Lecture 1

Budgetary and Other Constraints on Choice

Optimal Consumption Choice

A consumer is modeled as a rational agent who always chooses the most preferred consumption bundle available to her.

To model this optimization problem, we need to model:

- **the choice set (Today)**
- **preferences (Lec2)**

Consumption Choice Sets

A consumption choice set is the collection of all consumption choices available to the consumer.

消费选择集是消费者所能得到的所有消费选择的集合。

What constrains consumption choice?

- **Budgetary**, time and other resource limitations.

Budget Constraints

A **consumption bundle**（消费组合） containing x_1 units of commodity 1, x_2 units of commodity 2 and so on up to x_n units of commodity n is denoted by the vector $(x_1, x_2, \ldots, x_n)$.

Commodity prices are $p_1, p_2, \ldots, p_n$.

Budget Constraints

Q: When is a consumption bundle $(x_1, \ldots, x_n)$ affordable at given prices $p_1, \ldots, p_n$?

Budget Constraints

Q: When is a consumption bundle $(x_1, \ldots, x_n)$ affordable at given prices $p_1, \ldots, p_n$?

A: When

$$p_1x_1 + \ldots + p_nx_n \leq m$$

where *m* is the consumer's (disposable) income.

Budget Constraints

The consumer's budget set is the set of all affordable bundles;

$$B(p_1, \ldots, p_n, m) =$$
$$\{ (x_1, \ldots, x_n) \mid x_1 \geq 0, \ldots, x_n \geq 0 \text{ and } p_1x_1 + \ldots + p_nx_n \leq m \}$$

预算集是消费者可负担的所有商品组合的集合。

Budget Constraints

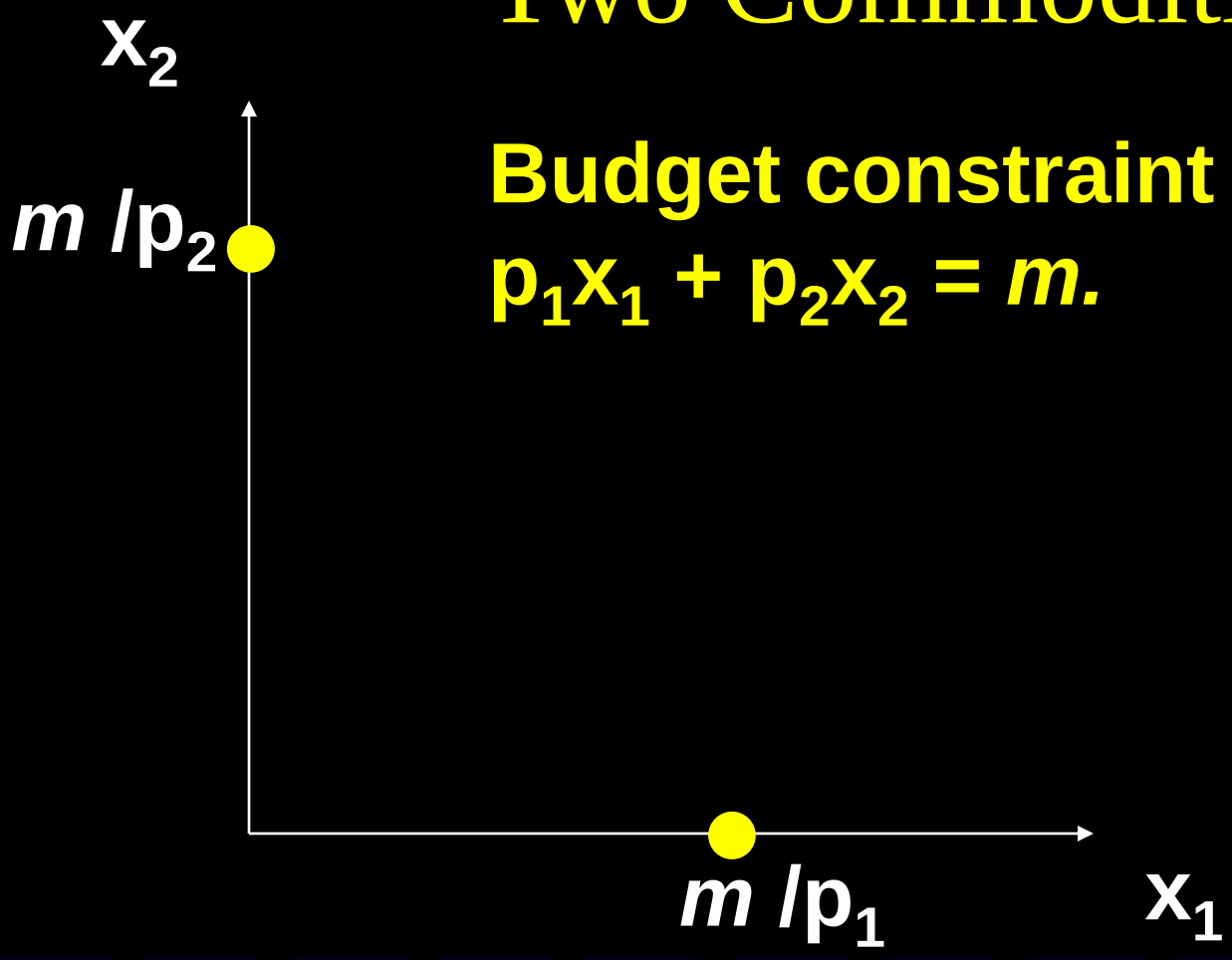
The bundles that are only just affordable form the consumer's budget constraint. This is the set

$$\{ (x_1,\ldots,x_n) \mid x_1 \geq 0, \ldots, x_n \geq 0 \text{ and } p_1x_1 + \ldots + p_nx_n = m \}.$$

The budget constraint is the upper boundary of the budget set.

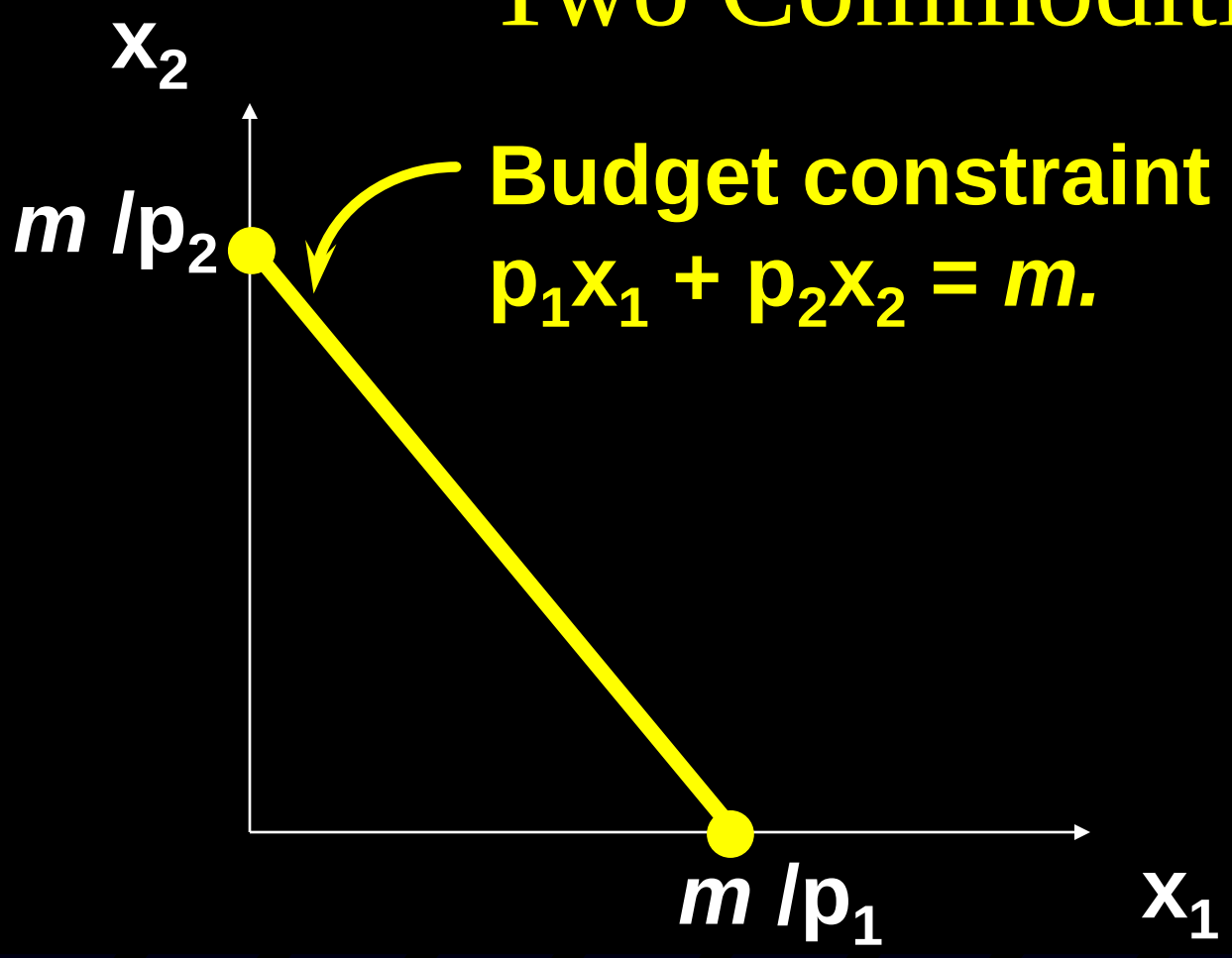
预算约束线是消费者恰好可负担的所有商品组合的集合。

Budget Set and Constraint for Two Commodities

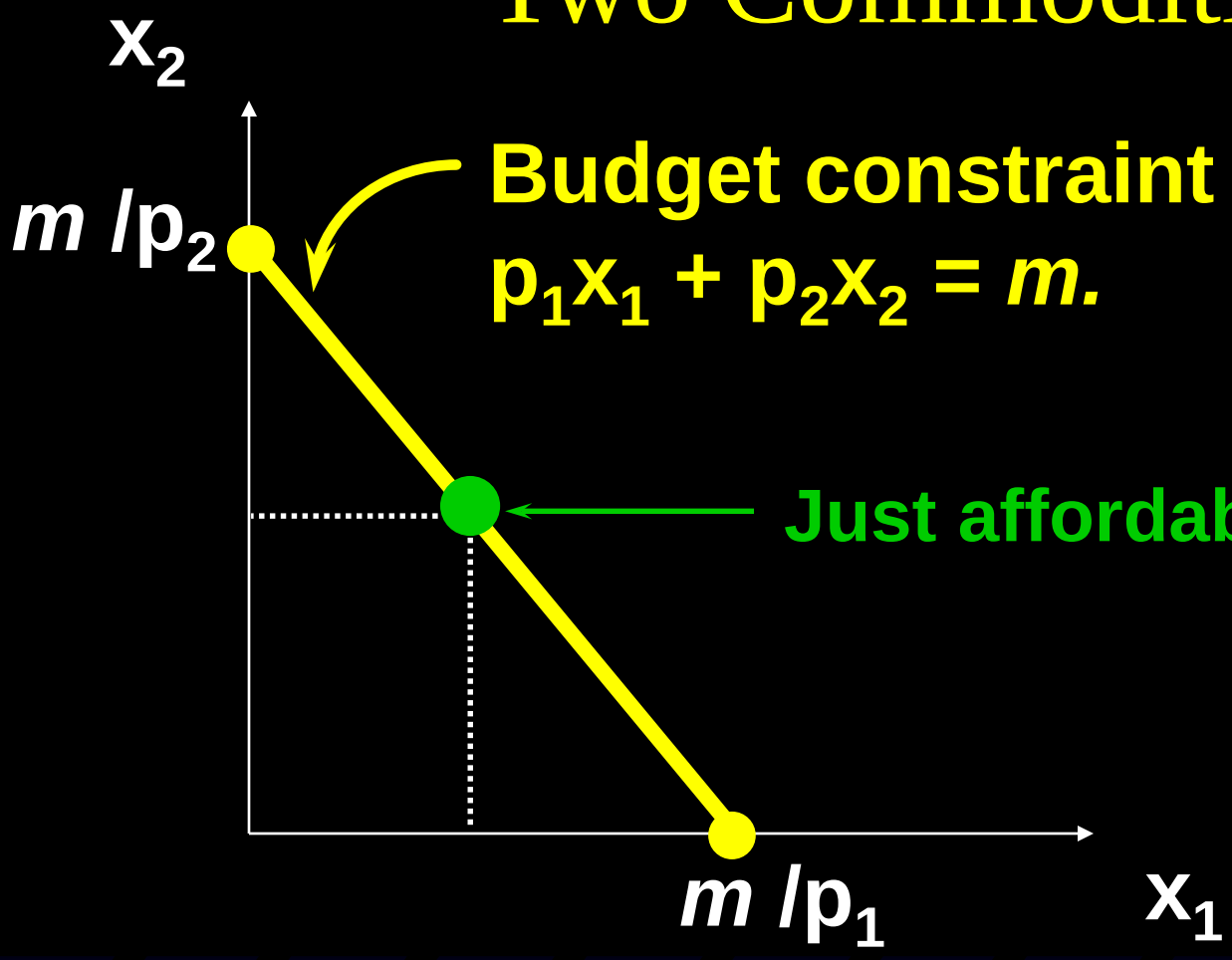


Budget constraint is
$p_1x_1 + p_2x_2 = m$.

Budget Set and Constraint for Two Commodities



Budget Set and Constraint for Two Commodities



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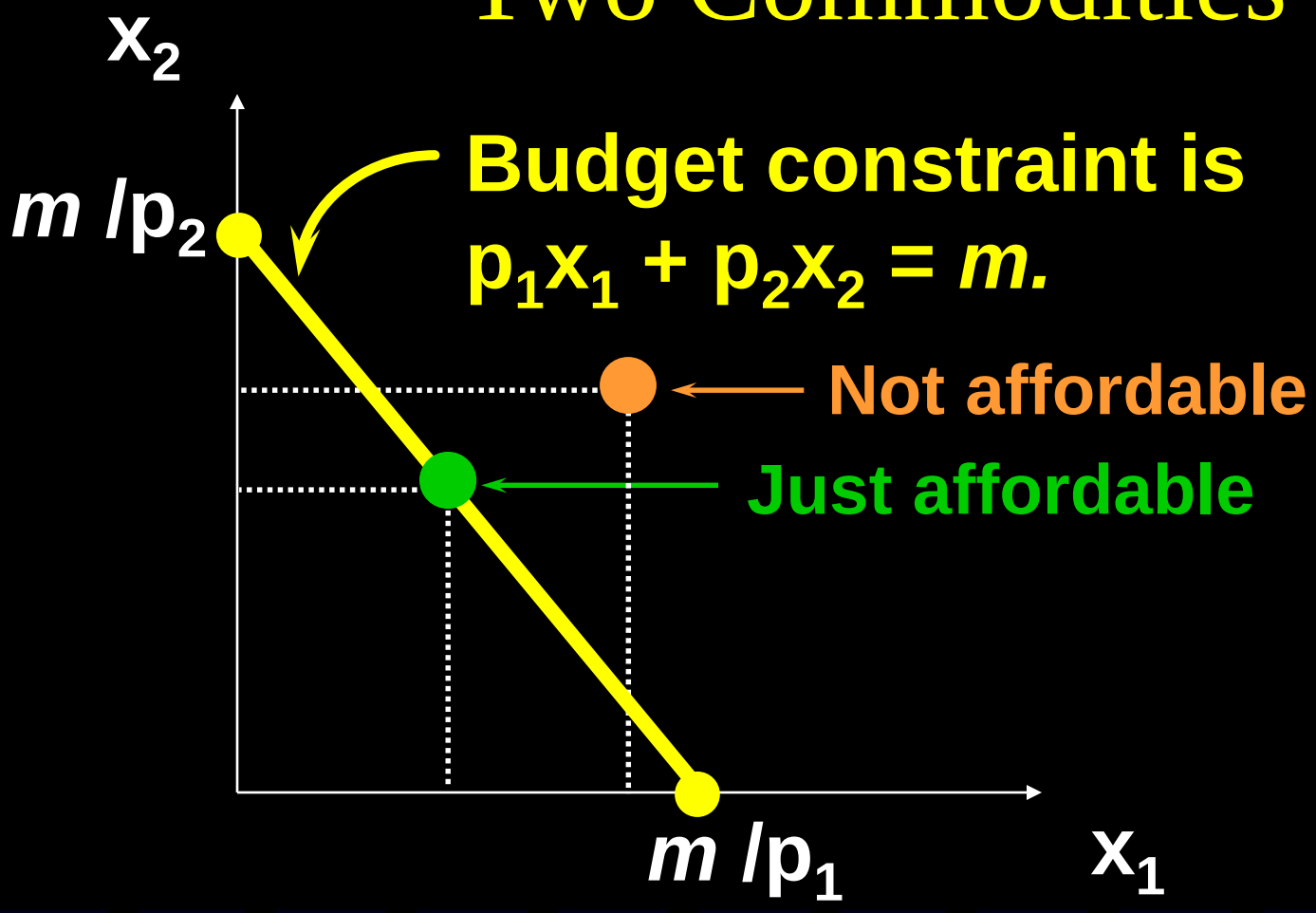

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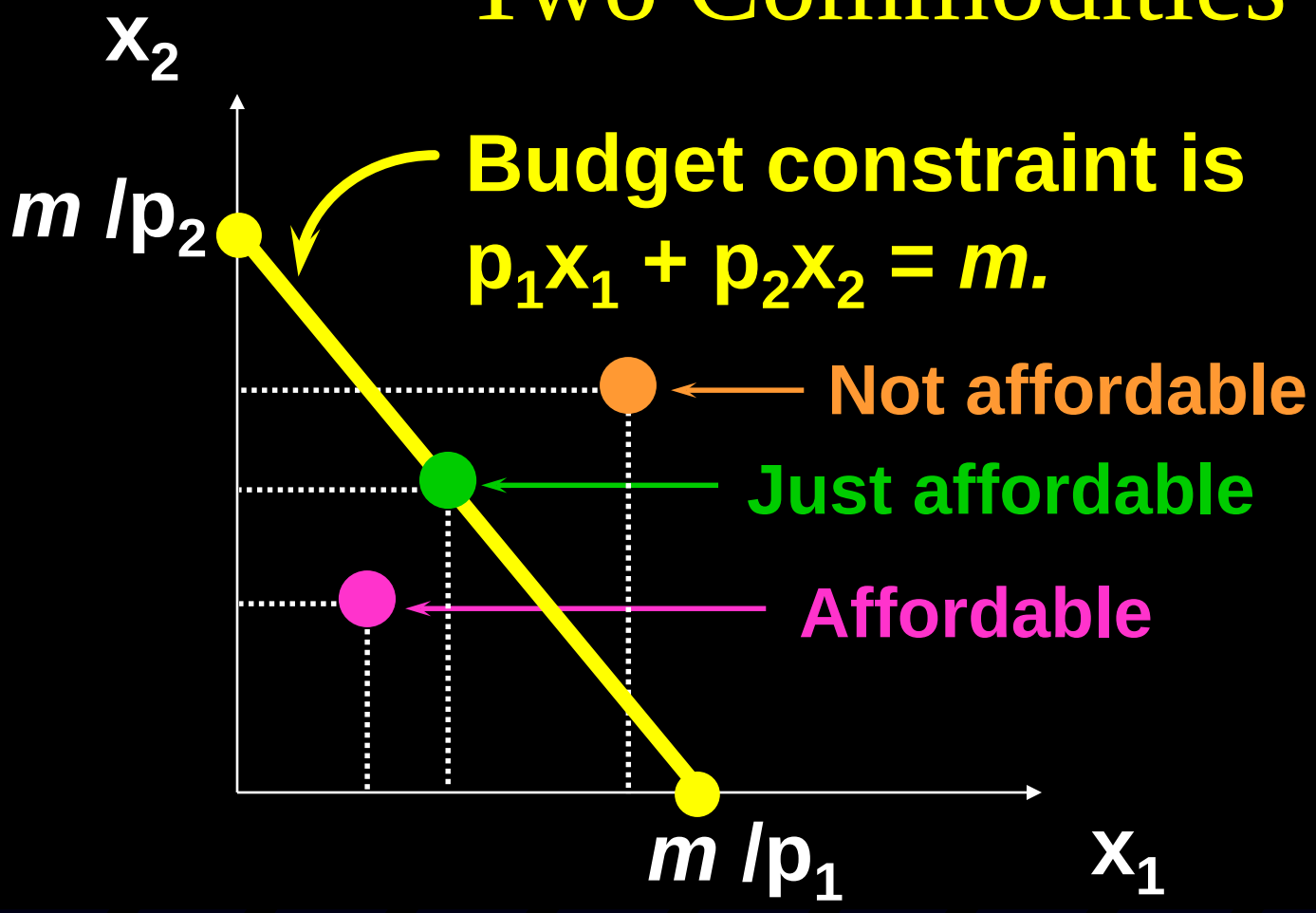

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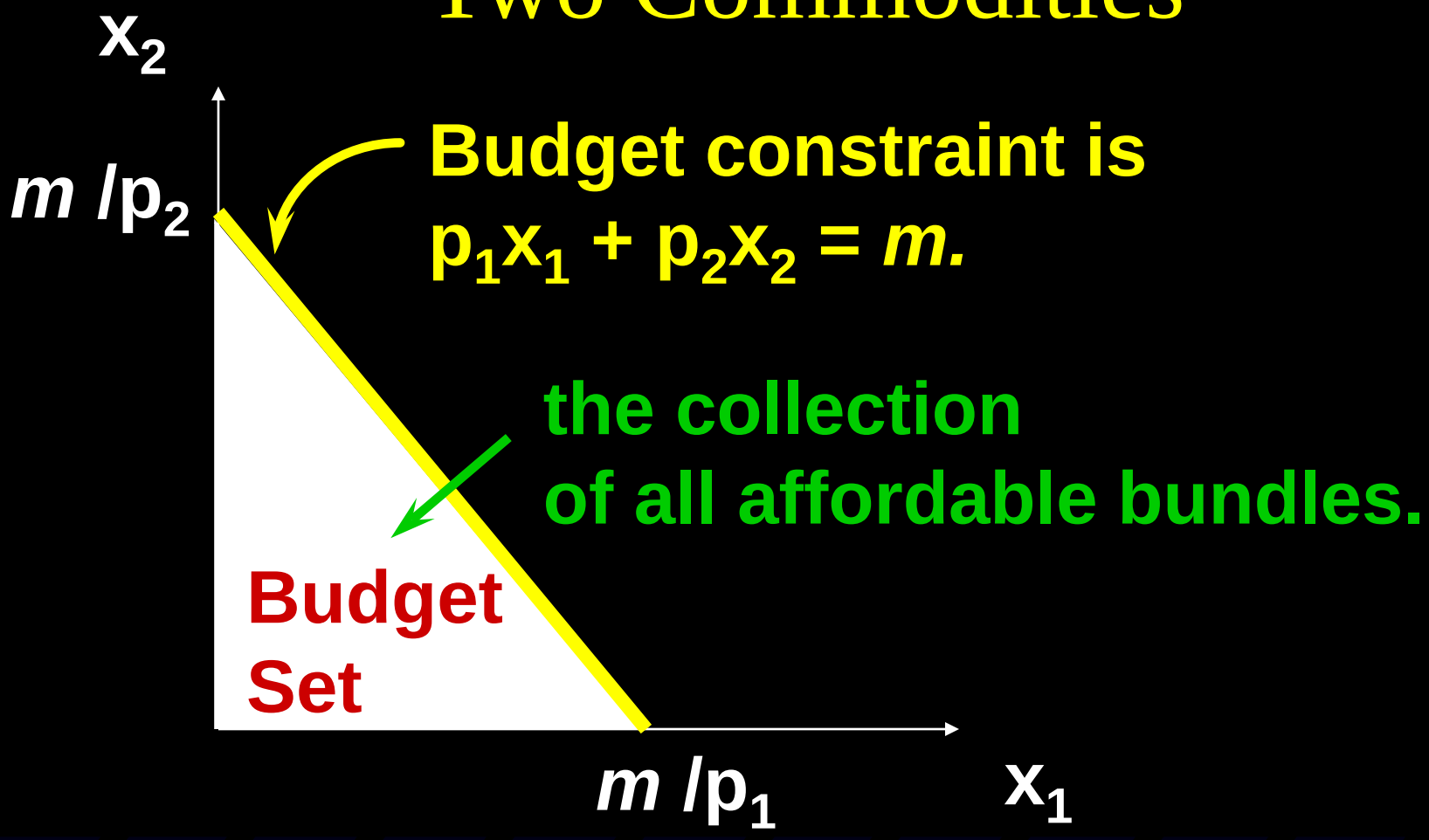
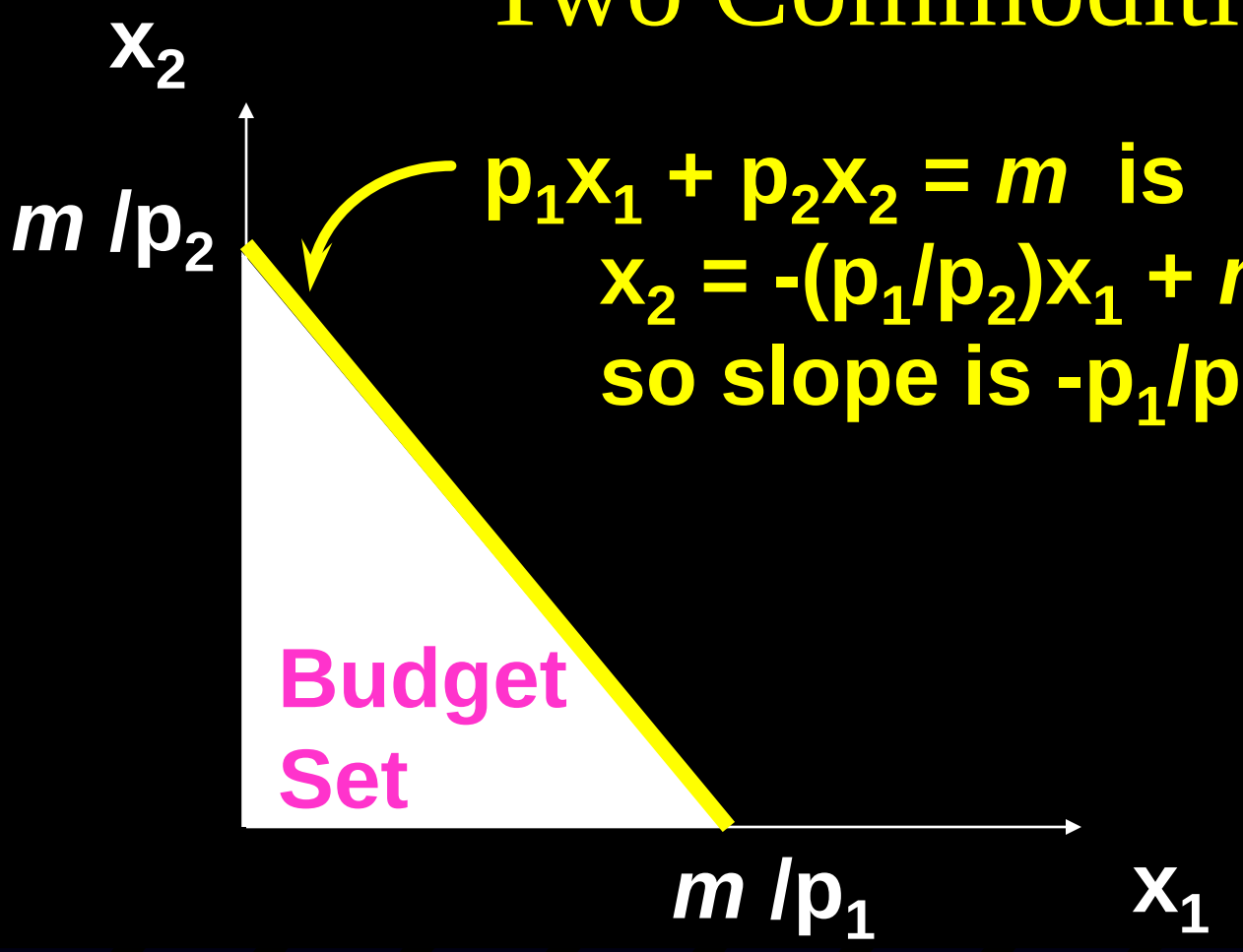


Budget Set and Constraint for Two Commodities



Budget Constraints

For n = 2 and x_1 on the horizontal axis, the constraint's slope is $-p_1/p_2$. What does it mean?

$$x_2 = -\frac{p_1}{p_2} x_1 + \frac{m}{p_2}$$

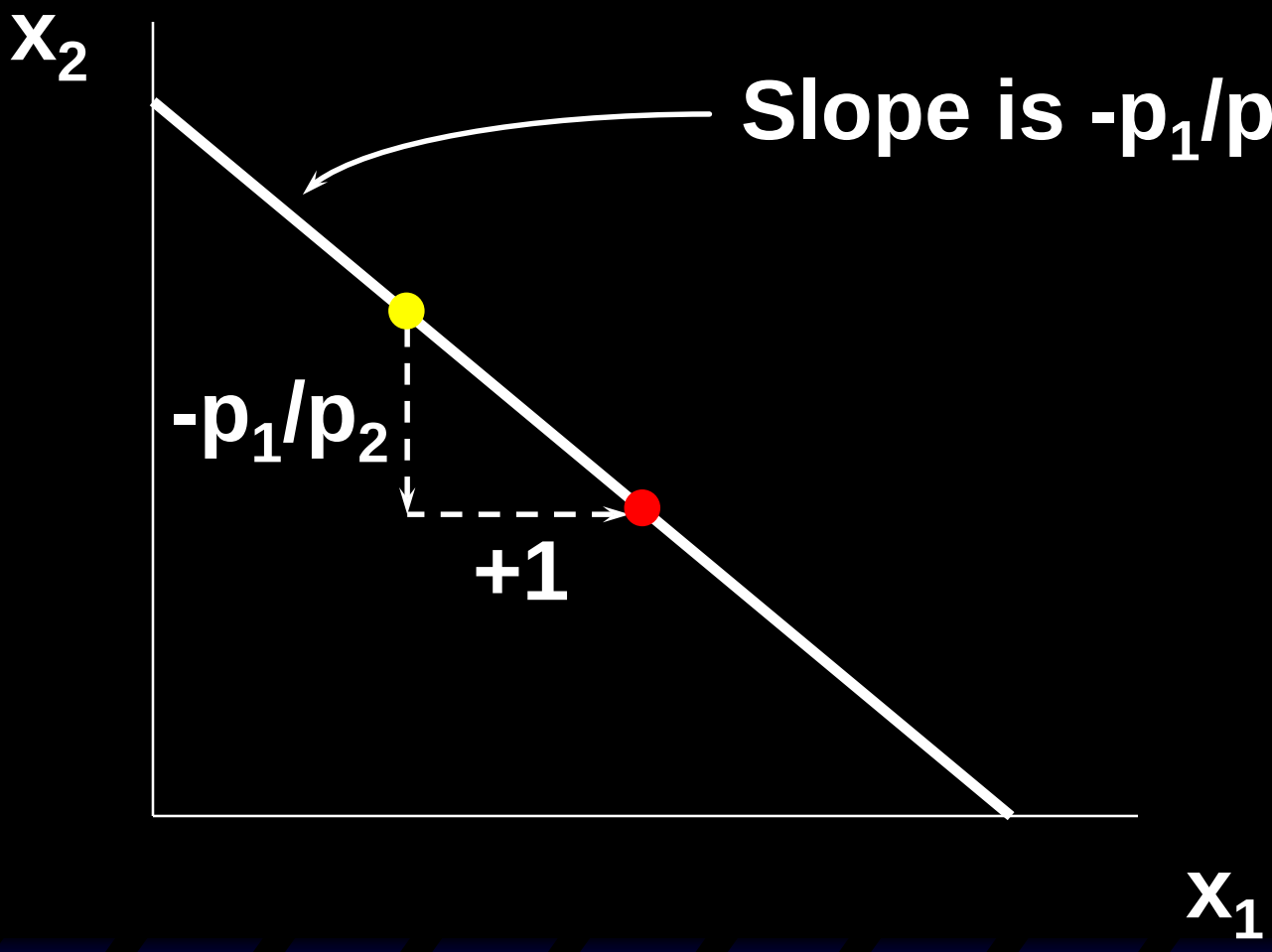
Budget Constraints

For n = 2 and x_1 on the horizontal axis, the constraint's slope is $-p_1/p_2$. What does it mean?

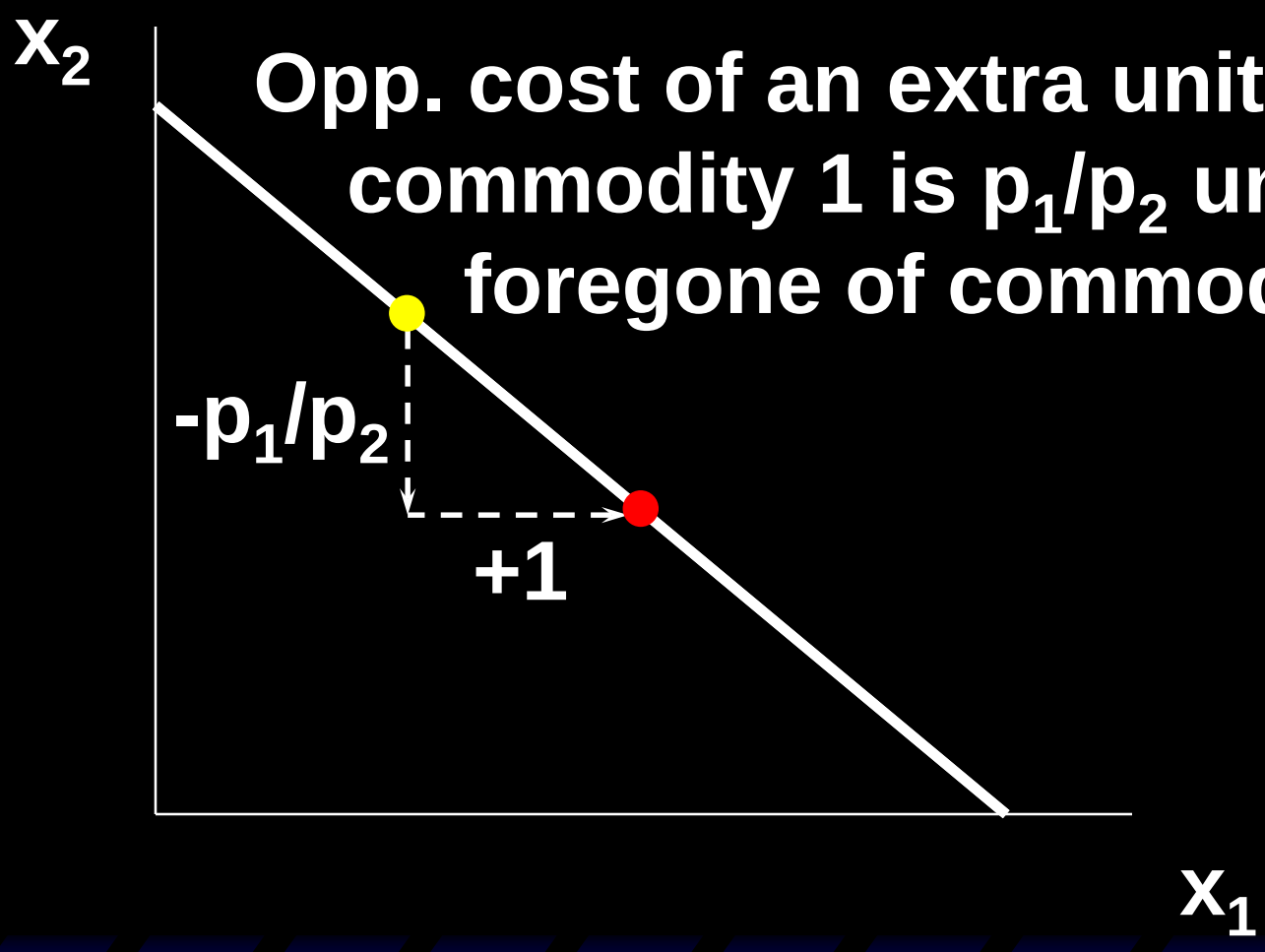
$$x_2 = -\frac{p_1}{p_2} x_1 + \frac{m}{p_2}$$

Increasing x_1 by 1 must reduce x_2 by p_1/p_2.

Budget Constraints



Budget Constraints



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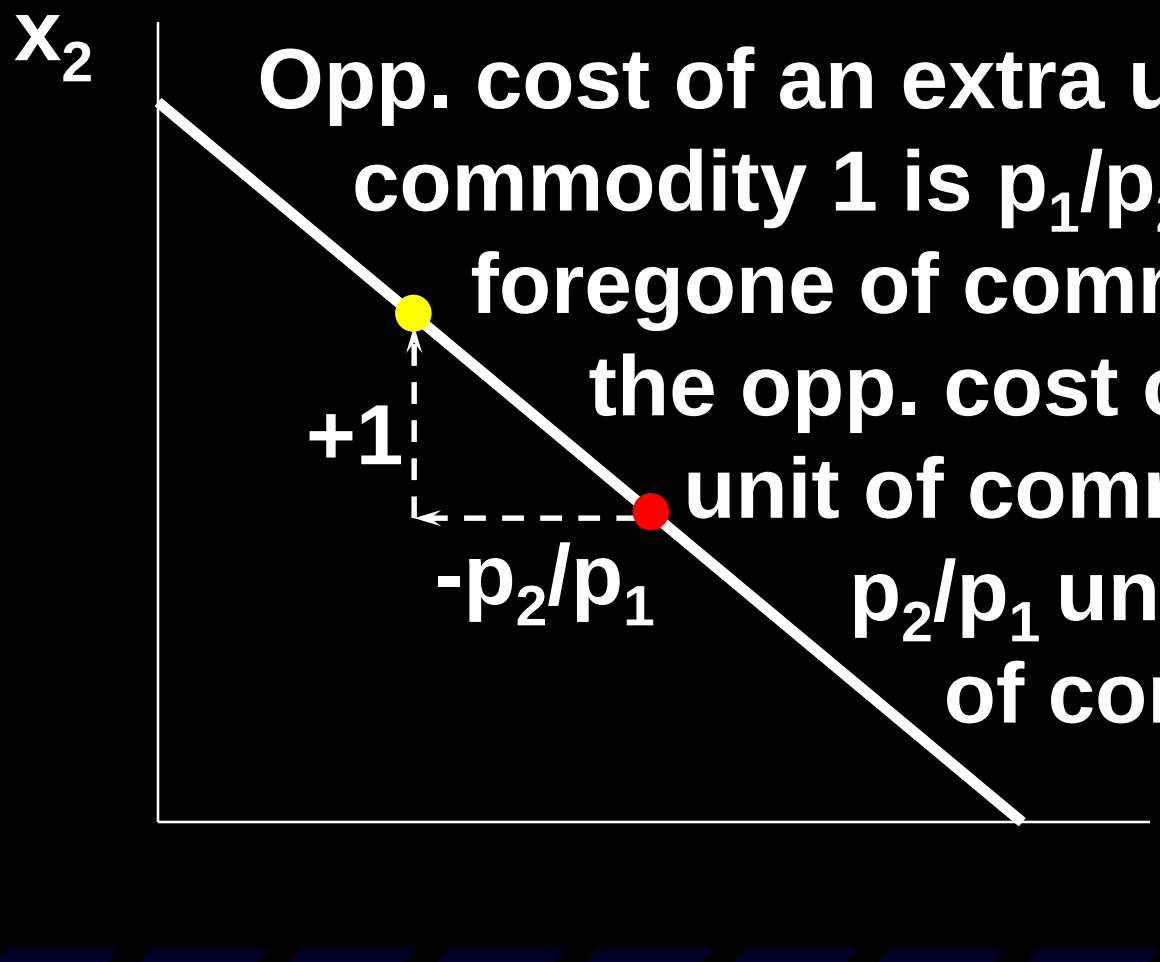


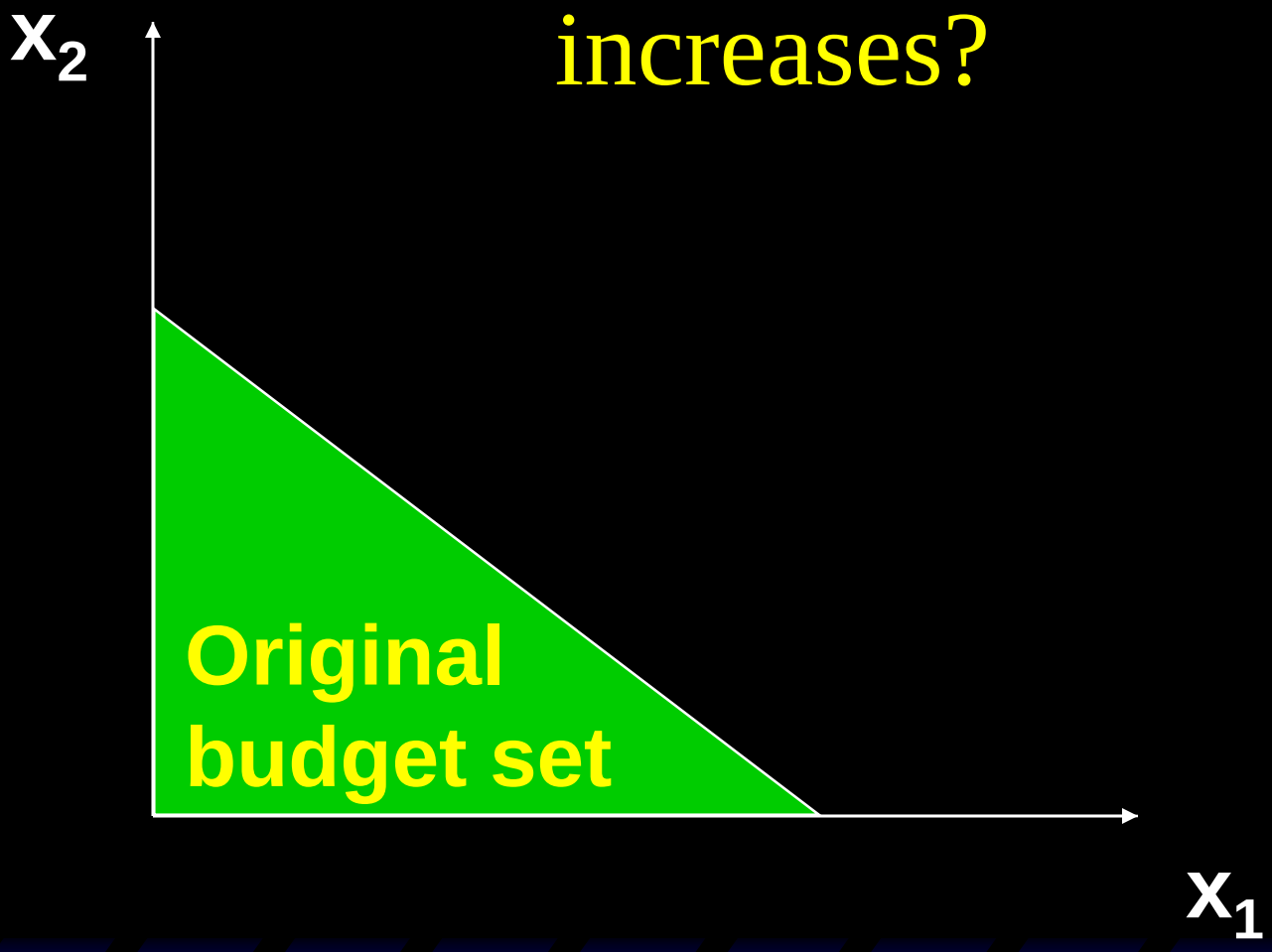
Opp. cost of an extra unit of commodity 1 is p_1/p_2 units foregone of commodity 2. And the opp. cost of an extra unit of commodity 2 is p_2/p_1 units foregone of commodity 1.

Budget Sets & Constraints; Income and Price Changes

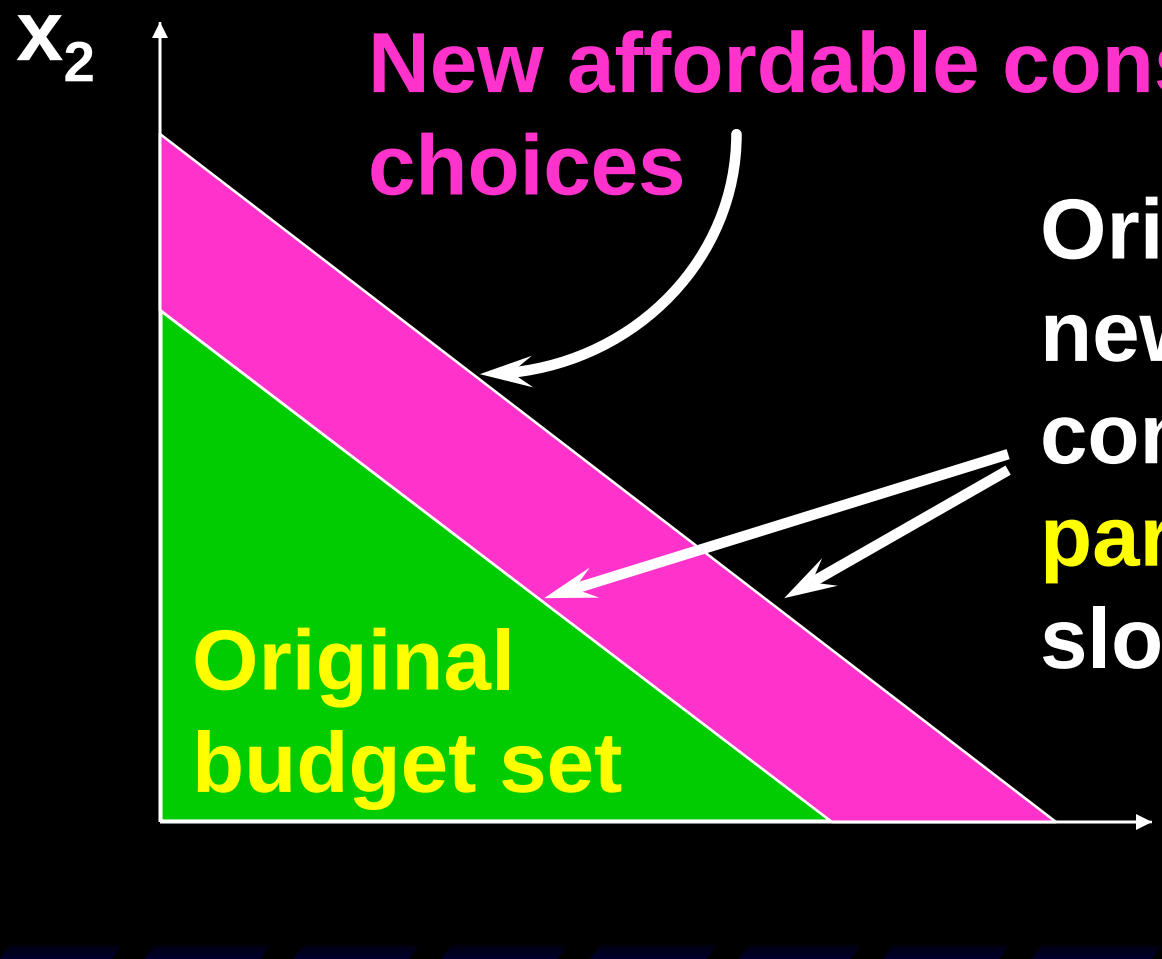
The budget constraint and budget set depend upon prices and income. What happens as prices or income change?

$$x_2 = -\frac{p_1}{p_2} x_1 + \frac{m}{p_2}$$

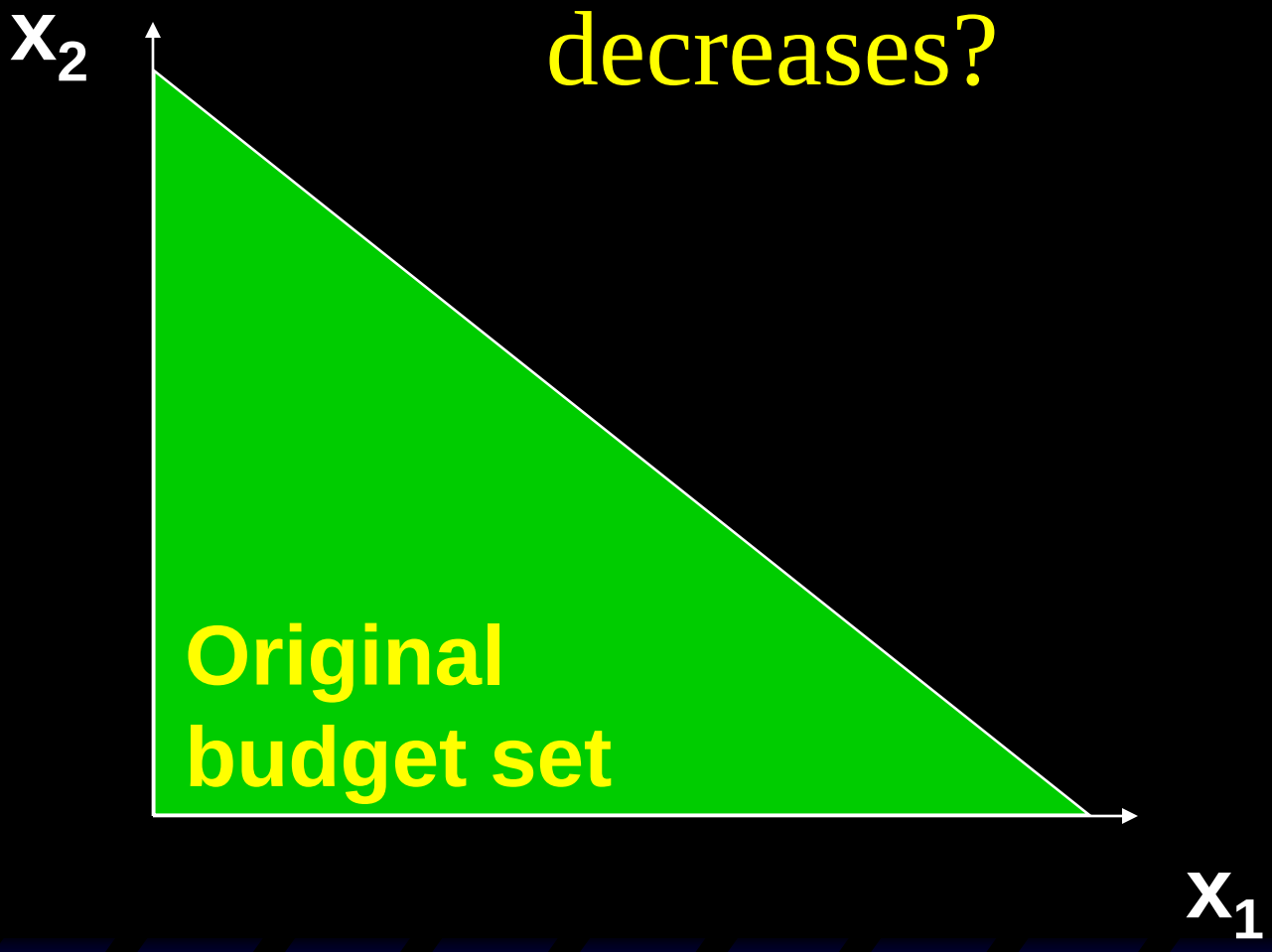
How do the budget set and budget constraint change as income *m* increases?



Higher income gives more choice



How do the budget set and budget constraint change as income *m* decreases?



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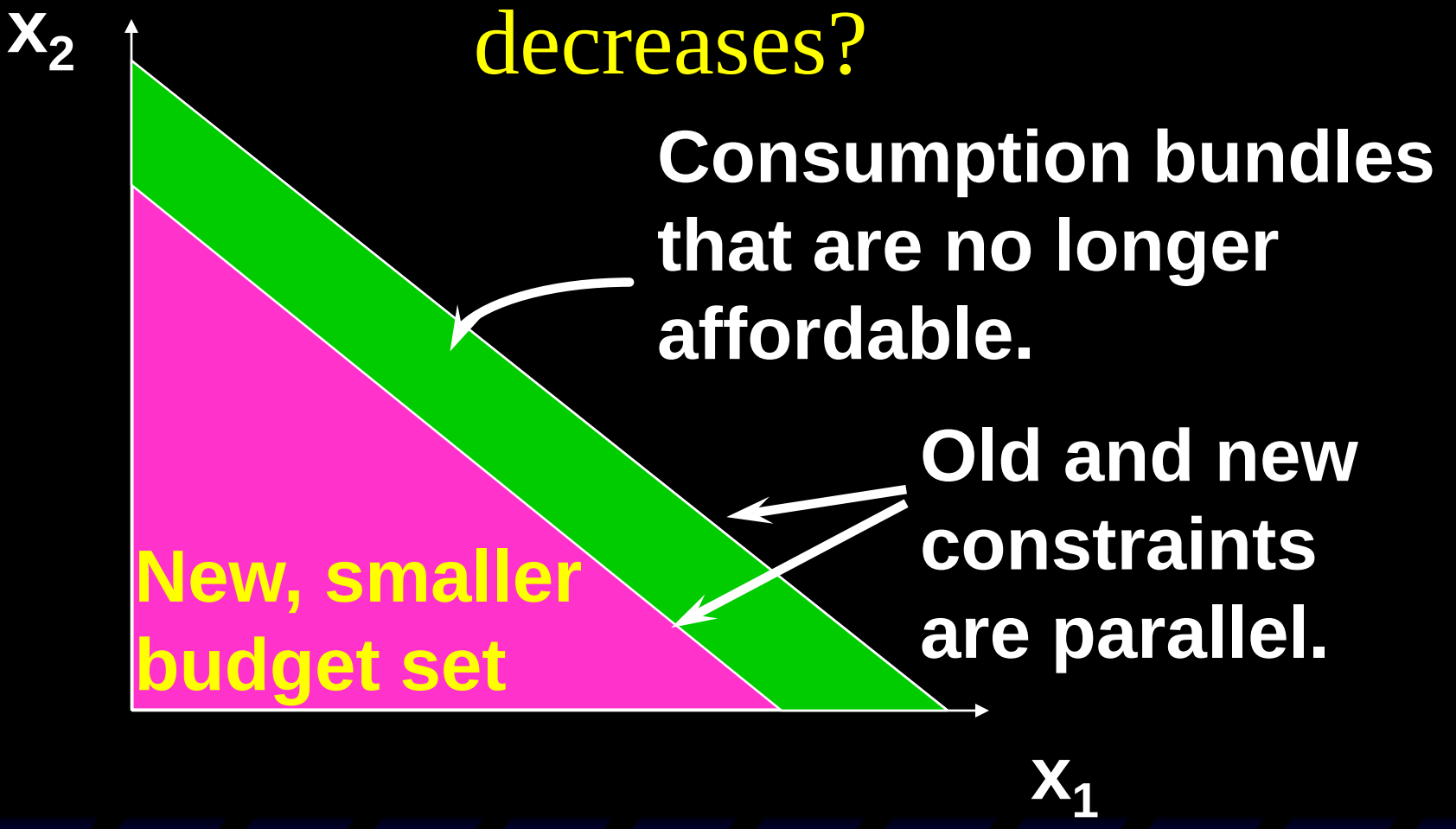


Budget Constraints - Income Changes

Increases in income *m* shift the constraint outward in a parallel manner, thereby enlarging the budget set and improving choice.

收入上升导致预算约束线向外平移，使消费者的预算集变大、选择增加。

Budget Constraints - Income Changes

Decreases in income *m* shift the constraint inward in a parallel manner, thereby shrinking the budget set and reducing choice.

收入下降导致预算约束线向内平移，使消费者的预算集变小、选择减少。

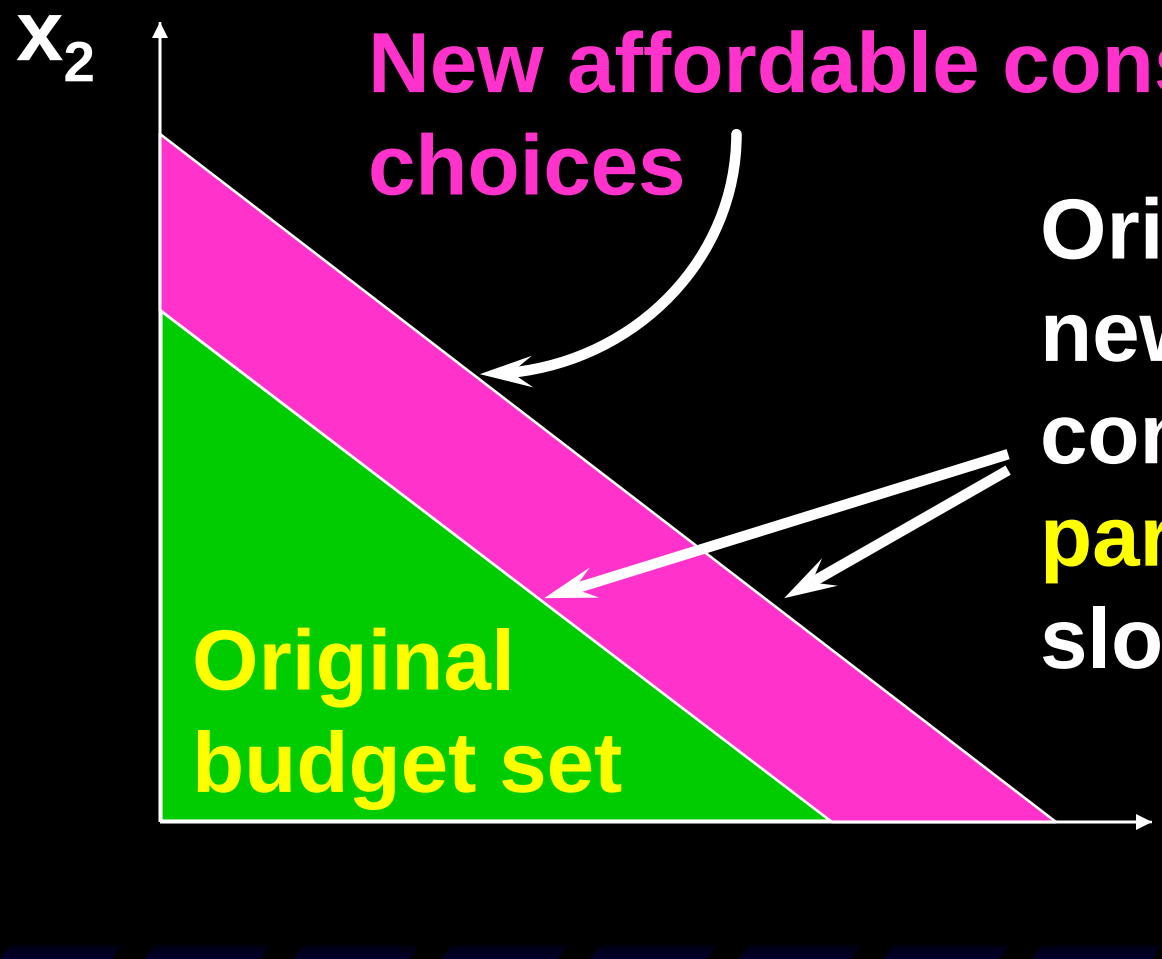
Budget Constraints - Income Changes

No original choice is lost and new choices are added when income increases, so higher income cannot make a consumer worse off.

收入上升给消费者带来了新选择、且没有减少消费者的初始选择，因此一定不会使消费者的福利下降。

An income decrease may (typically will) make the consumer worse off.

Higher income gives more choice



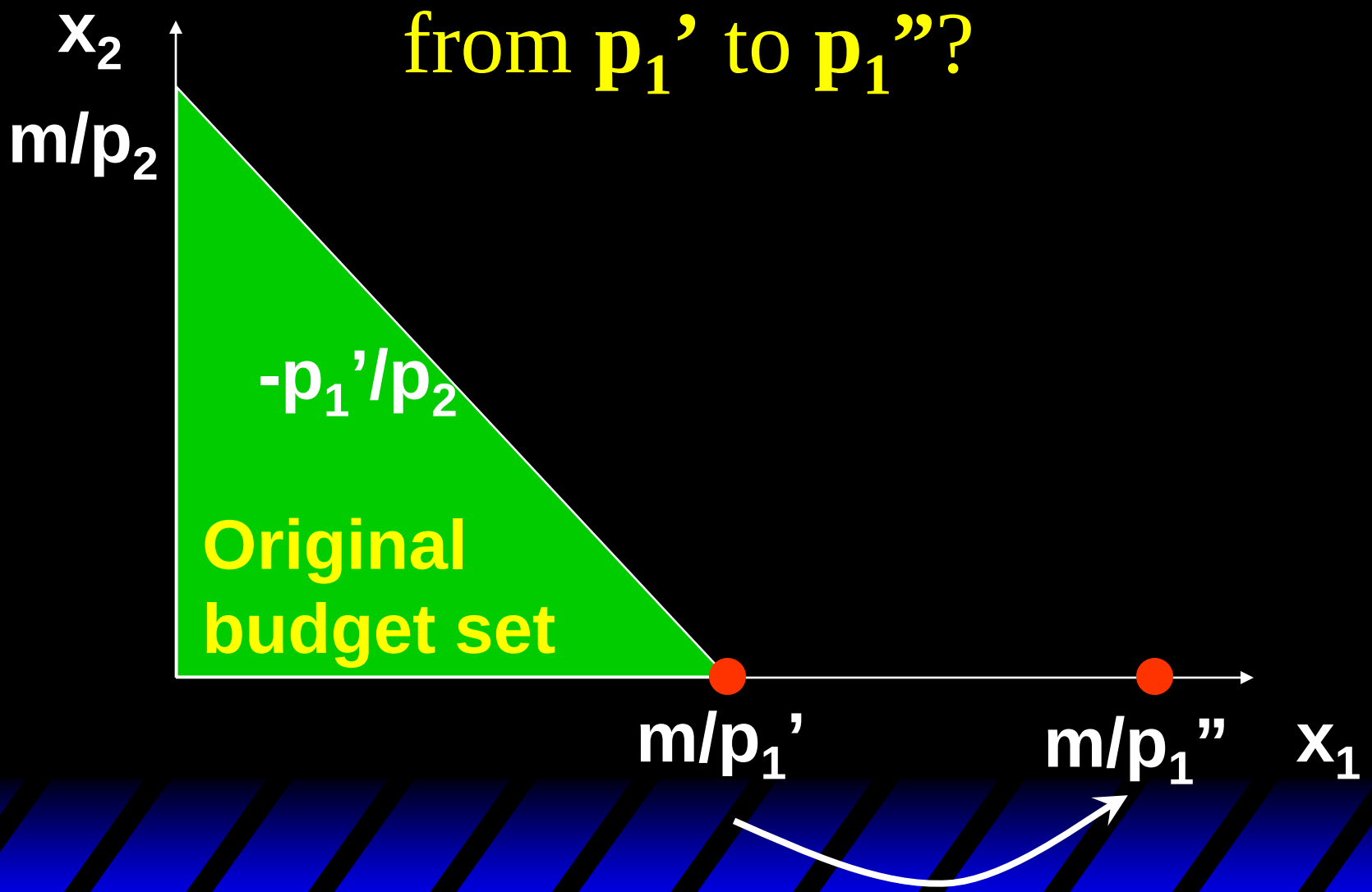
Budget Constraints - Price Changes

What happens if just one price decreases?

Suppose p_1 decreases.

$$x_2 = -\frac{p_1}{p_2} x_1 + \frac{m}{p_2}$$

How do the budget set and budget constraint change as p_1 decreases from p_1' to p_1"?



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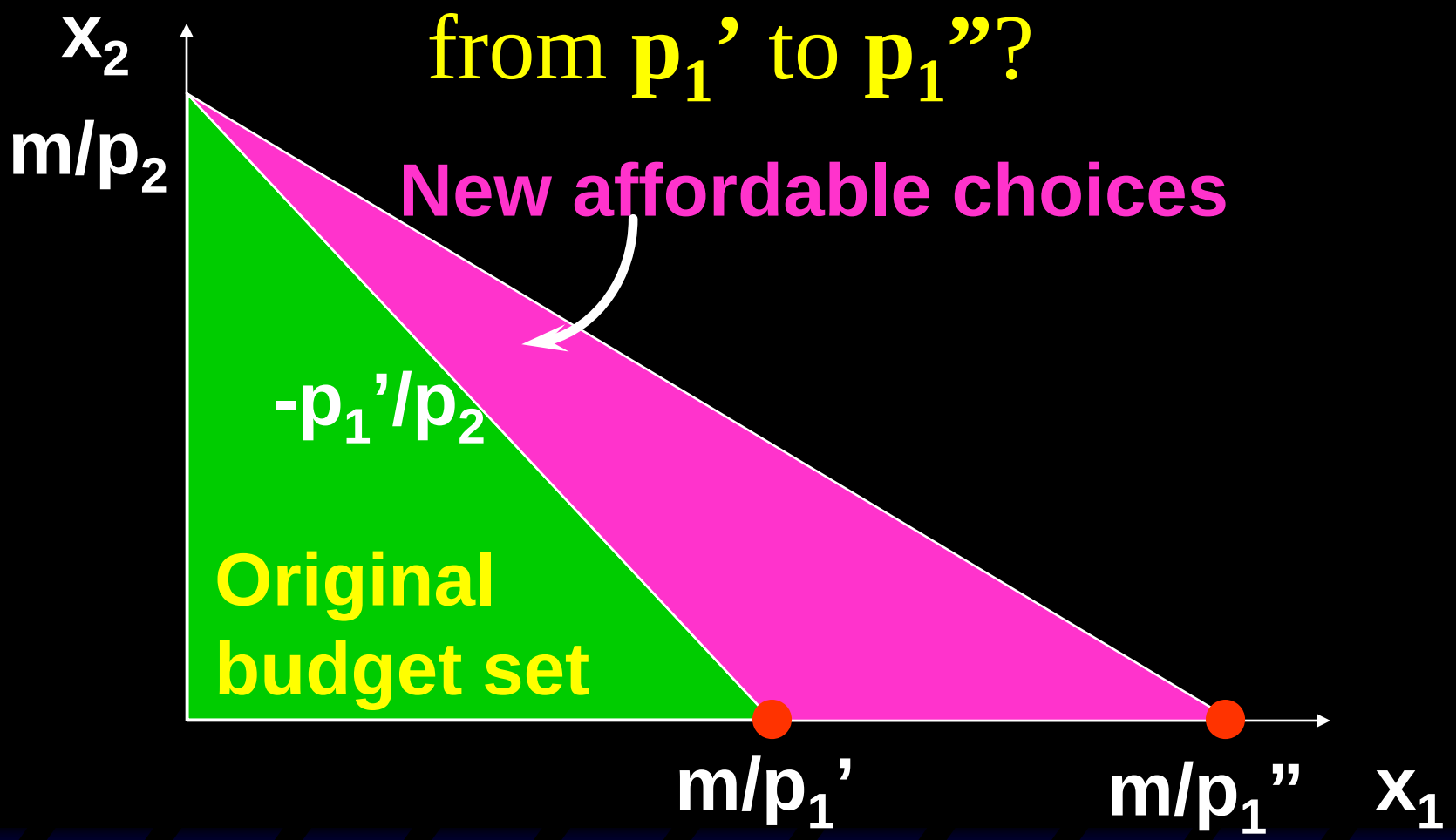

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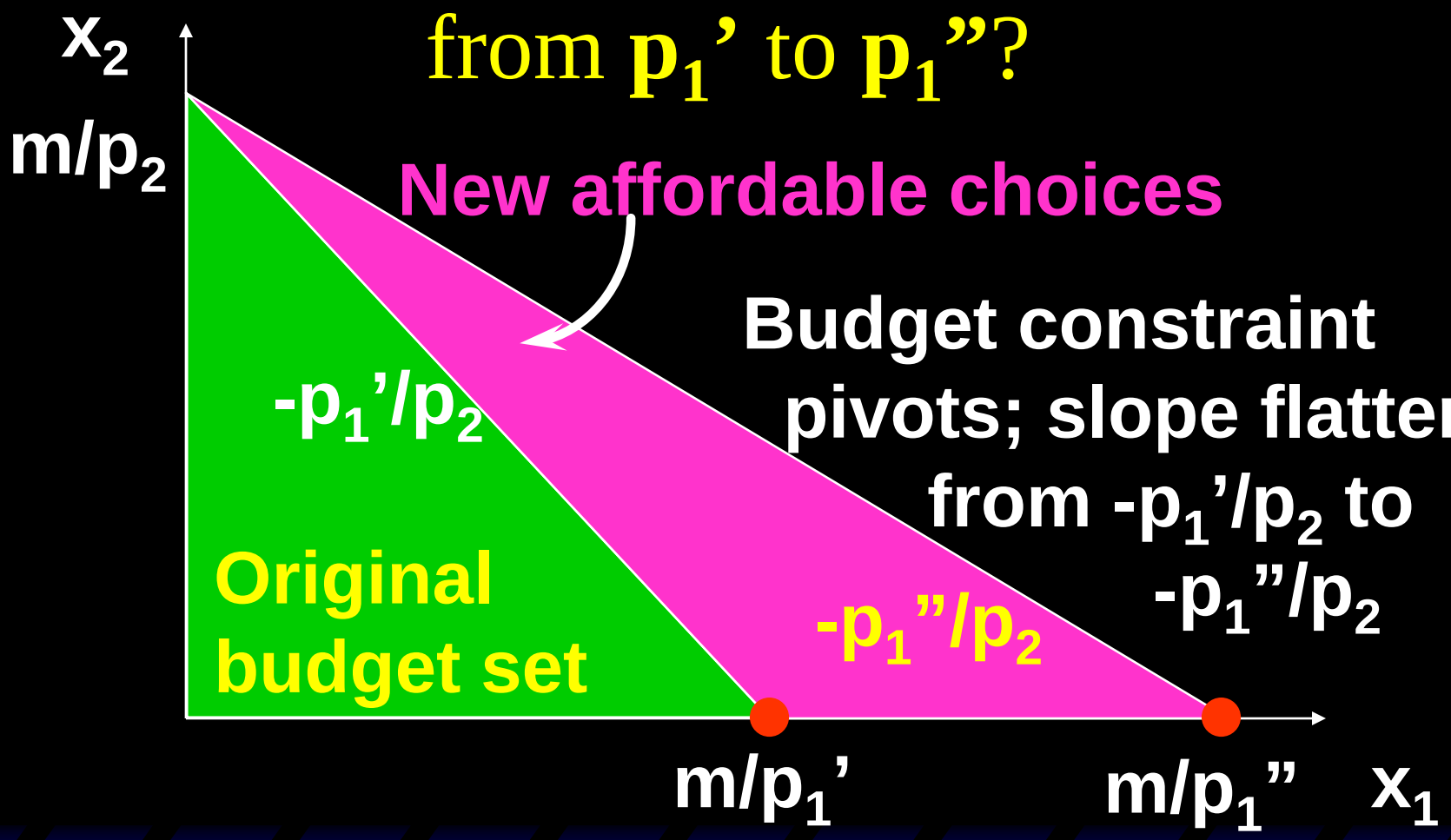


Budget Constraints - Price Changes

Reducing the price of one commodity pivots the constraint outward. No old choice is lost and new choices are added, so reducing one price cannot make the consumer worse off.

一种商品价格的下降使预算约束线向外旋转。

Budget Constraints - Price Changes

Similarly, increasing one price pivots the constraint inwards, reduces choice and may (typically will) make the consumer worse off.

一种商品价格的上升使预算约束线向内旋转。

Uniform *Ad Valorem* Sales Taxes

An *ad valorem* sales tax (从价税) levied at a rate of 5% increases all prices by 5%, from p to (1+0·05)p = 1·05p.

An *ad valorem* sales tax levied at a rate of *t* increases all prices by *t*p from p to (1+*t*)p.

A uniform sales tax is applied uniformly to all commodities.

Uniform *Ad Valorem* Sales Taxes

A uniform sales tax levied at rate *t* changes the constraint from

$$p_1x_1 + p_2x_2 = m$$

to

$$(1+t)p_1x_1 + (1+t)p_2x_2 = m$$

Uniform *Ad Valorem* Sales Taxes

A uniform sales tax levied at rate *t* changes the constraint from

$$p_1x_1 + p_2x_2 = m$$

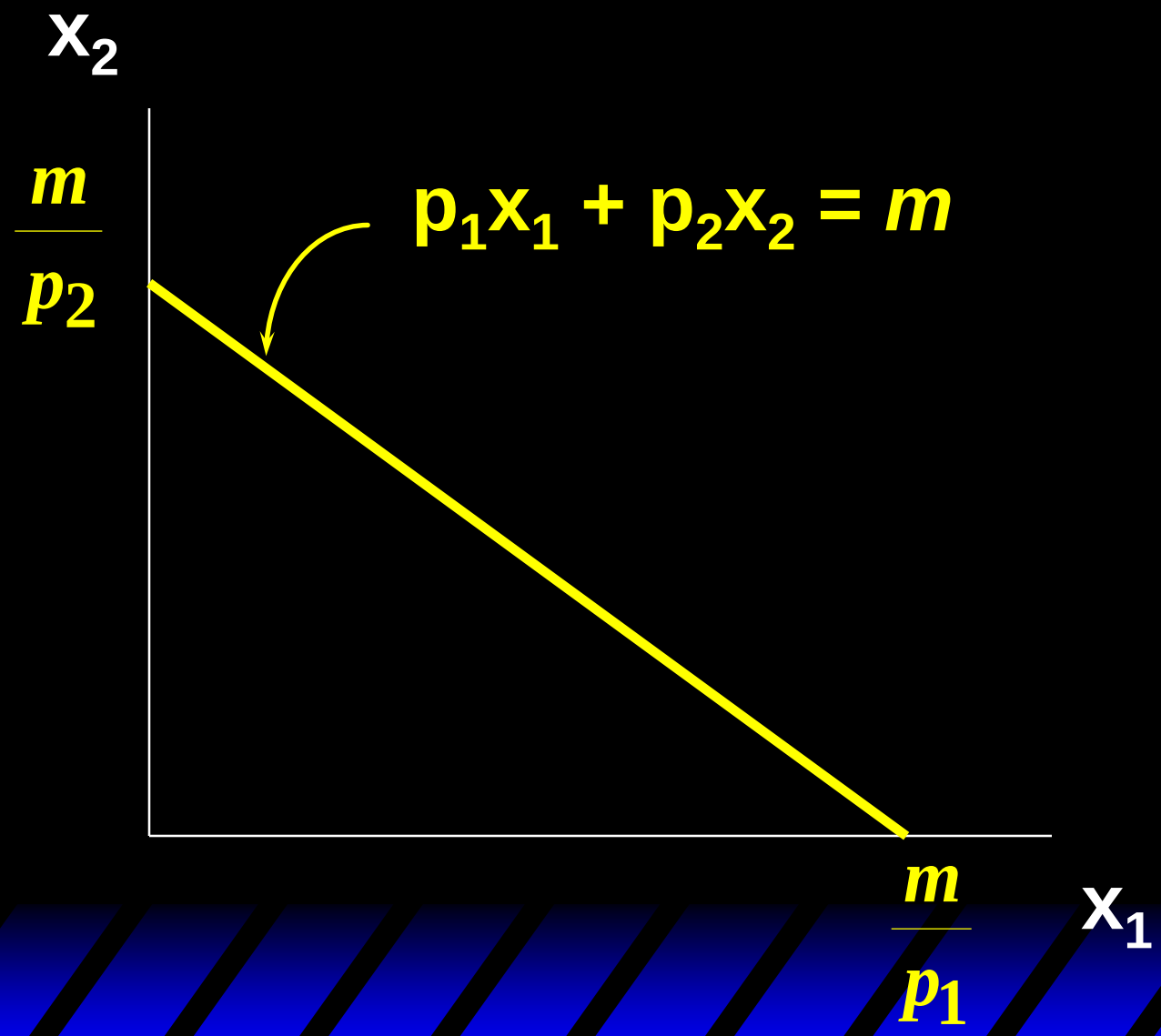
to

$$(1+t)p_1x_1 + (1+t)p_2x_2 = m$$

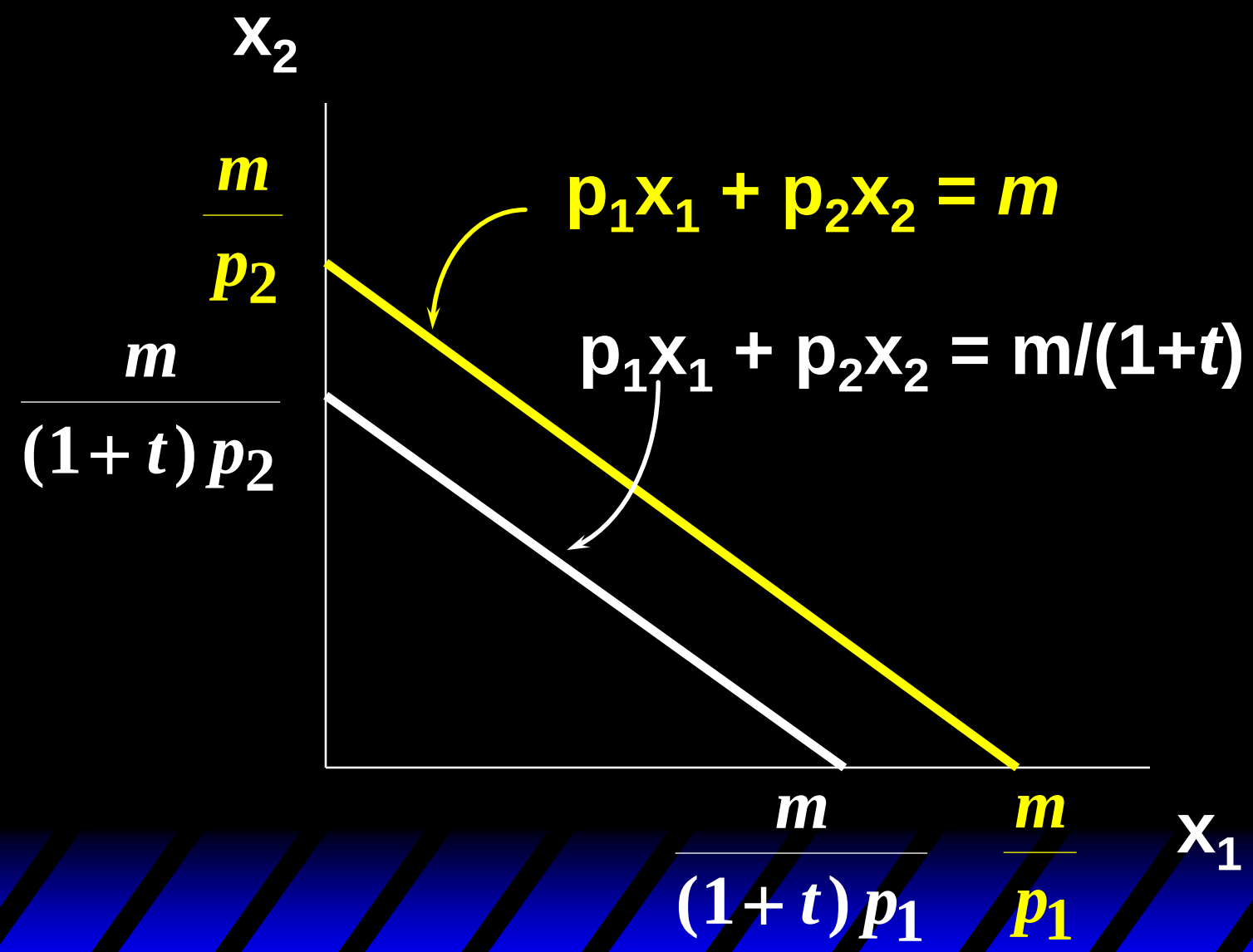
i.e.

$$p_1x_1 + p_2x_2 = m/(1+t).$$

Uniform *Ad Valorem* Sales Taxes



Uniform *Ad Valorem* Sales Taxes



Equivalent income tax

A uniform sales tax levied at rate *t* changes the constraint to

$$(1+t)p_1x_1 + (1+t)p_2x_2 = m$$

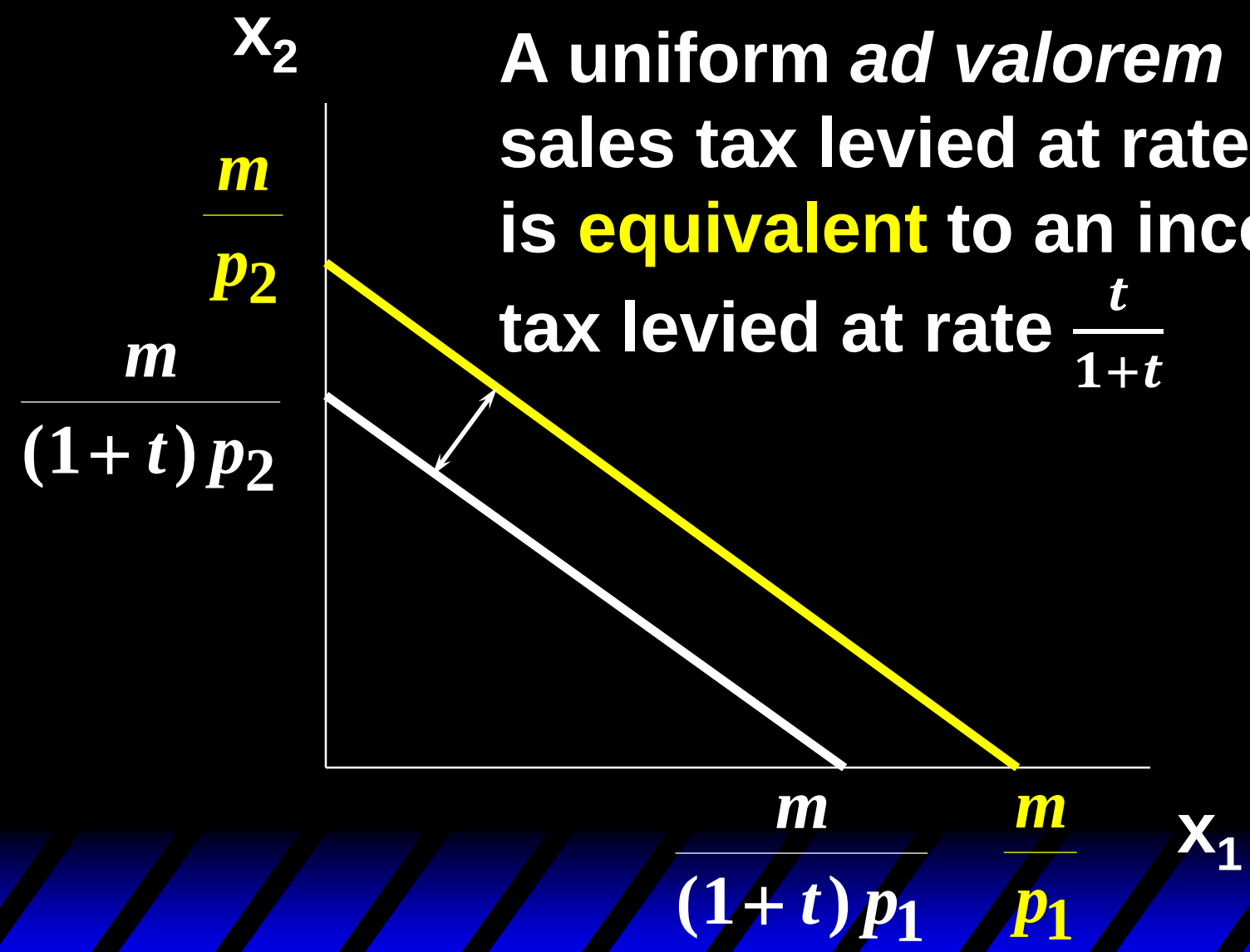
i.e.

$$p_1x_1 + p_2x_2 = m/(1+t).$$

An income tax levied at rate t/1+t changes the constraint to

$$p_1x_1 + p_2x_2 = m - \frac{t}{1+t}m = \frac{m}{1+t}$$

Equivalent income tax



A uniform *ad valorem* sales tax levied at rate t is equivalent to an income tax levied at rate $\frac{t}{1+t}$

The Food Stamp Program

Food stamps are coupons that can be legally exchanged only for food.

实名食品兑换券

How does a commodity-specific gift such as a food stamp alter a family's budget constraint?

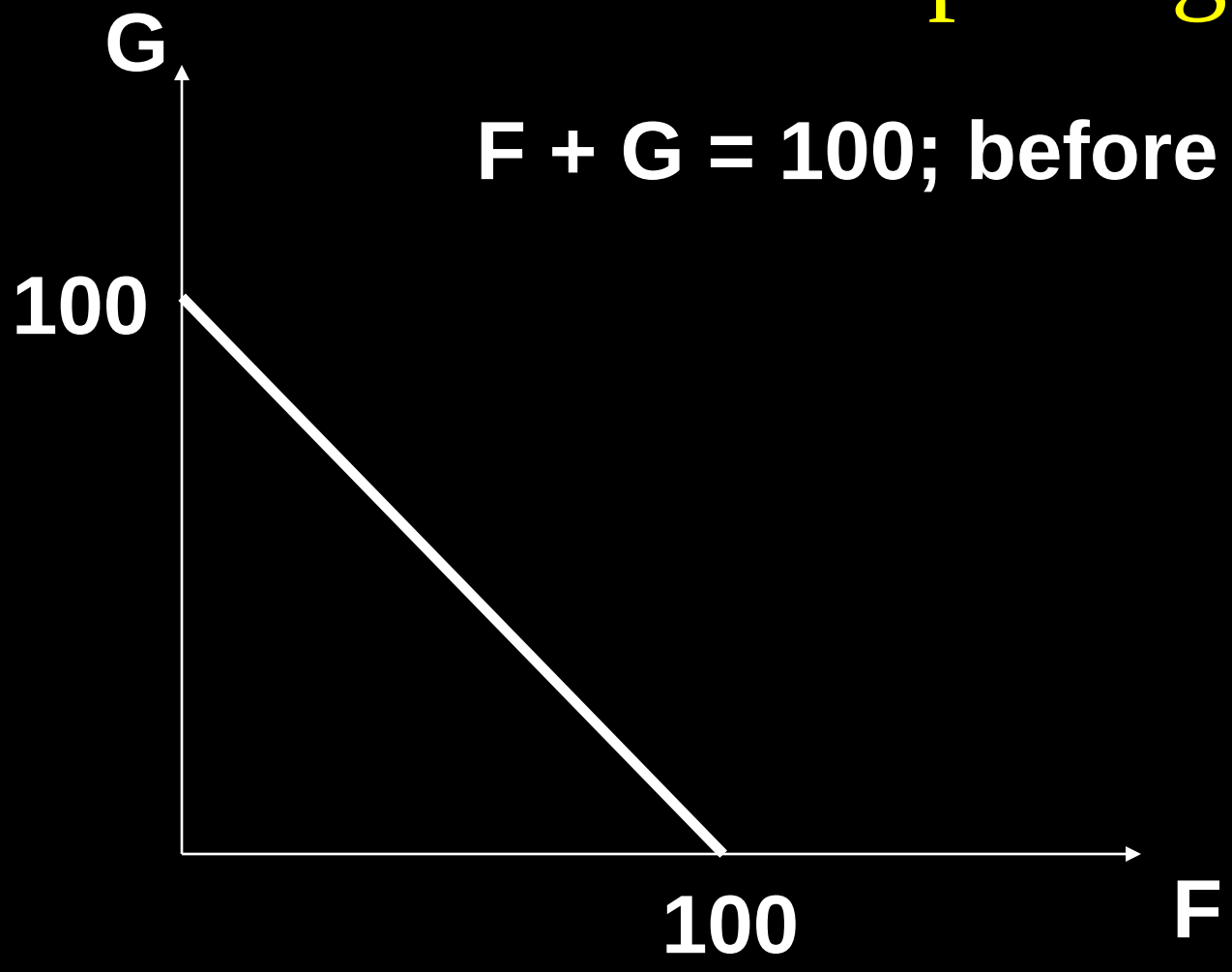
The Food Stamp Program

Suppose m = \$100, p_F = \$1 and the price of "other goods" is p_G = \$1.

The budget constraint is then

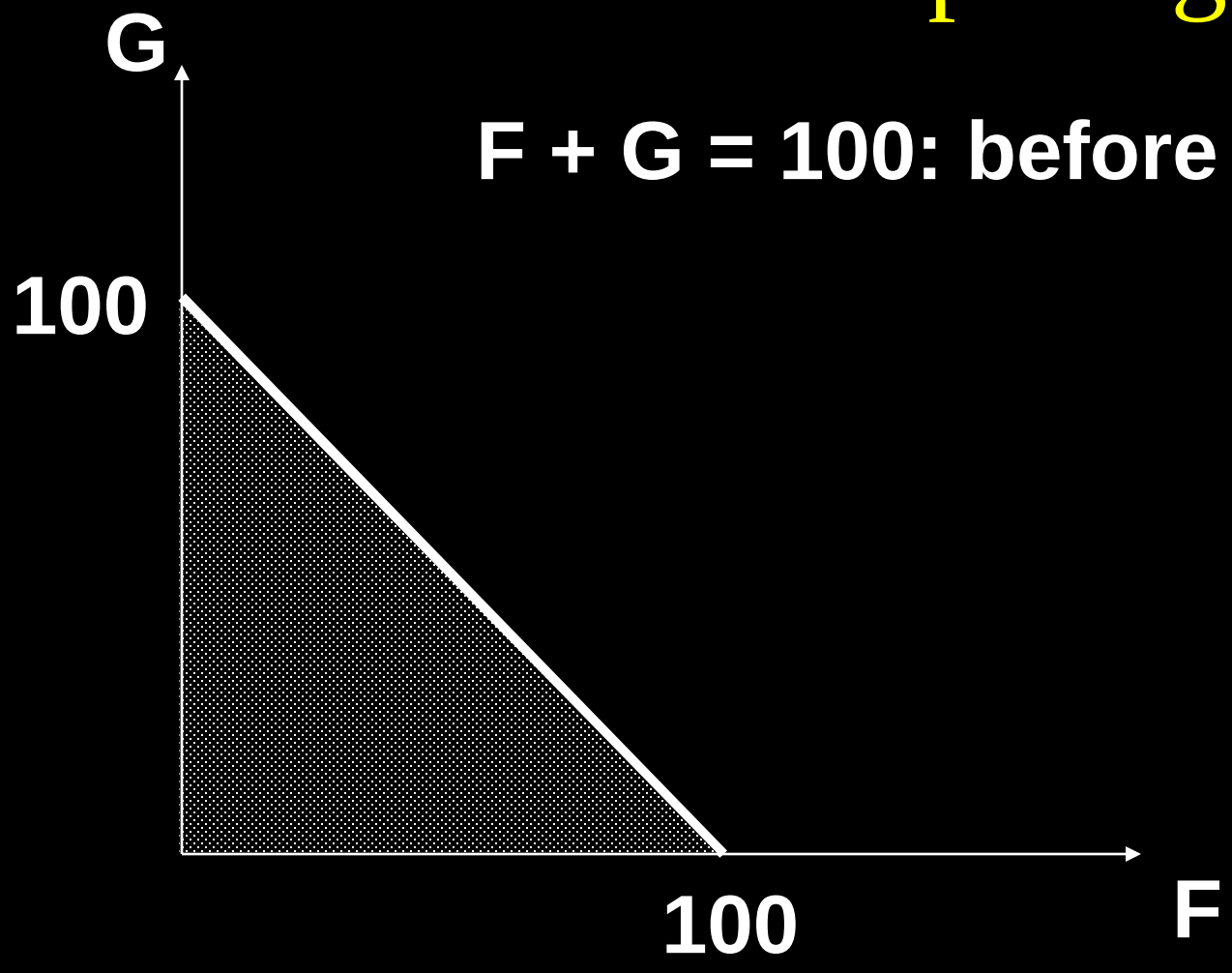
$$F + G = 100.$$

The Food Stamp Program



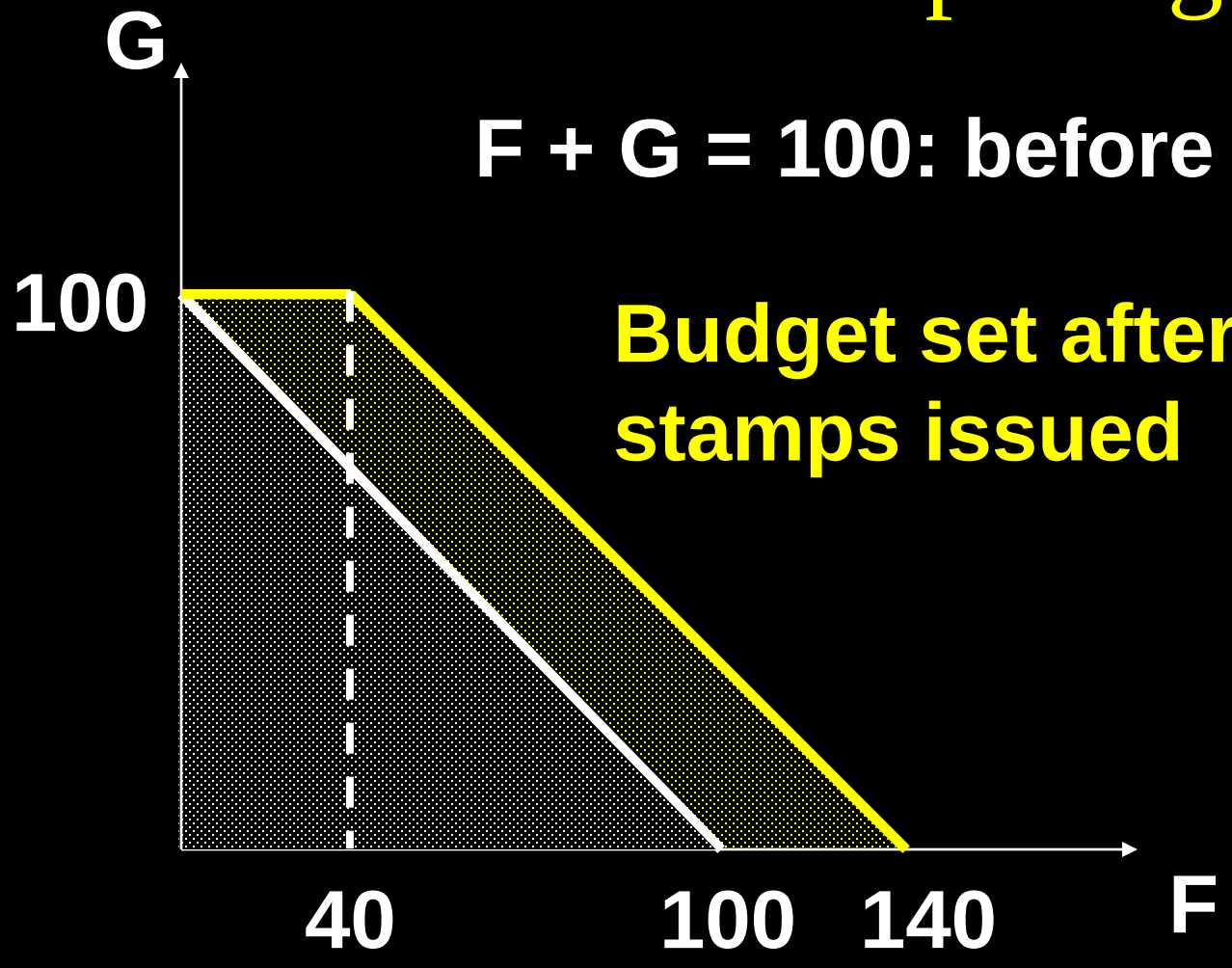
F + G = 100; before stamps.

The Food Stamp Program



F + G = 100: before stamps.

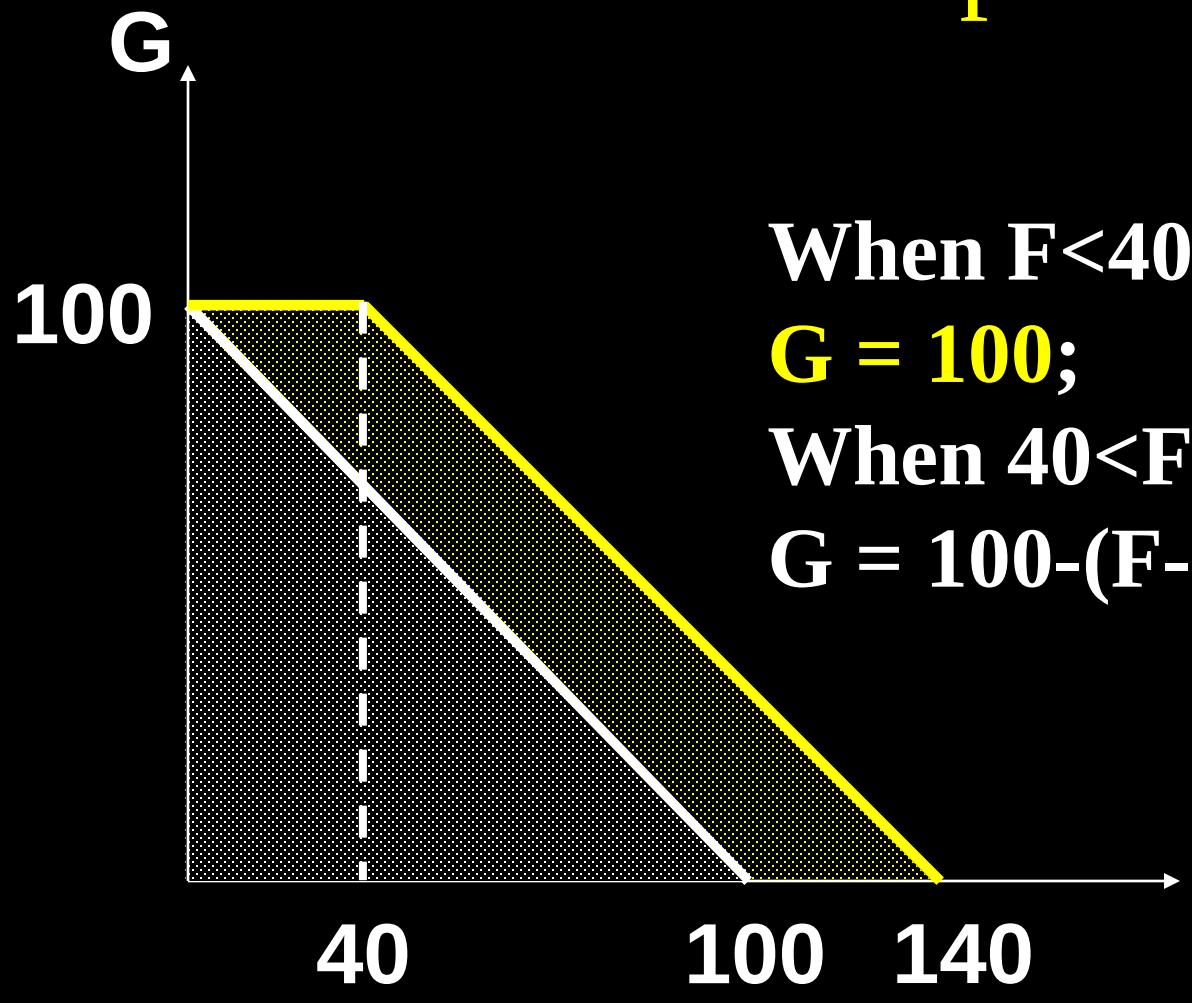
The Food Stamp Program



F + G = 100: before stamps.

Budget set after 40 food stamps issued

The Food Stamp Program



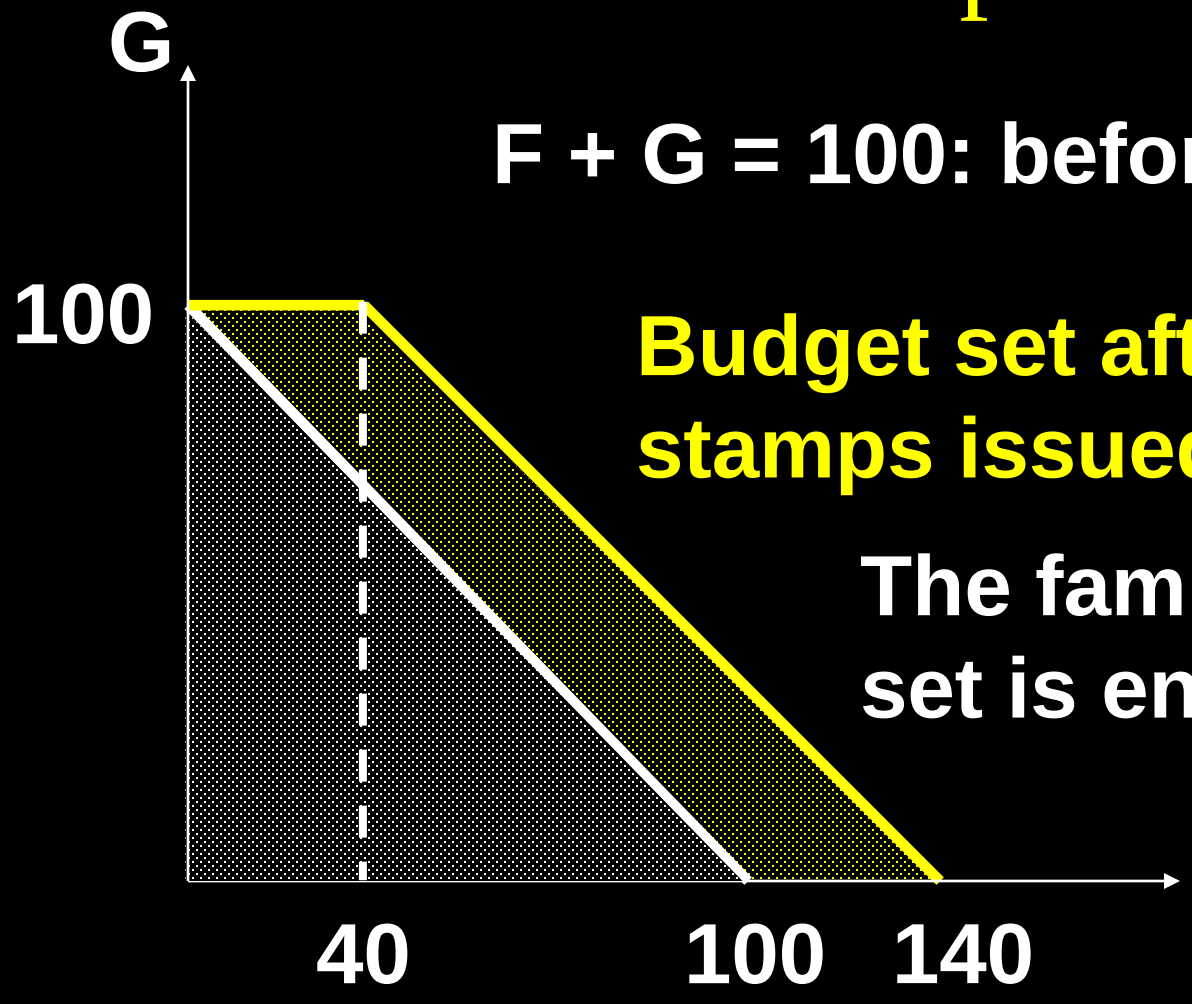
When F<40:

G = 100;

When 40<F<140:

G = 100-(F-40)=140-F

The Food Stamp Program



F + G = 100: before stamps.

Budget set after 40 food stamps issued

The family's budget set is enlarged.

The Food Stamp Program

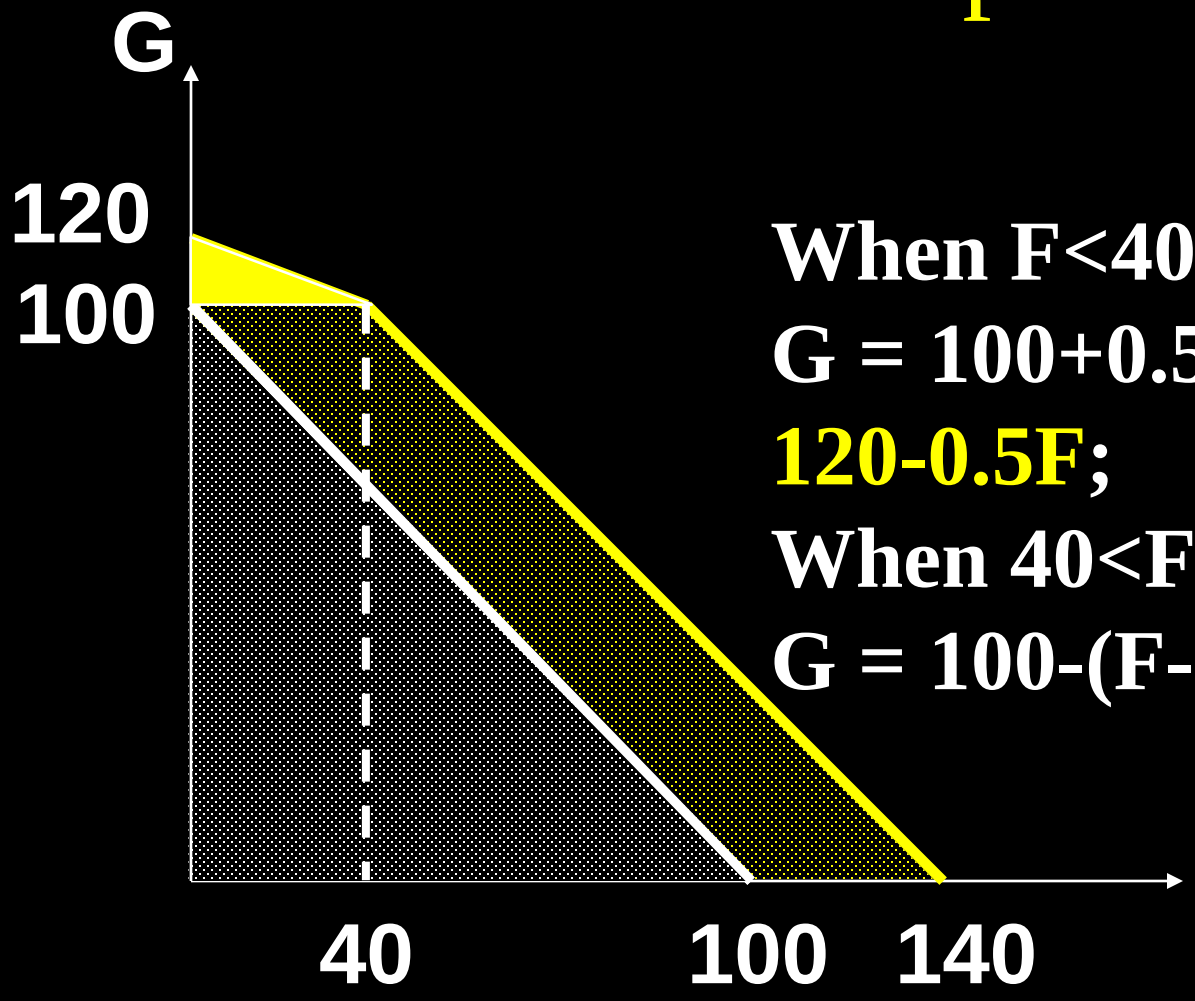
Question:

What if food stamps can be traded on a black market for $0.50 each?

The Food Stamp Program

When F<40:
G = 100+0.5(40-F) =
120-0.5F;
When 40<F<140:
G = 100-(F-40)=140-F

The Food Stamp Program



When F<40:

G = 100+0.5(40-F) =

120-0.5F;

When 40<F<140:

G = 100-(F-40)=140-F

Shapes of Budget Constraints

Q: What makes a budget constraint a straight line?

A: A straight line has a constant slope and the constraint is

$$p_1x_1 + \ldots + p_nx_n = m$$

so if prices are constants then a constraint is a straight line.

Shapes of Budget Constraints

But what if prices are not constants?

E.g. **bulk buying discounts, or price penalties for buying "too much".**

"第二件半价"，"仅限一件"

Then constraints will be curved.

Shapes of Budget Constraints - Quantity Discounts

Suppose p_2 is constant at \$1 but that p_1=\$2 for $0 \leq x_1 \leq 20$ and p_1=\$1 for $x_1>20$.

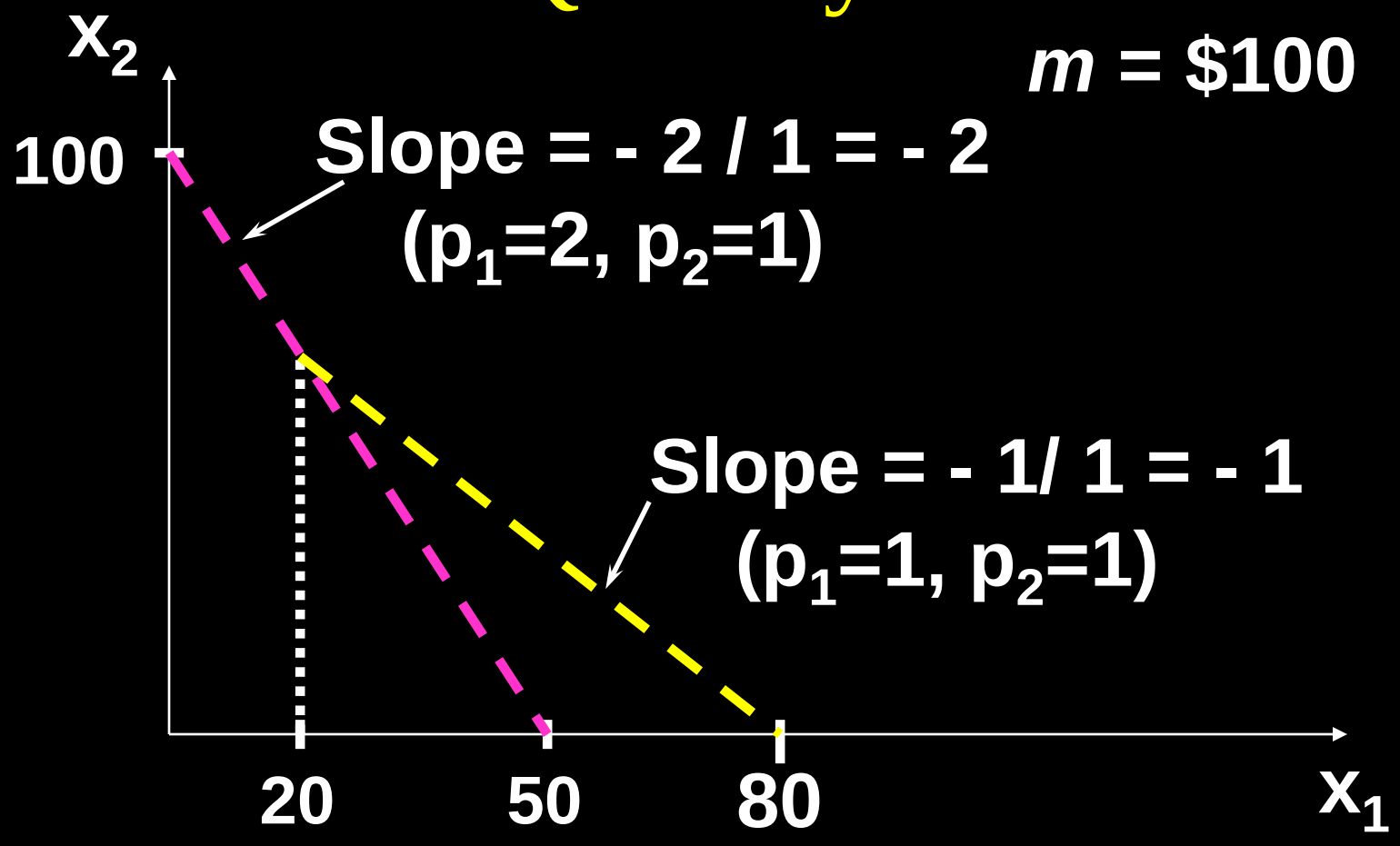
Shapes of Budget Constraints - Quantity Discounts

Suppose p_2 is constant at \$1 but that p_1=\$2 for $0 \leq x_1 \leq 20$ and p_1=\$1 for x_1>20. Then the constraint's slope is

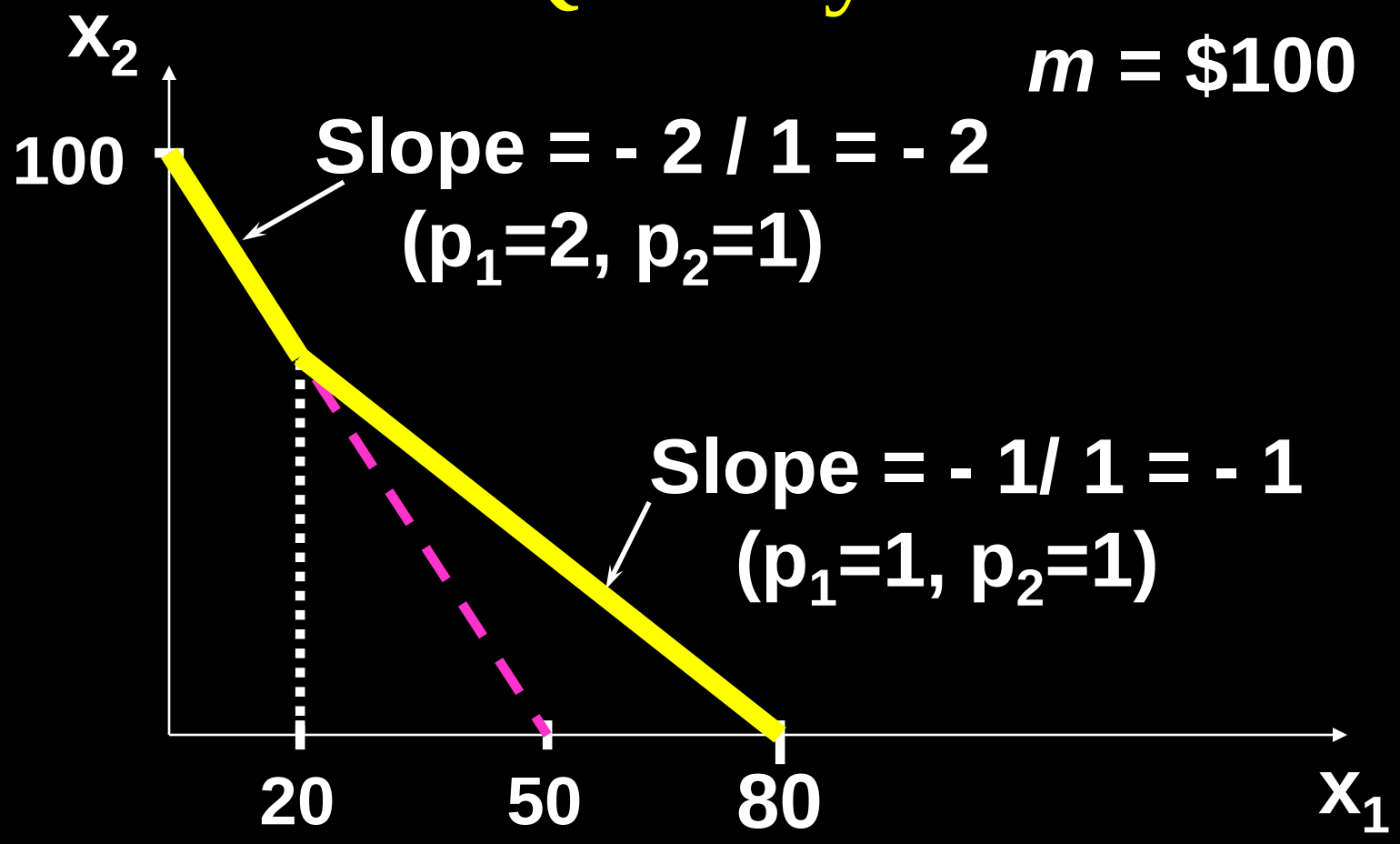
$$-p_1/p_2 = \begin{cases} -2, & \text{for } 0 \leq x_1 \leq 20 \\ -1, & \text{for } x_1 > 20 \end{cases}$$

and the constraint is

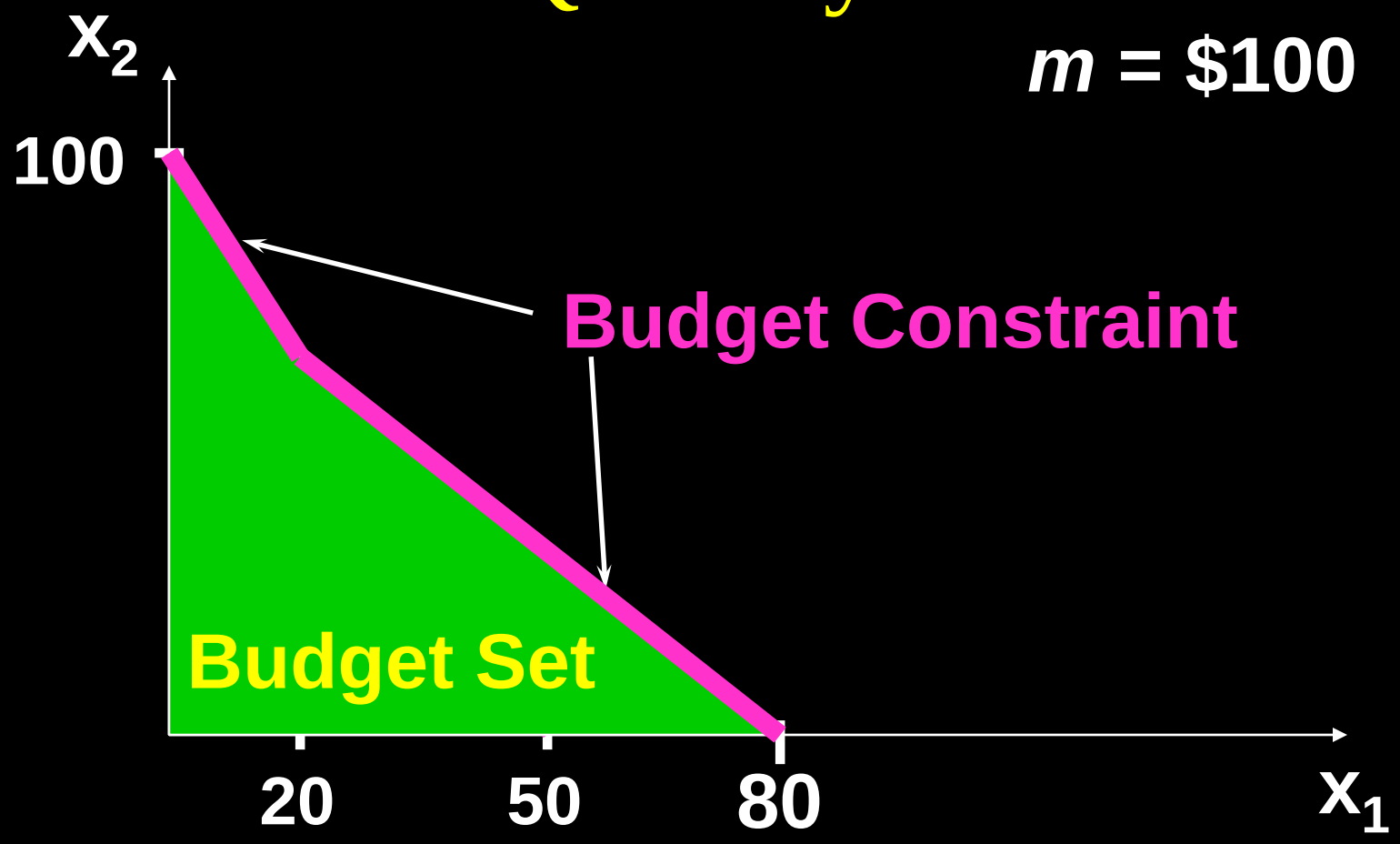
Shapes of Budget Constraints with a Quantity Discount



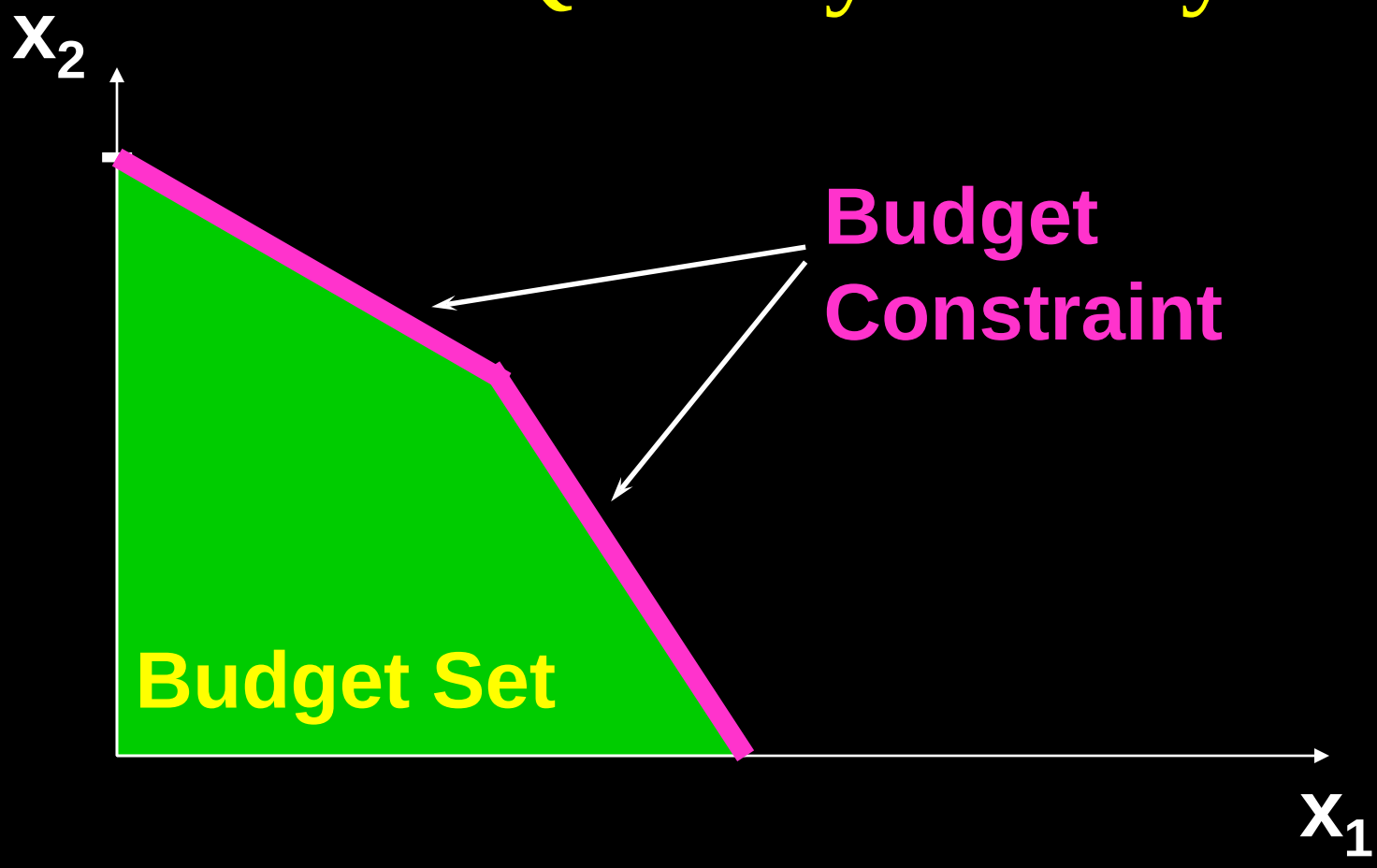
Shapes of Budget Constraints with a Quantity Discount



Shapes of Budget Constraints with a Quantity Discount



Shapes of Budget Constraints with a Quantity Penalty



Numeraire

"Numeraire" is a commodity whose price has been normalized to 1.

– "unit of account" (计价物)

Suppose prices and income are measured in dollars. Say p_1=\$2, p_2=\$3, m = \$12. Then the constraint is

$$2x_1 + 3x_2 = 12.$$

Numeraire

The original budget constraint:

$$2x_1 + 3x_2 = 12.$$

Dividing both sides by p_1 =>

$$1.x_1 + (3/2)x_2 = 6$$

Neither the budget constraint nor the budget set is affected.

3/2 is the price of commodity 2 relative to the price of commodity 1 (numeraire)

使用不同的计价物不影响预算约束的形状。