Lecture 5

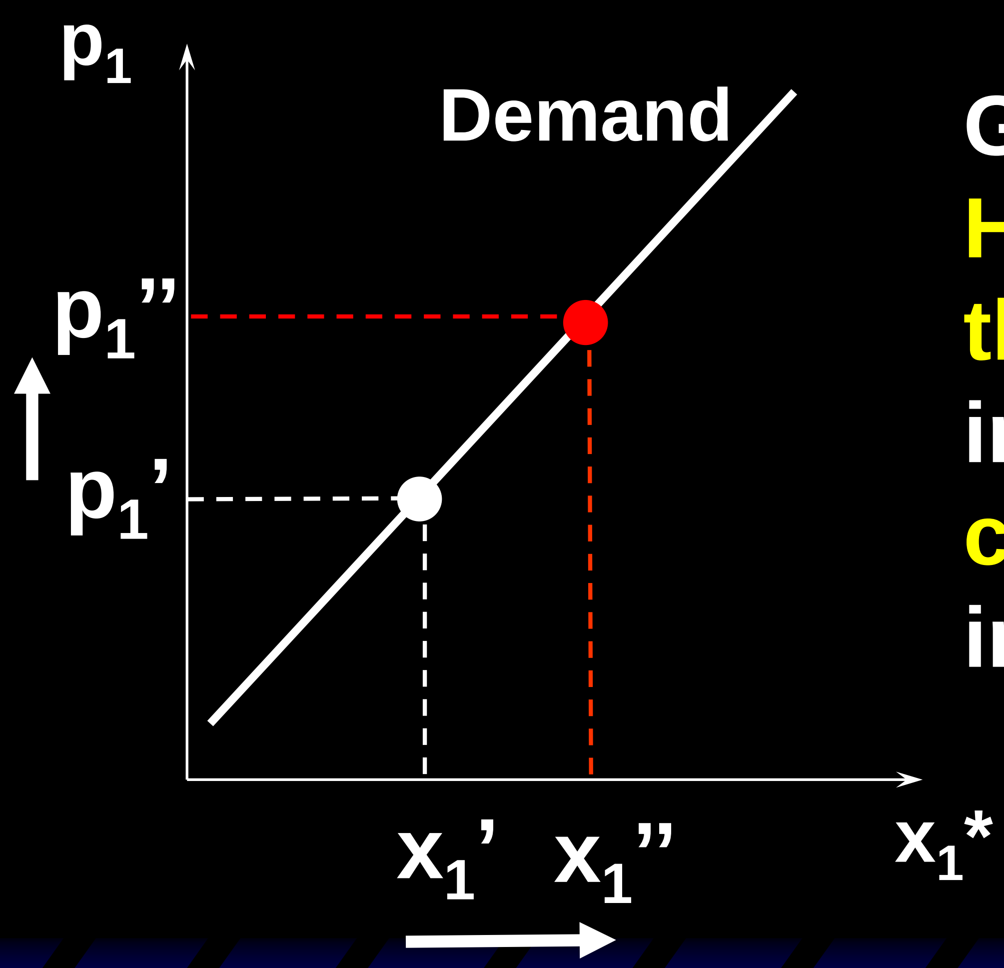
Revealed Preference

Review: Giffen Goods

If, for some values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called Giffen.

当一种商品的需求数量随自身价格的上升（下降）而上升（下降），这种商品被称为吉芬商品。

Review: Giffen Goods



Giffen goods: **Holding other things constant**, **increases in P** **cause** **increases in demand.**

隐含的前提：其它条件不变的情况下…

Review: Giffen Goods

The price of and demand for umbrellas both increase in rainy days.

Are umbrellas Giffen?

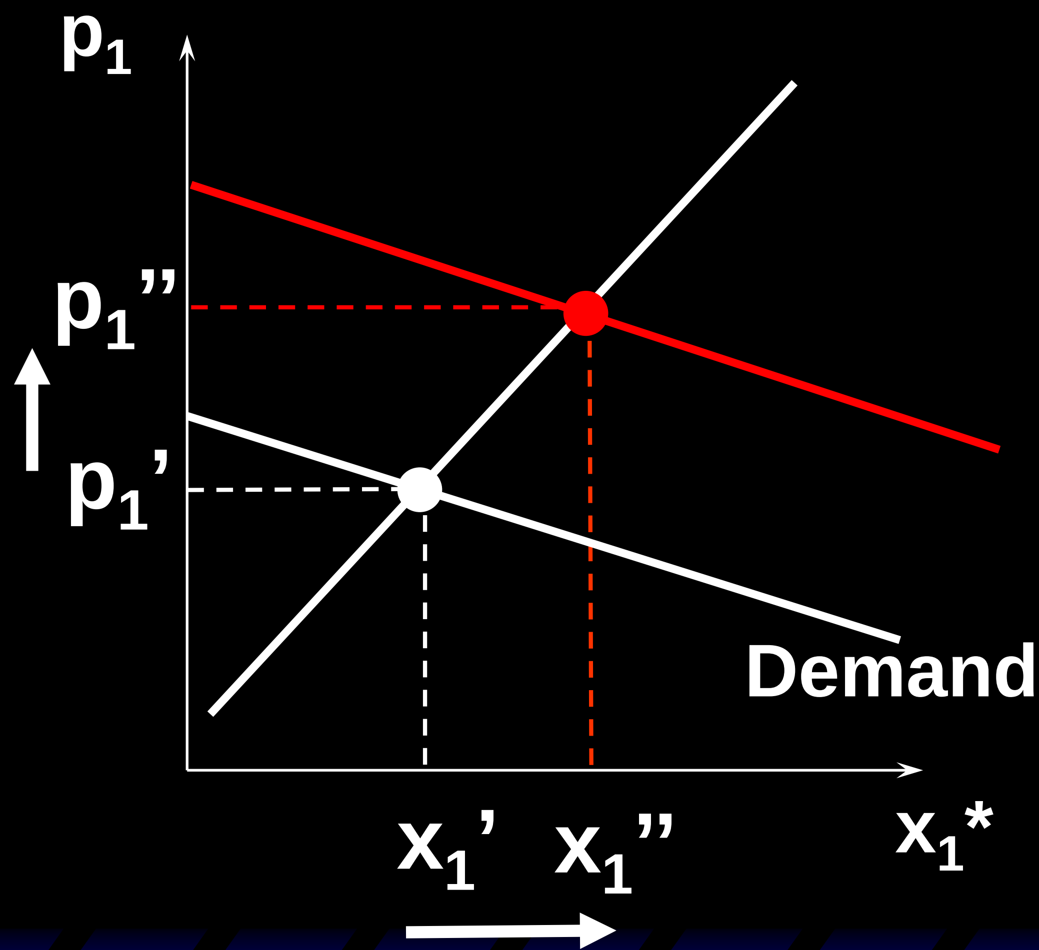
Review: Giffen Goods

The price of and demand for umbrellas both increase in rainy days.

Are umbrellas Giffen?

No. The increase in demand is not caused by the rise in price. Other things (e.g. weather conditions) are not constant.

Review: Giffen Goods



Increases in P and Q are both caused by the shift in demand curves.

Experimental Evidence for Giffen Goods

Table 3—Consumption Response to the Price Subsidy: Hunan

	Dependent variable: Rice							*Dependent variable: Meat*	
	Full sample (1)	Full sample (2)	ISCS ≤0.80 (3)	ISCS ≤0.80 (4)	ISCS >0.80 (5)	ISCS >0.80 (6)	ISCS 0.60–0.80 (7)	Full sample (8)	Initial intake >50g (9)
%ΔPrice(rice)	0.224	0.235*	0.451***	0.466***	−0.61**	−0.585**	0.640***	−0.325	−1.125*
	(0.149)	(0.140)	(0.170)	(0.159)	(0.296)	(0.262)	(0.192)	(0.472)	(0.625)
%Δ Earned		0.043***		0.047***		0.024	0.030	0.028	0.105
		(0.014)		(0.016)		(0.023)	(0.019)	(0.050)	(0.069)
%ΔUnearned		−0.044*		−0.038		−0.058	−0.053*	0.061	0.084
		(0.025)		(0.030)		(0.049)	(0.030)	(0.079)	(0.104)
%ΔPeople		0.89***		0.83***		1.16***	0.79***	−0.08	0.03
		(0.08)		(0.09)		(0.15)	(0.14)	(0.27)	(0.36)
Constant		4.1***		5.7***		−1.8	0.8	−12.3***	−49.0***
		(1.0)		(1.1)		(1.7)	(1.3)	(3.1)	(3.7)
Observations	1,258	1,258	997	997	261	261	513	997	452
R^2	0.08	0.19	0.09	0.20	0.15	0.33	0.24	0.09	0.28

Jensen and Miller (2008), "Giffen Behavior and Subsistence Consumption," *American Economic Review*.

Today's lecture:
Revealed Preference Analysis

Suppose we observe the demands (consumption choices) that a consumer makes for different budgets. This reveals information about the consumer's preferences. We can use this information to ...

Revealed Preference Analysis

- Test the behavioral hypothesis that a consumer chooses the most preferred bundle from those available.
- Discover the consumer's preference relation.

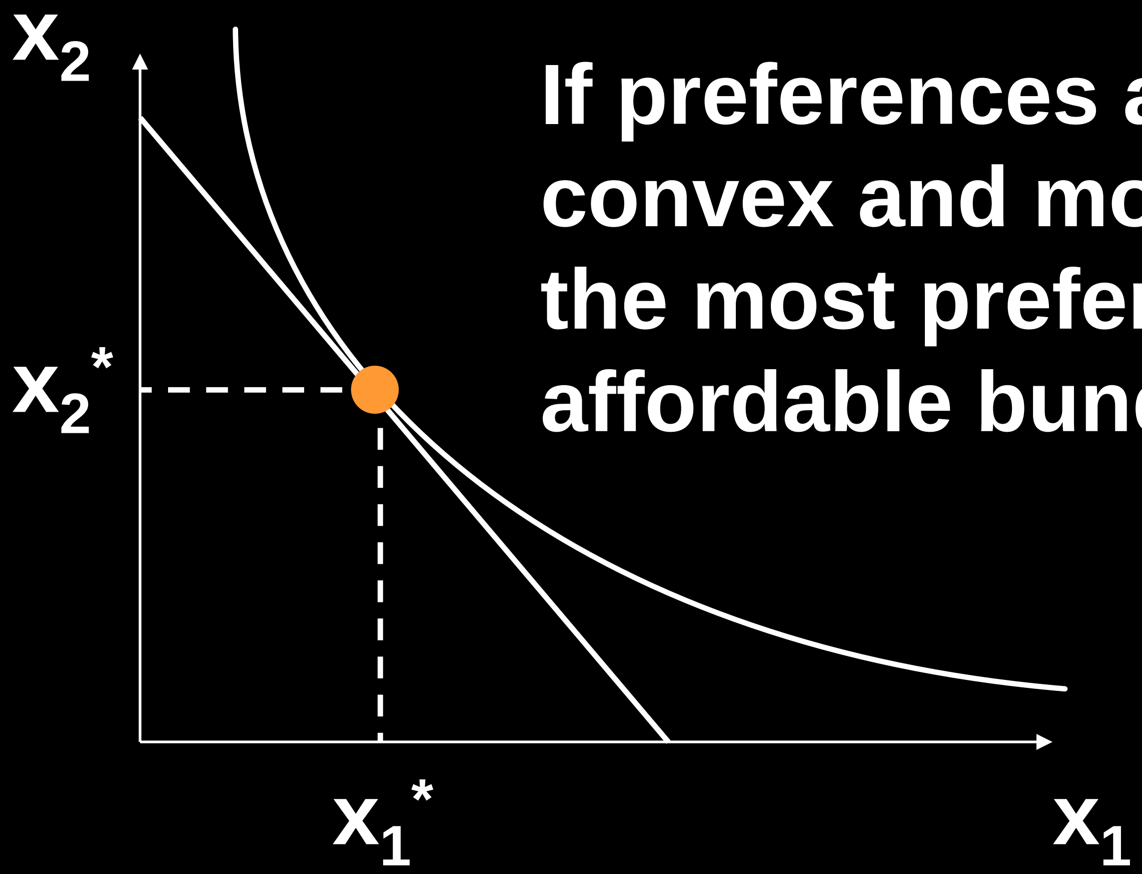
Assumptions on Preferences

Preferences

- do not change while the choice data are gathered.
- are strictly convex.
- are monotonic.

Together, convexity and monotonicity imply that the most preferred affordable bundle is unique.

Assumptions on Preferences



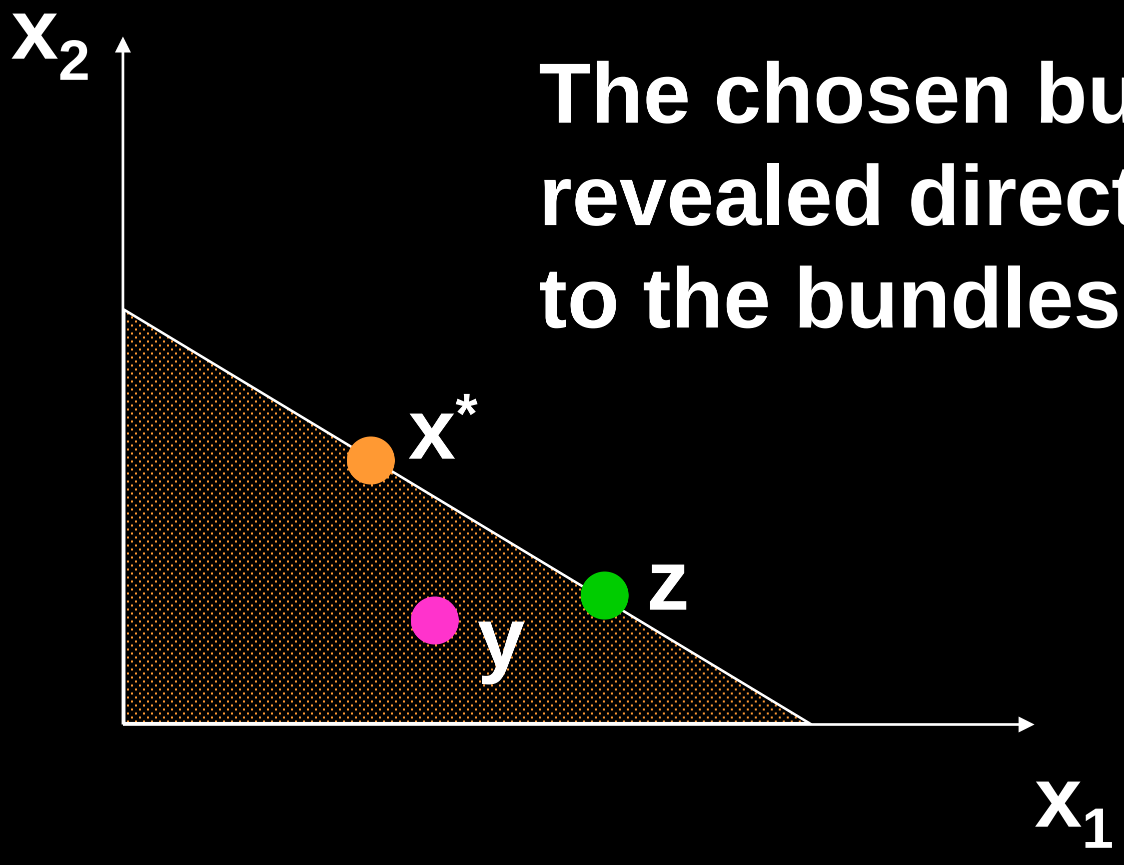
If preferences are strictly convex and monotonic then the most preferred affordable bundle is unique.

Direct Preference Revelation

Suppose that the bundle x^* is chosen when the bundle y is affordable. Then x* is revealed directly as preferred to y (otherwise y would have been chosen).

当y可得的时候，若消费者选择了x^*，我们则定义：x^*直接显示偏好于y

Direct Preference Revelation



The chosen bundle x^* is revealed directly as preferred to the bundles y and z.

Direct Preference Revelation

That x is revealed directly as preferred to y will be written as

$$\mathbf{x} \succ_{D} \mathbf{y}.$$

Indirect Preference Revelation

Suppose x is revealed directly preferred to y, and y is revealed directly preferred to z. Then, by transitivity, x is revealed indirectly as preferred to z. Write this as

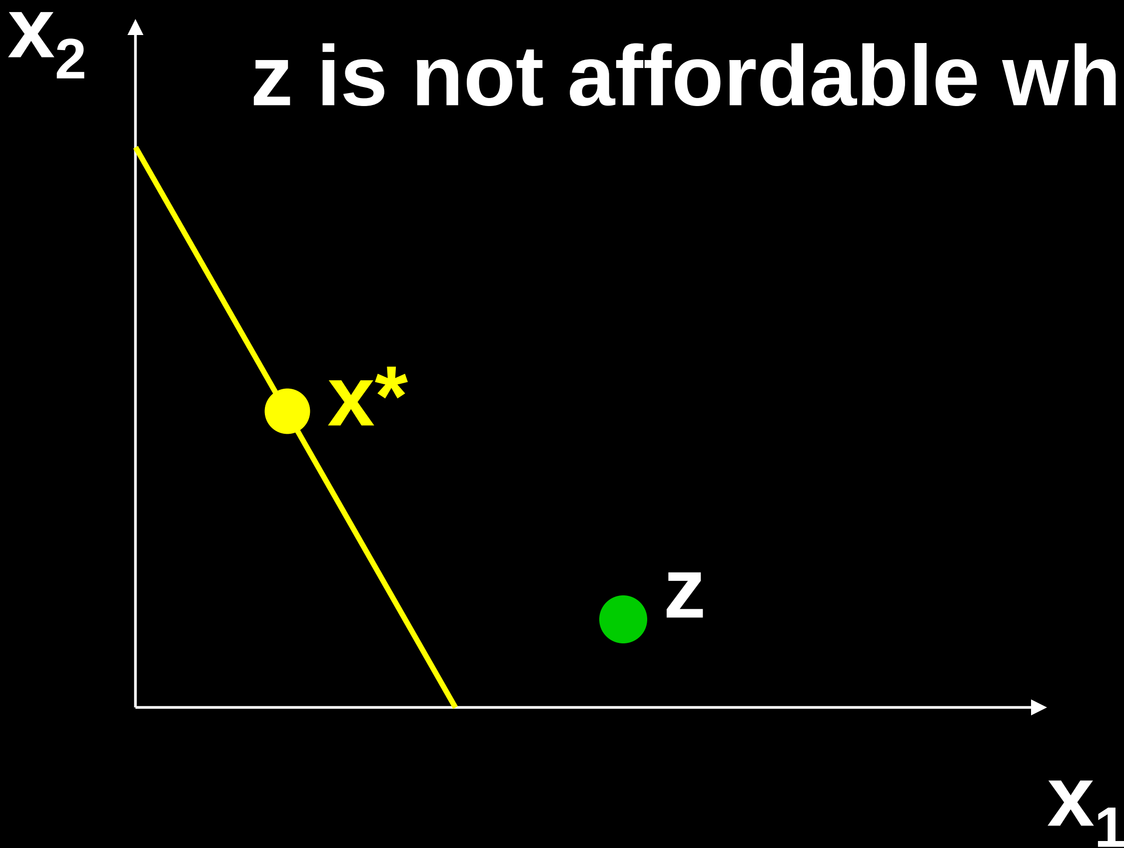
$$x \succ_I z$$

间接显示偏好于

so $x \succ_D y$ **and** $y \succ_D z \Rightarrow x \succ_I z.$

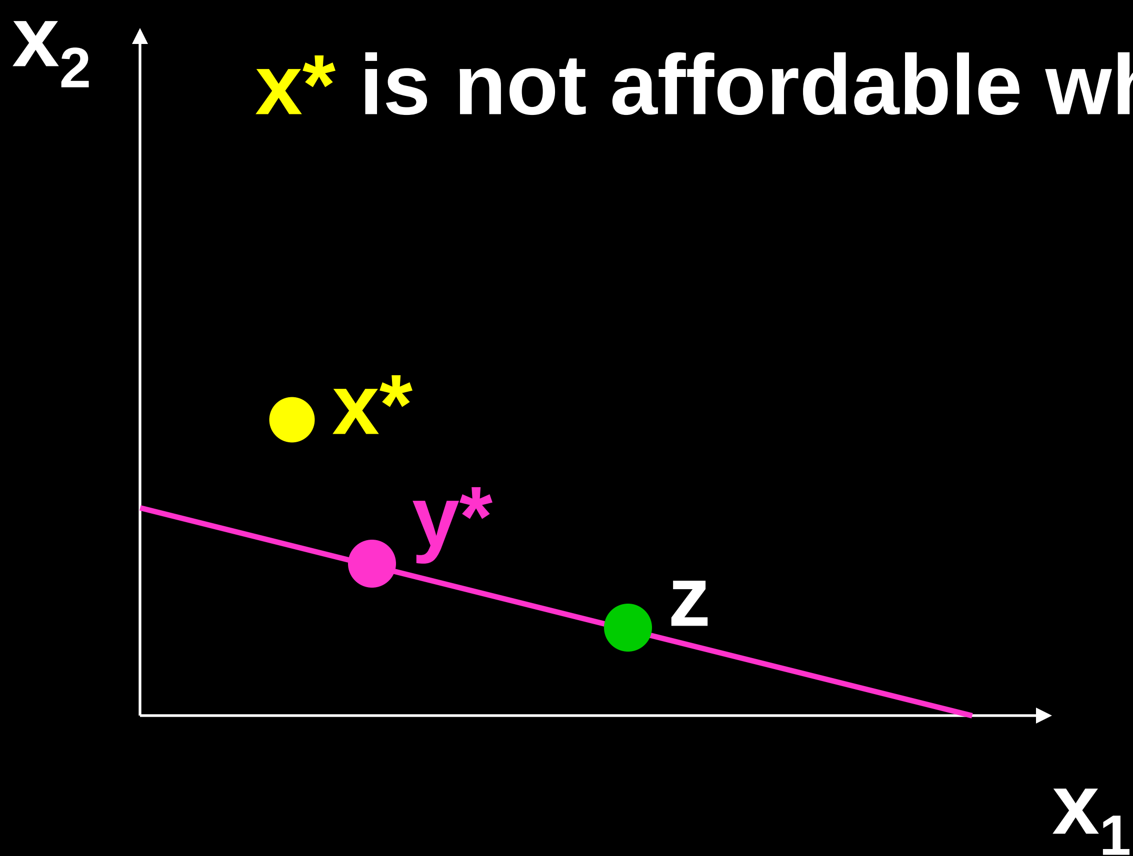
Indirect Preference Revelation

z is not affordable when x* is chosen.



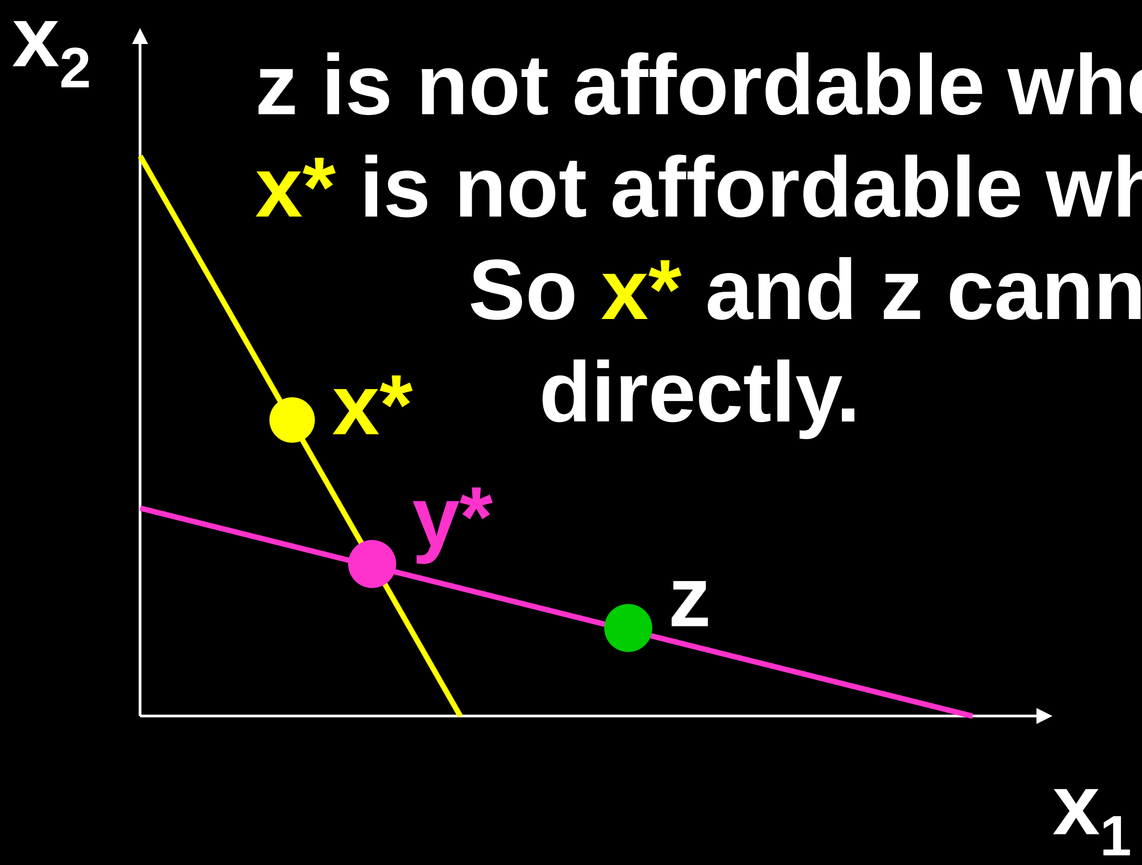
Indirect Preference Revelation

x* is not affordable when y* is chosen.



Indirect Preference Revelation

z is not affordable when x* is chosen.
x* is not affordable when y* is chosen.
So x* and z cannot be compared directly.



Indirect Preference Revelation

z is not affordable when x* is chosen.

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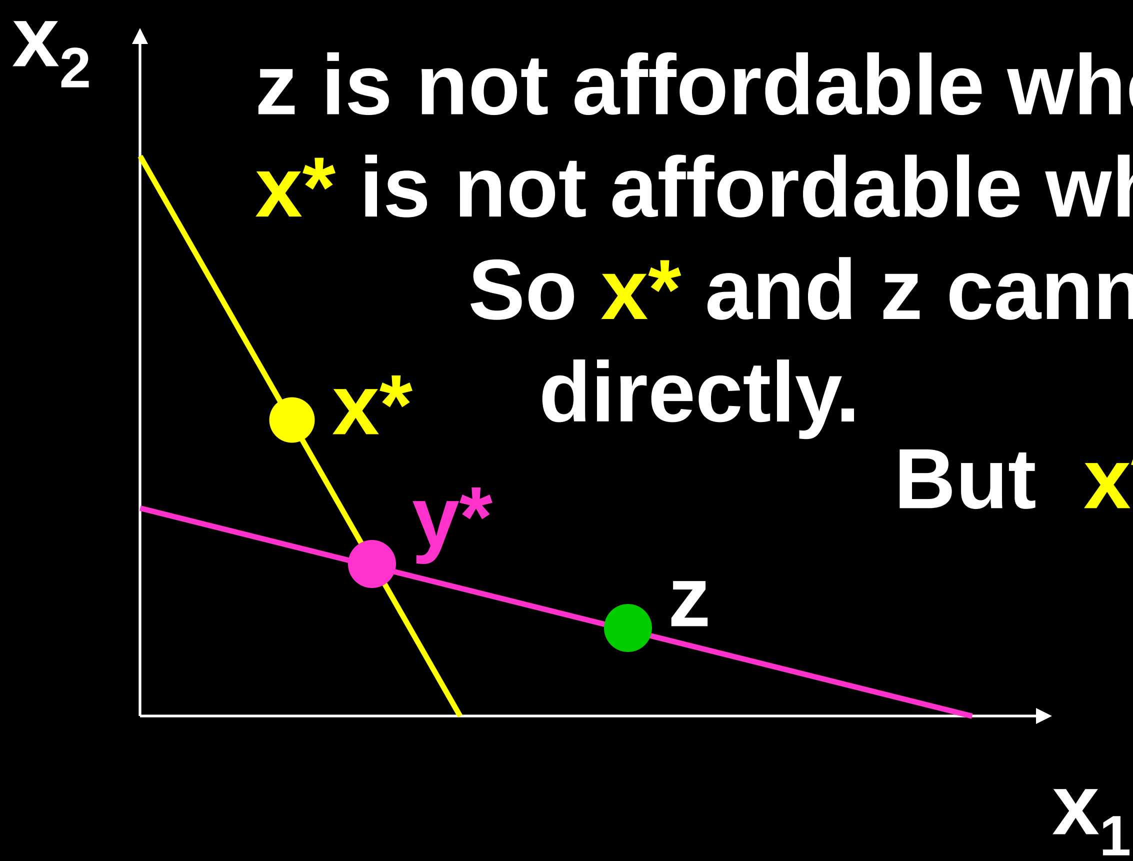
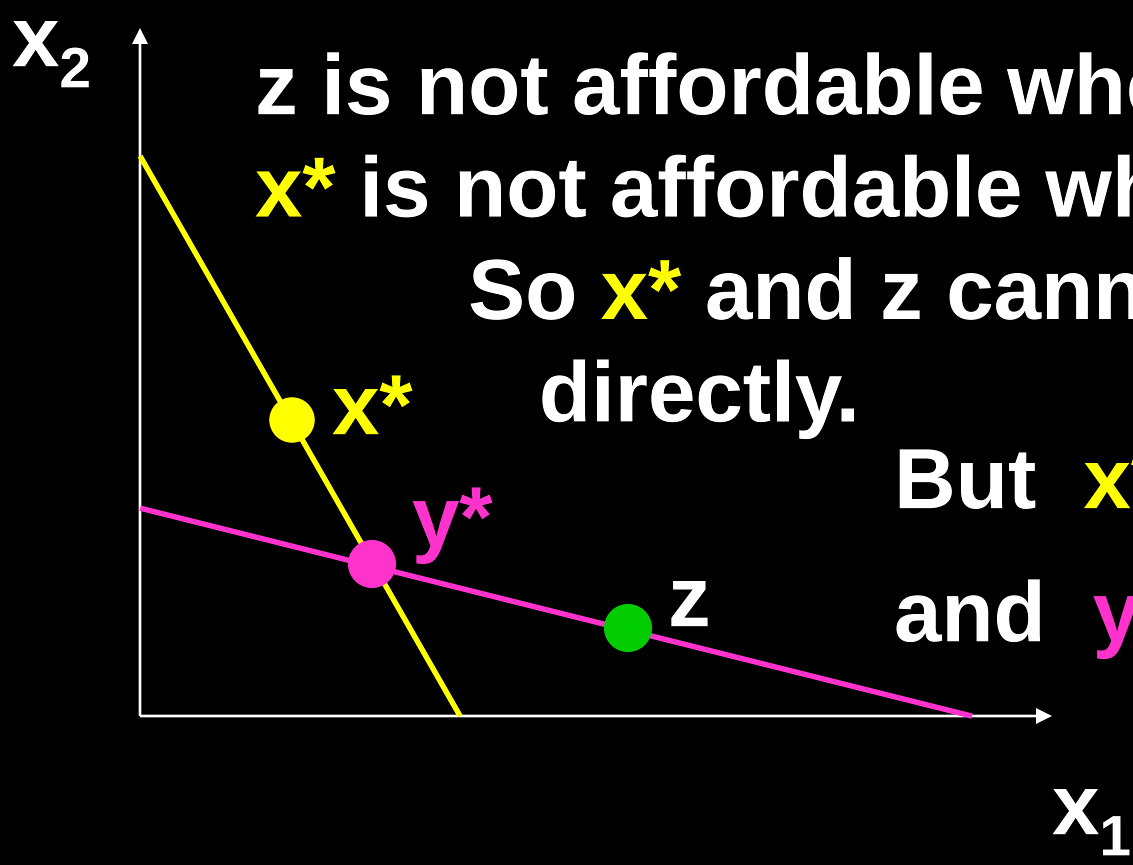
So x* and z cannot be compared directly.

But $x^* \succ_D y^*$



Indirect Preference Revelation



z is not affordable when x* is chosen.

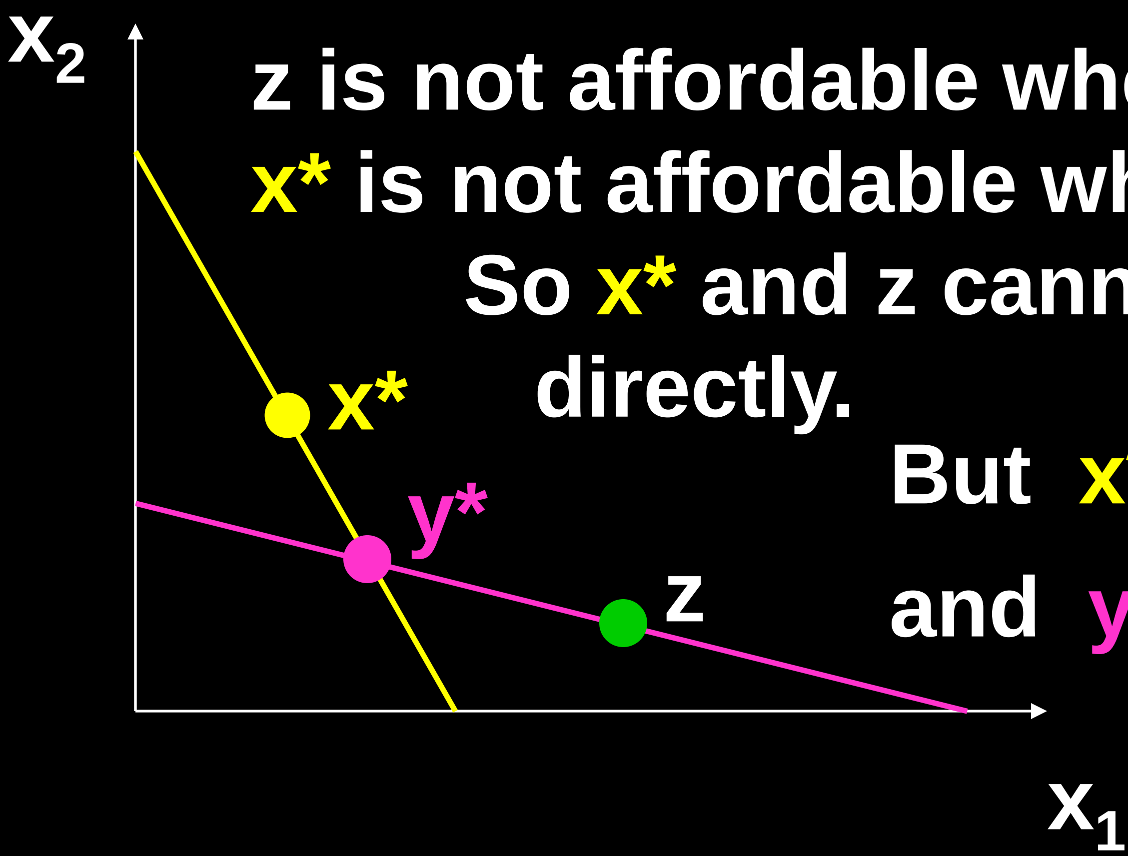
x* is not affordable when y* is chosen.

So x* and z cannot be compared directly.

But x* $\succ_D$ **y***

and y* $\succ_D$ **z**

Indirect Preference Revelation



z is not affordable when x* is chosen.

x* is not affordable when y* is chosen.

So x* and z cannot be compared directly.

But x* $\succ_D$ y*

and y* $\succ_D$ z

so x* $\succ_I$ z.

Two Axioms of Revealed Preference

To apply revealed preference analysis, choices must satisfy two criteria -- the Weak and the Strong Axioms of Revealed Preference.

The Weak Axiom of Revealed Preference (WARP)

If the bundle x is revealed directly as preferred to the bundle y then it is never the case that y is revealed directly as preferred to x; *i.e.*

$$x \succ_D y \Rightarrow \text{not } (y \succ_D x).$$

弱显示偏好公理：若x直显于y，则y不能直显于x

The Weak Axiom of Revealed Preference (WARP)

Choice data which violate the WARP are inconsistent with economic rationality.

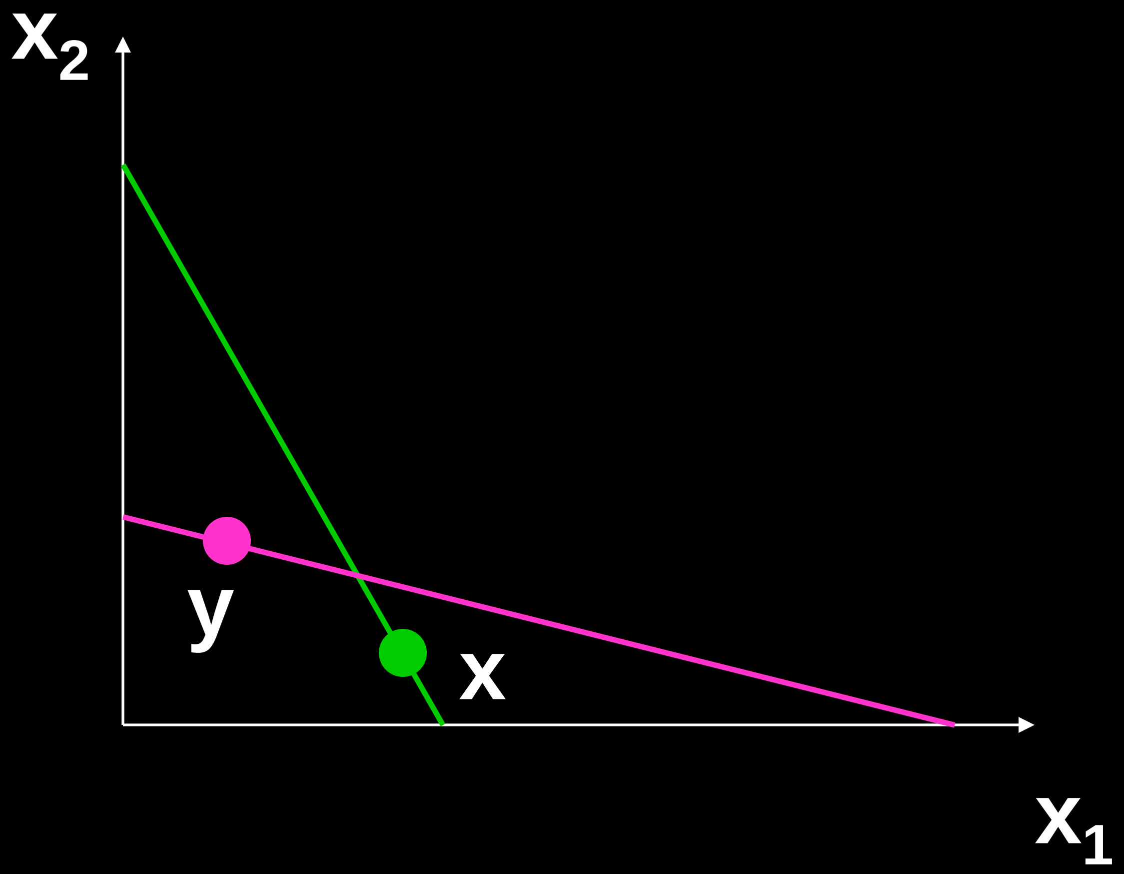
The WARP is a necessary condition for applying economic rationality to explain observed choices.

若消费者的选择不满足弱显示偏好公理，则无法进行显示偏好分析

The Weak Axiom of Revealed Preference (WARP)

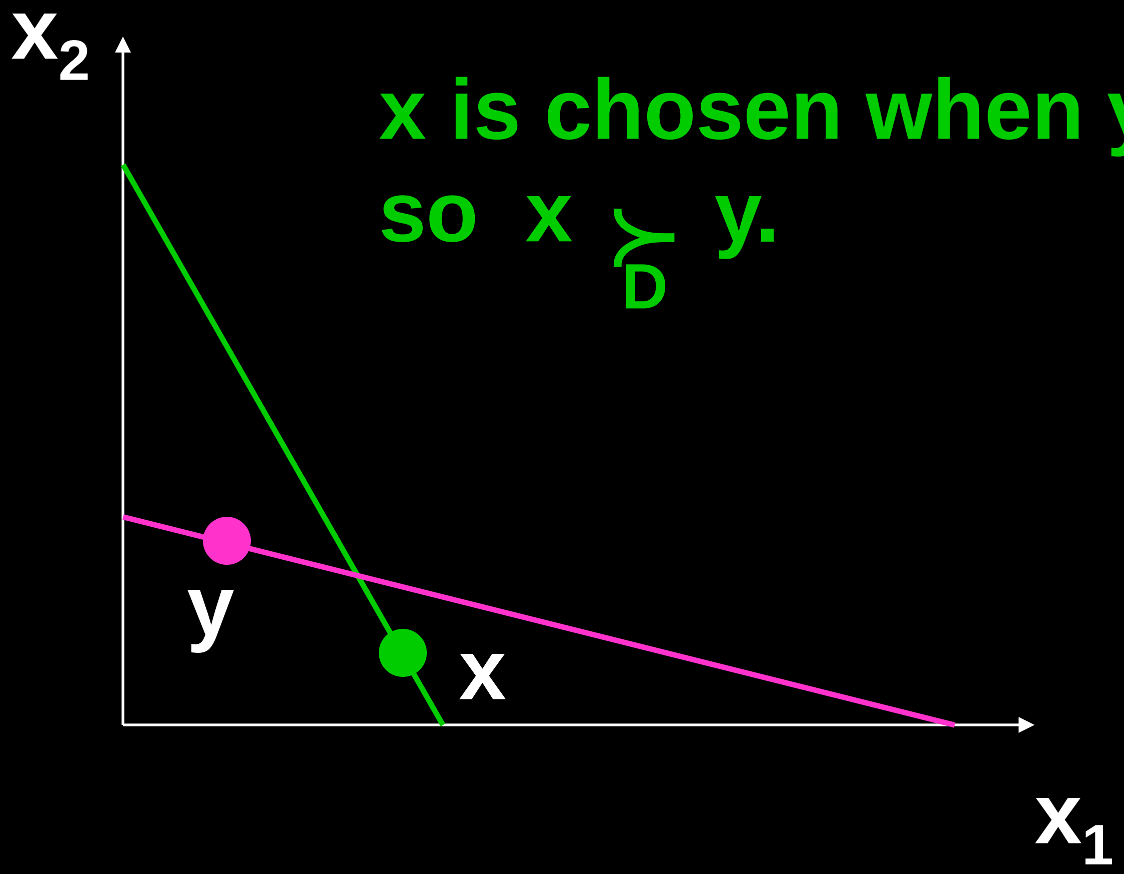
What choice data violate the WARP?

The Weak Axiom of Revealed Preference (WARP)

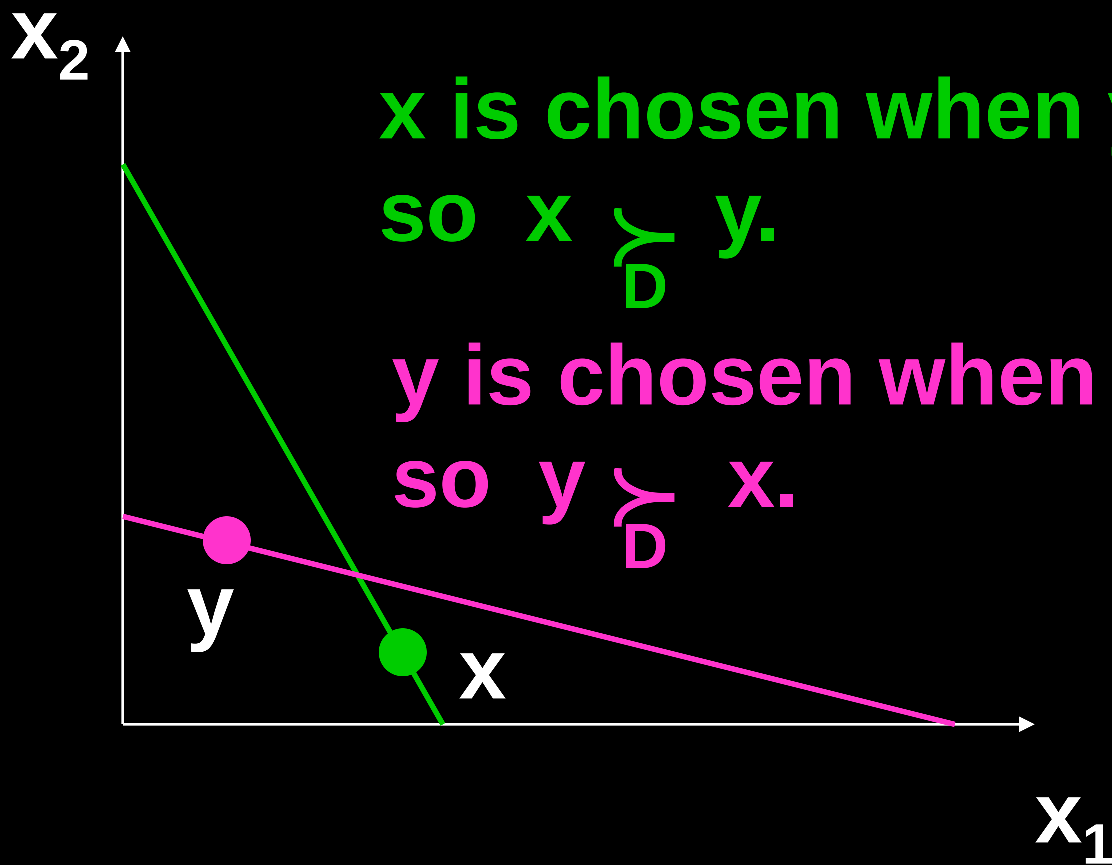


The Weak Axiom of Revealed Preference (WARP)

x is chosen when y is available so x $\succ_D$ y.



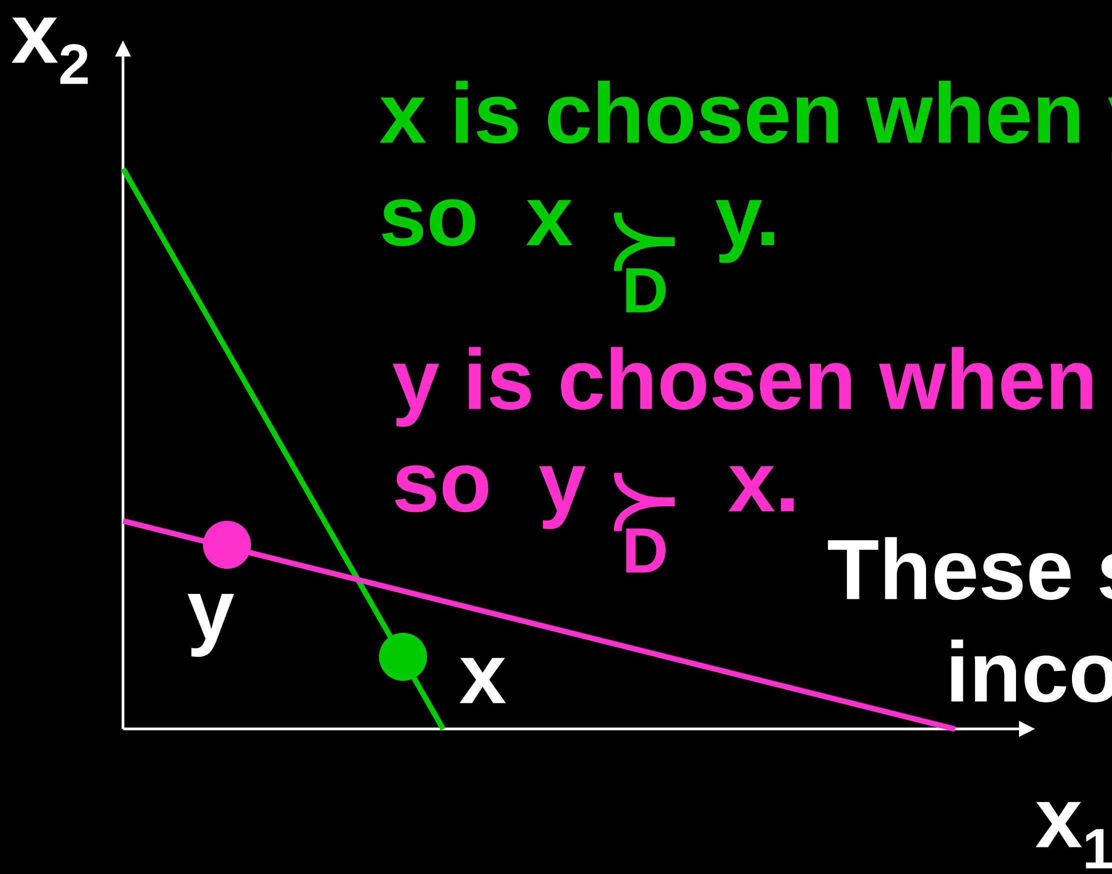
The Weak Axiom of Revealed Preference (WARP)



x is chosen when y is available so $x \succ_D y$.

y is chosen when x is available so $y \succ_D x$.

The Weak Axiom of Revealed Preference (WARP)



x is chosen when y is available so x $\succ_D$ y.

y is chosen when x is available so y $\succ_D$ x.

These statements are inconsistent with each other.

Checking if Data Violate the WARP

- A consumer makes the following choices:
  - At prices $(p_1,p_2)=(\$2,\$2)$ the choice was $(x_1,x_2) = (10,1)$.
  - At $(p_1,p_2)=(\$2,\$1)$ the choice was $(x_1,x_2) = (5,5)$.
  - At $(p_1,p_2)=(\$1,\$2)$ the choice was $(x_1,x_2) = (5,4)$.
- Is the WARP violated by these data?

Checking if Data Violate the WARP

Prices \ Choices	(10, 1)	(5, 5)	(5, 4)
(\$2, \$2)	\$22	\$20	\$18
(\$2, \$1)	\$21	\$15	\$14
(\$1, \$2)	\$12	\$15	\$13

Red numbers are costs of chosen bundles.

Checking if Data Violate the WARP

Prices \ Choices	(10, 1)	(5, 5)	(5, 4)
(\$2, \$2)	\$22	\$20	\$18
(\$2, \$1)	\$21	\$15	\$14
(\$1, \$2)	\$12	\$15	\$13

Circles surround affordable bundles that were not chosen.

Checking if Data Violate the WARP

Prices \ Choices	(10, 1)	(5, 5)	(5, 4)
($2, $2)	$22	$20	$18
($2, $1)	$21	$15	$14
($1, $2)	$12	$15	$13

Circles surround affordable bundles that were not chosen.

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Circles surround affordable bundles that were not chosen.

Checking if Data Violate the WARP

Choices / Prices	(10,1)	(5,5)	(5,4)
(\$2,\$2)	\$22	\$20	\$18
(\$2,\$1)	\$21	\$15	\$14
(\$1,\$2)	\$12	\$15	\$13

	(10, 1)	(5, 5)	(5, 4)
(10,1)		D	D
(5, 5)			D
(5 ,4)	D		

Checking if Data Violate the WARP

Choices / Prices	(10,1)	(5,5)	(5,4)
(\$2,\$2)	\$22	\$20	\$18
(\$2,\$1)	\$21	\$15	\$14
(\$1,\$2)	\$12	\$15	\$13

	(10, 1)	(5, 5)	(5, 4)
(10,1)		D	D
(5, 5)			D
(5 ,4)	D		

Checking if Data Violate the WARP

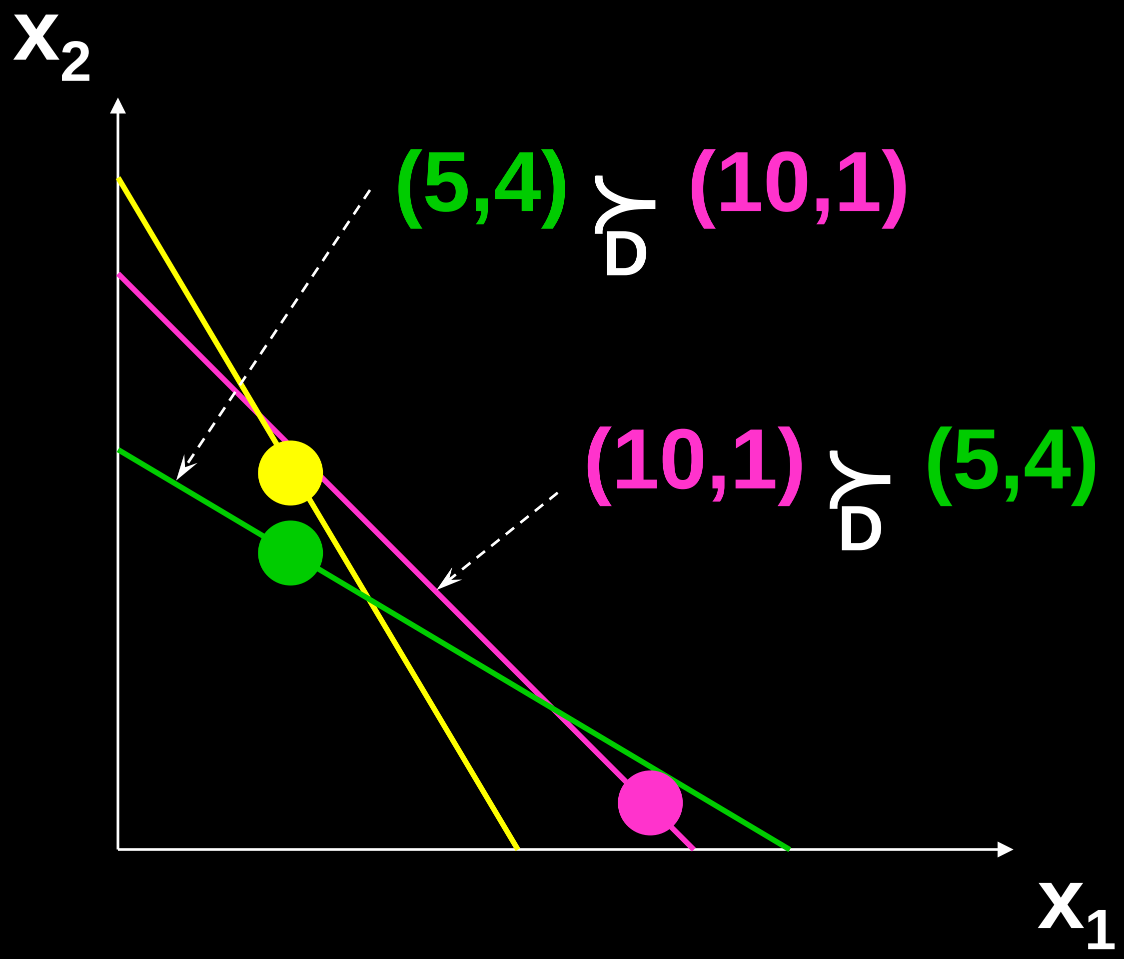
(10,1) is revealed directly as preferred to (5,4).

(5,4) is also revealed directly as preferred to (10,1).

⇒ WARP is violated by the data.

	(10, 1)	(5, 5)	(5, 4)
(10,1)		D	D
(5, 5)			D
(5 ,4)	D		

Checking if Data Violate the WARP



The Strong Axiom of Revealed Preference (SARP)

If the bundle x is revealed (**directly or indirectly**) as preferred to the bundle y and $x \neq y$, then it is never the case that the y is revealed (**directly or indirectly**) as preferred to x; *i.e.*

$$x \succ_D y \text{ or } x \succ_I y$$

$$\Rightarrow \text{not } (y \succ_D x \text{ or } y \succ_I x).$$

强显示偏好公理：若x直接或间接显示偏好于y，则y不能直接或间接显示偏好于x

The Strong Axiom of Revealed Preference

- What choice data would satisfy the WARP but violate the SARP?

The Strong Axiom of Revealed Preference

Consider the following data:

A: $(p_1,p_2,p_3) = (1,3,10)$ & $(x_1,x_2,x_3) = (3,1,4)$

B: $(p_1,p_2,p_3) = (4,3,6)$ & $(x_1,x_2,x_3) = (2,5,3)$

C: $(p_1,p_2,p_3) = (1,1,5)$ & $(x_1,x_2,x_3) = (4,4,3)$

The Strong Axiom of Revealed Preference

		(3,1,4)	(2,5,3)	(4,4,3)
	Choice / Prices	A	B	C
(\$1,\$3,\$10)	A	\$46	\$47	\$46
(\$4,\$3,\$6)	B	\$39	\$41	\$46
(\$1,\$1,\$5)	C	\$24	\$22	\$23

The Strong Axiom of Revealed Preference

Prices \ Choices	A	B	C
A	$46	$47	$46
B	$39	$41	$46
C	$24	$22	$23

The Strong Axiom of Revealed Preference

Prices \ Choices	A	B	C
A	$46	$47	$46
B	$39	$41	$46
C	$24	$22	$23

In situation A, bundle A is directly revealed preferred to bundle C;

$$A \succ_D C.$$

The Strong Axiom of Revealed Preference

Choices / Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

In situation B, bundle B is directly revealed preferred to bundle A;

$B \succ_D A.$

The Strong Axiom of Revealed Preference

Prices \ Choices	A	B	C
A	$46	$47	$46
B	$39	$41	$46
C	$24	$22	$23

In situation C, bundle C is directly revealed preferred to bundle B;

$C \succ_D B.$

The Strong Axiom of Revealed Preference

Choices / Prices	A	B	C
A	$46	$47	$46
B	$39	$41	$46
C	$24	$22	$23

	A	B	C
A			D
B	D		
C		D	

The Strong Axiom of Revealed Preference

Prices \ Choices	A	B	C
A	$46	$47	$46
B	$39	$41	$46
C	$24	$22	$23

	A	B	C
A			D
B	D		
C		D	

The data do not violate the WARP.

The Strong Axiom of Revealed Preference

We have that

$A \succ_D C$, $B \succ_D A$ and $C \succ_D B$

so,

$A \succ_I B$, $B \succ_I C$ and $C \succ_I A$.

	A	B	C
A			D
B	D		
C		D	

The data do not violate the WARP but ...

The Strong Axiom of Revealed Preference

We have that

$A \succ_D C$, $B \succ_D A$ and $C \succ_D B$

so,

$A \succ_I B$, $B \succ_I C$ and $C \succ_I A$.

	A	B	C
A		I	D
B	D		I
C	I	D	

The data do not violate the WARP but ...

The Strong Axiom of Revealed Preference

$B \succ_D A$ is inconsistent

with $A \succ_I B$.

	A	B	C
A		I	D
B	D		I
C	I	D	

The data do not violate the WARP but ...

The Strong Axiom of Revealed Preference

A $\succ_D$ C is inconsistent

with C $\succ_I$ A.

	A	B	C
A		I	D
B	D		I
C	I	D	

The data do not violate the WARP but ...

The Strong Axiom of Revealed Preference

$C \succ_D B$ is inconsistent

with $B \succ_I C$.

	A	B	C
A		I	D
B	D		I
C	I	D	

The data do not violate the WARP but ...

The Strong Axiom of Revealed Preference

The data do not violate the WARP but there are 3 violations of the SARP.

	A	B	C
A		I	D
B	D		I
C	I	D	

The Strong Axiom of Revealed Preference

That the observed choice data satisfy the SARP is a condition necessary and sufficient for there to be a well-behaved preference relation that "rationalizes" the data.

So our 3 data cannot be rationalized by a well-behaved preference relation.

Recovering Indifference Curves

Suppose we have the choice data satisfy the SARP.

Then we can discover approximately where are the consumer's indifference curves.

How?

Recovering Indifference Curves

Suppose we observe:

A: $(p_1,p_2) = (\$1,\$1)$ & $(x_1,x_2) = (15,15)$

B: $(p_1,p_2) = (\$2,\$1)$ & $(x_1,x_2) = (10,20)$

C: $(p_1,p_2) = (\$1,\$2)$ & $(x_1,x_2) = (20,10)$

D: $(p_1,p_2) = (\$2,\$5)$ & $(x_1,x_2) = (30,12)$

E: $(p_1,p_2) = (\$5,\$2)$ & $(x_1,x_2) = (12,30)$.

Where lies the indifference curve containing the bundle A = (15,15)?

Recovering Indifference Curves

Choices / Prices	A (15,15)	B (10,20)	C (20,10)	D (30,12)	E (12,30)
A ($1,$1)	$30	$30	$30	$42	$42
B ($2,$1)	$45	$40	$50	$72	$54
C ($1,$2)	$45	$50	$40	$54	$72
D ($2,$5)	$105	$120	$90	$120	$174
E ($5,$2)	$105	$90	$120	$174	$120

Recovering Indifference Curves

Choices / Prices	A (15,15)	B (10,20)	C (20,10)	D (30,12)	E (12,30)
A ($1,$1)	$30	$30	$30	$42	$42
B ($2,$1)	$45	$40	$50	$72	$54
C ($1,$2)	$45	$50	$40	$54	$72
D ($2,$5)	$105	$120	$90	$120	$174
E ($5,$2)	$105	$90	$120	$174	$120

Recovering Indifference Curves

The table showing direct preference revelations is:

Recovering Indifference Curves

	A	B	C	D	E
A		D	D		
B					
C					
D	D	D	D		
E	D	D	D		

Direct revelations only; the WARP is not violated by the data.

Recovering Indifference Curves

In this example, indirect preference revelations add no extra information, so the table showing both direct and indirect preference revelations is the same as the table showing only the direct preference revelations:

上述数据不存在额外的、有关间接显示偏好的有用信息；例如，虽然A直显于B，但B不直显于任何组合；虽然D直显于A，A直显于B，即D间显于B，但我们已经知道D直显于B。

Recovering Indifference Curves

	A	B	C	D	E
A		D	D		
B					
C					
D	D	D	D		
E	D	D	D		

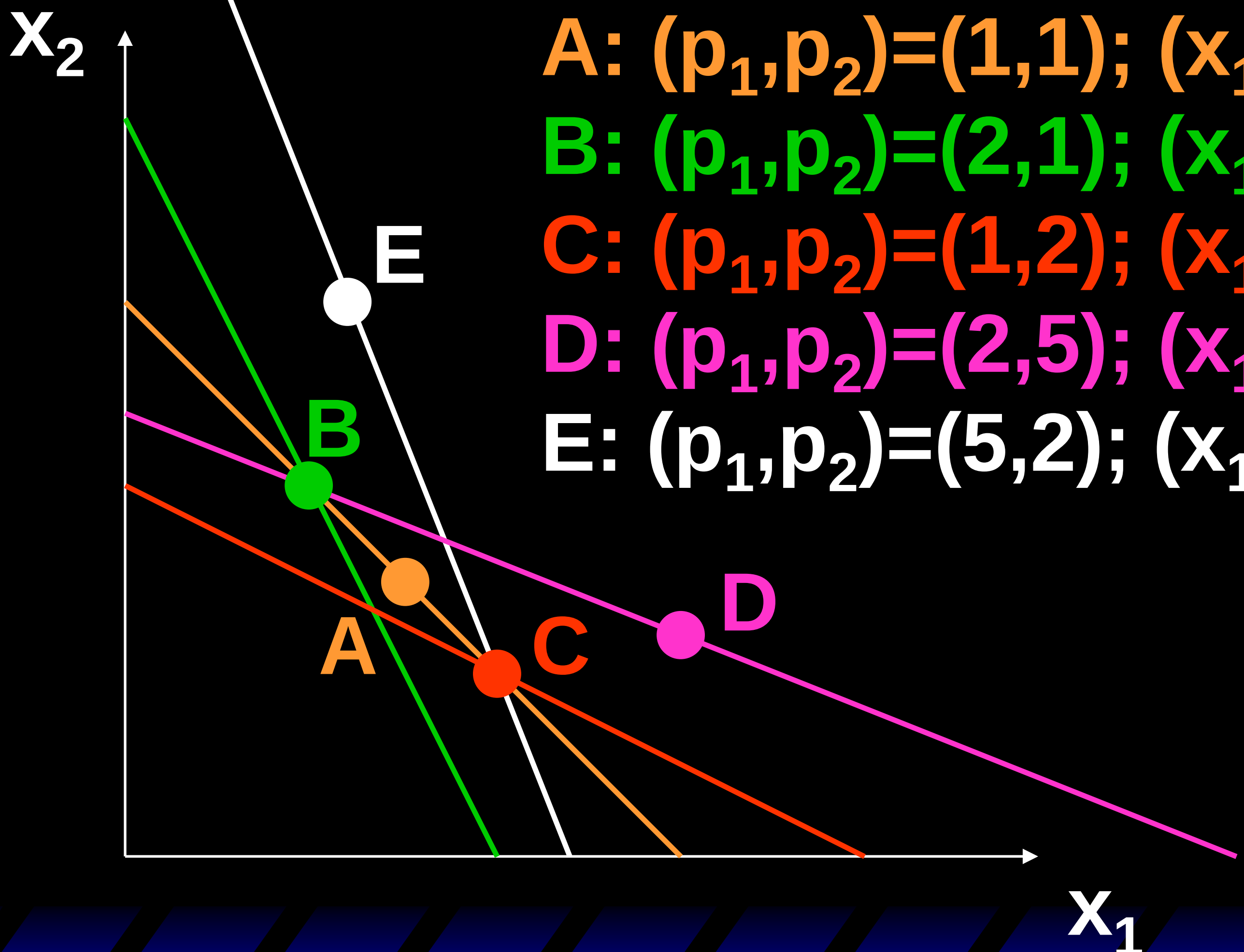
Both direct and indirect revelations; neither WARP nor SARP are violated by the data.

Recovering Indifference Curves

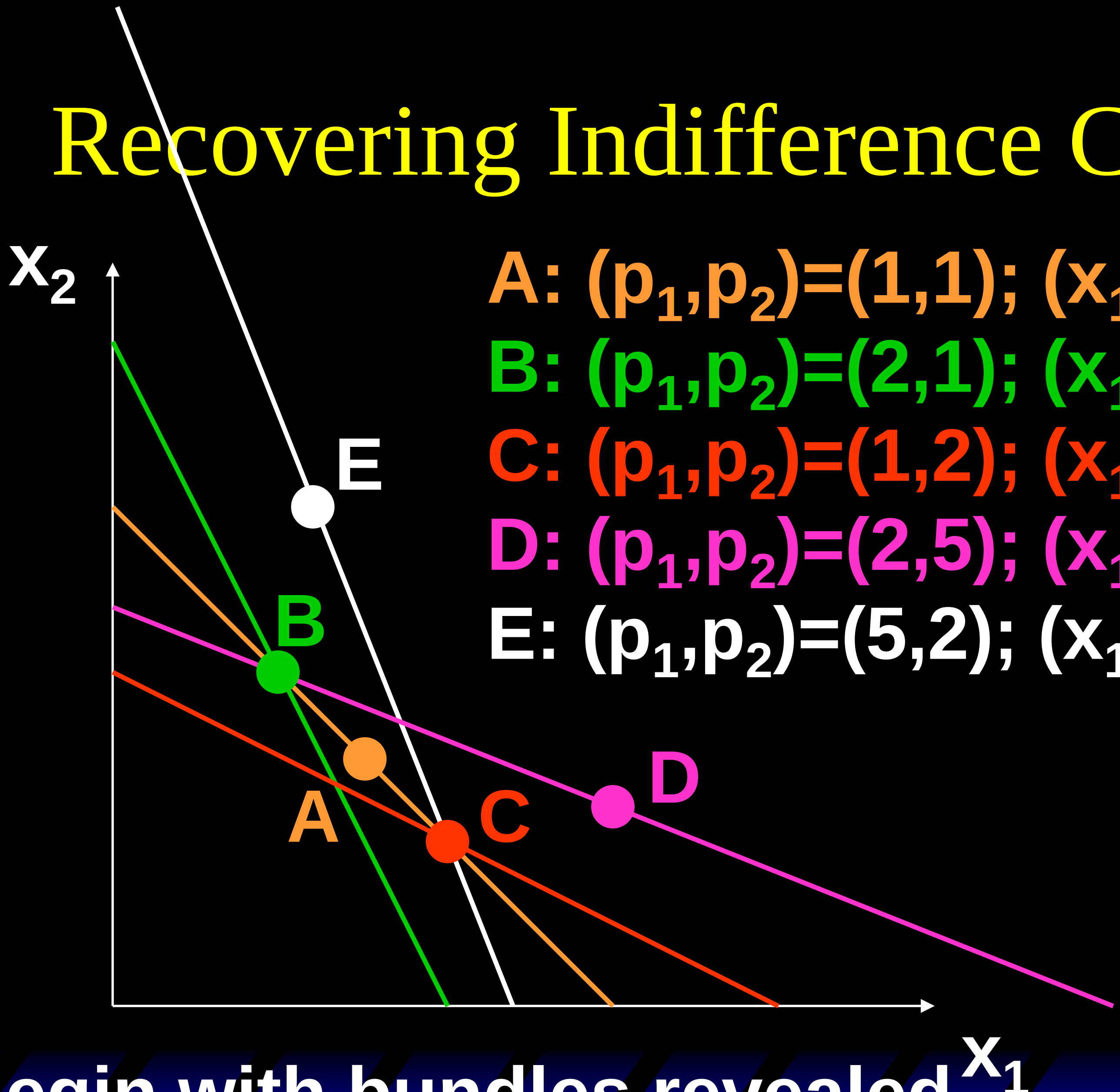
- Since the choices satisfy the SARP, there is a well-behaved preference relation that “rationalizes” the choices.

Recovering Indifference Curves

A: $(p_1,p_2)=(1,1)$; $(x_1,x_2)=(15,15)$
B: $(p_1,p_2)=(2,1)$; $(x_1,x_2)=(10,20)$
C: $(p_1,p_2)=(1,2)$; $(x_1,x_2)=(20,10)$
D: $(p_1,p_2)=(2,5)$; $(x_1,x_2)=(30,12)$
E: $(p_1,p_2)=(5,2)$; $(x_1,x_2)=(12,30)$.



Recovering Indifference Curves



A: $(p_1,p_2)=(1,1)$; $(x_1,x_2)=(15,15)$

B: $(p_1,p_2)=(2,1)$; $(x_1,x_2)=(10,20)$

C: $(p_1,p_2)=(1,2)$; $(x_1,x_2)=(20,10)$

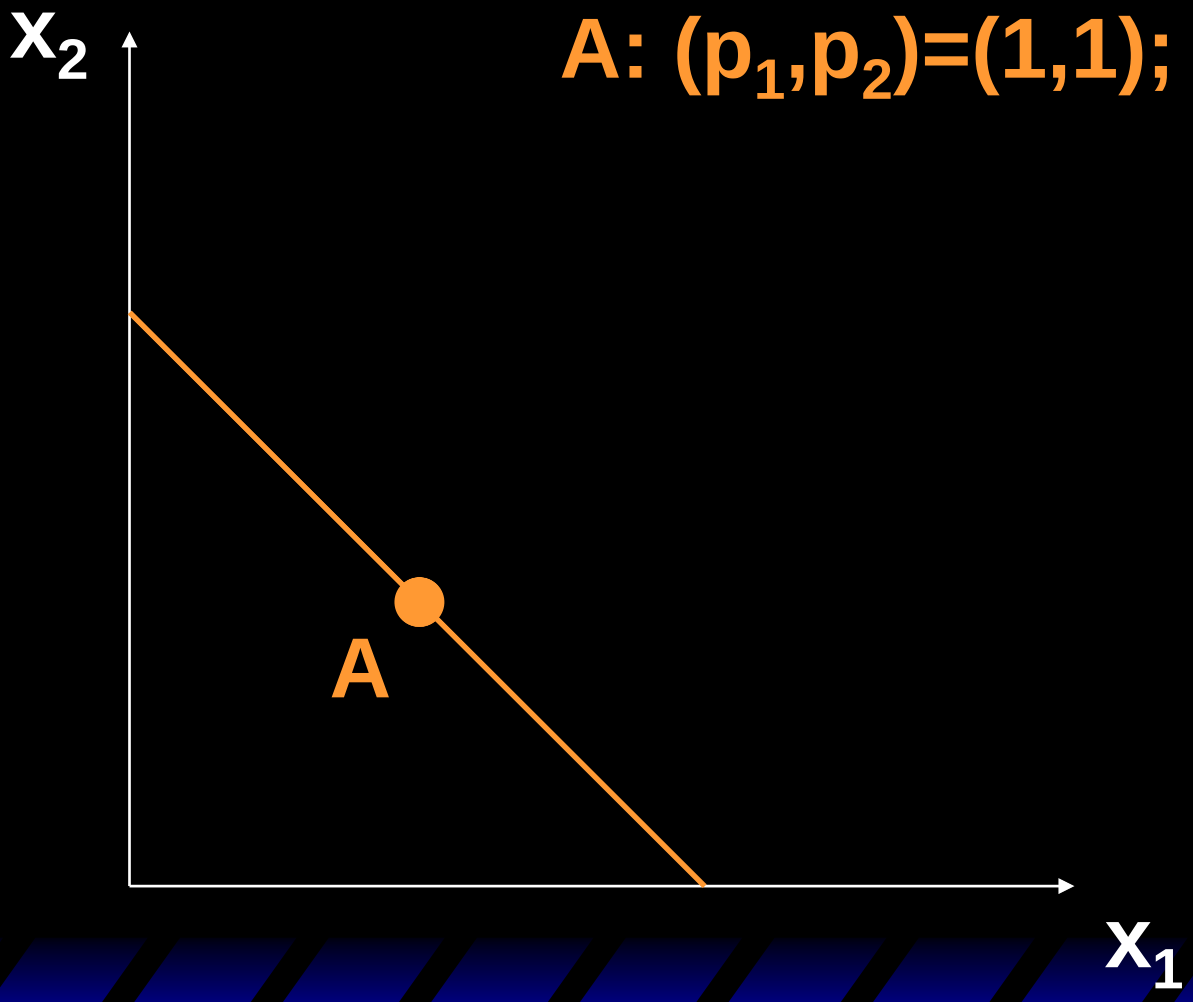
D: $(p_1,p_2)=(2,5)$; $(x_1,x_2)=(30,12)$

E: $(p_1,p_2)=(5,2)$; $(x_1,x_2)=(12,30)$.

Begin with bundles revealed to be less preferred than bundle A.

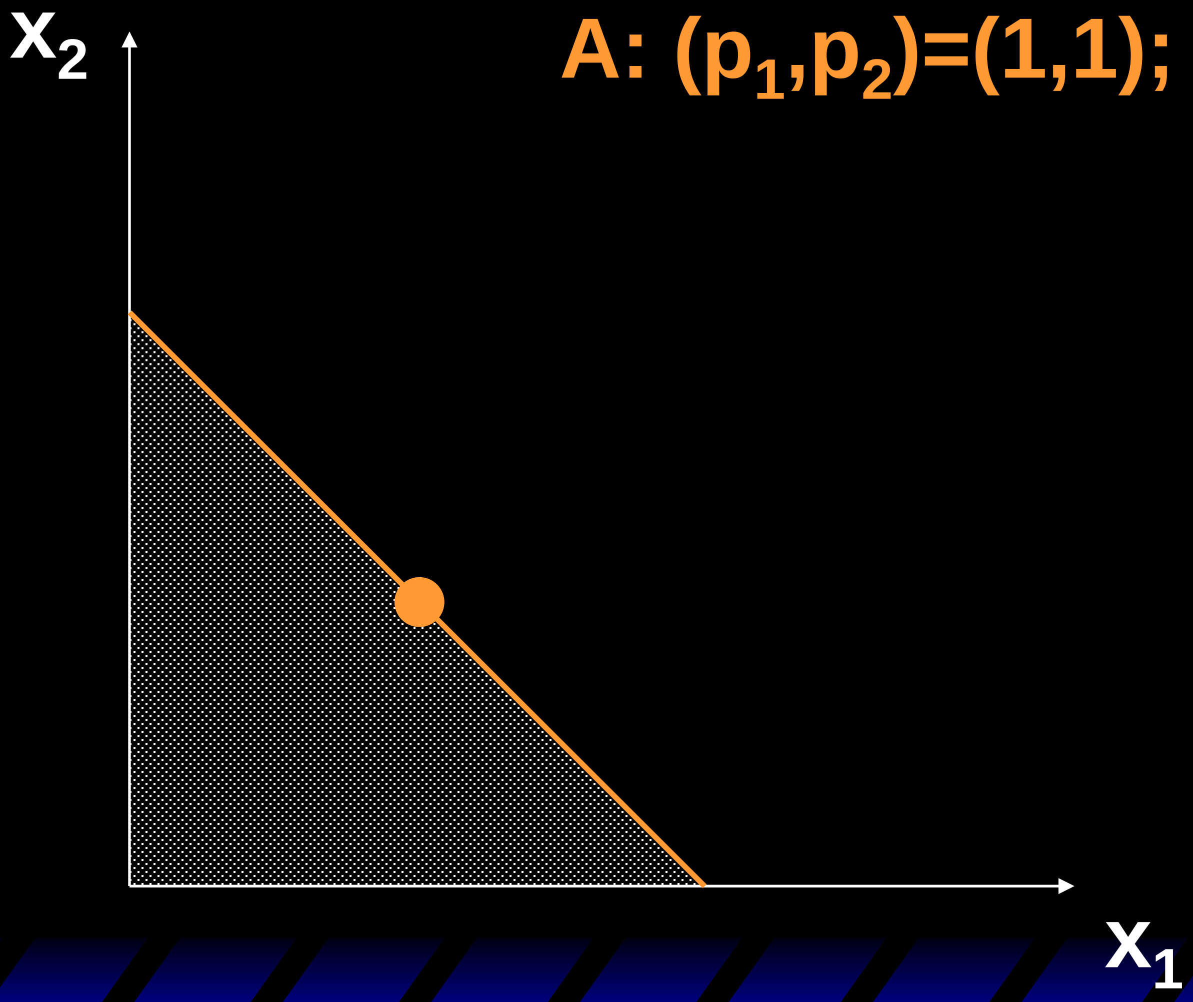
Recovering Indifference Curves

A: $(p_1,p_2)=(1,1)$; $(x_1,x_2)=(15,15)$.



Recovering Indifference Curves

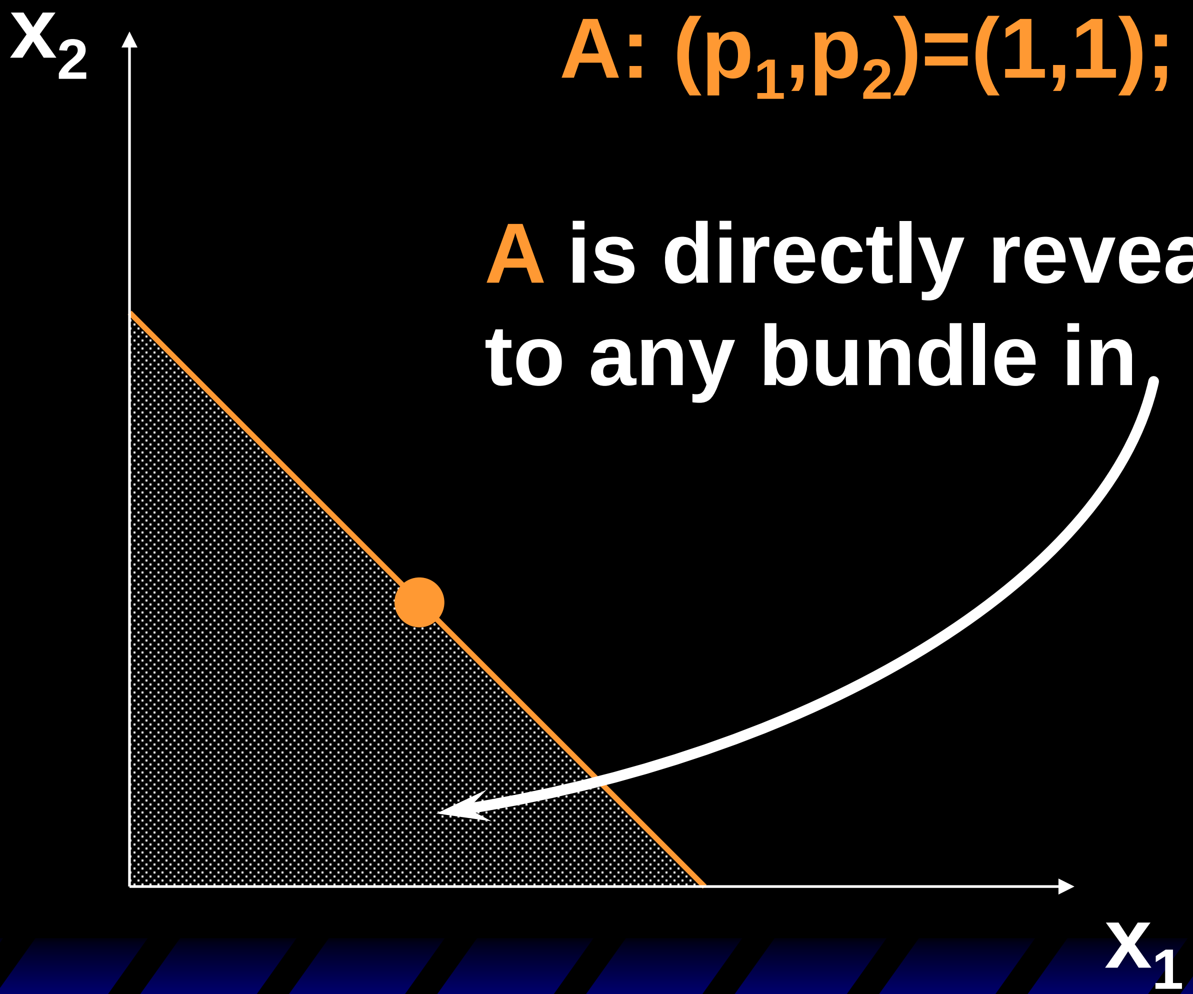
A: $(p_1,p_2)=(1,1)$; $(x_1,x_2)=(15,15)$.



Recovering Indifference Curves

A: $(p_1,p_2)=(1,1)$; $(x_1,x_2)=(15,15)$.

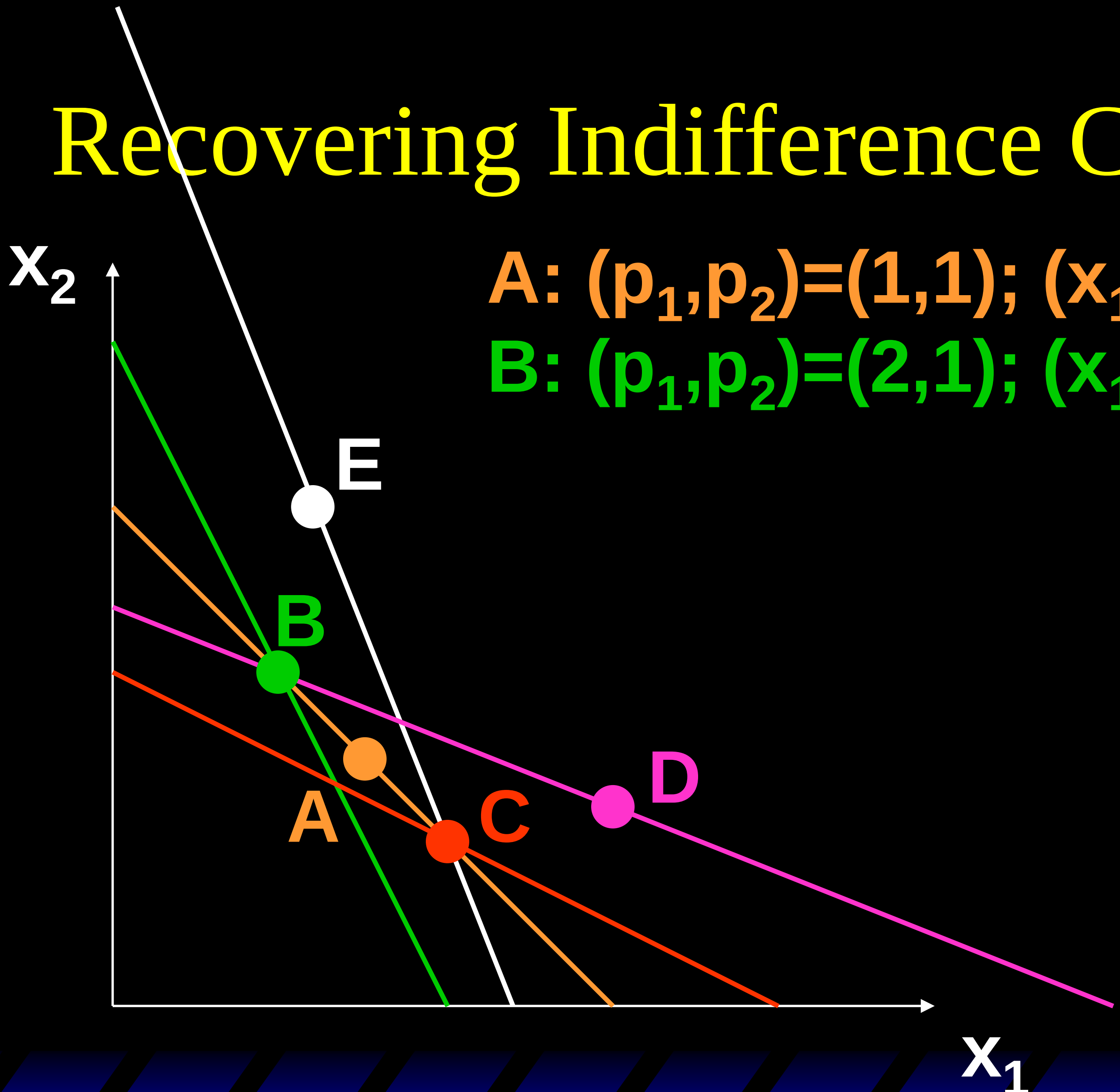
A is directly revealed preferred to any bundle in



Recovering Indifference Curves

A: $(p_1,p_2)=(1,1)$; $(x_1,x_2)=(15,15)$

B: $(p_1,p_2)=(2,1)$; $(x_1,x_2)=(10,20)$.



Recovering Indifference Curves

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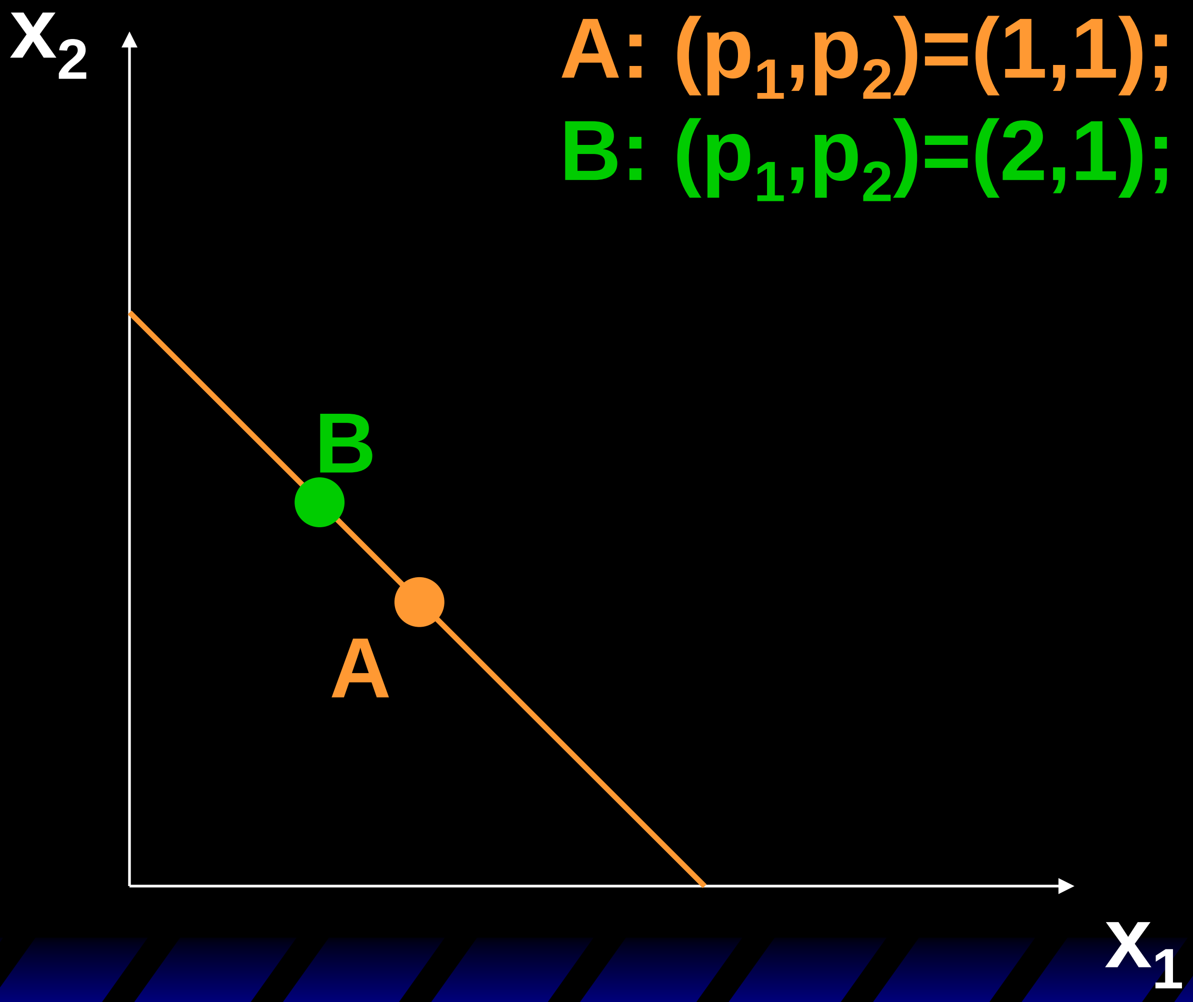
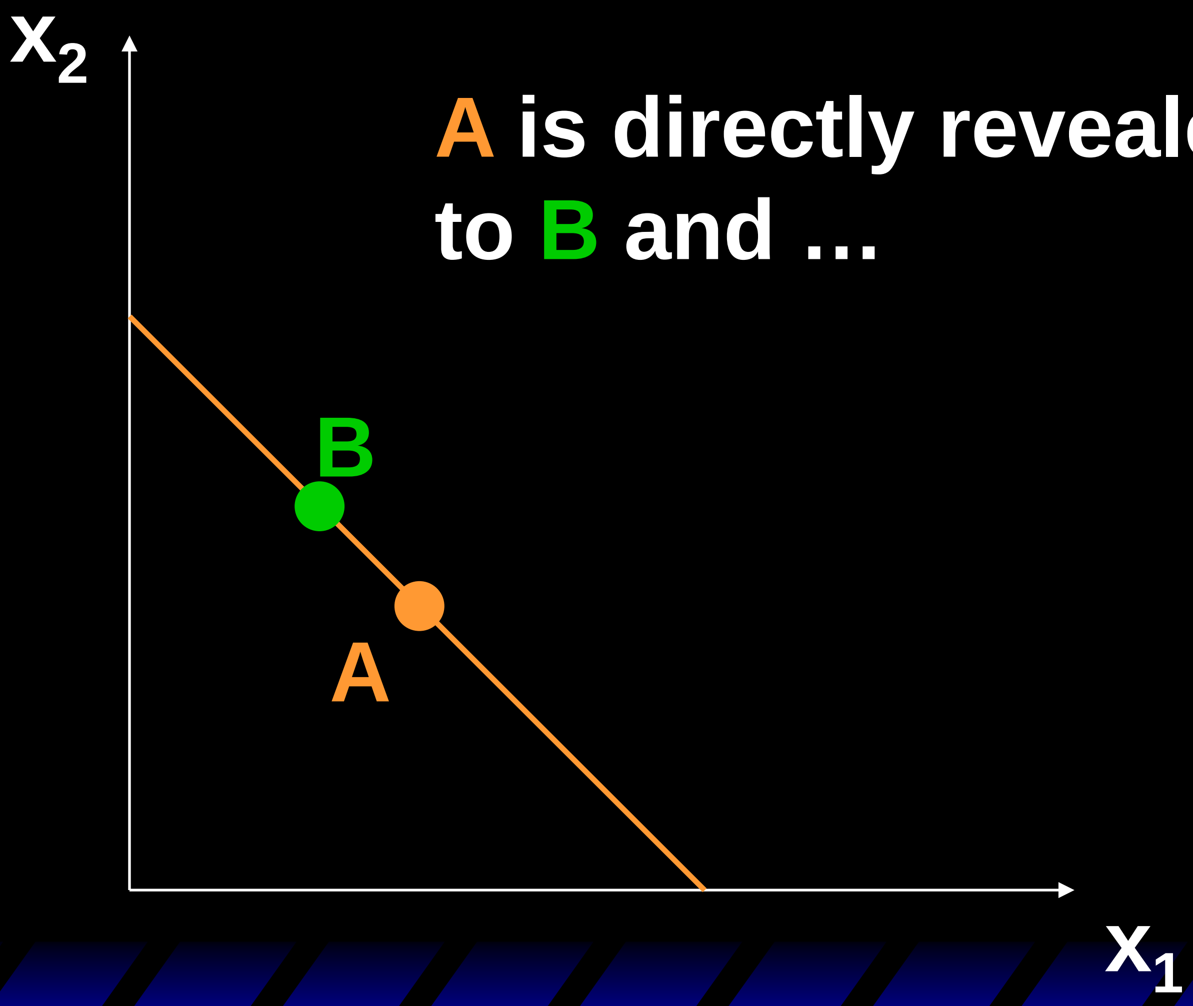
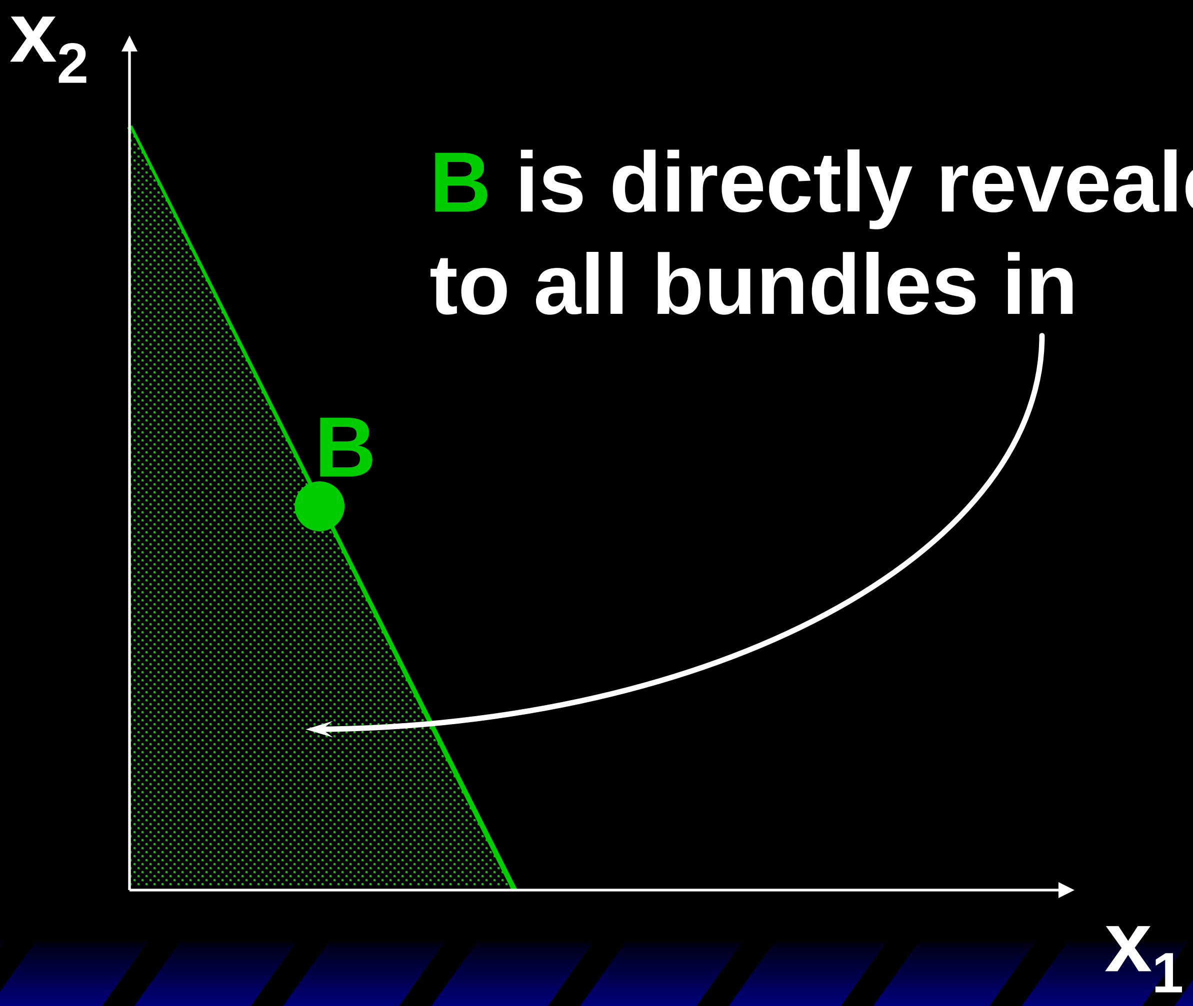


Recovering Indifference Curves



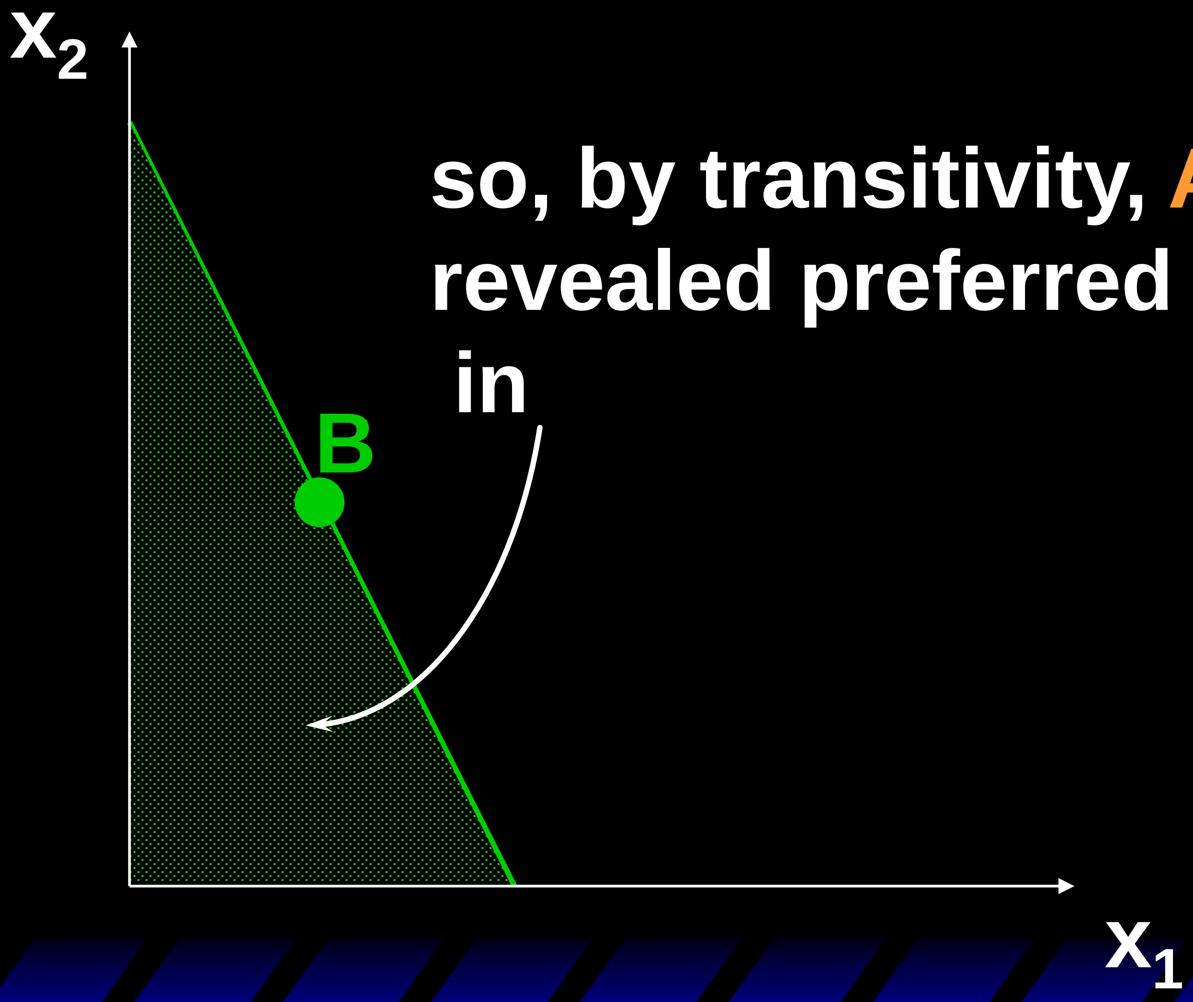
A is directly revealed preferred to B and …

Recovering Indifference Curves



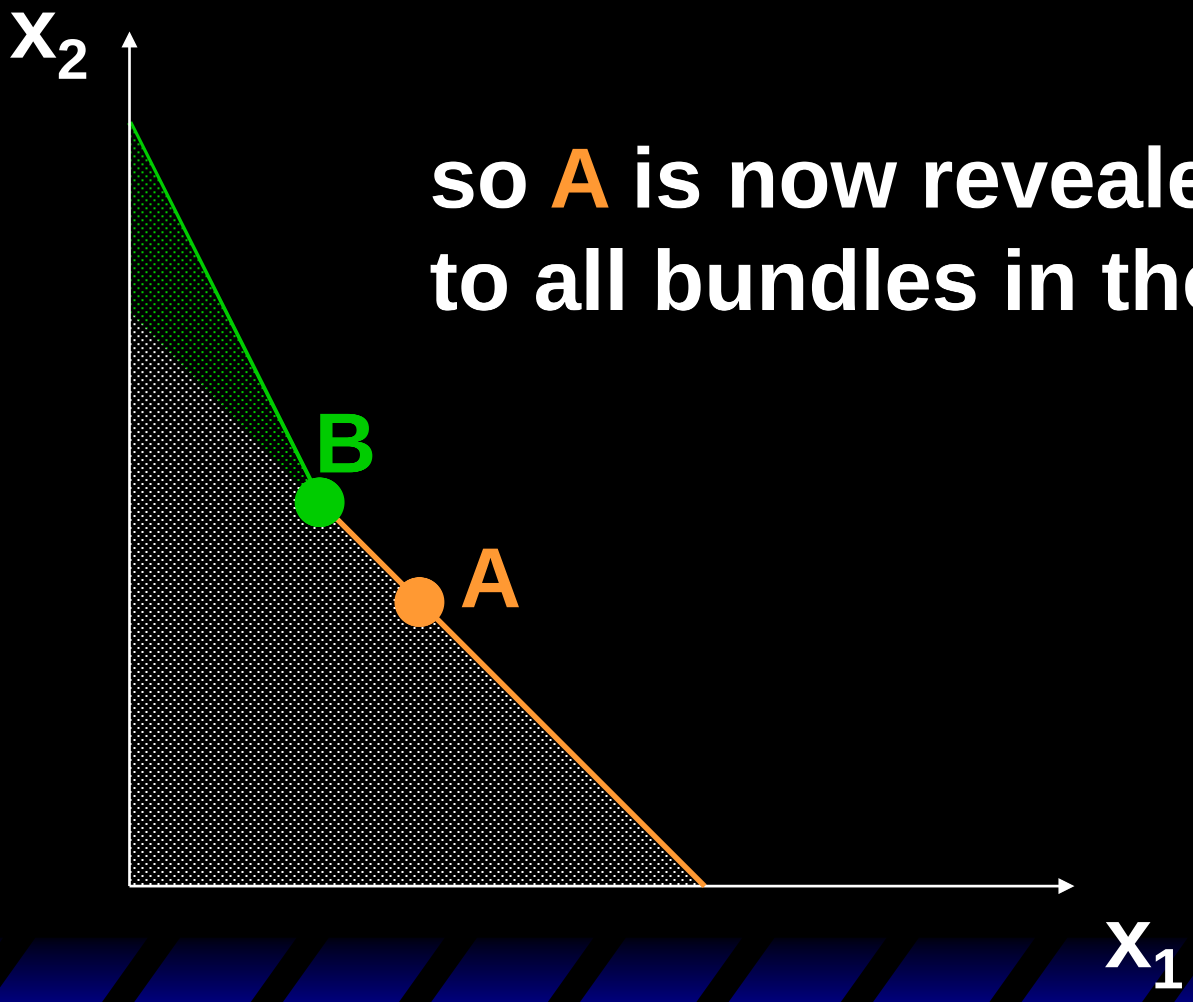
Recovering Indifference Curves

so, by transitivity, A is indirectly revealed preferred to all bundles in



Recovering Indifference Curves

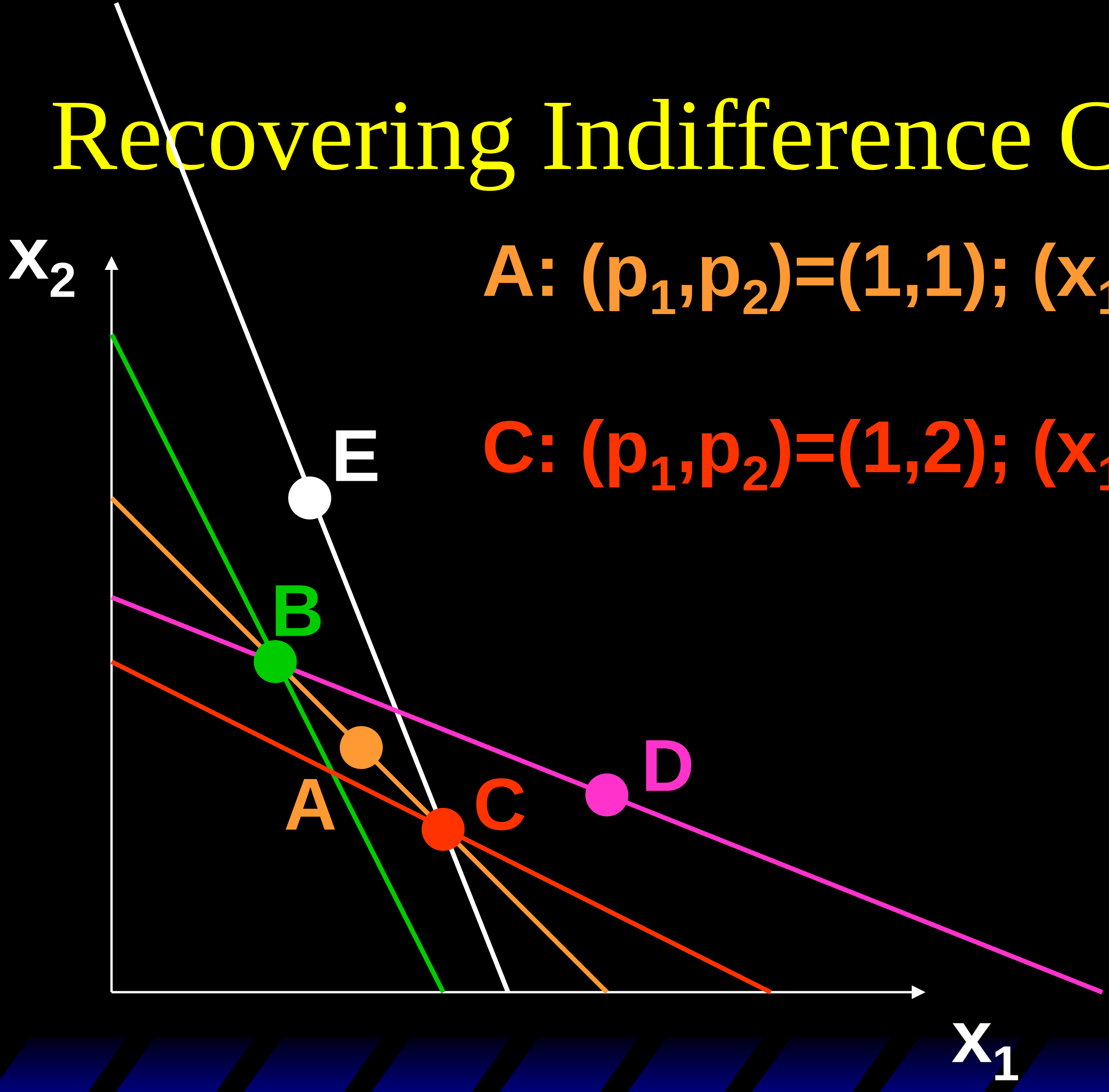
so A is now revealed preferred to all bundles in the union.



Recovering Indifference Curves

A: $(p_1,p_2)=(1,1)$; $(x_1,x_2)=(15,15)$

C: $(p_1,p_2)=(1,2)$; $(x_1,x_2)=(20,10)$.



Recovering Indifference Curves

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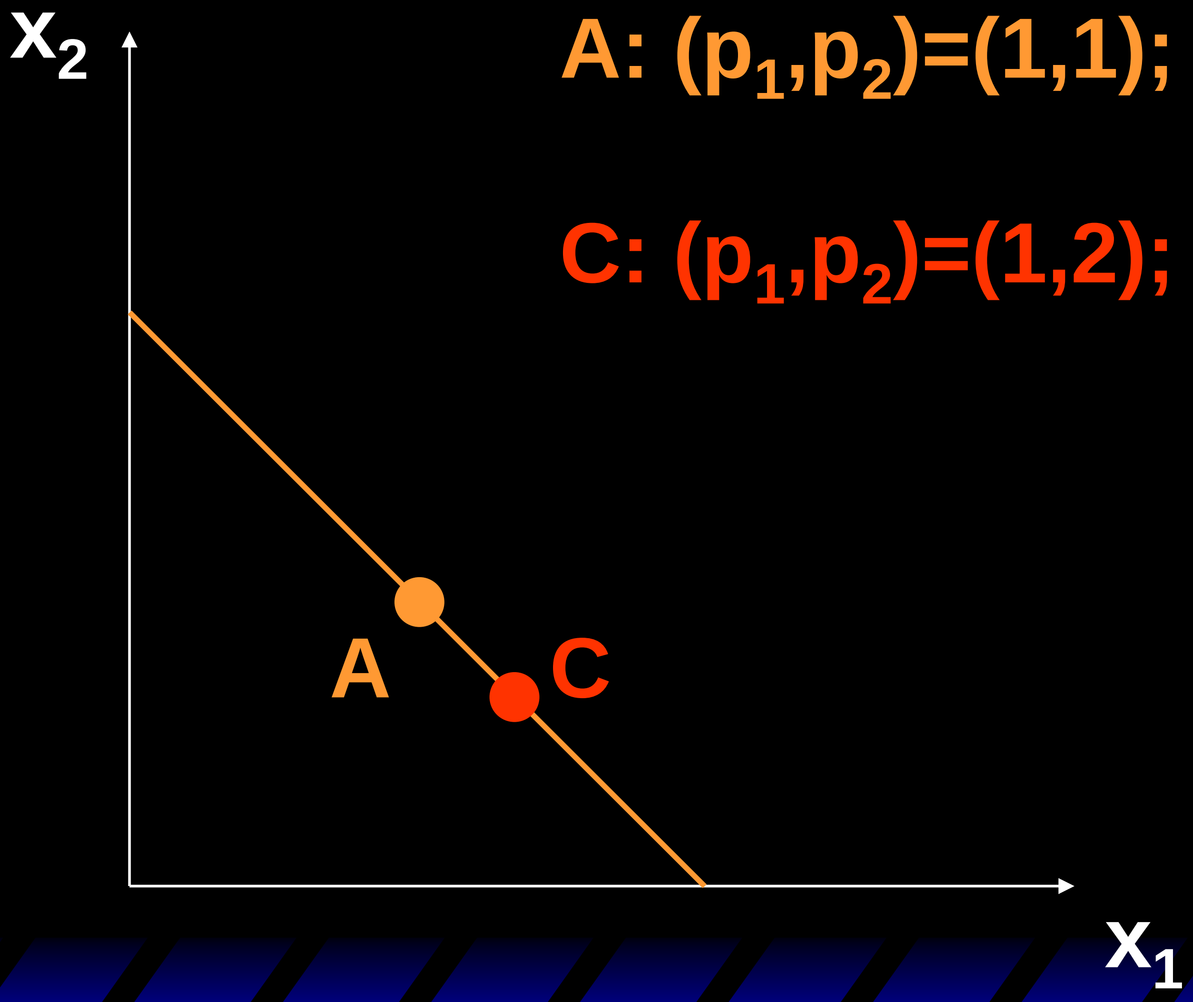
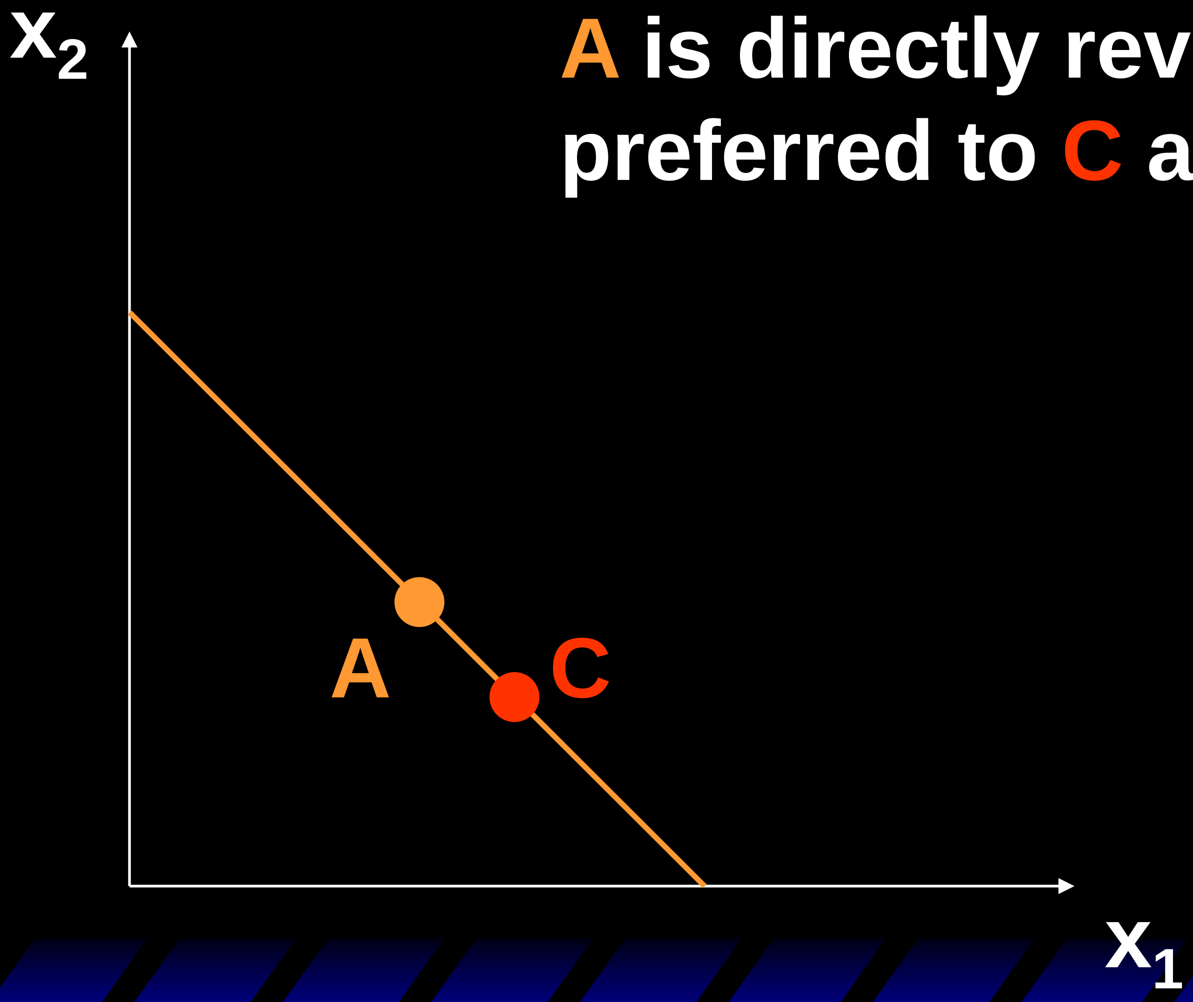


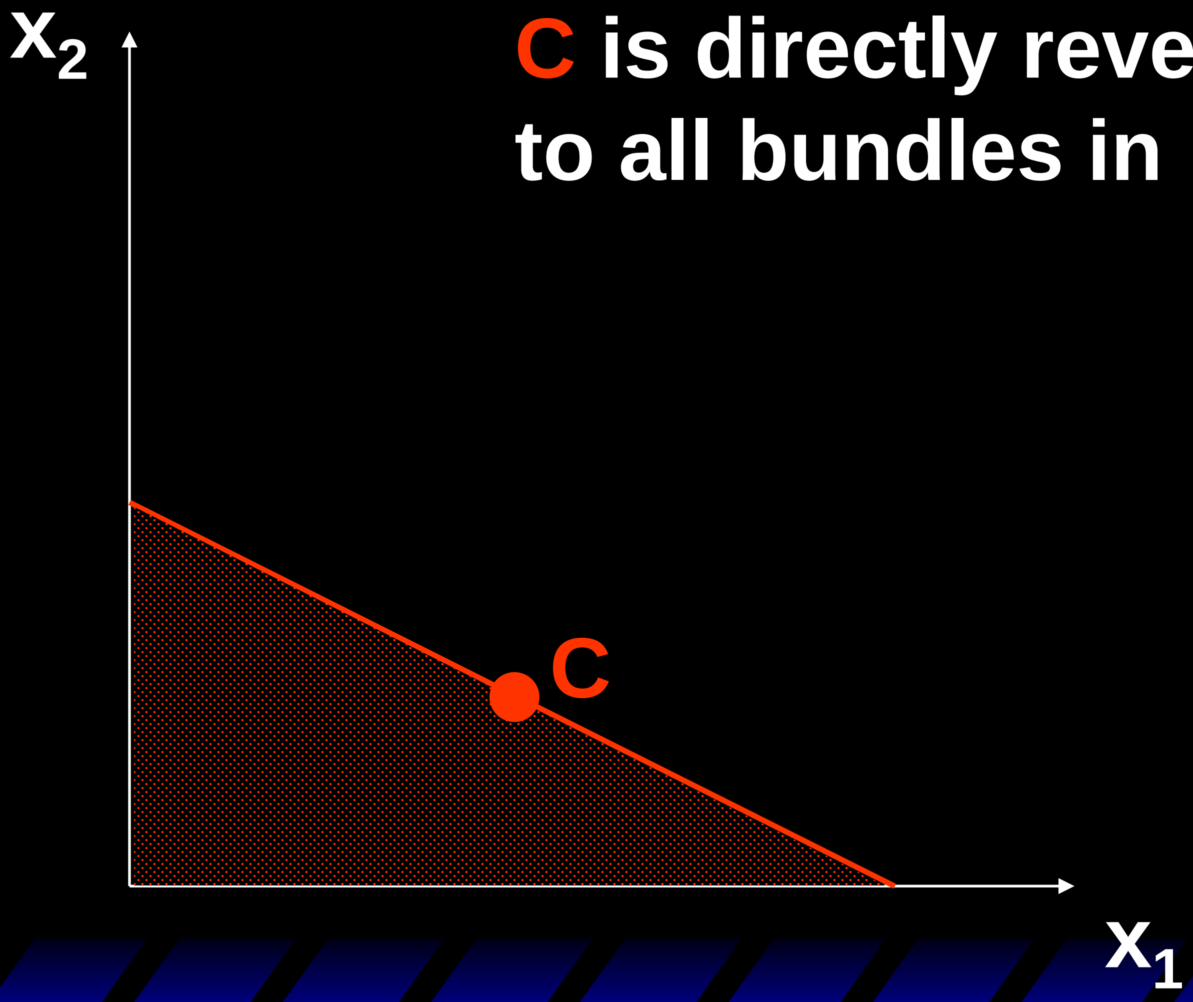
Recovering Indifference Curves



A is directly revealed preferred to C and ...

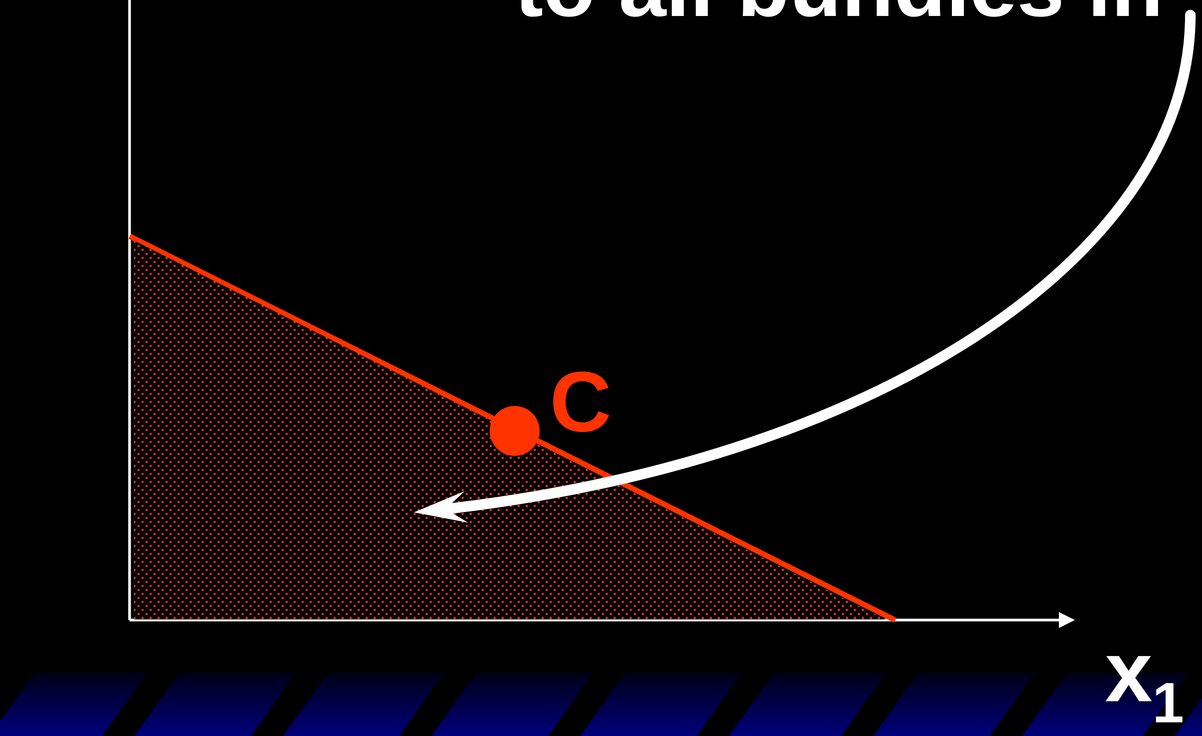
Recovering Indifference Curves

C is directly revealed preferred to all bundles in



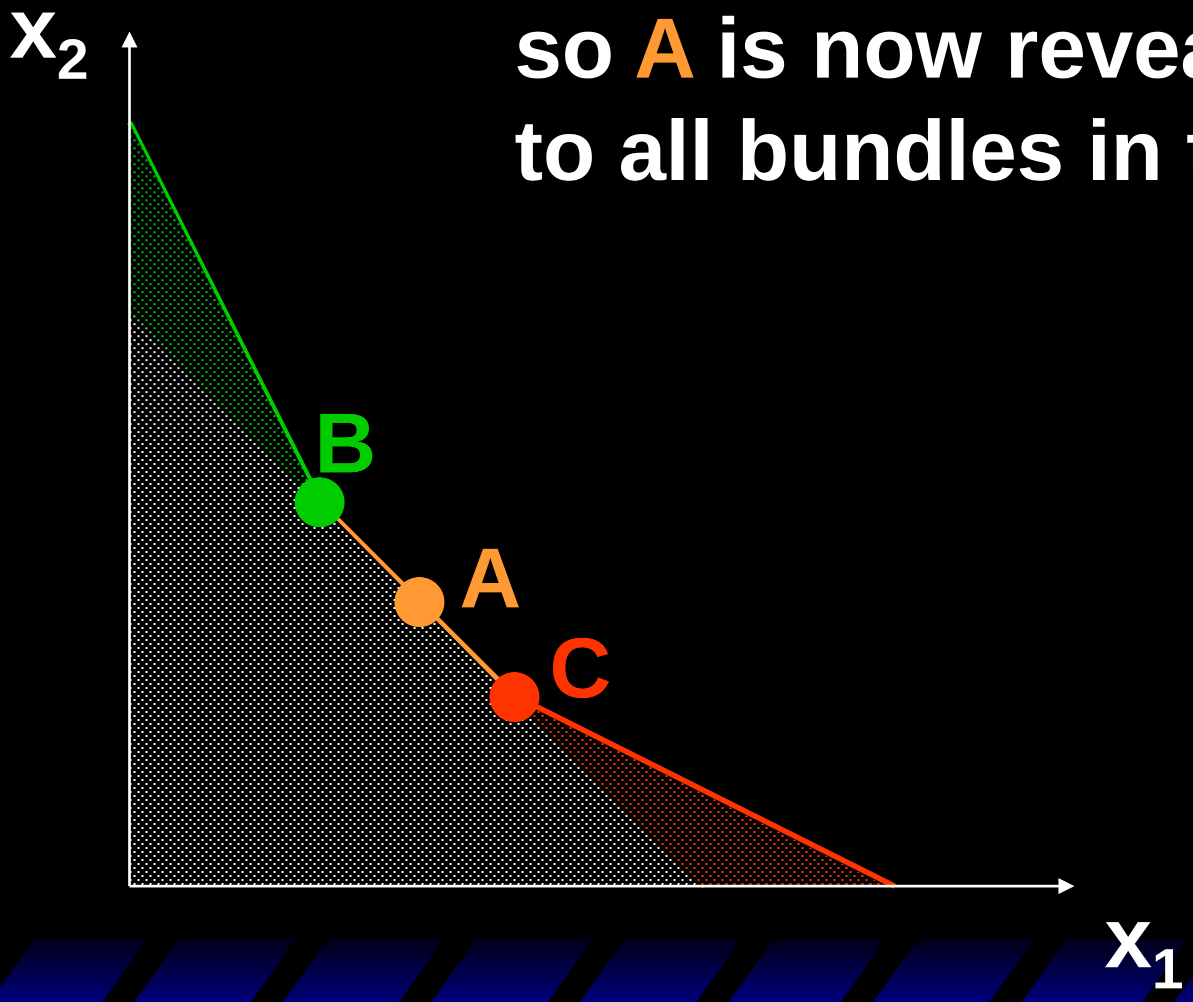
Recovering Indifference Curves

so, by transitivity, **A** is indirectly revealed preferred to all bundles in

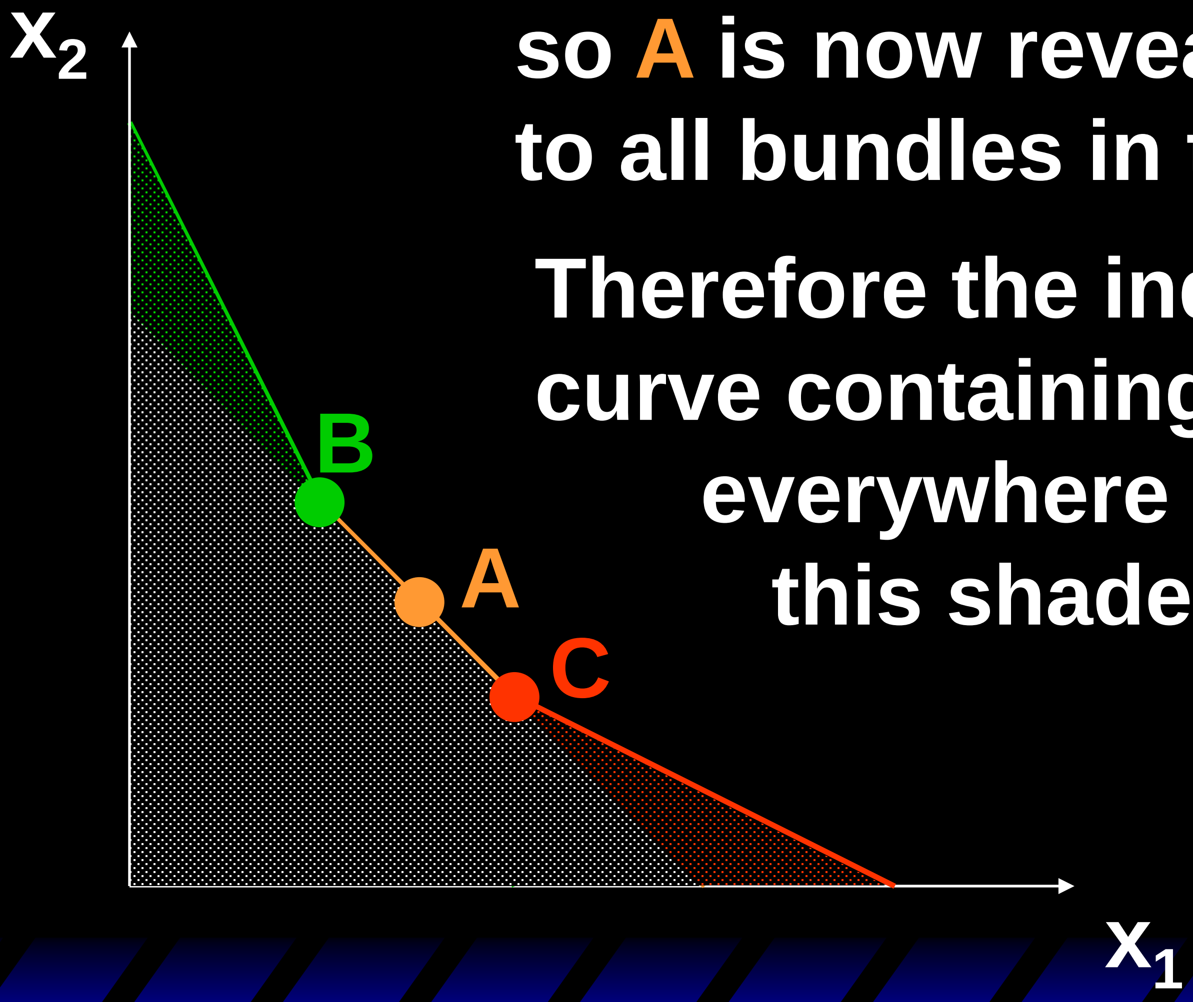


Recovering Indifference Curves

so A is now revealed preferred to all bundles in the union.



Recovering Indifference Curves



so A is now revealed preferred to all bundles in the union.

Therefore the indifference curve containing A must lie everywhere else above this shaded set.

Recovering Indifference Curves

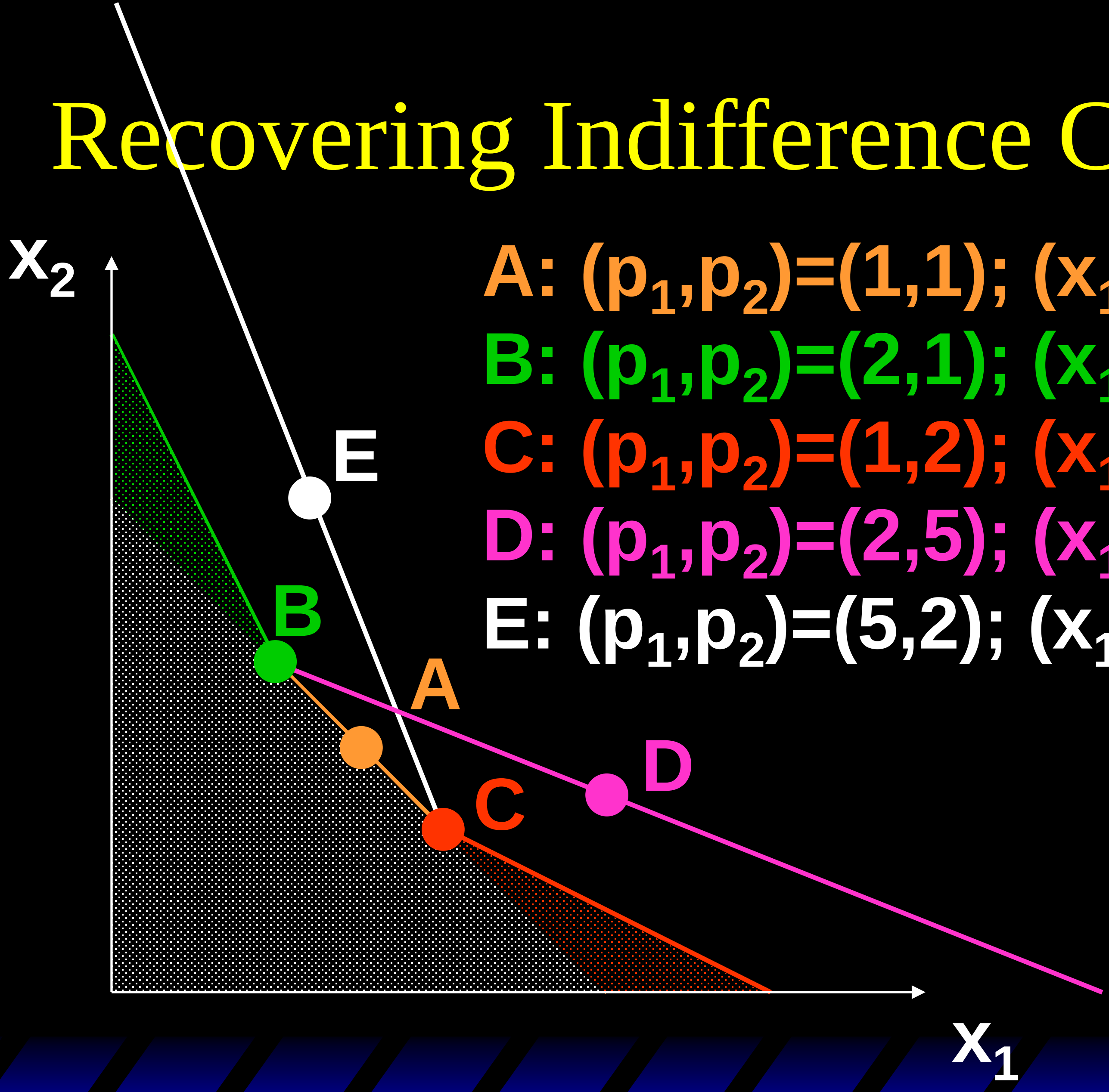
- Now, what about the bundles revealed as more preferred than A?

Recovering Indifference Curves

	A	B	C	D	E
A		D	D		
B					
C					
D	D	D	D		
E	D	D	D		

Both direct and indirect revelations; neither WARP nor SARP are violated by the data.

Recovering Indifference Curves



A: $(p_1,p_2)=(1,1)$; $(x_1,x_2)=(15,15)$

B: $(p_1,p_2)=(2,1)$; $(x_1,x_2)=(10,20)$

C: $(p_1,p_2)=(1,2)$; $(x_1,x_2)=(20,10)$

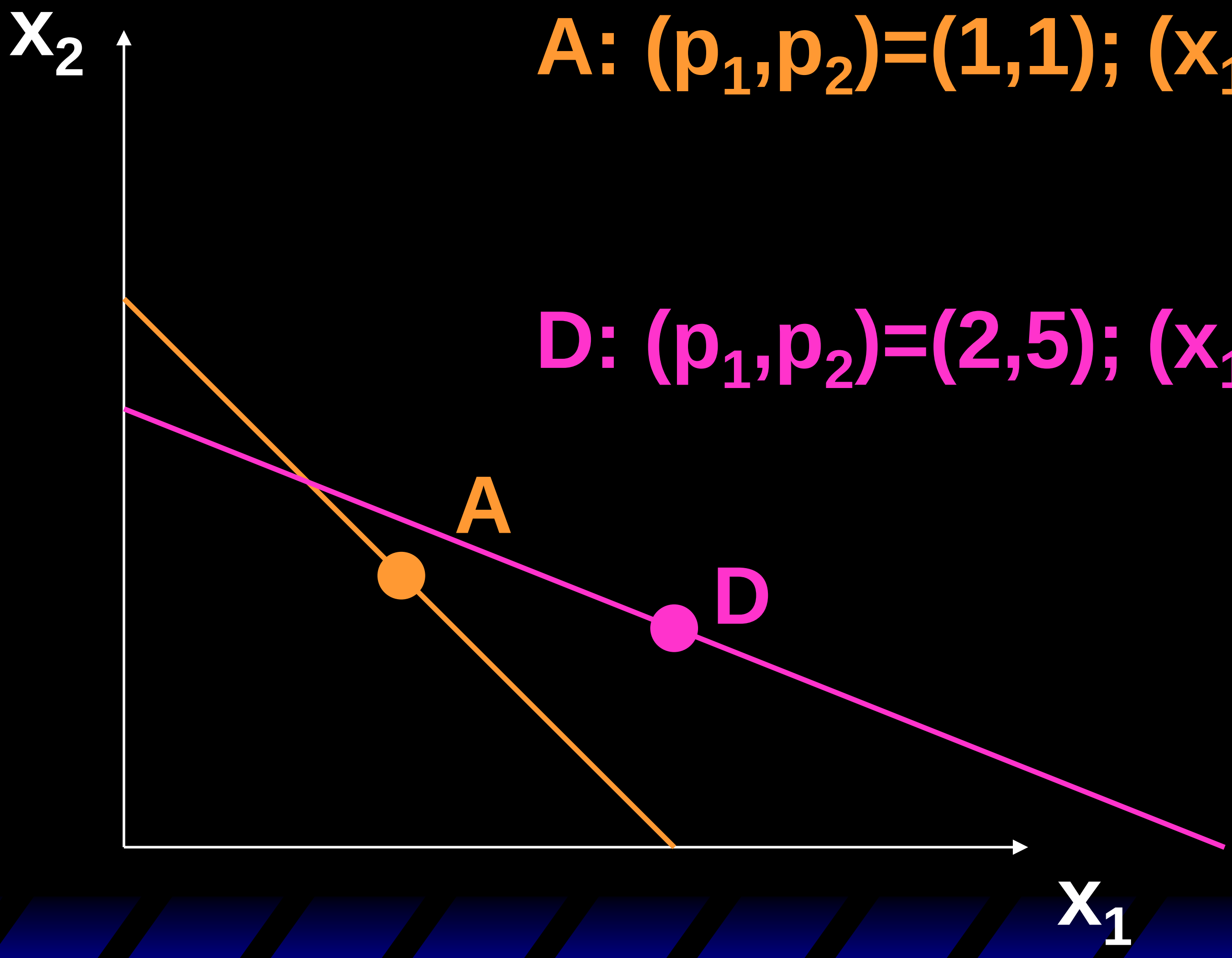
D: $(p_1,p_2)=(2,5)$; $(x_1,x_2)=(30,12)$

E: $(p_1,p_2)=(5,2)$; $(x_1,x_2)=(12,30)$.

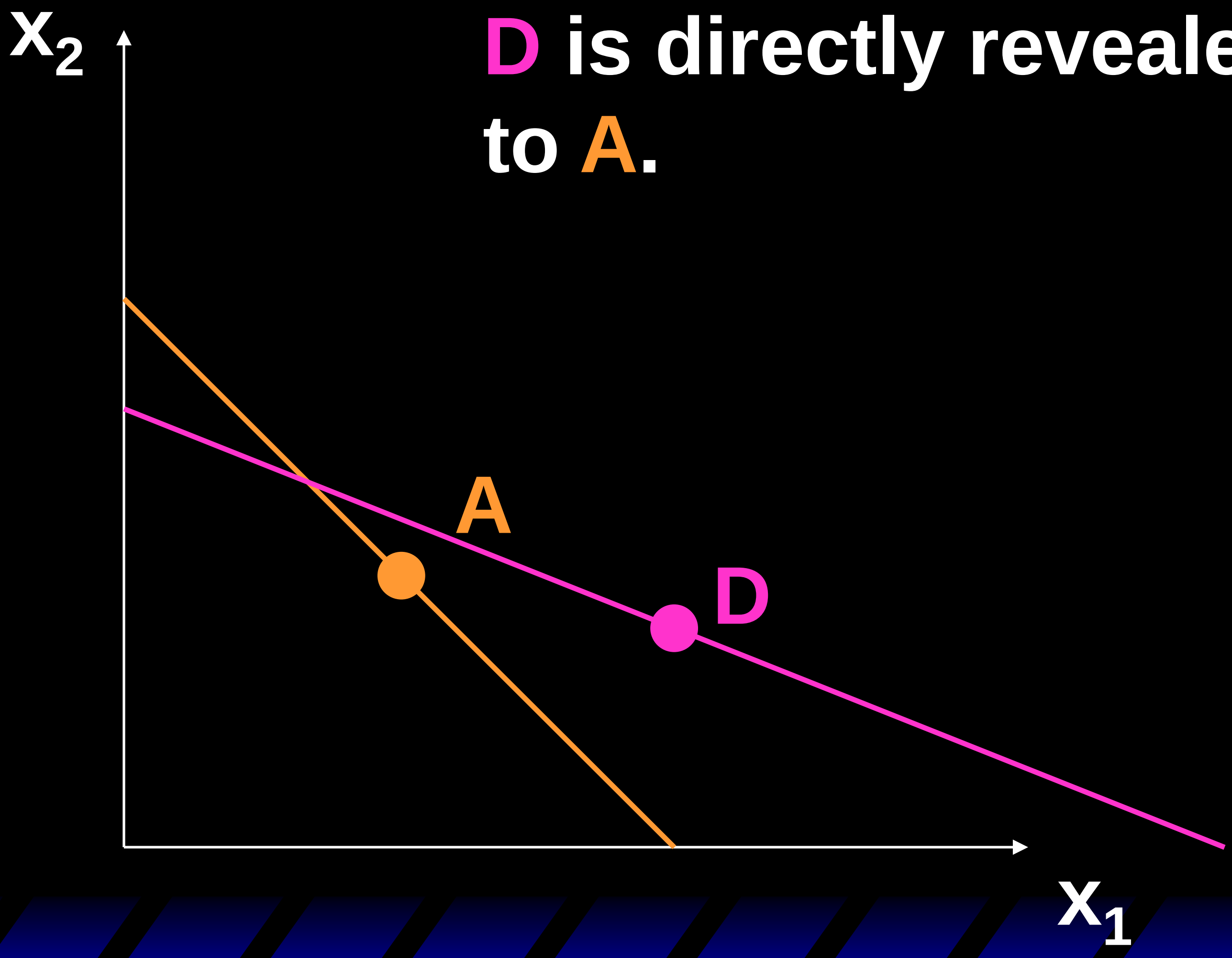
Recovering Indifference Curves

A: $(p_1,p_2)=(1,1)$; $(x_1,x_2)=(15,15)$

D: $(p_1,p_2)=(2,5)$; $(x_1,x_2)=(30,12)$.

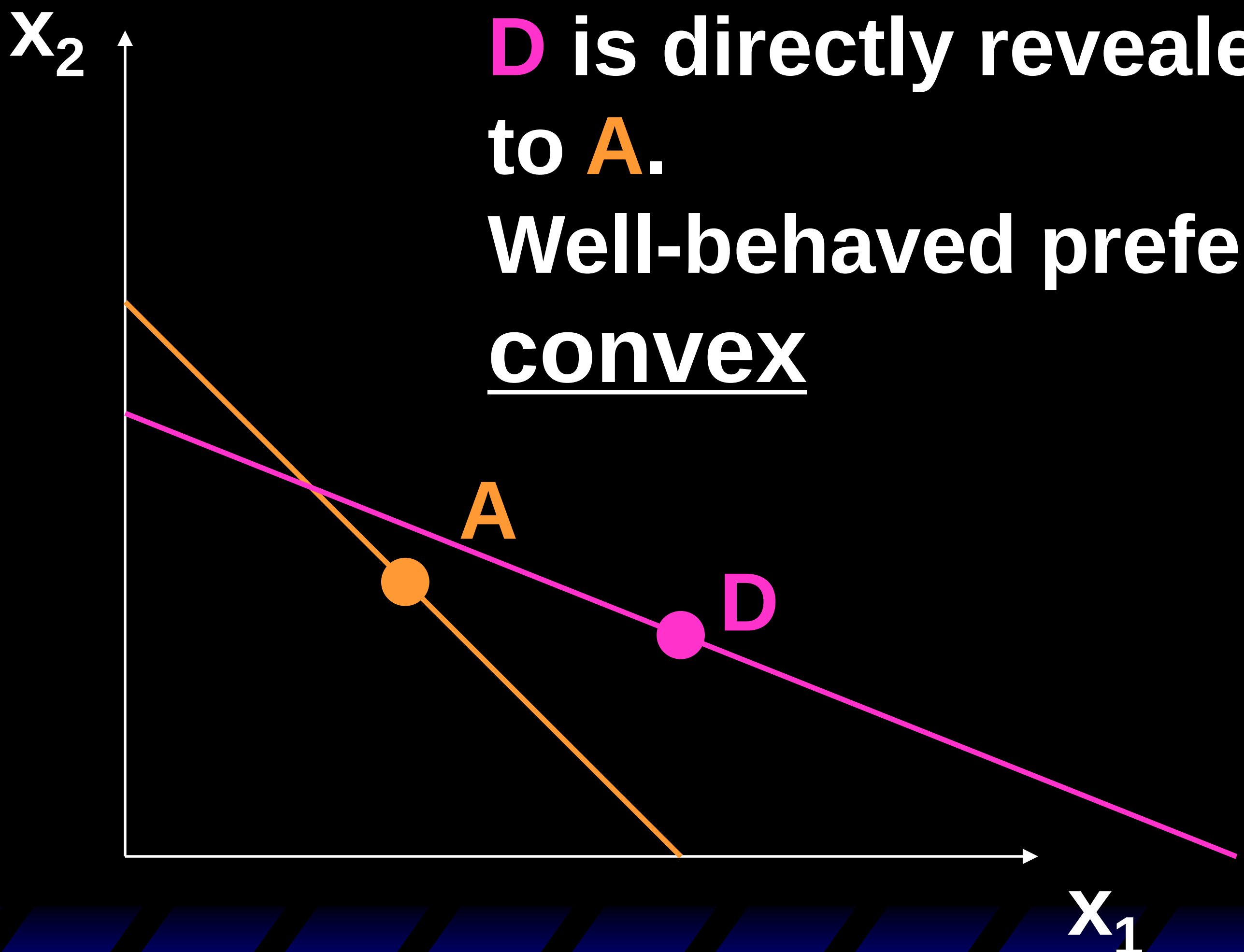


Recovering Indifference Curves



D is directly revealed preferred to A.

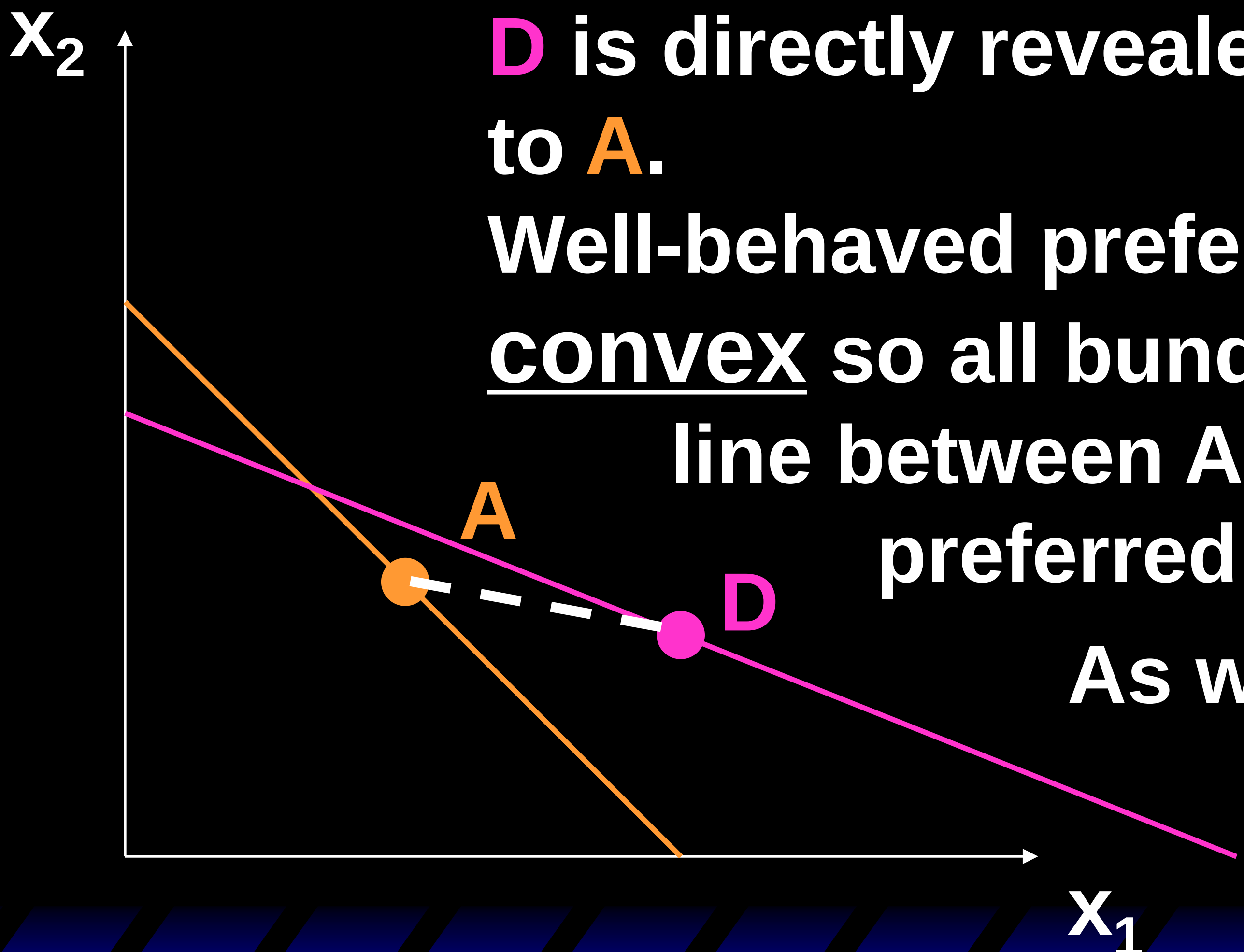
Recovering Indifference Curves



D is directly revealed preferred to A.

Well-behaved preferences are <u>convex</u>

Recovering Indifference Curves



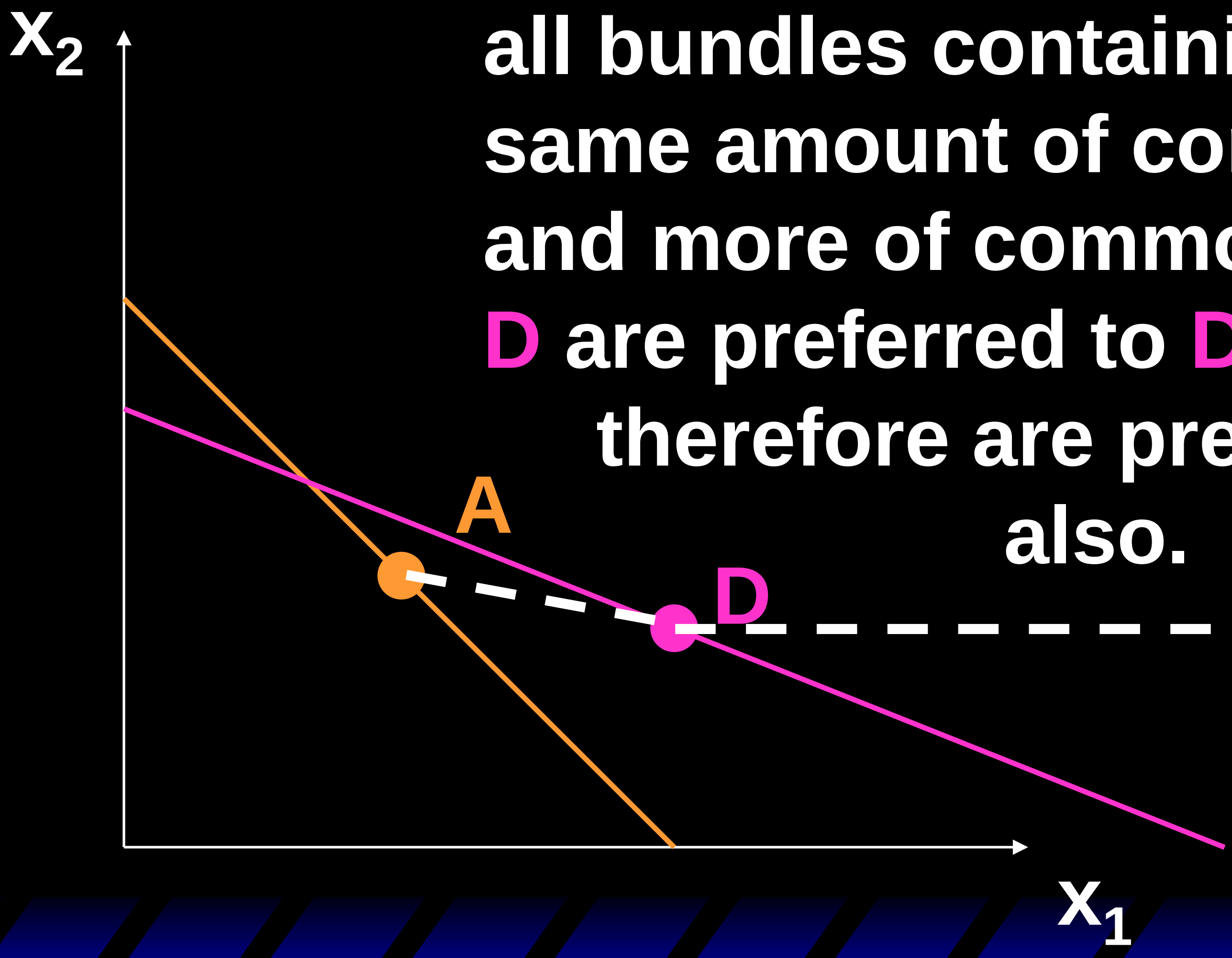
D is directly revealed preferred to A.

Well-behaved preferences are convex so all bundles on the line between A and D are preferred to A also.

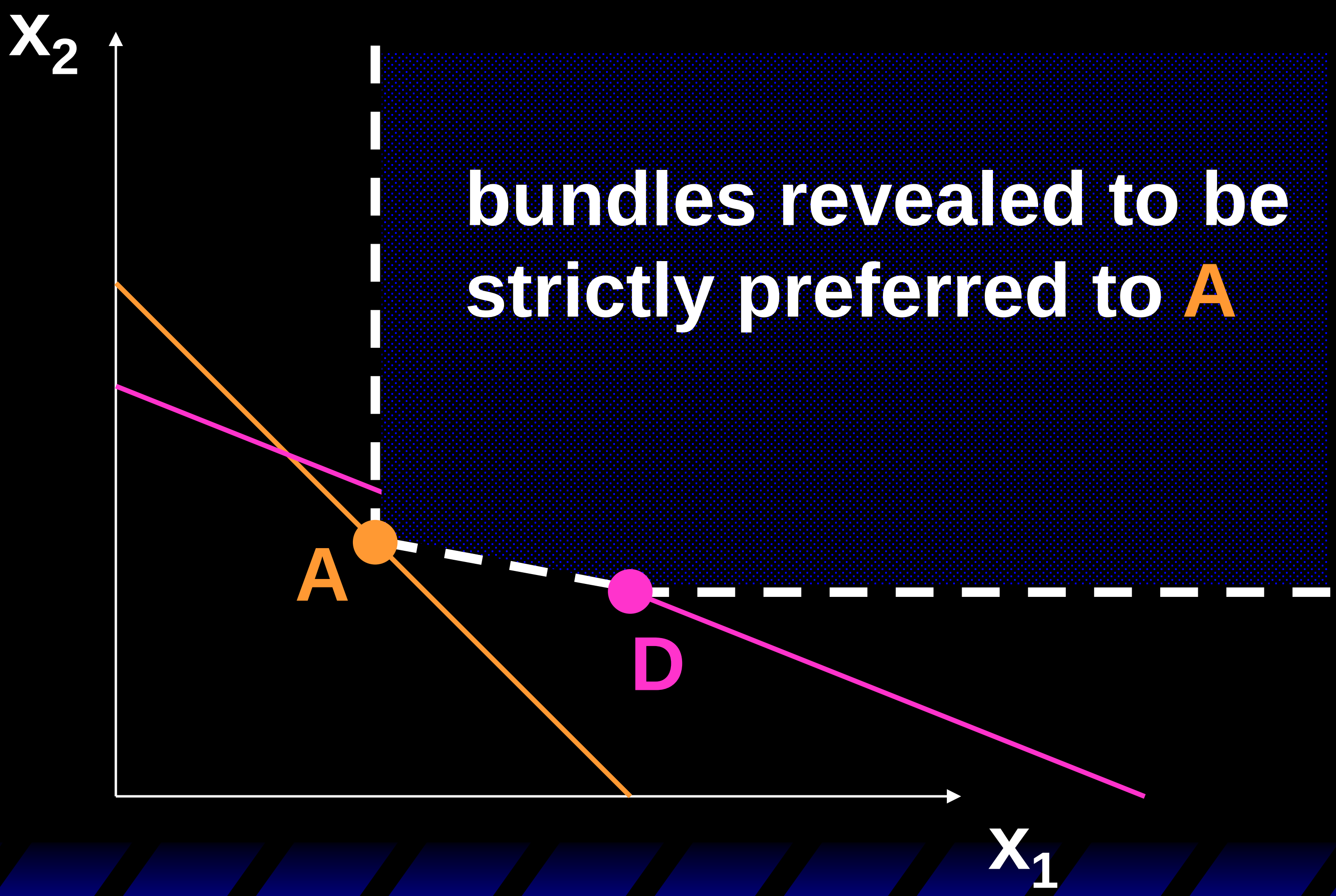
As well, ...

Recovering Indifference Curves

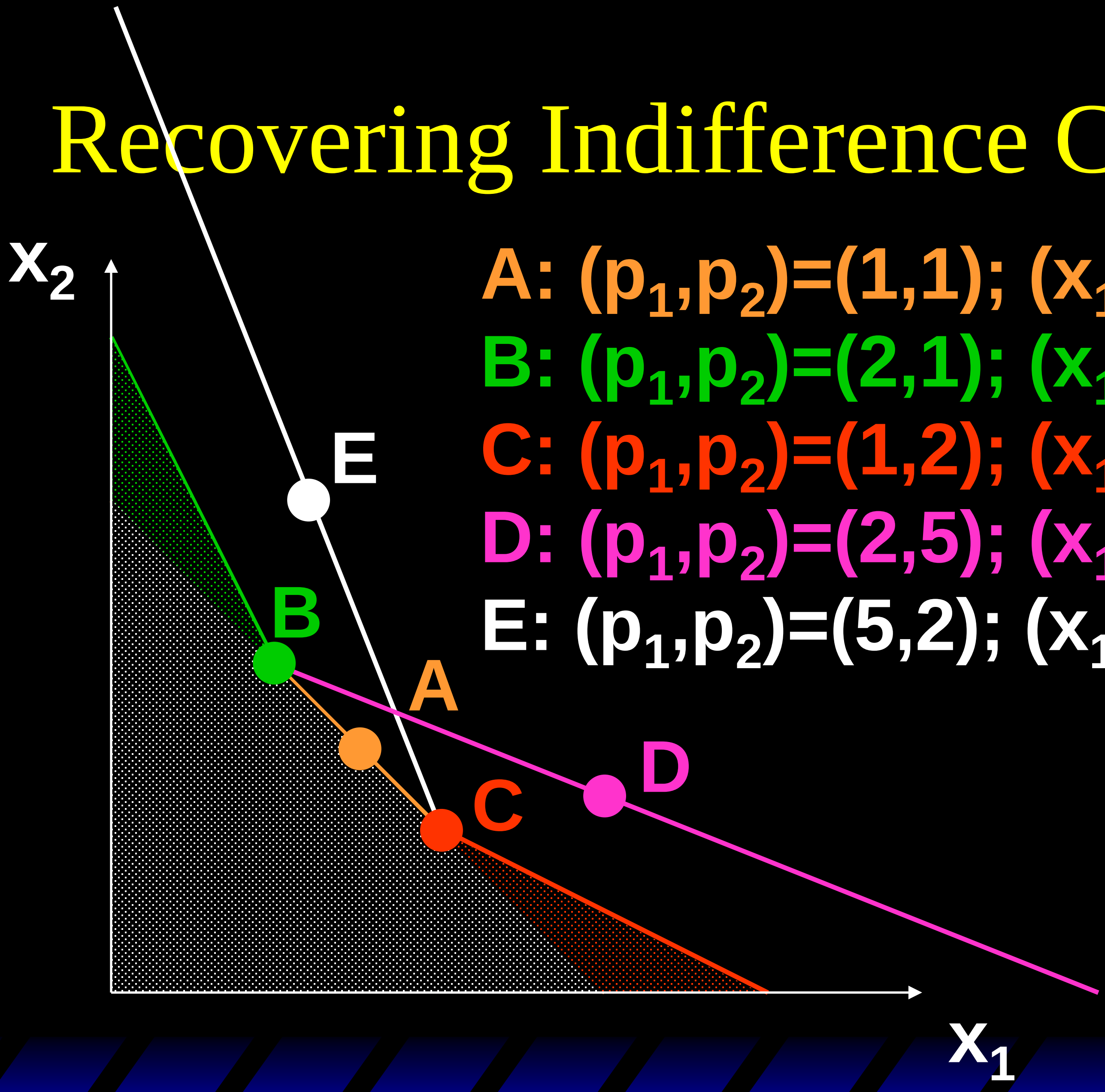
all bundles containing the same amount of commodity 2 and more of commodity 1 than D are preferred to D and therefore are preferred to A also.



Recovering Indifference Curves



Recovering Indifference Curves



A: $(p_1,p_2)=(1,1)$; $(x_1,x_2)=(15,15)$

B: $(p_1,p_2)=(2,1)$; $(x_1,x_2)=(10,20)$

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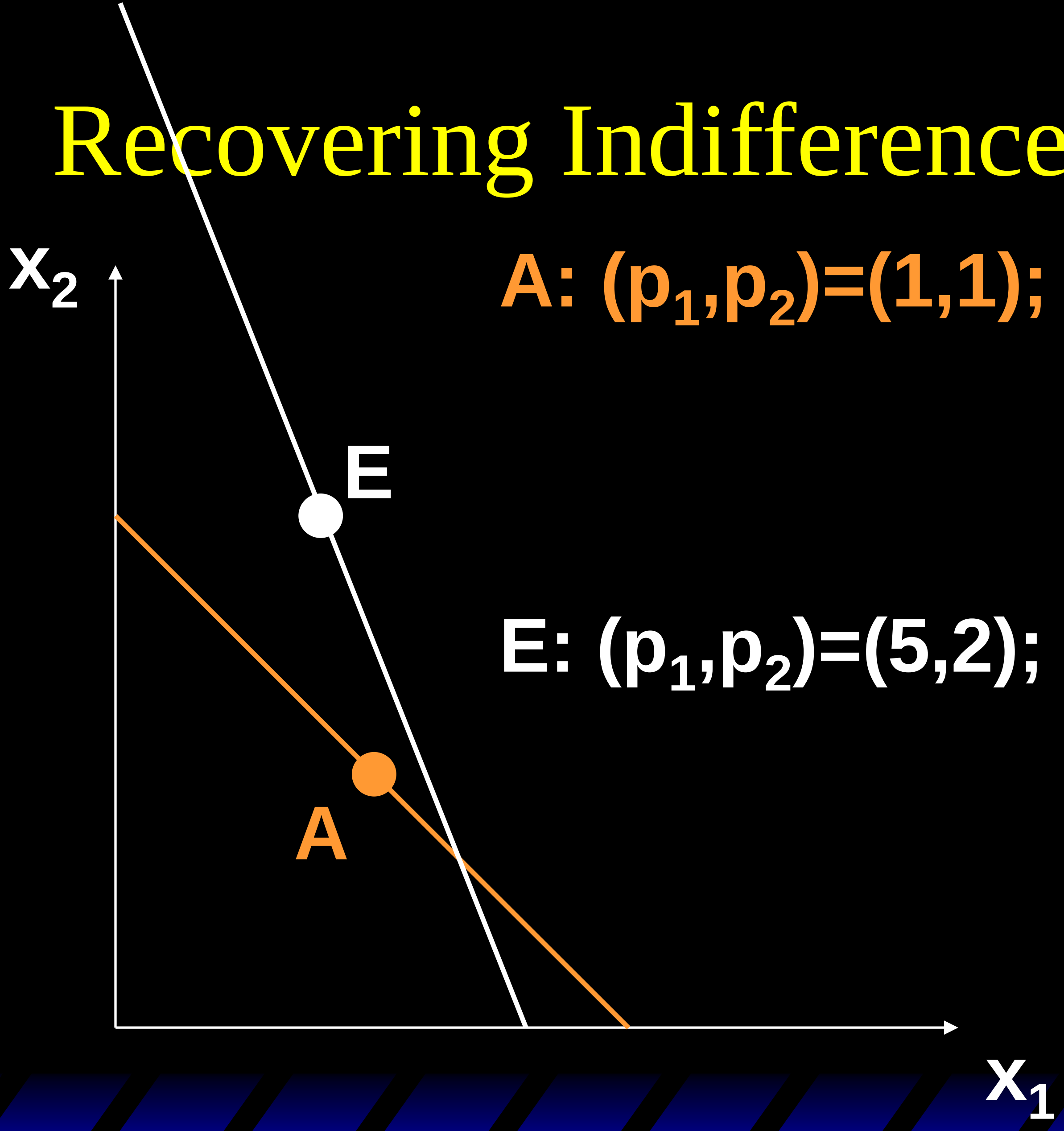
D: $(p_1,p_2)=(2,5)$; $(x_1,x_2)=(30,12)$

E: $(p_1,p_2)=(5,2)$; $(x_1,x_2)=(12,30)$.

Recovering Indifference Curves

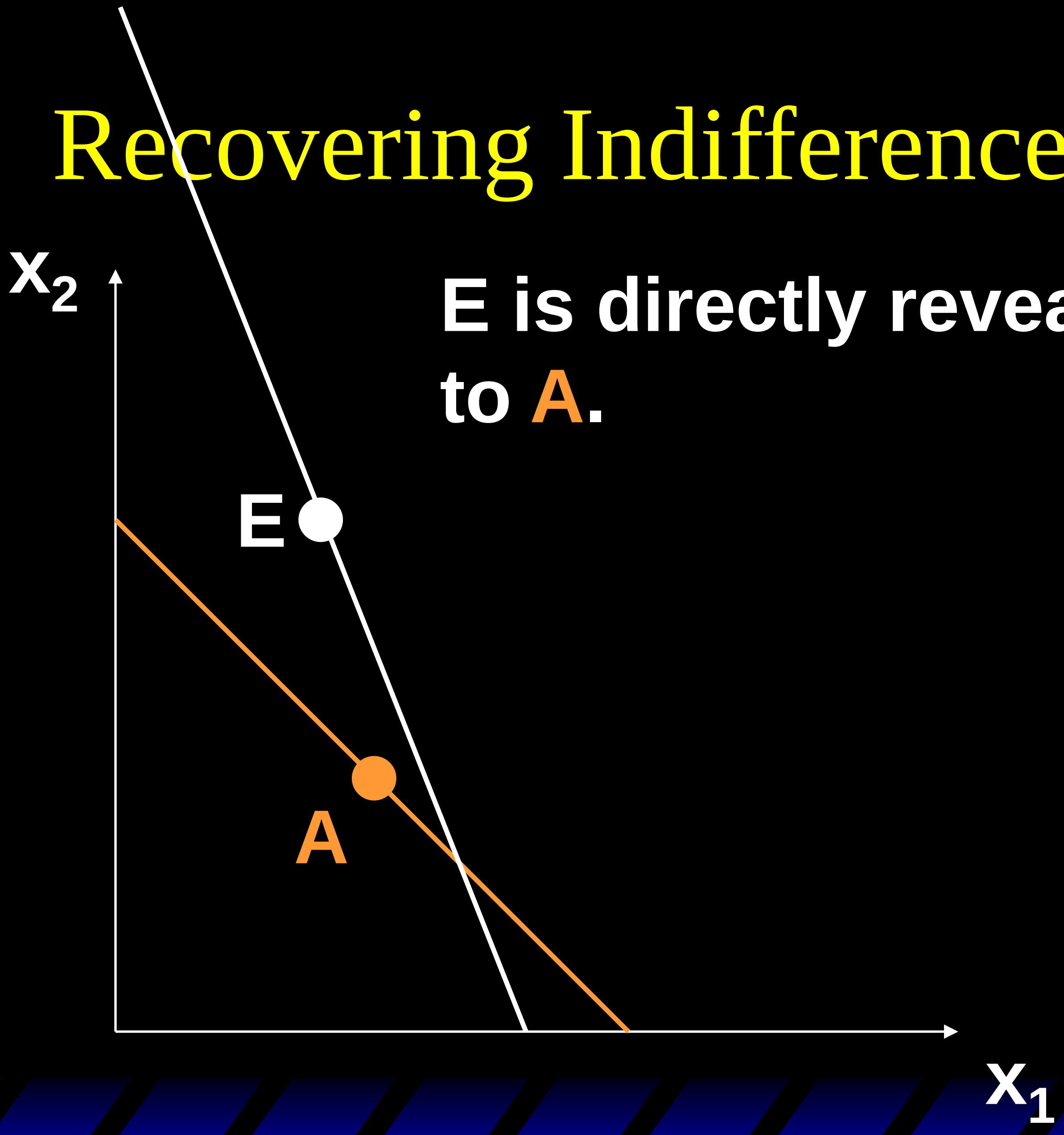
A: $(p_1,p_2)=(1,1)$; $(x_1,x_2)=(15,15)$

E: $(p_1,p_2)=(5,2)$; $(x_1,x_2)=(12,30)$.

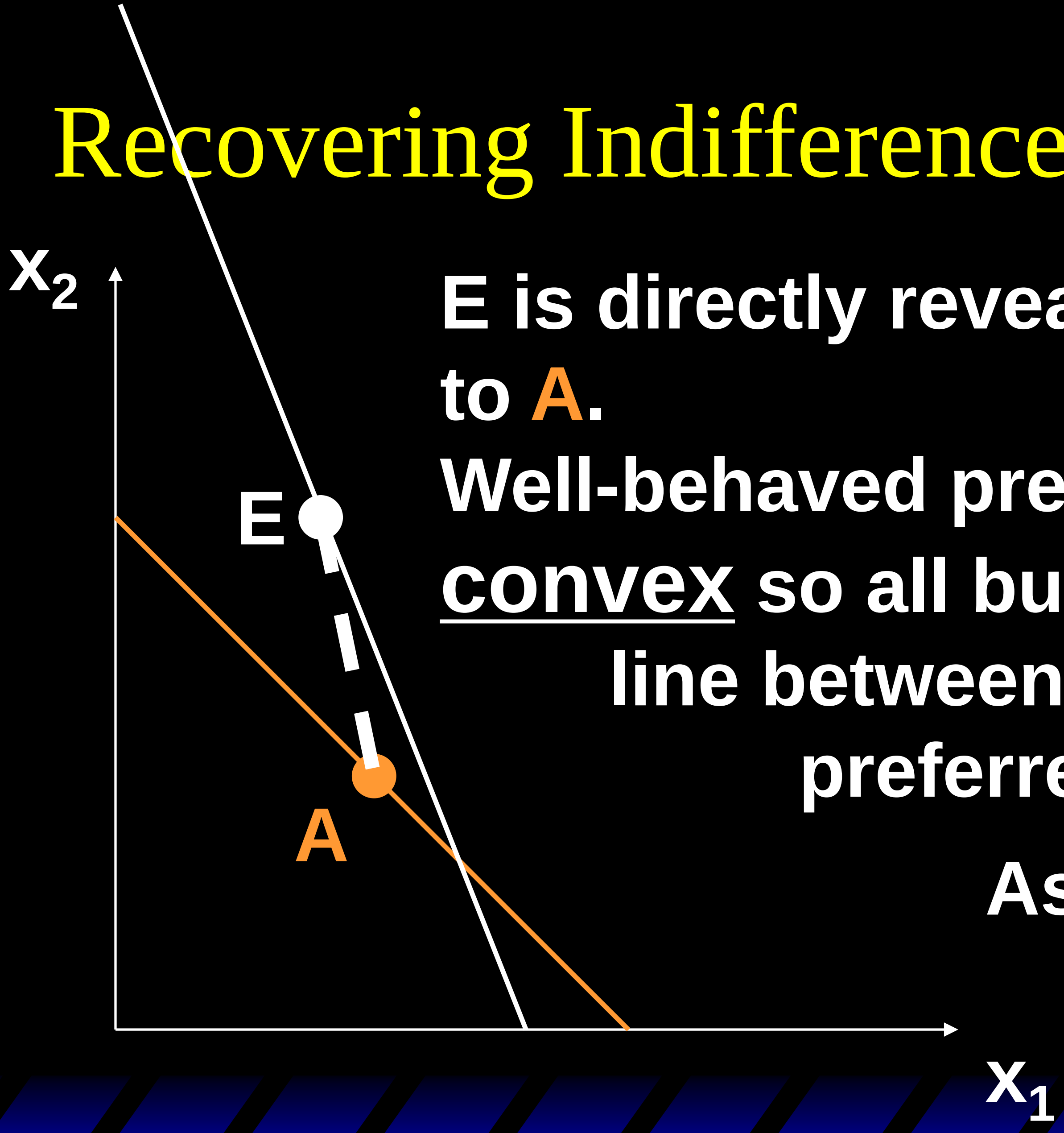


Recovering Indifference Curves

E is directly revealed preferred to A.



Recovering Indifference Curves

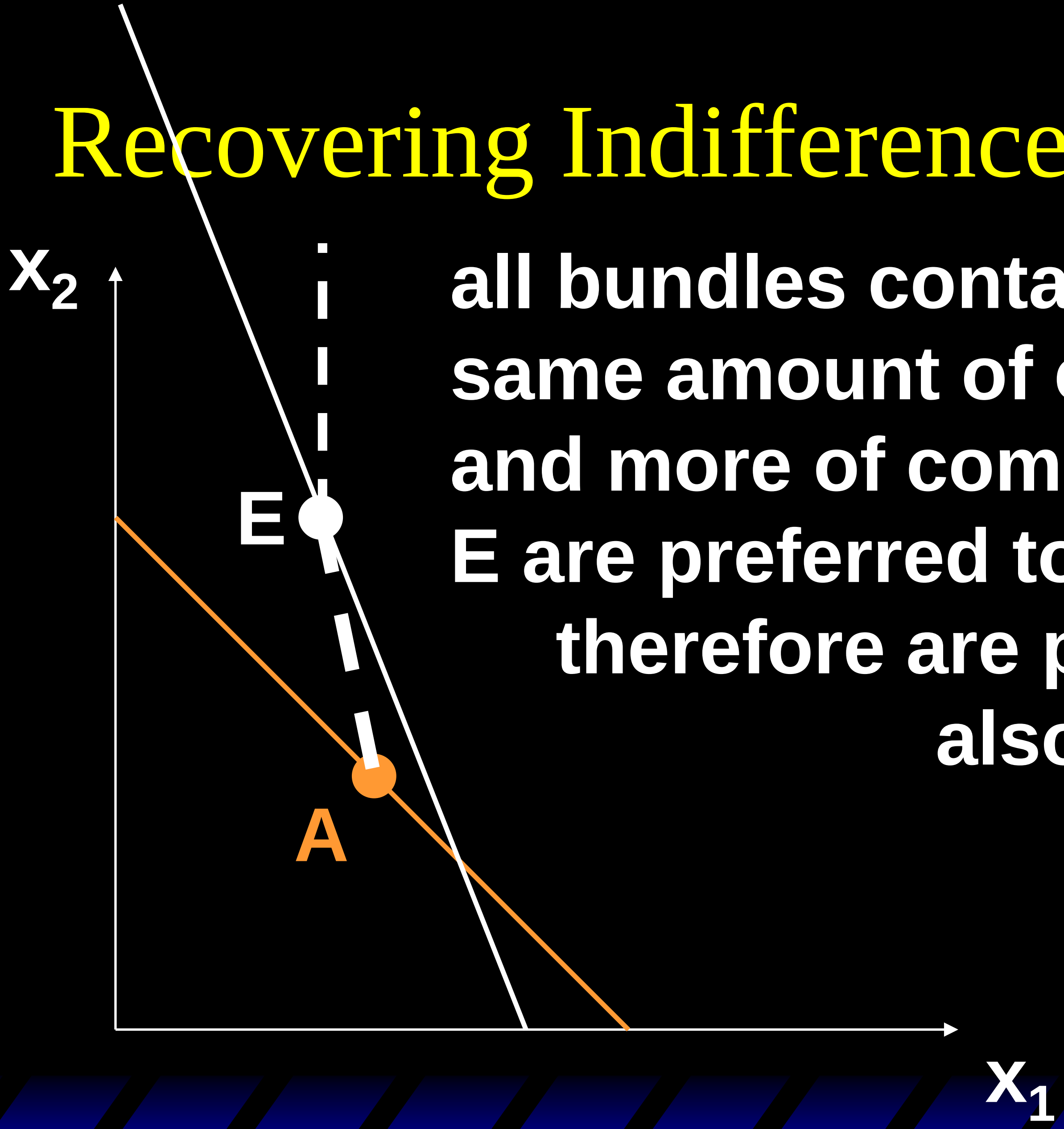


E is directly revealed preferred to A.

Well-behaved preferences are <u>convex</u> so all bundles on the line between A and E are preferred to A also.

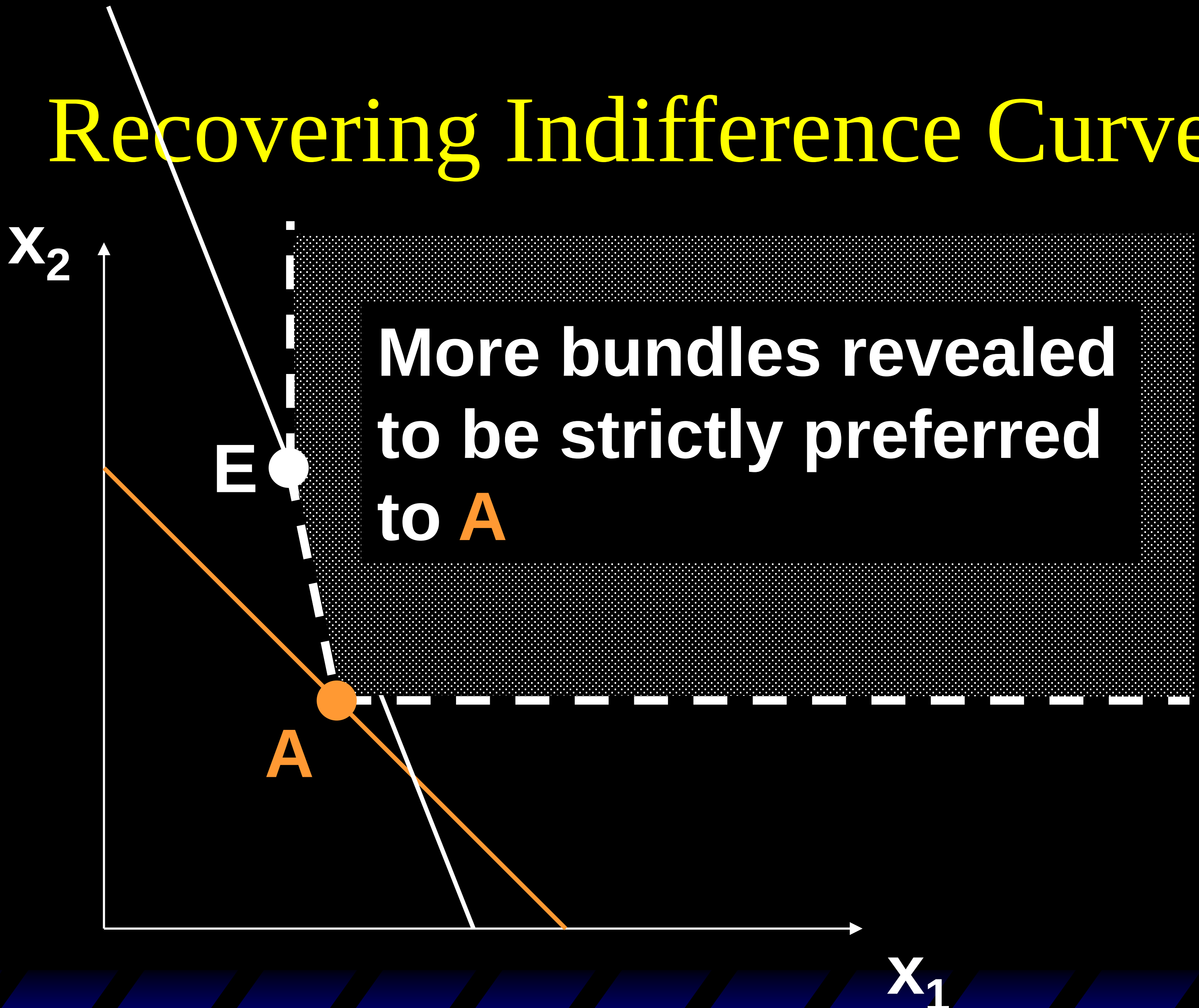
As well, ...

Recovering Indifference Curves

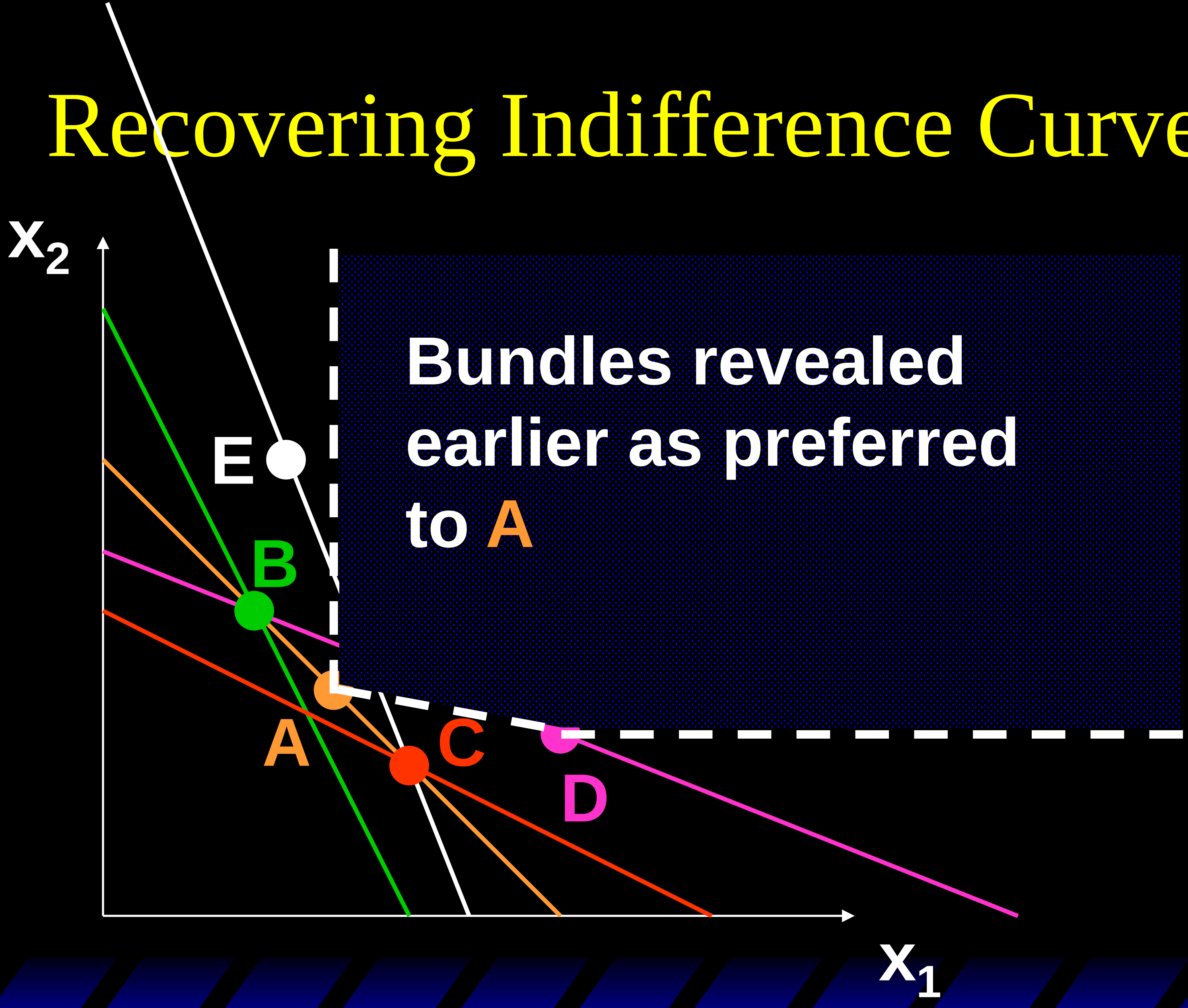


all bundles containing the same amount of commodity 1 and more of commodity 2 than E are preferred to E and therefore are preferred to A also.

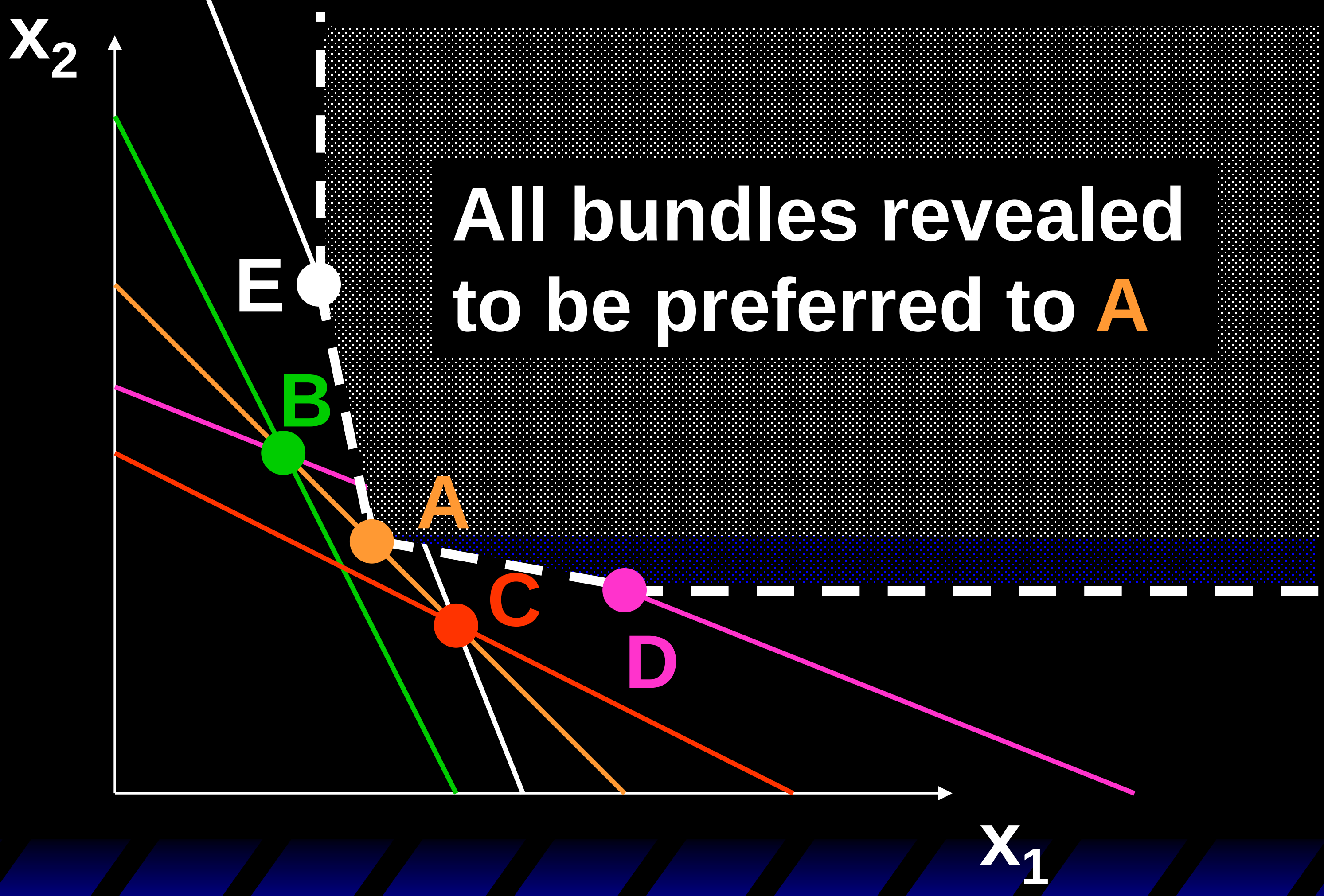
Recovering Indifference Curves



Recovering Indifference Curves



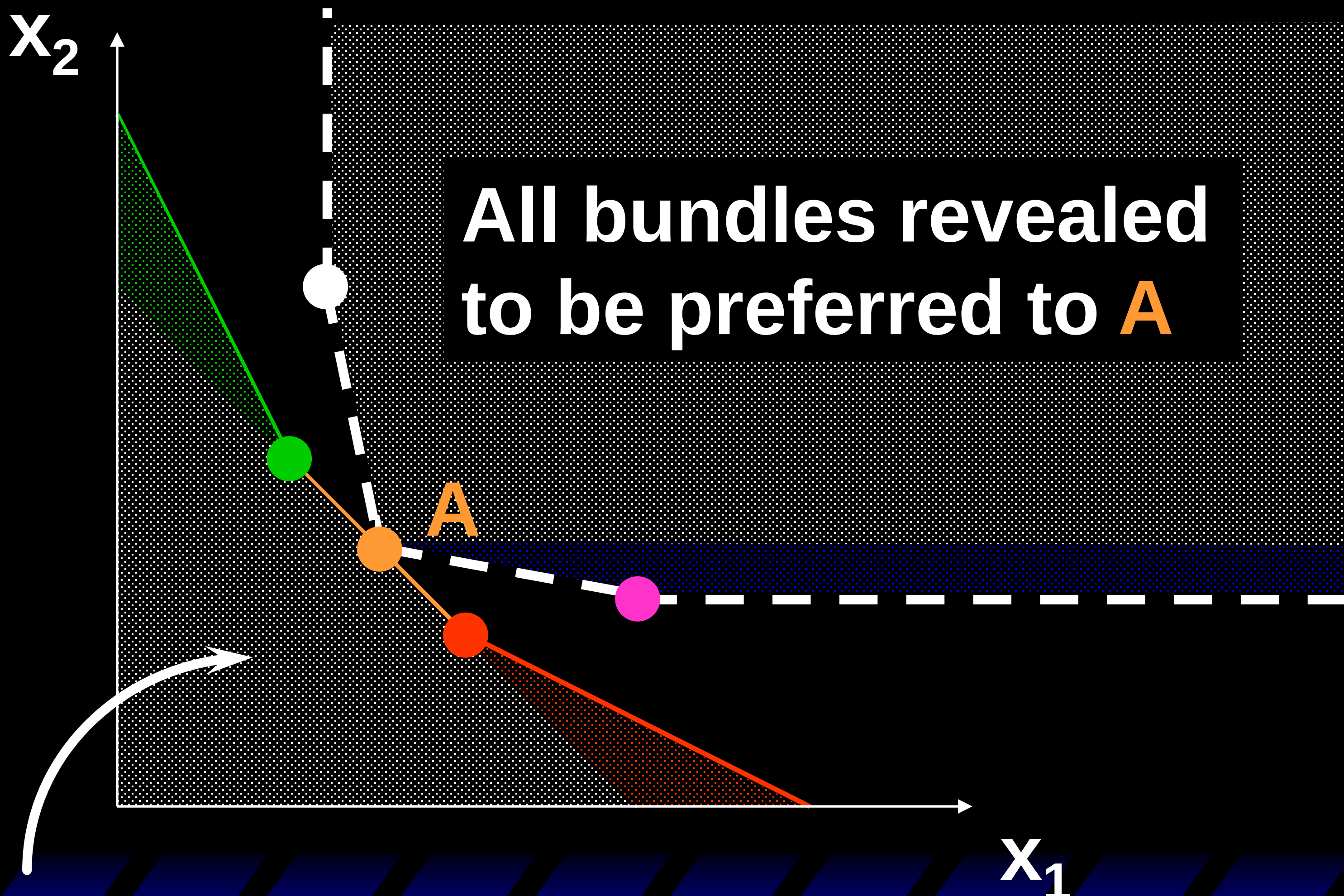
Recovering Indifference Curves



Recovering Indifference Curves

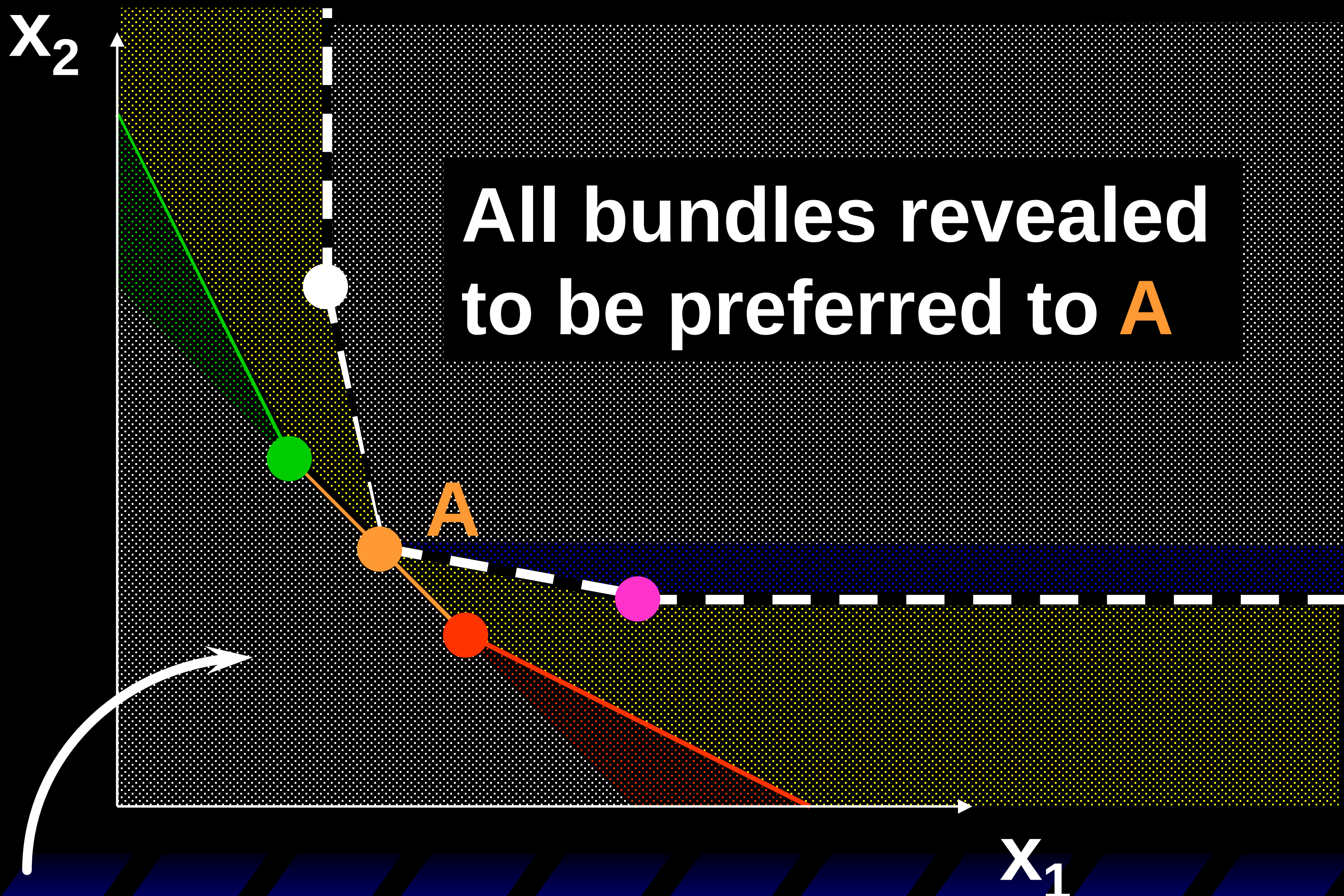
Now we have upper and lower bounds on where the indifference curve containing bundle A may lie.

Recovering Indifference Curves



All bundles revealed to be less preferred to **A**

Recovering Indifference Curves



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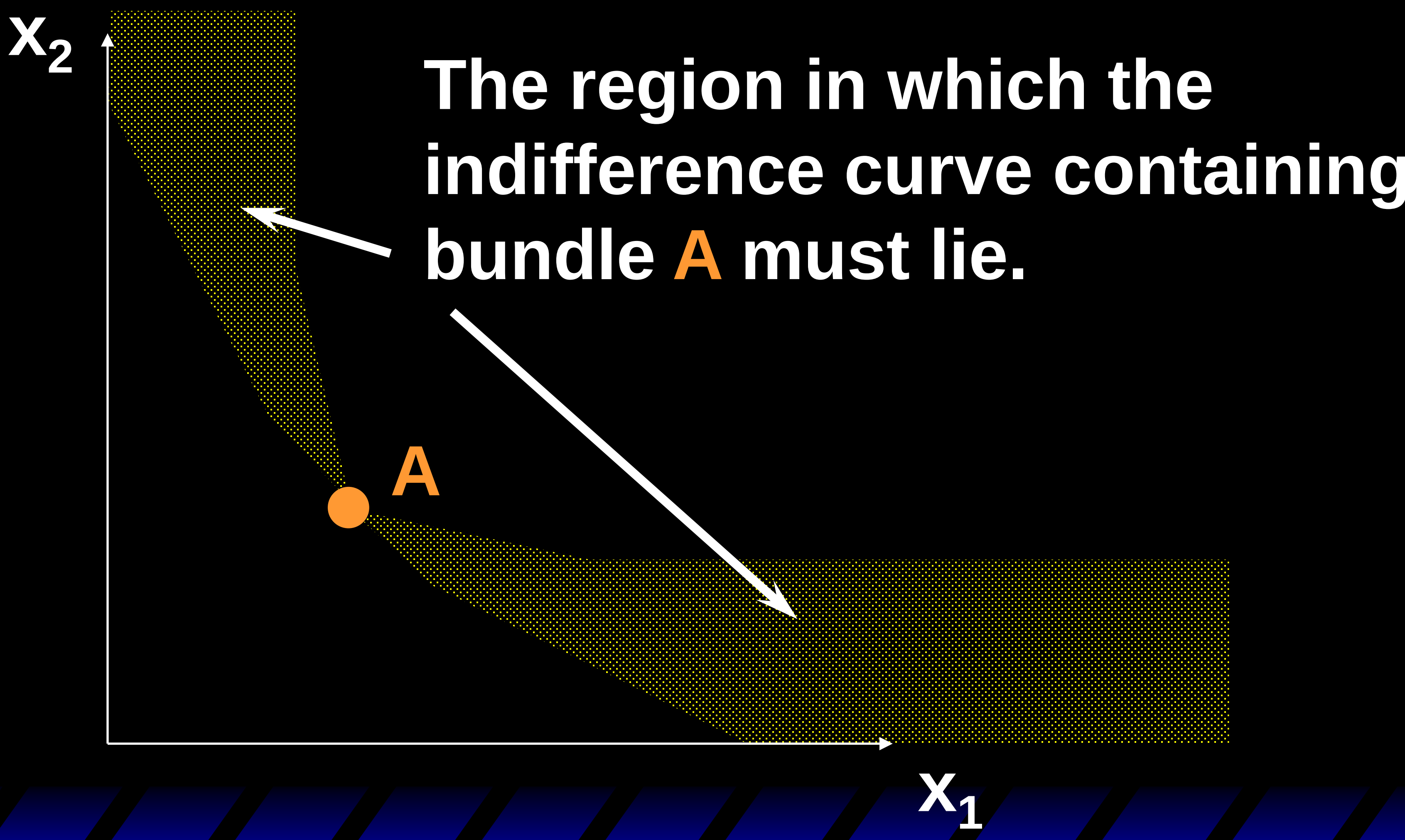
Recovering Indifference Curves



Index Numbers

- Over time, many prices change. Are consumers better or worse off "overall" as a consequence?
- Index numbers give approximate answers to such questions.

显示偏好分析的一个应用：从数量指数和价格指数中推断消费者整体福利的变化

Index Numbers

Two basic types of indices

- price indices, and
- quantity indices

Each index compares expenditures in a base period and in a current period by taking the ratio of expenditures.

Quantity Index Numbers

A quantity index is a price-weighted average of quantities demanded; *i.e.*

$$I_q = \frac{p_1 x_1^t + p_2 x_2^t}{p_1 x_1^b + p_2 x_2^b}$$

(p_1,p_2) can be base period prices (p_1^b,p_2^b) or current period prices (p_1^t,p_2^t).

数量指数：以价格为权重，对当期和基期的数量进行比较

Quantity Index Numbers

If $(p_1,p_2) = (p_1^b,p_2^b)$ then we have the Laspeyres quantity index;

$$L_q = \frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

以基期价格为权重的数量指数叫做拉氏数量指数

Quantity Index Numbers

If $(p_1,p_2) = (p_1^t,p_2^t)$ then we have the **Paasche** quantity index;

$$P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b}$$

以当期价格为权重的数量指数叫做帕氏数量指数

Quantity Index Numbers

How can quantity indices be used to make statements about changes in welfare?

Quantity Index Numbers

If $\quad L_q = \frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b} < 1 \quad$ then

$$p_1^b x_1^t + p_2^b x_2^t < p_1^b x_1^b + p_2^b x_2^b$$

so consumers overall were better off in the base period than they are now in the current period.

当期的组合在基期可被负担，但没有被选择，基期的组合直显于当期的组合。基期的福利水平更高。

Quantity Index Numbers

If $P_q = \dfrac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b} > 1$ then

$$p_1^t x_1^t + p_2^t x_2^t > p_1^t x_1^b + p_2^t x_2^b$$

so consumers overall are better off in the current period than in the base period.

基期的组合在当期可被负担，但没有被选择，当期的组合直显于基期的组合。当期的福利水平更高。

Price Index Numbers

- A price index is a quantity-weighted average of prices; *i.e.*

$$I_p = \frac{p_1^t x_1 + p_2^t x_2}{p_1^b x_1 + p_2^b x_2}$$

- (x_1,x_2) can be the base period bundle (x_1^b,x_2^b) or else the current period bundle (x_1^t,x_2^t).

Price Index Numbers

If $(x_1,x_2) = (x_1^b,x_2^b)$ then we have the **Laspeyres** price index;

$$L_p = \frac{p_1^t x_1^b + p_2^t x_2^b}{p_1^b x_1^b + p_2^b x_2^b}$$

Price Index Numbers

If $(x_1,x_2) = (x_1^t,x_2^t)$ then we have the Paasche price index;

$$P_p = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^t + p_2^b x_2^t}$$

Price Index Numbers

How can price indices be used to make statements about changes in welfare?

Define the expenditure ratio

$$M = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

Price Index Numbers

If

$$L_p = \frac{p_1^t x_1^b + p_2^t x_2^b}{p_1^b x_1^b + p_2^b x_2^b} \quad < \quad \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b} = M$$

then

$$p_1^t x_1^b + p_2^t x_2^b \quad < \quad p_1^t x_1^t + p_2^t x_2^t$$

so consumers overall are better off in the current period.

Price Index Numbers

But, if

$$P_p = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^t + p_2^b x_2^t} > \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b} = M$$

then

$$p_1^b x_1^t + p_2^b x_2^t < p_1^b x_1^b + p_2^b x_2^b$$

so consumers overall were better off in the base period.