



# Lecture 13

## Cost Curves



# Review: Returns-to-Scale and AC

The returns-to-scale properties of a firm's technology determine how **average production costs** change with **output level** (规模报酬决定平均成本如何随产出变化而变化).

# Review: Returns-to-Scale and AC

If a firm's technology exhibits **constant** returns-to-scale then average production cost does not change.

规模报酬**不变**时，平均成本**不随**产出变化而变化。

# Review: Returns-to-Scale and AC

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规模报酬**不变**时，平均成本**不随**产出变化而变化。

If  $(x_1, x_2)$  minimizes the cost of producing **y**, with minimized costs being **C**, then

$(kx_1, kx_2)$  minimizes the cost of producing **ky**, with min cost being **kC**  
(AC is constant)

# Review: Returns-to-Scale and AC

If a firm's technology exhibits **increasing** returns-to-scale then average production cost **decreases**.

规模报酬**递增**时，平均成本随产出的上升而**下降**。

# Review: Returns-to-Scale and AC

If a firm's technology exhibits **increasing** returns-to-scale then average production cost **decreases**.

规模报酬**递增**时，平均成本随产出的上升而**下降**。

If  $(x_1, x_2)$  minimizes the cost of producing **y**, with minimized costs being **C**, then

$(k'x_1, k'x_2)$  minimizes the cost of producing **ky**, with the min cost being  **$k'C < kC$**  ( $AC \downarrow$ , **under homotheticity**)

# Review: Returns-to-Scale and AC

If a firm's technology exhibits **decreasing** returns-to-scale then average production cost **increases**.

规模报酬**递减**时，平均成本随产出的上升而**上升**。

# Review: Returns-to-Scale and AC

If a firm's technology exhibits **decreasing** returns-to-scale then average production cost **increases**.

规模报酬**递减**时，平均成本随产出的上升而**上升**。

If  $(x_1, x_2)$  minimizes the cost of producing **y**, with minimized costs being **C**, then

$(k'x_1, k'x_2)$  minimizes the cost of producing **ky**, with the min cost being  $k'C > kC$  ( $AC \uparrow$ , **under homotheticity**)



# Review: Returns-to-Scale and AC

规模报酬递减时，平均成本随产出的上升而上升。

**Proof (without homotheticity):**

Suppose  $(x_1, x_2)$  **minimizes** the cost of producing **y** output units, with the minimized cost being **C**.

**Assume**  $(x'_1, x'_2)$  minimizes the cost of producing **ky** output units, with the min cost being **C' ≤ kC** (i.e. AC ↓)

# Review: Returns-to-Scale and AC

**Assume**  $(x'_1, x'_2)$  minimizes the cost of producing  **$ky$**  output units, with the minimized cost being  **$C' \leq kC$**  (i.e. AC  $\downarrow$ )

Then the cost of  $\left(\frac{x'_1}{k}, \frac{x'_2}{k}\right)$  is  $\frac{C'}{k} \leq C$ .

Because the technology exhibits DRS, we know  $f\left(\frac{x'_1}{k}, \frac{x'_2}{k}\right) > y$ .

# Review: Returns-to-Scale and AC

The cost of  $\left(\frac{x'_1}{k}, \frac{x'_2}{k}\right)$  is  $\frac{C'}{k} \leq C$ .

Because the technology exhibits DRS,  
 $f\left(\frac{x'_1}{k}, \frac{x'_2}{k}\right) > y$ .

They contradict the fact that  $C$  is the minimized costs of producing  $y$  output units.

# Review: Returns-to-Scale and AC

Therefore,

if  $(x_1, x_2)$  **minimizes** the cost of producing **y** output units, with the minimized cost being **C**

And  $(x'_1, x'_2)$  minimizes the cost of producing **ky** output units.

The cost of  $(x'_1, x'_2)$  must be **> kC** (i.e. AC must **increase** as y increases)

# Review: Cost Minimization

**Given  $y$** , the firm is to find the optimal input bundle that **minimizes** the production costs

$$\min_{x_1, x_2} C = \omega_1 x_1 + \omega_2 x_2$$

$$\text{s.t. } f(x_1, x_2) = y$$

The least-costly input bundle  $x_1^*(\omega_1, \omega_2, y)$  and  $x_2^*(\omega_1, \omega_2, y)$  are the firm's **conditional demands for inputs 1 and 2**

# Review: Cost Minimization

**Given  $y$** , the firm is to find the optimal input bundle that **minimizes** the production costs

$$\min_{x_1, x_2} C = \omega_1 x_1 + \omega_2 x_2$$

$$\text{s.t. } f(x_1, x_2) = y$$

The **total cost function** is the smallest possible total cost for producing  $y$  output units

$$C^* = \omega_1 x_1^* + \omega_2 x_2^* = C(y)$$

# Review: Cost Minimization

In the short run,  $x_2 = \tilde{x}_2$ , the firm's problem becomes

$$\min_{x_1} C = \omega_1 x_1 + \omega_2 \tilde{x}_2$$

$$\text{s.t. } f(x_1, \tilde{x}_2) = y$$

The **total cost function** can be expressed as

$$\underbrace{C(y)}_{\text{总成本}} = \omega_1 x_1^*(y) + \omega_2 \tilde{x}_2 = \underbrace{C_v(y)}_{\text{可变成本}} + \underbrace{F}_{\text{固定成本}}$$

# Types of Cost Curves

A **total cost curve** is the graph of a firm's total cost function 总成本曲线,  $C(y)$

A **variable cost curve** is the graph of a firm's variable cost function 可变成本曲线,  $C_v(y)$

A **fixed cost curve** is the graph of a firm's fixed cost function 固定成本曲线,  $FC(y) = F$



# Types of Cost Curves

**average total cost curve** 平均成本曲线,  
 $ATC(y) = C(y)/y$

**average variable cost curve** 平均可变成本曲线,  $AVC(y) = C_v(y)/y$

**average fixed cost curve** 平均固定成本曲线,  $AFC(y) = F/y$

**marginal cost curve** 边际成本曲线,

$$MC(y) = \frac{\partial C(y)}{\partial y} = \frac{\partial [F + C_v(y)]}{\partial y} = \frac{\partial C_v(y)}{\partial y}$$

# Types of Cost Curves

**How are these cost curves related to each other?**

**How are a firm's long-run and short-run cost curves related?**

\$

固定成本曲线是一条水平线

F

y



\$

可变成本曲线是一条通过原点的、向上倾斜的曲线

$c_v(y)$

$y$

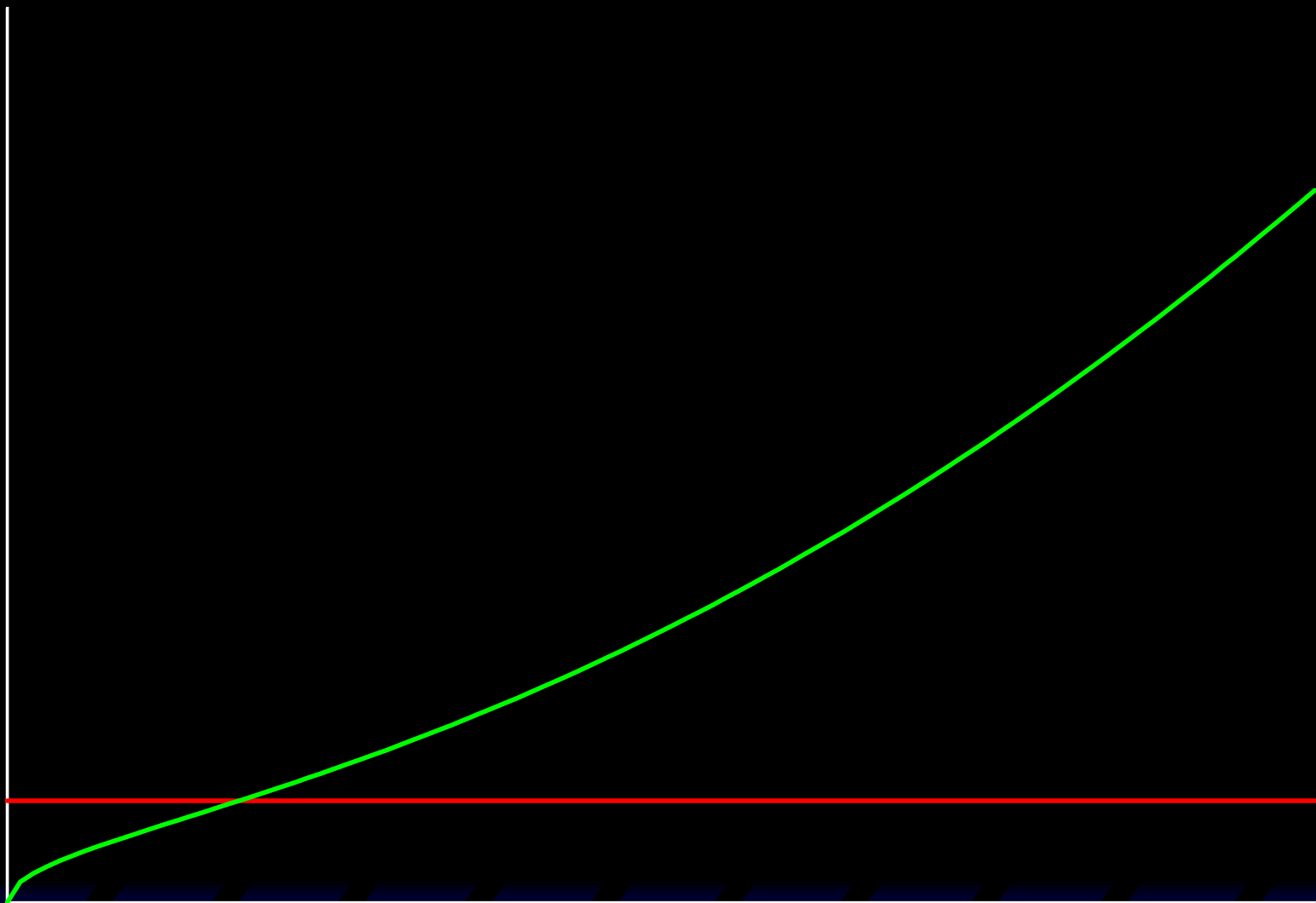


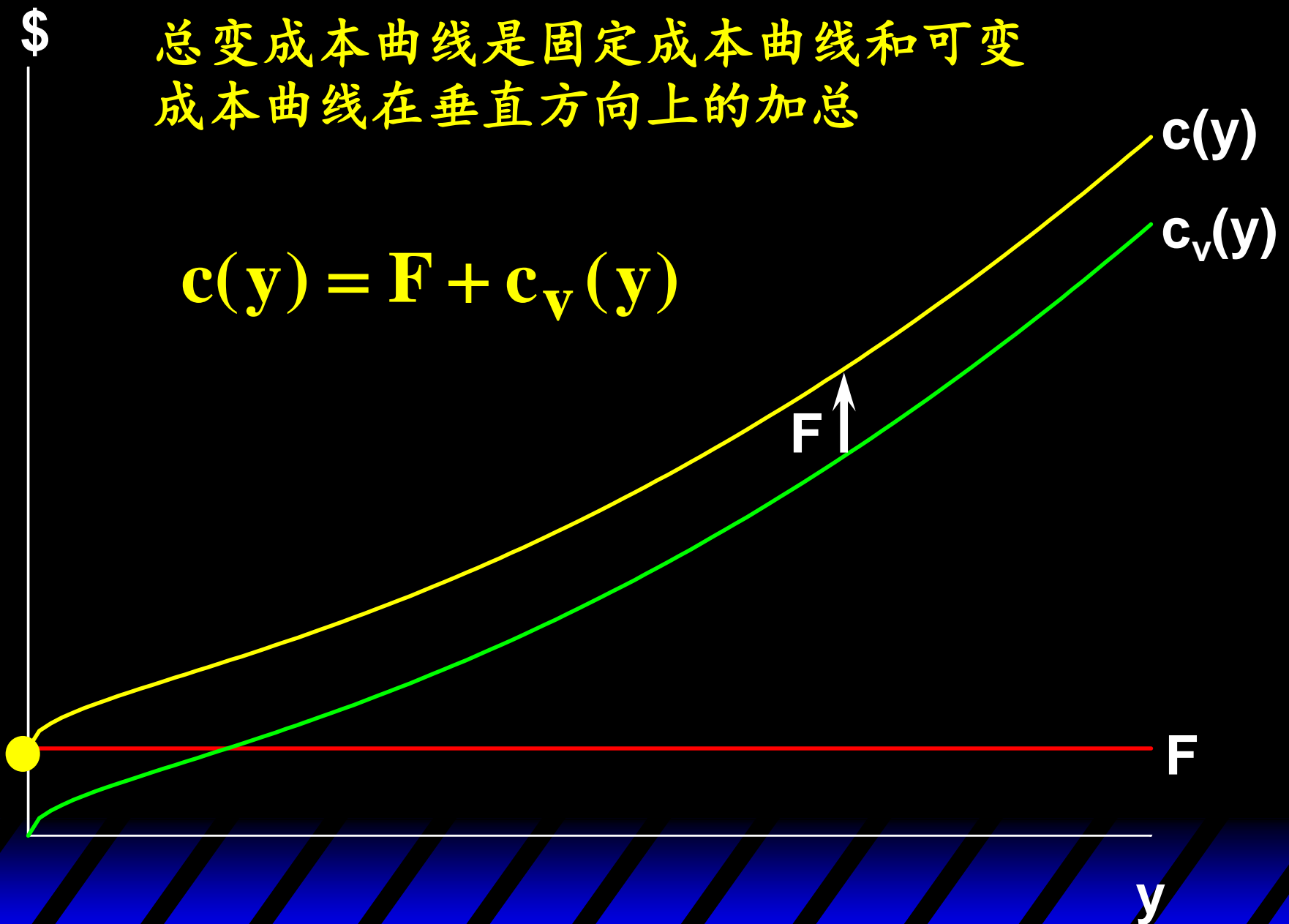
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$c_v(y)$

$F$

$y$





# Av. Fixed, Av. Variable & Av. Total Cost Curves

The firm's total cost function is

$$c(y) = F + c_v(y).$$

For  $y > 0$ , the firm's average total cost function is

$$\begin{aligned} AC(y) &= \frac{F}{y} + \frac{c_v(y)}{y} \\ &= AFC(y) + AVC(y). \end{aligned}$$

# Av. Fixed, Av. Variable & Av. Total Cost Curves

What does an average fixed cost curve look like?

$$AFC(y) = \frac{F}{y}$$

AFC(y) is a rectangular hyperbola (双曲线) so its graph looks like ...



**\$/output unit**

平均固定成本随着产量上升而下降，最终趋近为0.

$$\text{AFC}(y) \rightarrow \infty \text{ as } y \rightarrow 0$$

$$\text{AFC}(y) \rightarrow 0 \text{ as } y \rightarrow \infty$$

**AFC(y)**

**0**

**y**

# Av. Fixed, Av. Variable & Av. Total Cost Curves

In a short-run with a fixed amount of at least one input, the Law of Diminishing (Marginal) Returns must apply, causing the firm's **average variable cost** of production to increase **eventually**.

当某种要素的数量固定时，生产额外一单位产品所需的其它可变要素（最终）会越来越多，因此平均可变成本（最终）会随着 $y$ 上升而上升。

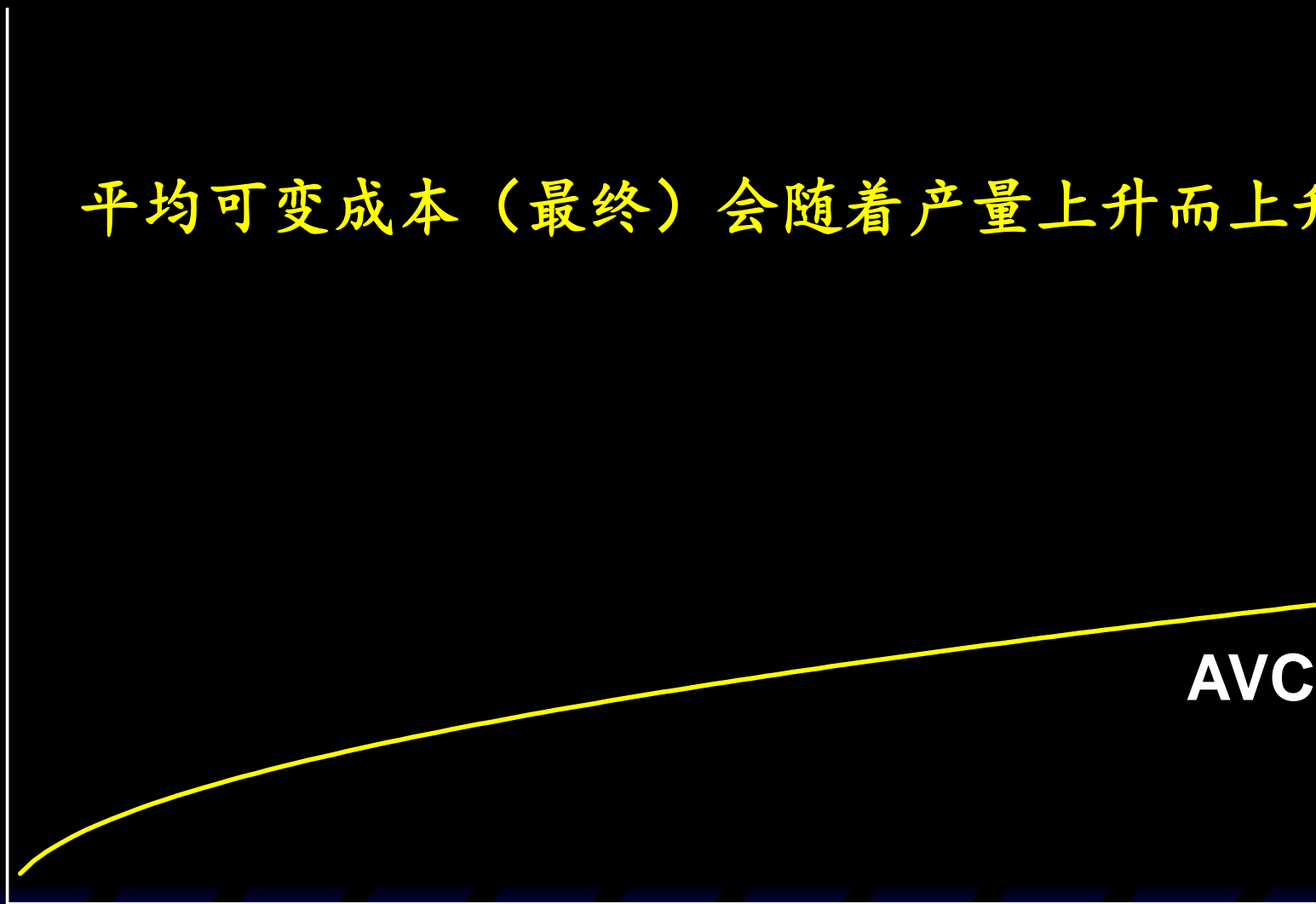
\$/output unit

平均可变成本（最终）会随着产量上升而上升。

$AVC(y)$

0

$y$



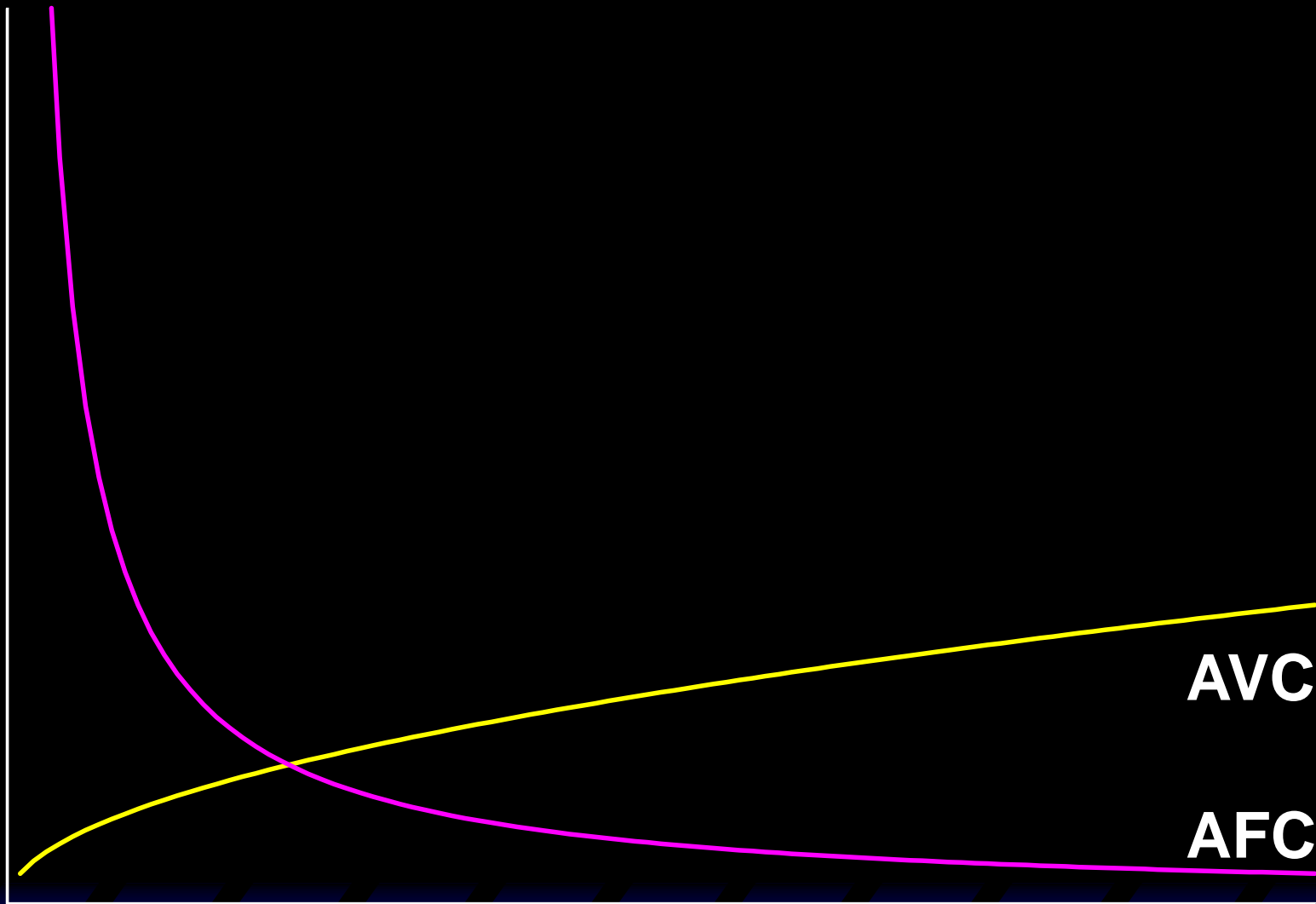
**\$/output unit**

**$AVC(y)$**

**$AFC(y)$**

**0**

**$y$**



# Av. Fixed, Av. Variable & Av. Total Cost Curves

$$TC(y) = F + C_v(y)$$

$$\frac{TC(y)}{y} = \frac{F}{y} + \frac{C_v(y)}{y}$$

$$ATC(y) = AFC(y) + AVC(y)$$


**\$/output unit**

$$\text{ATC}(y) = \text{AFC}(y) + \text{AVC}(y)$$

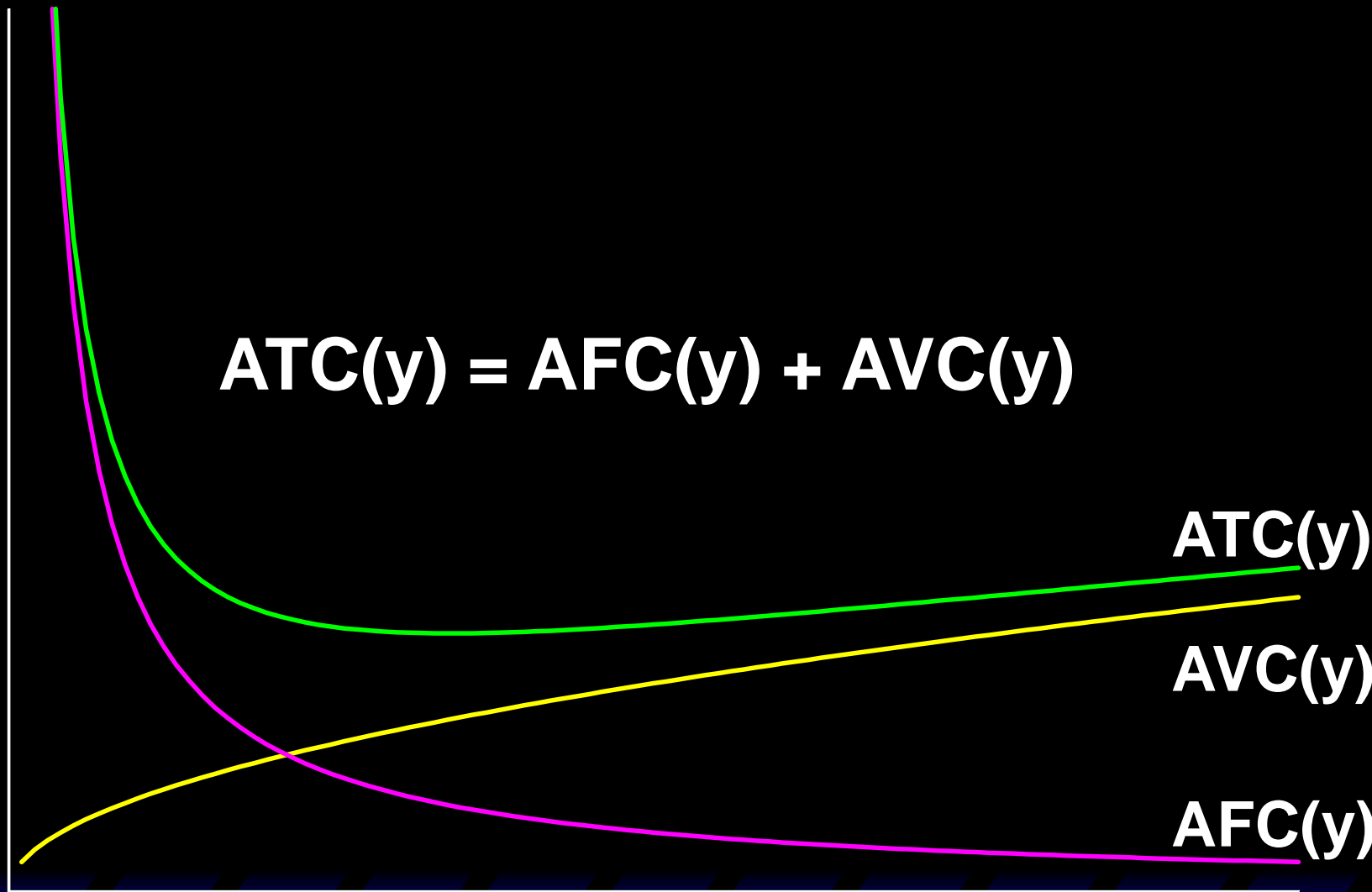
**ATC(y)**

**AVC(y)**

**AFC(y)**

**0**

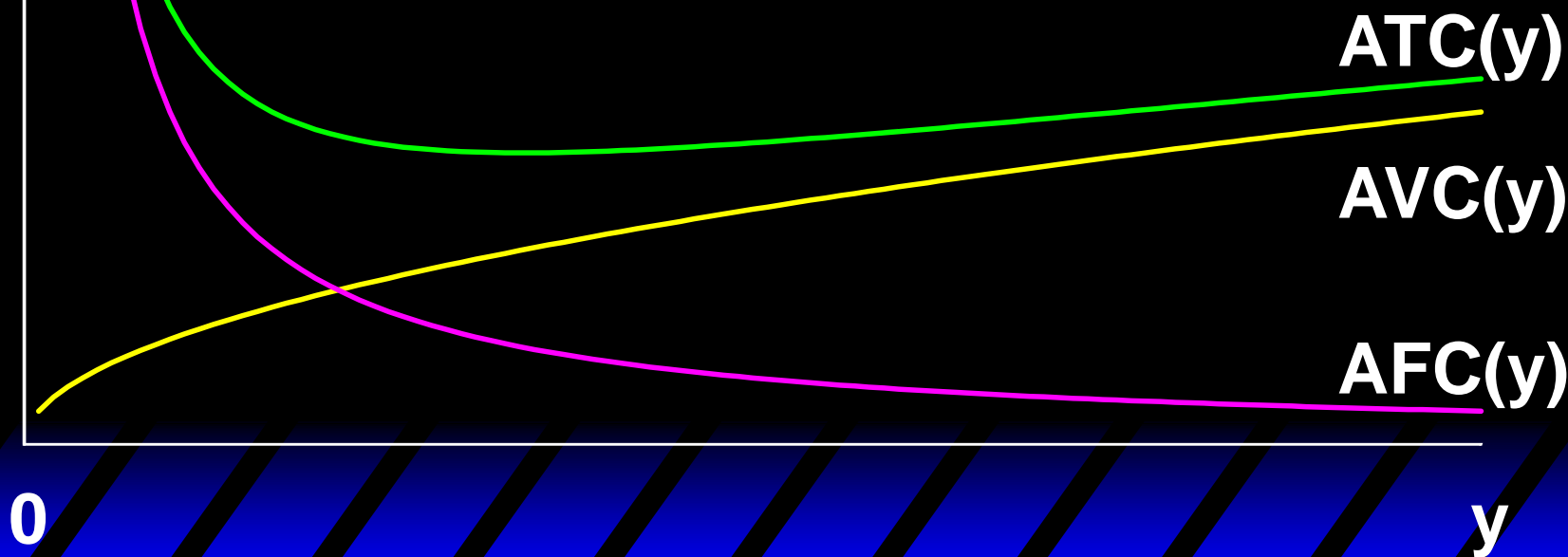
**y**



\$/output unit

Since  $AFC(y) \rightarrow 0$  as  $y \rightarrow \infty$ ,  
 **$ATC(y) \rightarrow AVC(y)$  as  $y \rightarrow \infty$ .**

ATC是AFC和AVC的垂直加总；  
y较小时AFC下降较快，因此ATC呈下降；  
y趋于无穷时，AFC趋于0，ATC趋于AVC，  
呈现上升。



# Marginal Cost Function

The firm's total cost function is

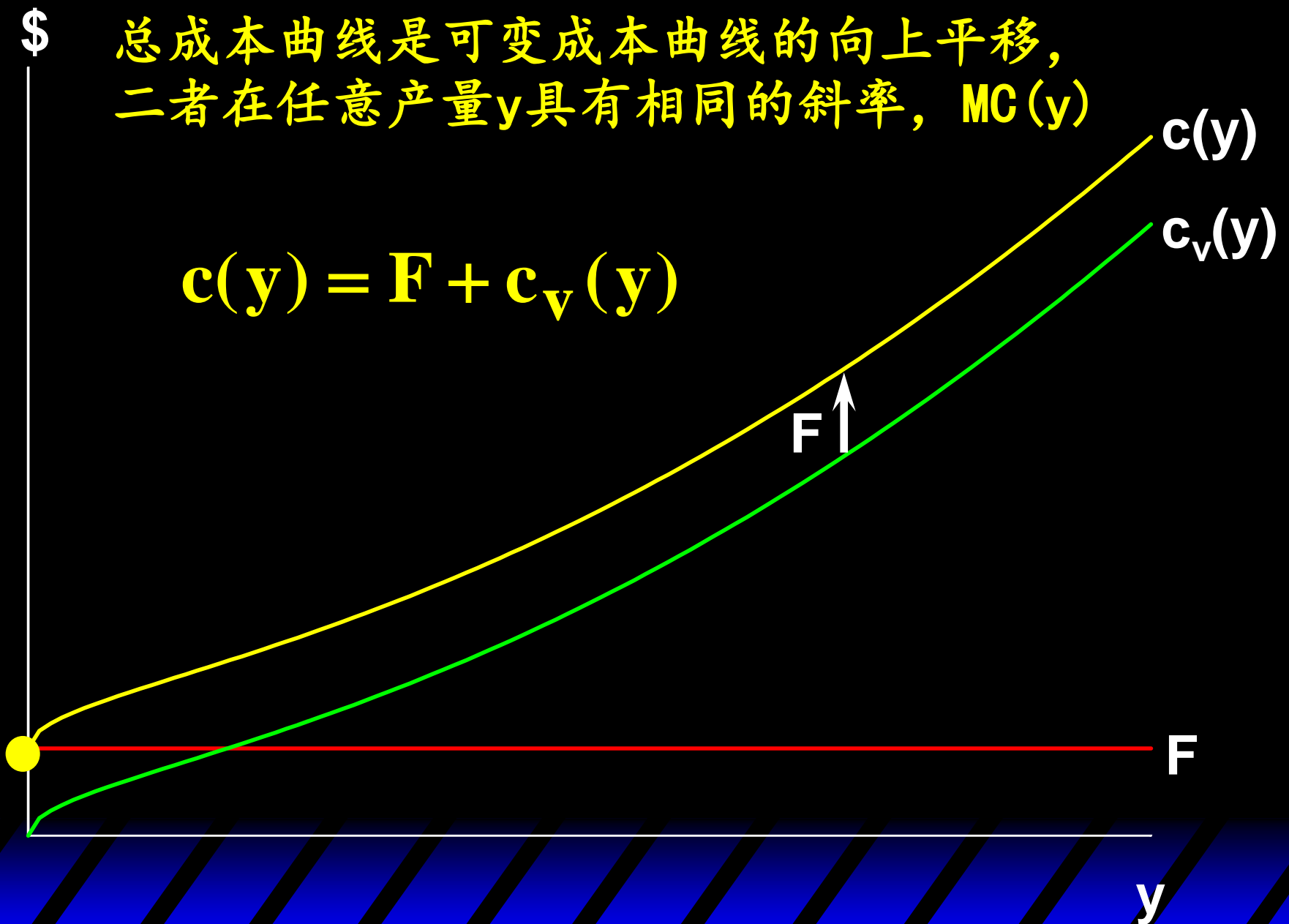
$$c(y) = F + c_v(y)$$

and the fixed cost  $F$  does not change with the output level  $y$ , so

$$MC(y) = \frac{\partial c_v(y)}{\partial y} = \frac{\partial c(y)}{\partial y}.$$

**MC** is the slope of both the variable cost and the total cost functions.





# Marginal and Variable Cost Functions

Since  $MC(y)$  is the derivative of  $c_v(y)$ ,  
 $c_v(y)$  must be the integral of  $MC(y)$ .

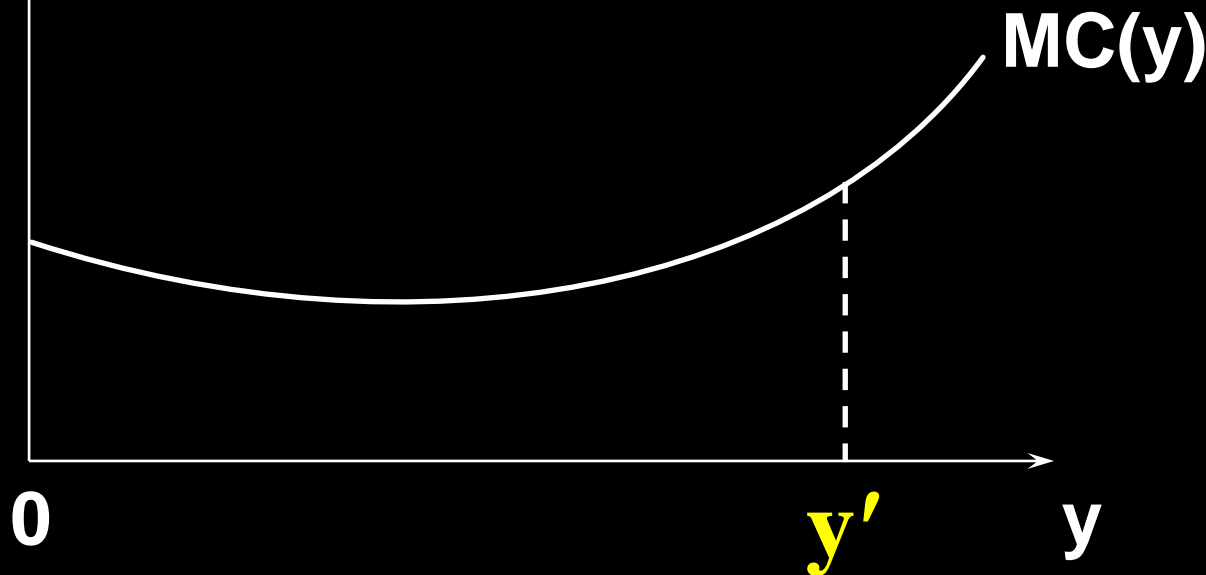
That is,

$$MC(y) = \frac{\partial c_v(y)}{\partial y}$$
$$\Rightarrow c_v(y) = \int_0^y MC(z) dz.$$

# Marginal and Variable Cost Functions

\$/output unit

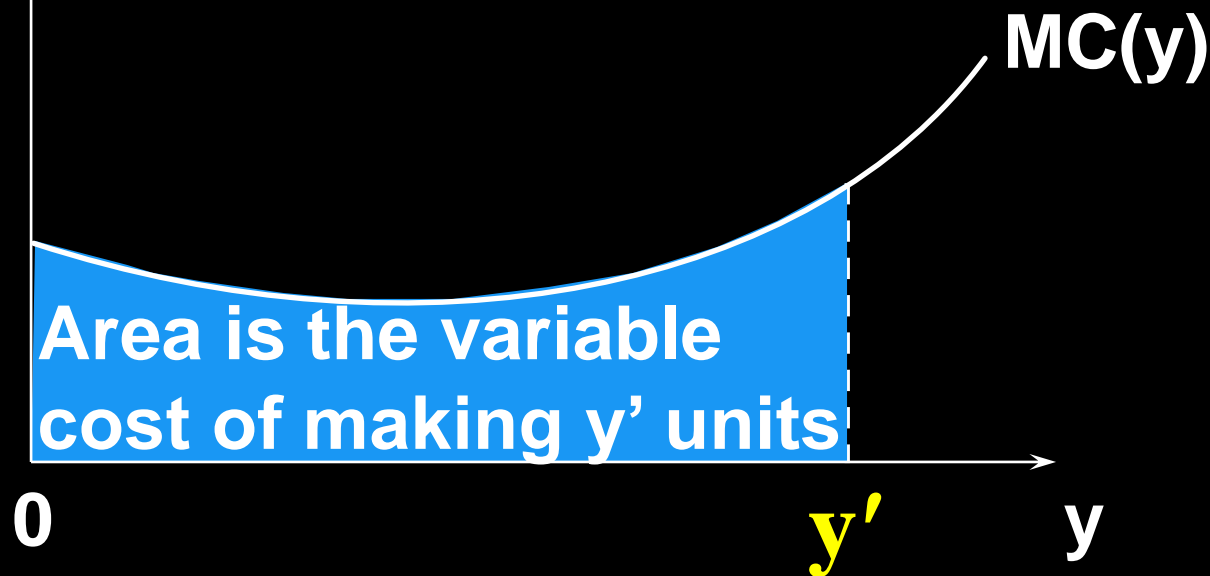
$$c_v(y') = \int_0^{y'} MC(z) dz$$



# Marginal and Variable Cost Functions

\$/output unit

$$c_v(y') = \int_0^{y'} MC(z) dz$$



# Marginal & Average Cost Functions

**How is marginal cost related to average variable cost?**

# Marginal & Average Cost Functions

Since  $AVC(y) = \frac{c_v(y)}{y},$

$$\frac{\partial AVC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c_v(y)}{y^2}.$$

# Marginal & Average Cost Functions

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$$\frac{\partial AVC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c_v(y)}{y^2}.$$

Therefore,

$$\frac{\partial AVC(y)}{\partial y} \begin{matrix} > \\ = 0 \\ < \end{matrix} \quad \text{as} \quad y \times MC(y) \begin{matrix} > \\ = \\ < \end{matrix} c_v(y).$$

# Marginal & Average Cost Functions

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$$\frac{\partial AVC(y)}{\partial y} \begin{matrix} > \\ = 0 \\ < \end{matrix} \quad \text{as} \quad MC(y) \begin{matrix} > \\ = \\ < \end{matrix} \frac{c_v(y)}{y} = AVC(y).$$



# Marginal & Average Cost Functions

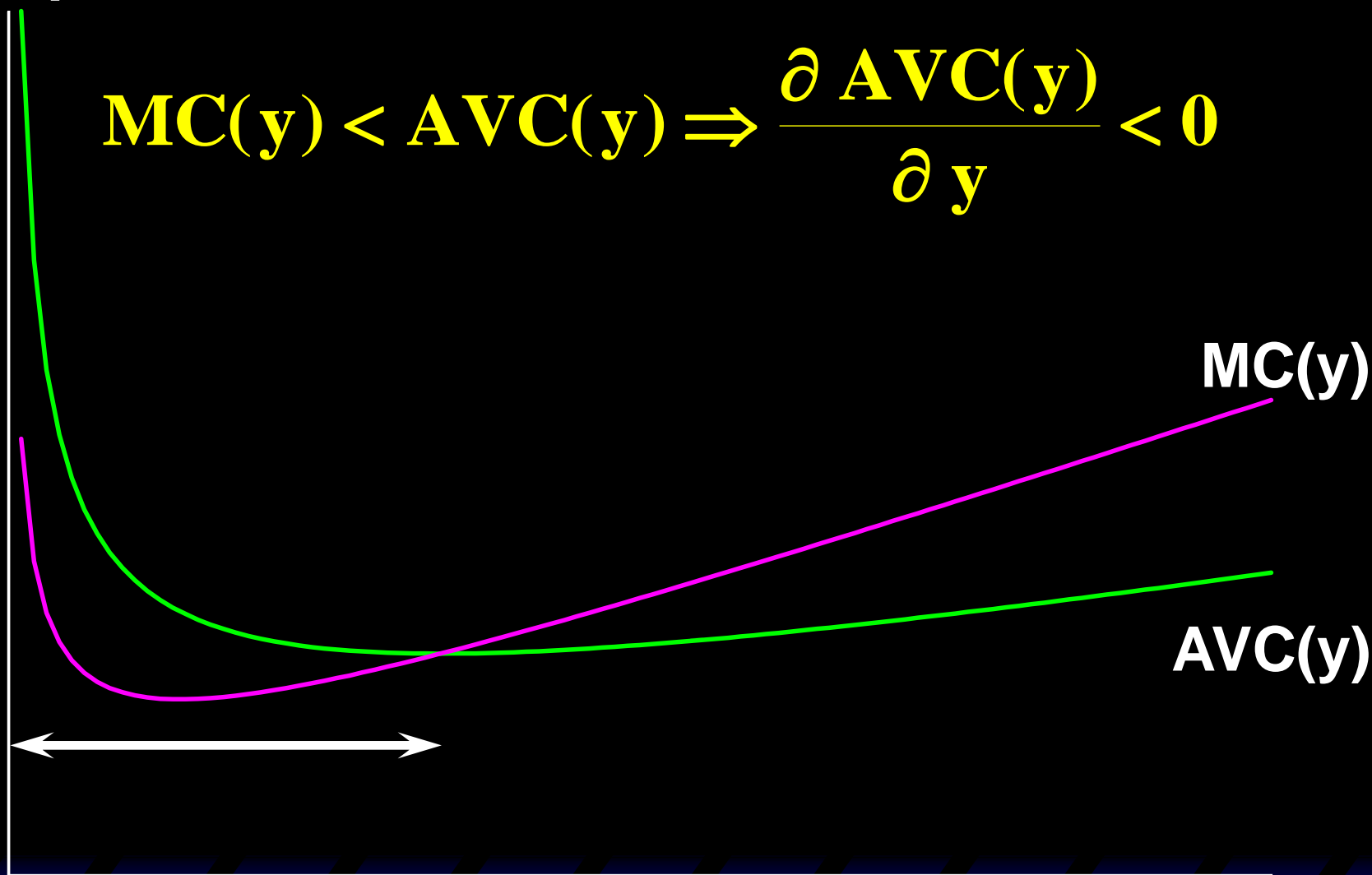
$$\frac{\partial \text{AVC}(y)}{\partial y} \begin{matrix} > \\ = 0 \\ < \end{matrix} \text{ as } \text{MC}(y) \begin{matrix} > \\ = \\ < \end{matrix} \text{AVC}(y).$$

当边际成本**高于**平均可变成本时，平均可变成本随产量上升而上升；

当边际成本**低于**平均可变成本时，平均可变成本随产量上升而下降

\$/output unit

$$MC(y) < AVC(y) \Rightarrow \frac{\partial AVC(y)}{\partial y} < 0$$

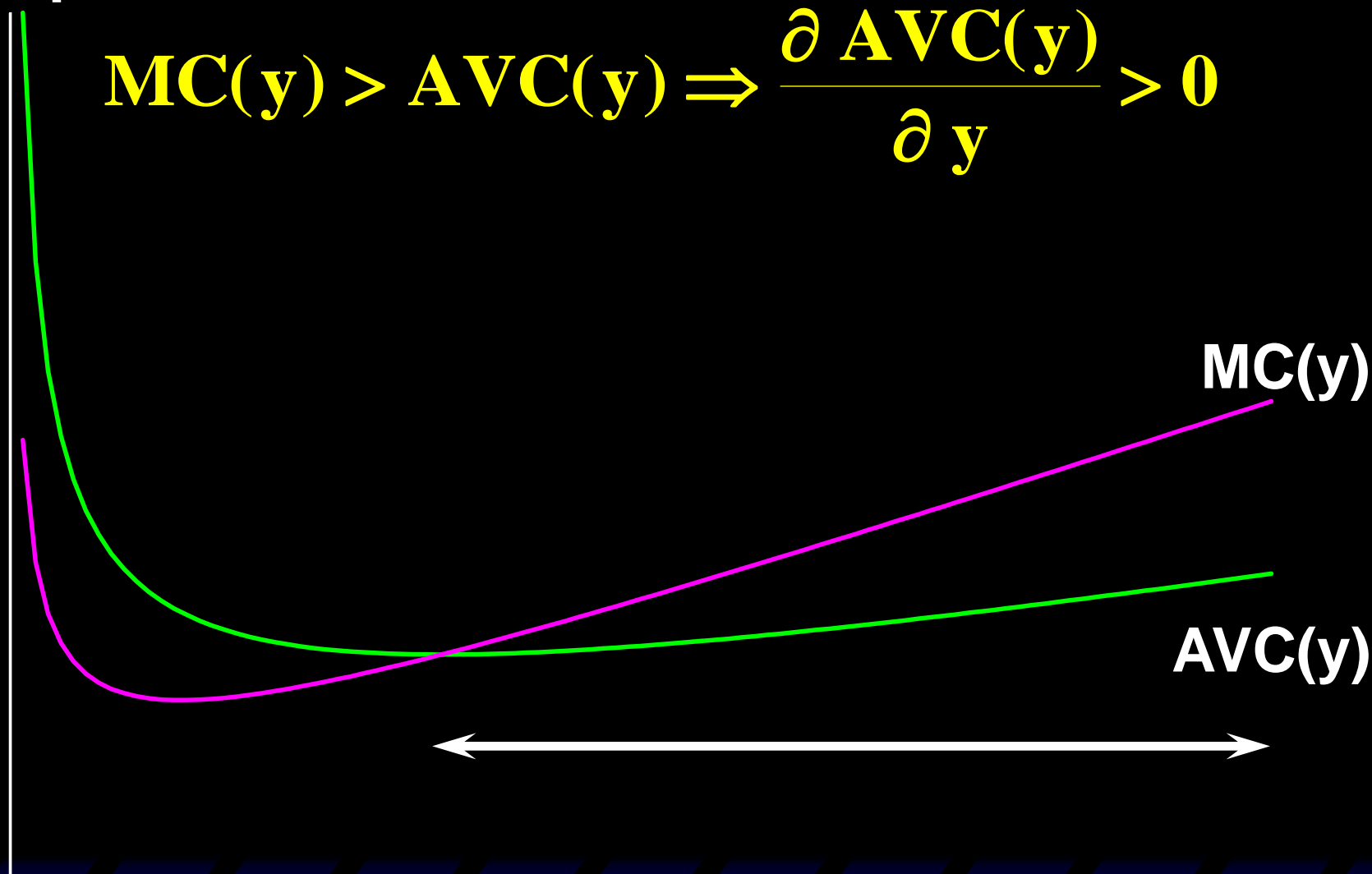


边际成本低于平均可变成本，平均可变成本下降

y

\$/output unit

$$MC(y) > AVC(y) \Rightarrow \frac{\partial AVC(y)}{\partial y} > 0$$

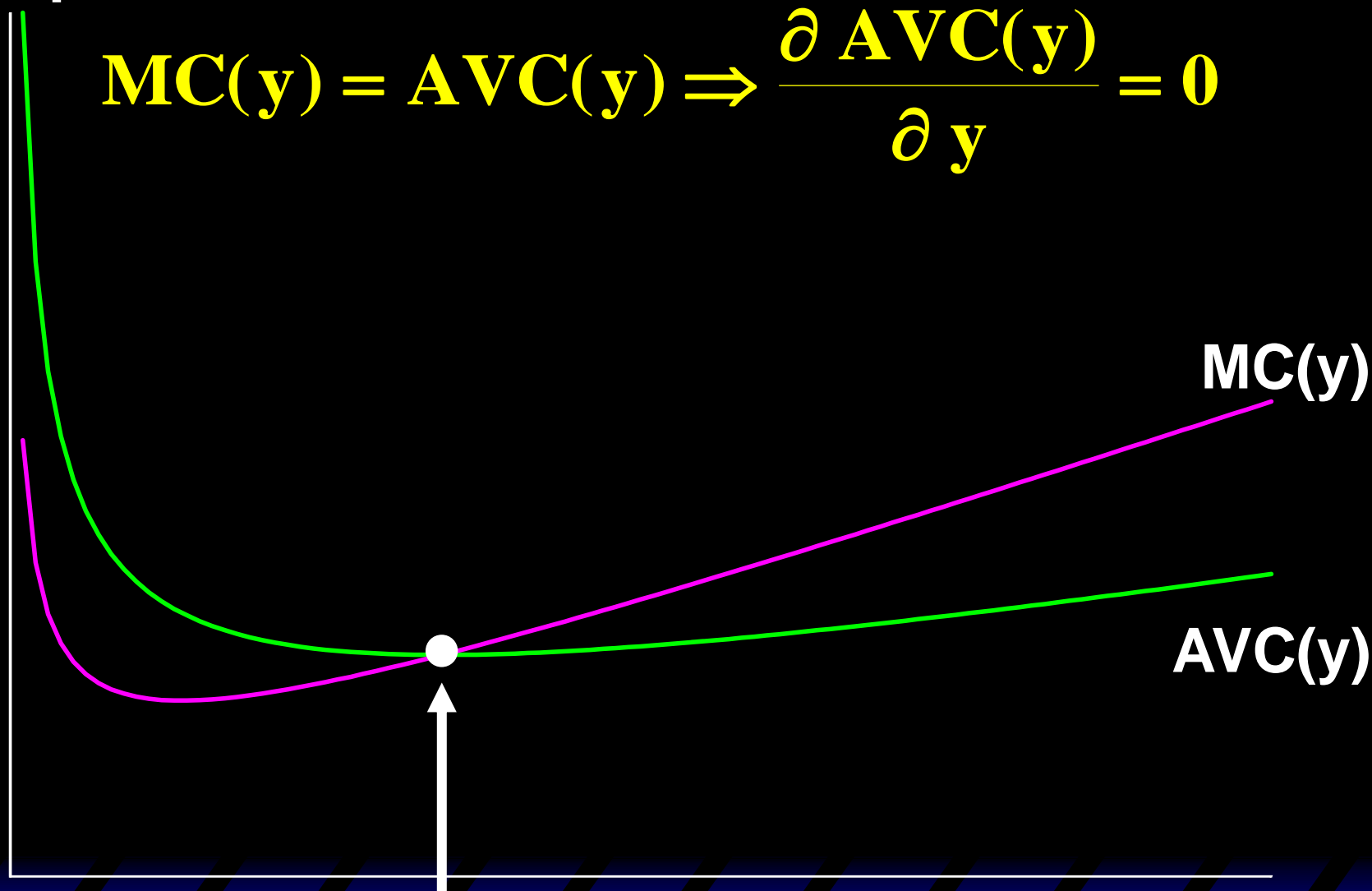


边际成本高于平均可变成本，平均可变成本上升

y

\$/output unit

$$MC(y) = AVC(y) \Rightarrow \frac{\partial AVC(y)}{\partial y} = 0$$



边际成本等于平均可变成本时，  
平均可变成本在最低点

y

\$/output unit

$$MC(y) = AVC(y) \Rightarrow \frac{\partial AVC(y)}{\partial y} = 0$$

The short-run MC curve intersects the short-run AVC curve **from below** at the AVC curve's minimum.

MC(y)  
AVC(y)

边际成本曲线从下往上、与平均可变成本曲线相交于其最低点

y

# Marginal & Average Cost Functions

Similarly, since  $ATC(y) = \frac{c(y)}{y}$ ,

$$\frac{\partial ATC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c(y)}{y^2}.$$

# Marginal & Average Cost Functions

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$$\frac{\partial ATC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c(y)}{y^2}.$$

Therefore,

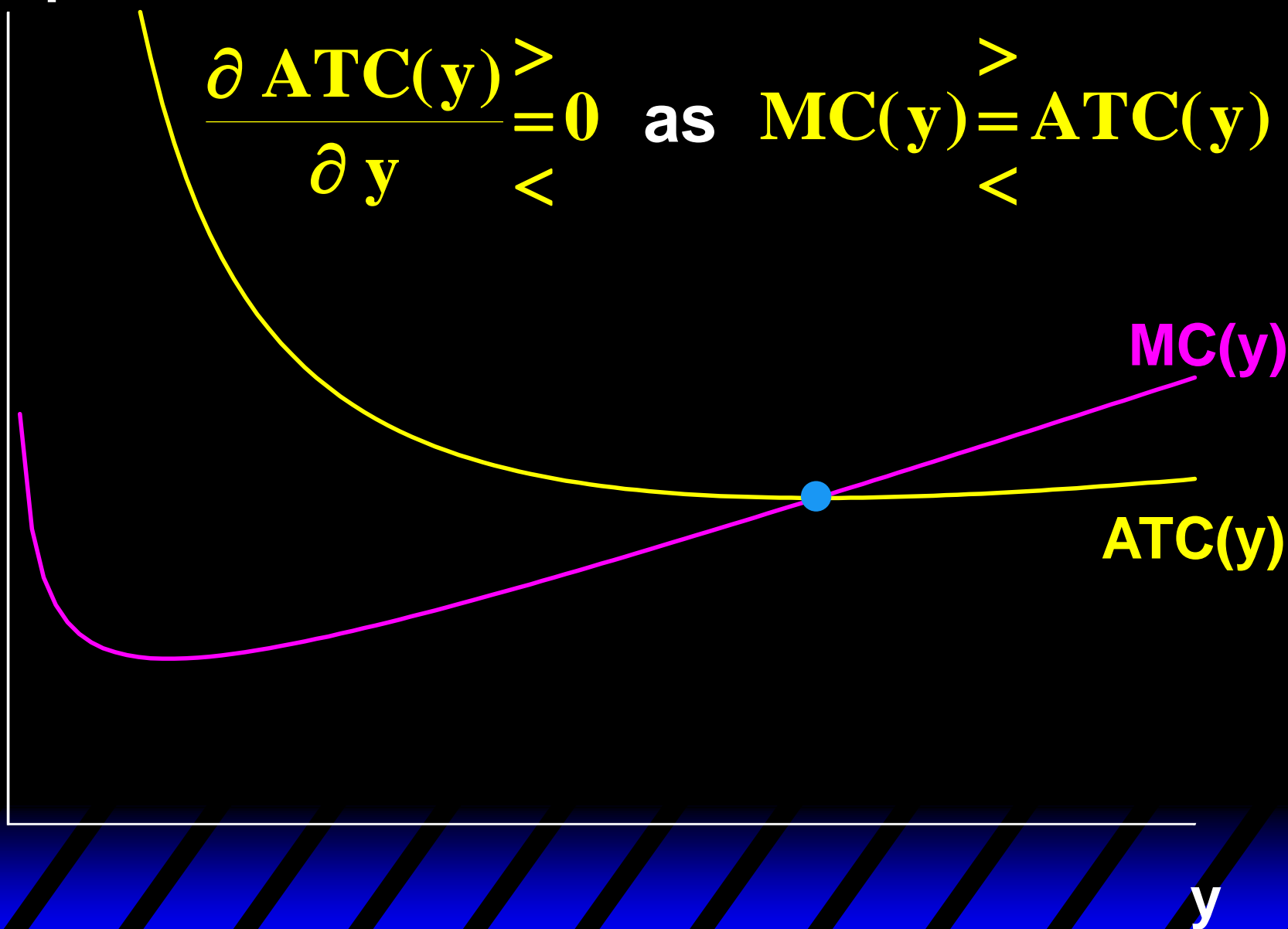
$$\frac{\partial ATC(y)}{\partial y} \begin{matrix} > \\ = 0 \\ < \end{matrix} \quad \text{as} \quad y \times MC(y) \begin{matrix} > \\ = \\ < \end{matrix} c(y).$$

$$\frac{\partial ATC(y)}{\partial y} \begin{matrix} > \\ = 0 \\ < \end{matrix} \quad \text{as} \quad MC(y) \begin{matrix} > \\ = \\ < \end{matrix} \frac{c(y)}{y} = ATC(y).$$



\$/output unit

$$\frac{\partial ATC(y)}{\partial y} \begin{matrix} > \\ = 0 \\ < \end{matrix} \text{ as } MC(y) \begin{matrix} > \\ = \\ < \end{matrix} ATC(y)$$



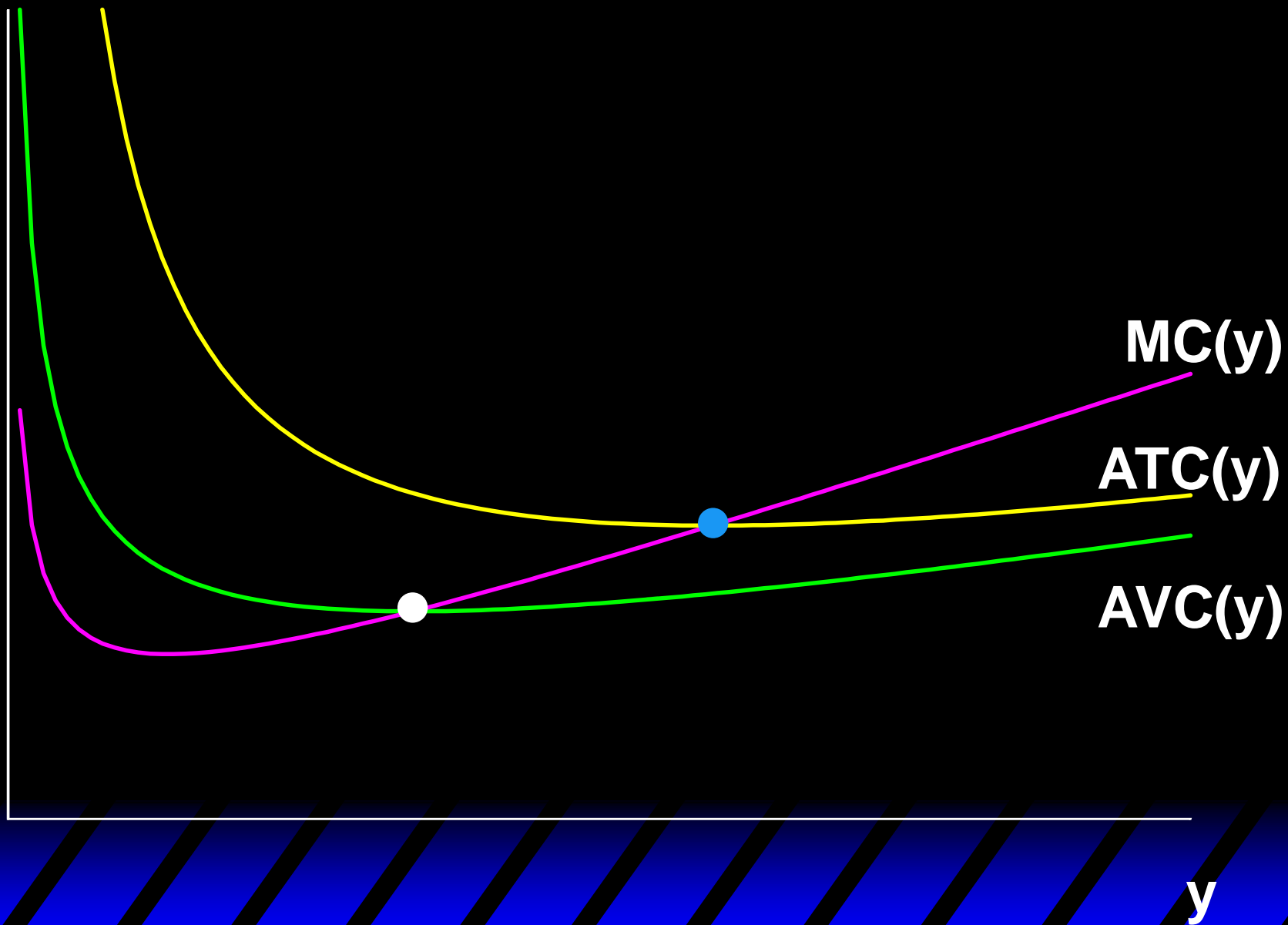
# Marginal & Average Cost Functions

The short-run MC curve intersects the short-run AVC curve from below at the AVC curve's minimum.

And, similarly, the short-run MC curve intersects the short-run ATC curve **from below** at the ATC curve's minimum.

边际成本曲线从下往上、与平均成本曲线相交于其最低点

**\$/output unit**



# Marginal & Average Cost Functions

**Cost function:**

$$C(y) = 1 + y^2$$

$$ATC(y) = \frac{1}{y} + y$$

$$\frac{\partial ATC(y)}{\partial y} = -\frac{1}{y^2} + 1$$

**When  $MC(y) = 2y < ATC(y) = \frac{1}{y} + y$ ,**

$$y < \frac{1}{y}, \text{ } y < 1 \Rightarrow \frac{\partial ATC(y)}{\partial y} = -\frac{1}{y^2} + 1 < 0$$

**$\Rightarrow$  ATC 递减**

# Marginal & Average Cost Functions

**Cost function:**

$$C(y) = 1 + y^2$$

$$ATC(y) = \frac{1}{y} + y$$

$$\frac{\partial ATC(y)}{\partial y} = -\frac{1}{y^2} + 1$$

**When  $MC(y) = 2y > ATC(y) = \frac{1}{y} + y$ ,**

$$y > \frac{1}{y}, \mathbf{y > 1} \Rightarrow \frac{\partial ATC(y)}{\partial y} = -\frac{1}{y^2} + 1 > \mathbf{0}$$

**$\Rightarrow$  ATC 递增**

# Marginal & Average Cost Functions

**Cost function:**

$$C(y) = 1 + y^2$$

$$ATC(y) = \frac{1}{y} + y$$

$$\frac{\partial ATC(y)}{\partial y} = -\frac{1}{y^2} + 1$$

$$\text{When } MC(y) = 2y = ATC(y) = \frac{1}{y} + y,$$

$$y = 1 \Rightarrow \frac{\partial ATC(y)}{\partial y} = -\frac{1}{y^2} + 1 = 0 \Rightarrow ATC$$

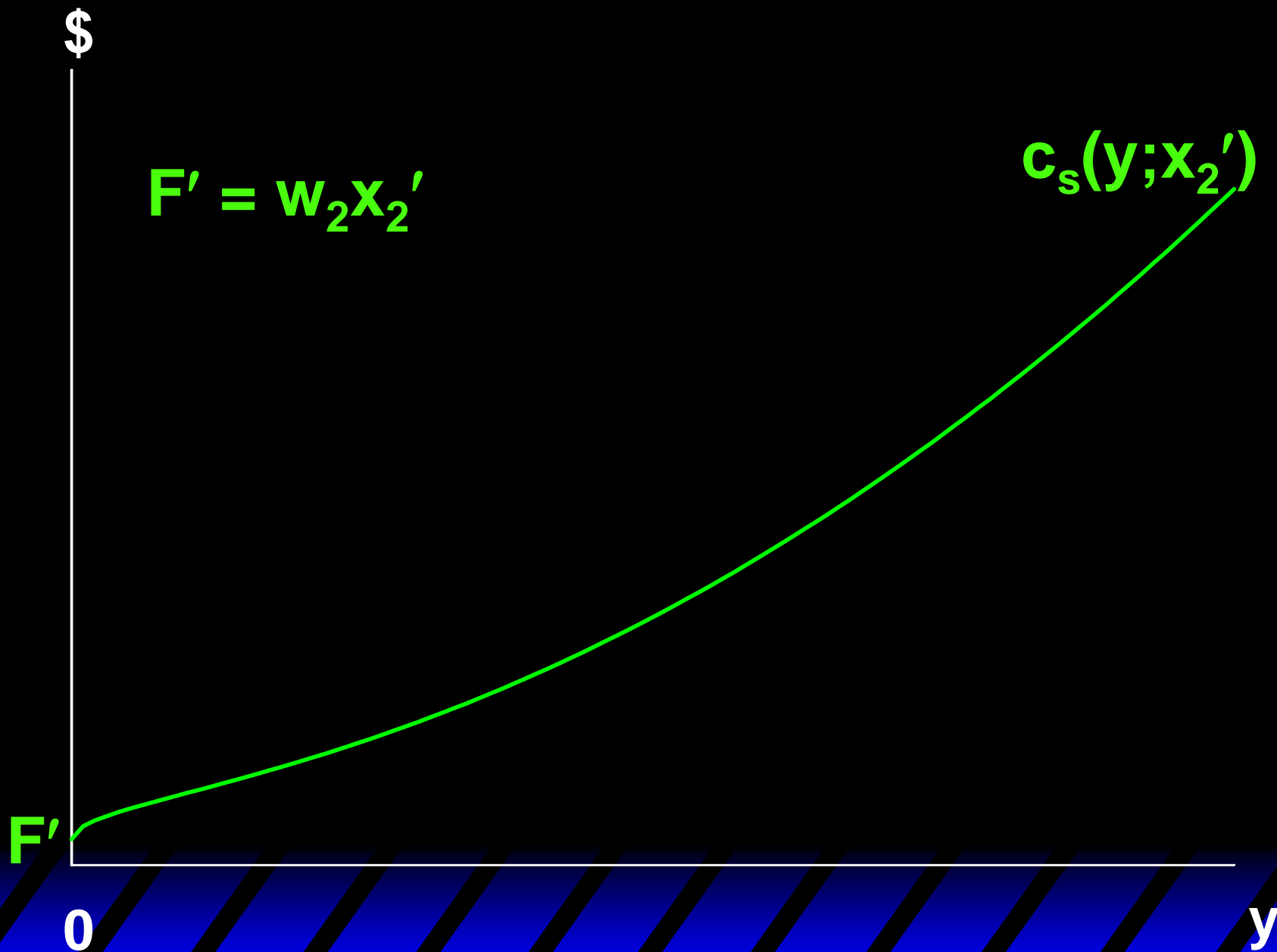
最小

# Short-Run & Long-Run Total Cost Curves

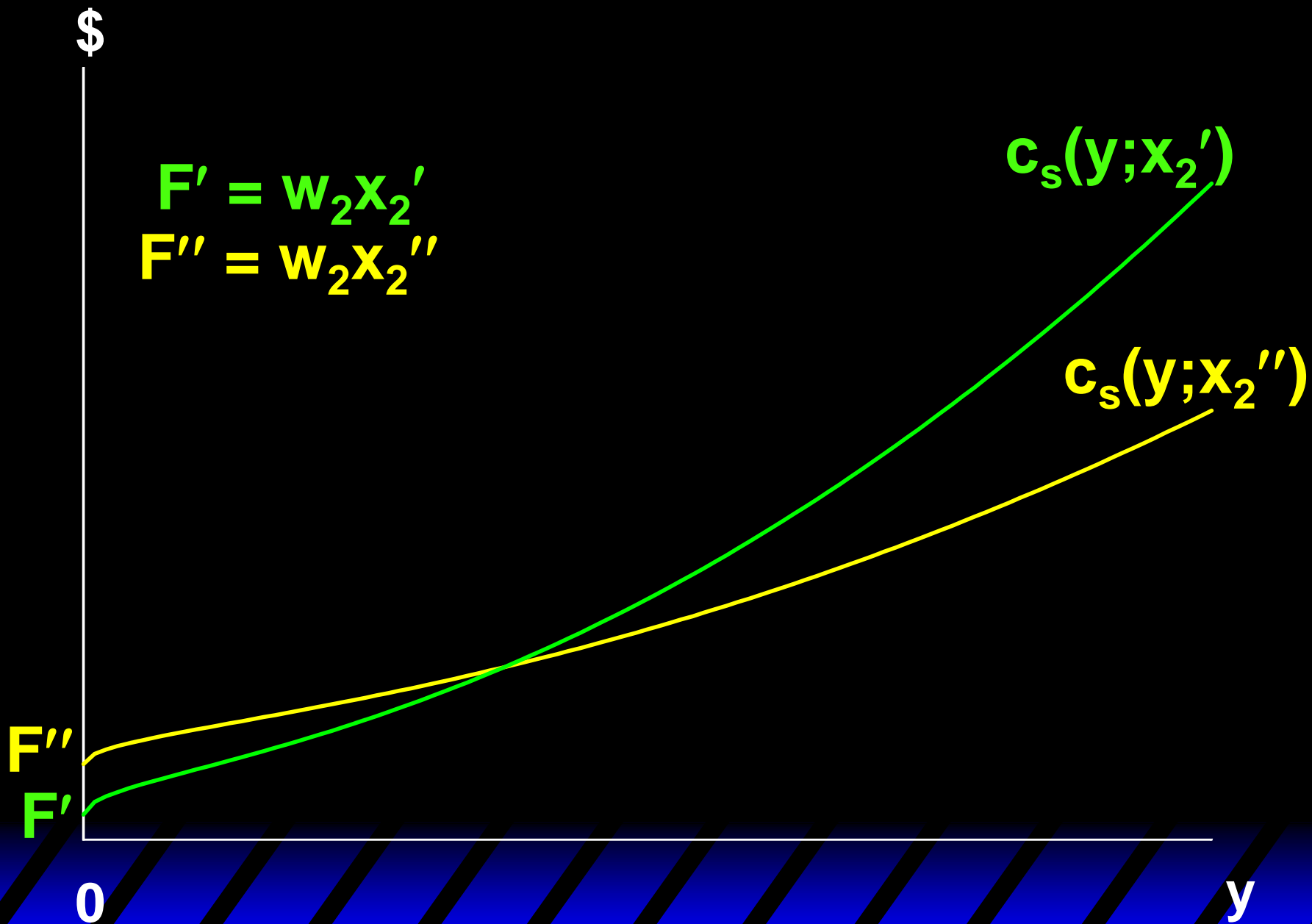
A firm has a different short-run total cost curve for each possible short-run circumstance.

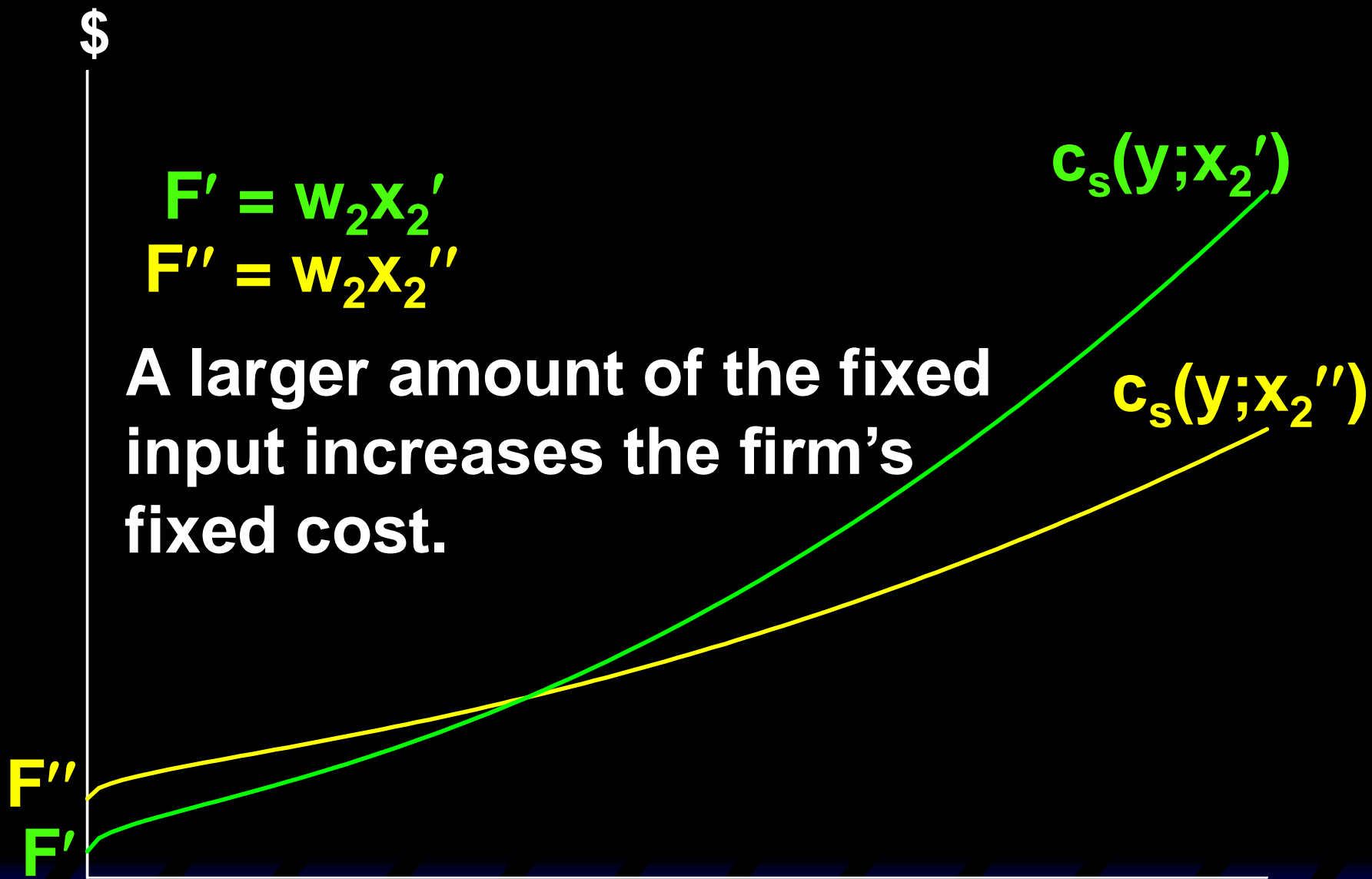
Suppose the firm can be in one of just three short-runs;

$$\begin{array}{lll} & x_2 = x_2' & \\ \text{or} & x_2 = x_2'' & x_2' < x_2'' < x_2'''. \\ \text{or} & x_2 = x_2''' & \end{array}$$



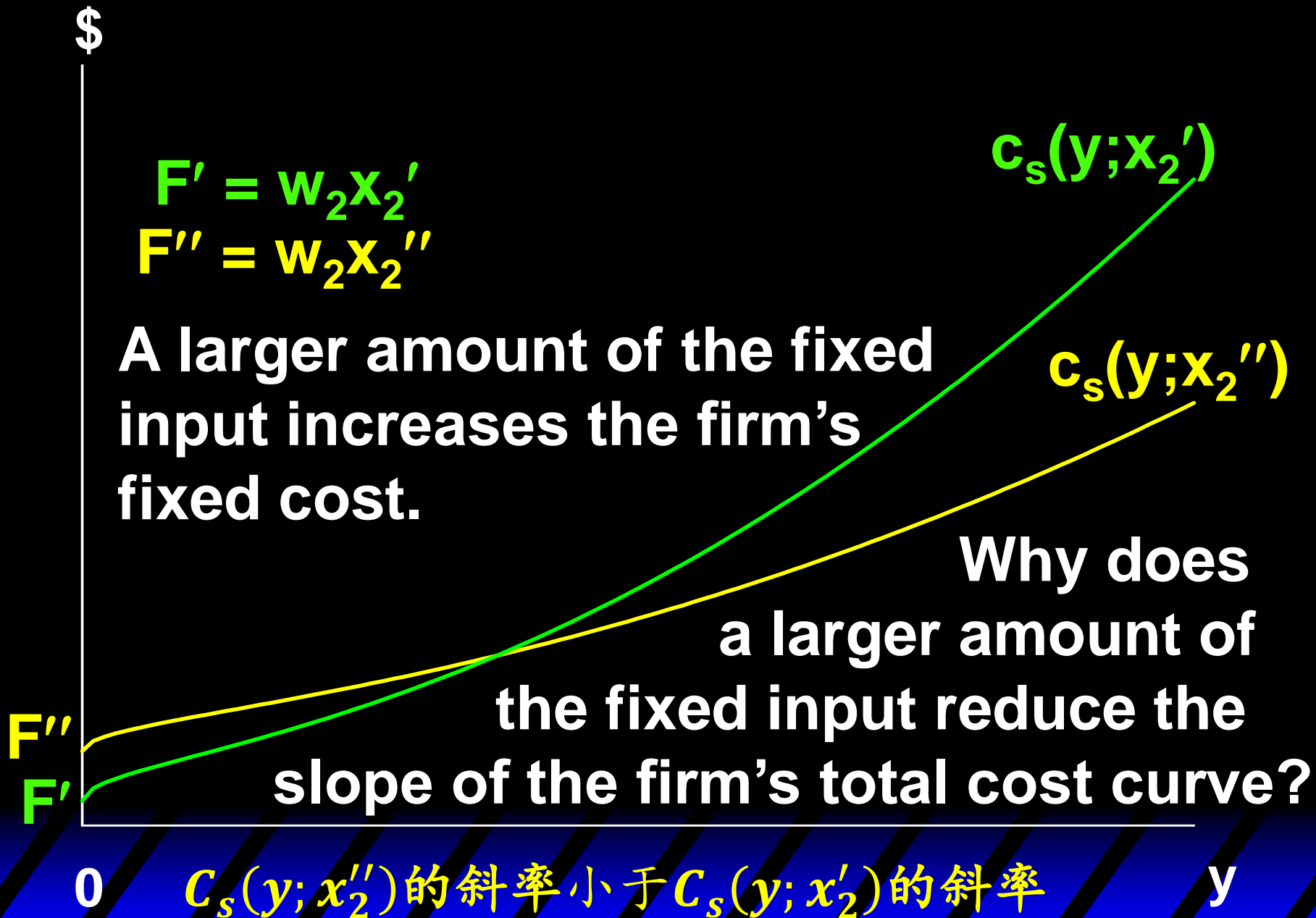






0  $c_s(y; x_2'')$ 的纵截距高于 $c_s(y; x_2')$ 的纵截距

$y$



# Short-Run & Long-Run Total Cost Curves

$MP_1$  is the marginal physical productivity of the variable input 1, so one extra unit of input 1 gives  $MP_1$  extra output units.

Therefore, the extra amount of input 1 needed for 1 extra output unit is

$1/MP_1$  units of input 1.

$C_s(y; x_2'')$ 的斜率是边际成本，即额外1单位产出的花费；

额外1单位要素1能生产 $MP_1$ ，即额外1单位产出需要 $1/MP_1$ 单位要素1，那么边际成本 =  $\omega_1/MP_1$

# Short-Run & Long-Run Total Cost Curves

$MP_1$  is the marginal physical productivity of the variable input 1, so one extra unit of input 1 gives  $MP_1$  extra output units.

Therefore, the extra amount of input 1 needed for 1 extra output unit is

$1/MP_1$  units of input 1.

Each unit of input 1 costs  $w_1$ , so the firm's extra cost from producing one extra unit of output is

$$MC = \frac{w_1}{MP_1}.$$

# Short-Run & Long-Run Total Cost Curves

$MC = \frac{w_1}{MP_1}$  is the slope of the firm's total cost curve.

$MP_1 = \frac{\partial f(x_1, x_2)}{\partial x_1}$  随  $x_2$  增加而增加

即:  $MP_1(x_1, x_2'') > MP_1(x_1, x_2')$

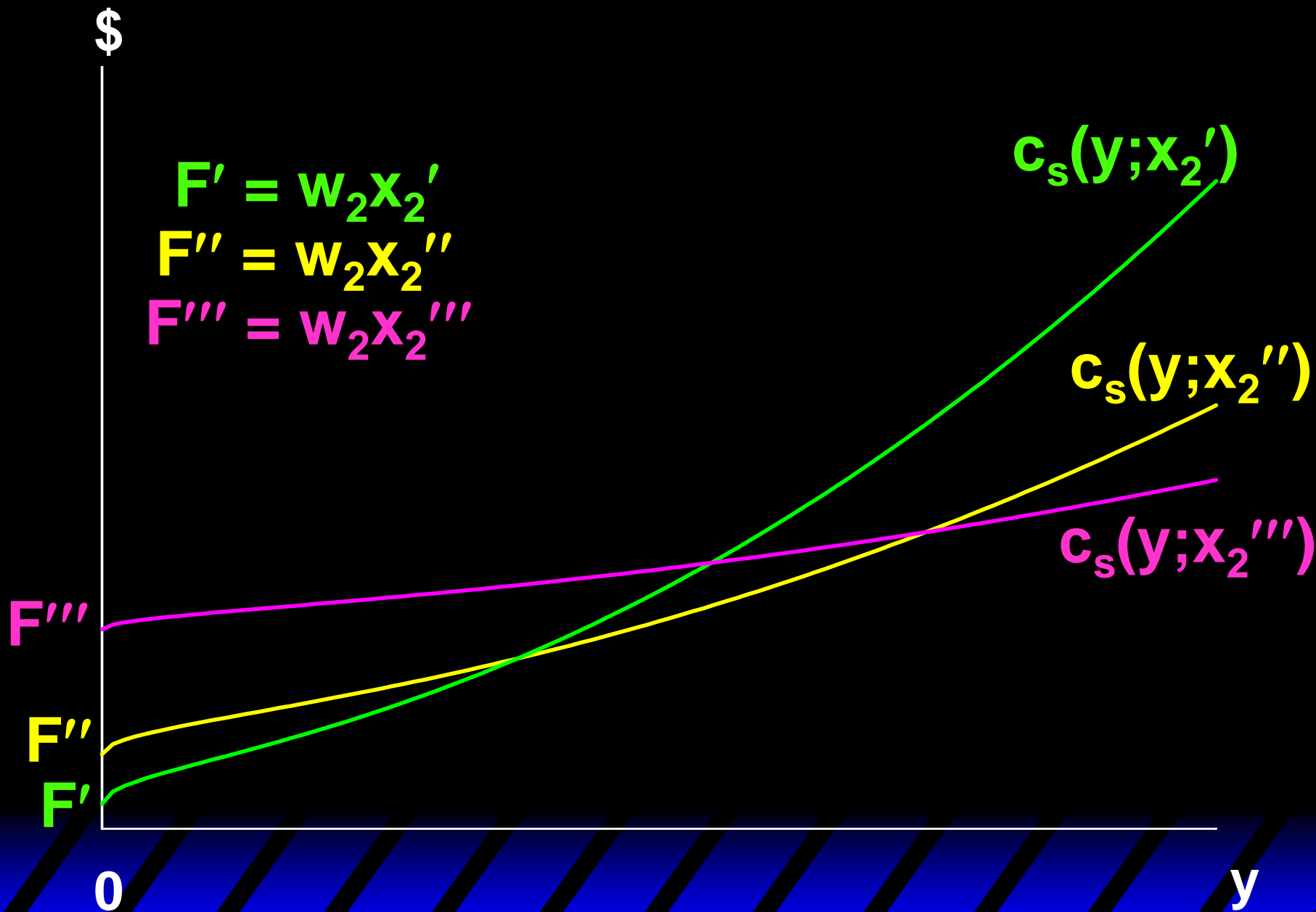
$MC(x_1, x_2'') < MC(x_1, x_2')$

# Short-Run & Long-Run Total Cost Curves

$MC = \frac{w_1}{MP_1}$  is the slope of the firm's total cost curve.

That is, a short-run total cost curve starts higher and has a lower slope if  $x_2$  is larger.

不变要素的固定数量越大，对应的短期成本曲线纵截距越高、斜率越小



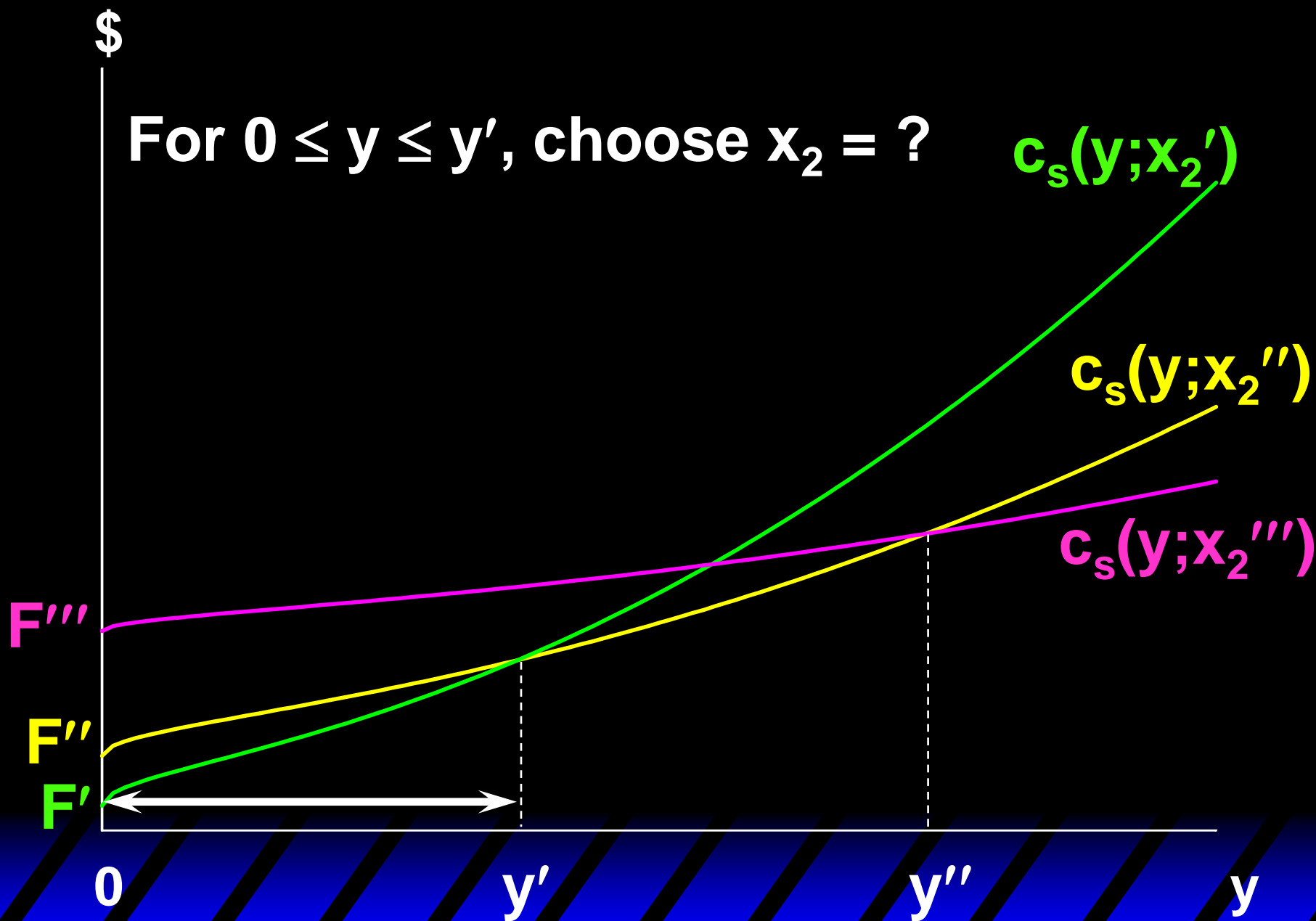


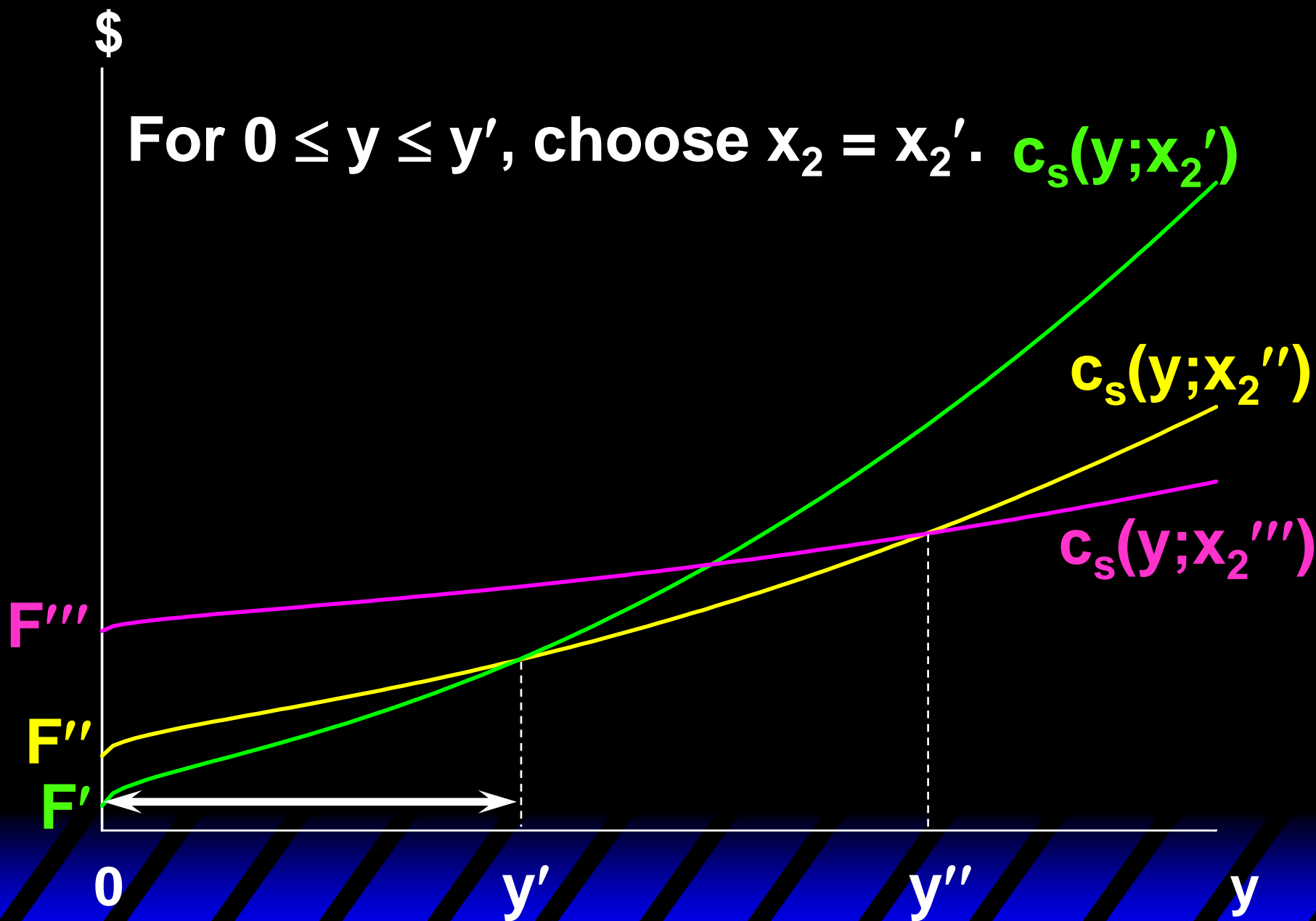
# Short-Run & Long-Run Total Cost Curves

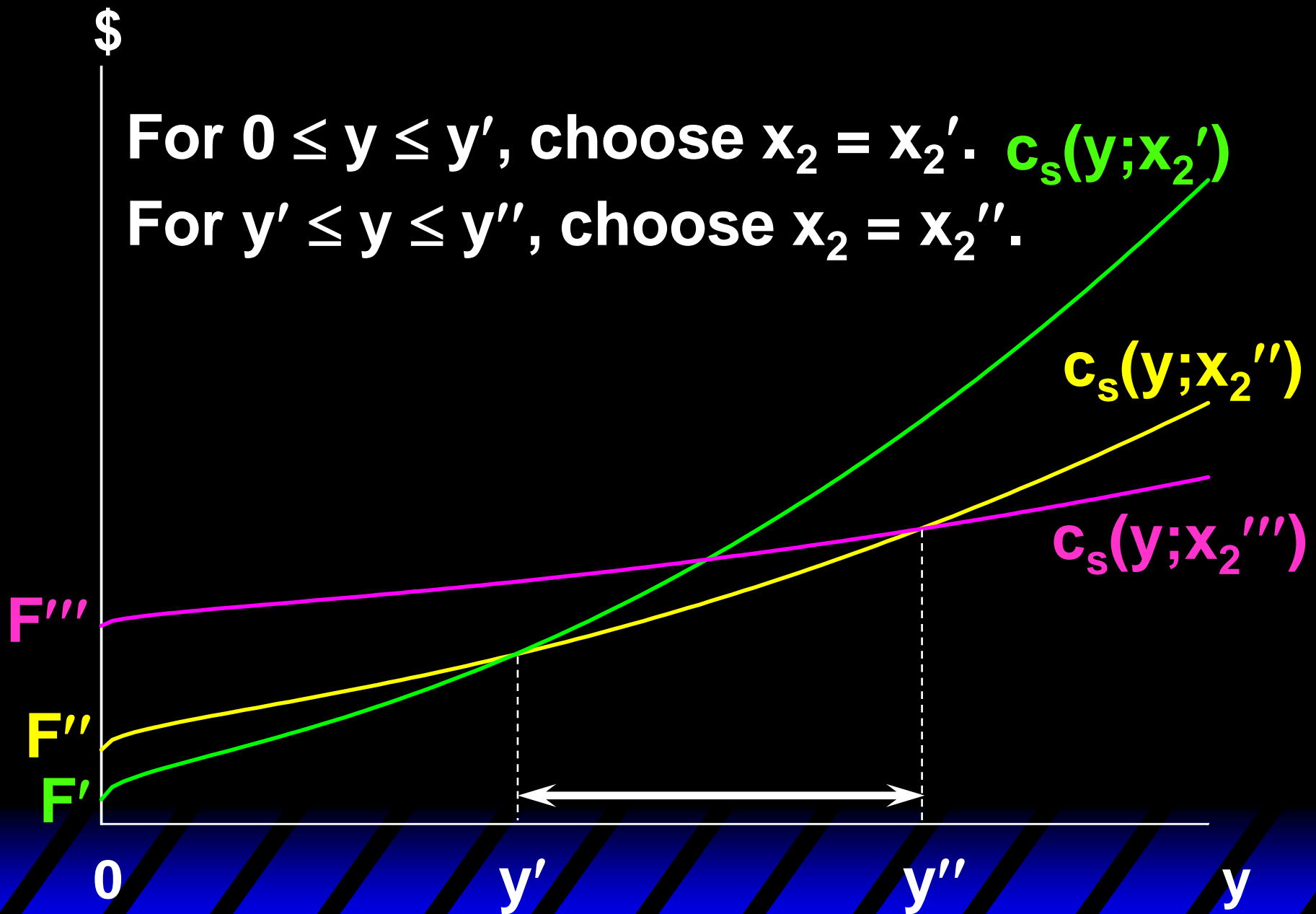
The firm has **three** short-run total cost curves.

In the long-run the firm is free to choose amongst these three since it is free to select  $x_2$  equal to any of  $x_2'$ ,  $x_2''$ , or  $x_2'''$ .

How does the firm make this choice?





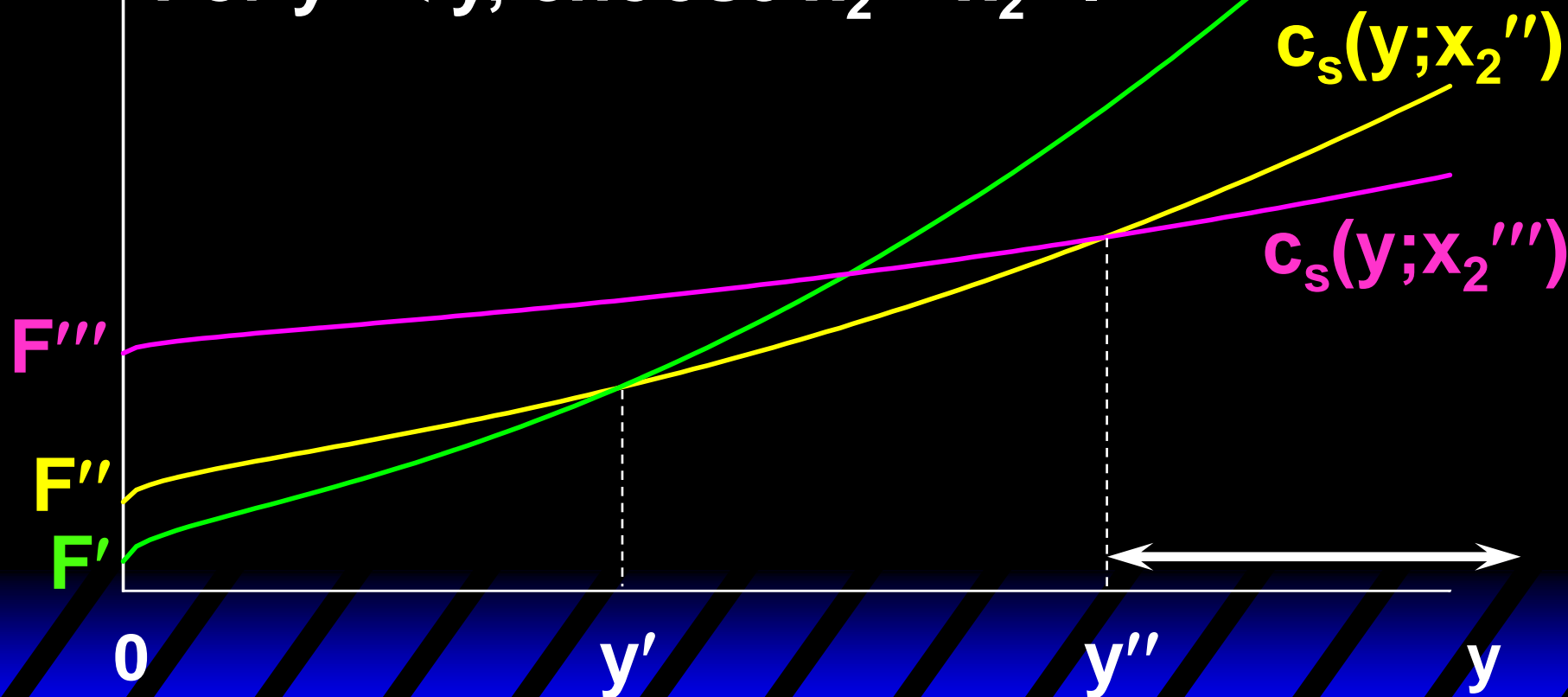


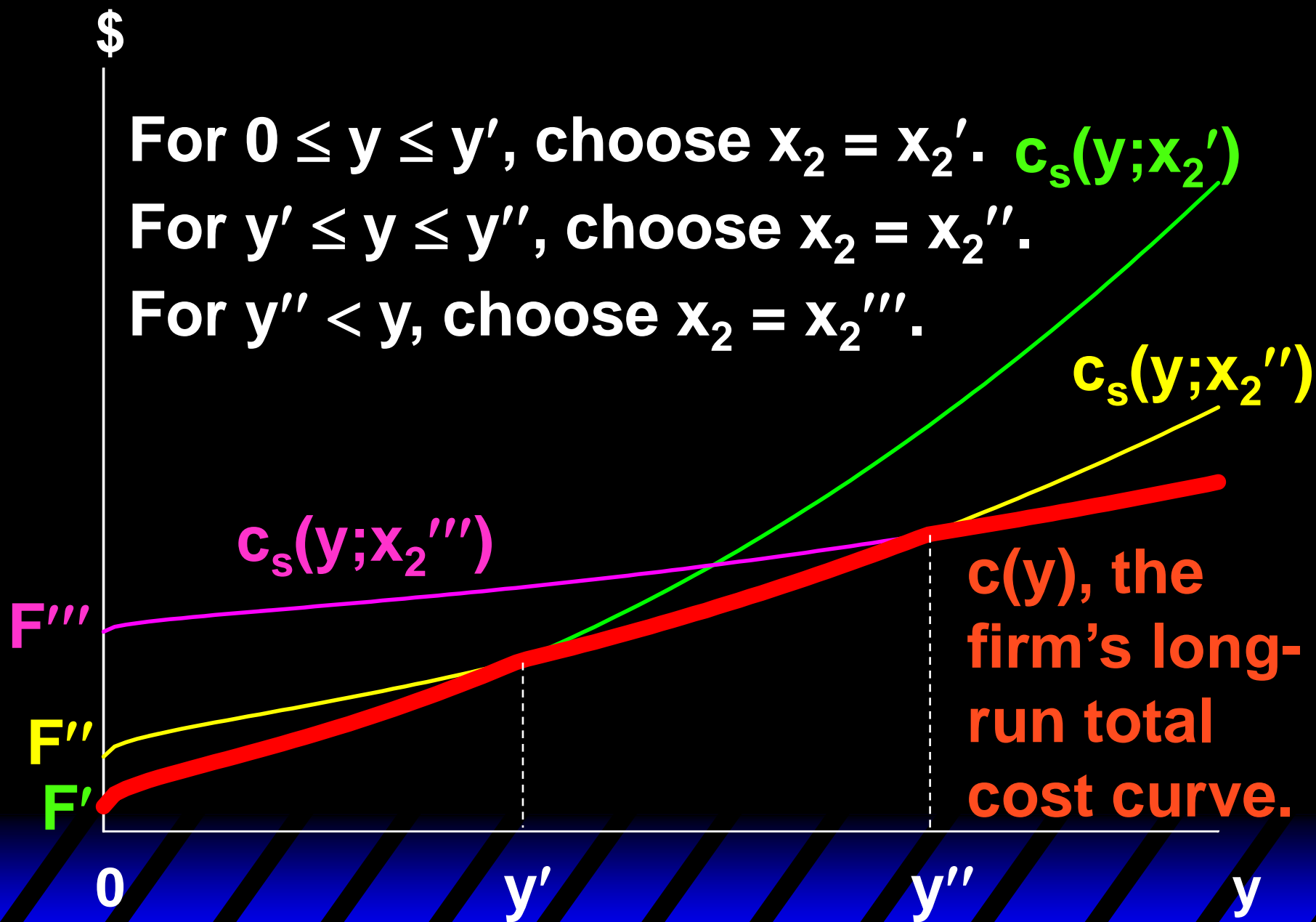
\$

For  $0 \leq y \leq y'$ , choose  $x_2 = x_2'$ .  $c_s(y; x_2')$

For  $y' \leq y \leq y''$ , choose  $x_2 = x_2''$ .

For  $y'' < y$ , choose  $x_2 = x_2'''$ .





# Short-Run & Long-Run Total Cost Curves

The firm's long-run total cost curve consists of the lowest parts of the short-run total cost curves. The long-run total cost curve is the **lower envelope** of the short-run total cost curves.

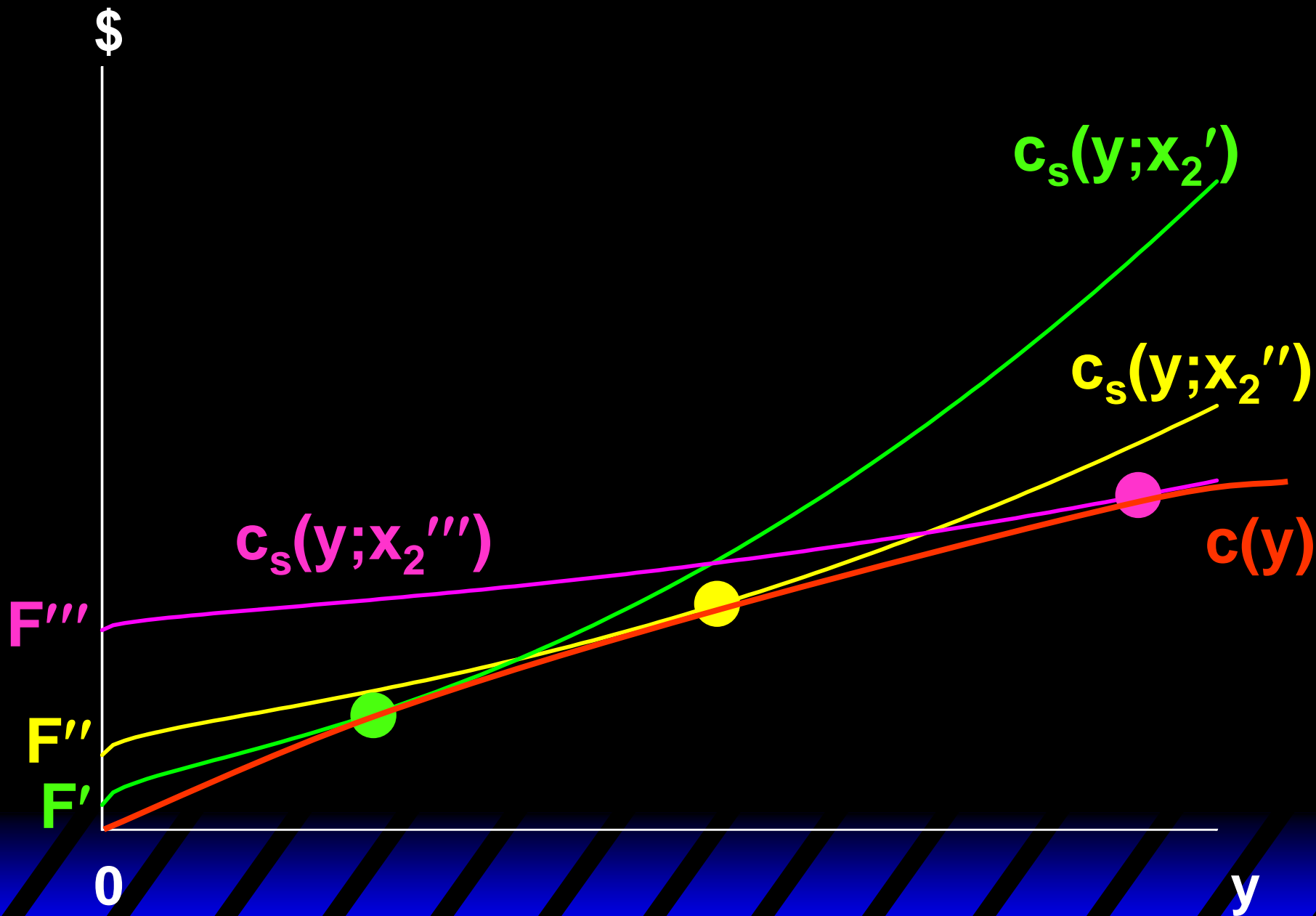
长期总成本曲线是短期总成本曲线的**下包络线**。

# Short-Run & Long-Run Total Cost Curves

If input 2 is available in **continuous** amounts then there is an infinity of short-run total cost curves but the long-run total cost curve is still the lower envelope of all of the short-run total cost curves.

不变要素的数量是连续的时候，有无数条短期总成本曲线。每一条都位于长期总成本曲线之上、并恰好相交于一点。(Lec 12)





# Short-Run & Long-Run Average Total Cost Curves

**For any output level  $y$ , the long-run total cost curve always gives the lowest possible total production cost.**

给定任意一个 $y$ ，每一种短期情境都对应一个总成本；在长期，厂商选择使生产 $y$ 的总成本最低的那个短期情境。

# Short-Run & Long-Run Average Total Cost Curves

Therefore, the long-run av. total cost curve must always give the lowest possible av. total production cost.

换句话说，给定任意一个 $y$ ，每一种短期情境都对应一个平均成本；在长期，厂商选择使生产 $y$ 的平均成本最低的那个短期情境。

The long-run av. total cost curve must be the **lower envelope** of all of the firm's short-run av. total cost curves.

由此可知，长期平均成本线也是短期平均成本线的下包络线。

# Short-Run & Long-Run Average Total Cost Curves

E.g. suppose again that the firm can be in one of just three short-runs;

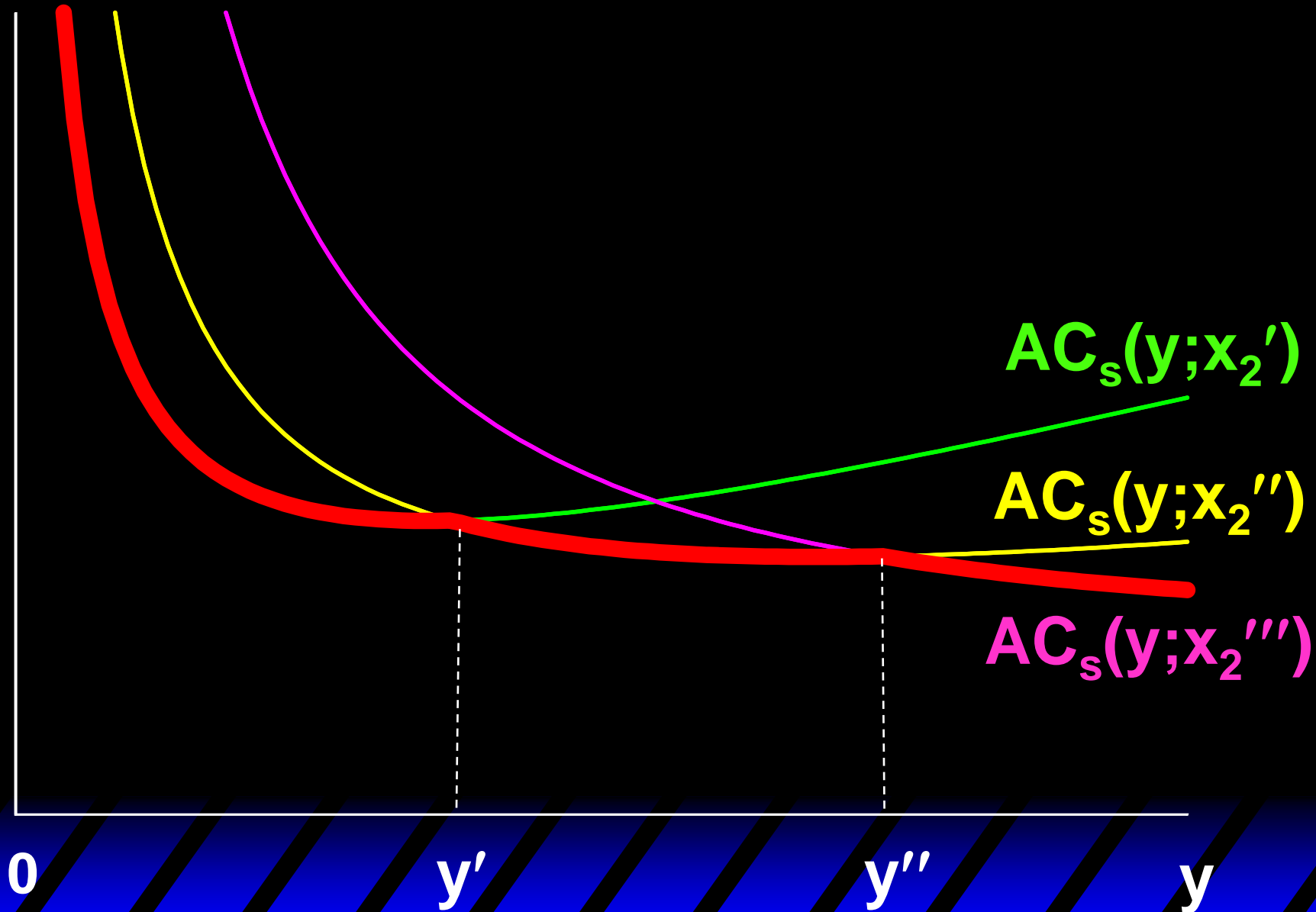
$$x_2 = x_2'$$

or  $x_2 = x_2''$  ( $x_2' < x_2'' < x_2'''$ )

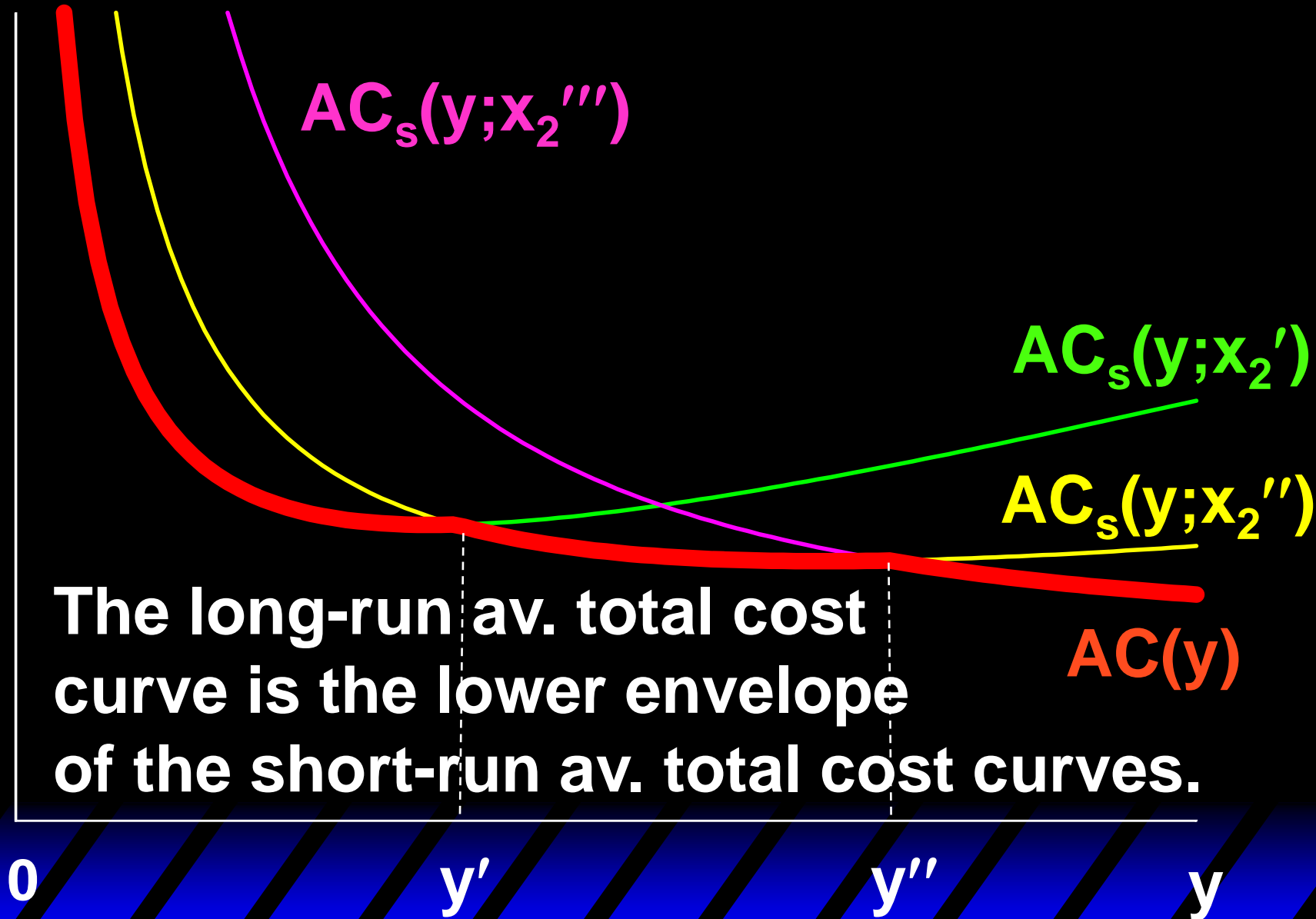
or  $x_2 = x_2'''$

then the firm's three short-run average total cost curves are ...

**\$/output unit**



\$/output unit



# Short-Run & Long-Run Marginal Cost Curves

**Q: Is the long-run marginal cost curve the lower envelope of the firm's short-run marginal cost curves?**

# Short-Run & Long-Run Marginal Cost Curves

**Q:** Is the long-run **marginal cost curve** the lower envelope of the firm's short-run marginal cost curves?

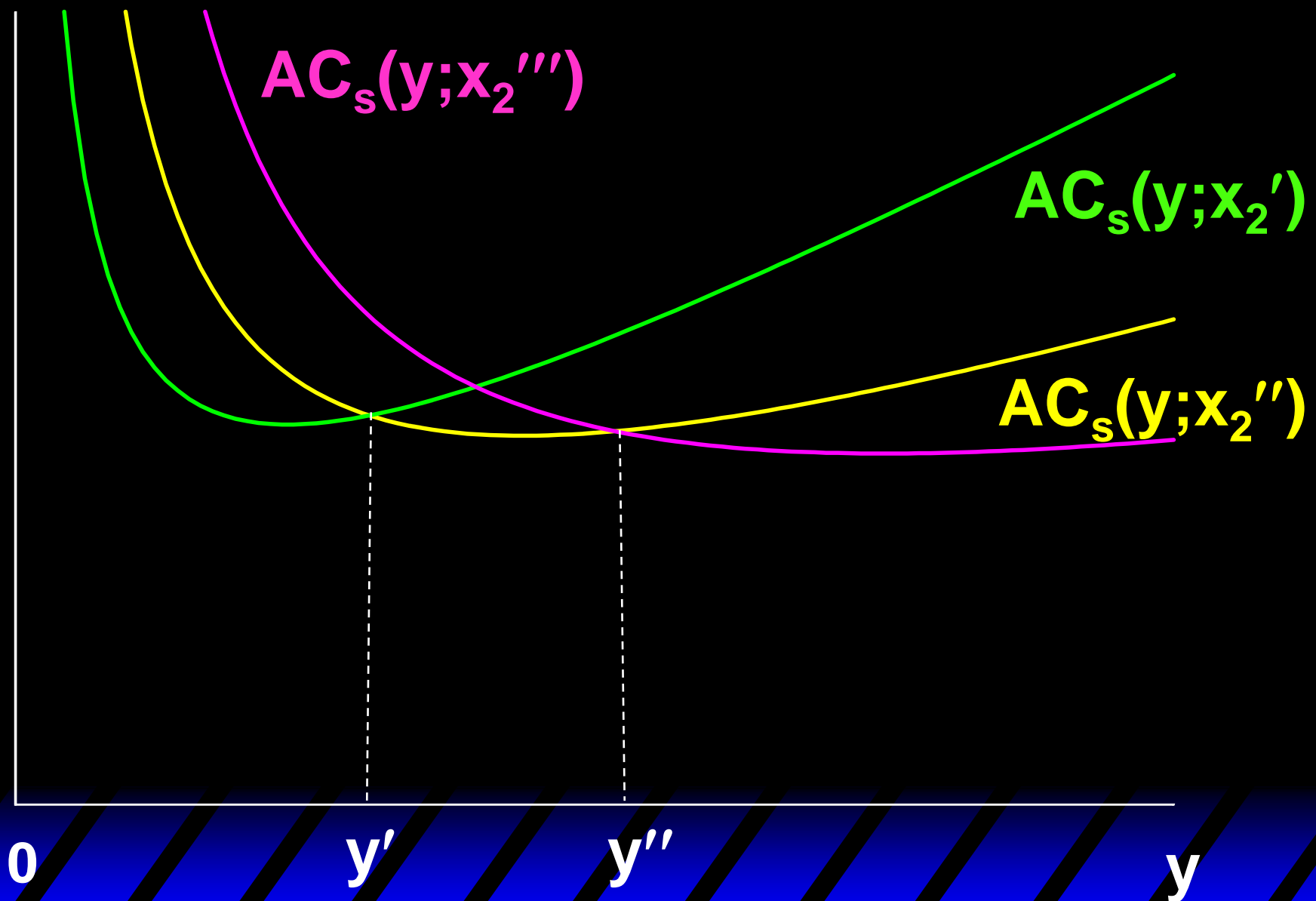
**A:** No.



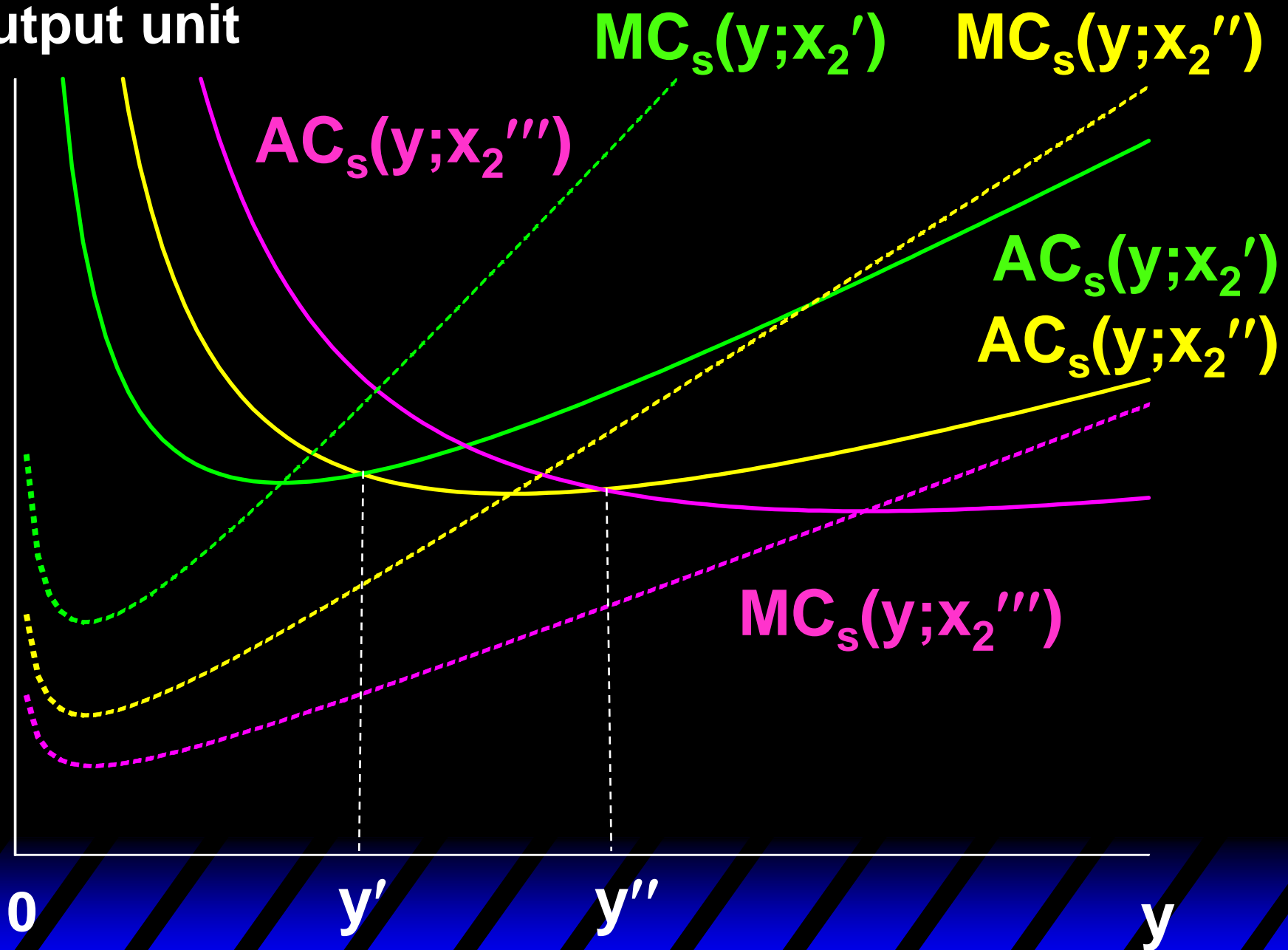
# Short-Run & Long-Run Marginal Cost Curves

**The firm's three short-run average total cost curves are ...**

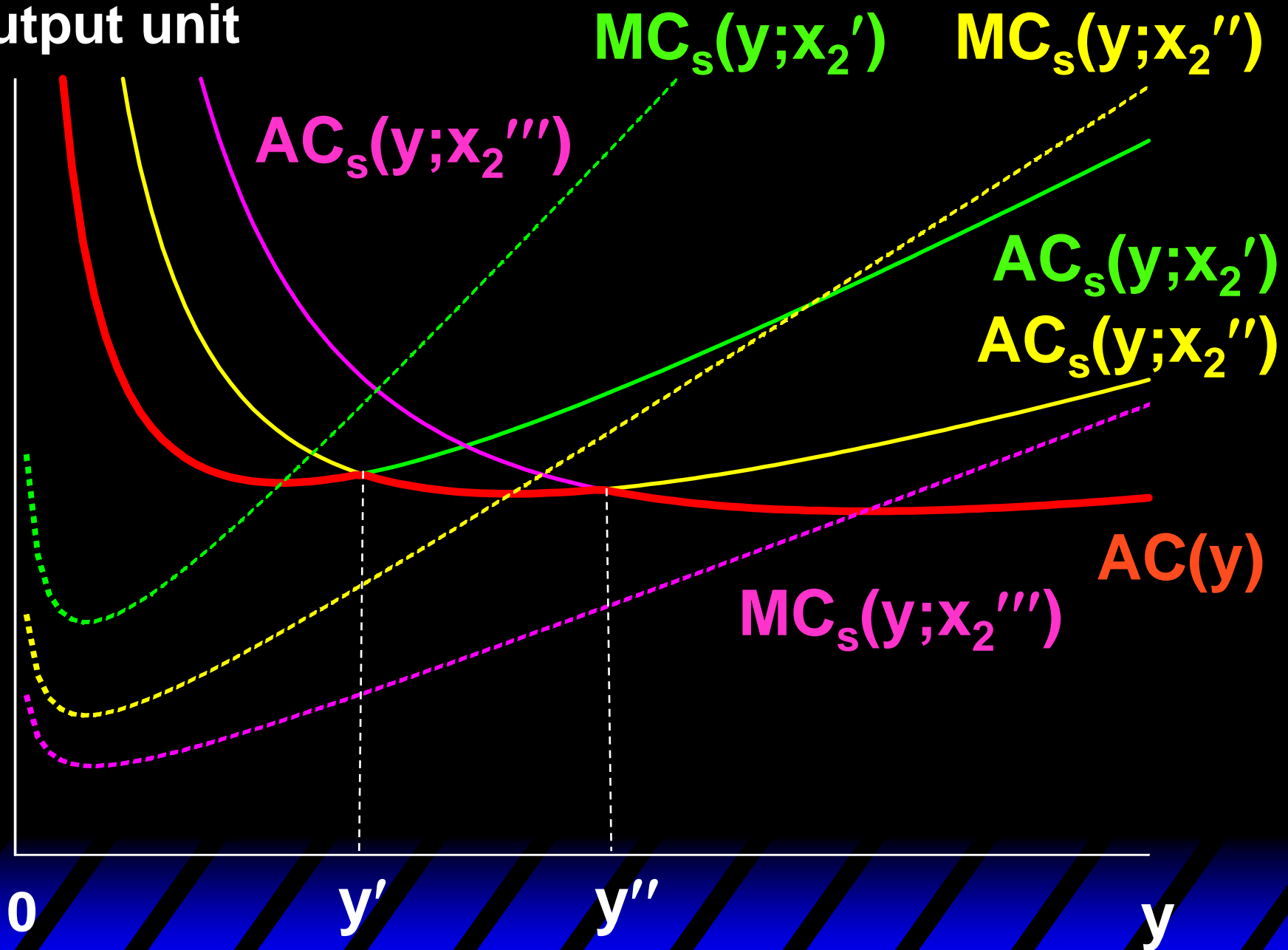
**\$/output unit**



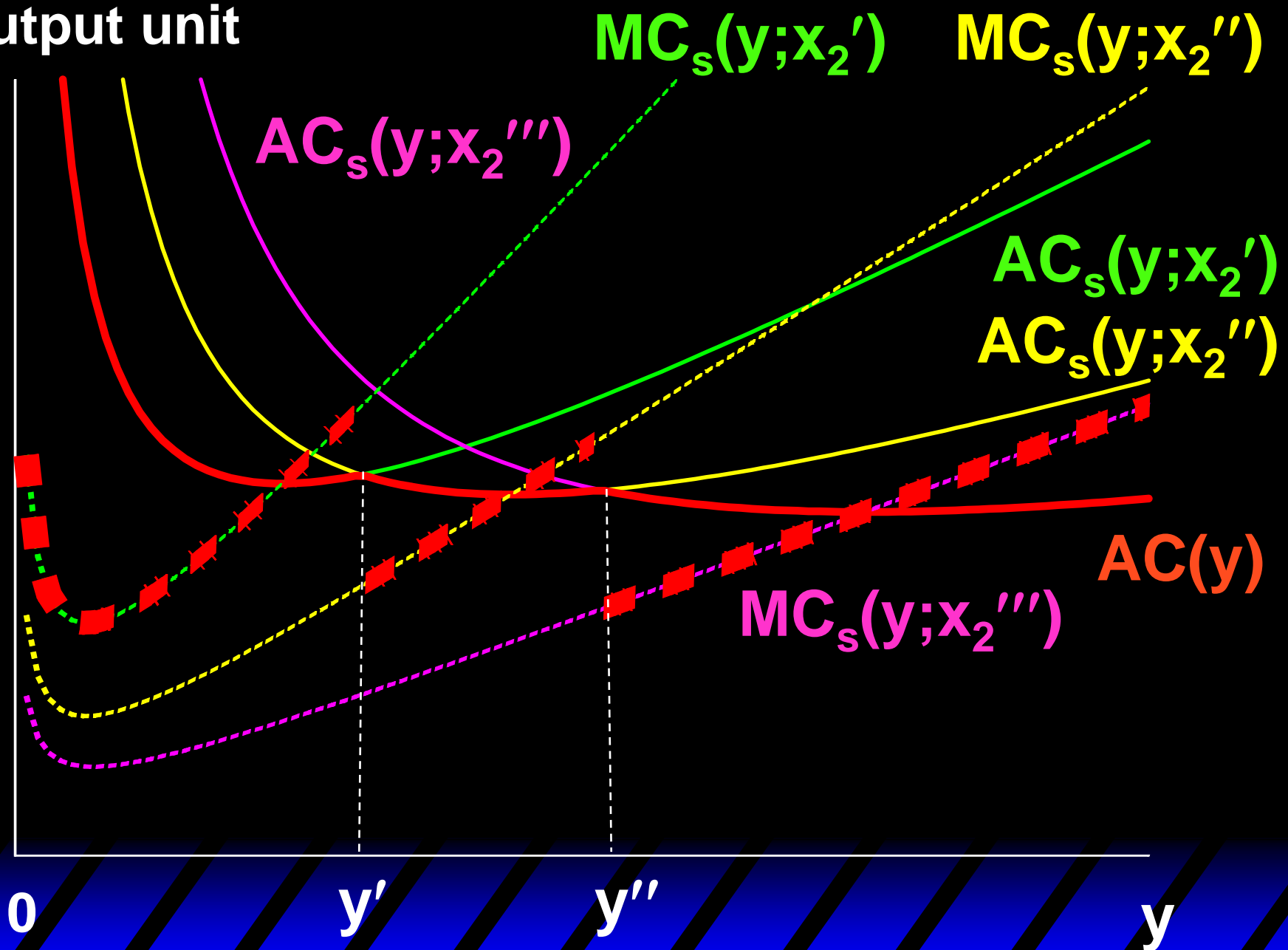
\$/output unit



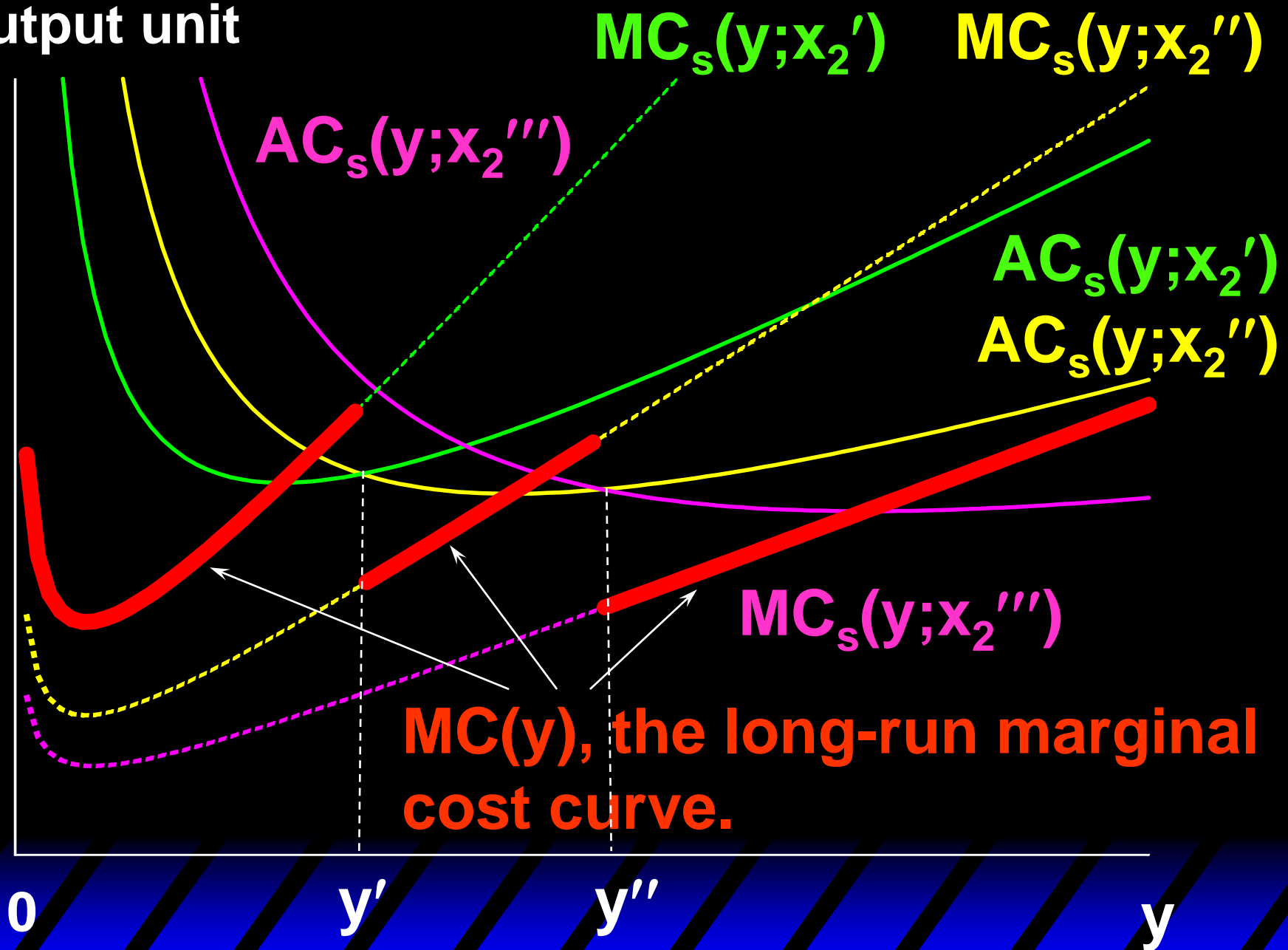
\$/output unit



\$/output unit



\$/output unit



# Short-Run & Long-Run Marginal Cost Curves

For any output level  $y > 0$ , the long-run marginal cost is the marginal cost for the short-run chosen by the firm (任意产量水平处的长期MC都是企业选定的最优短期情境下的MC).

This is always true, no matter how many and which short-run circumstances exist for the firm.

# Short-Run & Long-Run Marginal Cost Curves

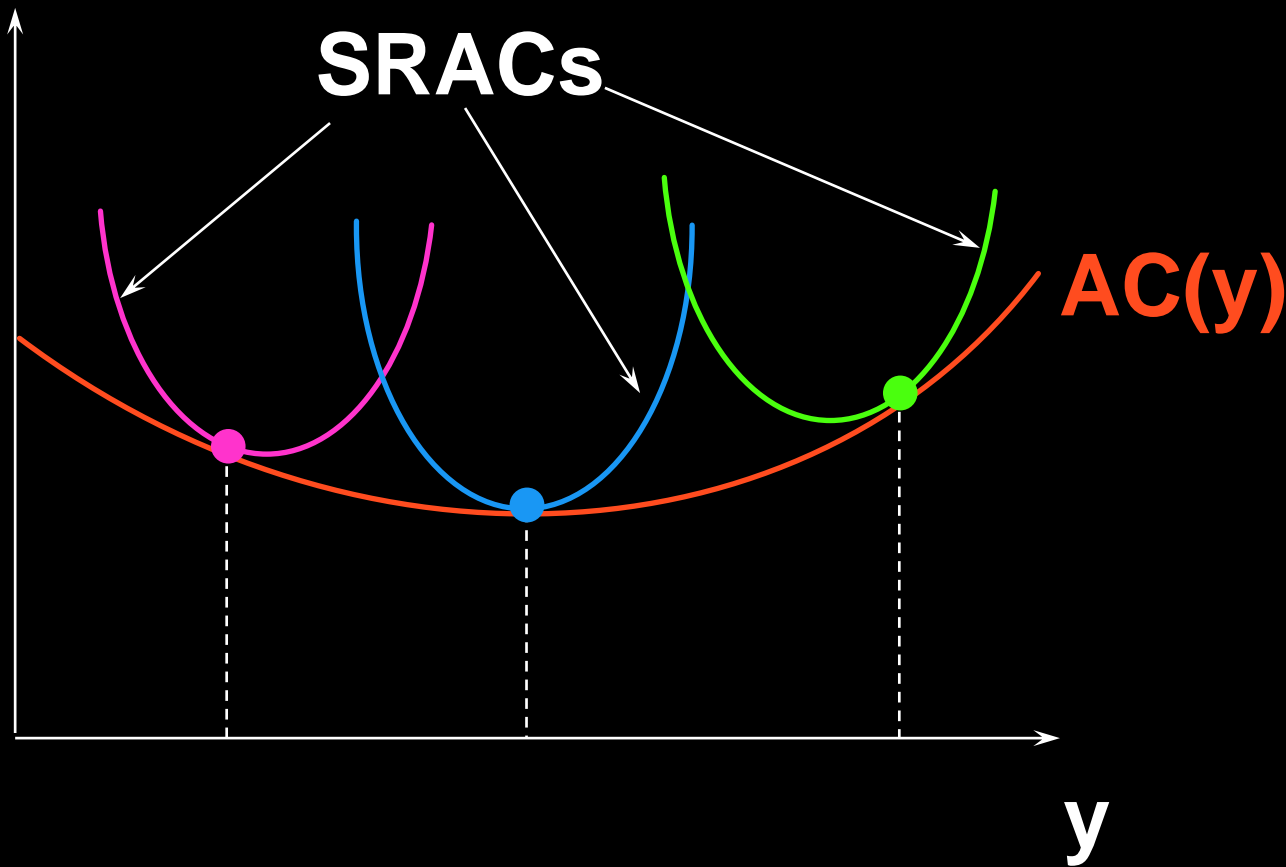
**So for the continuous case, where  $x_2$  can be fixed at any value of zero or more, the relationship between the long-run marginal cost and all of the short-run marginal costs is ...**

**在连续情况下，长期平均成本线、边际成本线和短期平均成本线、边际成本线的关系如下：**



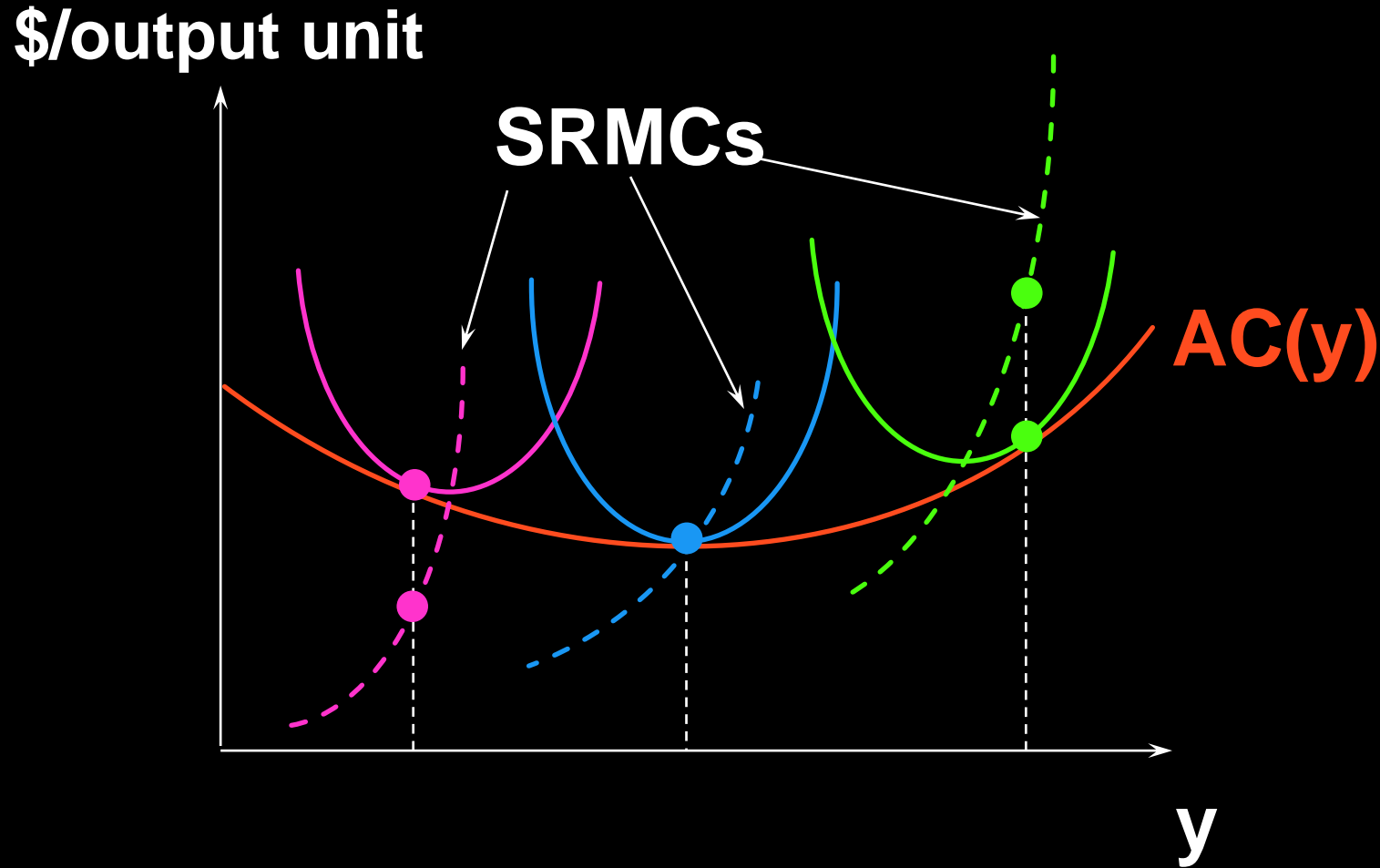
# Short-Run & Long-Run Marginal Cost Curves

\$/output unit



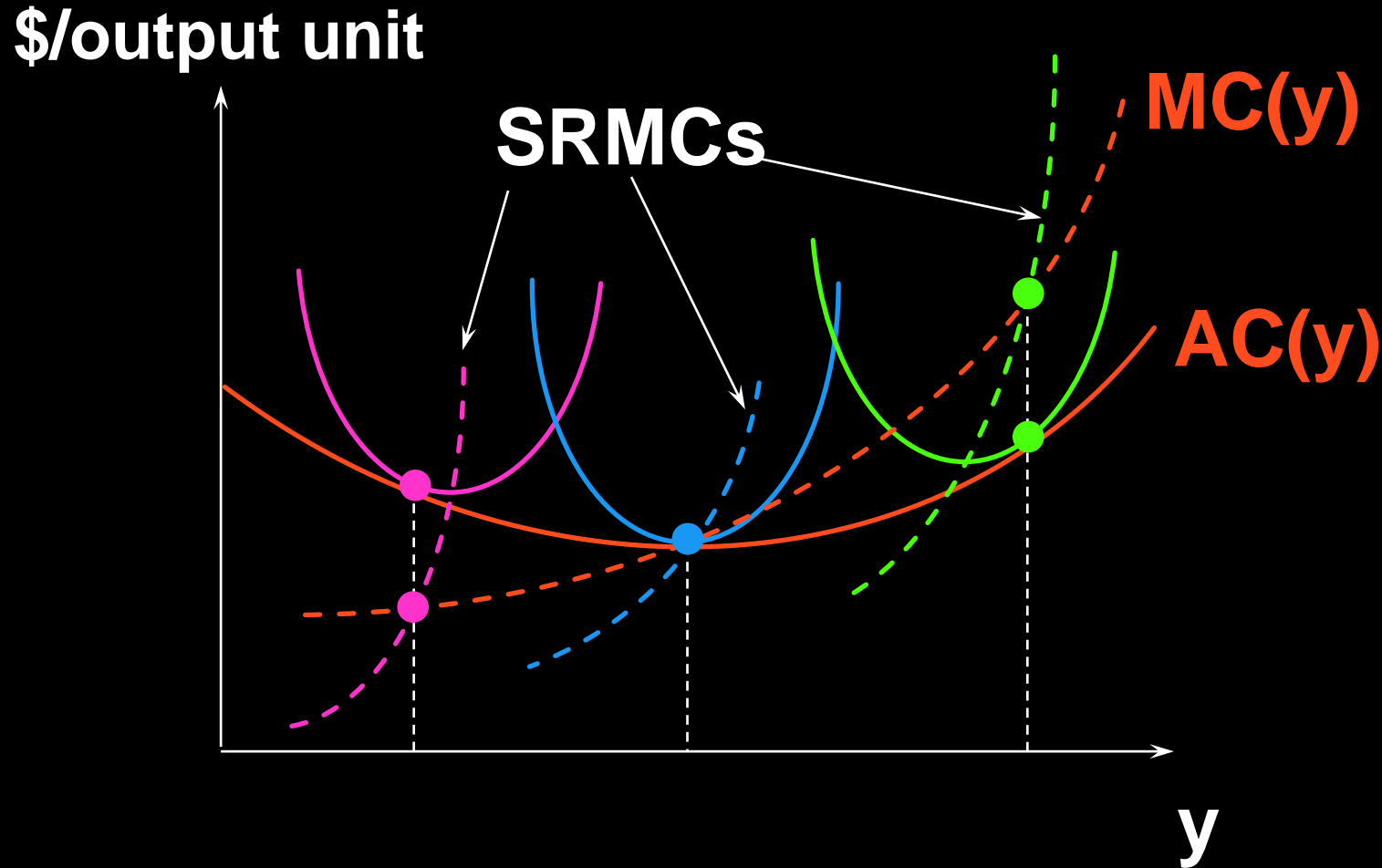
每条短期平均成本线都与长期平均成本线相切于一点

# Short-Run & Long-Run Marginal Cost Curves



每个切点对应的MC即长期MC

# Short-Run & Long-Run Marginal Cost Curves



For each  $y > 0$ , the long-run MC equals the MC for the short-run chosen by the firm.