Lecture 18

Oligopoly

Oligopoly

- A monopoly is an industry consisting a single firm.
- An **oligopoly** is an industry consisting of a few firms. Particularly, each firm’s own price or output decisions affect its competitors’ profits.
- Today’s lecture: strategic interactions of two firms supplying the same product -- **duopoly** (双头垄断)

Oligopoly

Simultaneous moves:
- Quantity setting – **Cournot** model
- Price setting – **Bertrand** model

Sequential moves:
- Quantity setting – **Stackelberg** model
- Price setting – **price leader** model

Collusion (合谋)

Quantity Competition

- Assume that firms compete by **simultaneously** choosing **output** levels.
- If firm 1 produces y_1 units and firm 2 produces y_2 units then total quantity supplied is $y_1 + y_2$. The market price will be $p(y_1 + y_2)$.
- The firms' total cost functions are $c_1(y_1)$ and $c_2(y_2)$.

Quantity Competition

Suppose firm 1 takes firm 2's output level choice y_2 as given. Then firm 1 sees its profit function as

$$\Pi_1(y_1;y_2) = p(y_1 + y_2)y_1 - c_1(y_1).$$

Given y_2, what output level y_1 maximizes firm 1's profit?

Quantity Competition; An Example

Suppose that the market inverse demand function is

$$p(y_T) = 60 - y_T = 60 - y_1 - y_2$$

and that the firms' total cost functions are

$c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2$.

Quantity Competition; An Example

- Then, for given y_2, firm 1's profit function is

$$\Pi(y_1;y_2) = (60 - y_1 - y_2)y_1 - y_1^2.$$

Quantity Competition; An Example

Then, for given y_2, firm 1's profit function is

$$\Pi(y_1;y_2) = (60 - y_1 - y_2)y_1 - y_1^2.$$

So, given y_2, firm 1's profit-maximizing output level solves

$$\frac{\partial \Pi}{\partial y_1} = 60 - 2y_1 - y_2 - 2y_1 = 0.$$

I.e. firm 1's **best response** to y_2 is

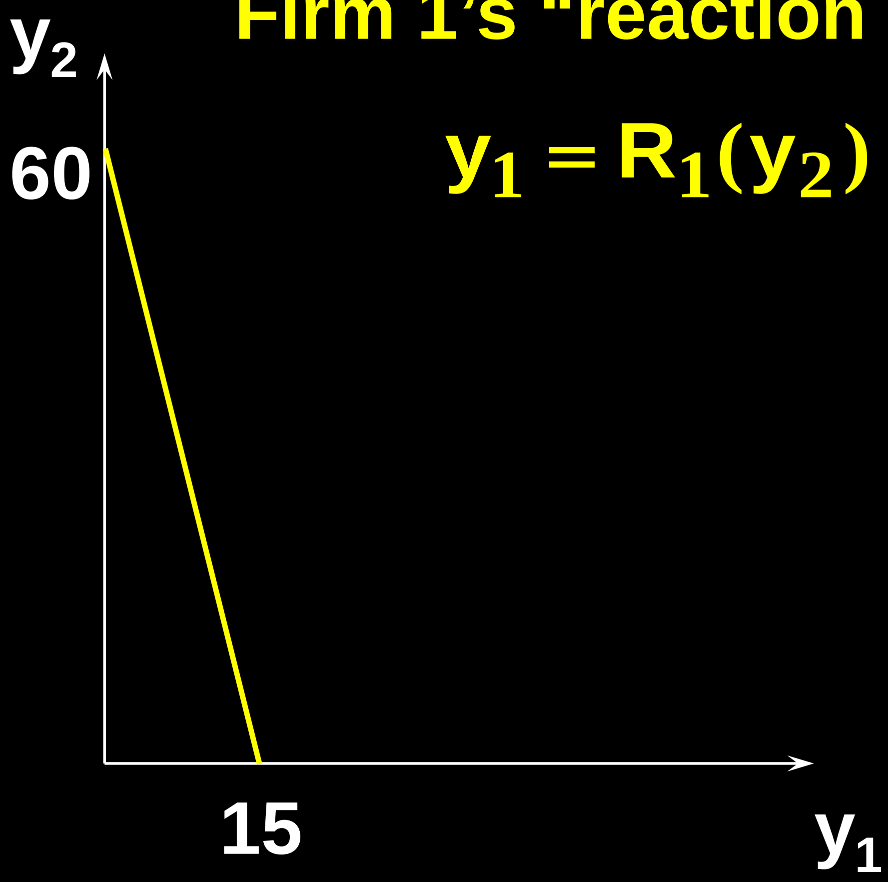
$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

企业1的最优反应函数：给定企业2的产量y_2，使企业1利润最大化的产量$y_1 = R_1(y_2)$

Quantity Competition; An Example

Firm 1's "reaction curve"

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$



Quantity Competition; An Example

Similarly, given y_1, firm 2's profit function is

$$\Pi(y_2; y_1) = (60 - y_1 - y_2)y_2 - 15y_2 - y_2^2.$$

Quantity Competition; An Example

Similarly, given y_1, firm 2's profit function is

$$\Pi(y_2;y_1) = (60 - y_1 - y_2)y_2 - 15y_2 - y_2^2.$$

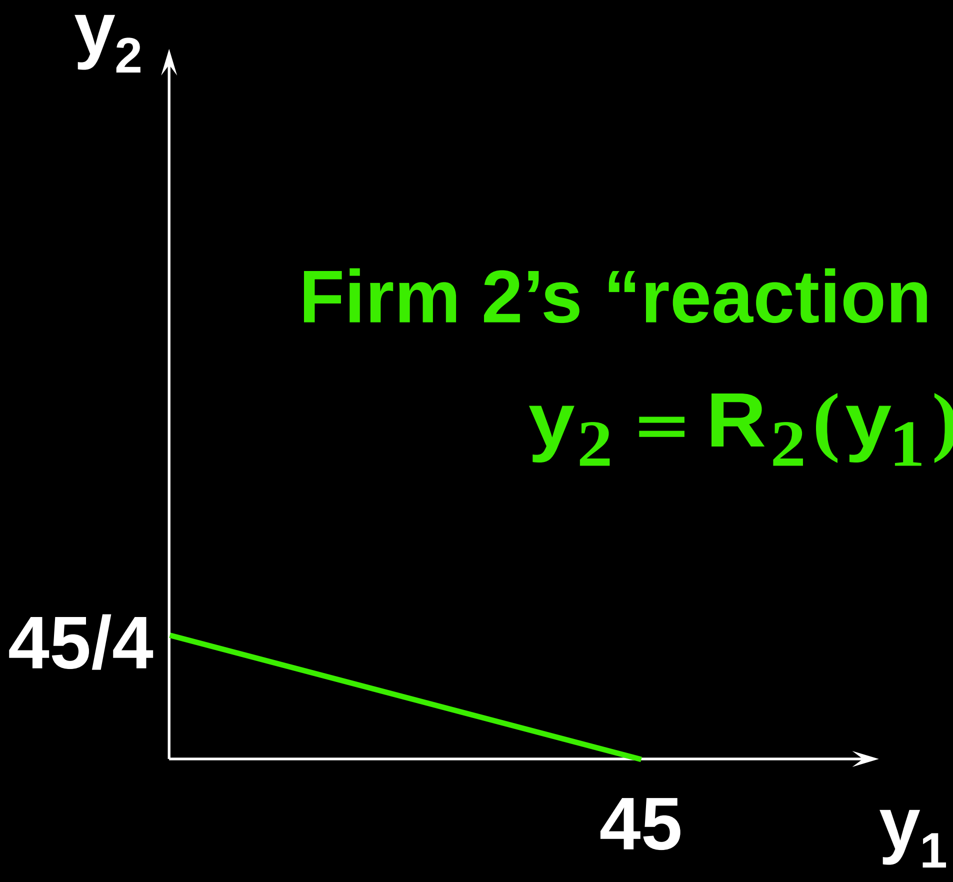
So, given y_1, firm 2's profit-maximizing output level solves

$$\frac{\partial \Pi}{\partial y_2} = 60 - y_1 - 2y_2 - 15 - 2y_2 = 0.$$

I.e. firm 1's best response to y_2 is

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Quantity Competition; An Example



Firm 2's "reaction curve"

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Quantity Competition; An Example

- An equilibrium is when each firm's output level is a best response to the other firm's output level, for then neither wants to deviate from its output level.
- A pair of output levels (y_1^*, y_2^*) is a **Cournot-Nash equilibrium** if

$$y_1^* = R_1(y_2^*) \quad \text{and} \quad y_2^* = R_2(y_1^*).$$

互为最优的产量组合(y_1^*, y_2^*)被称为古诺-纳什均衡；在古诺均衡处，厂商没有单方面偏离的动机。

Quantity Competition; An Example

$$y_1^* = R_1(y_2^*) = 15 - \frac{1}{4}y_2^*$$ and $$y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}.$$

Quantity Competition; An Example

$$y_1^* = R_1(y_2^*) = 15 - \frac{1}{4}y_2^*$$ and $$y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}.$$

Substitute for y_2* to get

$$y_1^* = 15 - \frac{1}{4}\left(\frac{45 - y_1^*}{4}\right) \quad \Rightarrow \quad y_1^* = 13$$

Quantity Competition; An Example

$$y_1^* = R_1(y_2^*) = 15 - \frac{1}{4}y_2^* \text{ and } y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}.$$

Substitute for y_2^* to get

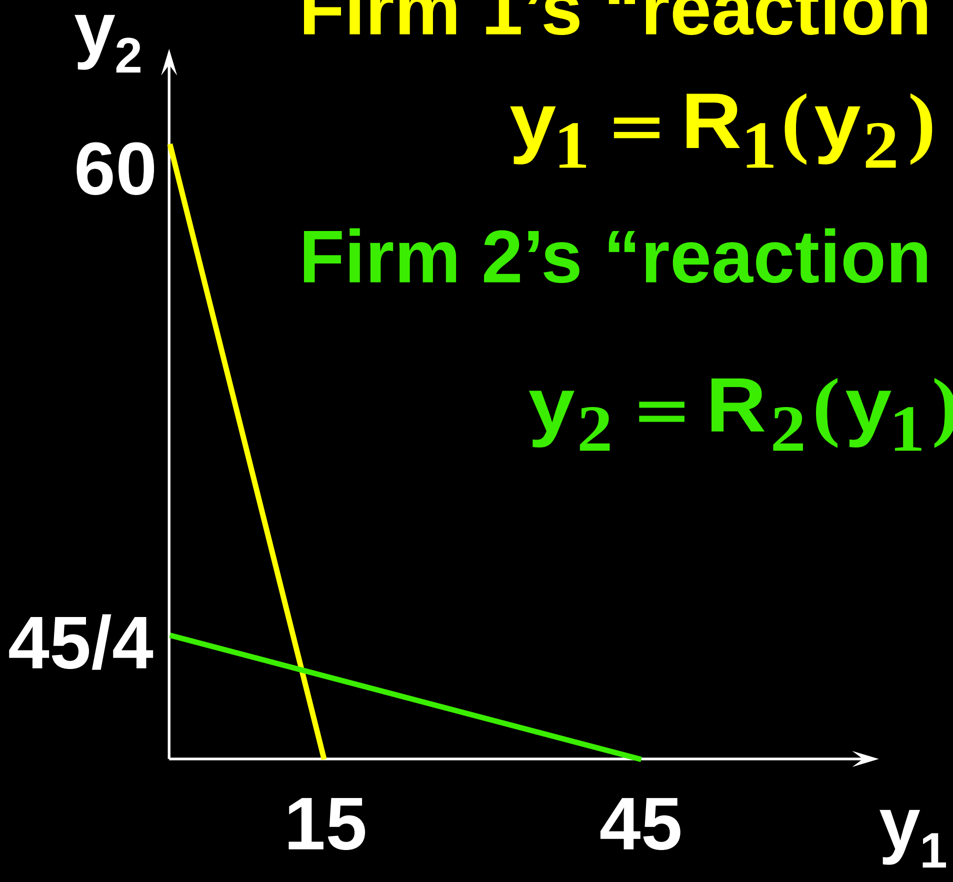
$$y_1^* = 15 - \frac{1}{4}\left(\frac{45 - y_1^*}{4}\right) \quad \Rightarrow \quad y_1^* = 13$$

Hence $$y_2^* = \frac{45 - 13}{4} = 8.$$

So the Cournot-Nash equilibrium is

$$(y_1^*, y_2^*) = (13,8).$$

Quantity Competition; An Example



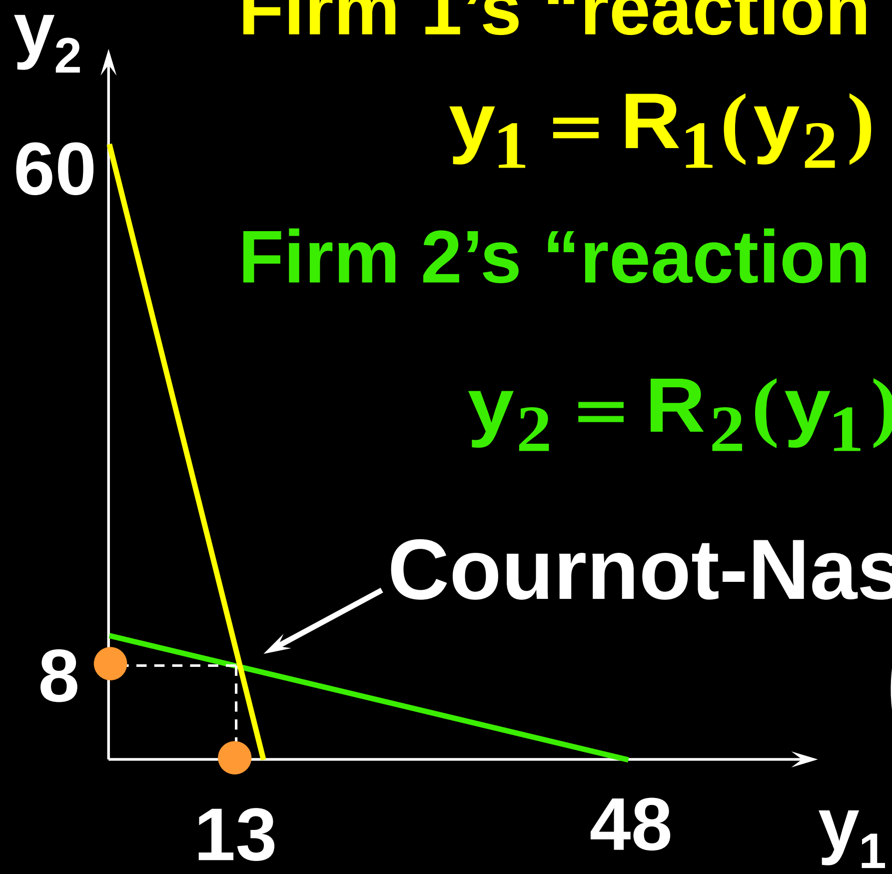
Firm 1's "reaction curve"

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

Firm 2's "reaction curve"

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Quantity Competition; An Example



Firm 1's "reaction curve"

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$

Firm 2's "reaction curve"

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Cournot-Nash equilibrium

$$\left(y_1^*, y_2^*\right) = (13, 8).$$

Quantity Competition; An Example

$$y_1^* = 13,\ y_2^* = 8$$

$$y_T^* = 21$$

$$p^* = 60 - y_1^* - y_2^* = 39$$

$$\Pi_1 = py_1^* - (y_1)^2 = 39 \times 13 - 13^2 = 338$$

$$\Pi_2 = py_2^* - 15y_2 - (y_2)^2 = 39 \times 8 - 120 - 64$$
$$= 128$$

$$\Pi_\mathrm{T} = 466$$

Quantity Competition

Generally, given firm 2's chosen output level y_2, firm 1's profit function is

$$\Pi_1(y_1;y_2) = p(y_1 + y_2)y_1 - c_1(y_1)$$

and the profit-maximizing value of y_1 solves

$$\frac{\partial \Pi_1}{\partial y_1} = p(y_1 + y_2) + y_1 \frac{\partial p(y_1 + y_2)}{\partial y_1} - c_1'(y_1) = 0.$$

The solution, $y_1 = R_1(y_2)$, is firm 1's Cournot-Nash reaction to y_2.

Quantity Competition

- Similarly, given firm 1's chosen output level y_1, firm 2's profit function is

$$\Pi_2(y_2; y_1) = p(y_1 + y_2)y_2 - c_2(y_2)$$

and the profit-maximizing value of y_2 solves

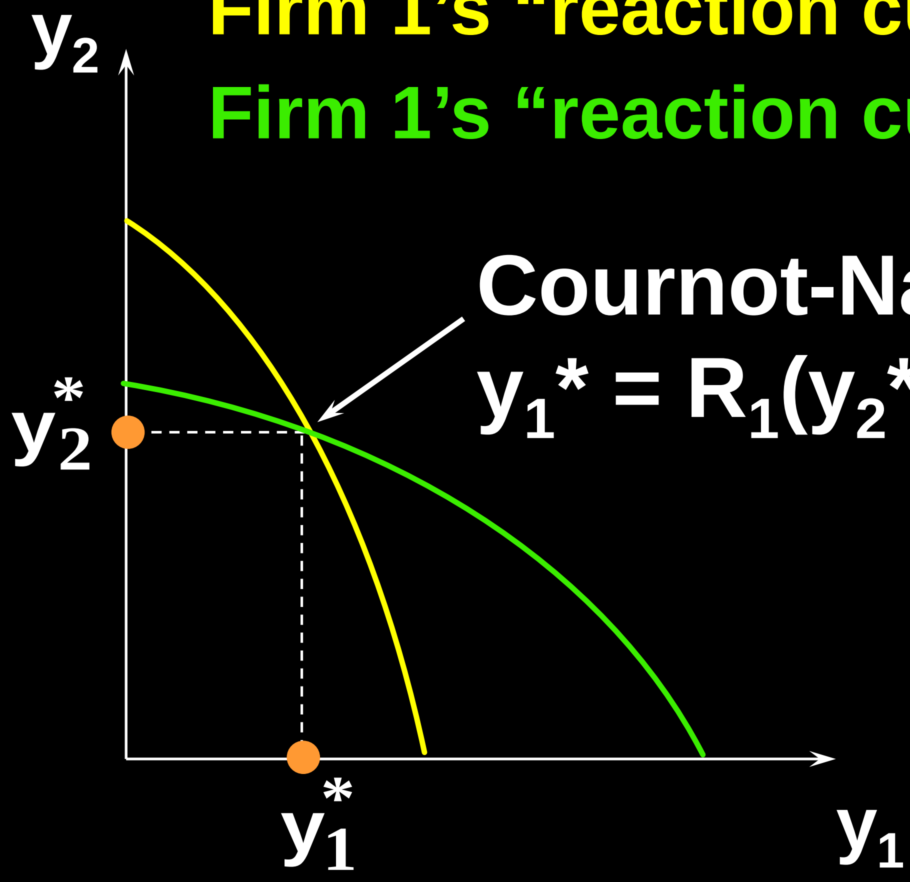
$$\frac{\partial \Pi_2}{\partial y_2} = p(y_1 + y_2) + y_2 \frac{\partial p(y_1 + y_2)}{\partial y_2} - c_2{}'(y_2) = 0.$$

- The solution, $y_2 = R_2(y_1)$, is firm 2's Cournot-Nash reaction to y_1.

Quantity Competition

Firm 1's "reaction curve" $y_1 = R_1(y_2)$.

Firm 1's "reaction curve" $y_2 = R_2(y_1)$.



Cournot-Nash equilibrium
$y_1^* = R_1(y_2^*)$ and $y_2^* = R_2(y_1^*)$

Collusion

- Instead of conducting quantity competition, firms may "**cooperate**" to maximize their total profit by lowering their output levels (e.g. OPEC, De Beers)
- This is **collusion** (合谋)
- Firms that collude are said to have formed a **cartel** (卡特尔)
- If firms form a cartel, how should they do it?

Collusion

Suppose the two firms want to maximize their total profit and divide it between them. Their goal is to choose cooperatively output levels y_1 and y_2 that maximize

$$\Pi^{m}(y_1, y_2) = p(y_1 + y_2)[y_1 + y_2] - C(y_1) - C(y_2)$$

合谋的企业共同决定y_1和y_2来最大化它们的总利润

Collusion

The firms **cannot do worse** by colluding since they can cooperatively choose their Cournot-Nash equilibrium output levels and so earn their Cournot-Nash equilibrium profits. So collusion must provide profits **at least as large as** their Cournot-Nash equilibrium profits.

合谋企业的总利润往往高于古诺均衡下的总利润，因此企业有合谋的动机。

Collusion

Cournot competition:

Firm 1 chooses $\boldsymbol{y_1^*}$ to max $\Pi_1 = p(y_1 + y_2)y_1 - C(y_1)$

Firm 2 chooses $\boldsymbol{y_2^*}$ to max $\Pi_2 = p(y_1 + y_2)y_2 - C(y_2)$

Firms' total profit in Cournot:

$$\mathrm{p}(\mathrm{y}_1^* + \mathrm{y}_2^*)[\mathrm{y}_1^* + \mathrm{y}_2^*] - \mathrm{C}(\mathrm{y}_1^*) - \mathrm{C}(\mathrm{y}_2^*)$$

Collusion:

$$\max_{\mathrm{y}_1, \mathrm{y}_2} \mathrm{p}(\mathrm{y}_1 + \mathrm{y}_2)[\mathrm{y}_1 + \mathrm{y}_2] - \mathrm{C}(\mathrm{y}_1) - \mathrm{C}(\mathrm{y}_2)$$

Collusion

Colluding firms can make at least the Cournot equilibrium profits by producing at the Cournot equilibrium quantities (y_1^*, y_2^*)

如果愿意，合谋企业总可以选取古诺均衡产量，因此至少可以获得古诺均衡处的总利润。

Collusion: An Example

Again, the market inverse demand function is

$$p(y_T) = 60 - y_T = 60 - y_1 - y_2$$

and that the firms' total cost functions are

$c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2.$

Collusion: An Example

Two firms cooperatively determine $(\mathbf{y_1}, \mathbf{y_2})$ to max

$$\mathbf{\Pi^m = p(y_1 + y_2)[y_1 + y_2] - C(y_1) - C(y_2)}$$

$$= \mathbf{(60 - y_1 - y_2)}(\boldsymbol{y_1} + \boldsymbol{y_2}) - (\boldsymbol{y_1})^2 - \mathbf{15}\boldsymbol{y_2} - (\boldsymbol{y_2})^2$$

Collusion: An Example

Two firms cooperatively determine $(\mathbf{y_1}, \mathbf{y_2})$ to max

$$\Pi^m = p(y_1 + y_2)[y_1 + y_2] - C(y_1) - C(y_2)$$

$$= (60 - y_1 - y_2)[y_1 + y_2] - (y_1)^2 - 15y_2 - (y_2)^2$$

$$\frac{\partial \Pi^m}{\partial y_1} = 60 - 2(y_1 + y_2) - 2y_1 = 0 \Rightarrow$$

$$4y_1 + 2y_2 = 60$$

Collusion: An Example

$$\Pi^{\mathrm{m}} = (60 - \mathrm{y}_1 - \mathrm{y}_2)(y_1 + y_2) - (y_1)^2 - 15y_2 - (y_2)^2$$

$$\frac{\partial \Pi^{\mathrm{m}}}{\partial y_1} = 60 - 2(y_1 + y_2) - 2y_1 = 0 \Rightarrow$$

$$4y_1 + 2y_2 = 60$$

$$\frac{\partial \Pi^{\mathrm{m}}}{\partial y_2} = 60 - 2(y_1 + y_2) - 15 - 2y_2 = 0 \Rightarrow$$

$$2y_1 + 4y_2 = 45$$

Collusion: An Example

$$\begin{cases} 4y_1 + 2y_2 = 60 \\ 2y_1 + 4y_2 = 45 \end{cases}$$

$$y_1^{**} = 12.5,\ y_2^{**} = 5$$

$$y_T^{**} = 17.5 < y_T^{*} = 21$$

$$p^{**} = 42.5 > p^{*} = 39$$

$$\Pi^{**} = p[y_1 + y_2] - (y_1)^2 - 15y_2 - (y_2)^2$$
$$= 487.5 > \Pi^{*} = 466$$

Collusion

- Is such a cartel stable?
- Does one firm have an incentive to cheat on the other?
- I.e. if firm 1 continues to produce y_1^m units, is it profit-maximizing for firm 2 to continue to produce y_2^m units?

Collusion

Firm 2's profit-maximizing response to $y_1 = y_1^m$ is $y_2 = R_2(y_1^m)$.

假定厂商1按照约定的合谋产量y_1^m进行生产。使厂商2利润最大化的产量由反应函数给出，为$\mathrm{R}_2(y_1^m)$. 这一产量与约定的合谋产量y_2^m不同，因此厂商2有偏离合谋产量、增加自身利润的动机。

Collusion

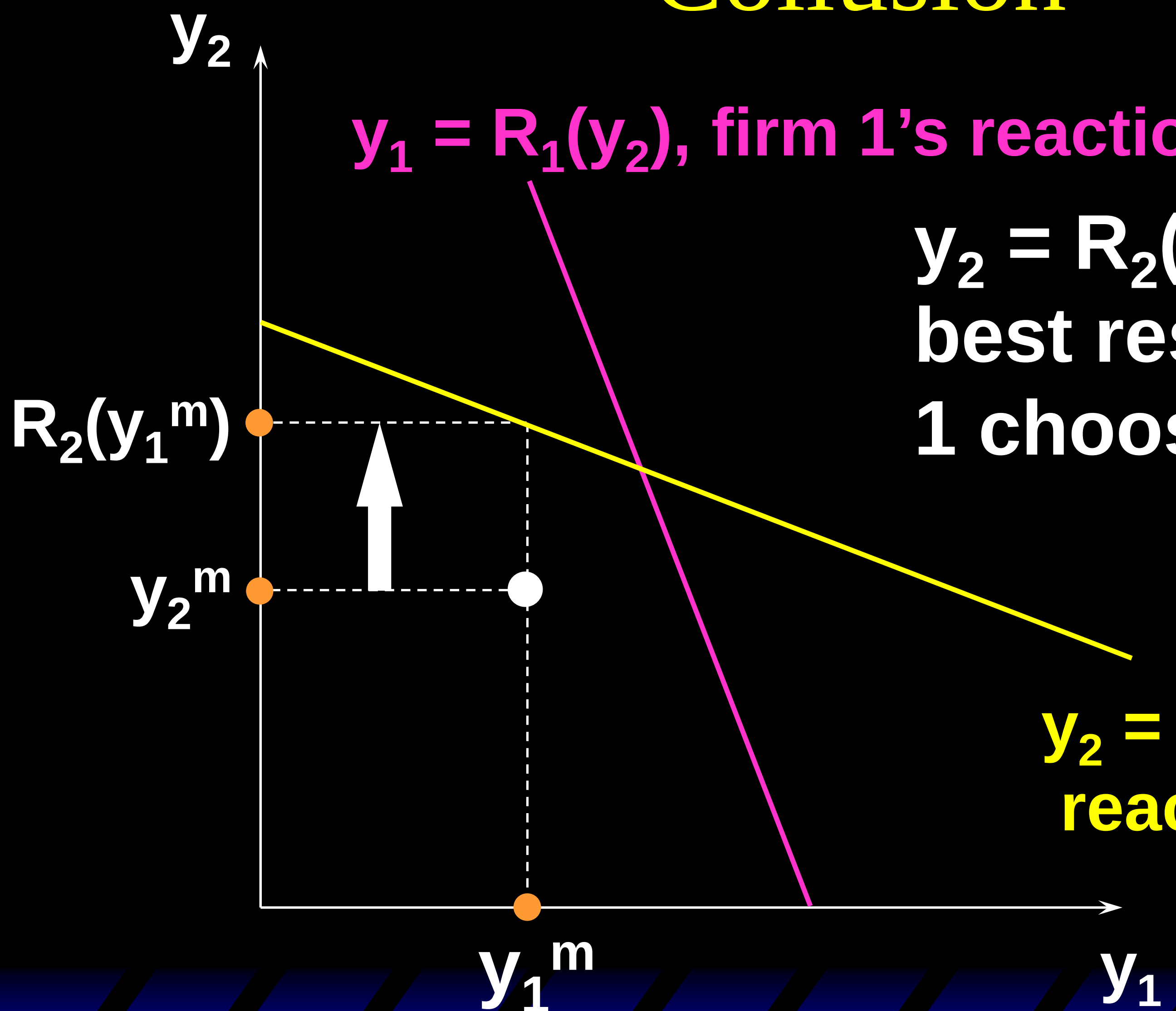
Firm 2's profit-maximizing response to $y_1 = y_1^m$ is $y_2 = R_2(y_1^m) > y_2^m$.

Firm 2's profit increases if it cheats on firm 1 by increasing its output level from y_2^m to $R_2(y_1^m)$.

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

给定厂商1在约定产量生产，$y_1^m = 12.5$，厂商2的利润最大化产量是$R_2(y_1^m) = \frac{45-12.5}{4} = 8.125$，大于约定产量$y_2^m = 5$，因此有偏离的动机

Collusion

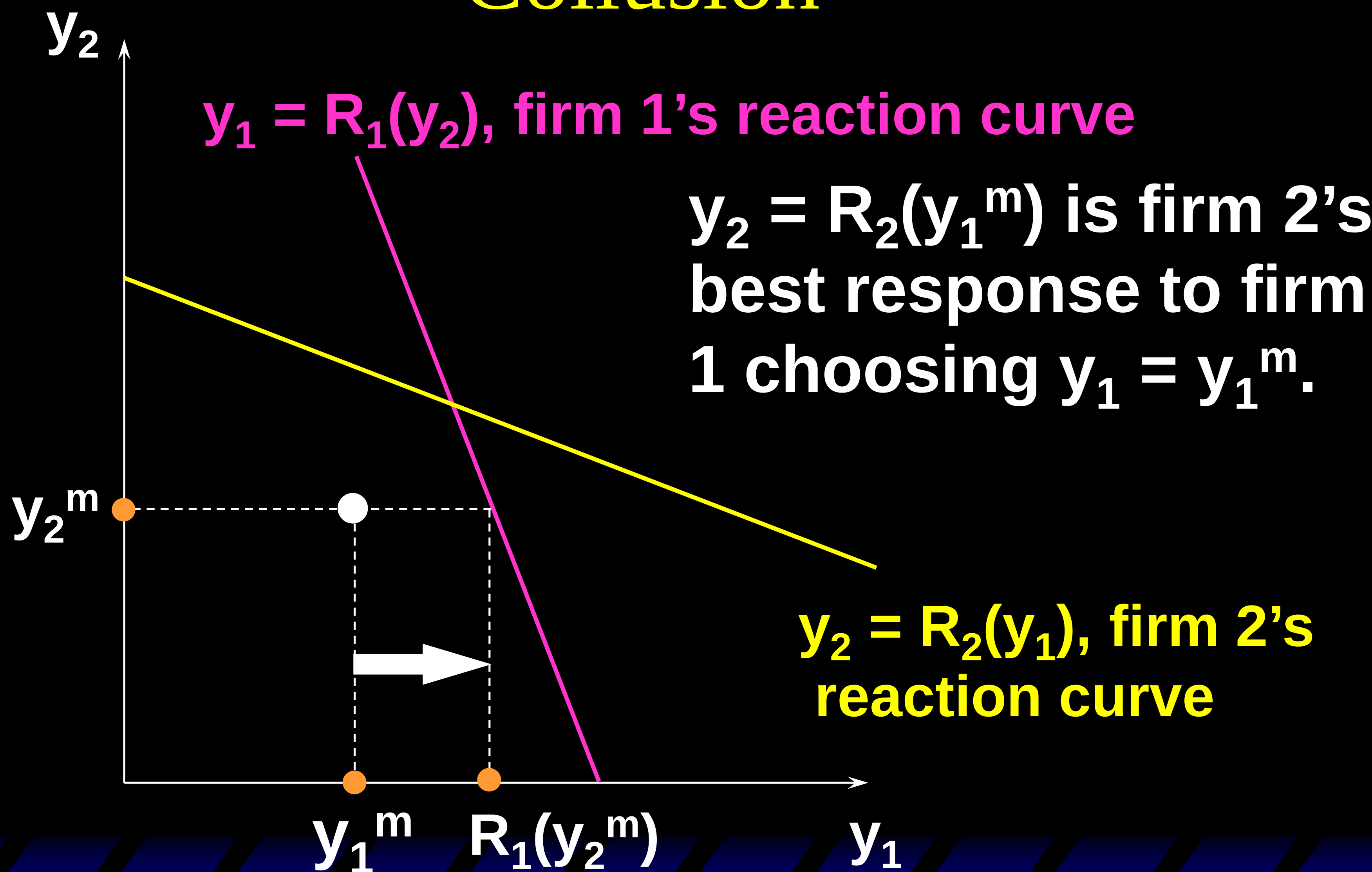


$y_2 = R_2(y_1^m)$ is firm 2's best response to firm 1 choosing $y_1 = y_1^m$.

Collusion

Similarly, firm 1’s profit increases if it cheats on firm 2 by increasing its output level from y_1^m to $R_1(y_2^m)$.

Collusion



y_2
$y_1 = R_1(y_2)$, firm 1's reaction curve
$y_2 = R_2(y_1^m)$ is firm 2's best response to firm 1 choosing $y_1 = y_1^m$.
y_2^m
$y_2 = R_2(y_1)$, firm 2's reaction curve
y_1^m
$R_1(y_2^m)$
y_1

Collusion

So a profit-seeking cartel in which firms cooperatively set their output levels is fundamentally unstable.

E.g. OPEC's broken agreements.

合谋均衡不是一个稳定的均衡。

The Order of Play

- **So far it has been assumed that firms choose their output levels simultaneously.**
- **The competition between the firms is then a simultaneous play game (同时行动博弈) in which the output levels are the strategic variables.**

The Order of Play

- What if firm 1 chooses its output level **first** and then firm 2 responds to this choice?
- Firm 1 is then a **leader** (领导者) Firm 2 is a **follower** (跟随者)
- The competition is a **sequential game** (序贯博弈) in which the output levels are the strategic variables.

The Order of Play

Such games are Stackelberg games.

斯塔克伯格产量竞争模型

Is it better to be the leader?

Or is it better to be the follower?

Stackelberg Games

Q: What is the best response that follower firm 2 can make to the choice y_1 already made by the leader, firm 1?

Stackelberg Games

Q: What is the best response that follower firm 2 can make to the choice y_1 already made by the leader, firm 1?

A: Choose $y_2 = R_2(y_1)$.

Firm 1 knows this and so perfectly anticipates firm 2's reaction to any y_1 chosen by firm 1.

Stackelberg Games

This makes the leader's profit function

$$\Pi_1^s(y_1) = p(y_1 + R_2(y_1))y_1 - c_1(y_1).$$

Stackelberg Games

This makes the leader's profit function

$$\Pi_1^s(y_1) = p(y_1 + R_2(y_1))y_1 - c_1(y_1).$$

The leader chooses y_1 to maximize its profit.

Q: Will the leader make a profit at least as large as its Cournot-Nash equilibrium profit?

Stackelberg Games

A: Yes. The leader could choose its Cournot-Nash output level, knowing that the follower would then also choose its C-N output level. The leader's profit would then be its C-N profit. But the leader does not have to do this, so its profit must be at least as large as its C-N profit.

如果市场领导者选择古诺均衡时的产量，市场跟随者也会选择古诺均衡时的产量 $y_2 = R_2(y_1^N) = y_2^N$. 此时市场领导者的利润和古诺竞争时的利润一样高。

Stackelberg Games; An Example

The market inverse demand function is $p = 60 - y_T$. The firms' cost functions are $c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2$.

Firm 2 is the follower. Its reaction function is

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Stackelberg Games; An Example

The leader's profit function is therefore

$$\Pi_1^s(y_1) = (60 - y_1 - R_2(y_1))y_1 - y_1^2$$

$$= (60 - y_1 - \frac{45 - y_1}{4})y_1 - y_1^2$$

$$= \frac{195}{4}y_1 - \frac{7}{4}y_1^2.$$

Stackelberg Games; An Example

The leader's profit function is therefore

$$\Pi_1^s(y_1) = (60 - y_1 - R_2(y_1))y_1 - y_1^2$$
$$= (60 - y_1 - \frac{45 - y_1}{4})y_1 - y_1^2$$
$$= \frac{195}{4}y_1 - \frac{7}{4}y_1^2.$$

For a profit-maximum,

$$\frac{195}{4} = \frac{7}{2}y_1 \;\Rightarrow\; y_1^s = 13 \cdot 9.$$

Stackelberg Games; An Example

Q: What is firm 2's response to the leader's choice $y_1^s = 13 \cdot 9$?

Stackelberg Games; An Example

Q: What is firm 2's response to the leader's choice $y_1^s = 13 \cdot 9$?

A: $y_2^s = R_2(y_1^s) = \dfrac{45 - 13 \cdot 9}{4} = 7 \cdot 8.$

The C-N output levels are $(y_1^*, y_2^*) = (13,8)$ so the leader produces **more** than its C-N output and the follower produces **less** than its C-N output. This is true generally.

相比于古诺均衡，市场领导者的产量上升、跟随者的产量下降。

Price Competition

- What if firms compete using only **price-setting** strategies, instead of using only quantity-setting strategies?
- Games in which firms use only price strategies and play simultaneously are **Bertrand** games.

Bertrand Games

Each firm's marginal production cost is constant at c.

All firms **simultaneously** set their prices.

Q: Is there a Nash equilibrium?

Bertrand Games

- Each firm's marginal production cost is constant at c.
- All firms **simultaneously** set their prices.
- Q: Is there a Nash equilibrium?
- A: Yes. Exactly one. All firms set their prices equal to the marginal cost c. Why?

Bertrand Games

Suppose one firm sets its price **higher** than another firm’s price.

Then the higher-priced firm would have no customers.

Hence, at an equilibrium, all firms must set the **same** price.

Bertrand Games

- Suppose the common price set by all firm is higher than marginal cost c.
- Then one firm can just slightly lower its price and sell to all the buyers, thereby increasing its profit.
- The only common price which prevents undercutting is c. Hence this is the only Nash equilibrium.

Bertrand Games

What if the marginal costs $c_1 < c_2$?

All firms simultaneously set their prices.

Q: Is there a Nash equilibrium?

Sequential Price Games

- What if, instead of simultaneous play in pricing strategies, one firm decides its price ahead of the others.
- This is a sequential game in pricing strategies called a **price-leadership game** (价格领导者博弈)
- The firm which sets its price ahead of the other firms is the price-leader.

Sequential Price Games

- Think of one large firm (the leader) and many competitive small firms (the followers).
- The small firms are **price-takers** and so their collective supply reaction to a market price p is their aggregate supply function $Y_f(p)$.

价格领导者制定价格，跟随者接受价格。

Sequential Price Games

The market demand function is D(p).

So the leader knows that if it sets a price p the quantity demanded from it will be the residual demand 剩余需求

$$L(p) = D(p) - Y_f(p).$$

Hence the leader's profit function is

$$\Pi_L(p) = p(D(p) - Y_f(p)) - c_L(D(p) - Y_f(p)).$$

Sequential Price Games

The leader's profit function is

$$\Pi_L(p) = p(D(p) - Y_f(p)) - c_L(D(p) - Y_F(p))$$

so the leader chooses the price level p* for which profit is maximized.

The followers collectively supply $Y_f(p^*)$ units and the leader supplies the residual quantity $D(p^*) - Y_f(p^*)$.

Sequential Price Games: an example

Suppose that the market inverse demand function is

$$p(y_T) = 60 - y_T = 60 - y_1 - y_2$$

Firm 1 is the **price leader** and firm 2 is the **follower**. Their total cost functions are given by

$c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2.$

Sequential Price Games: an example

The follower takes p as given and produces where

$$\mathbf{p = MC(y_2) = 15 + 2y_2}$$

Its supply function is given by

$$\mathbf{y_2 = Y_f(p) = \frac{p - 15}{2}}$$

Sequential Price Games: an example

The follower takes p as given and produces where

$$p = MC(y_2) = 15 + 2y_2$$

Its supply function is given by

$$y_2 = Y_f(p) = \frac{p-15}{2}$$

Residual demand for firm 1 is

$$L(p) = D(p) - Y_f(p) = 60 - p - \frac{p-15}{2}$$

$$= 67.5 - \frac{3}{2}p$$

Sequential Price Games: an example

Residual demand for firm 1 is

$$\mathbf{L(p) = D(p) - Y_f(p) = 60 - p - \frac{p - 15}{2}}$$

$$\mathbf{= 67.5 - \frac{3}{2}p = y_1}$$

=> $\mathbf{p = \frac{2}{3}(67.5 - y_1)}$

Sequential Price Games: an example

Residual demand for firm 1 is

$$\mathbf{L(p) = D(p) - Y_f(p) = 60 - p - \frac{p - 15}{2}}$$

$$\mathbf{= 67.5 - \frac{3}{2}p = y_1}$$

Firm 1, the price leader, then chooses $\mathbf{p}$ to max its profit

$$\mathbf{\Pi_L = p * L(p) - c(y_1) = p\left(67.5 - \frac{3}{2}p\right) - (y_1)^2}$$

Sequential Price Games: an example

- Firm 1, the price leader, then chooses y_1 to max its profit

$$\Pi_L = p * L(p) - c(y_1) = p\left(67.5 - \frac{3}{2}p\right) - (y_1)^2$$

$$= p\left(67.5 - \frac{3}{2}p\right) - \left(67.5 - \frac{3}{2}p\right)^2$$

Sequential Price Games: an example

- Firm 1, the price leader, then chooses y_1 to max its profit

$$\Pi_L = p * L(p) - c(y_1) = p\left(67.5 - \frac{3}{2}p\right) - (y_1)^2$$

$$= p\left(67.5 - \frac{3}{2}p\right) - \left(67.5 - \frac{3}{2}p\right)^2$$

$$\frac{\partial \Pi_L}{\partial p} = 67.5 - 3p + 3\left(67.5 - \frac{3}{2}p\right) = 0$$

$$p^* = 36$$

Sequential Price Games: an example

- **Firm 1, the price leader, then chooses y_1 to max its profit**

$$\Pi_L = p * L(p) - c(y_1) = p\left(67.5 - \frac{3}{2}p\right) - (y_1)^2$$

$$= p\left(67.5 - \frac{3}{2}p\right) - \left(67.5 - \frac{3}{2}p\right)^2$$

$$\frac{\partial \Pi_L}{\partial p} = 67.5 - 3p + 3\left(67.5 - \frac{3}{2}p\right) = 0$$

$$p^* = 36$$

$$y_1^* = L(p^*) = 67.5 - 3/2p = 13.5$$

$$y_2^* = Y_f(p) = \frac{p^* - 15}{2} = 10.5$$