Lecture 4

Demand

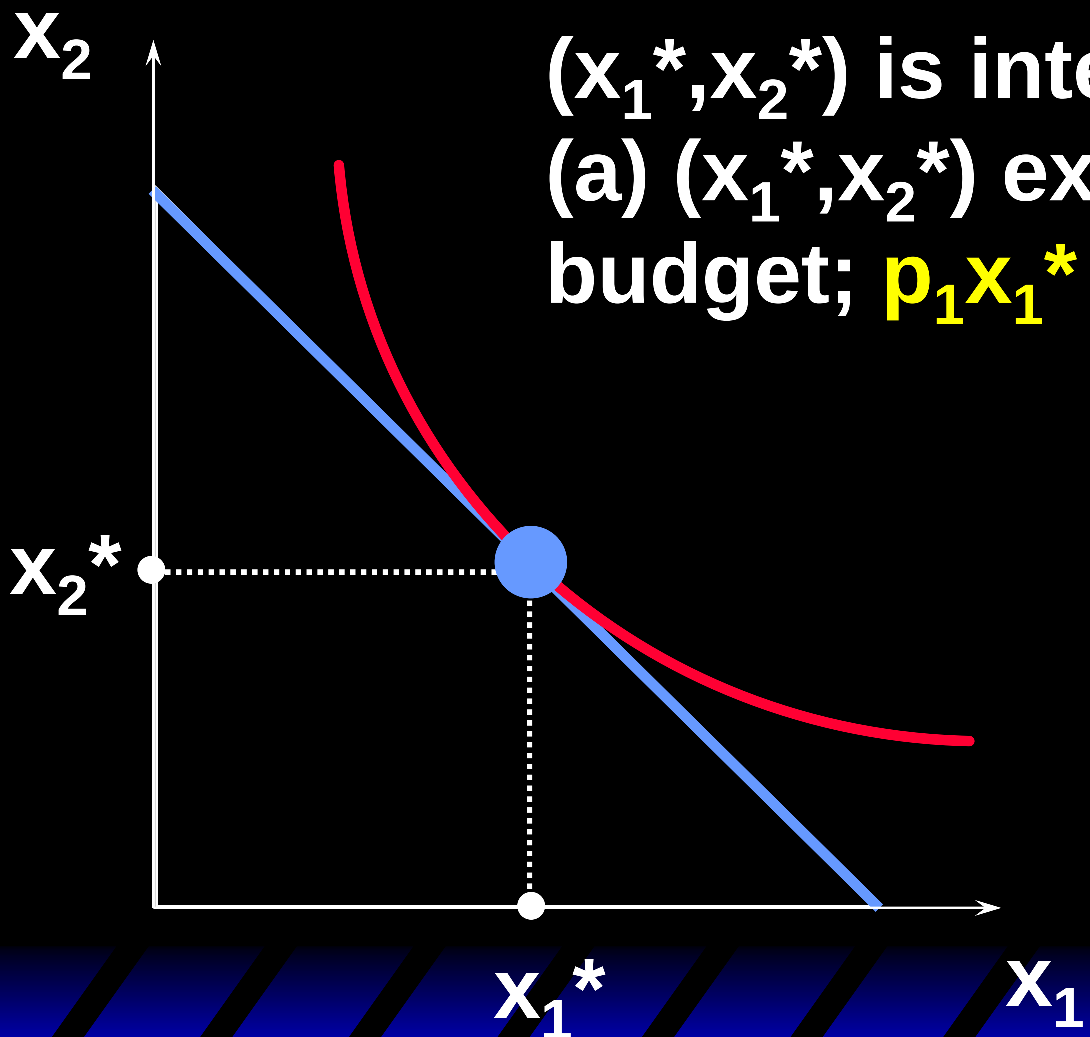
Review

Given preferences and budget constraint, the optimal consumption bundle can be expressed as functions of prices and income

$$(x_1^*(p_1,p_2,y),\ x_2^*(p_1,p_2,y))$$

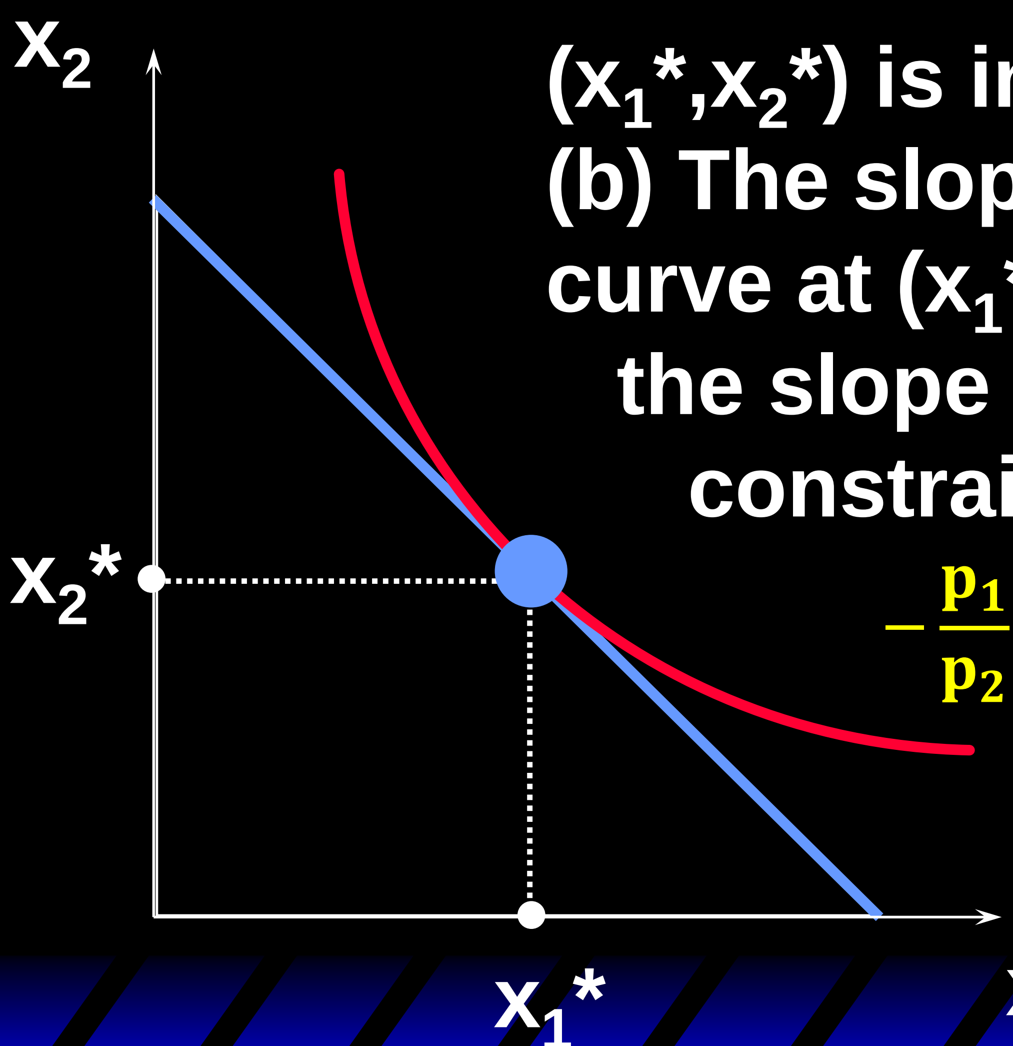
- Ordinary demand function

Review



(x_1^*,x_2^*) is interior.
(a) (x_1^*,x_2^*) exhausts the budget; $p_1x_1^* + p_2x_2^* = y$.

Review



(x_1^*,x_2^*) is interior .
(b) The slope of the indiff. curve at (x_1^*,x_2^*) equals the slope of the budget constraint.

$$-\frac{p_1}{p_2} = MRS = -\frac{MU_1}{MU_2}$$

Review: A Cobb-Douglas Example

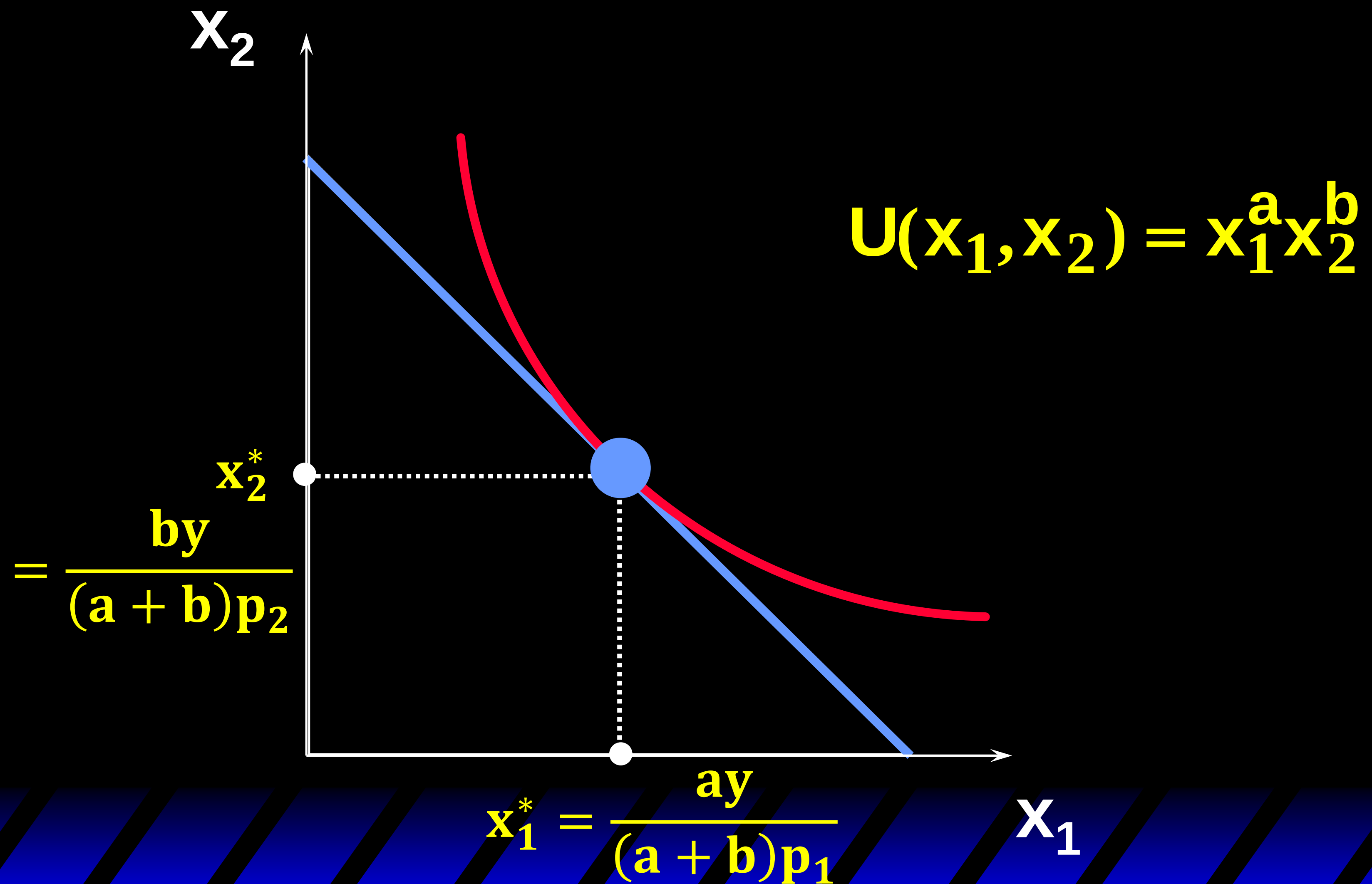
When $U(x_1, x_2) = x_1^a x_2^b$, we have

$$-\frac{ax_2^*}{bx_1^*} = -\frac{p_1}{p_2} \qquad \text{(A)}$$

$$p_1 x_1^* + p_2 x_2^* = m. \qquad \text{(B)}$$

Solving for x_1^* and x_2^* =>

Review: A Cobb-Douglas Example



Today's Lecture

Properties of Demand Functions

Comparative statics analysis of ordinary demand functions -- the study of how ordinary demands $x_1^*(p_1,p_2,y)$ and $x_2^*(p_1,p_2,y)$ change as prices p_1, p_2 and income y change.

比较静态分析：研究所关心的变量如何随某一参数的变化而变化的分析方法。

Own-Price Changes

How does $x_1^*(p_1,p_2,y)$ change as p_1 changes, holding p_2 and y **constant**?

Suppose only p_1 increases, from p_1' to p_1'' and then to p_1'''.

Own-Price Changes

The curve containing all the utility-maximizing bundles traced out as p_1 changes, with p_2 and y constant, is the p_1- price offer curve.

价格提供曲线：收入和其它价格不变时，最优商品组合随某一商品价格变化的轨迹线。

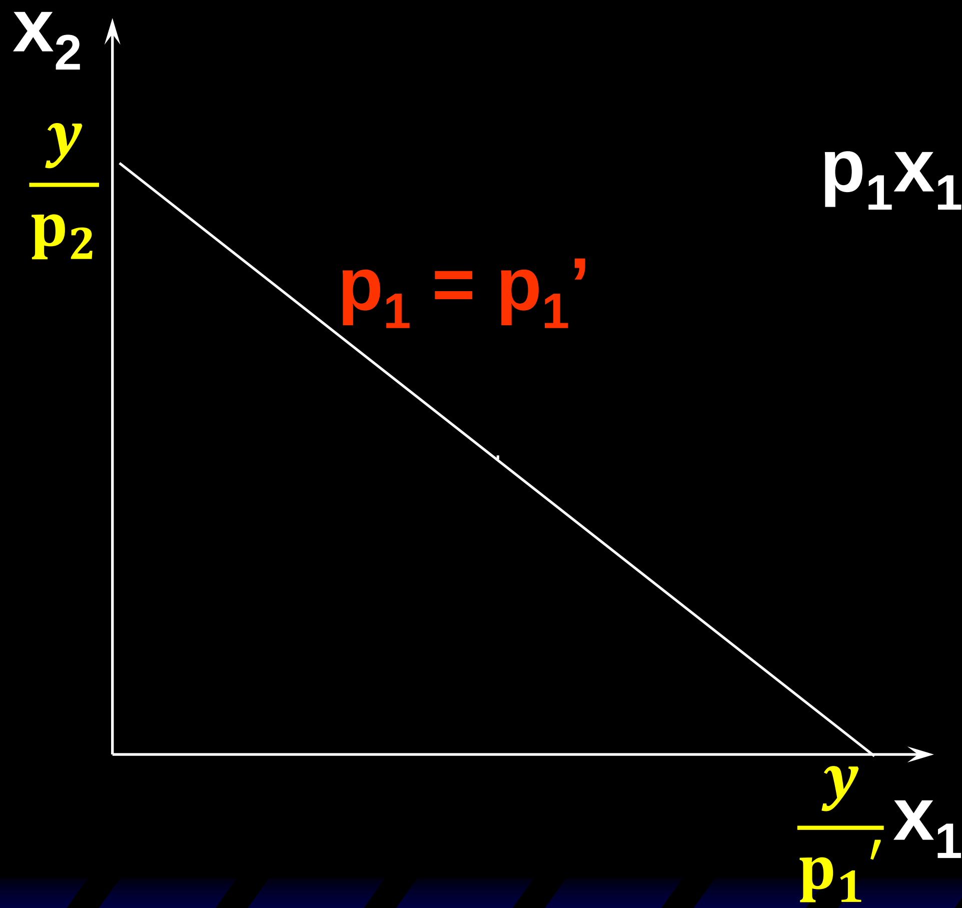
Own-Price Changes

The plot of the x_1-coordinate of the p_1- price offer curve against p_1 is the **ordinary** demand curve for commodity 1.

普通需求曲线：收入和其它价格不变时，描述某种商品的最优消费数量和自身价格关系的曲线。

Own-Price Changes

Fixed p_2 and y.

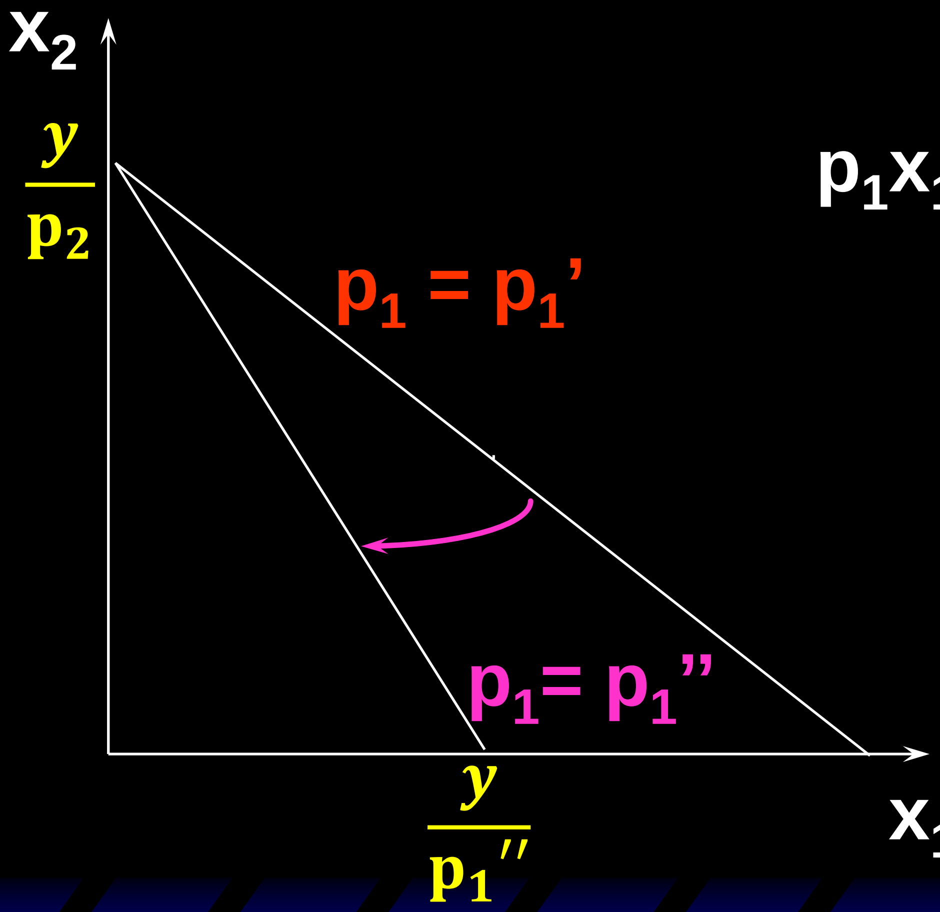


$p_1x_1 + p_2x_2 = y$

$$\text{slope} = -\frac{p_1}{p_2}$$

Own-Price Changes

Fixed p_2 and y.



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Own-Price Changes

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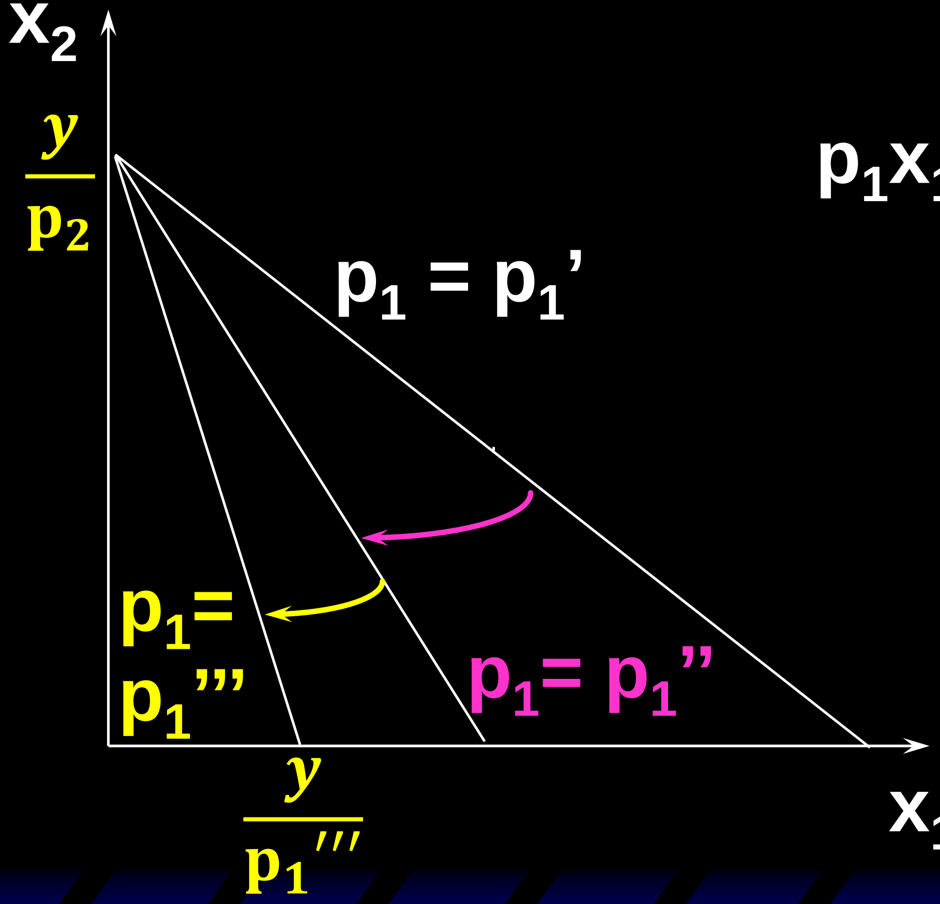

Own-Price Changes

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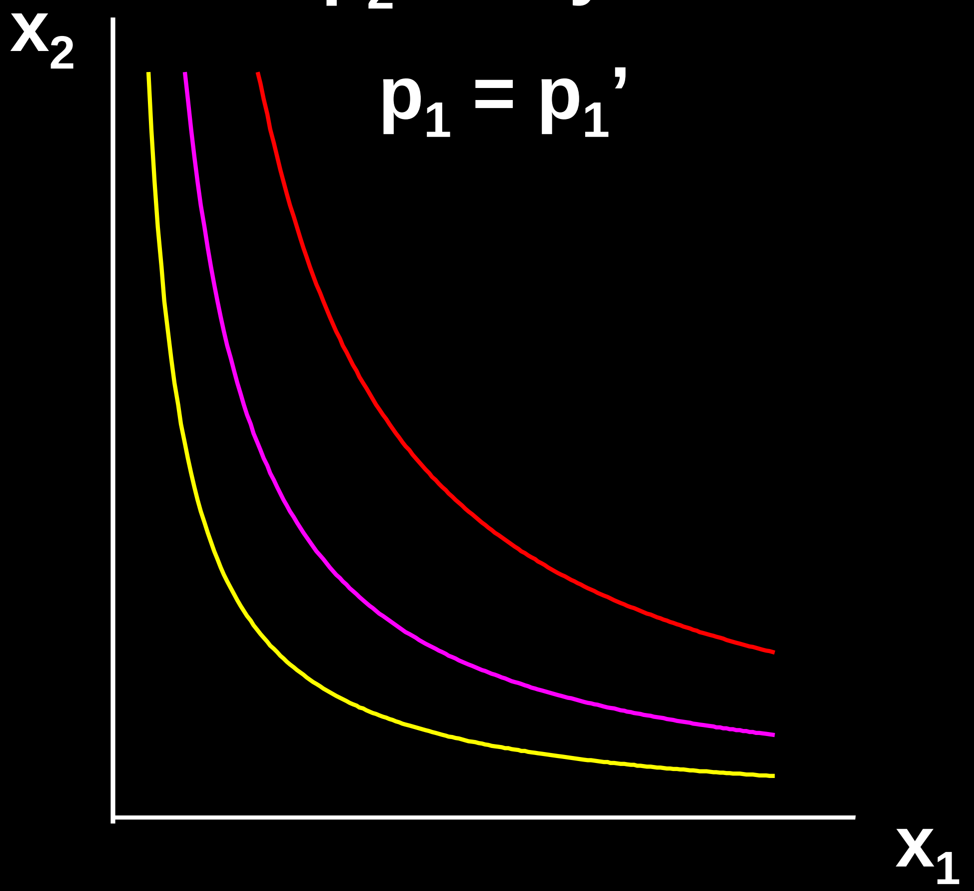

Own-Price Changes

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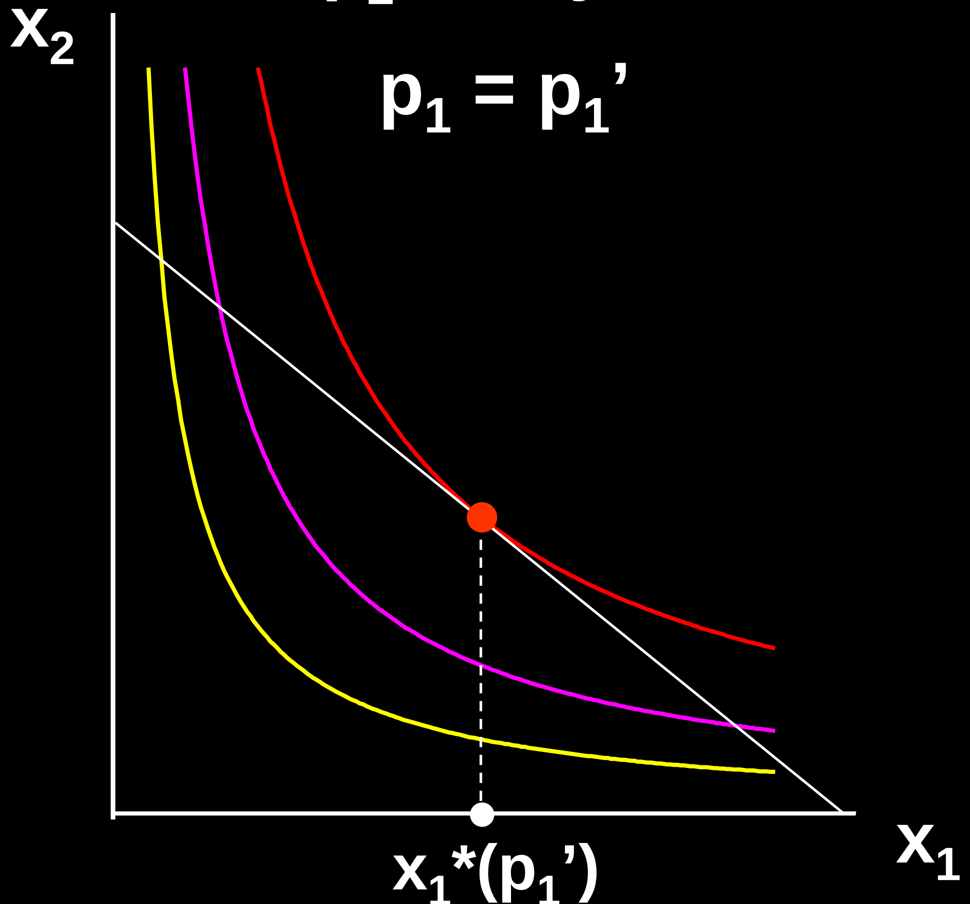

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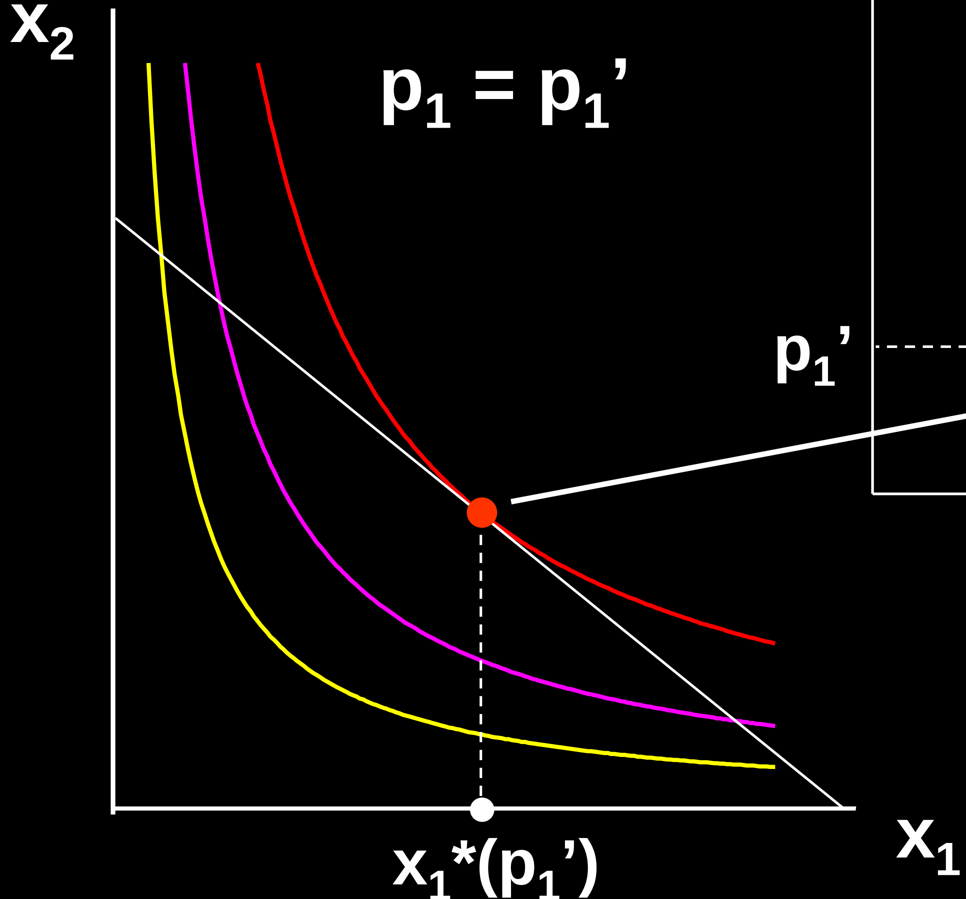
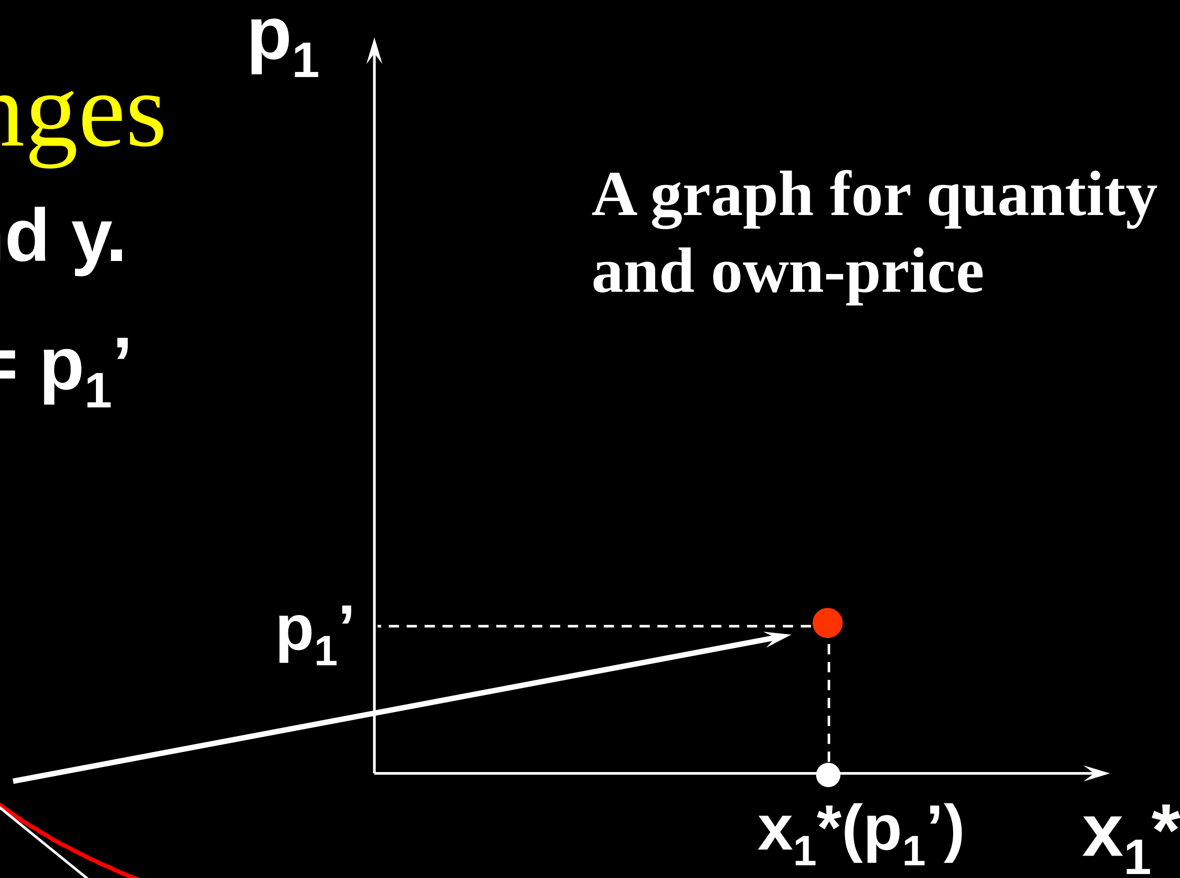


A graph for quantity and own-price



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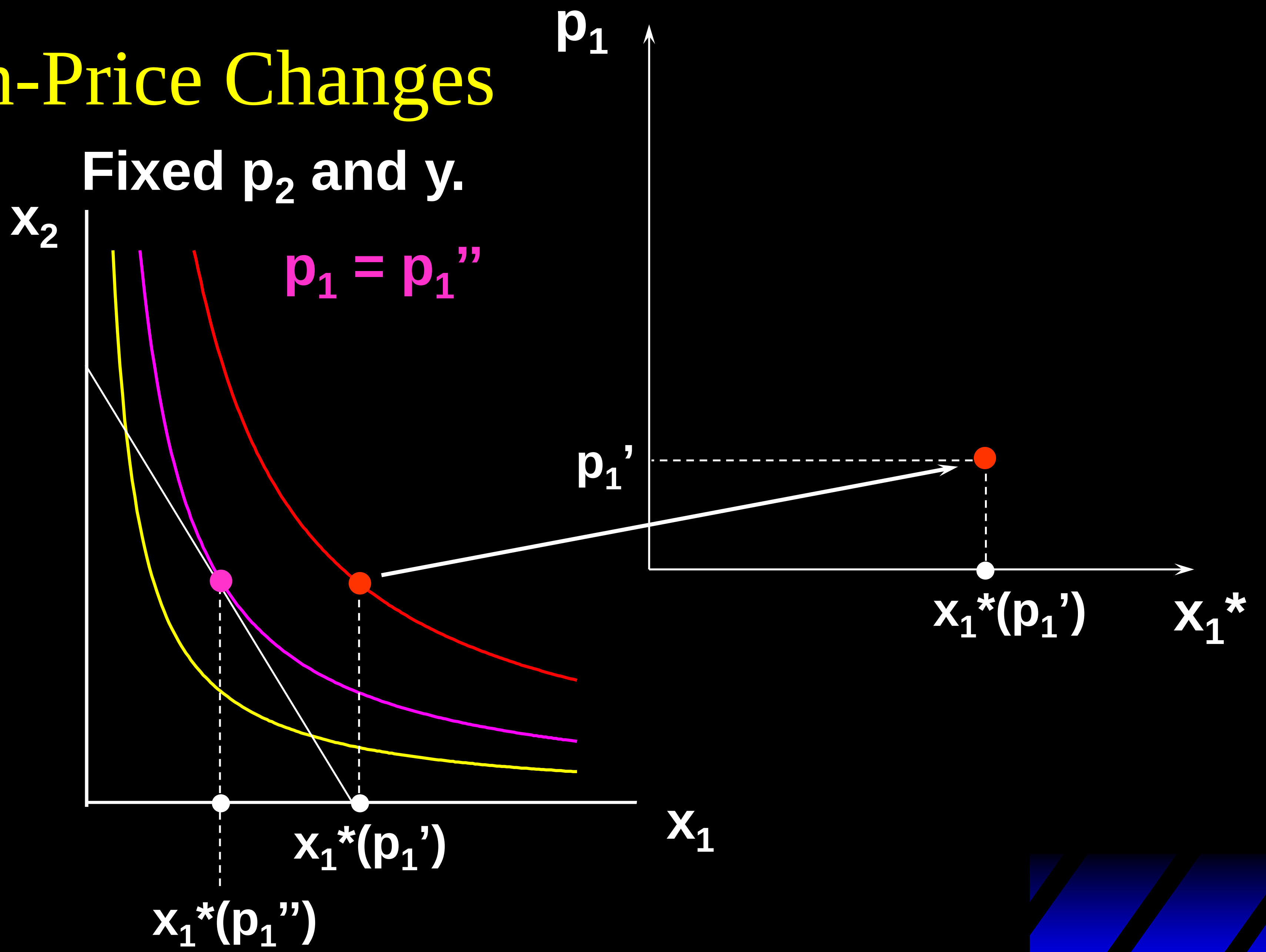

Own-Price Changes

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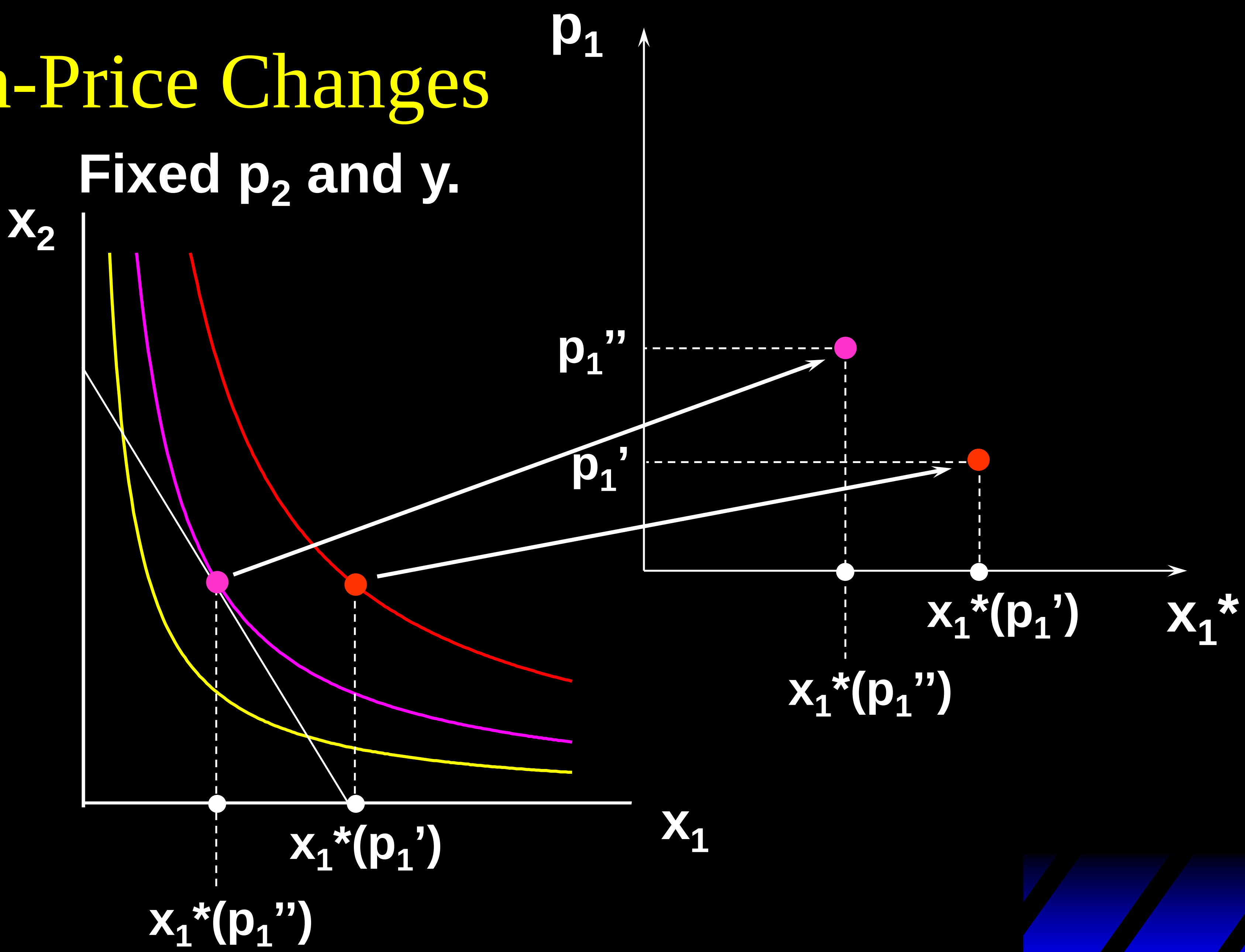
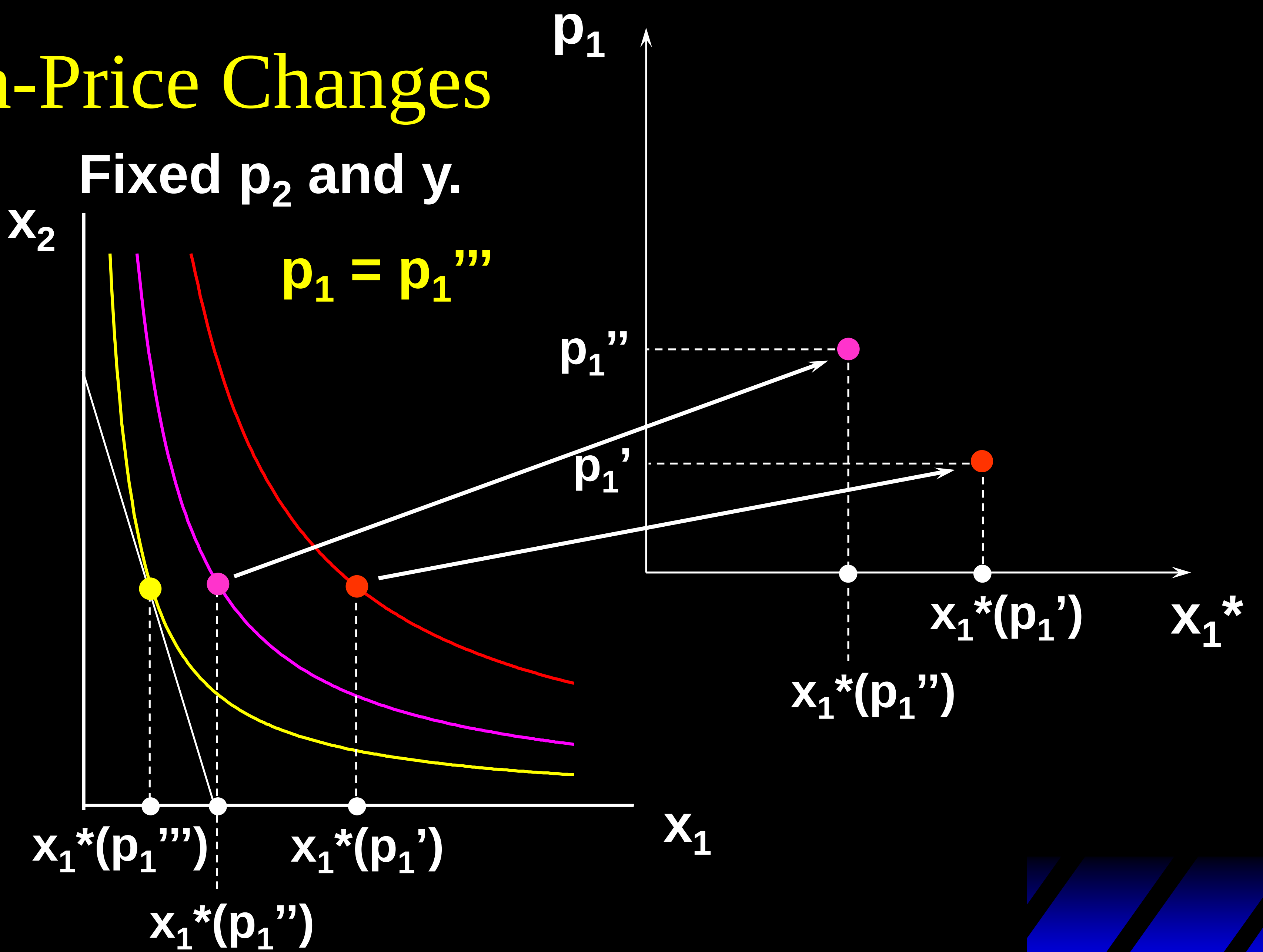


Own-Price Changes



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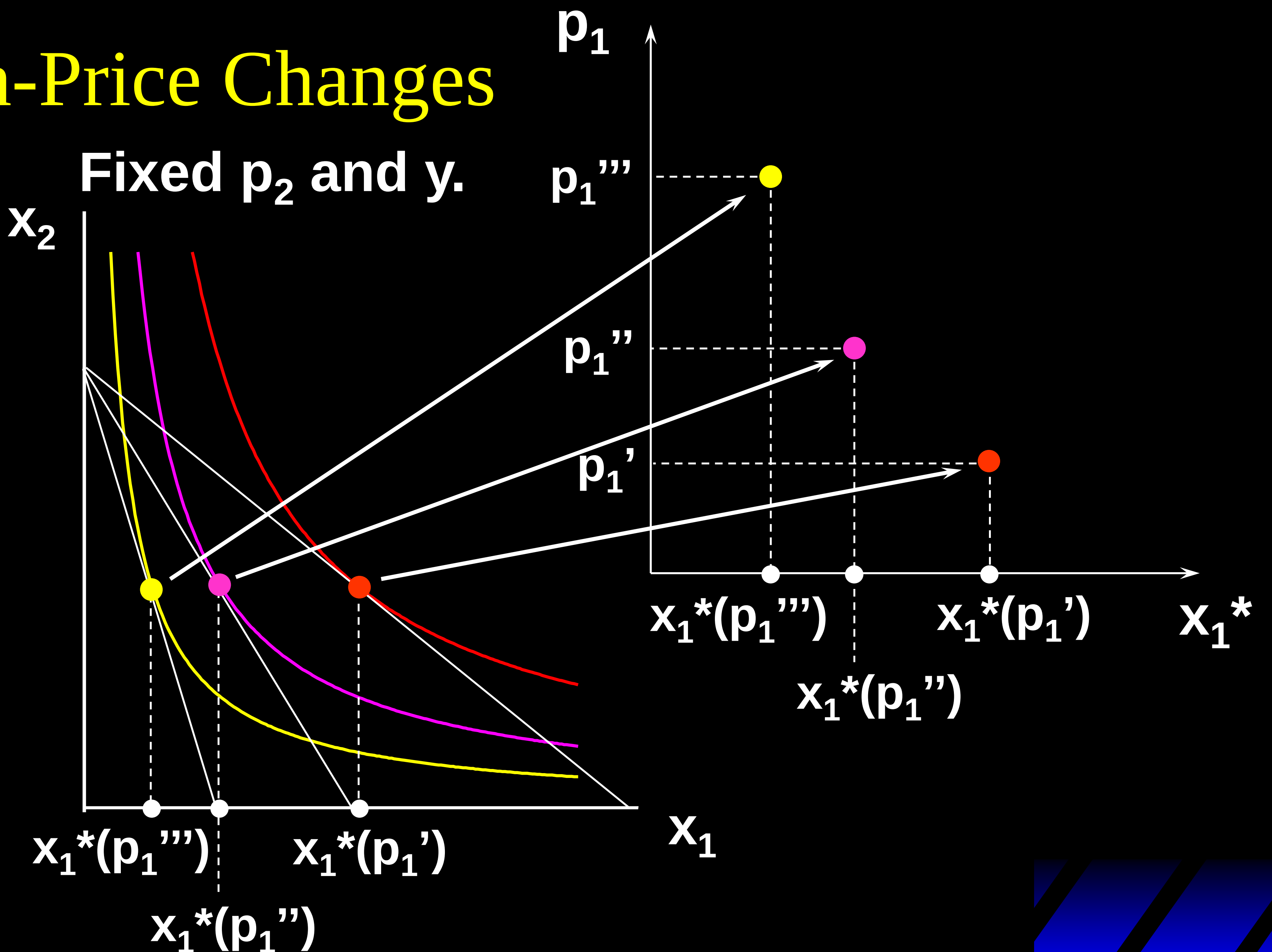

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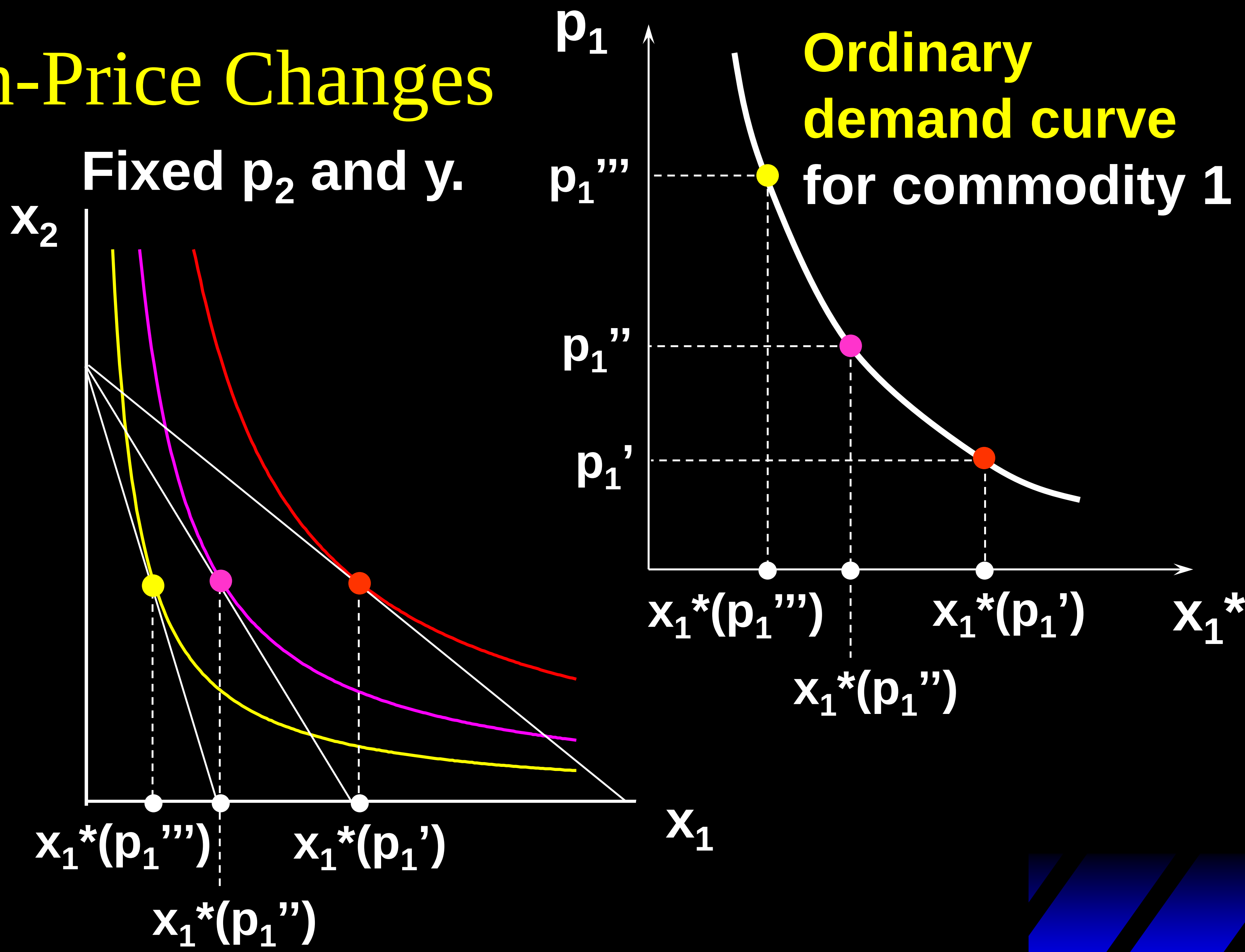
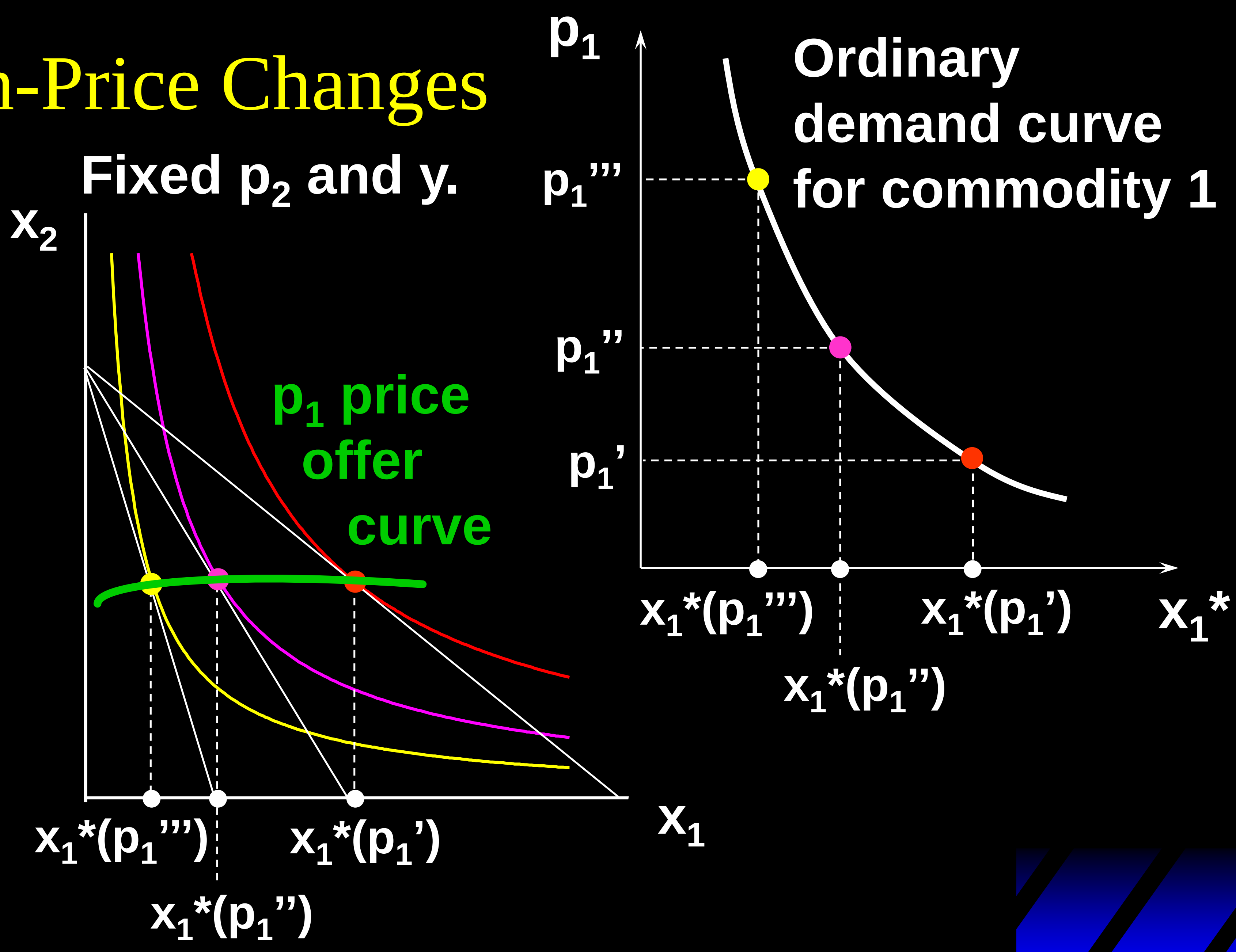


Own-Price Changes



Own-Price Changes

What does a p_1 price-offer curve look like for Cobb-Douglas preferences?

Own-Price Changes

What does a p_1 price-offer curve look like for Cobb-Douglas preferences?

Take

$$U(x_1, x_2) = x_1^a x_2^b.$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

$$U(x_1, x_2) = x_1^a x_2^b.$$

Recall that under C-D utility function, the consumer always spends $\frac{a}{a+b}y$ on x_1 and $\frac{b}{a+b}y$ on x_2

$$p_1 x_1^* = \frac{a}{a+b}y$$

$$p_2 x_2^* = \frac{b}{a+b}y$$

Own-Price Changes

$$x_1^*(p_1,p_2,y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

and

$$x_2^*(p_1,p_2,y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

Own-Price Changes

$$x_1^*(p_1,p_2,y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

and

$$x_2^*(p_1,p_2,y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

Notice that x_2* does not vary with p_1 so the p_1 price offer curve is flat

Own-Price Changes

$$x_1^*(p_1,p_2,y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

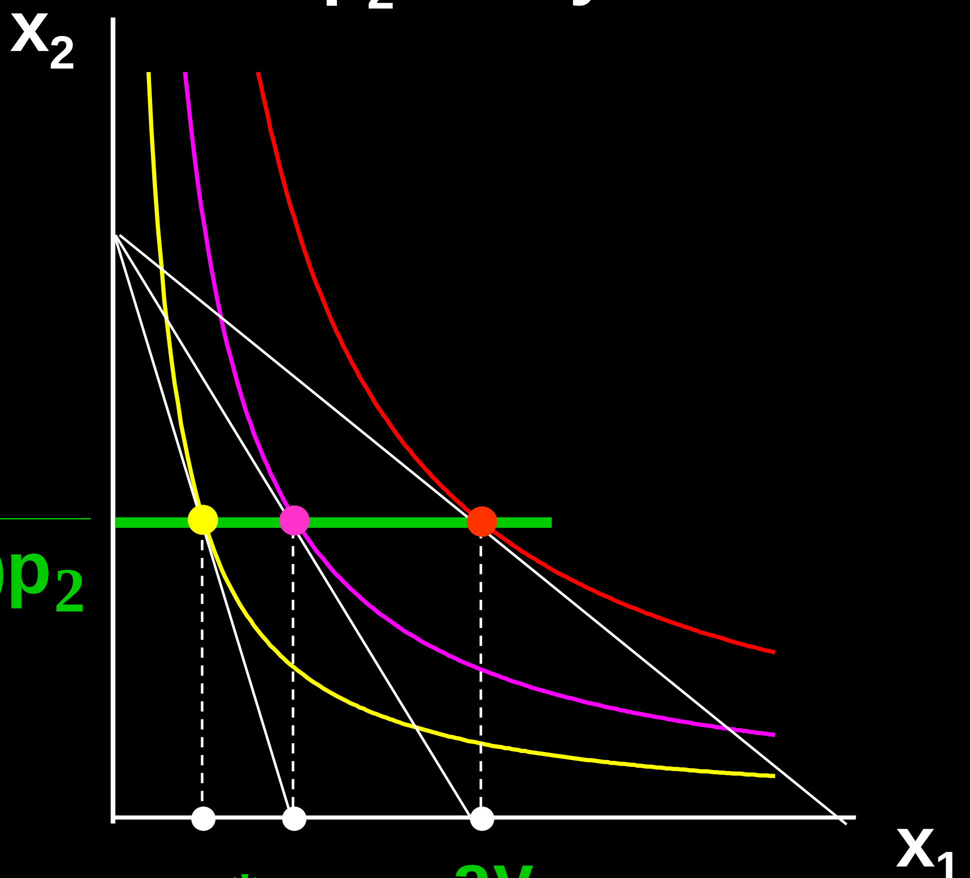
and

$$x_2^*(p_1,p_2,y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

Notice that x_2* does not vary with p_1 so the p_1 price offer curve is **flat** and the ordinary demand curve for commodity 1 is a **rectangular hyperbola**.

Own-Price Changes

Fixed p_2 and y.

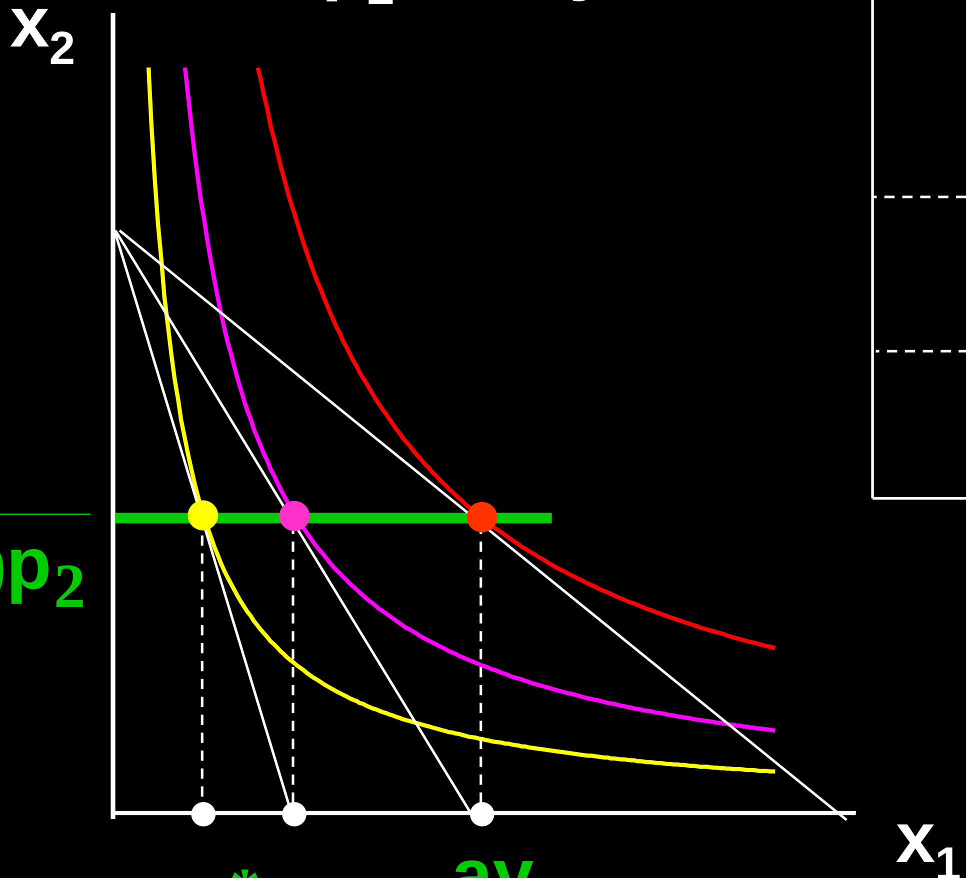


$$x_2^* = \frac{by}{(a+b)p_2}$$

$$x_1^* = \frac{ay}{(a+b)p_1}$$

Own-Price Changes

Fixed p_2 and y.



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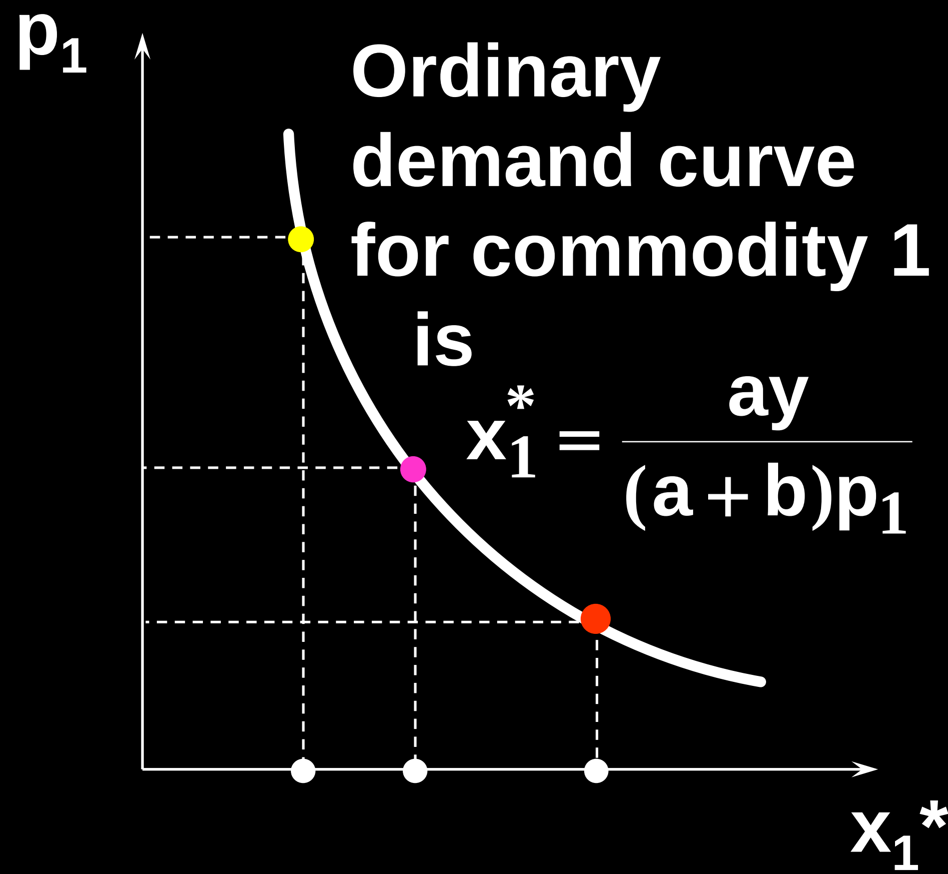


Ordinary demand curve for commodity 1 is

$$x_1^* = \frac{ay}{(a+b)p_1}$$

Own-Price Changes

What does a p_1 price-offer curve look like for a perfect-complements utility function?

Own-Price Changes

What does a p_1 price-offer curve look like for a perfect-complements utility function?

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

$x_1^* = x_2^*$ **(A)**

$p_1 x_1^* + p_2 x_2^* = y$ **(B)**

$$x_1^*(p_1, p_2, y) = x_2^*(p_1, p_2, y) = \frac{y}{p_1 + p_2}.$$

Own-Price Changes

$$x_1^*(p_1, p_2, y) = x_2^*(p_1, p_2, y) = \frac{y}{p_1 + p_2}.$$

With p_2 and y fixed, higher p_1 causes smaller x_1* and x_2*.

Own-Price Changes

$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1+p_2}.$$

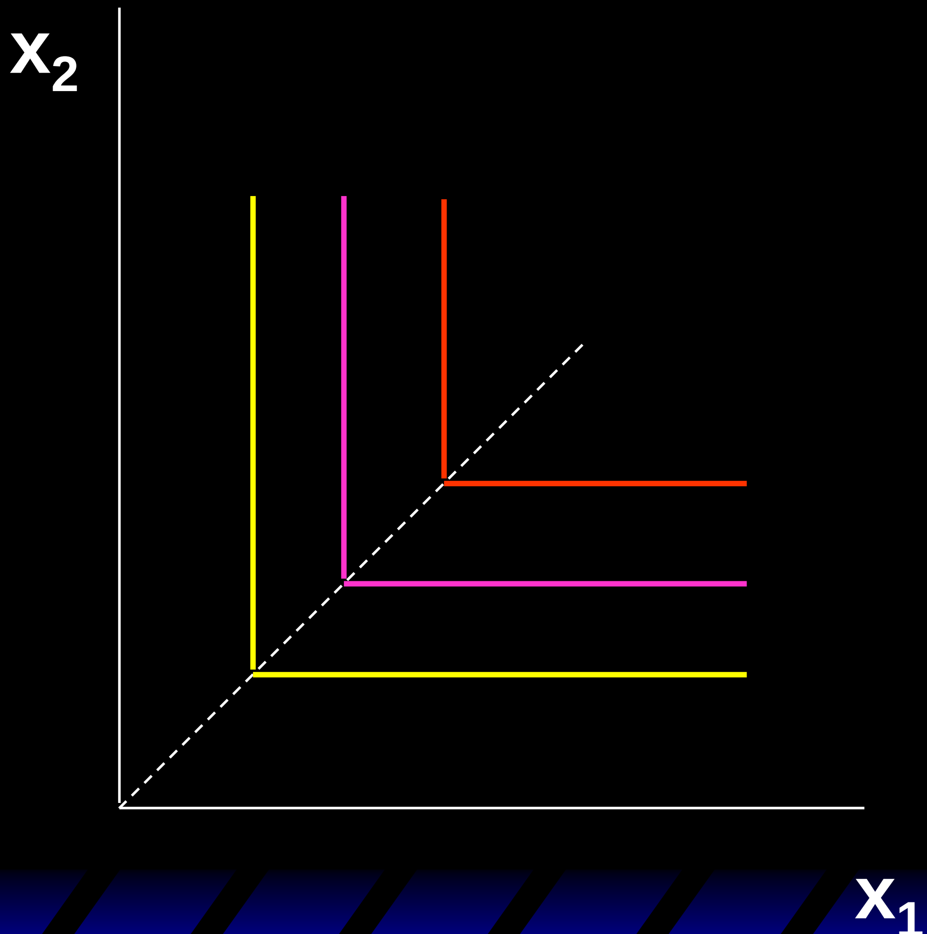
With p_2 and y fixed, higher p_1 causes smaller x_1* and x_2*.

As $p_1 \to 0, \quad x_1^* = x_2^* \to \dfrac{y}{p_2}.$

As $p_1 \to \infty, \quad x_1^* = x_2^* \to 0.$

Own-Price Changes

Fixed p_2 and y.



Own-Price Changes

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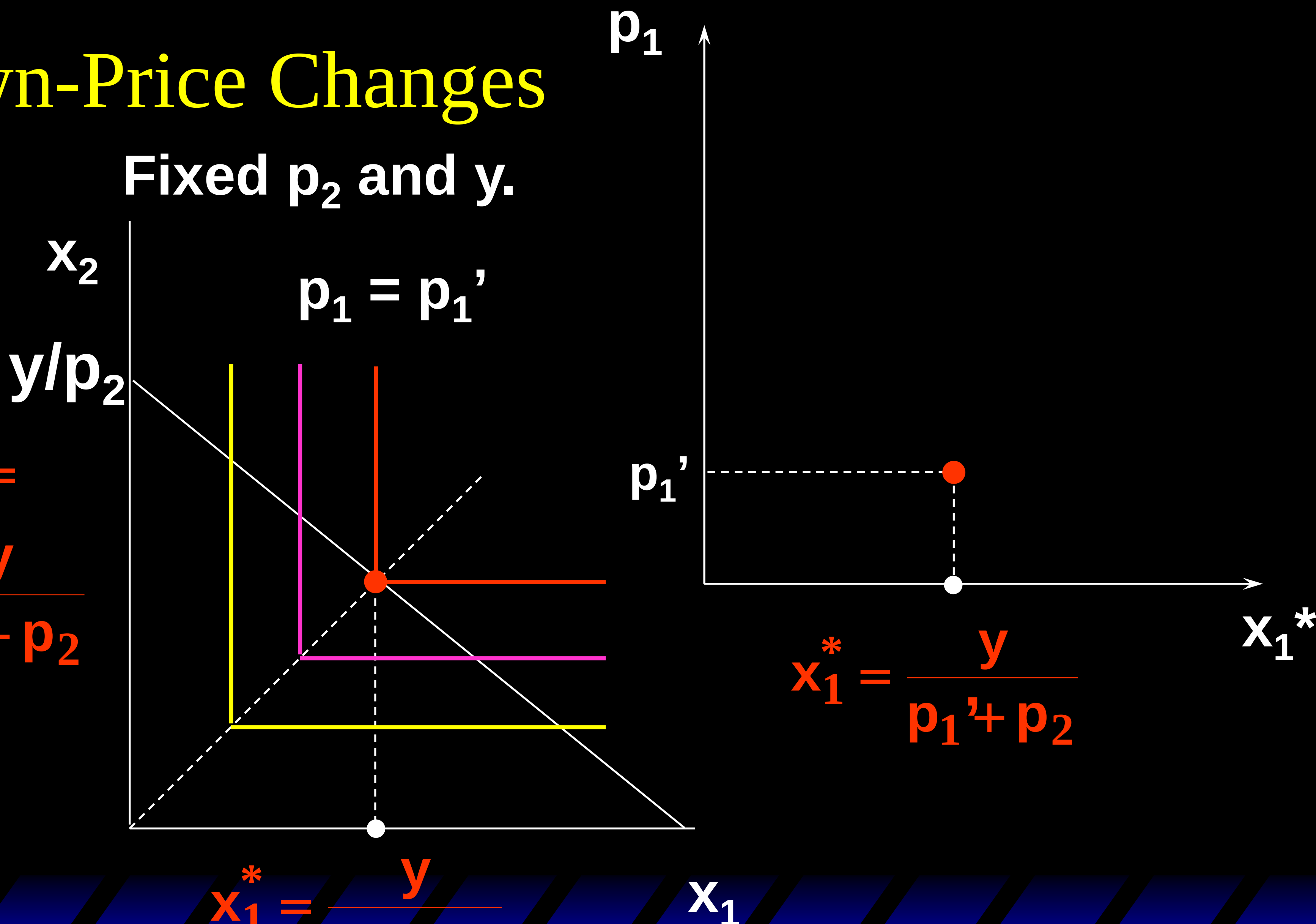

$$x_2^* = \frac{y}{p_1' + p_2}$$

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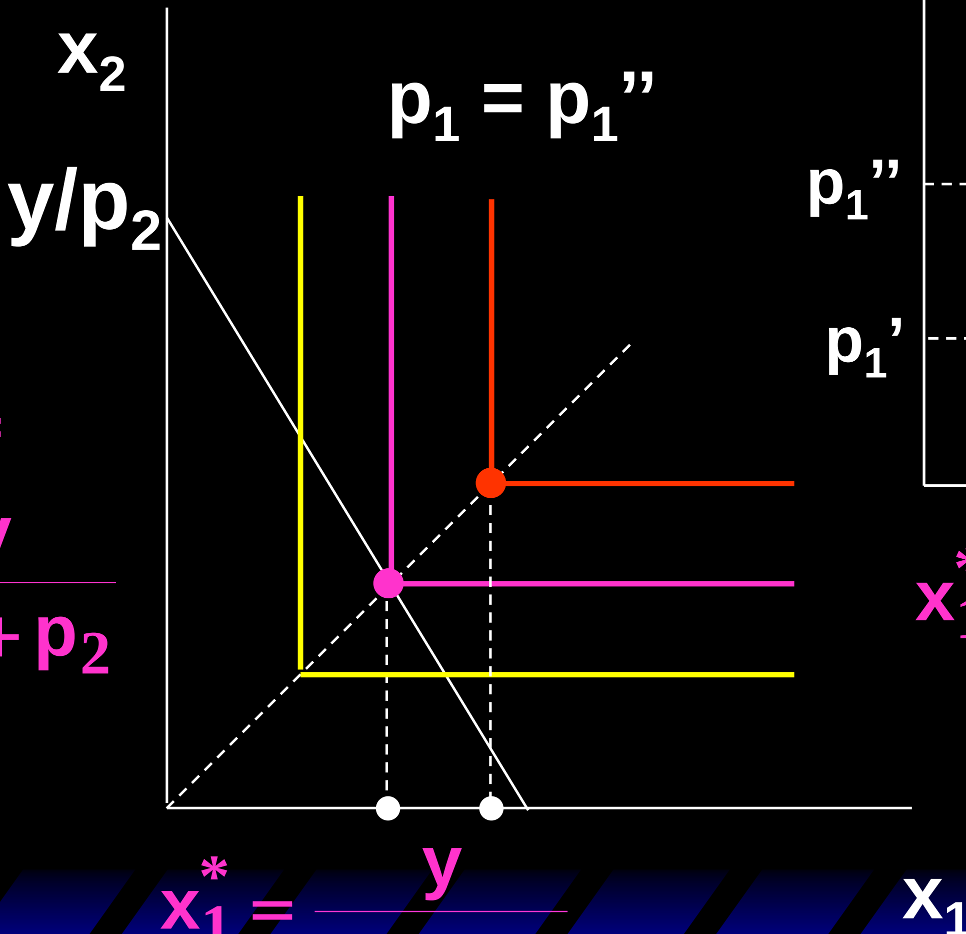
Own-Price Changes

Fixed p_2 and y.



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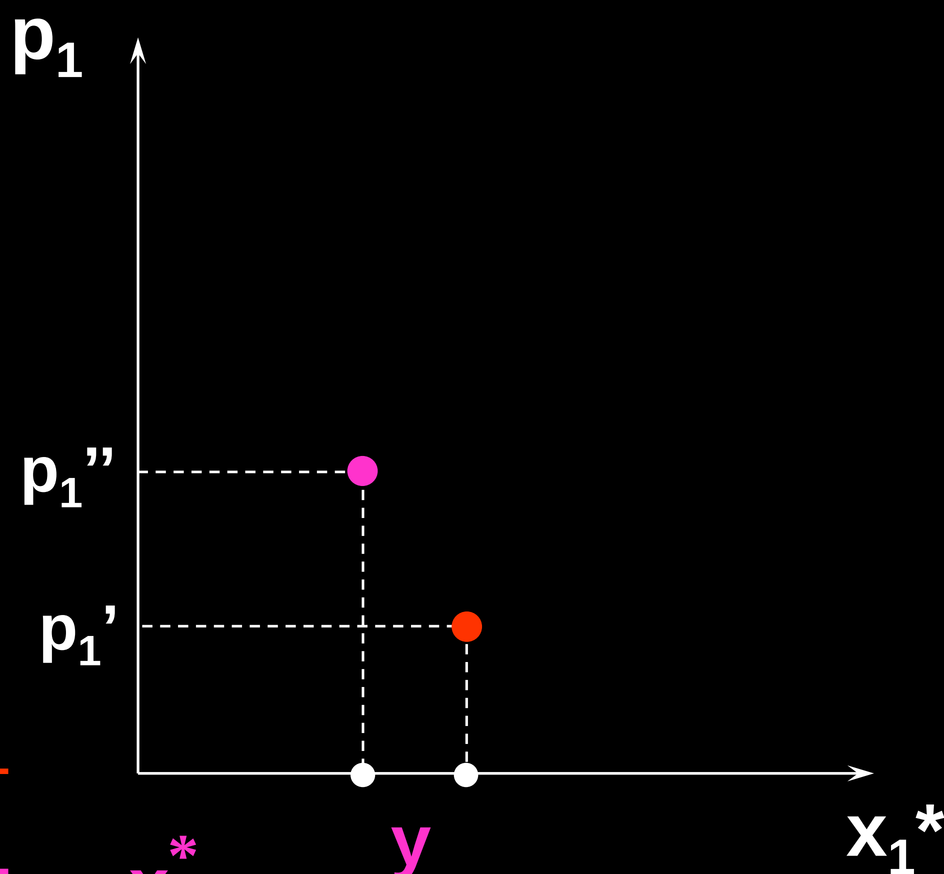

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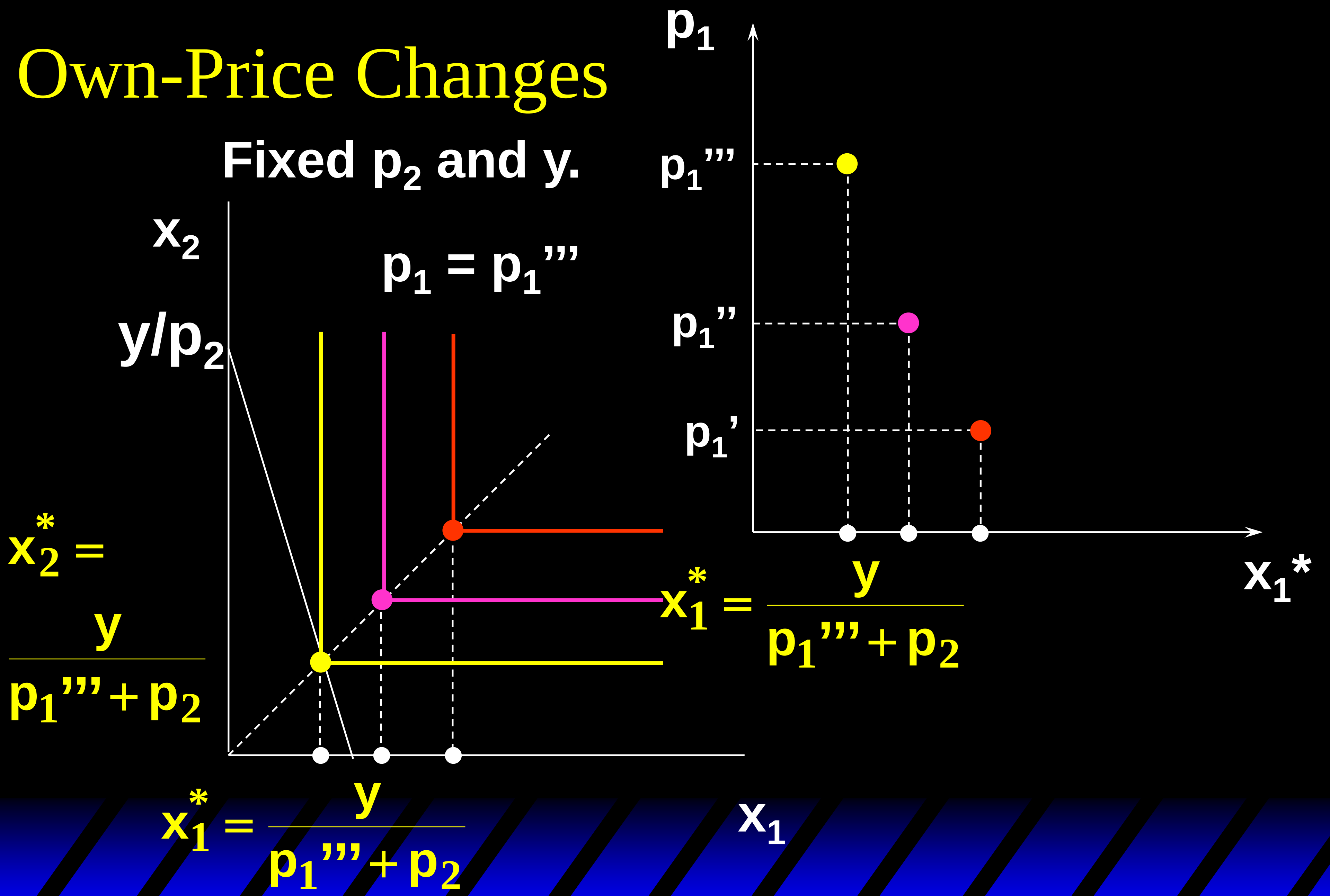

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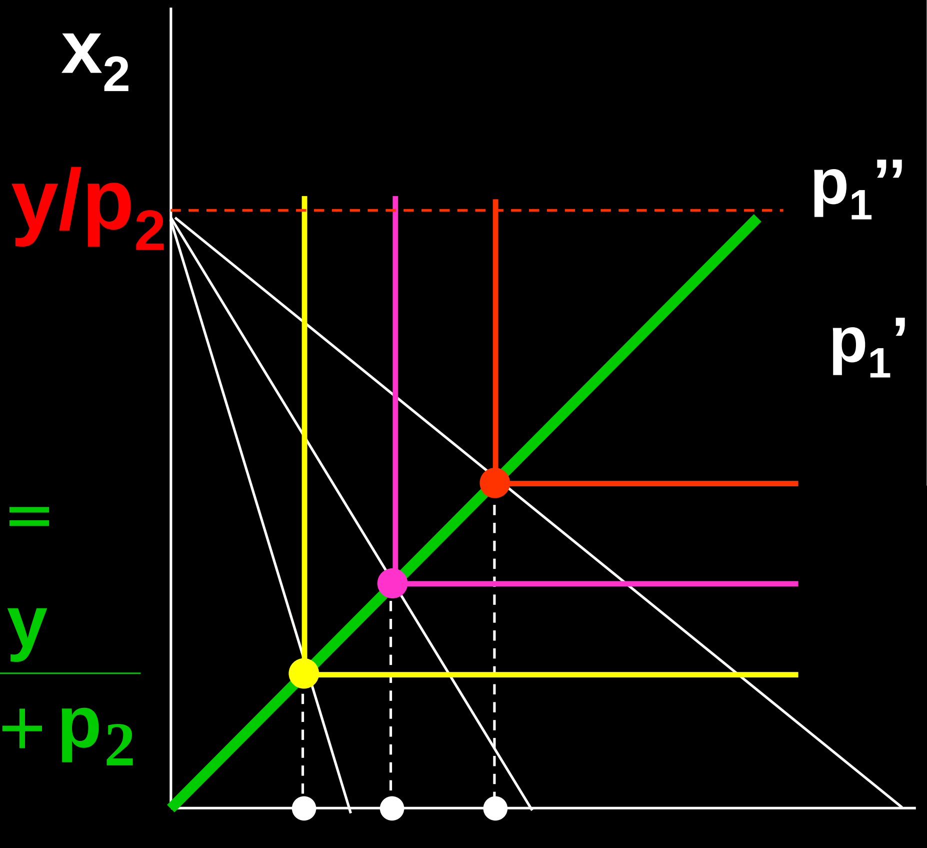

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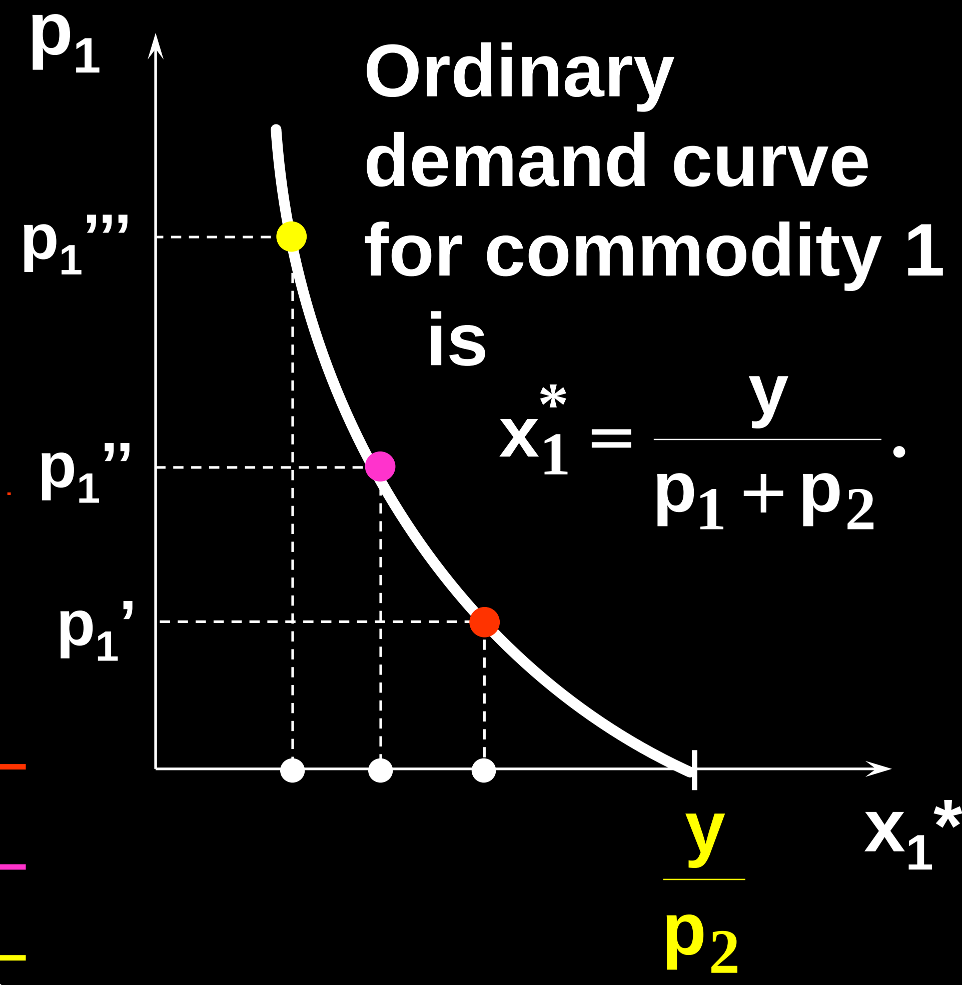


Ordinary demand curve for commodity 1 is

$$x_1^* = \frac{y}{p_1 + p_2}.$$

Own-Price Changes

What does a p_1 price-offer curve look like for a perfect-substitutes utility function?

$$U(x_1, x_2) = x_1 + x_2.$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & \text{, if } p_1 > p_2 \\ y/p_1 & \text{, if } p_1 < p_2 \end{cases}$$

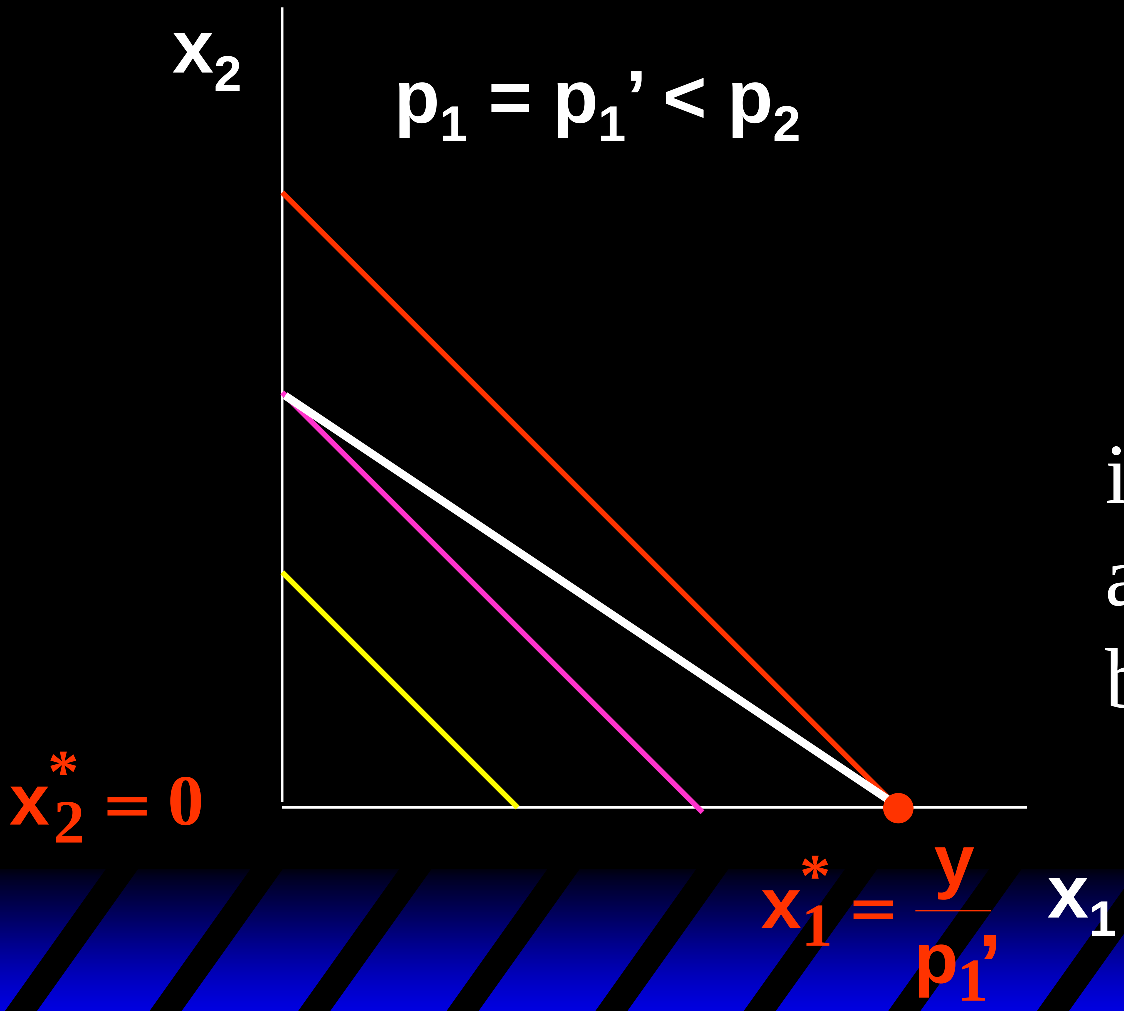
and

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & \text{, if } p_1 < p_2 \\ y/p_2 & \text{, if } p_1 > p_2. \end{cases}$$

What if $p_1 = p_2$?

Own-Price Changes

Fixed p_2 and y.

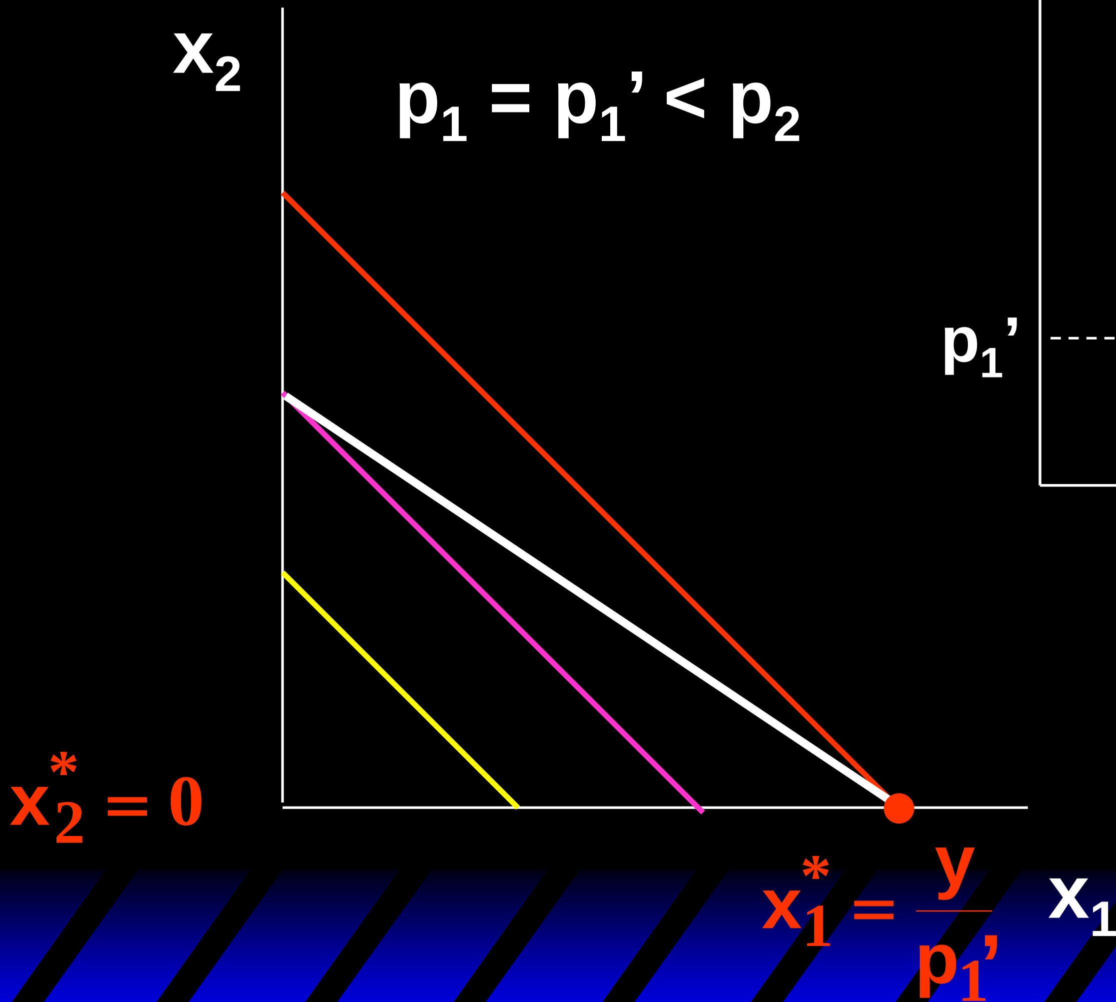


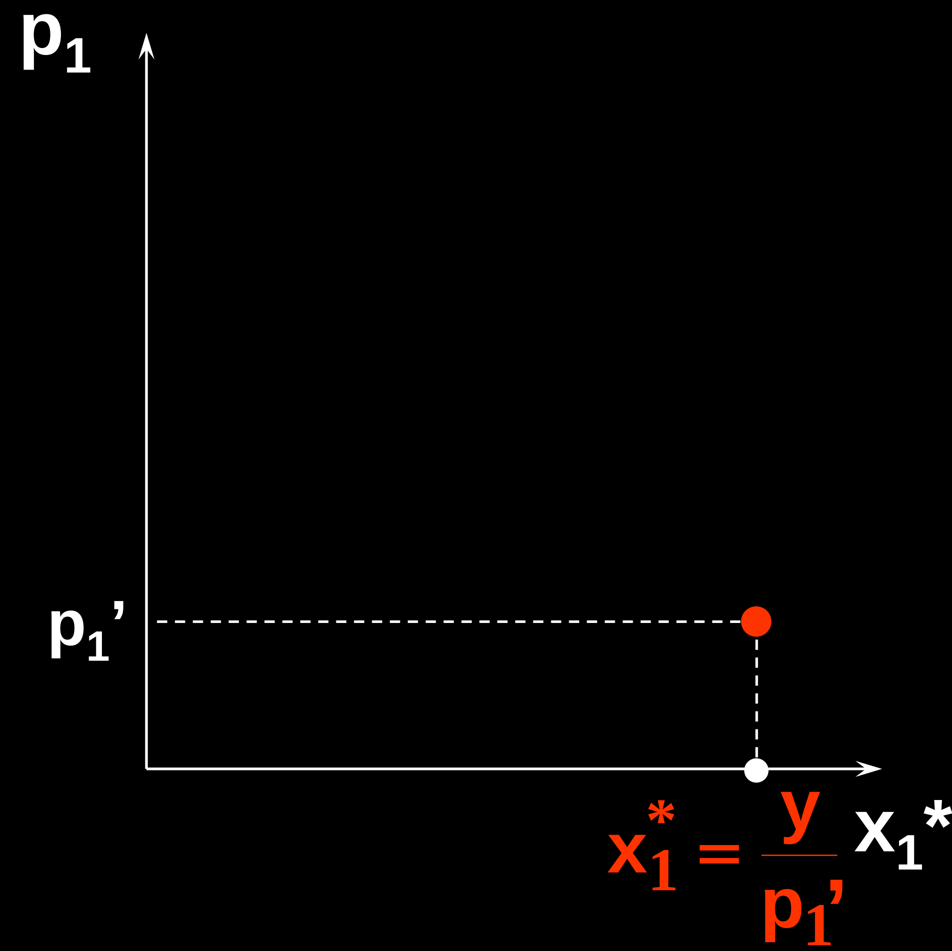
$$\frac{p_1}{p_2} < 1 = \frac{MU_1}{MU_2}$$

i.e. indiff. curves are steeper than the budget constraint

Own-Price Changes

Fixed p_2 and y.





Own-Price Changes

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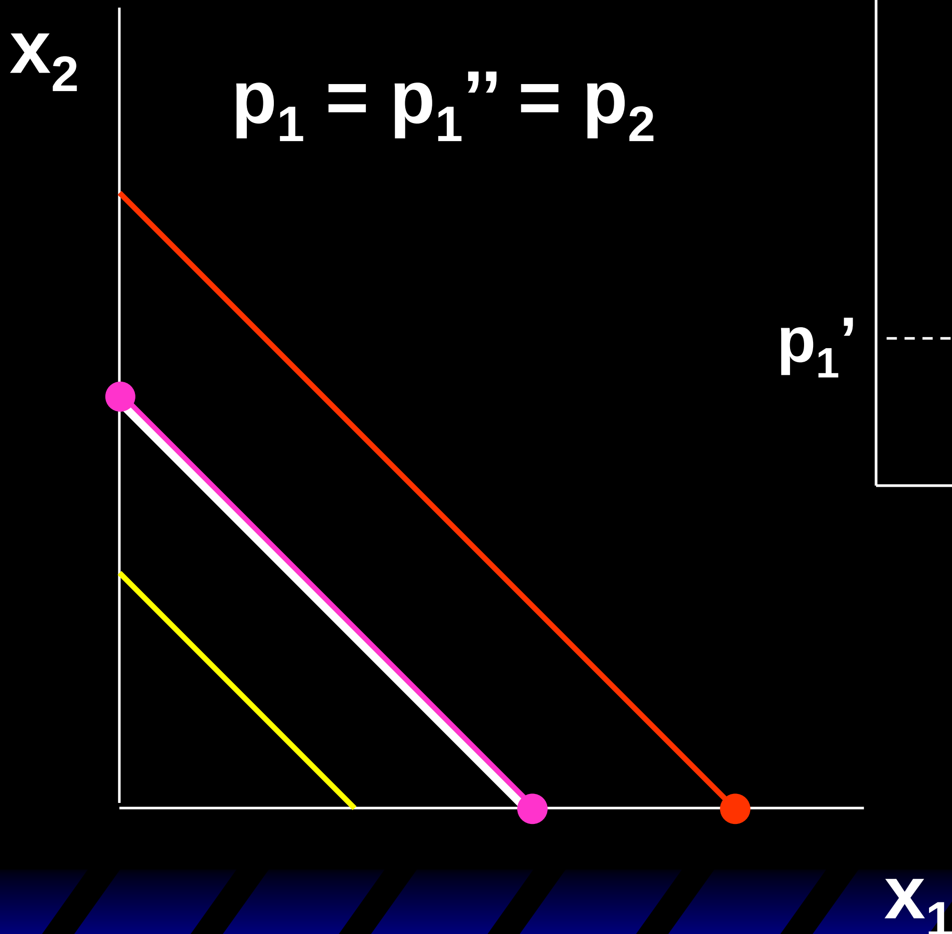
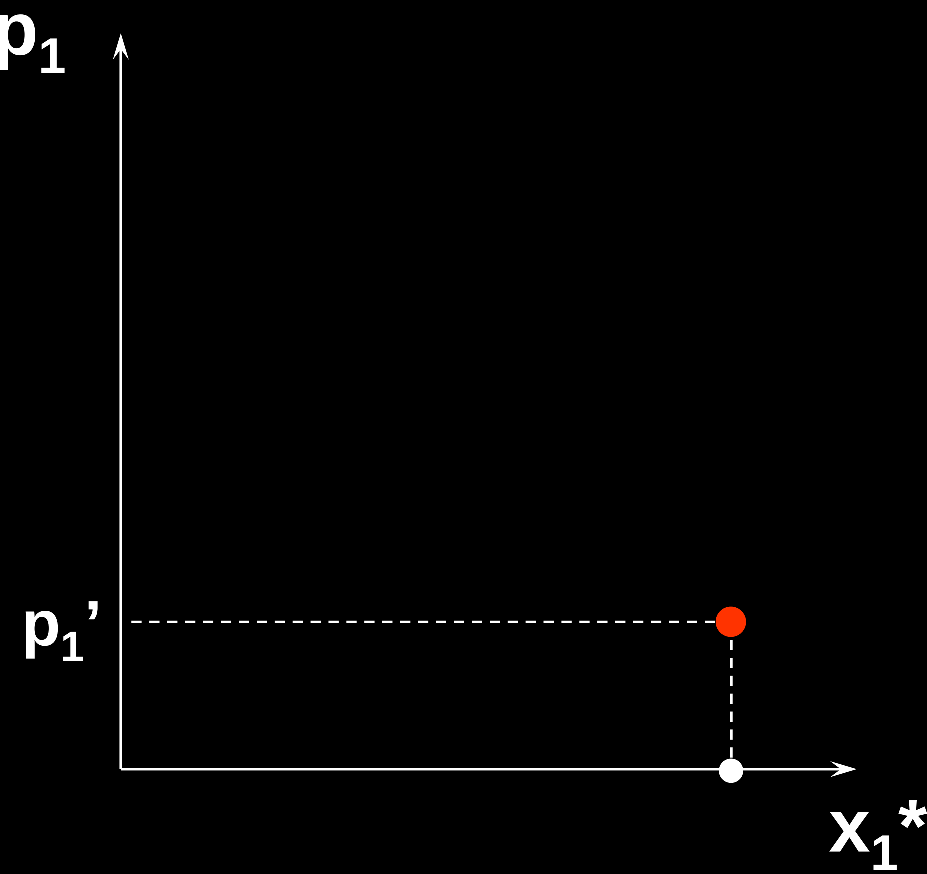
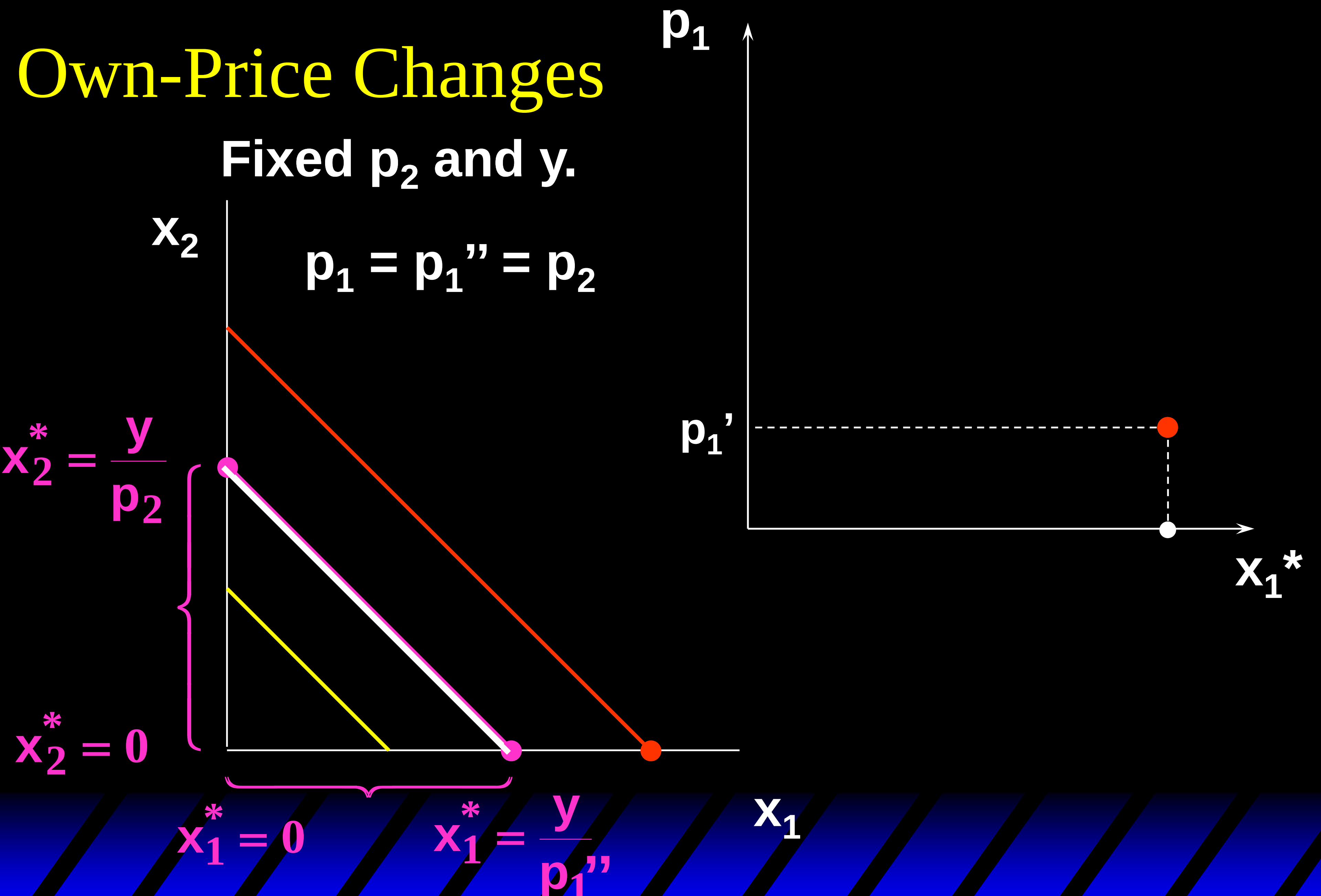




All bundles on the budget line are optimal

Own-Price Changes

Fixed p_2 and y.



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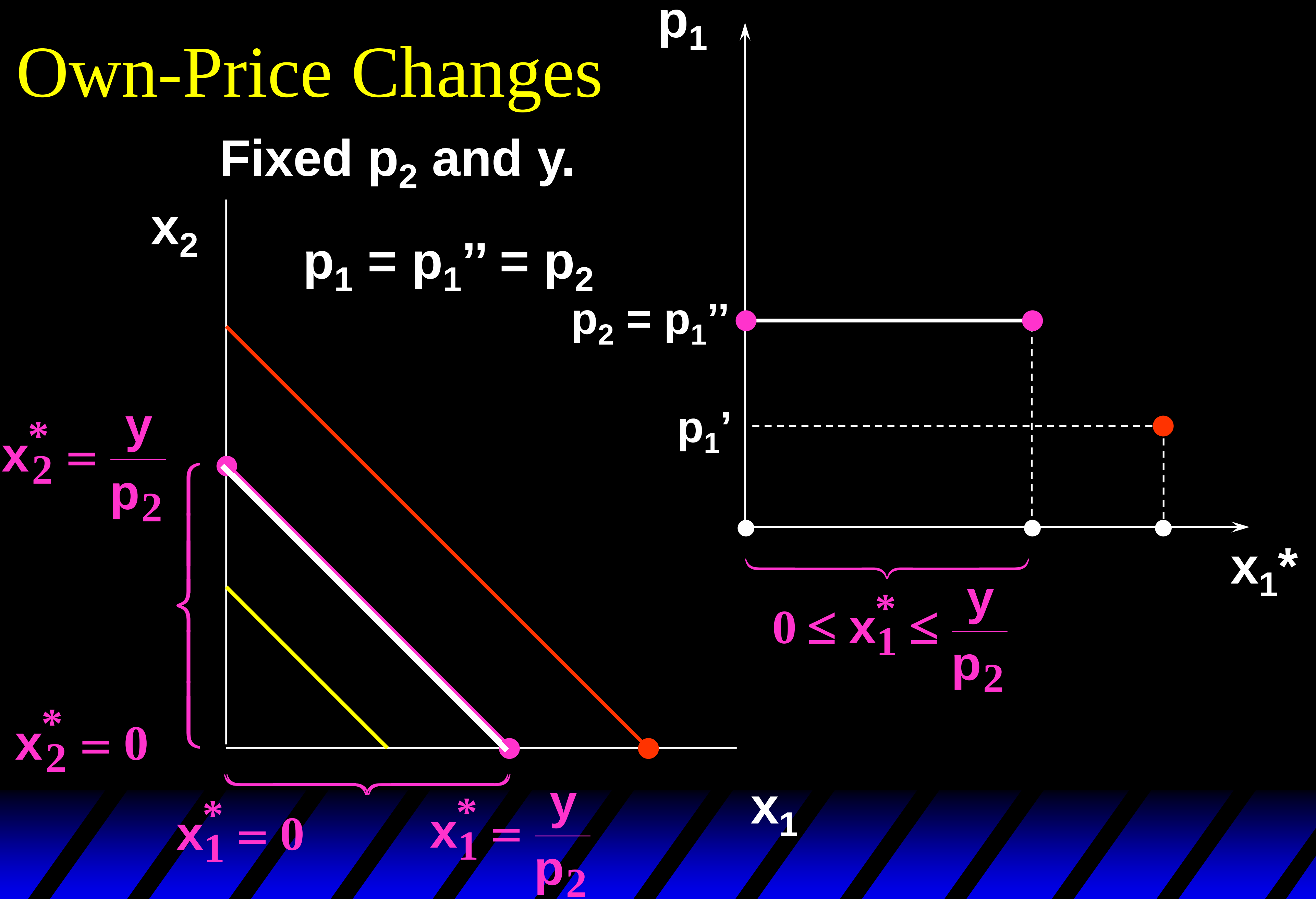

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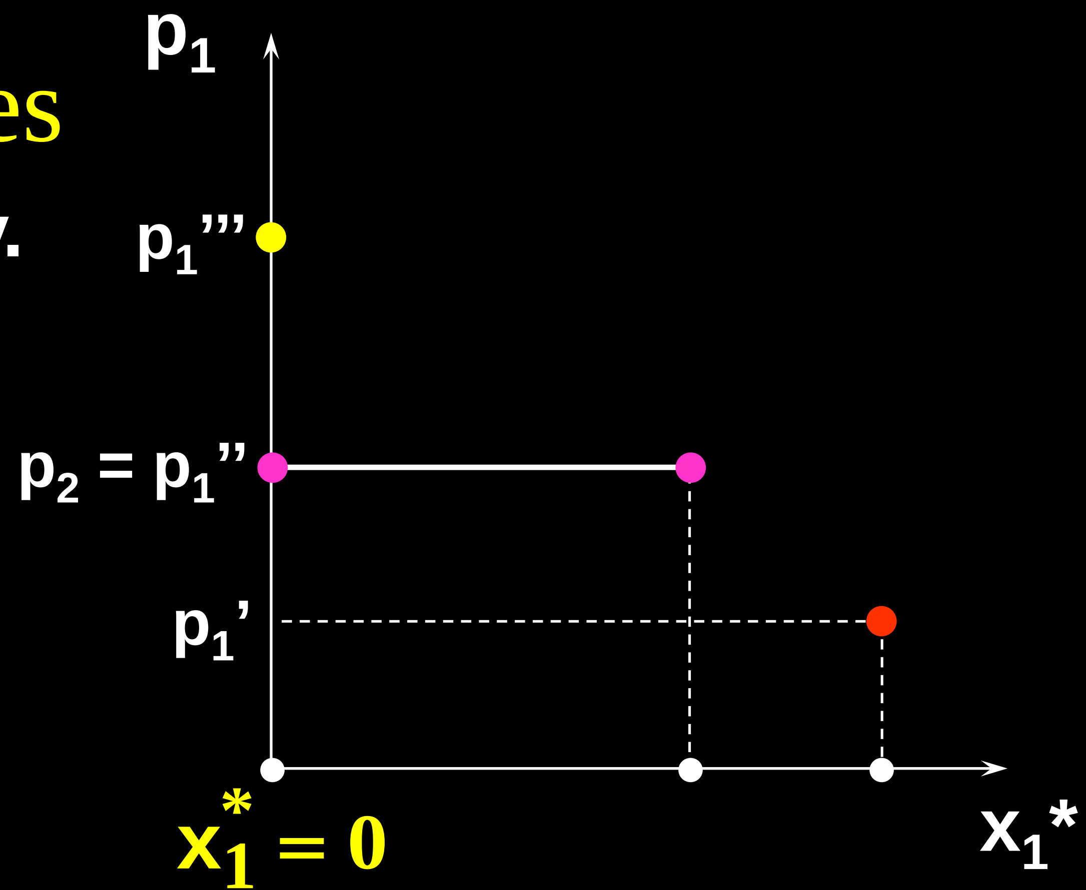
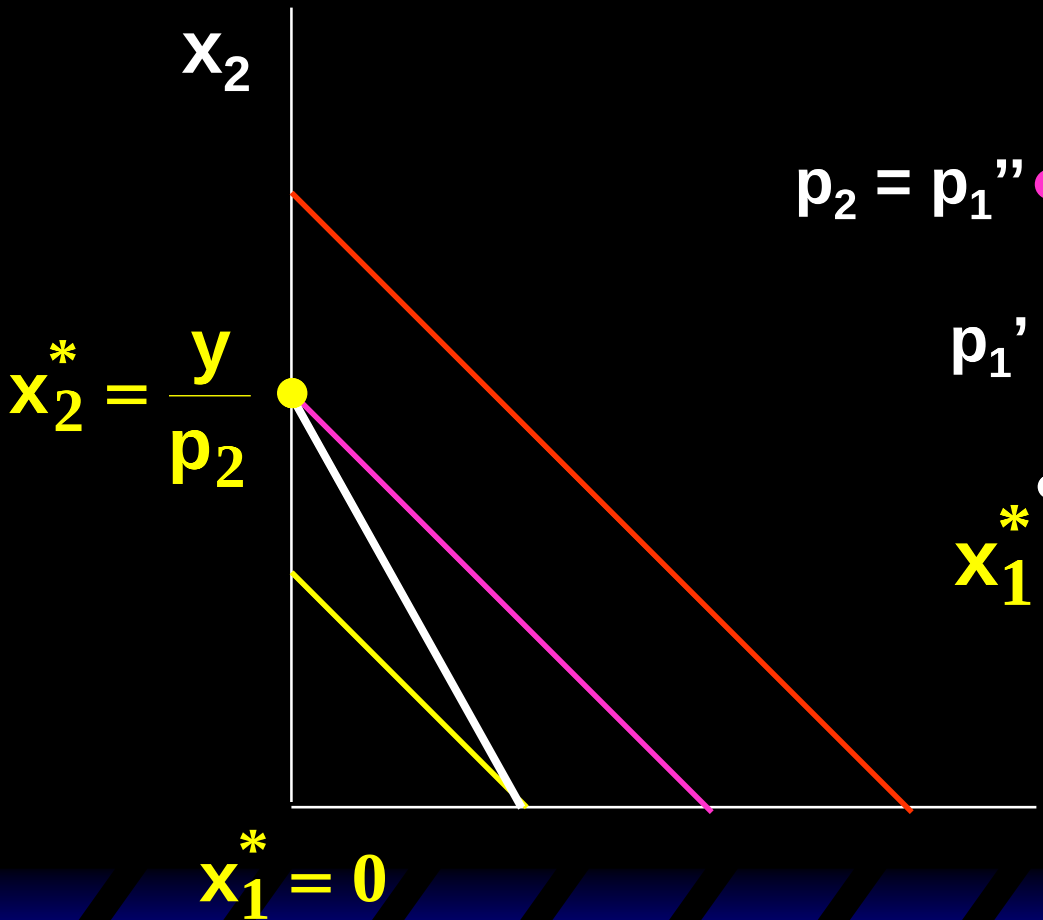
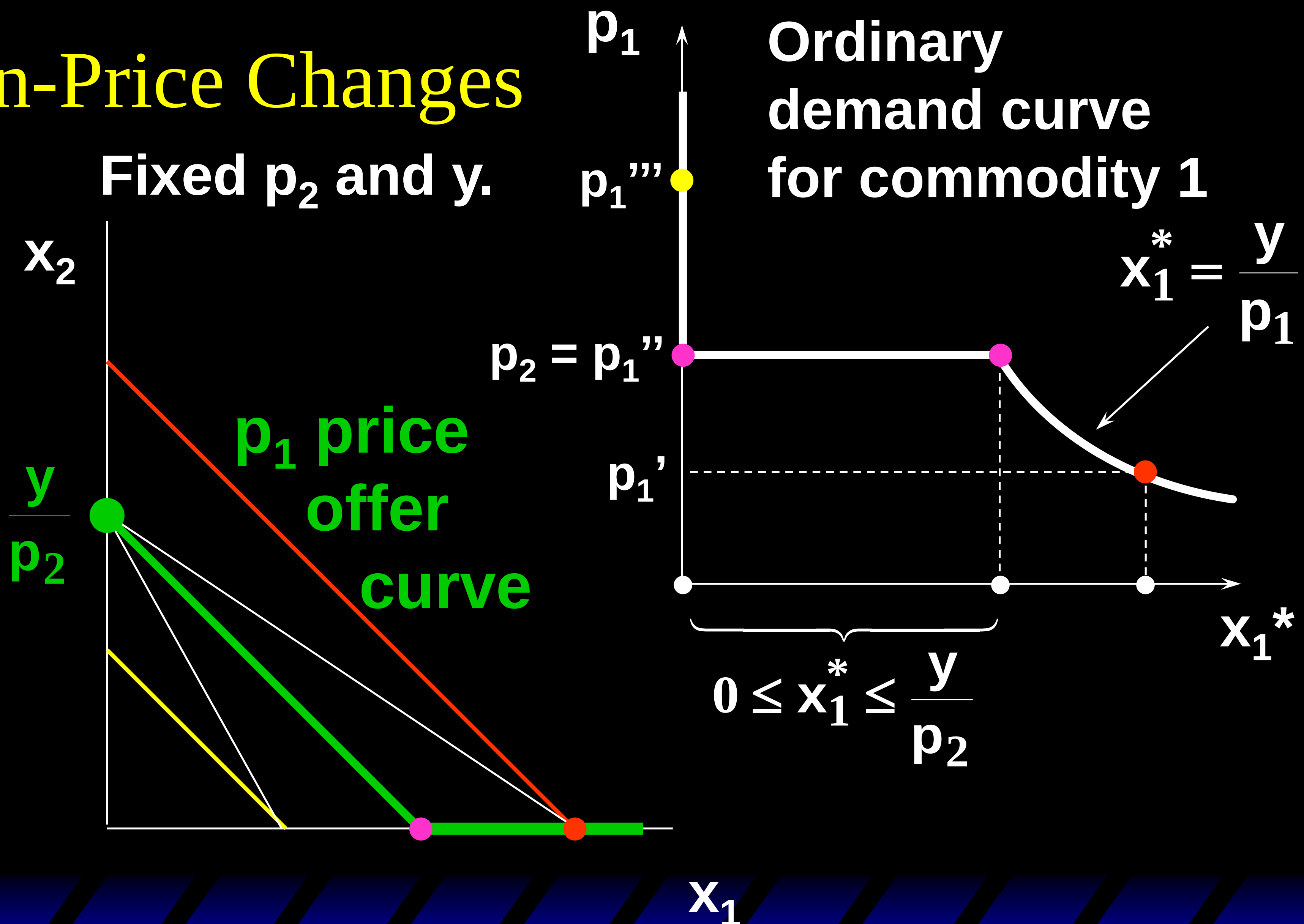




$$\frac{p_1}{p_2} > 1 = \frac{MU_1}{MU_2}$$

Own-Price Changes



Ordinary and Inverse Demand

A Cobb-Douglas example:

$$x_1^* = \frac{ay}{(a+b)p_1}$$

is the ordinary demand function and

$$p_1 = \frac{ay}{(a+b)x_1^*}$$

is the inverse demand function.

Ordinary and Inverse Demand

A perfect-complements example:

$$x_1^* = \frac{y}{p_1 + p_2}$$

is the ordinary demand function and

$$p_1 = \frac{y}{x_1^*} - p_2$$

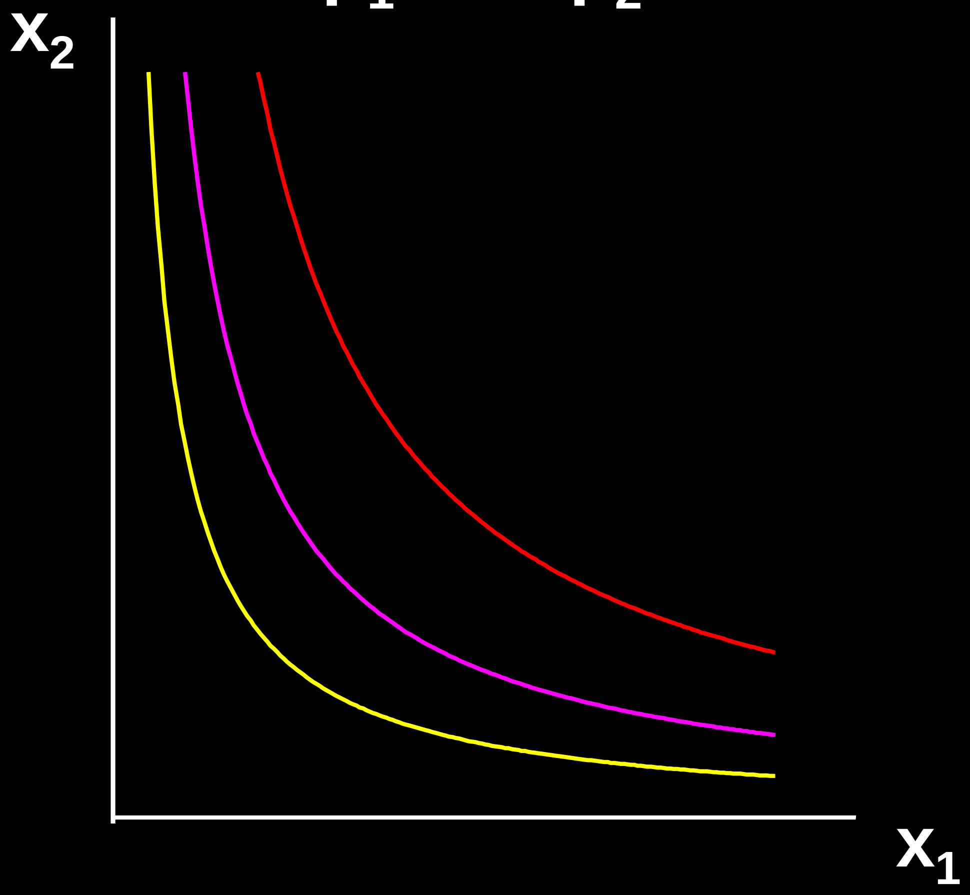
is the inverse demand function.

Income Changes

How does the value of $x_1^*(p_1,p_2,y)$ change as y changes, holding both p_1 and p_2 constant?

Income Changes

Fixed p_1 and p_2.



Income Changes

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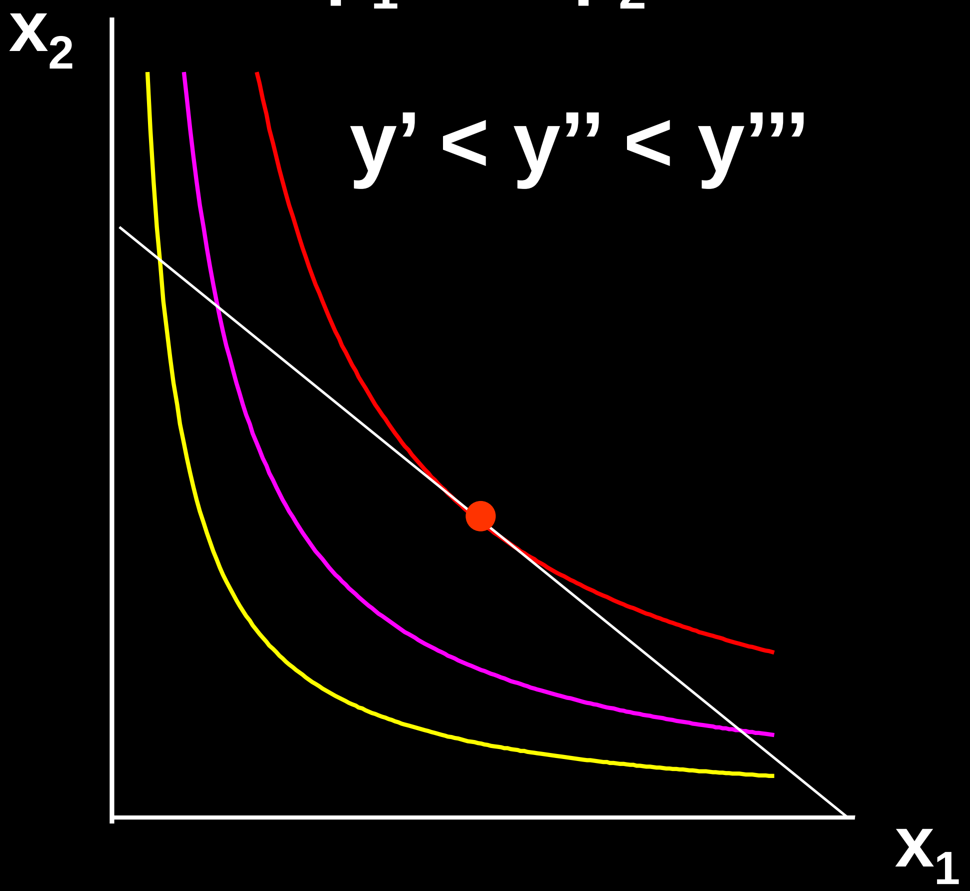

$p_1x_1 + p_2x_2 = y$

Income Changes

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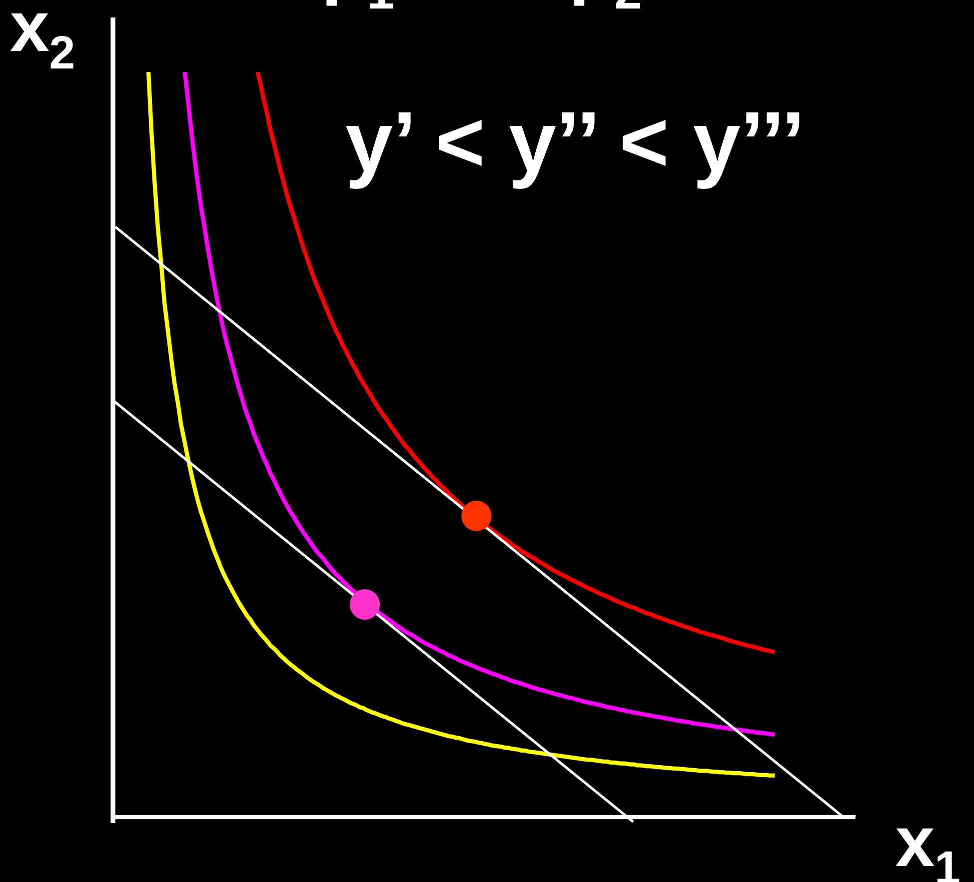

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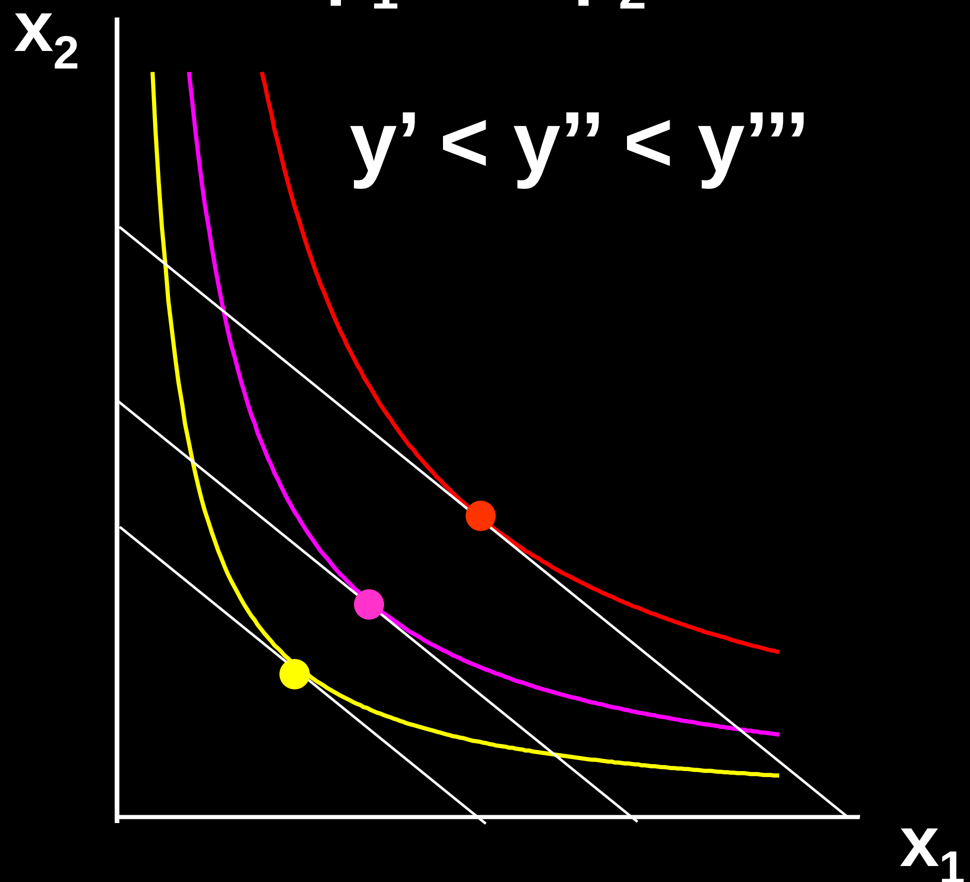

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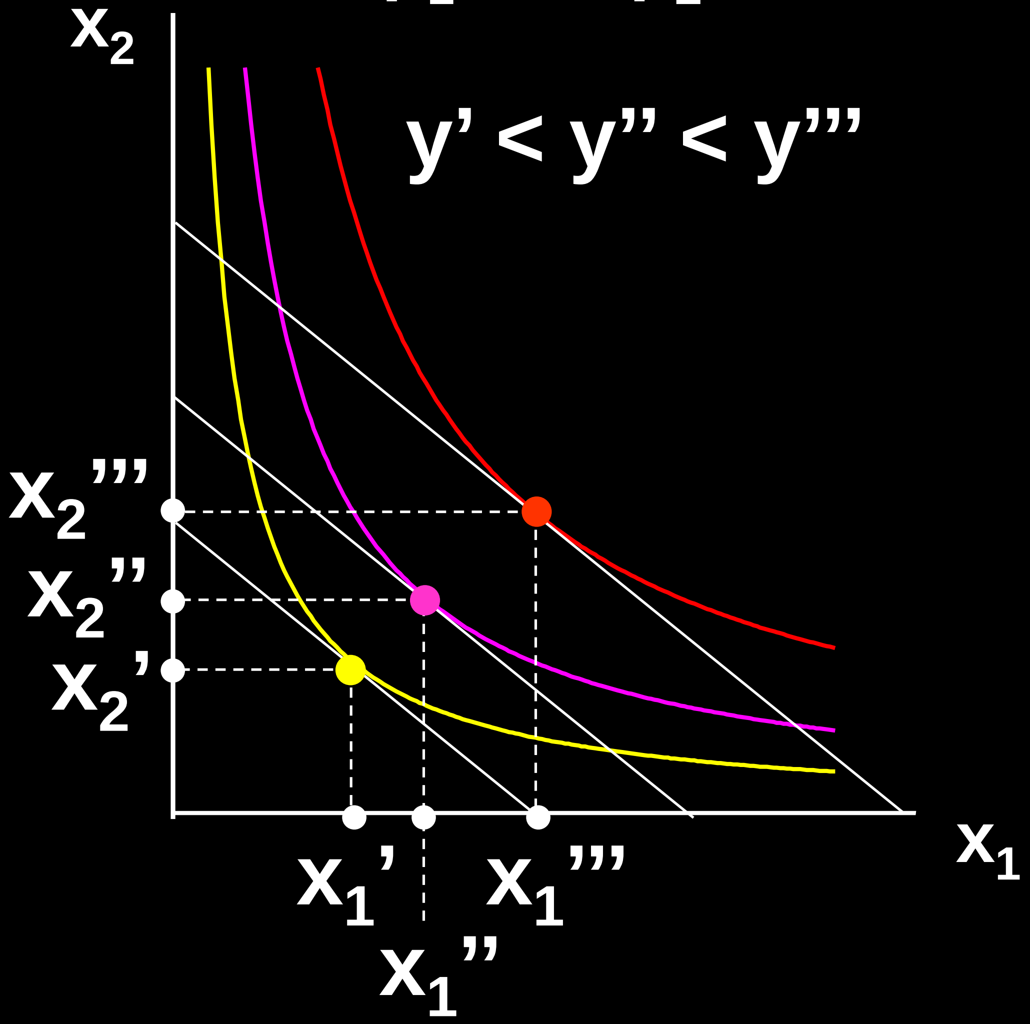
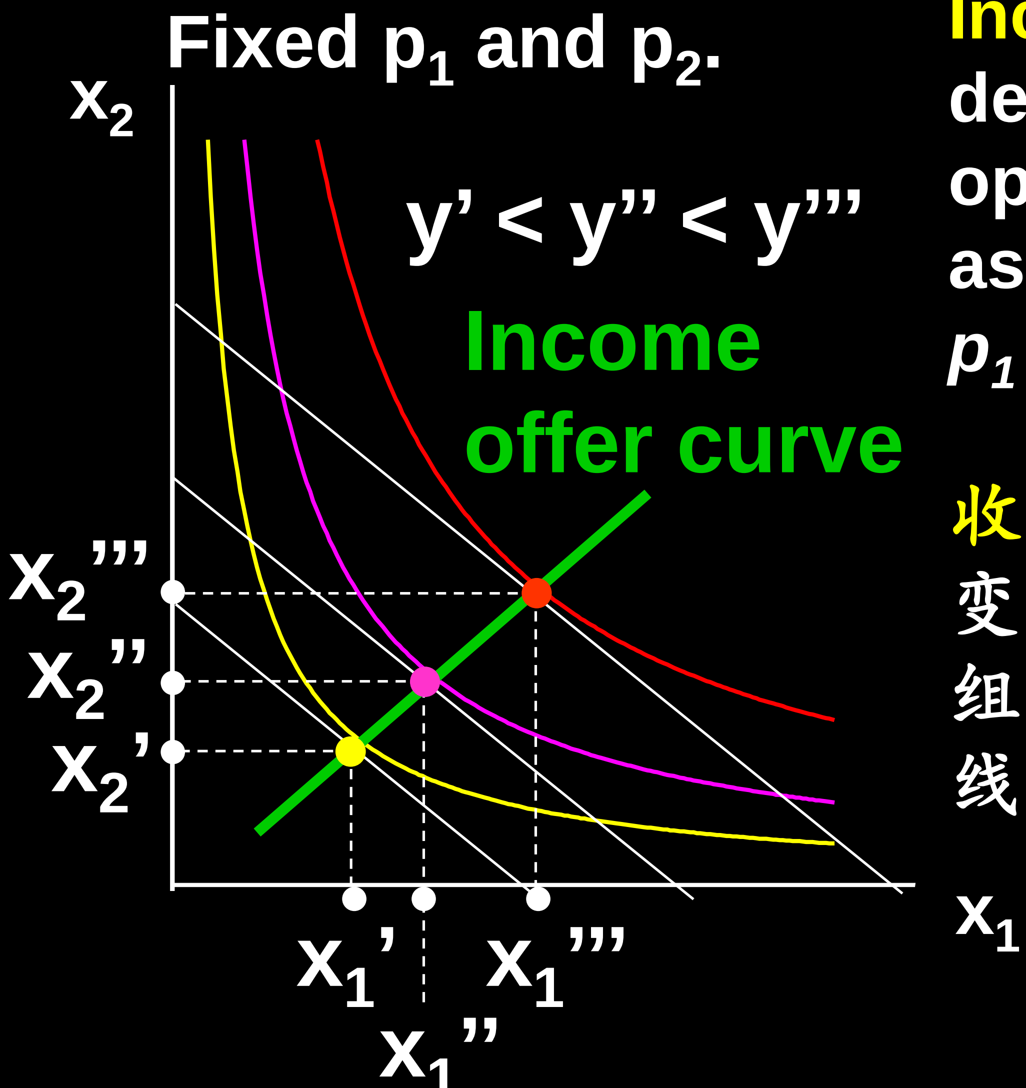


Income Changes



Income offer curve describes how the optimal bundle changes as y changes, *holding p_1 and p_2 constant*.

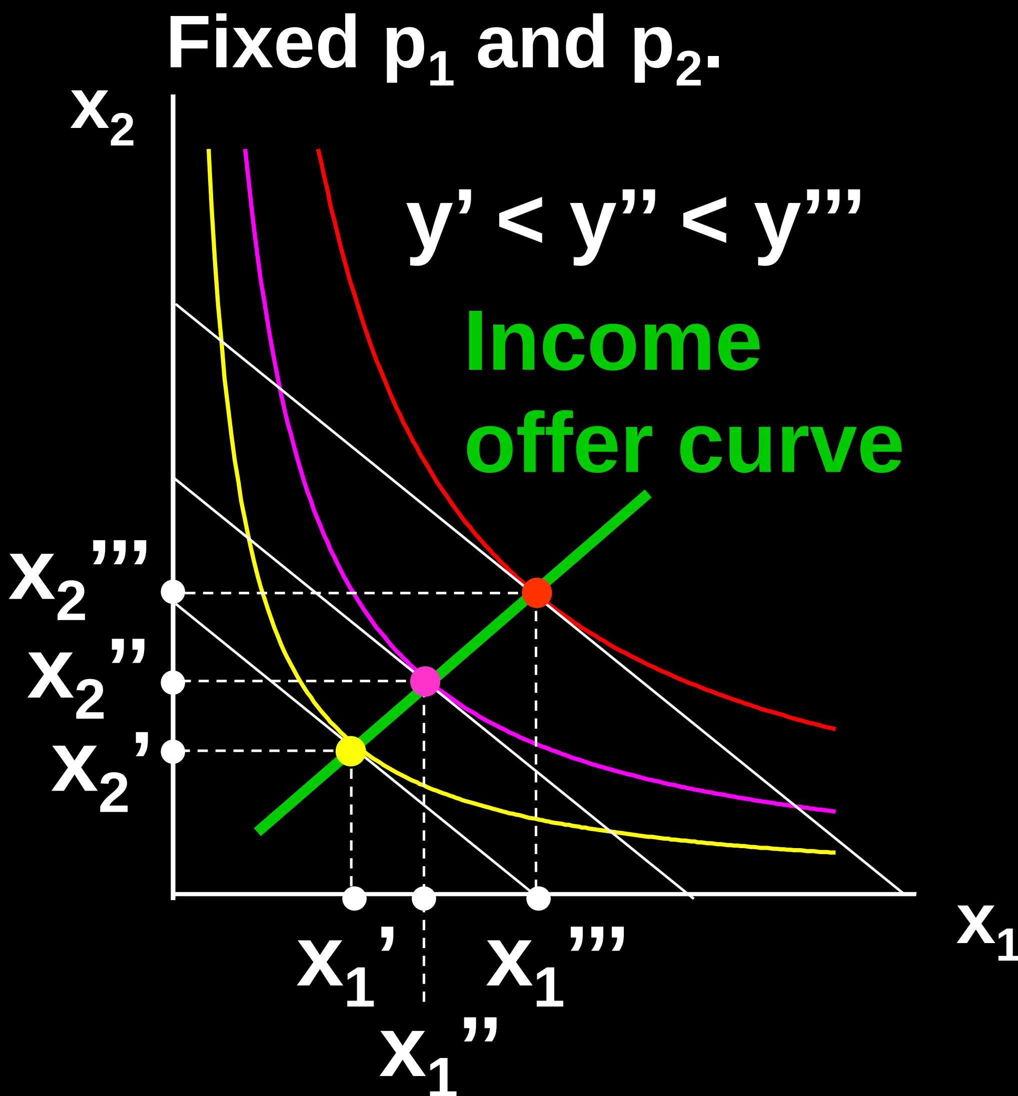
收入提供曲线：价格不变的情况下，最优商品组合随收入变化的轨迹线。

Income Changes

A plot of quantity demanded against income is called an Engel curve.

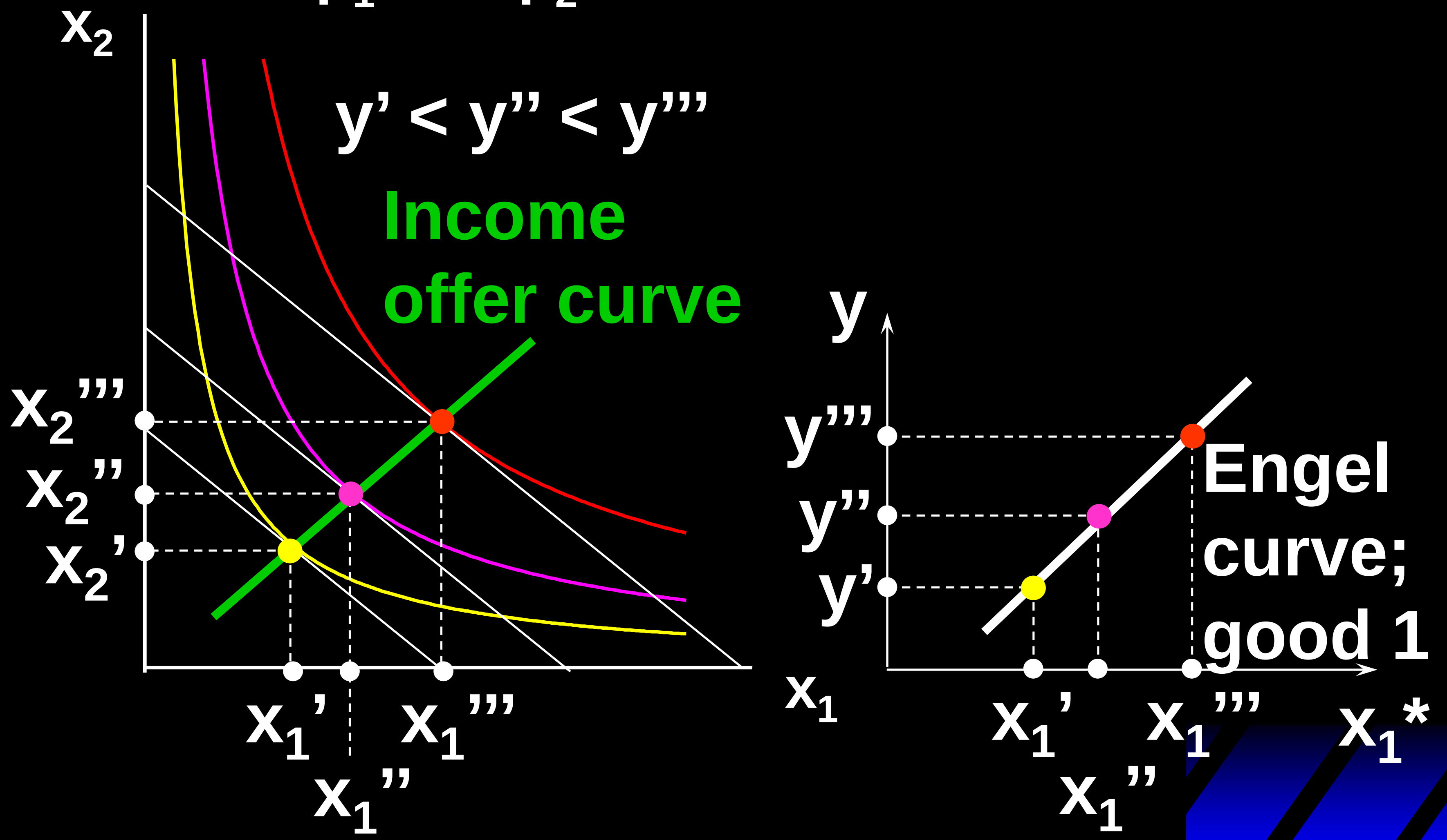
描述某种商品的最优消费数量与收入关系的曲线叫做恩格尔曲线。

Income Changes



Income Changes

Fixed p_1 and p_2.



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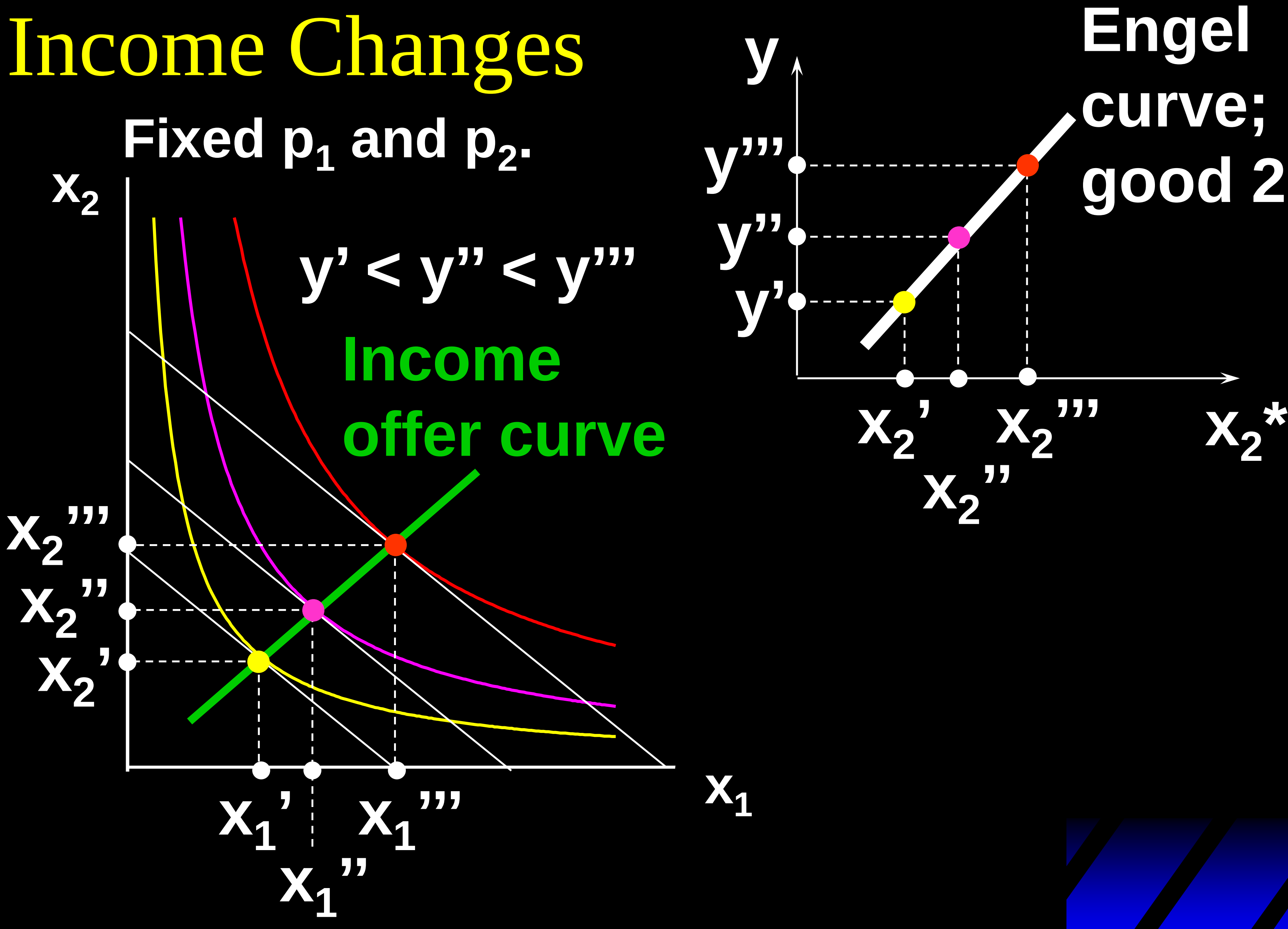


Income Changes and Cobb-Douglas Preferences

An example of computing the equations of Engel curves; the Cobb-Douglas case.

$$U(x_1, x_2) = x_1^a x_2^b.$$

The ordinary demand equations are

$$x_1^* = \frac{ay}{(a+b)p_1}; \quad x_2^* = \frac{by}{(a+b)p_2}.$$

Income Changes and Cobb-Douglas Preferences

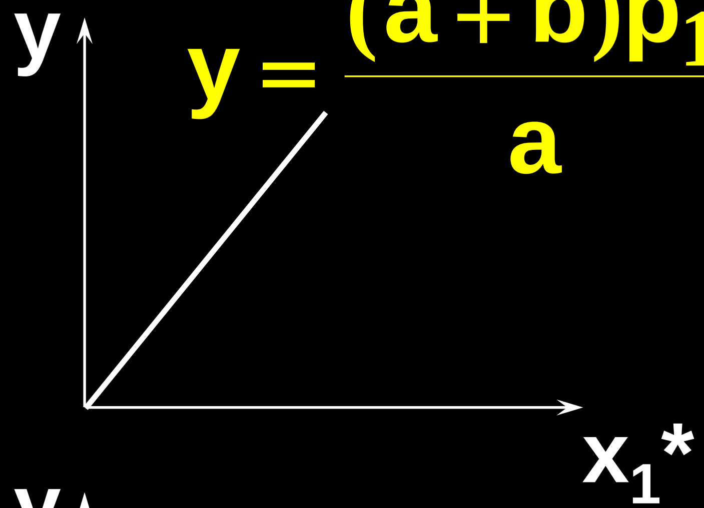
$$x_1^* = \frac{ay}{(a+b)p_1}; \quad x_2^* = \frac{by}{(a+b)p_2}.$$

Rearranged to isolate y, these are:

$$y = \frac{(a+b)p_1}{a} x_1^*$$ **Engel curve for good 1**

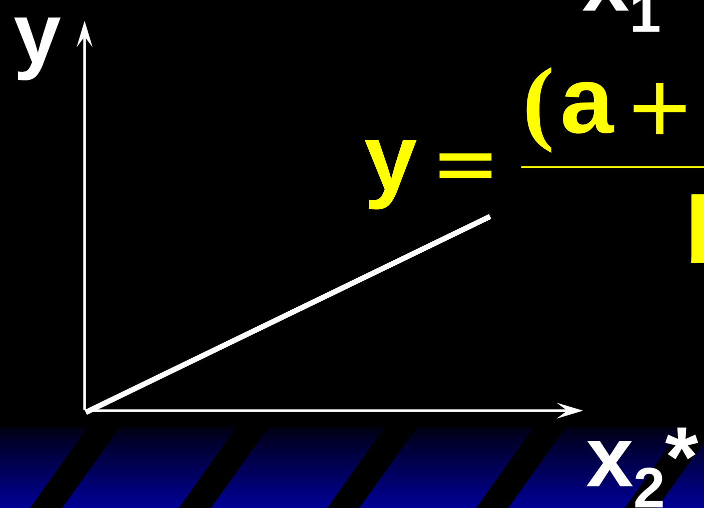
$$y = \frac{(a+b)p_2}{b} x_2^*$$ **Engel curve for good 2**

Income Changes and Cobb-Douglas Preferences



$$y = \frac{(a+b)p_1}{a} x_1^*$$

Engel curve for good 1



$$y = \frac{(a+b)p_2}{b} x_2^*$$

Engel curve for good 2

Income Changes and Perfectly-Complementary Preferences

Another example of computing the equations of Engel curves; the perfectly-complementary case.

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

The ordinary demand equations are

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$

Income Changes and Perfectly-Complementary Preferences

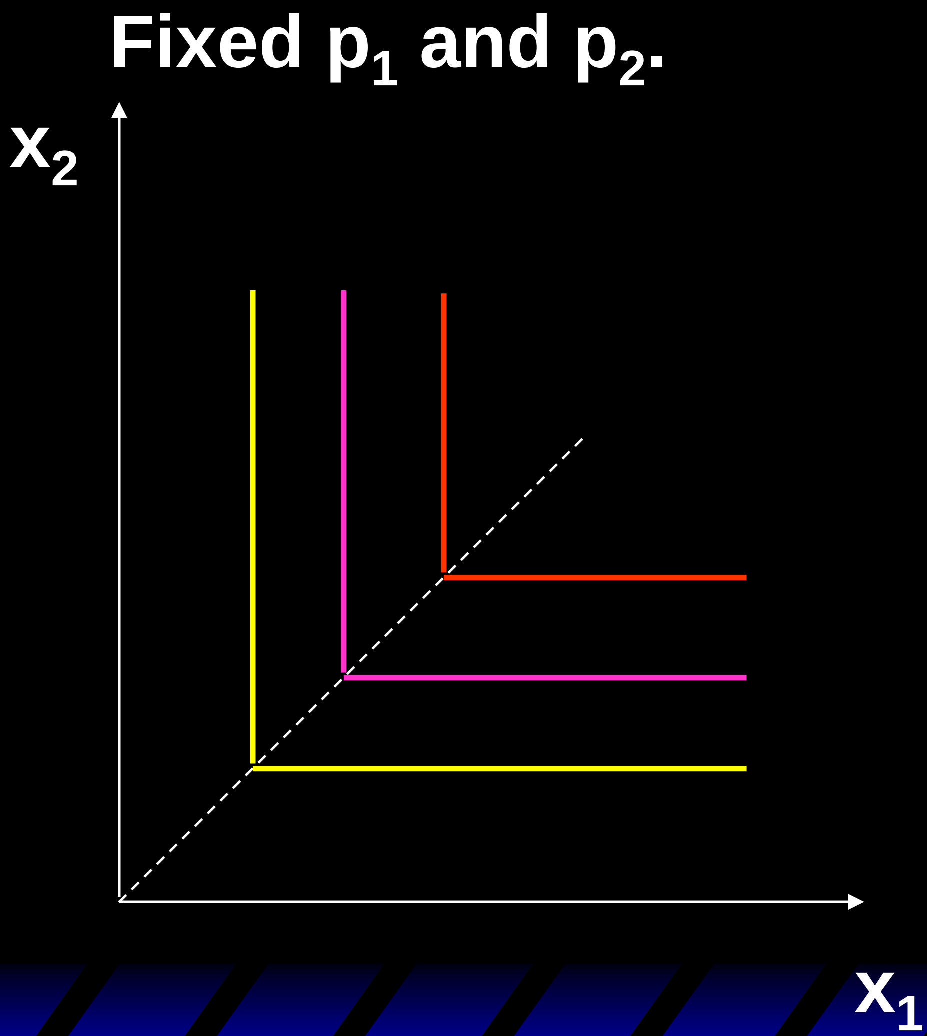
$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$

Rearranged to isolate y, these are:

$$y = (p_1 + p_2)x_1^*$$ **Engel curve for good 1**

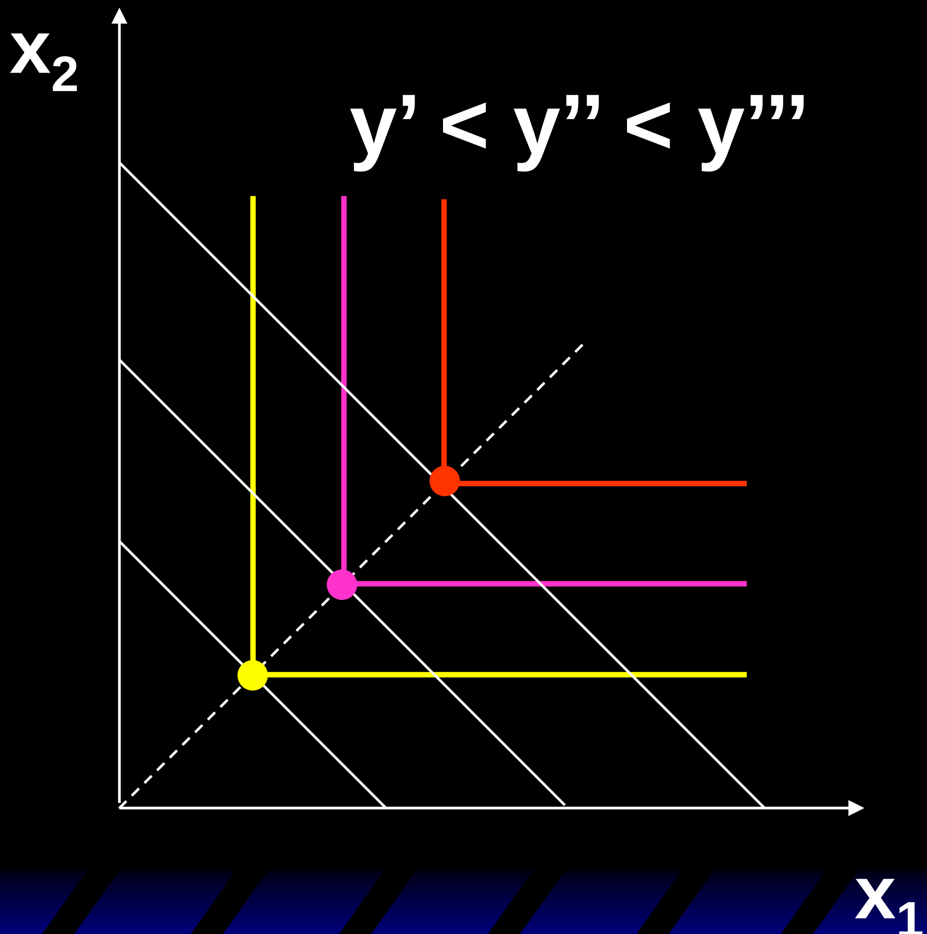
$$y = (p_1 + p_2)x_2^*$$ **Engel curve for good 2**

Income Changes



Income Changes

Fixed p_1 and p_2.



Income Changes

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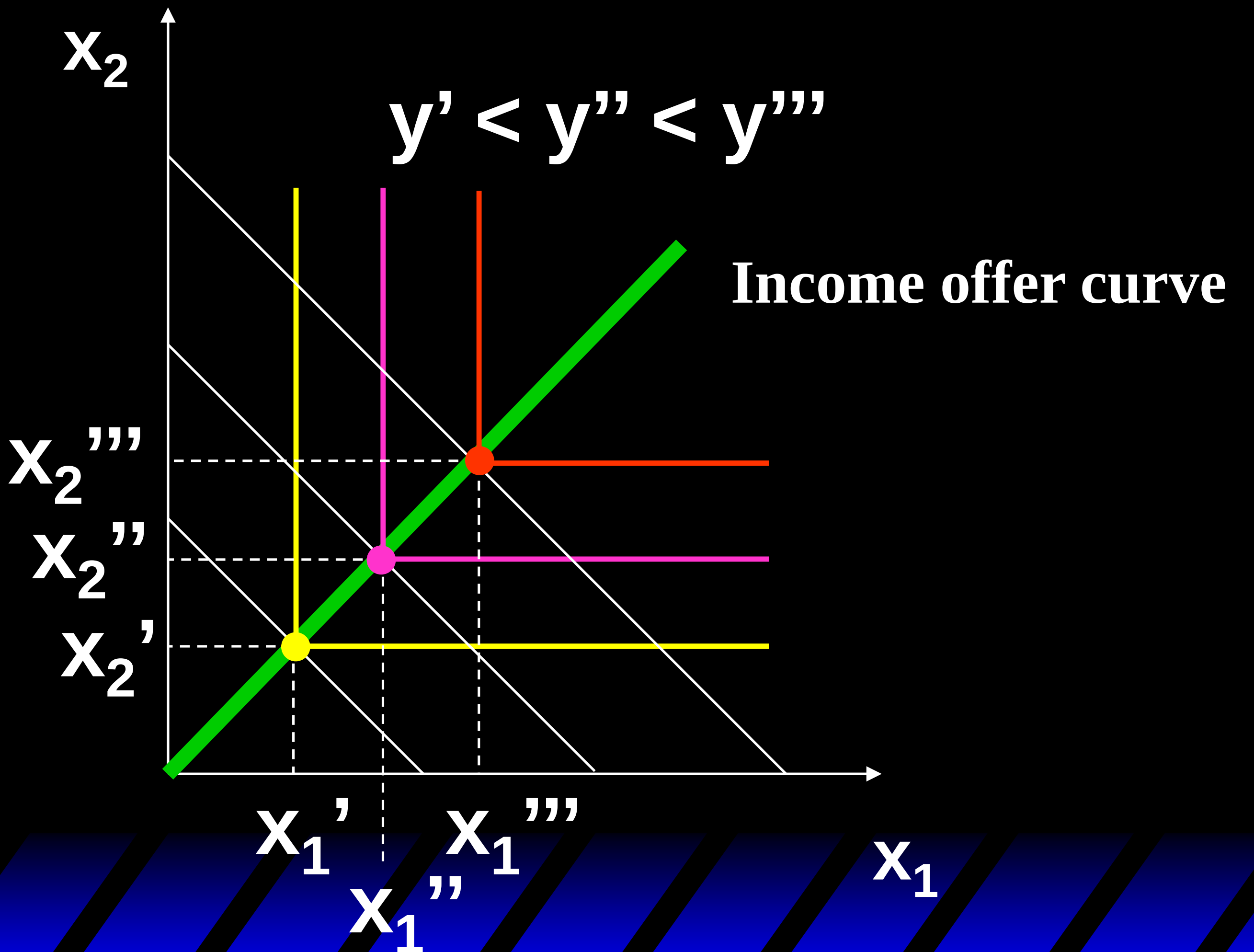
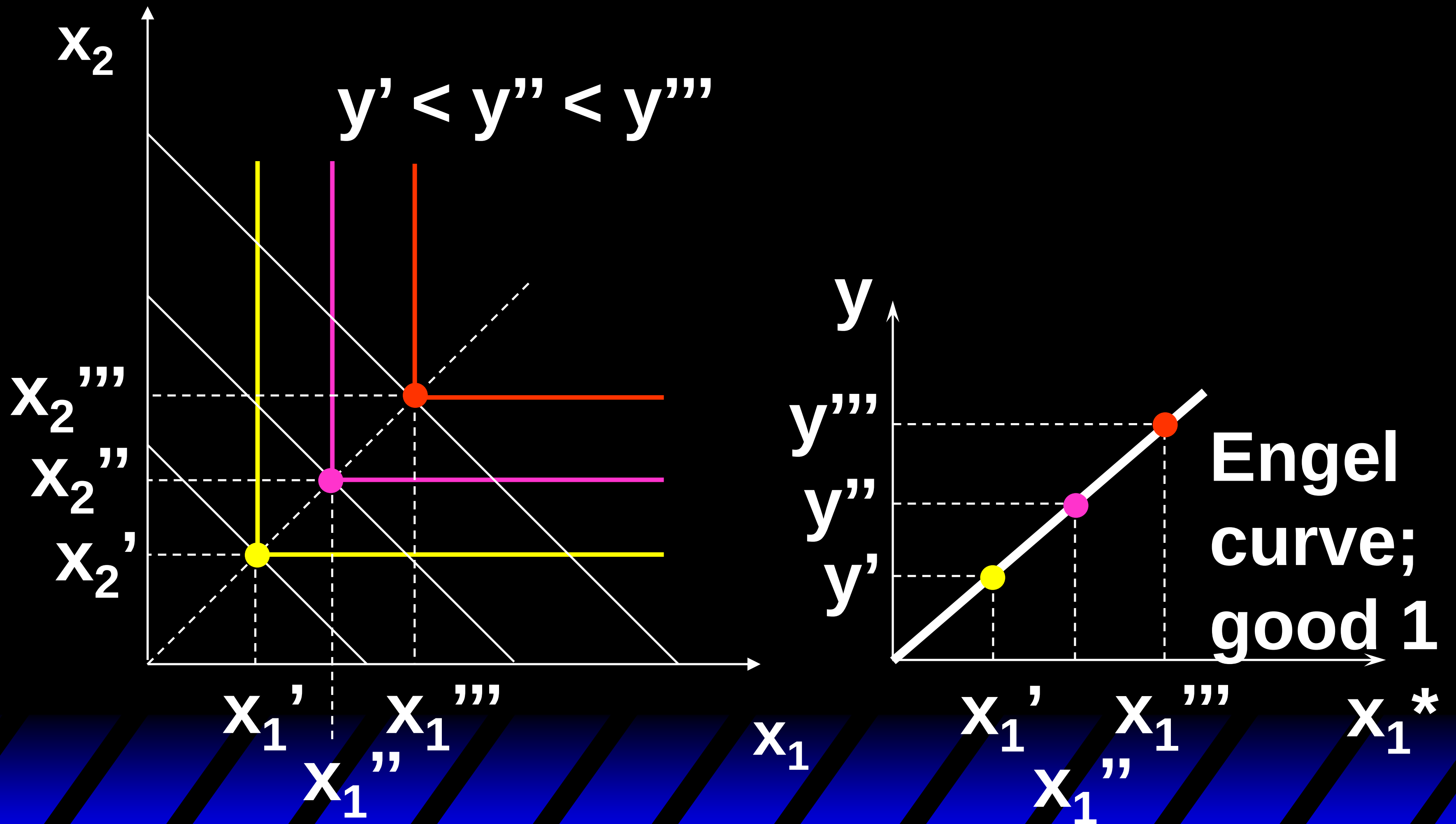


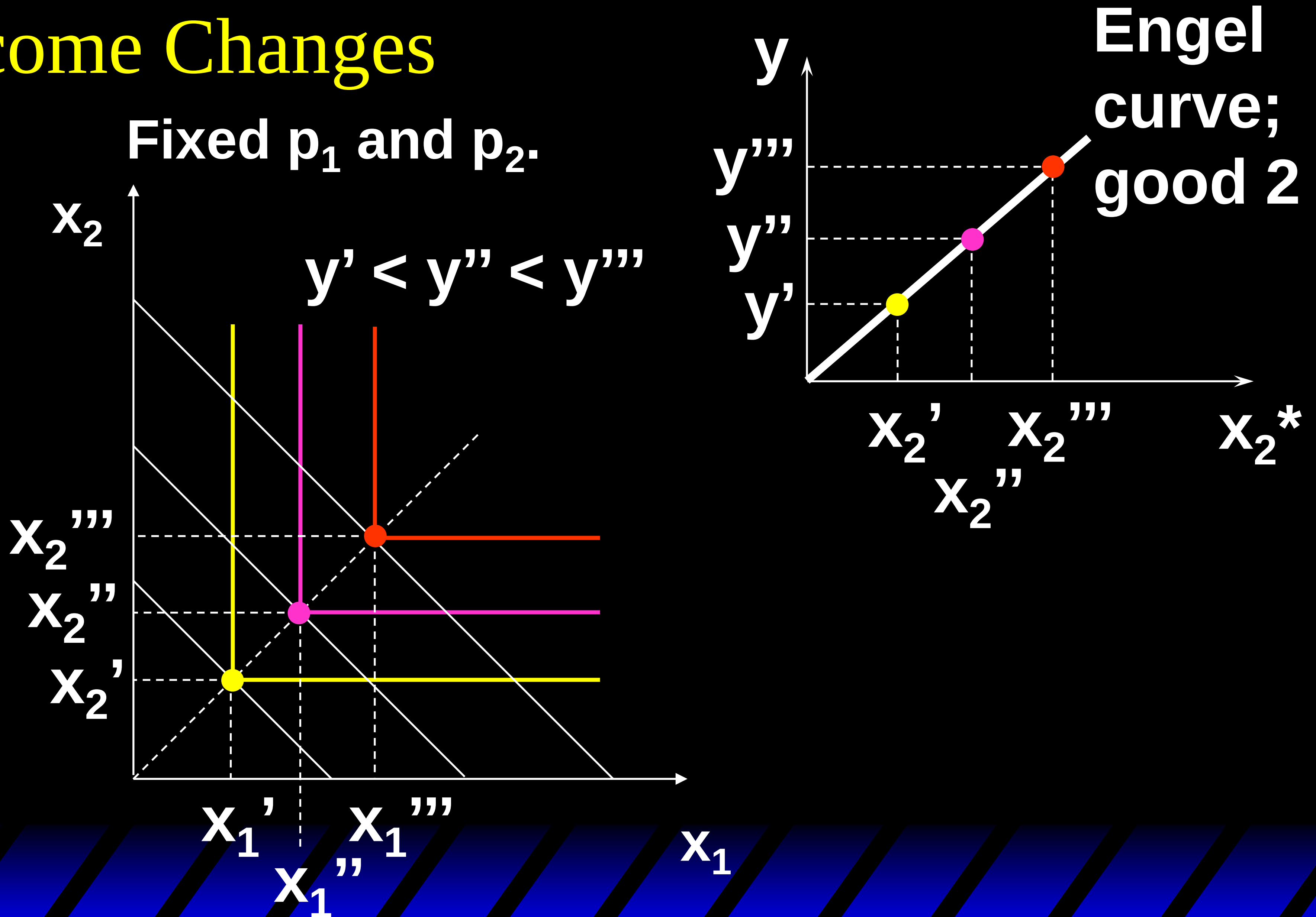
Income Changes

Fixed p_1 and p_2.



Income Changes

Fixed p_1 and p_2.

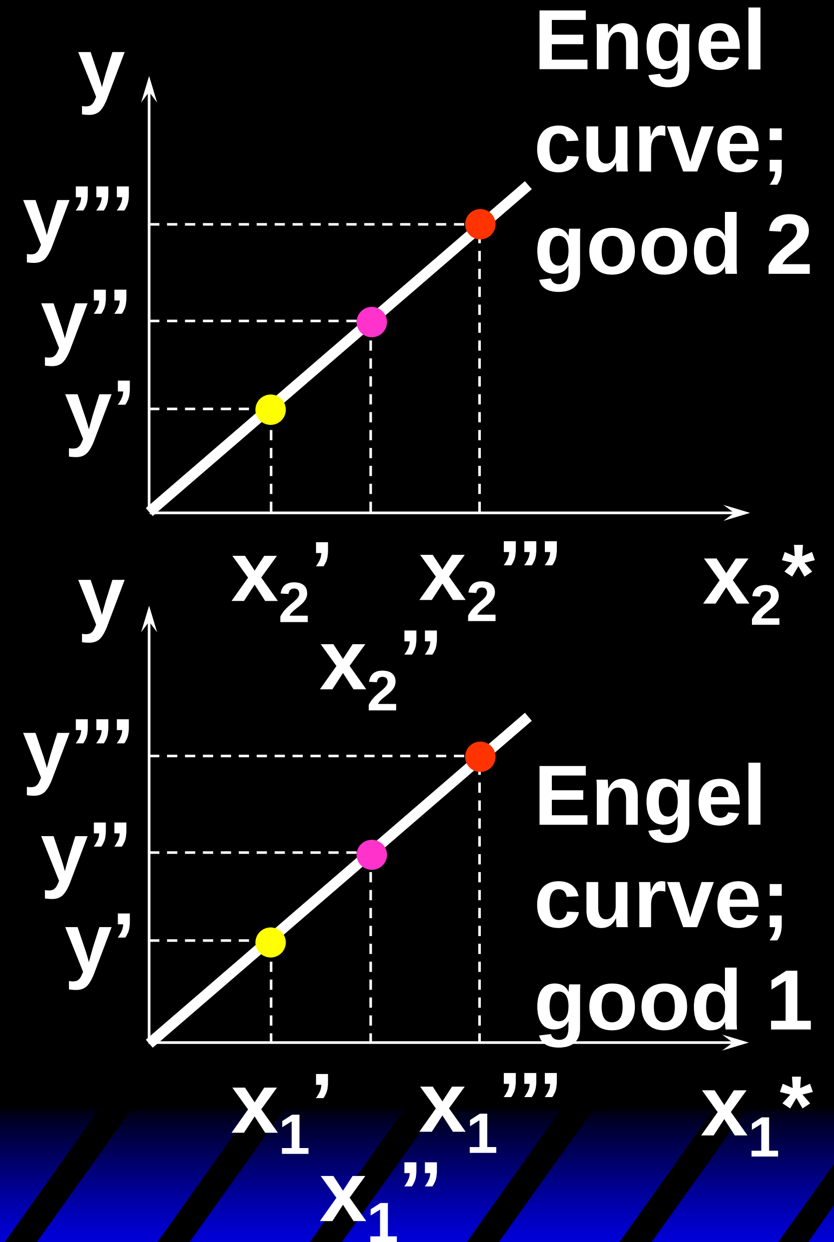


Income Changes

Fixed p_1 and p_2.

$$y = (p_1 + p_2)x_2^*$$

$$y = (p_1 + p_2)x_1^*$$



Income Changes and Perfectly-Substitutable Preferences

- **Another example of computing the equations of Engel curves; the perfectly-substitution case.**

$$U(x_1, x_2) = x_1 + x_2.$$

- **The ordinary demand equations are**

Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ y/p_1 & , \text{if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 < p_2 \\ y/p_2 & , \text{if } p_1 > p_2. \end{cases}$$

Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & \text{, if } p_1 > p_2 \\ y/p_1 & \text{, if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & \text{, if } p_1 < p_2 \\ y/p_2 & \text{, if } p_1 > p_2. \end{cases}$$

Suppose $p_1 < p_2$. Then $x_1^* = \dfrac{y}{p_1}$ and $x_2^* = 0$

Income Changes and Perfectly-Substitutable Preferences

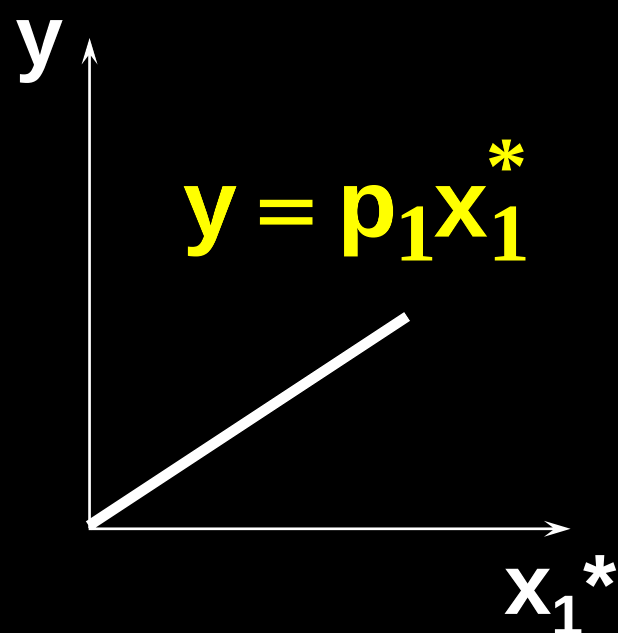
$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ y/p_1 & , \text{if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 < p_2 \\ y/p_2 & , \text{if } p_1 > p_2. \end{cases}$$

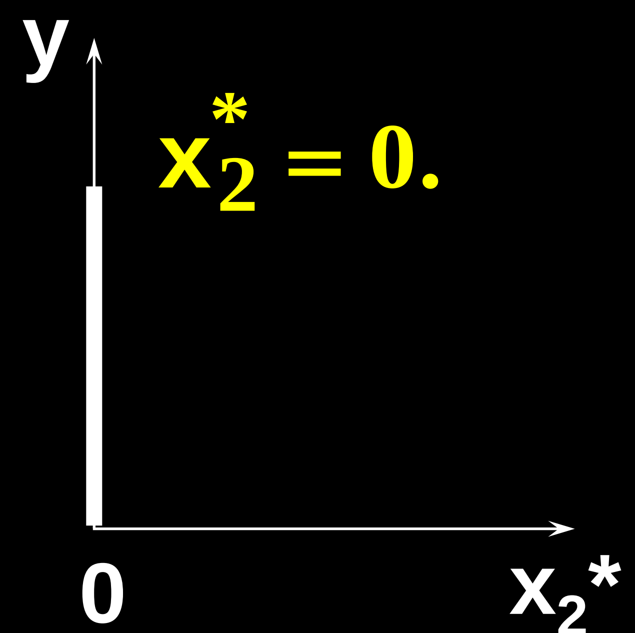
Suppose $p_1 < p_2$. Then $x_1^* = \dfrac{y}{p_1}$ and $x_2^* = 0$

$\Rightarrow$ $y = p_1 x_1^*$ and $x_2^* = 0.$

Income Changes and Perfectly-Substitutable Preferences



Engel curve for good 1



Engel curve for good 2

Income Changes

In every example so far the Engel curves have all been straight lines?
Q: Is this true in general?

A: No. Engel curves are straight lines if the consumer's preferences are homothetic.

消费者的偏好满足位似性时，恩格尔曲线是一条直线。

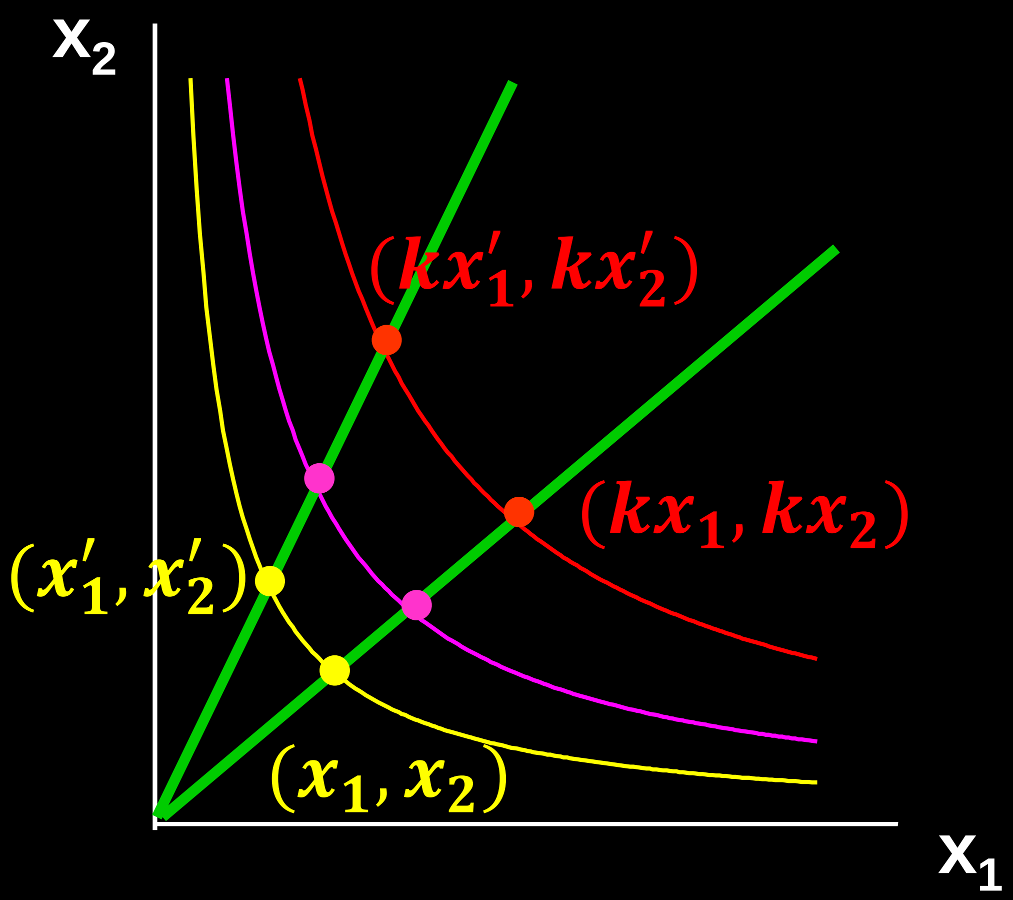
Homotheticity

A consumer's preferences are homothetic if and only if

$$(x_1,x_2) \prec (y_1,y_2) \Leftrightarrow (kx_1,kx_2) \prec (ky_1,ky_2)$$

for every k > 0.

Homotheticity



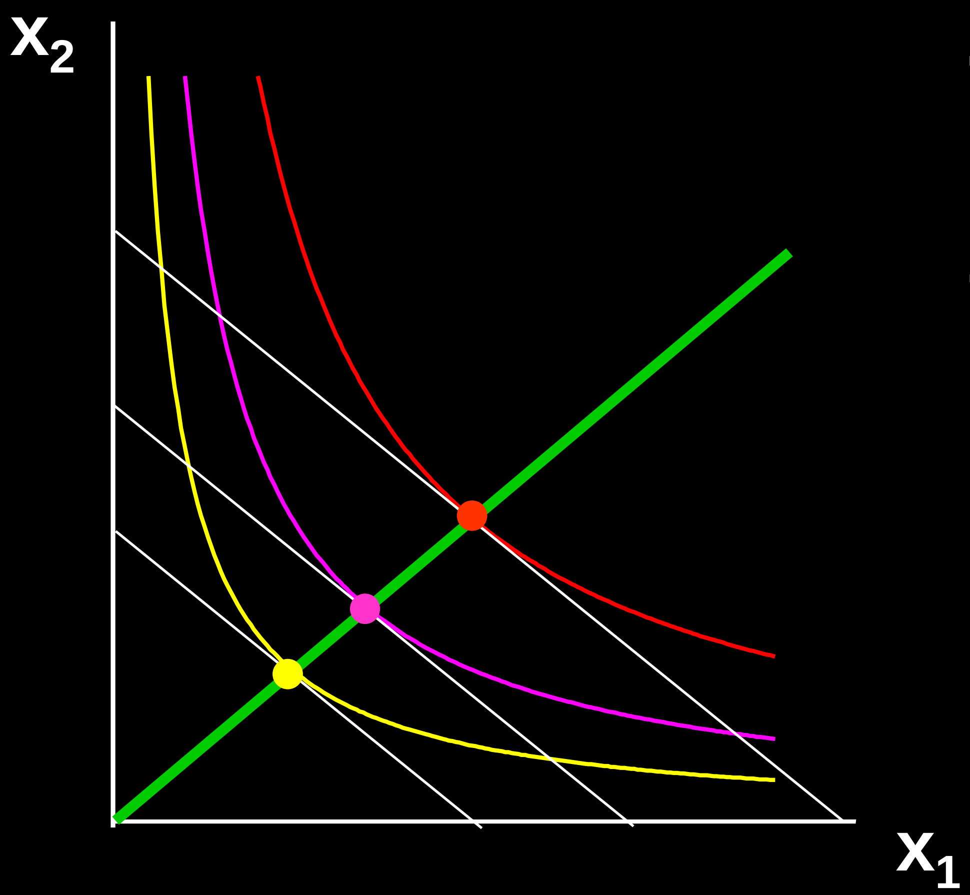
Homotheticity

A consumer's preferences are homothetic if and only if

the consumer's MRS is the same anywhere on a straight line drawn from the origin.

若偏好具有位似性，则经过原点的任意一条射线上的所有点具有相同的MRS。

Homotheticity



The indiff. curves have the same slope at any intersection.

Homotheticity

An example of C-D utility:

$$U = x_1^a x_2^b$$

Suppose $(x_1, x_2) \prec (y_1, y_2)$**. Then**

$$U(x_1, x_2) = x_1^a x_2^b < U(y_1, y_2) = y_1^a y_2^b$$

$\forall t > 0,$

$$U(tx_1, tx_2) = (tx_1)^a (tx_2)^b = t^{a+b} x_1^a x_2^b$$

$$U(ty_1, ty_2) = (ty_1)^a (ty_2)^b = t^{a+b} y_1^a y_2^b$$

$$U(tx_1, tx_2) < U(ty_1, ty_2)$$

Homotheticity

$$U = x_1^a x_2^b$$

If $(x_1, x_2) \prec (y_1, y_2)$,

$\forall t > 0$,

$$(tx_1, tx_2) \prec (ty_1, ty_2)$$

By definition, the preferences are *homothetic*.

Homotheticity

An example of C-D utility:

$$U = x_1^a x_2^b$$

$$\mathrm{MRS} = -\frac{MU_1}{MU_2} = -\frac{a x_1^{a-1} x_2^b}{b x_1^a x_2^{b-1}} = -\frac{a x_2}{b x_1}$$

Consider another point on the same ray from the origin: (tx_1, tx_2)

$$\mathrm{MRS}' = -\frac{a(tx_2)}{b(tx_1)} = -\frac{a x_2}{b x_1}$$

Homotheticity

When $U = x_1^a x_2^b$,

- any two points along the same ray from the origin, (x_1, x_2) and (tx_1, tx_2), have the same MRS.

By definition, … *homothetic*.

Income Effects -- A Nonhomothetic Example

Quasilinear preferences are not homothetic.

$$U(x_1, x_2) = f(x_1) + x_2.$$

For example,

$$U(x_1, x_2) = \ln(x_1) + x_2$$

Income Effects -- A Nonhomothetic Example

$$\mathbf{U(x_1, x_2) = \ln(x_1) + x_2}$$

$$\mathbf{MRS} = -\frac{\mathbf{MU_1}}{MU_2} = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

Income Effects -- A Nonhomothetic Example

$$\mathbf{U(x_1, x_2) = \ln(x_1) + x_2}$$

$$\mathbf{MRS} = -\frac{1}{x_1}$$

MRS is $-\frac{1}{x_1}$ **at** (x_1, x_2)

MRS is $-\frac{1}{tx_1}$ **at** (tx_1, tx_2)

Non-homothetic

Income Effects -- A Nonhomothetic Example

$$\mathbf{U(x_1, x_2) = ln(x_1) + x_2}$$

$$\mathbf{MRS} = -\frac{1}{x_1}$$

Optimal consumption choice?

Income Effects -- A Nonhomothetic Example

$$\mathbf{U(x_1, x_2) = ln(x_1) + x_2}$$

$$\mathbf{MRS} = -\frac{1}{x_1} = -\frac{p_1}{p_2} \qquad \textbf{(A)}$$

$$p_1x_1 + p_2x_2 = y \qquad \textbf{(B)}$$

Income Effects -- A Nonhomothetic Example

$$\mathbf{MRS} = -\frac{1}{x_1} = -\frac{p_1}{p_2} \qquad \textbf{(A)}$$

$$p_1x_1 + p_2x_2 = y \qquad \textbf{(B)}$$

(A)=> $x_1^* = \frac{p_2}{p_1}$

Substitute into (B),

$$p_1\left(\frac{p_2}{p_1}\right) + p_2x_2 = y$$

$$x_2^* = \frac{y - p_2}{p_2} = \frac{y}{p_2} - 1$$

Income Effects -- A Nonhomothetic Example

When $\frac{\mathrm{y}}{\mathrm{p}_2} > \mathbf{1}\ (\mathbf{y} > \mathbf{p}_2)$**,**

$$x_1^* = \frac{p_2}{p_1}$$

$$x_2^* = \frac{y - p_2}{p_2} = \frac{y}{p_2} - 1 > 0$$

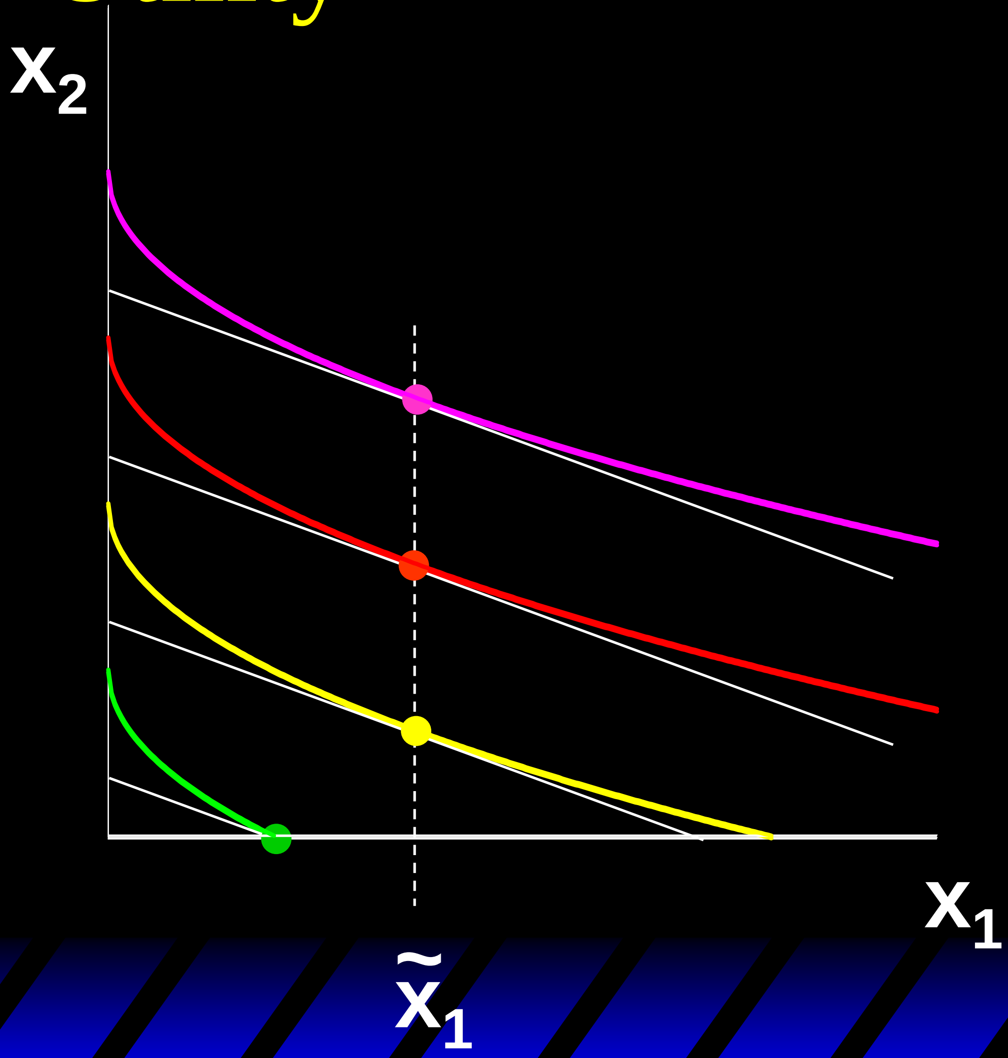
Income Effects -- A Nonhomothetic Example

When $\frac{y}{p_2} < 1 \ (y < p_2)$,

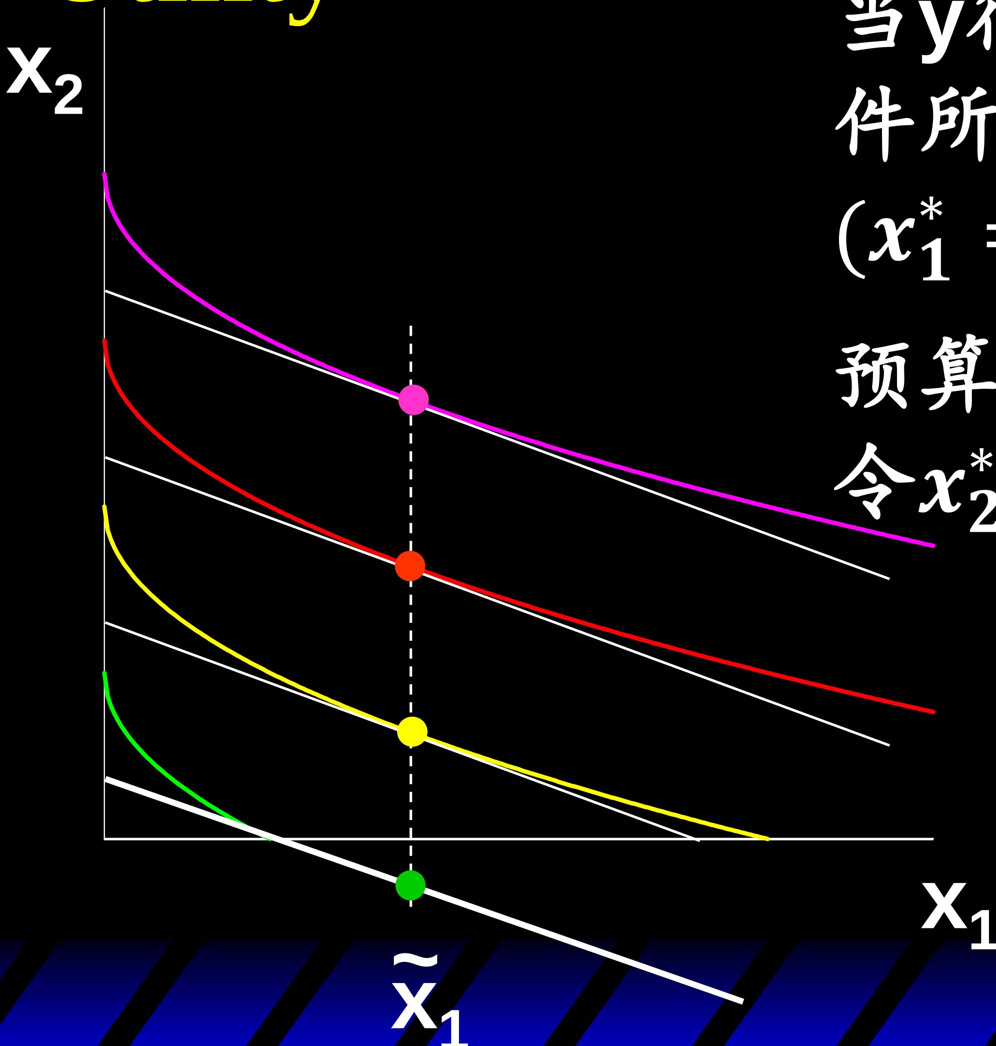
$$x_2^* = 0 \text{ (corner solution)}$$

$$x_1^* = \frac{y}{p_2}$$

Income Changes; Quasilinear Utility



Income Changes; Quasilinear Utility



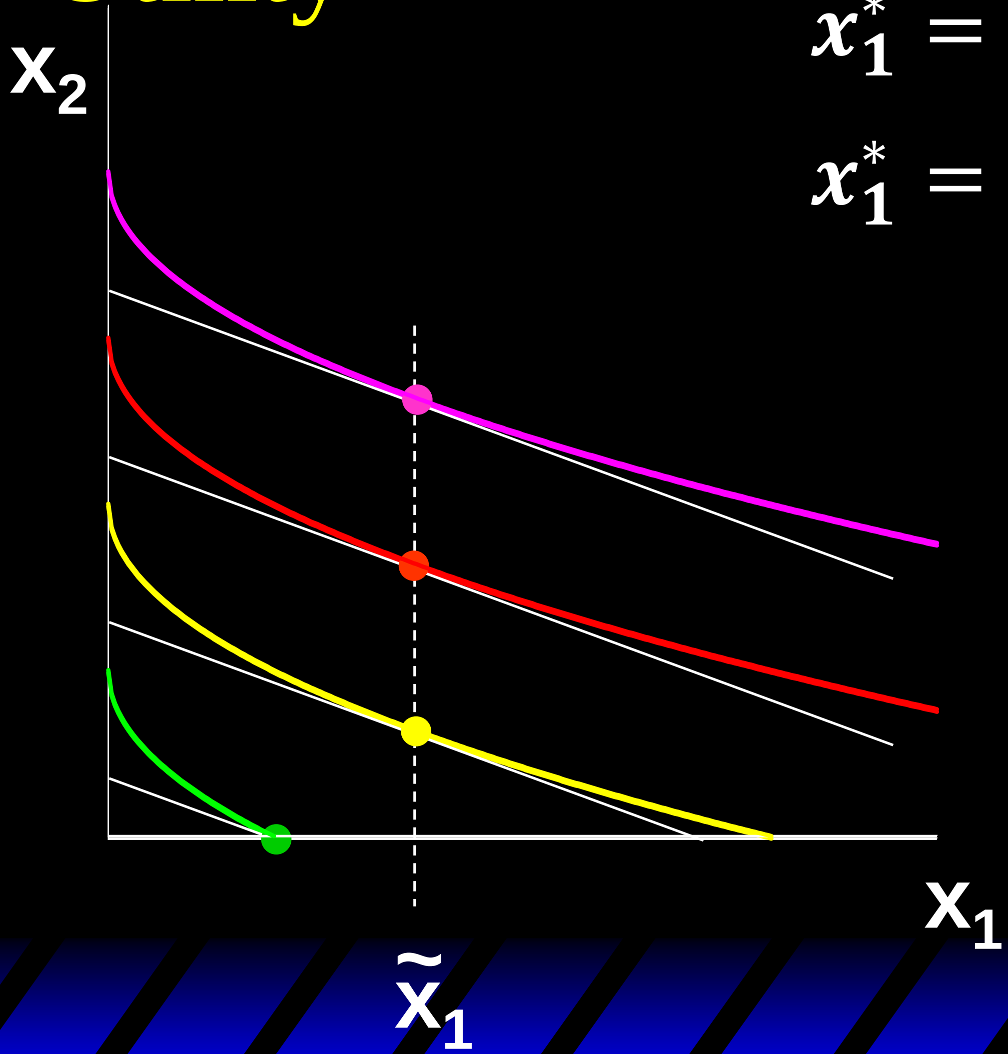
当y很小时，使用相切条件所求得的最优商品组合$\left(x_1^* = \frac{p_2}{p_1}, x_2^* = \frac{y}{p_2} - 1\right)$在预算集之外，不可得。

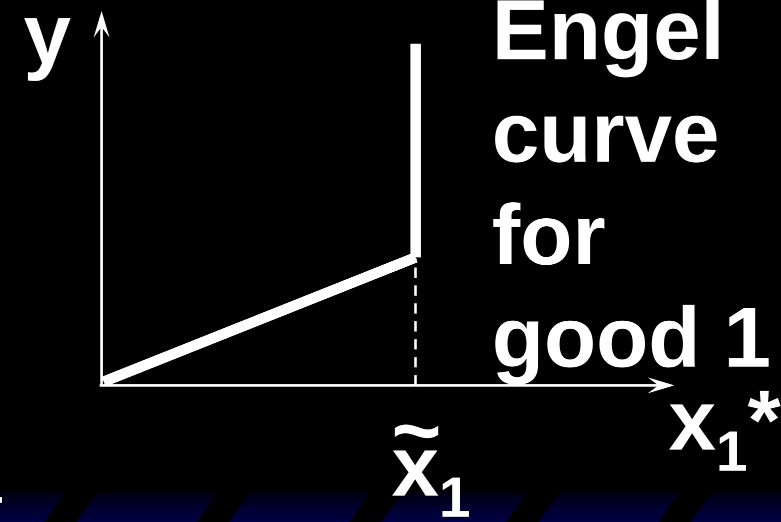
令$x_2^* = 0$，求得$x_1^* = \frac{y}{p_1}$

Income Changes; Quasilinear Utility

$x_1^* = \frac{y}{p_2}$ **when y is small**

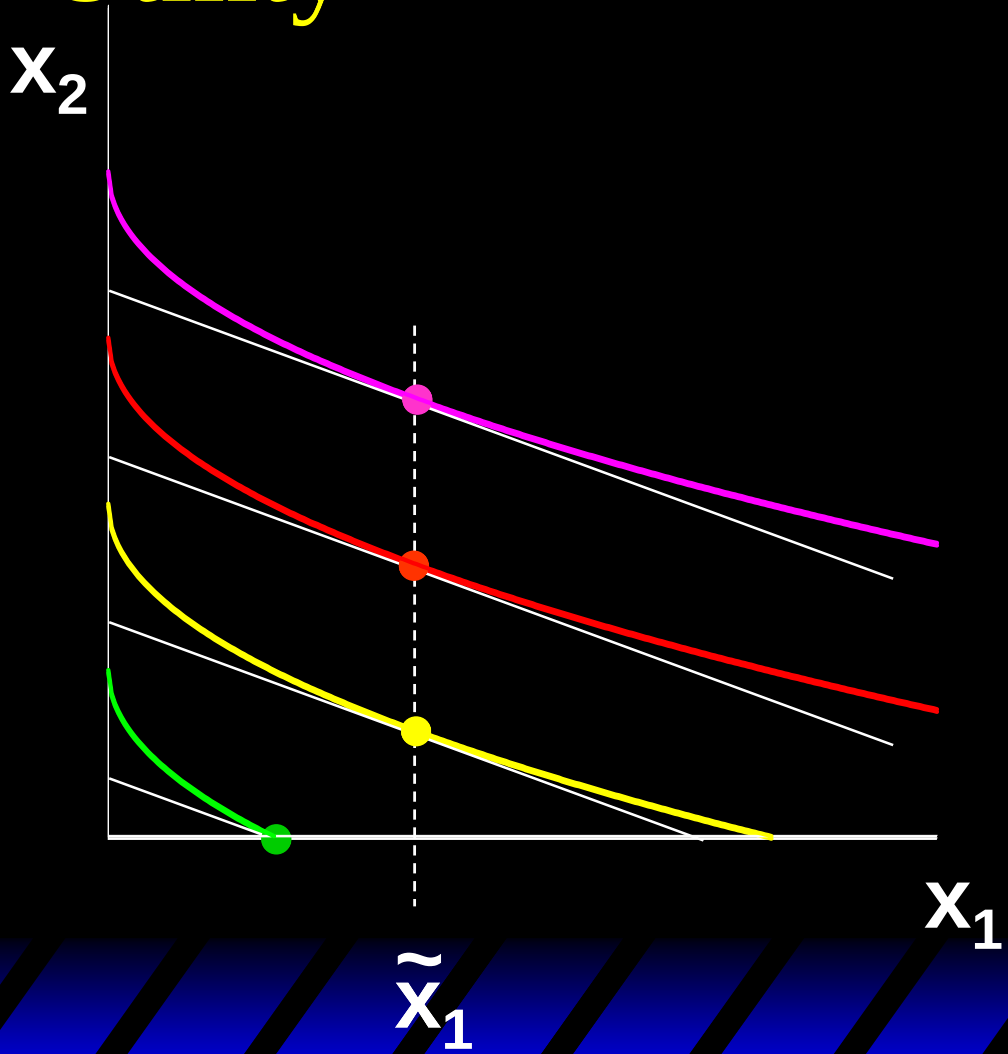
$x_1^* = \frac{p_2}{p_1}$ **when y is big**

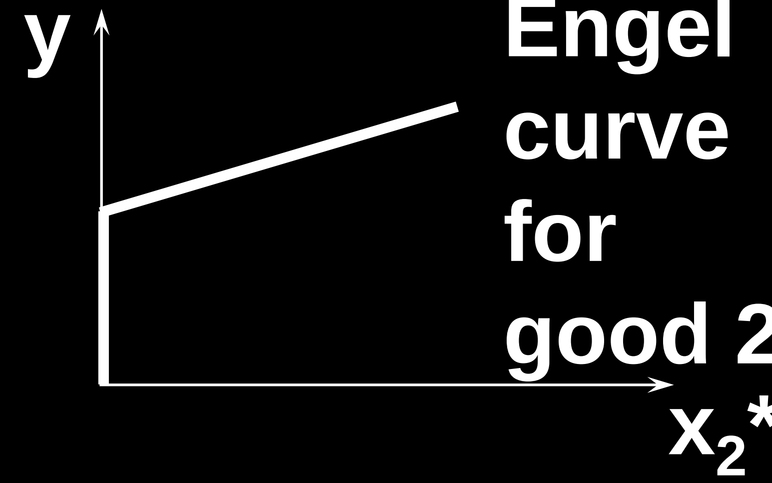




Engel curve for good 1

Income Changes; Quasilinear Utility

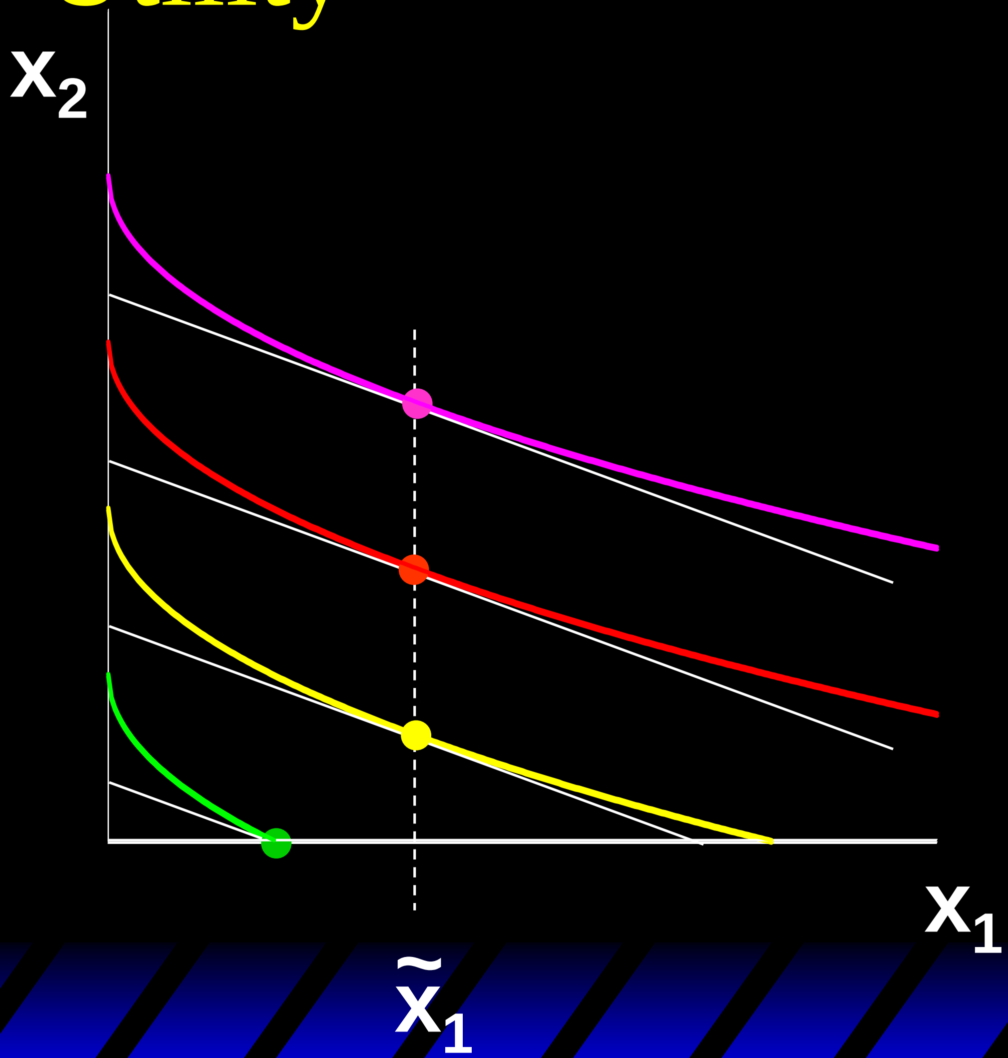


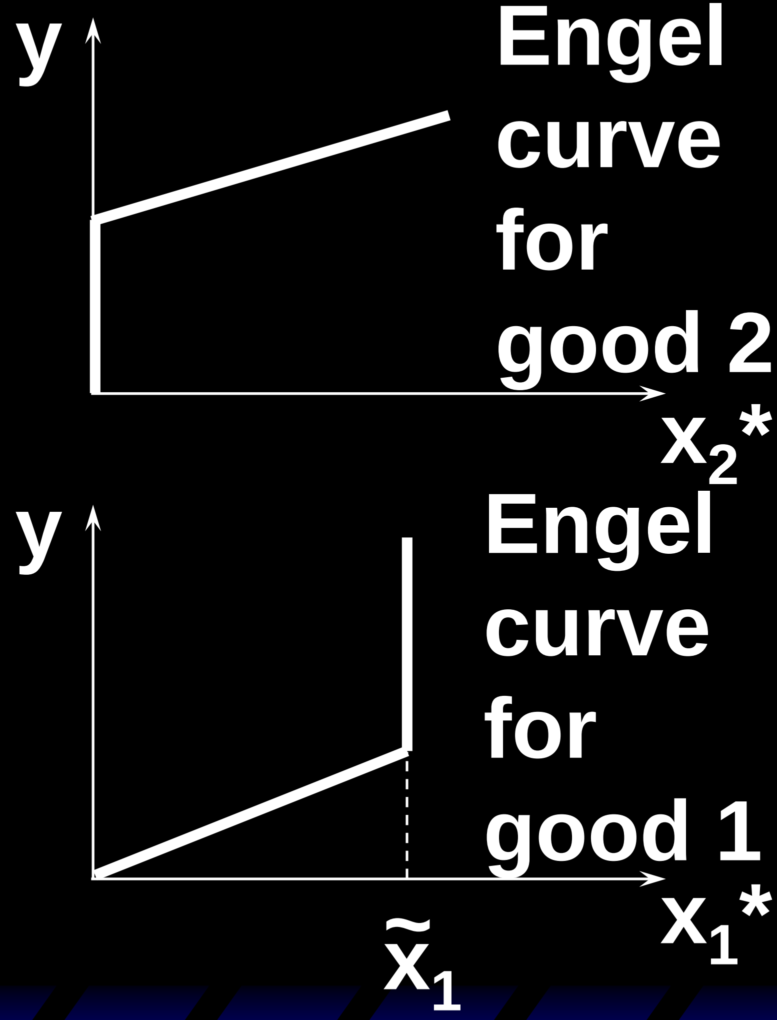


$x_2^* = 0$ if $y < p_2$

$x_2^* = \frac{y}{p_2} - 1$ if $y > p_2$

Income Changes; Quasilinear Utility





Income Effects

A good for which quantity demanded rises with income is called normal.

Therefore a normal good's Engel curve is positively sloped.

若需求数量随收入的上升而上升（价格不变），则该商品为正常品。恩格尔曲线斜率为正。

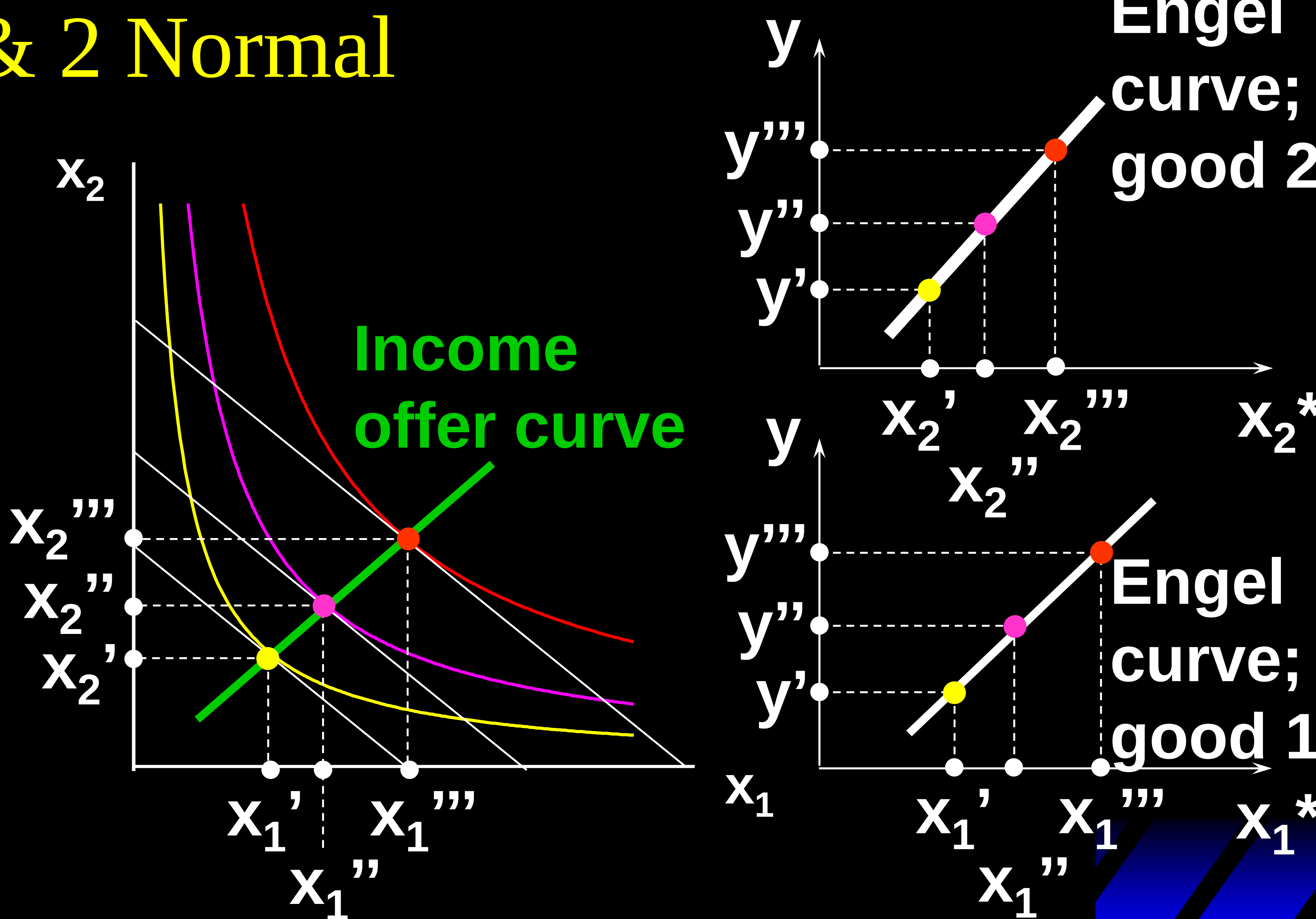
Income Effects

A good for which quantity demanded falls as income increases is called **income inferior**.

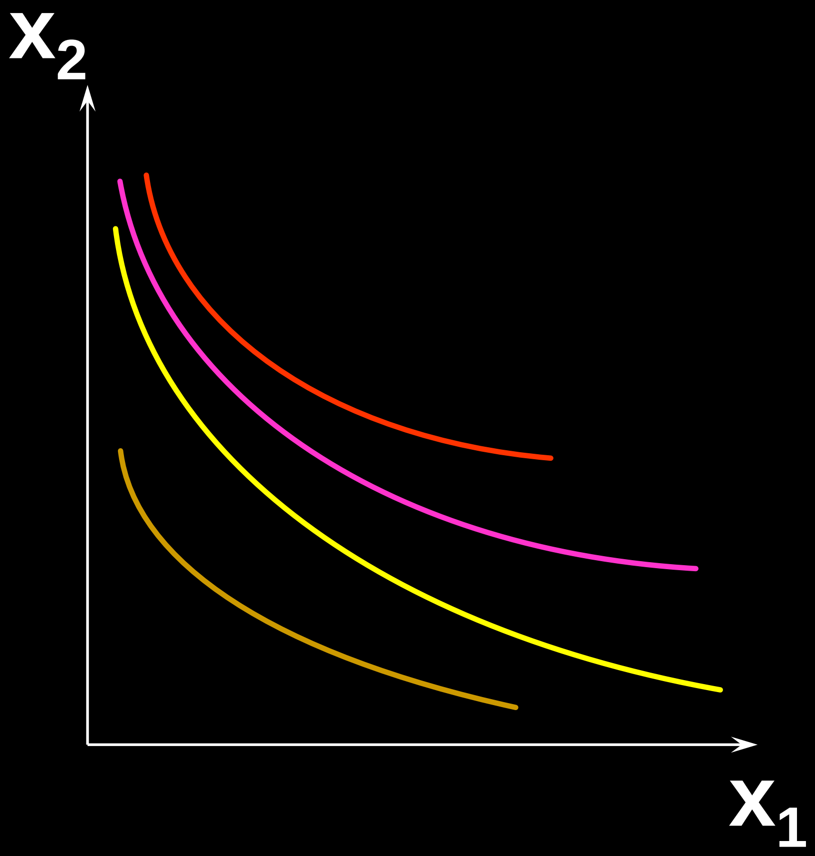
Therefore an income inferior good's Engel curve is negatively sloped.

若需求数量随收入的上升而下降（价格不变），则该商品为低档品。恩格尔曲线斜率为负。

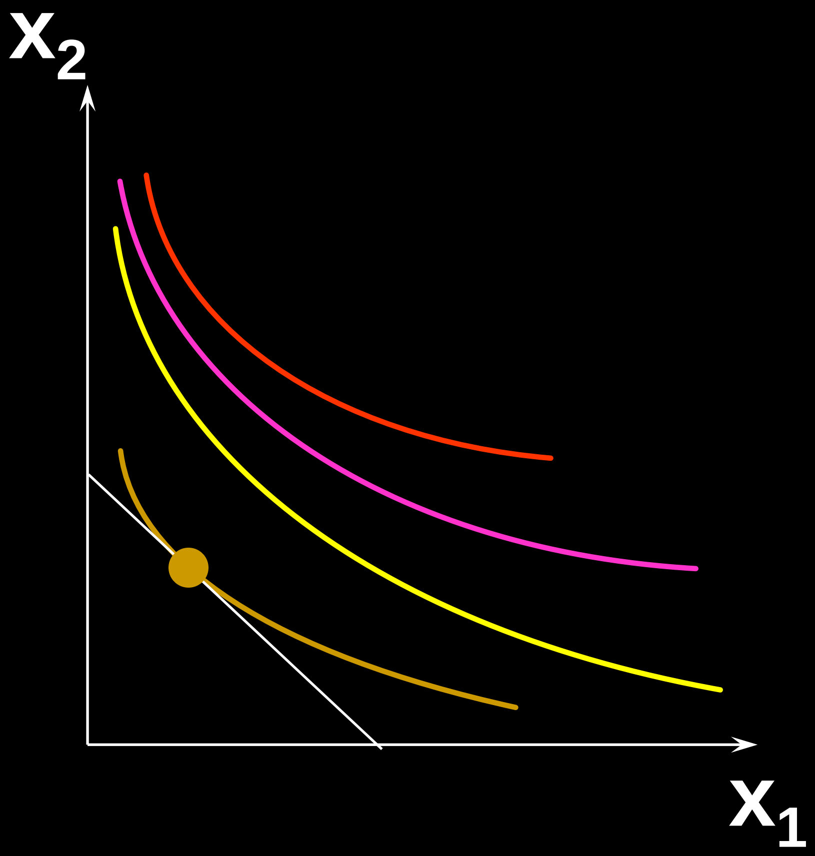
Income Changes; Goods 1 & 2 Normal



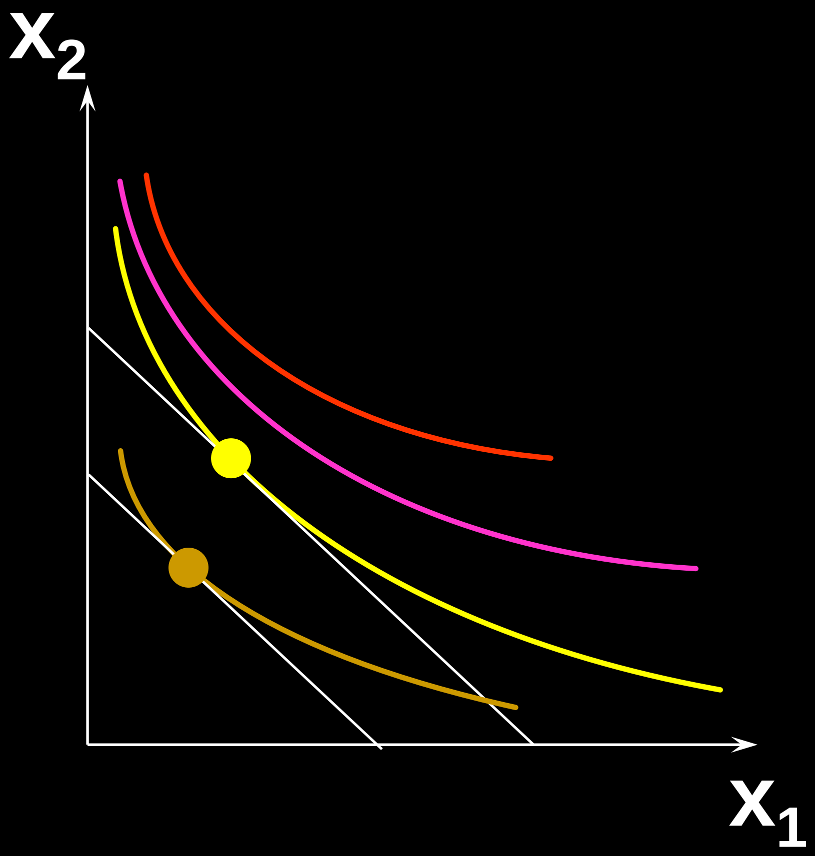
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



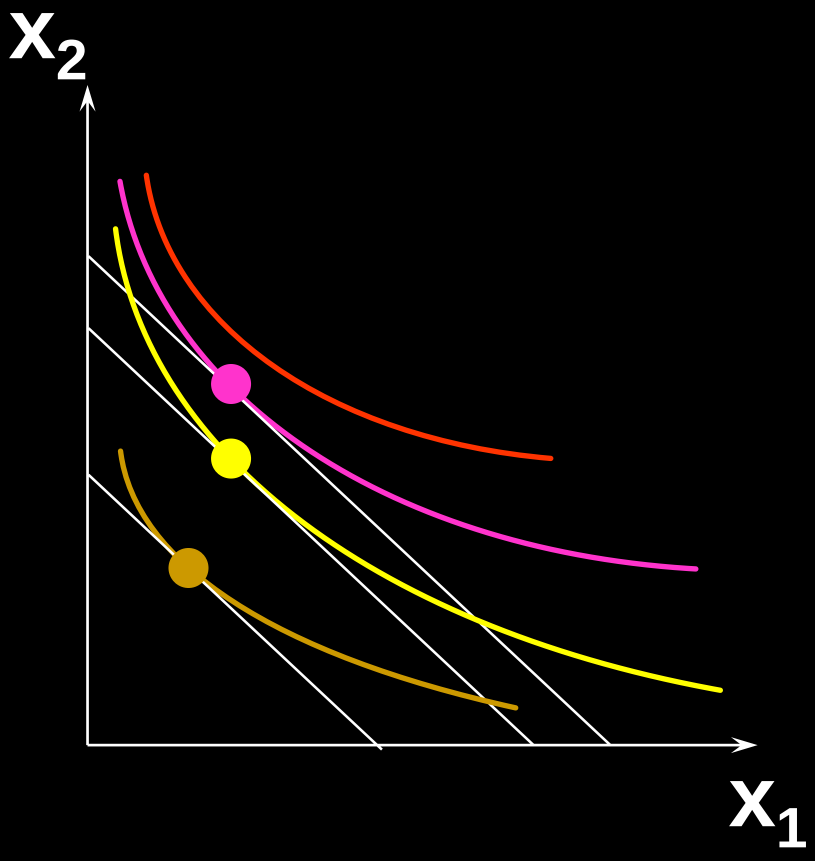
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



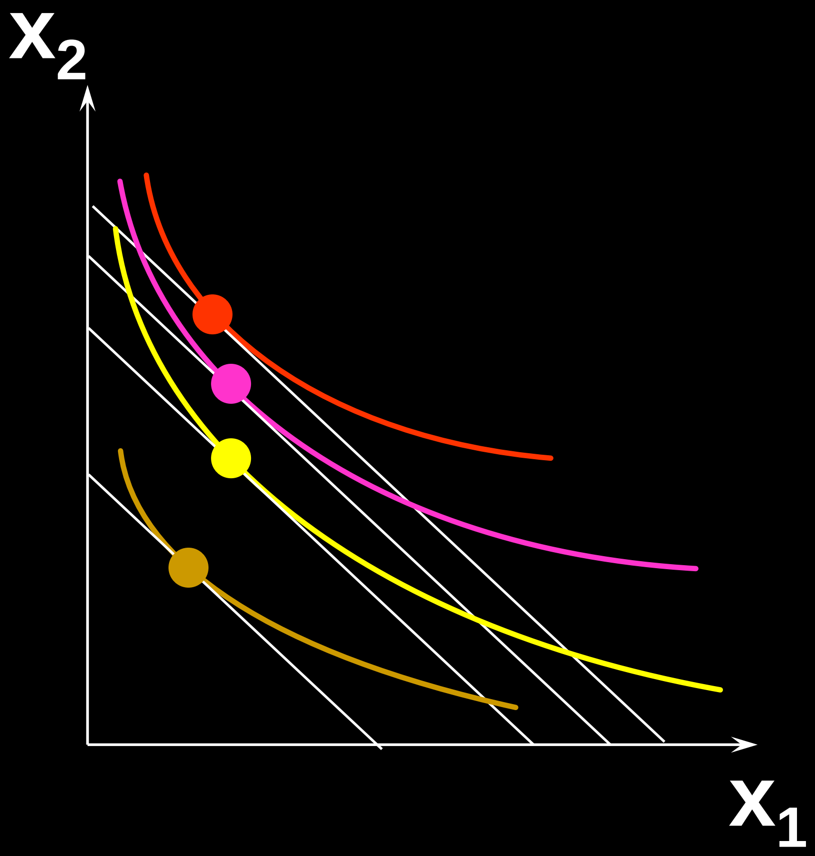
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



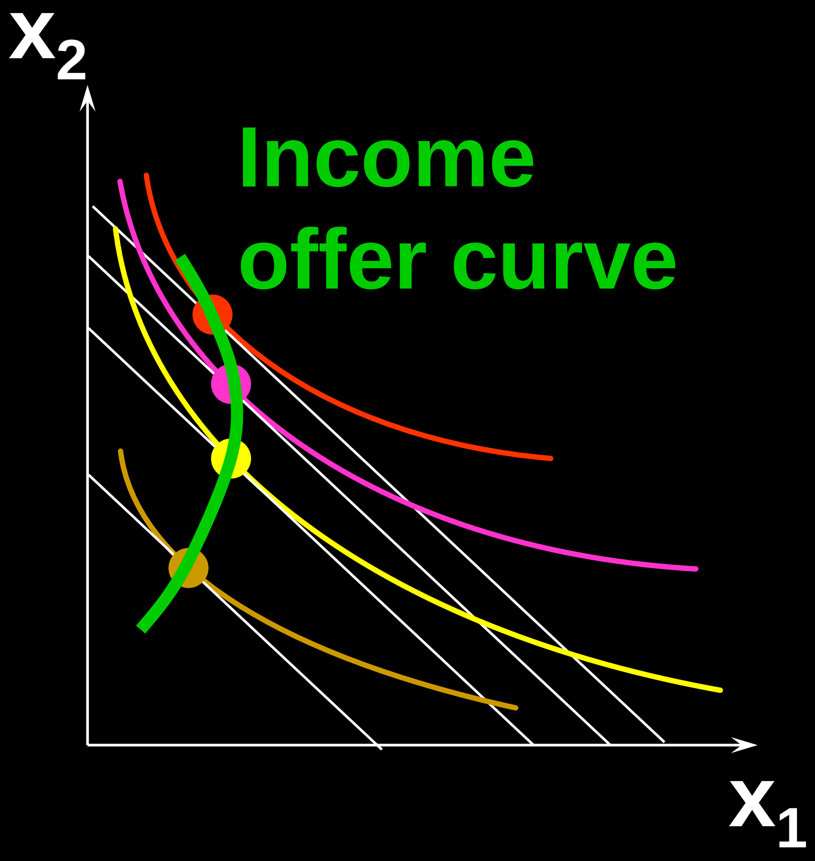
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



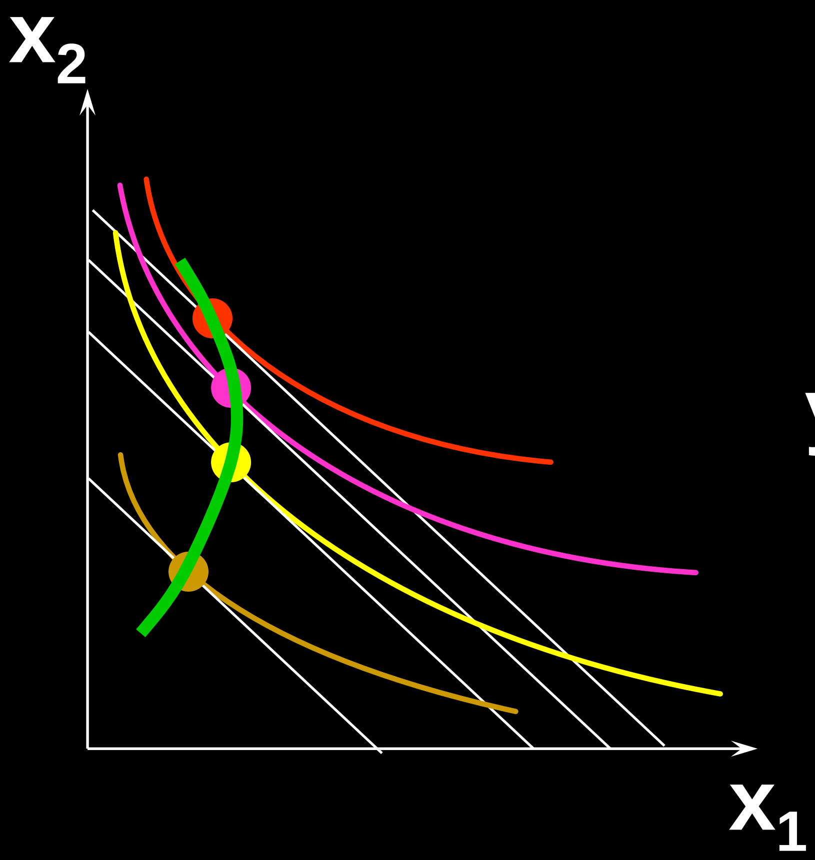
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior

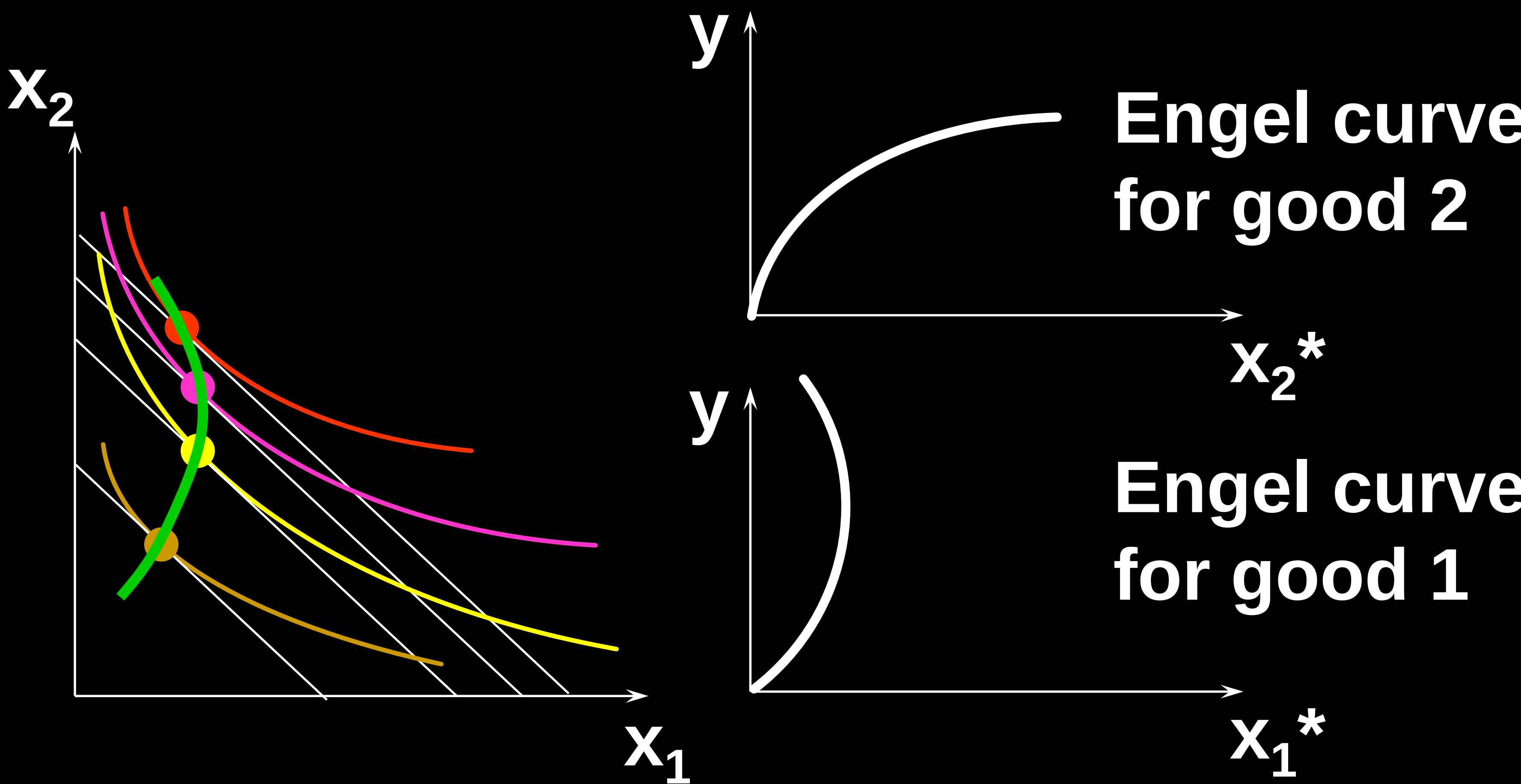


Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Engel curve for good 1

Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



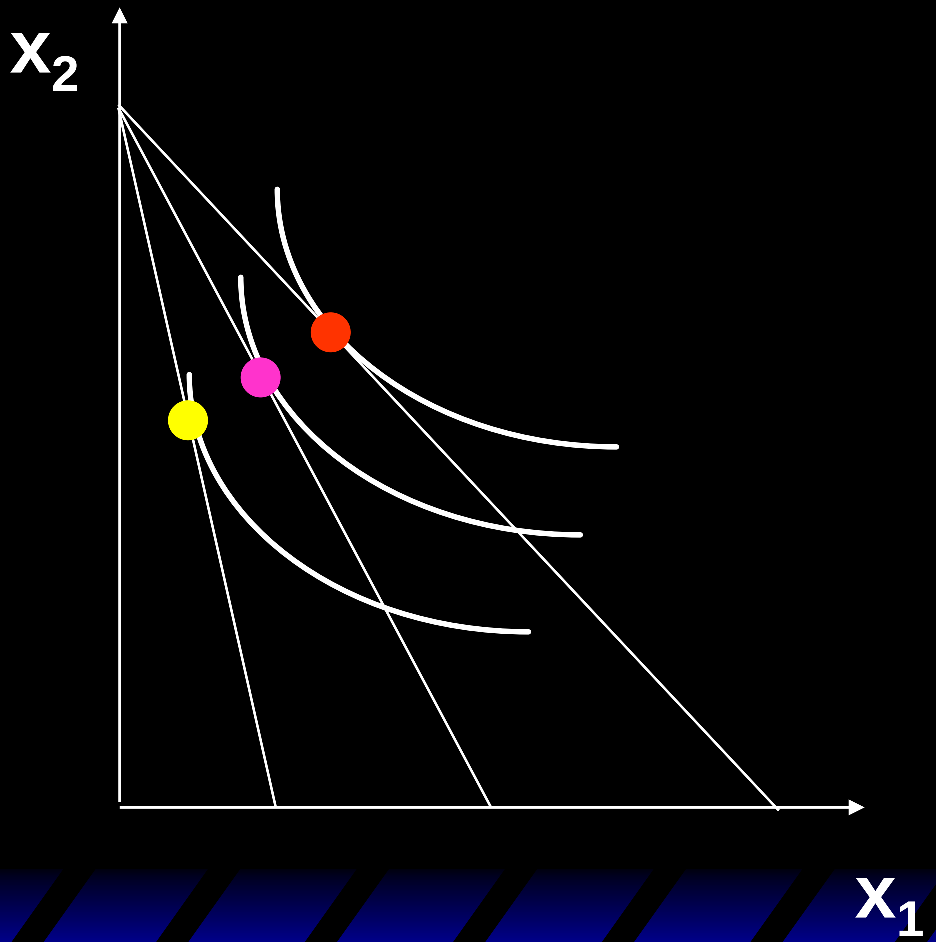
Ordinary Goods

A good is called ordinary if the quantity demanded of it always increases as its own price decreases.

若需求数量随自身价格的上升而下降，该商品被称为普通商品。

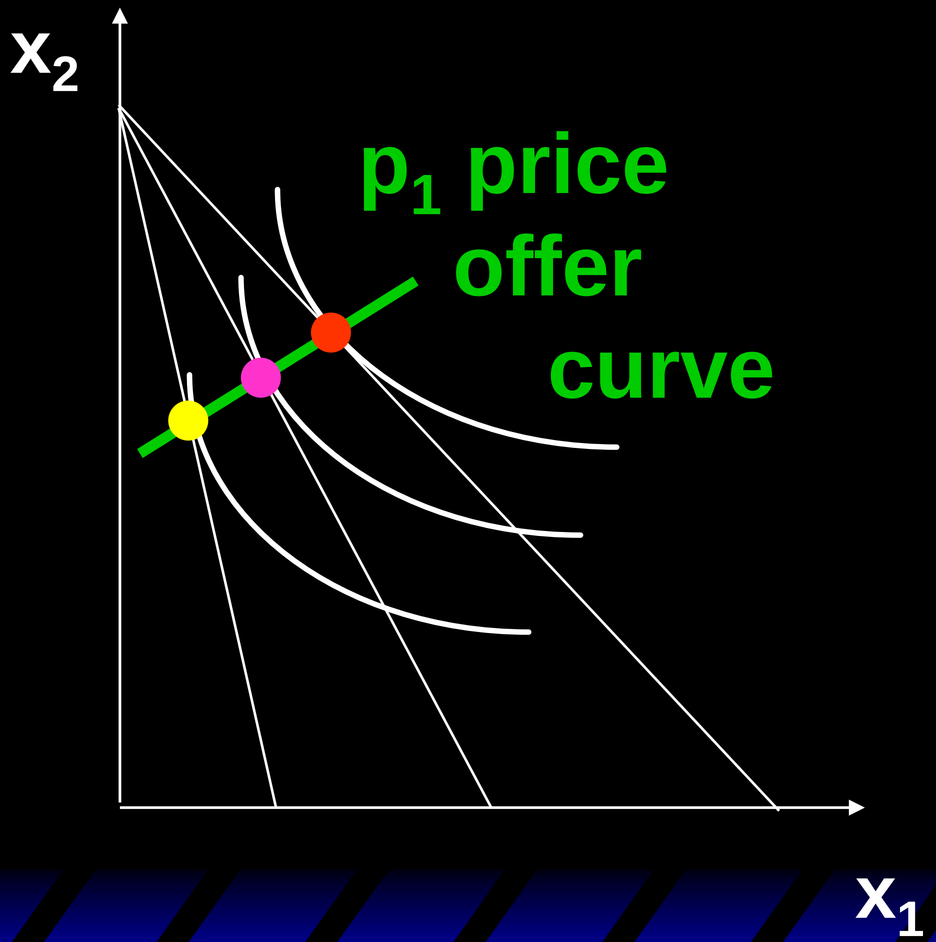
Ordinary Goods

Fixed p_2 and y.

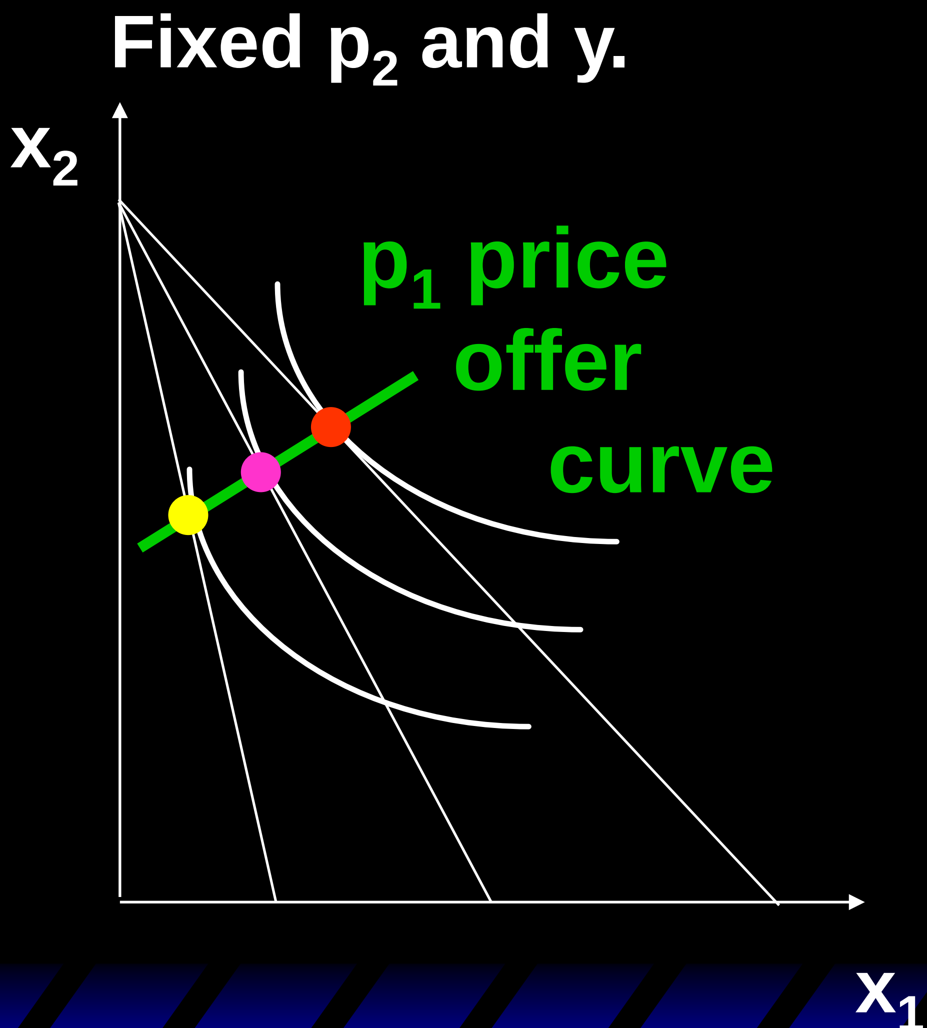


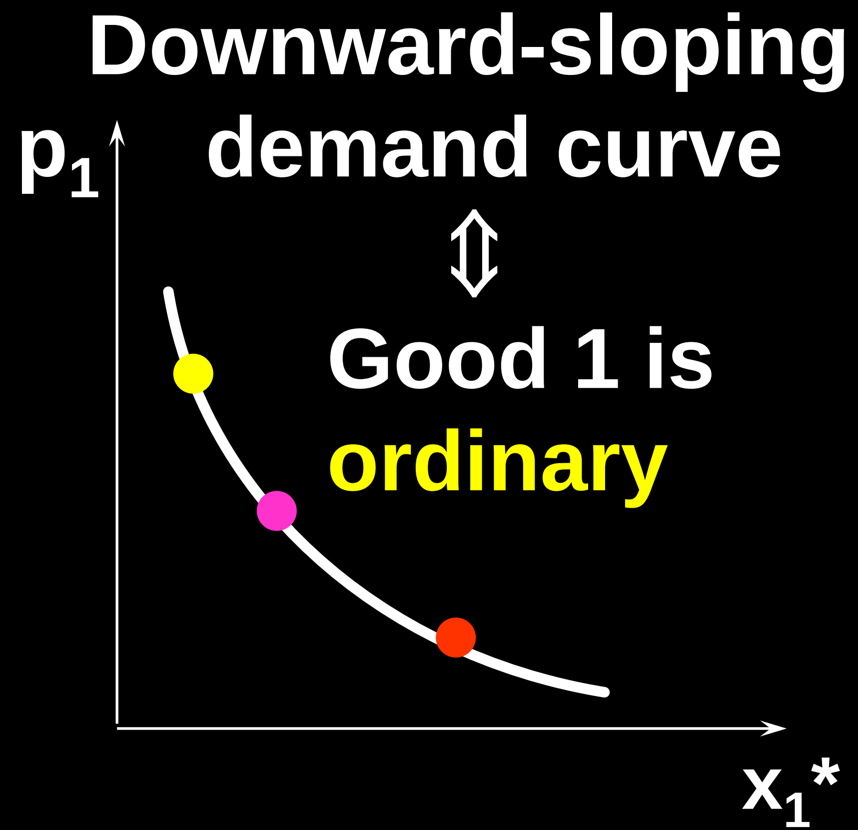
Ordinary Goods

Fixed p_2 and y.



Ordinary Goods





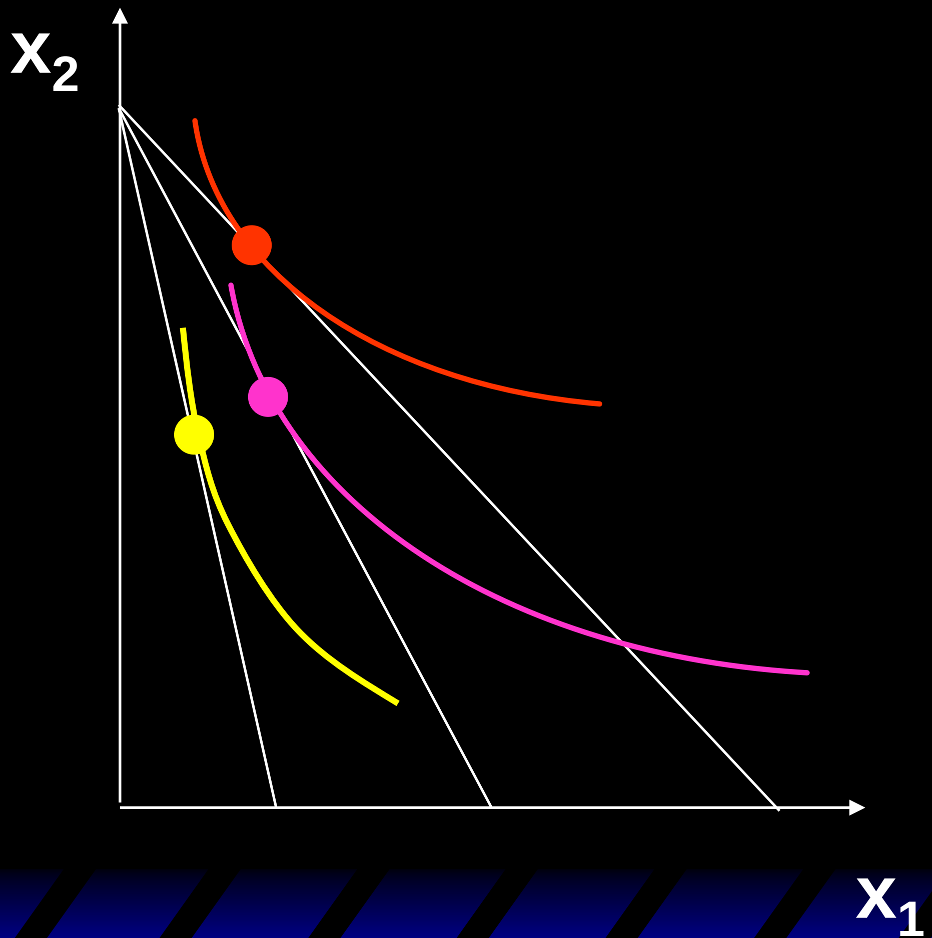
Giffen Goods

If, for some values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called Giffen.

若存在一个价格区间，使得需求数量随自身价格的上升而上升，该商品被称为吉芬商品。

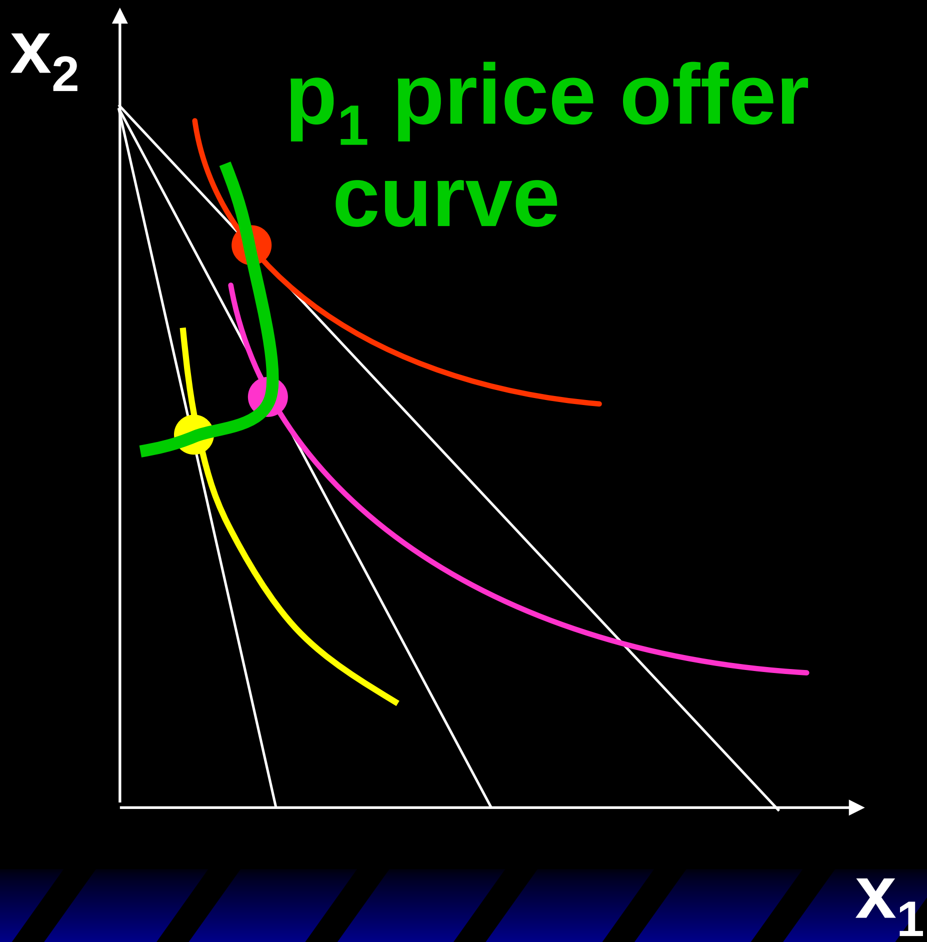
Ordinary Goods

Fixed p_2 and y.



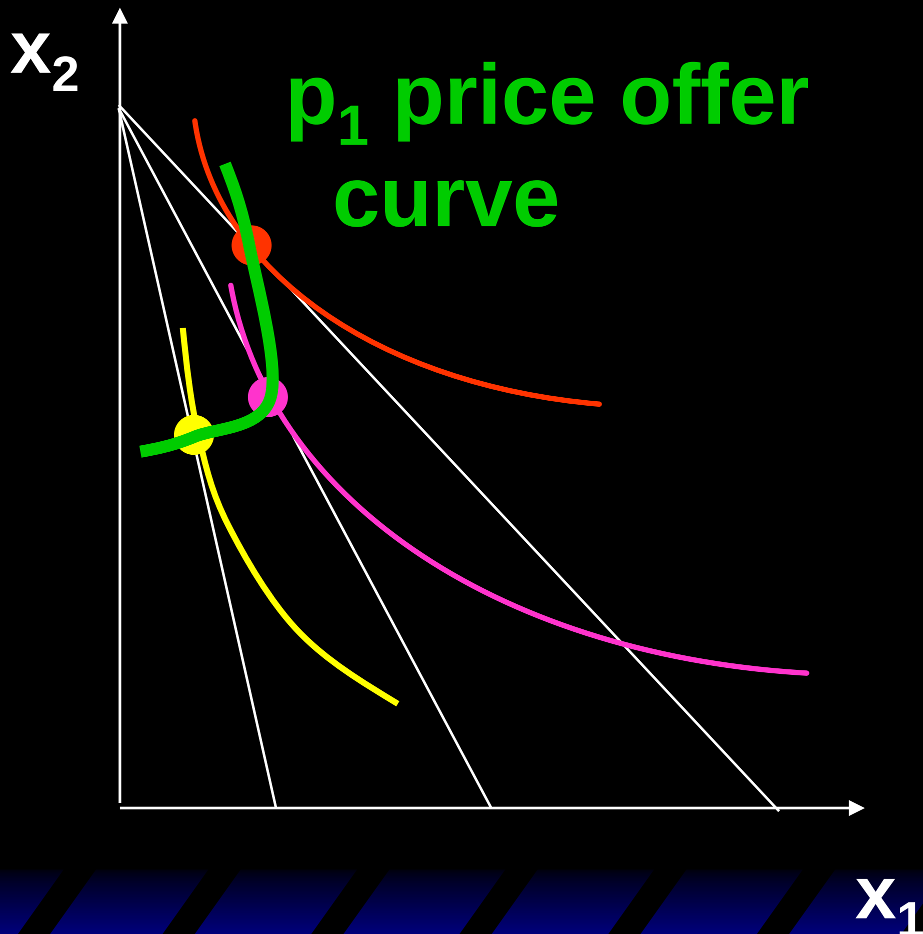
Ordinary Goods

Fixed p_2 and y.



Ordinary Goods

Fixed p_2 and y.



Demand curve has a positively sloped part

$\Updownarrow$

Good 1 is Giffen

Cross-Price Effects

If an increase in p_2

- **increases** demand for commodity 1 $\left(\frac{\partial x_1}{\partial p_2} > 0\right)$, then commodity 1 is a **gross substitute** for commodity 2
- **reduces** demand for commodity 1 $\left(\frac{\partial x_1}{\partial p_2} < 0\right)$, then commodity 1 is a **gross complement** for commodity 2.

Cross-Price Effects

A perfect-complements example:

$$x_1^* = \frac{y}{p_1 + p_2}$$

so

$$\frac{\partial x_1^*}{\partial p_2} = -\frac{y}{(p_1 + p_2)^2} < 0.$$

Therefore commodity 2 is a gross complement for commodity 1.

Cross-Price Effects

A Cobb- Douglas example:

$$x_2^* = \frac{by}{(a+b)p_2}$$

so

$$\frac{\partial x_2^*}{\partial p_1} = 0.$$

Therefore commodity 1 is neither a gross complement nor a gross substitute for commodity 2.