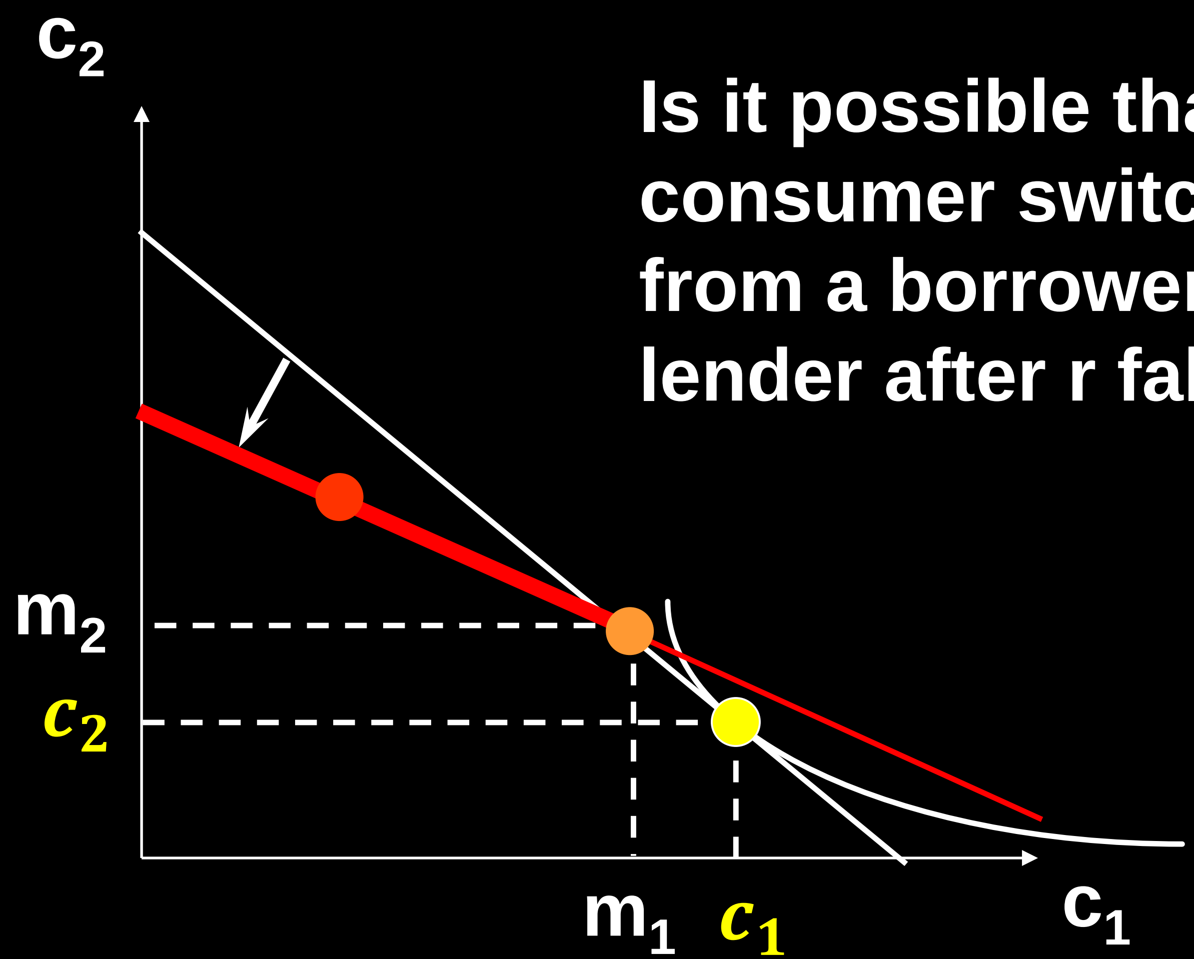
**$c_1$ increases, and the consumer borrows more.**
**The welfare is increased.**

# Comparative Statics




**Is it possible that the consumer switches from a borrower to a lender after r falls?**

# Comparative Statics




**Is it possible that the consumer switches from a borrower to a lender after r falls?**

**No. Otherwise WARP will be violated.**