# Chapter 8:
# Economic Growth I：
# Capital Accumulation and Population Growth

# Learning objectives

- Understand Solow's economic growth model.
- See how a country's standard of living depends on its saving and population growth rates.
- Learn how to use the "Golden Rule" to find the optimal savings rate for a country and the resulting capital stock

# *The importance of economic growth*

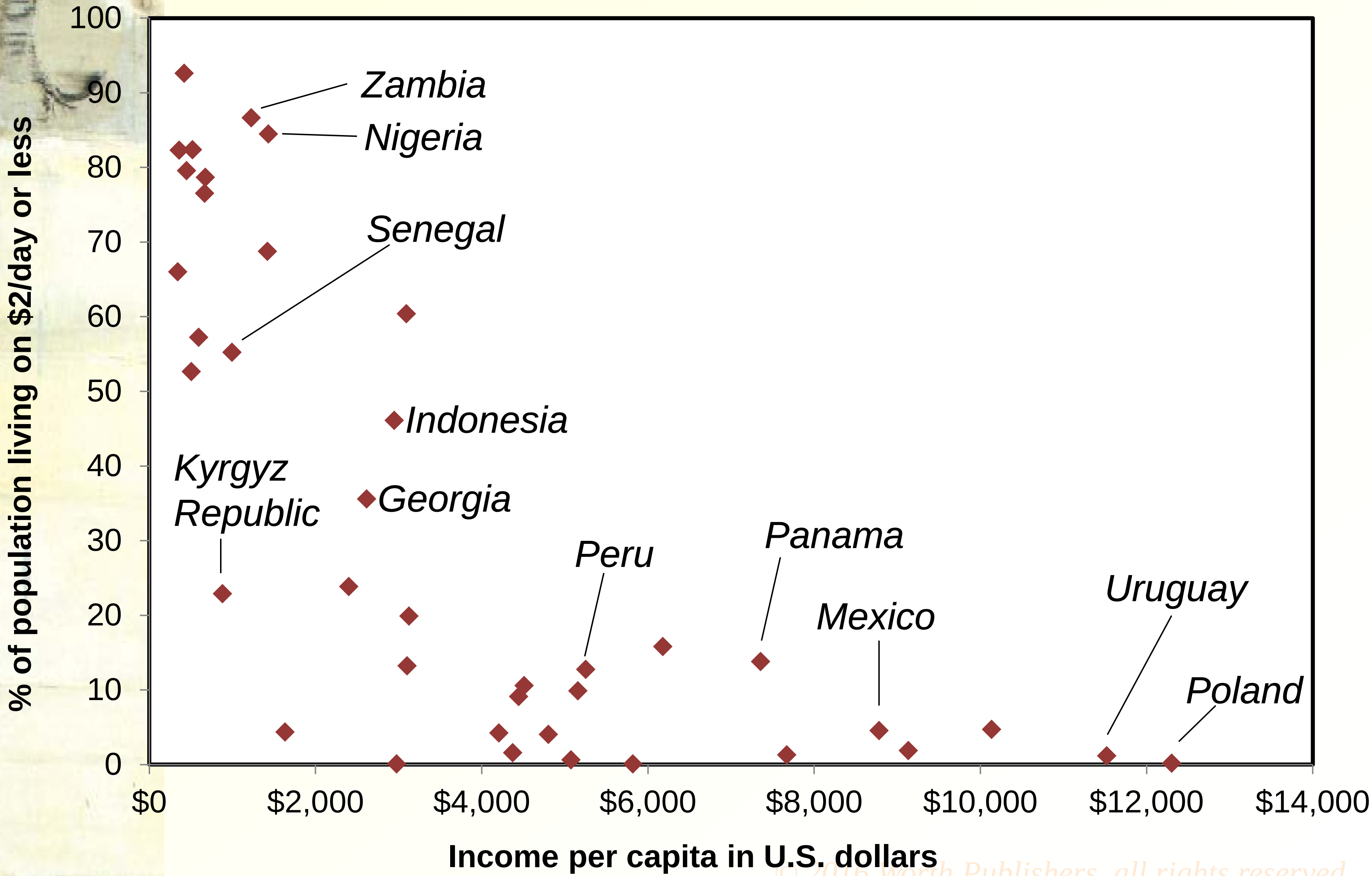
*…for poor countries*

# Why growth matters

- Data on infant mortality rates:
  - – 20% in the poorest 1/5 of all countries
  - – 0.4% in the richest 1/5
- In Pakistan, 85% of people live on less than $2/d
- One-fourth of the poorest countries have had famines during the past 3 decades.
- Poverty is associated with oppression of women and minorities.

***Economic growth raises living standards and reduces poverty….***

# Income and poverty in the world
## selected countries, 2010



# *The importance of economic growth*

*…for poor countries*

*…for rich countries*

# *Huge effects from small differences*

| annual growth rate of income per capita | percentage increase in standard of living after… | | |
|---|---|---|---|
| | …25 years | …50 years | …100 years |
| 1.5% | 45.1% | 110.5% | 343.2% |
| 2.0% | 64.1% | 169.2% | 624.5% |

# *The lessons of growth theory*

*...can make a positive difference in the lives of hundreds of millions of people.*

These lessons help us

- understand why poor countries are poor
- design policies that can help them grow
- learn how our own growth rate is affected by shocks and government's policies

# The Solow Model

- due to Robert Solow,
  won Nobel Prize for contributions to the study of economic growth
- a major paradigm:
  - widely used in policy making
  - benchmark against which most recent growth theories are compared
- looks at the determinants of economic growth and the standard of living in the long run

# How Solow model is different from Chapter 3's model

1. ***K*** is no longer fixed:
   investment causes it to grow,
   depreciation causes it to shrink.

2. ***L*** is no longer fixed:
   population growth causes grow of the labor force.

3. The consumption function is simpler.

# How Solow model is different from Chapter 3's model

4. No ***G*** or ***T***
   (only to simplify presentation;
   we can still do fiscal policy experiments)

5. Other minor differences.

# The production function

- In aggregate terms: $\boldsymbol{Y = F(K, L)}$
- Define: $\boldsymbol{y = Y/L}$ = output per worker

  $\boldsymbol{k = K/L}$ = capital per worker

- Assume constant returns to scale:

  $\boldsymbol{zY = F(zK, zL)}$ for any $\boldsymbol{z} > 0$

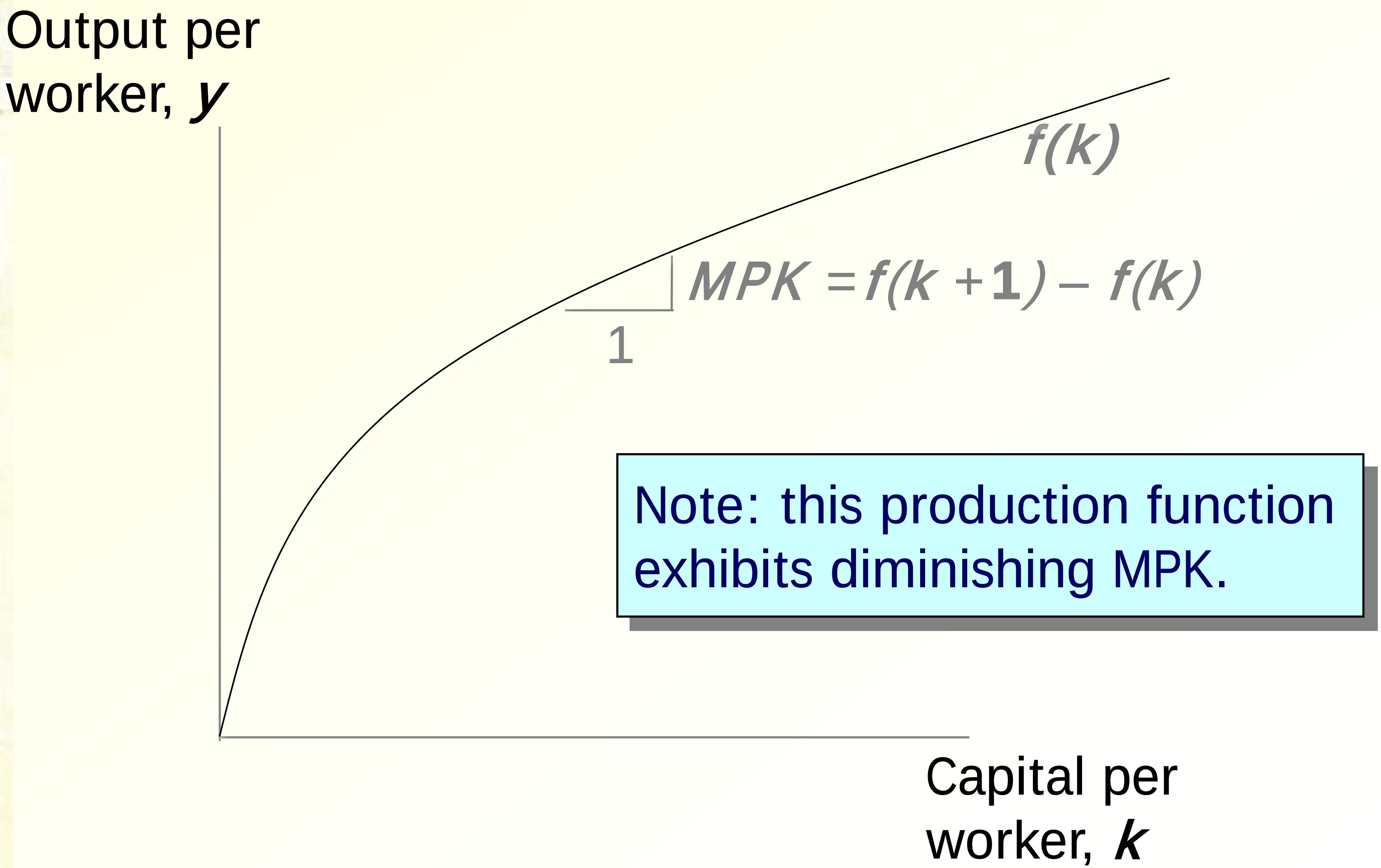
- Pick $\boldsymbol{z = 1/L}$. Then

$$\boldsymbol{Y/L = F(K/L, 1)}$$

$$\boldsymbol{y = F(k, 1)}$$

$$\boldsymbol{y = f(k)} \qquad \text{where } \boldsymbol{f(k) = F(k, 1)}$$

# The production function



# The national income identity

- $\boldsymbol{Y = C + I}$ (*remember, no* ***G*** )
- In "per worker" terms:

$$\boldsymbol{y = c + i}$$

where $\boldsymbol{c = C/L}$ and $\boldsymbol{i = I/L}$

# The consumption function

- ***s*** = the saving rate,
  the fraction of income that is saved
  (***s*** is an exogenous parameter)

**Note: *s* is the only lower case variable
that is not equal to
its upper case version divided by *L***

- Consumption function: $\boldsymbol{c = (1 - s)y}$
  *(per worker)*

# Saving and investment

- saving (per worker) $= \boldsymbol{y} - \boldsymbol{c}$

  $= \boldsymbol{y} - (\mathbf{1}-\boldsymbol{s})\boldsymbol{y}$

  $= \boldsymbol{sy}$

- National income identity is $\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{i}$

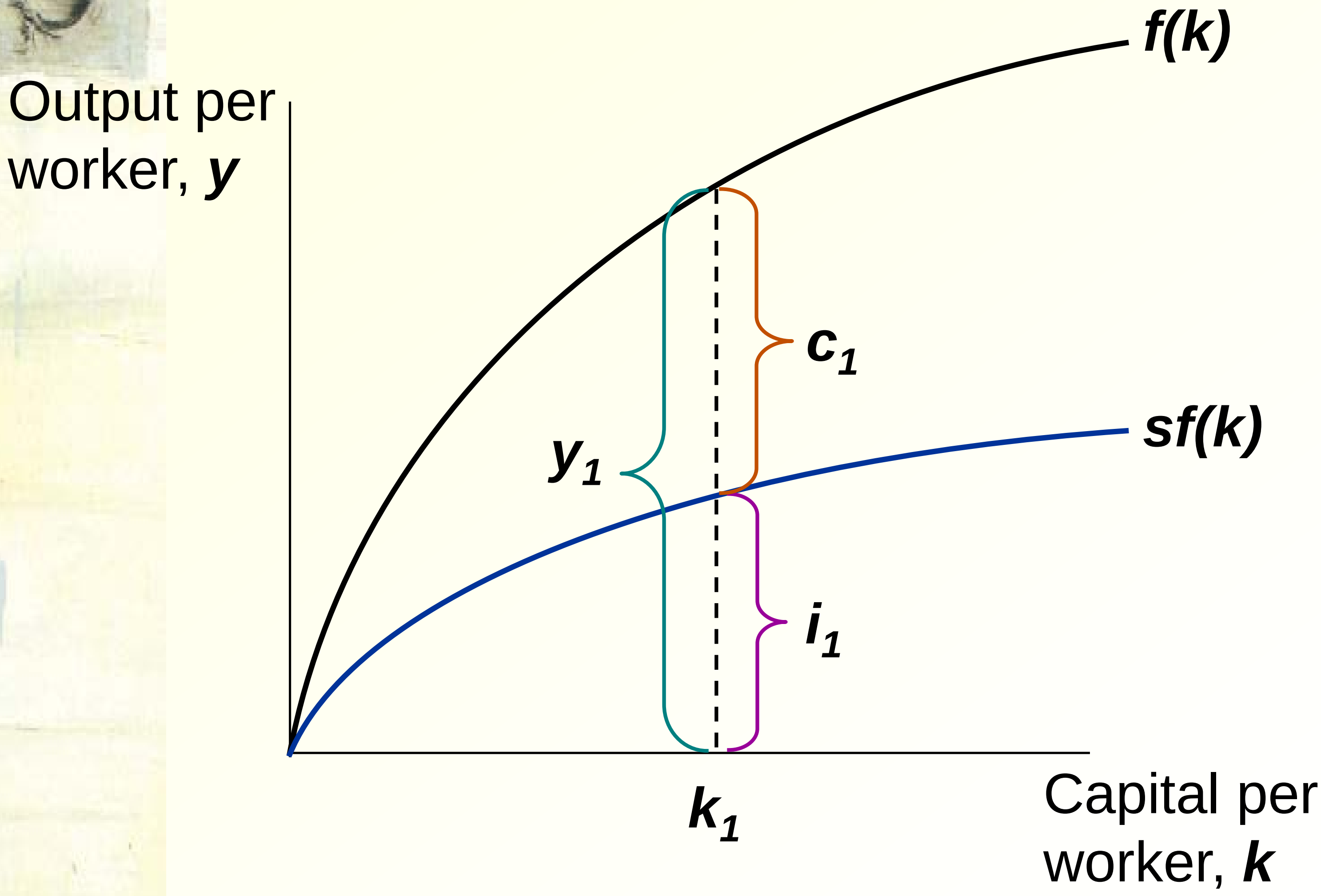
  Rearrange to get: $\boldsymbol{i} = \boldsymbol{y} - \boldsymbol{c} = \boldsymbol{sy}$

  *(investment = savings, like in chap. 3!)*

- Using the results above,

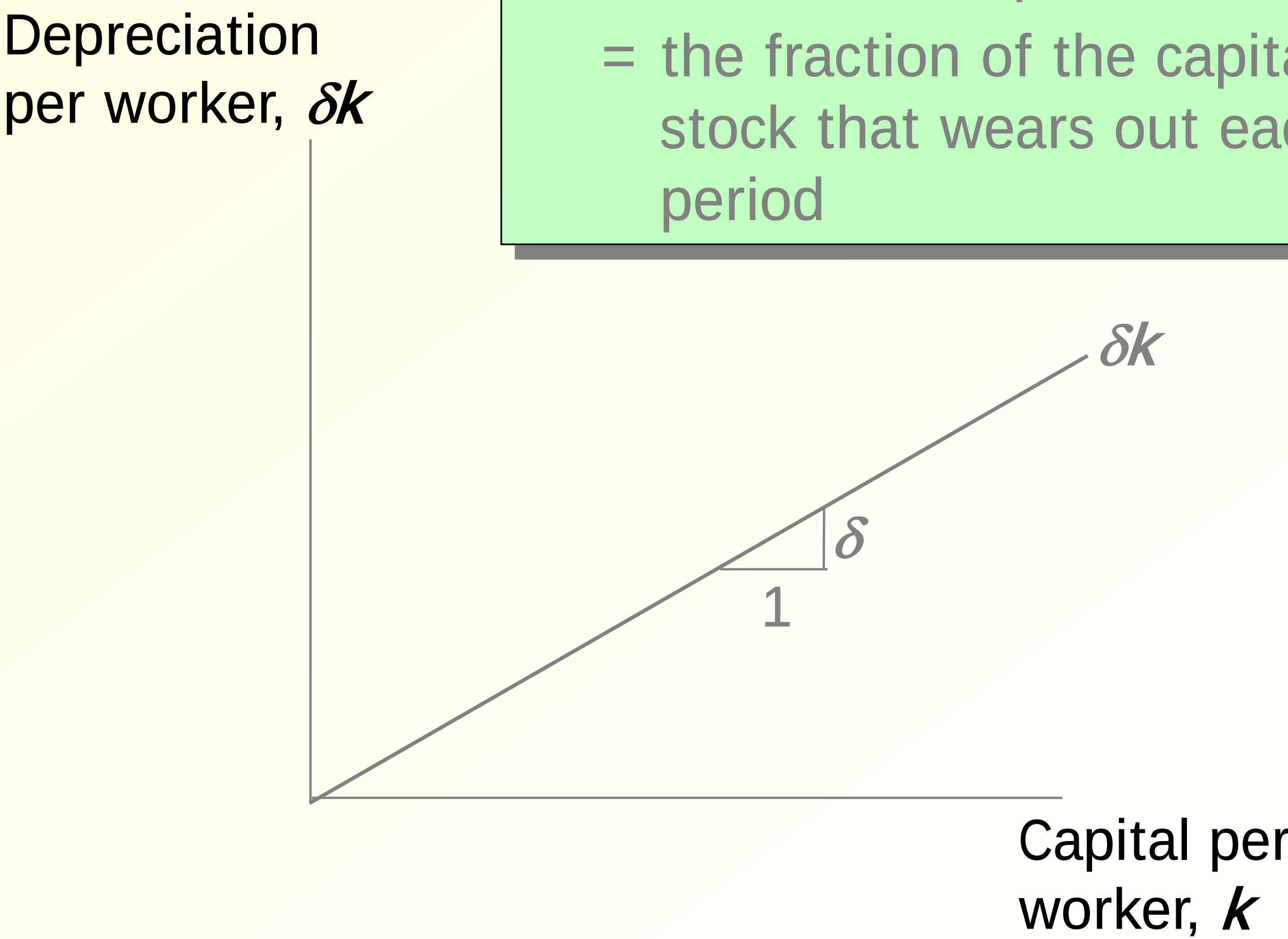
$$\boldsymbol{i} = \boldsymbol{sy} = \boldsymbol{sf(k)}$$

# Output, consumption, and investment



# Depreciation

δ = the rate of depreciation
= the fraction of the capital stock that wears out each period



# Capital accumulation

**The basic idea:**

***Investment makes the capital stock bigger, depreciation makes it smaller.***

# Capital accumulation

Change in capital stock = investment – depreciation

$$\Delta K = I - \delta K$$

Since $i = sf(k)$ , this becomes:

$$\boxed{\Delta k = s\,f(k) - \delta k}$$

# The law of motion of *k*

$$\boxed{\Delta k = s\,f(k) - \delta k}$$

- the Solow model's central equation
- Determines behavior of capital over time…
- …which, in turn, determines behavior of all of the other endogenous variables because they all depend on ***k***. E.g.,

  income per person: $y = f(k)$

  consumption per person: $c = (1-s)\,f(k)$

# The steady state

$$\boxed{\Delta k = s\,f(k) - \delta k}$$

If investment is just enough to cover depreciation

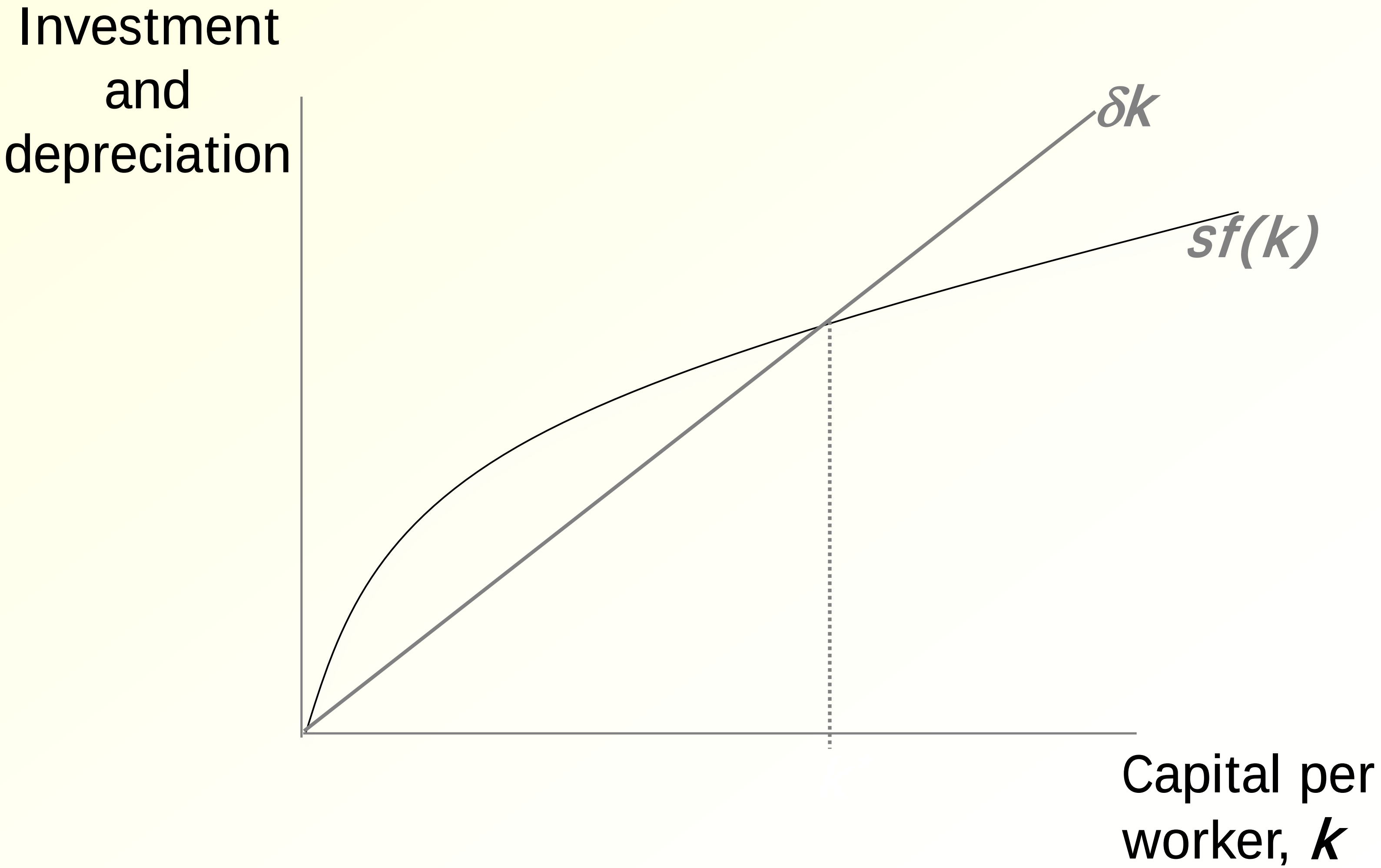
[$sf(k) = \delta k$ ],

then capital per worker will remain constant:

$$\Delta k = 0.$$

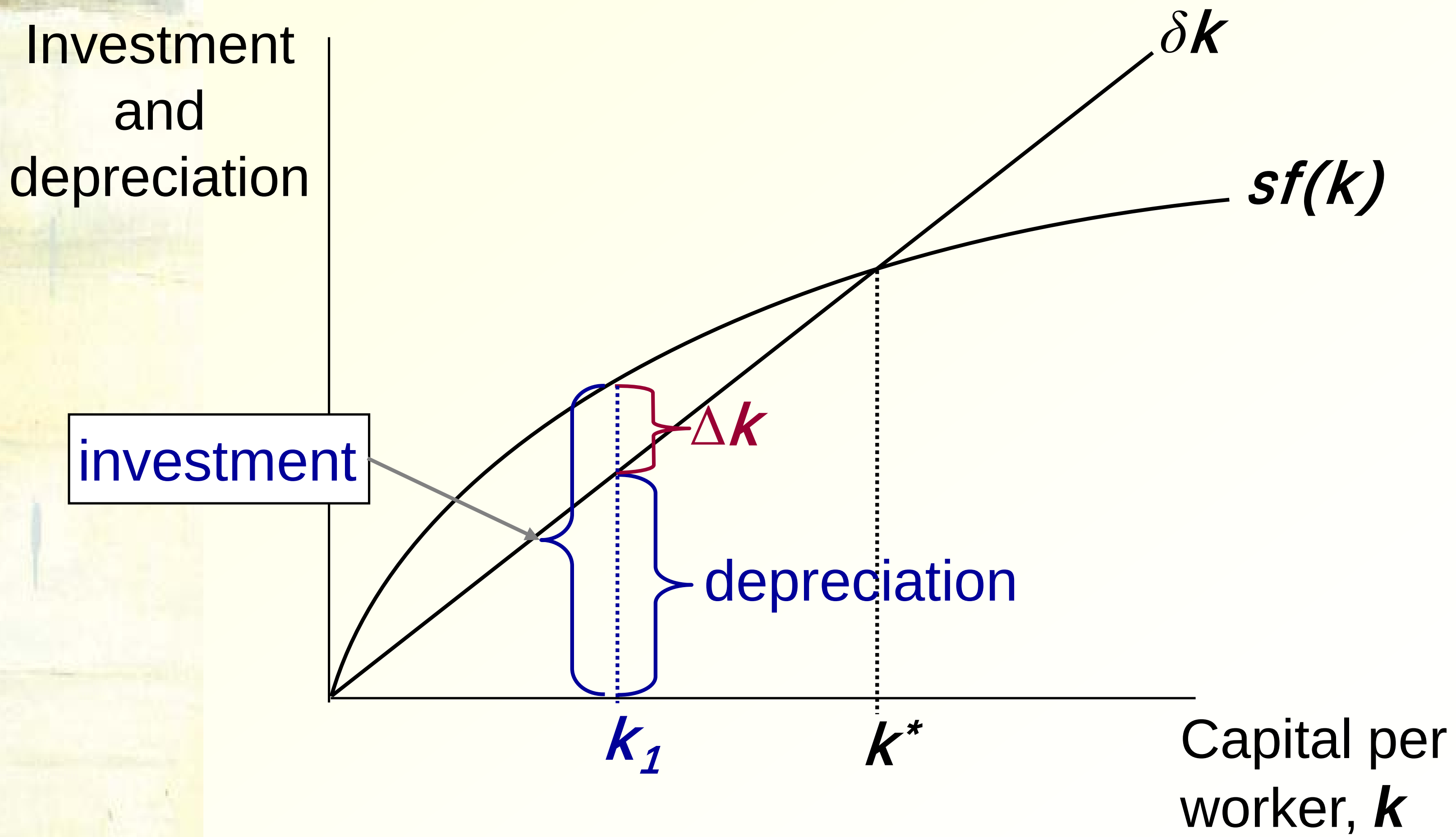
This constant value, denoted $k^*$, is called the ***steady state capital stock***.

# The steady state



# Moving toward the steady state

$$\Delta k = sf(k) - \delta k$$



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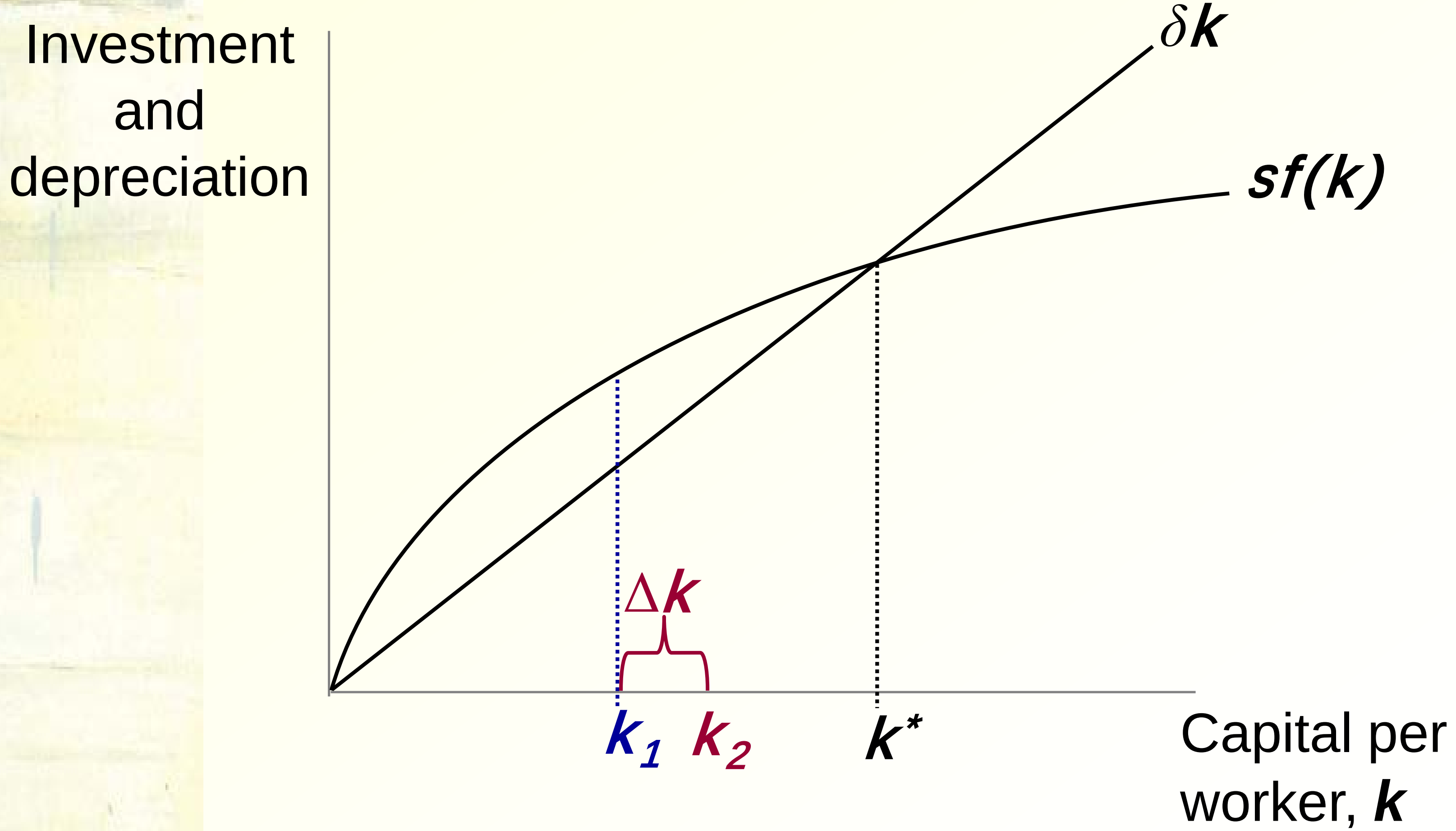

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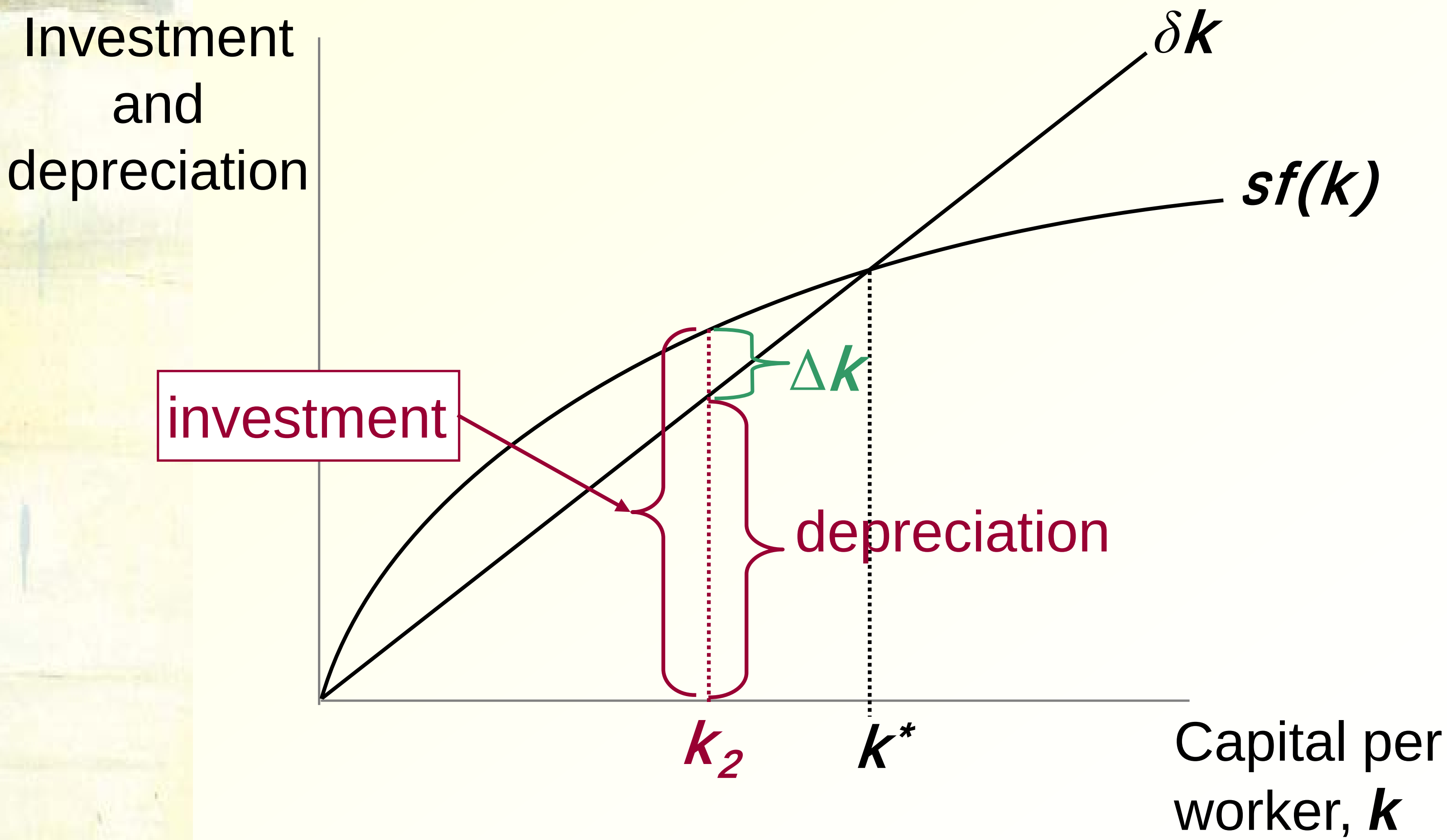

# Moving toward the steady state

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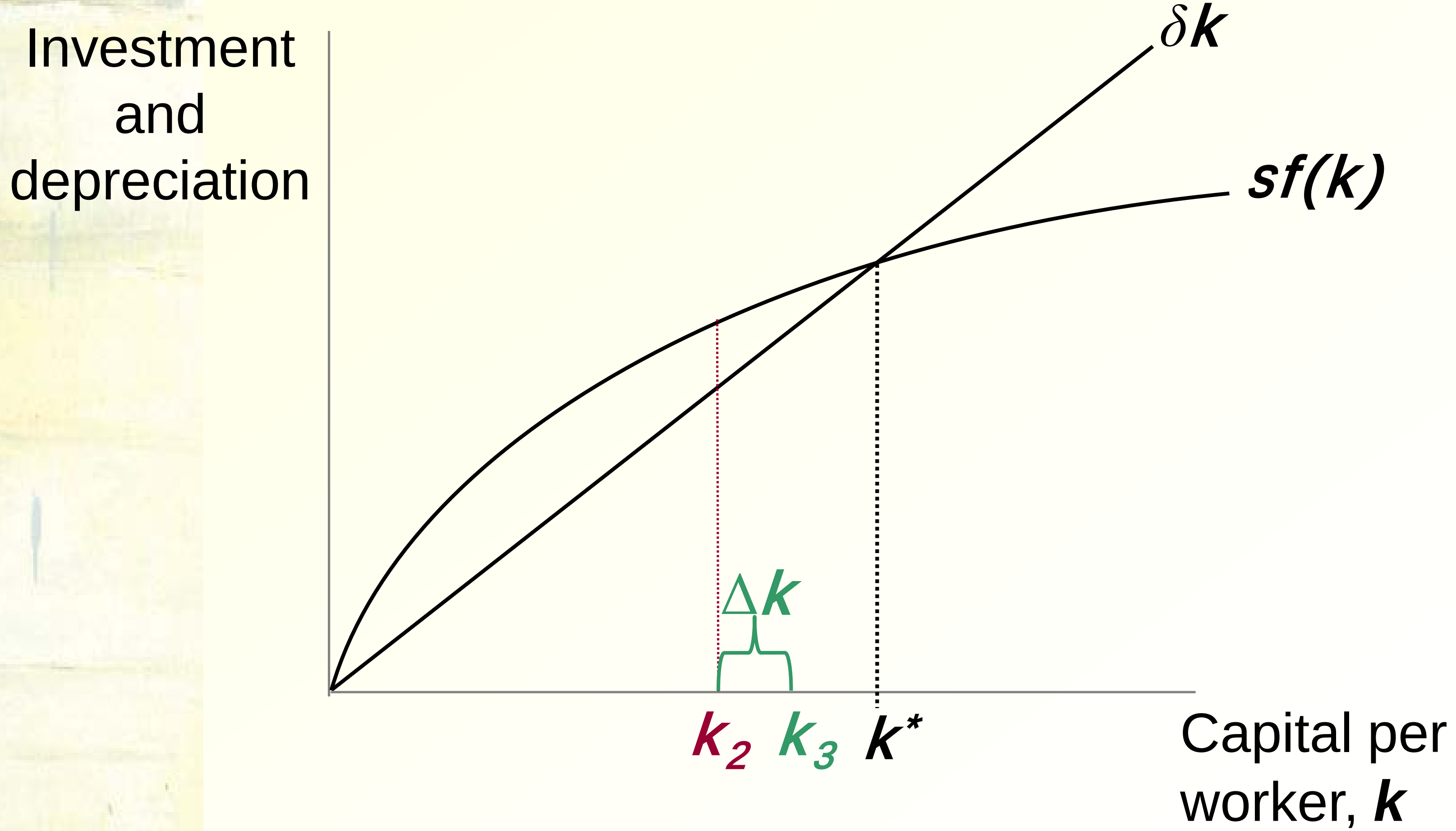
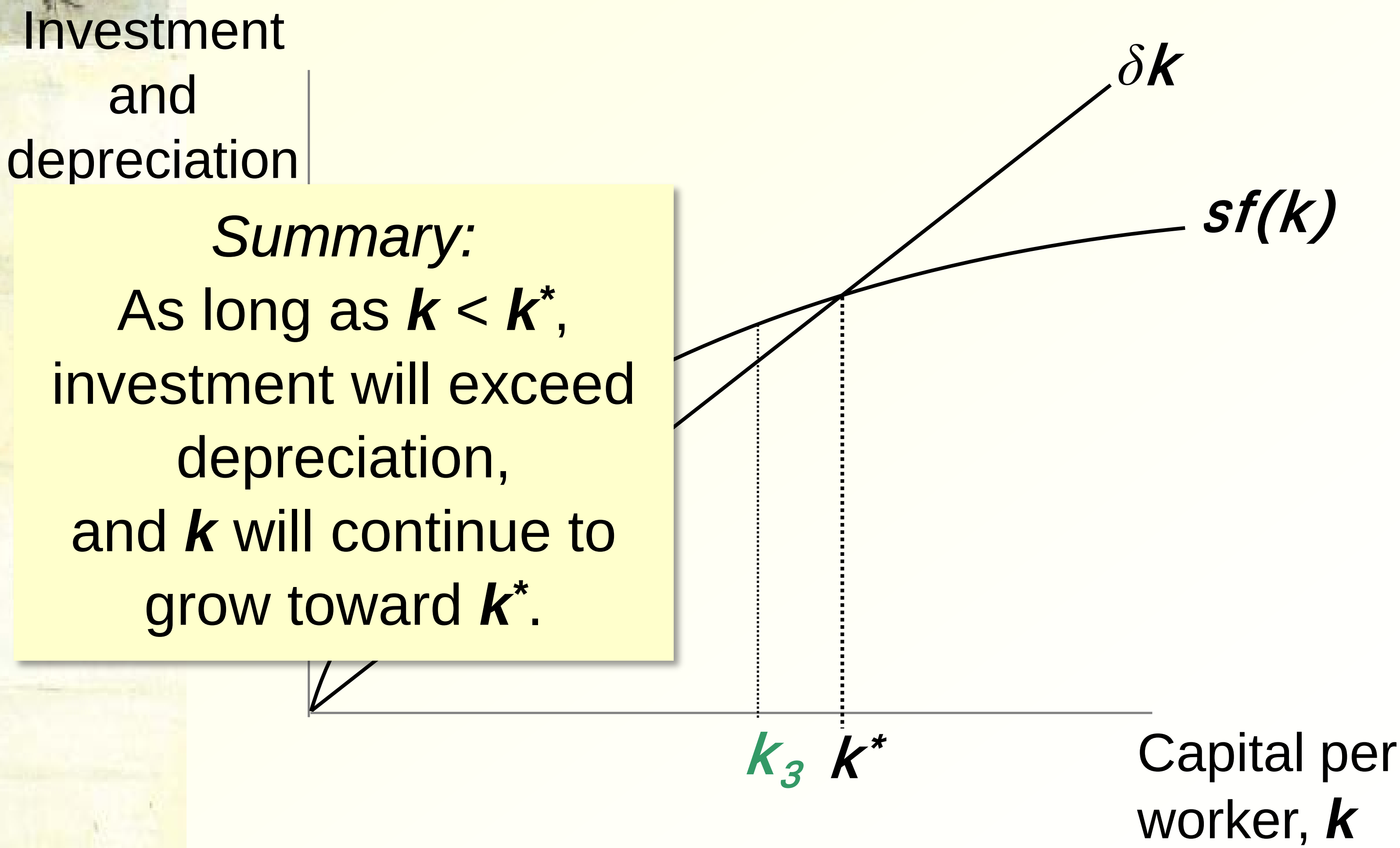


# Moving toward the steady state

$$\Delta k = sf(k) - \delta k$$




*Summary:*
As long as $k < k^*$, investment will exceed depreciation, and $k$ will continue to grow toward $k^*$.

# Now you try:

Draw the Solow model diagram, labeling the steady state $\boldsymbol{k^*}$.

On the horizontal axis, pick a value greater than $\boldsymbol{k^*}$ for the economy's initial capital stock.  Label it $\boldsymbol{k_1}$.

Show what happens to $\boldsymbol{k}$ over time. Does $\boldsymbol{k}$ move toward the steady state or away from it?

# A numerical example

Production function (aggregate):

$$\boldsymbol{Y = F(K, L) = \sqrt{K \times L} = K^{1/2} L^{1/2}}$$

To derive the per-worker production function, divide through by $L$:

$$\boldsymbol{\frac{Y}{L} = \frac{K^{1/2} L^{1/2}}{L} = \left(\frac{K}{L}\right)^{1/2}}$$

Then substitute $y$ = $Y/L$ and $k$ = $K/L$ to get

$$\boldsymbol{y = f(k) = k^{1/2}}$$

# A numerical example, *cont.*

Assume:

- $\boldsymbol{s} = 0.3$
- $\delta = 0.1$
- initial value of $\boldsymbol{k} = 4.0$

# A numerical example, *cont.*

- $y = f(k)$
- $c = (1 - s) f(k)$
- $i = s f(k)$
- $\Delta k = s\,f(k) - \delta k$

# Approaching the Steady State: A Numerical Example

**Assumptions:** $y = \sqrt{k}$; $s = 0.3$; $\delta = 0.1$; initial $k = 4.0$

| Year | $k$ | $y$ | $c$ | $i$ | $\delta k$ | $\Delta k$ |
|---|---|---|---|---|---|---|
| 1 | 4.000 | 2.000 | 1.400 | 0.600 | 0.400 | 0.200 |
| 2 | 4.200 | 2.049 | 1.435 | 0.615 | 0.420 | 0.195 |
| 3 | 4.395 | 2.096 | 1.467 | 0.629 | 0.440 | 0.189 |

# Approaching the Steady State: A Numerical Example

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| 2 | 4.200 | 2.049 | 1.435 | 0.615 | 0.420 | 0.195 |
| 3 | 4.395 | 2.096 | 1.467 | 0.629 | 0.440 | 0.189 |
| 4 | 4.584 | 2.141 | 1.499 | 0.642 | 0.458 | 0.184 |
| … | | | | | | |
| 10 | 5.602 | 2.367 | 1.657 | 0.710 | 0.560 | 0.150 |
| … | | | | | | |
| 25 | 7.351 | 2.706 | 1.894 | 0.812 | 0.732 | 0.080 |
| … | | | | | | |
| 100 | 8.962 | 2.994 | 2.096 | 0.898 | 0.896 | 0.002 |
| … | | | | | | |
| $\infty$ | 9.000 | 3.000 | 2.100 | 0.900 | 0.900 | 0.000 |

# Exercise: solve for the steady state

Continue to assume

$$s = 0.3, \quad \delta = 0.1, \quad \text{and} \quad y = k^{1/2}$$

Use the "law of motion"

$$\Delta k = s\,f(k) - \delta k$$

to solve for the steady-state values of ***k***, ***y***, and ***c***.

# Solution to exercise:

$\Delta \boldsymbol{k} = 0$ def. of steady state

$\boldsymbol{s}\,\boldsymbol{f}(\boldsymbol{k}^*) = \delta \boldsymbol{k}^*$ eq'n of motion with $\Delta \boldsymbol{k} = 0$

$0.3\sqrt{\boldsymbol{k}^*} = 0.1\boldsymbol{k}^*$ using assumed values

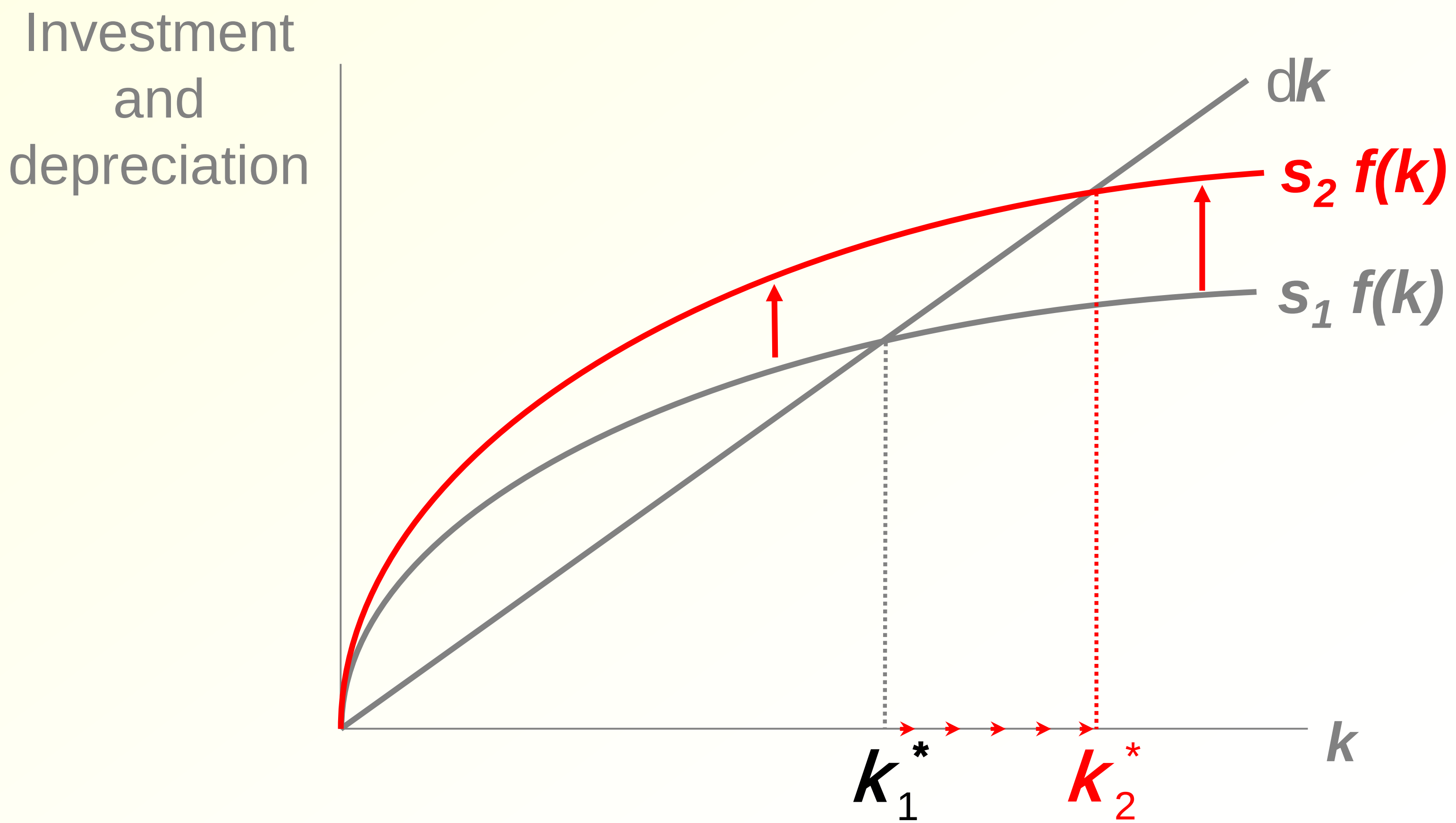
$$3 = \frac{\boldsymbol{k}^*}{\sqrt{\boldsymbol{k}^*}} = \sqrt{\boldsymbol{k}^*}$$

Solve to get: $\boldsymbol{k}^* = 9$ and $\boldsymbol{y}^* = \sqrt{\boldsymbol{k}^*} = 3$

Finally, $\boldsymbol{c}^* = (1 - \boldsymbol{s})\boldsymbol{y}^* = 0.7 \times 3 = 2.1$

# An increase in the saving rate

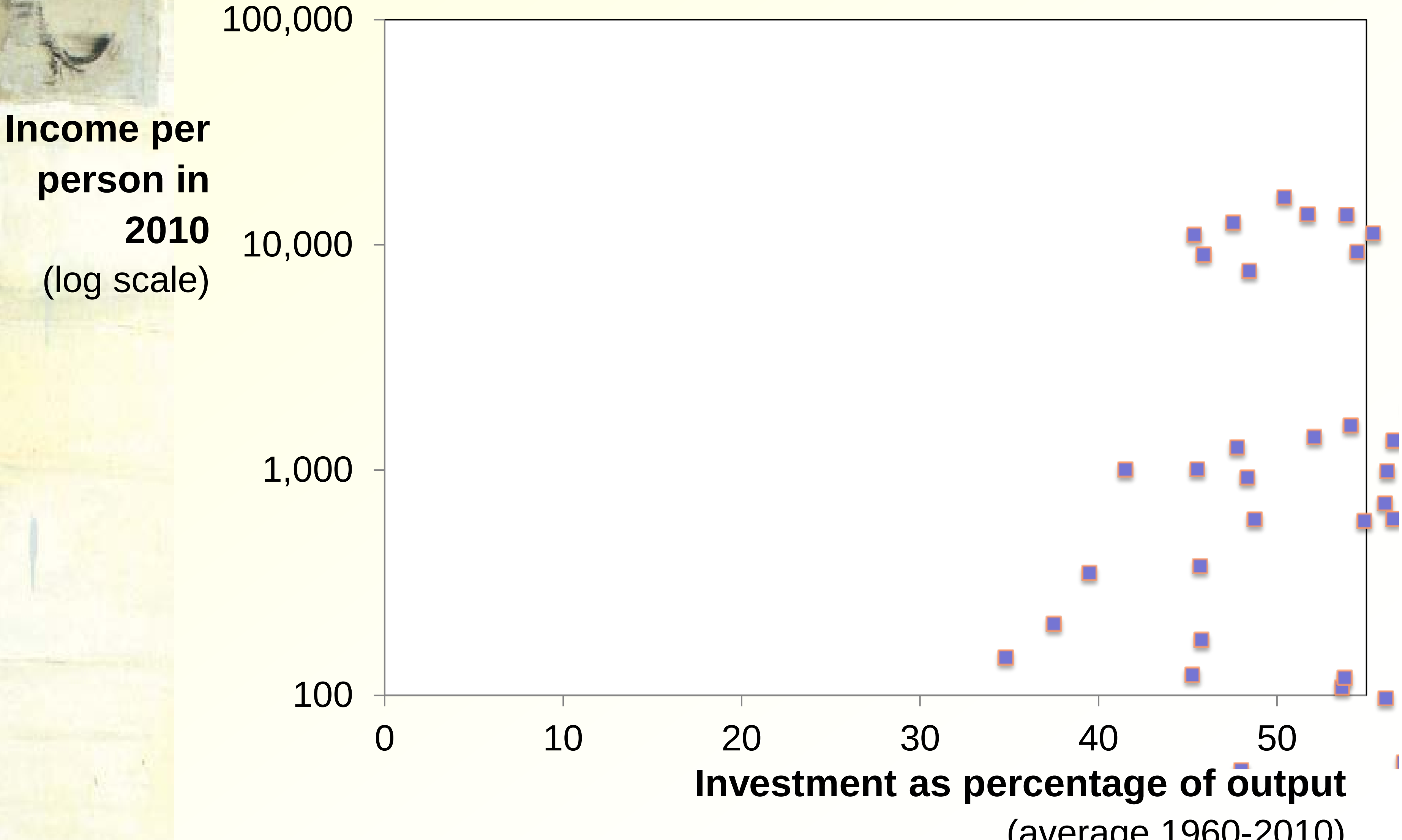
An increase in the savings rate raises investment...
...causing the capital stock to grow toward a new steady state:



# Prediction:

- Higher $\boldsymbol{s}$ $\Rightarrow$ higher $\boldsymbol{k^*}$.
- And since $\boldsymbol{y} = \boldsymbol{f(k)}$ ,
  higher $\boldsymbol{k^*}$ $\Rightarrow$ higher $\boldsymbol{y^*}$ .
- Thus, the Solow model predicts that countries with higher rates of saving and investment
  will have higher levels of capital and income per worker **in the long run**.

# International evidence on investment rates and income per person



# The Golden Rule:  introduction

- Different values of $\boldsymbol{s}$ lead to different steady states. How do we know which is the "best" steady state?
- Economic well-being depends on consumption, so the "best" steady state has the highest possible value of consumption per person:   $\boldsymbol{c^* = (1 - s)\, f(k^*)}$
- An increase in $\boldsymbol{s}$
  - leads to higher $\boldsymbol{k^*}$ and $\boldsymbol{y^*}$, which may raise $\boldsymbol{c^*}$
  - reduces consumption's share of income $(\boldsymbol{1-s})$, which may lower $\boldsymbol{c^*}$
- So, how do we find the $\boldsymbol{s}$ and $\boldsymbol{k^*}$ that maximize $\boldsymbol{c^*}$?

# The Golden Rule Capital Stock

$k^*_{gold}$ = the **Golden Rule level of capital,** the steady state value of $k$ that maximizes consumption.

To find it, first express $c^*$ in terms of $k^*$:

$$\begin{aligned} c^* &= y^* - i^* \\ &= f(k^*) - i^* \\ &= f(k^*) - \delta k^* \end{aligned}$$

In general:

$$i = \Delta k + \delta k$$

In the steady state:

$$i^* = \delta k^*$$

because $\Delta k = 0$.

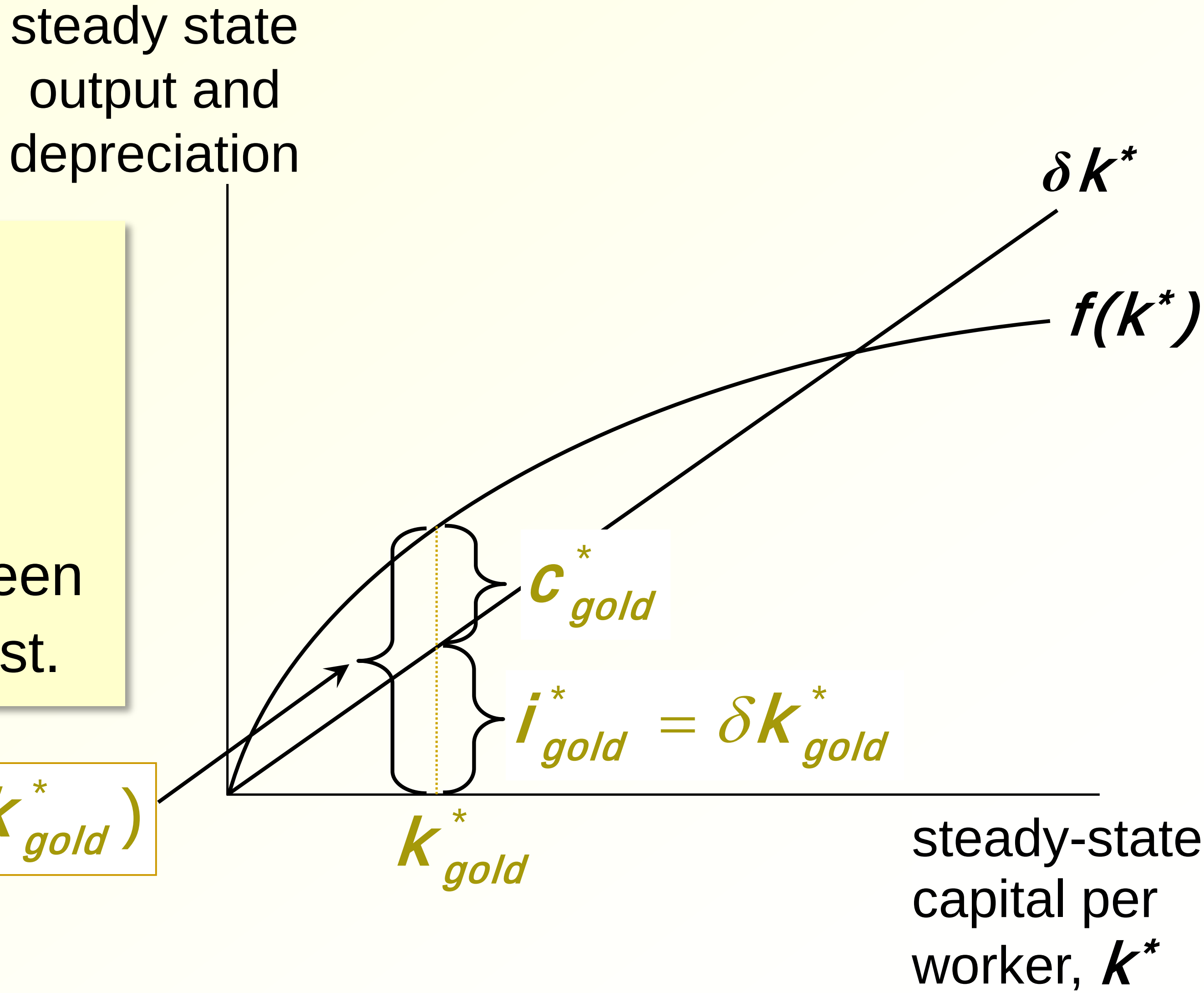
# The Golden Rule Capital Stock

- $f'(k) = \delta$

# The Golden Rule capital stock

Then, graph ***f*(*k**)** and ***δk****, look for the point where the gap between them is biggest.

$$y^*_{gold} = f(k^*_{gold})$$

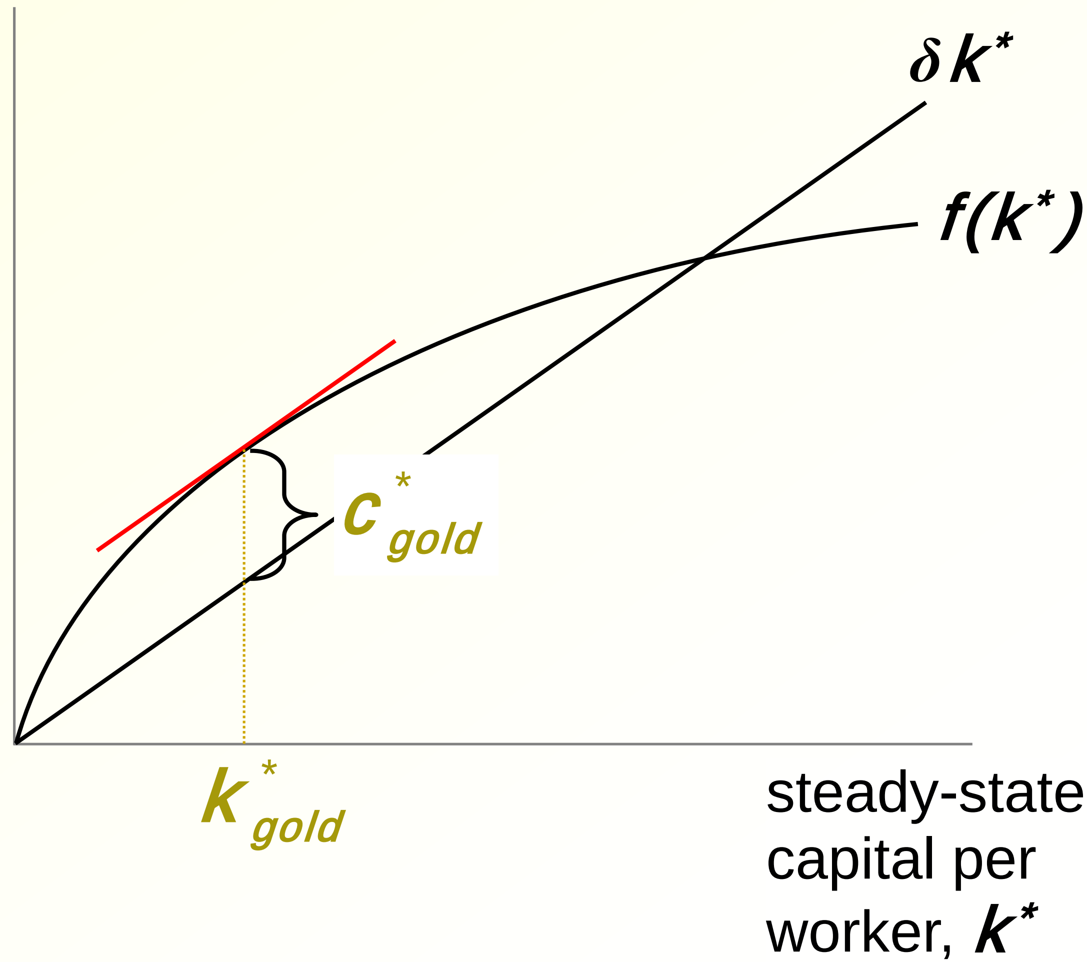


# The Golden Rule capital stock

$c^* = f(k^*) - \delta k^*$

is biggest where the slope of the production function equals the slope of the depreciation line:

$$\text{MPK} = \delta$$



# The transition to the Golden Rule Steady State

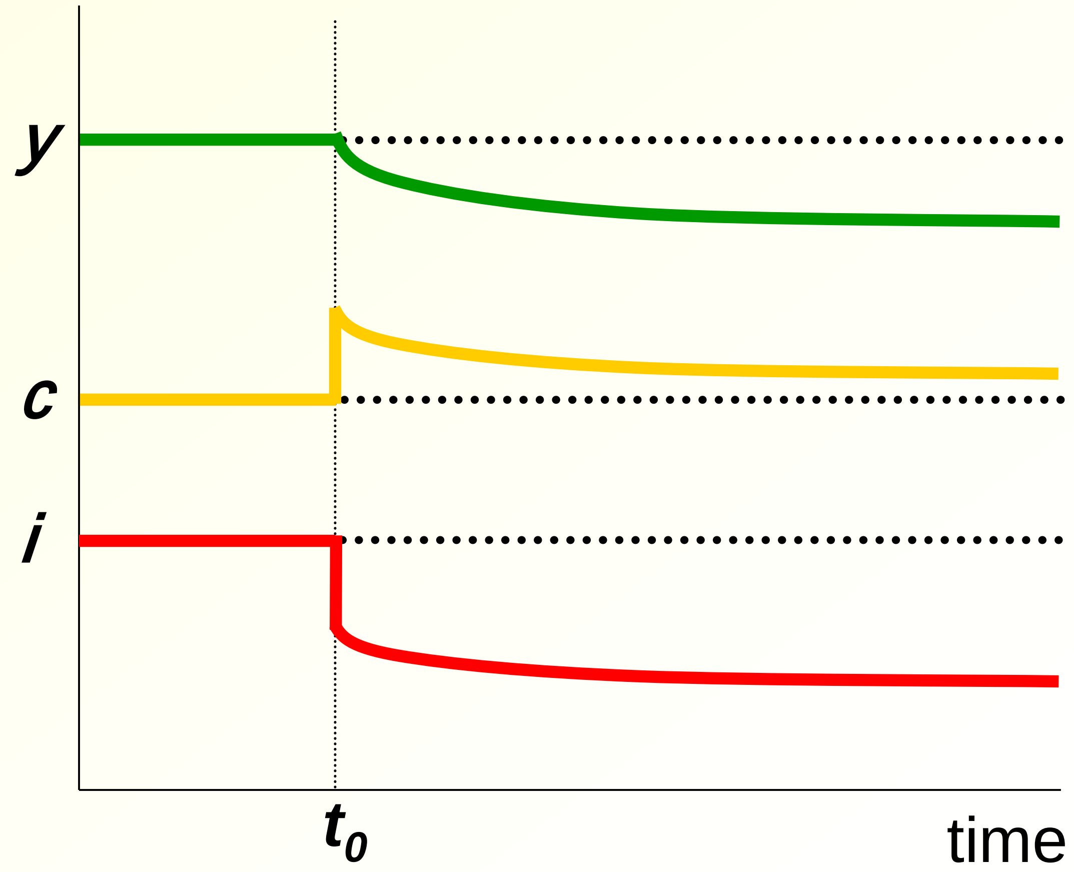
- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust ***s***.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

# Starting with too much capital

If $k^* > k^*_{gold}$

then increasing $c^*$ requires a fall in $s$.

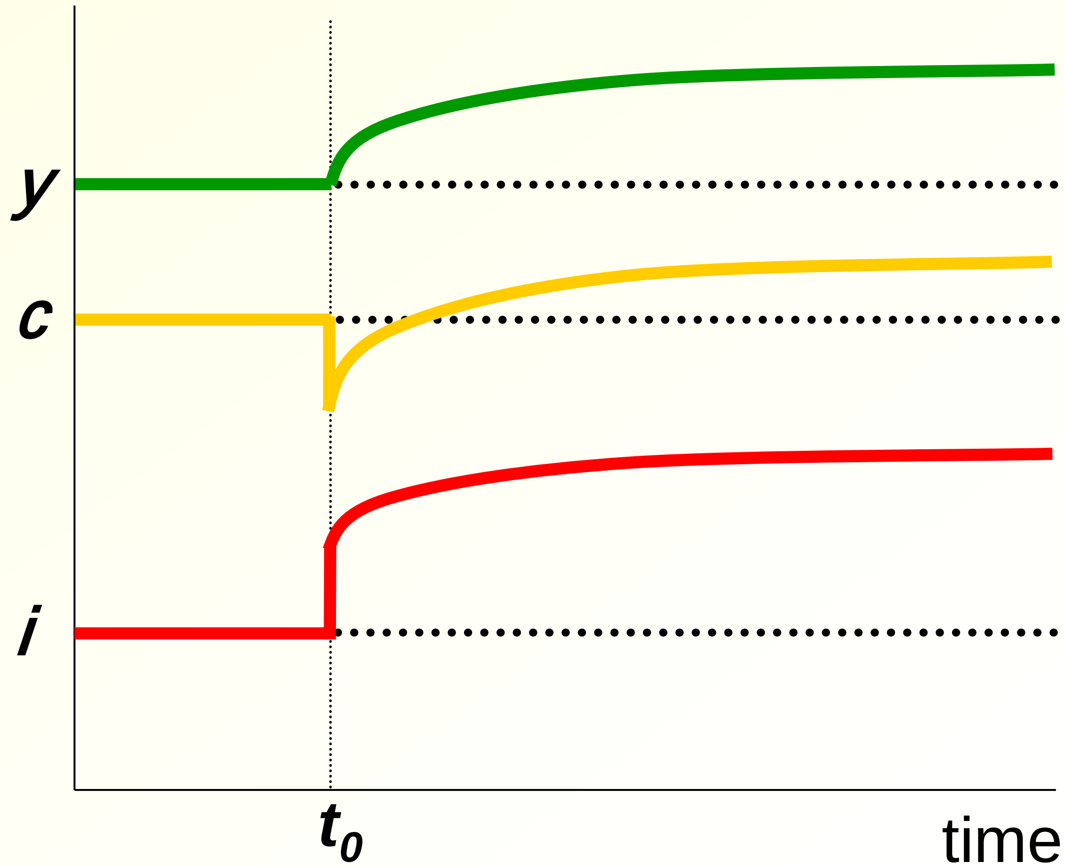
In the transition to the Golden Rule, consumption is higher at all points in time.



# Starting with too little capital

If $k^* < k^*_{gold}$ then increasing $c^*$ requires an increase in $s$.

Future generations enjoy higher consumption, but the current one experiences an initial drop in consumption.



# Population Growth

- Assume that the population--and labor force-- grow at rate ***n***.    (***n*** is exogenous)

$$\frac{\Delta \boldsymbol{L}}{\boldsymbol{L}} = \boldsymbol{n}$$

- EX:  Suppose ***L*** = 1000 in year 1 and the population is growing at 2%/year (***n*** = 0.02).

Then   $\Delta \boldsymbol{L} = \boldsymbol{n}\, \boldsymbol{L} = 0.02 \times 1000 = 20$, so ***L*** = 1020 in year 2.

# Break-even investment

$(\delta + n)k$ = **break-even investment**,
the amount of investment necessary to keep $k$ constant.

Break-even investment includes:

- $\delta k$ to replace capital as it wears out
- $nk$ to equip new workers with capital
  (*otherwise,* $k$ *would fall as the existing capital stock would be spread more thinly over a larger population of workers*)

# The law of motion for *k*

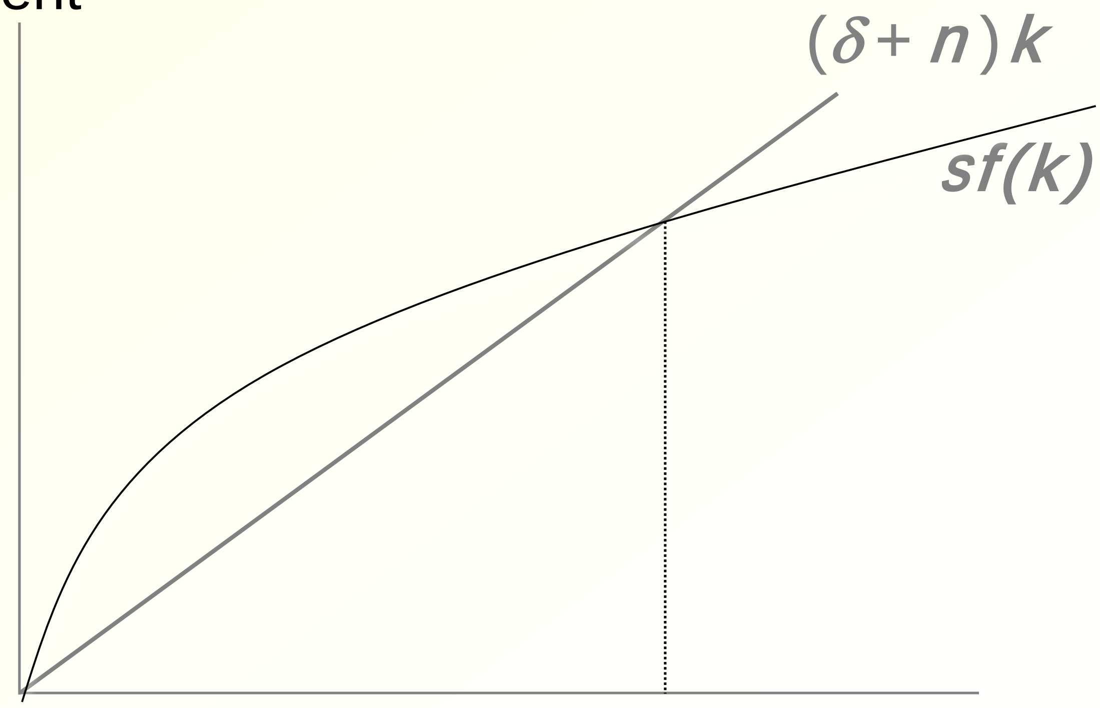
- With population growth, the law of motion for ***k*** becomes

$$\Delta k = \underbrace{s\,f(k)}_{\text{actual investment}} - \underbrace{(\delta + n)\,k}_{\text{break-even investment}}$$

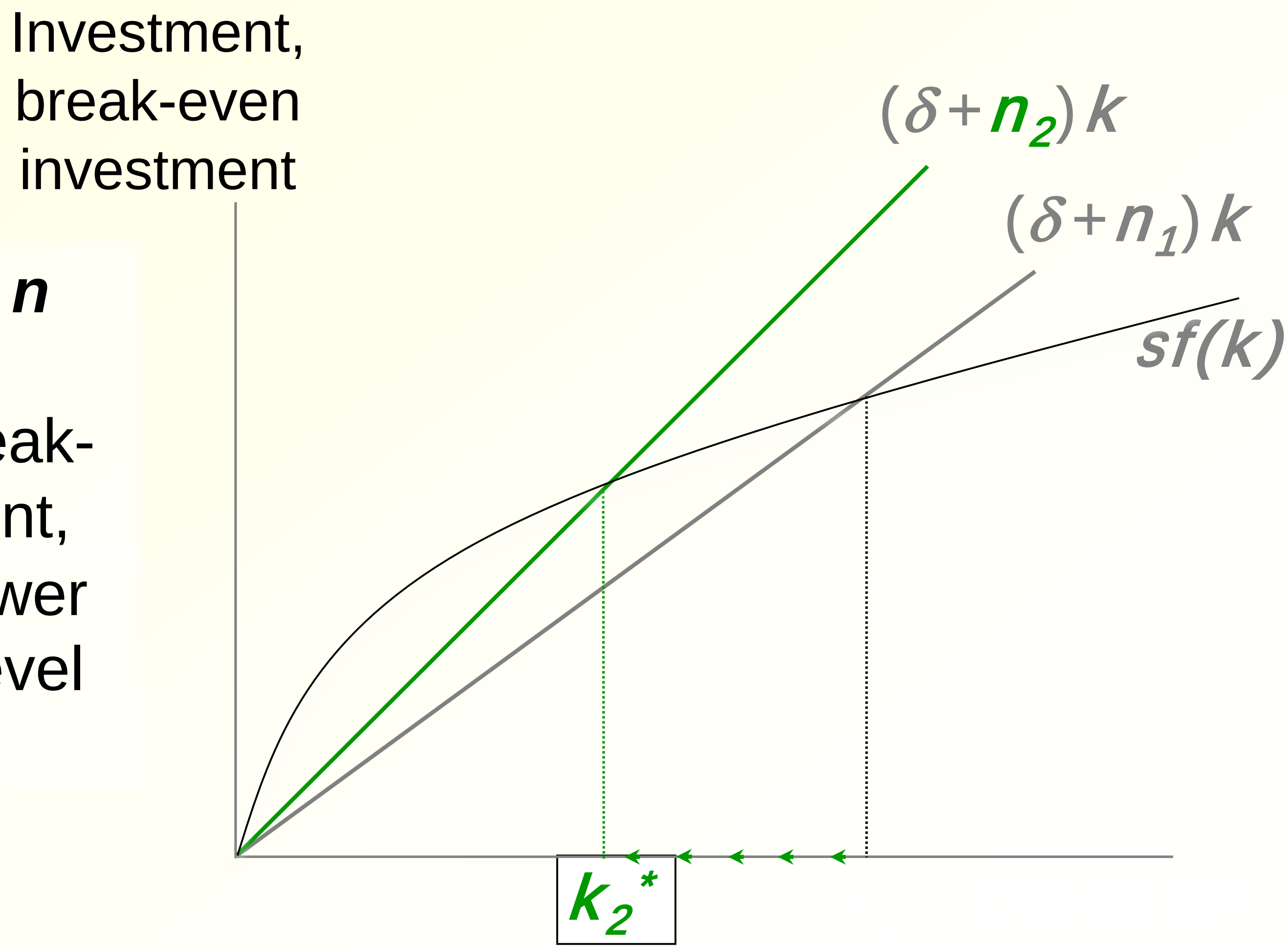
# The Solow Model diagram

$$\Delta k = s\,f(k) - (\delta + n)k$$

Investment,
break-even
investment



# The impact of population growth

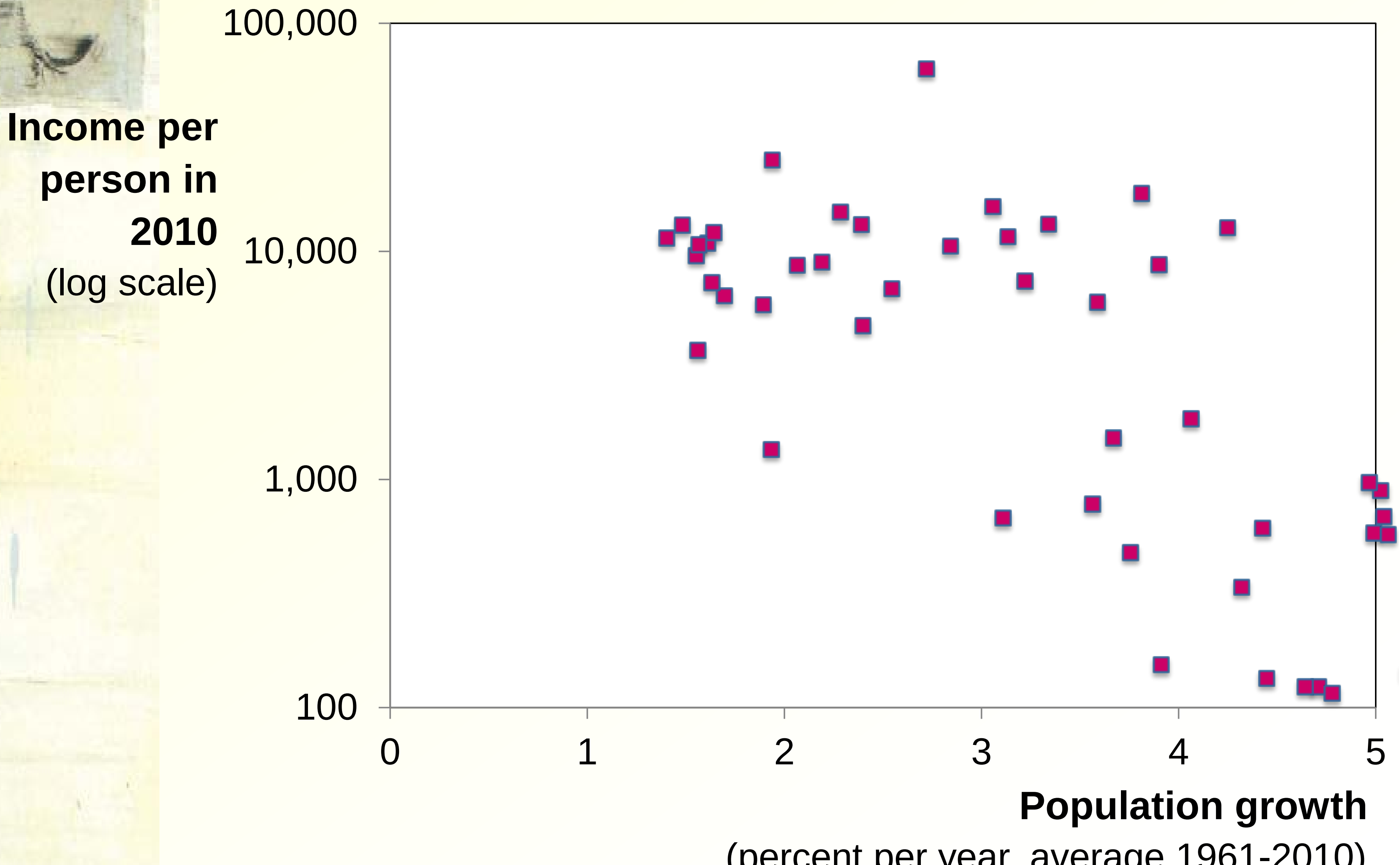



An increase in ***n*** causes an increase in break-even investment, leading to a lower steady-state level of ***k***.

# Prediction:

- Higher $\boldsymbol{n} \Rightarrow$ lower $\boldsymbol{k}^*$.
- And since $\boldsymbol{y} = \boldsymbol{f(k)}$ ,
  lower $\boldsymbol{k}^* \Rightarrow$ lower $\boldsymbol{y}^*$ .
- Thus, the Solow model predicts that in the long run countries with higher population growth rates will have lower levels of capital and income per worker.

# International evidence on population growth and income per person



# The Golden Rule with Population Growth

To find the Golden Rule capital stock, we again express $\boldsymbol{c}^*$ in terms of $\boldsymbol{k}^*$:

$$\boldsymbol{c}^* = \boldsymbol{y}^* - \boldsymbol{i}^*$$

$$= \boldsymbol{f}(\boldsymbol{k}^*) - (\delta + \boldsymbol{n})\,\boldsymbol{k}^*$$

$\boldsymbol{c}^*$ is maximized when

$$\text{MPK} = \delta + \boldsymbol{n}$$

or equivalently,

$$\boxed{\text{MPK} - \delta = \boldsymbol{n}}$$

In the Golden Rule Steady State, the marginal product of capital net of depreciation equals the population growth rate.

# Alternative perspectives on population growth

## The Malthusian Model (1798)

- Predicts population growth will outstrip the Earth’s ability to produce food, leading to the impoverishment of humanity.
- Since Malthus, world population has increased sixfold, yet living standards are higher than eve
- Malthus neglected the effects of technological progress.

# Alternative perspectives on population growth

## The Kremerian Model (1993)

- Posits that population growth contributes to economic growth.
- More people = more geniuses, scientists & engineers, so faster technological progress.
- Evidence, from very long historical periods:
  - As world pop. growth rate increased, so did rate of growth in living standards
  - Historically, regions with larger populations have enjoyed faster growth.

# Chapter Summary

1. The Solow growth model shows that, in the long run, a country's standard of living depends
   - positively on its saving rate.
   - negatively on its population growth rate.
2. An increase in the saving rate leads to
   - higher output in the long run
   - faster growth temporarily
   - but not faster steady state growth.

# Chapter Summary

3. If the economy has more capital than the Golden Rule level, then reducing saving will increase consumption at all points in time, making all generations better off.

   If the economy has less capital than the Golden Rule level, then increasing saving will increase consumption for future generations, but reduce consumption for the present generation.