Lecture 8

Consumer's Surplus

Monetary Measures of Gains-to-Trade

- You can buy as much gasoline as you wish at $1 per gallon once you enter the gasoline market.
- Q: How many units would you purchase in the market?
- Q: How much would you **gain** from the trade?
- Q: How much would you **gain** or **lose** as a result of price changes?

Monetary Measures of Gains-to-Trade

- Three such measures are:
  - Consumer's Surplus (消费者剩余)
  - Compensating Variation (补偿变化)
  - Equivalent Variation (等价变化)

$ Equivalent Utility Gains

- Use r_1 to denote the **most** a single consumer would pay for a 1st gallon -- call this her **reservation price** for the 1st gallon.

保留价格是消费者愿意为某一单位商品支付的**最高价格**，或者说，是使消费者在消费和不消费该单位商品之间无差异的价格

$ Equivalent Utility Gains

- **The consumer purchases gasoline (x) and other commodities (y)**
- **Utility is quasi-linear:**

$$U(x, y) = v(x) + y$$

y is the amount of $ spent on other commodities.

$$v'(x) > 0, v''(x) < 0$$

Total income is m.

\$ Equivalent Utility Gains

- **Utility is quasi-linear:**

$$U(x, y) = v(x) + y$$

- **If 0 unit of gasoline,** $U(0, m) = v(0) + m$;
- **If 1 unit of gasoline,** $U(1, m - p_1) = v(1) + m - p_1$
- **The reservation price for the 1st unit** r_1 **satisfies:**

$$v(1) + m - r_1 = v(0) + m$$

$$r_1 = v(1) - v(0)$$

$ Equivalent Utility Gains

- The reservation price for the 1st unit

$$r_1 = v(1) - v(0)$$

- r_1 is the **dollar equivalent** of the marginal utility of the 1st gallon.

第1单位汽油带来的额外效用为$v(1) - v(0)$;由于每一单位货币的边际效用总是1，第1单位汽油所带来的效用等价于$v(1) - v(0)$单位货币。消费者对第1单位汽油的最高支付意愿是$v(1) - v(0)$元

$ Equivalent Utility Gains

- Now that she has one gallon, use r_2 to denote the most she would pay for a 2nd gallon -- this is her reservation price for the 2nd gallon.

$ Equivalent Utility Gains

- **If 1 unit of gasoline,** $U(1, m - p_1) = v(1) + m - p_1$
- **If 2 units of gasoline,** $U(2, m - p_1 - p_2) = v(2) + m - p_1 - p_2$
- **The reservation price for the 2nd unit** r_2 **satisfies:**

$$v(1) + m - p_1 = v(2) + m - p_1 - r_2$$

$$r_2 = v(2) - v(1)$$

$ Equivalent Utility Gains

- The reservation price for the 2nd unit
$$r_2 = v(2) - v(1)$$
- r_2 is the dollar equivalent of the marginal utility of the 2nd gallon.

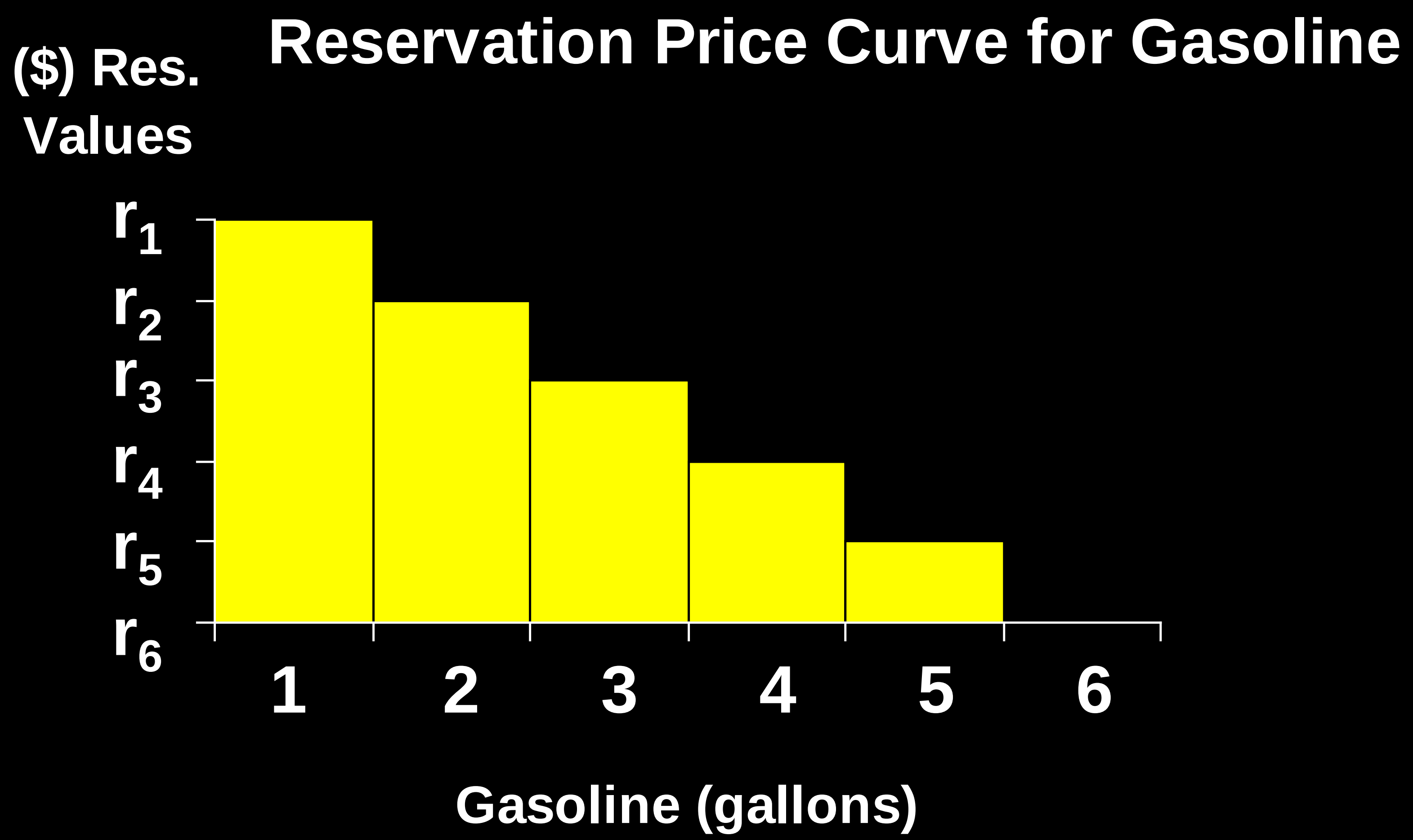
消费者对第2单位汽油的最高支付意愿(保留价格)是第2单位汽油所带来的边际效用的等价货币量：$v(2) - v(1)$元

$ Equivalent Utility Gains

- Generally, if she already has n-1 gallons of gasoline then r_n denotes the most she will pay for an nth gallon.
- r_n is the dollar equivalent of the marginal utility of the nth gallon.

$$r_n = v(n) - v(n-1)$$

$ Equivalent Utility Gains

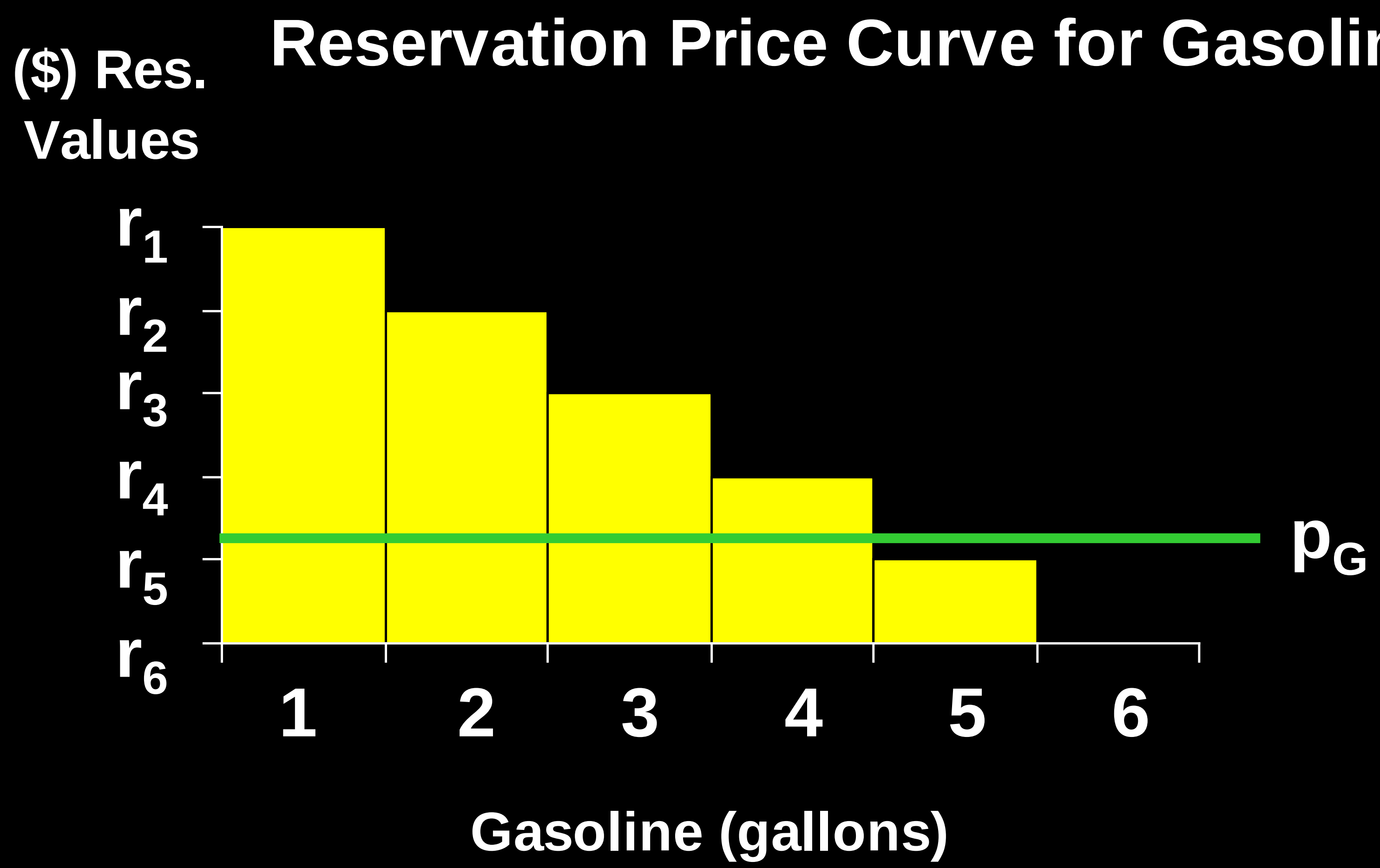


$v(x)$随x递减;保留价格随数量上升而下降

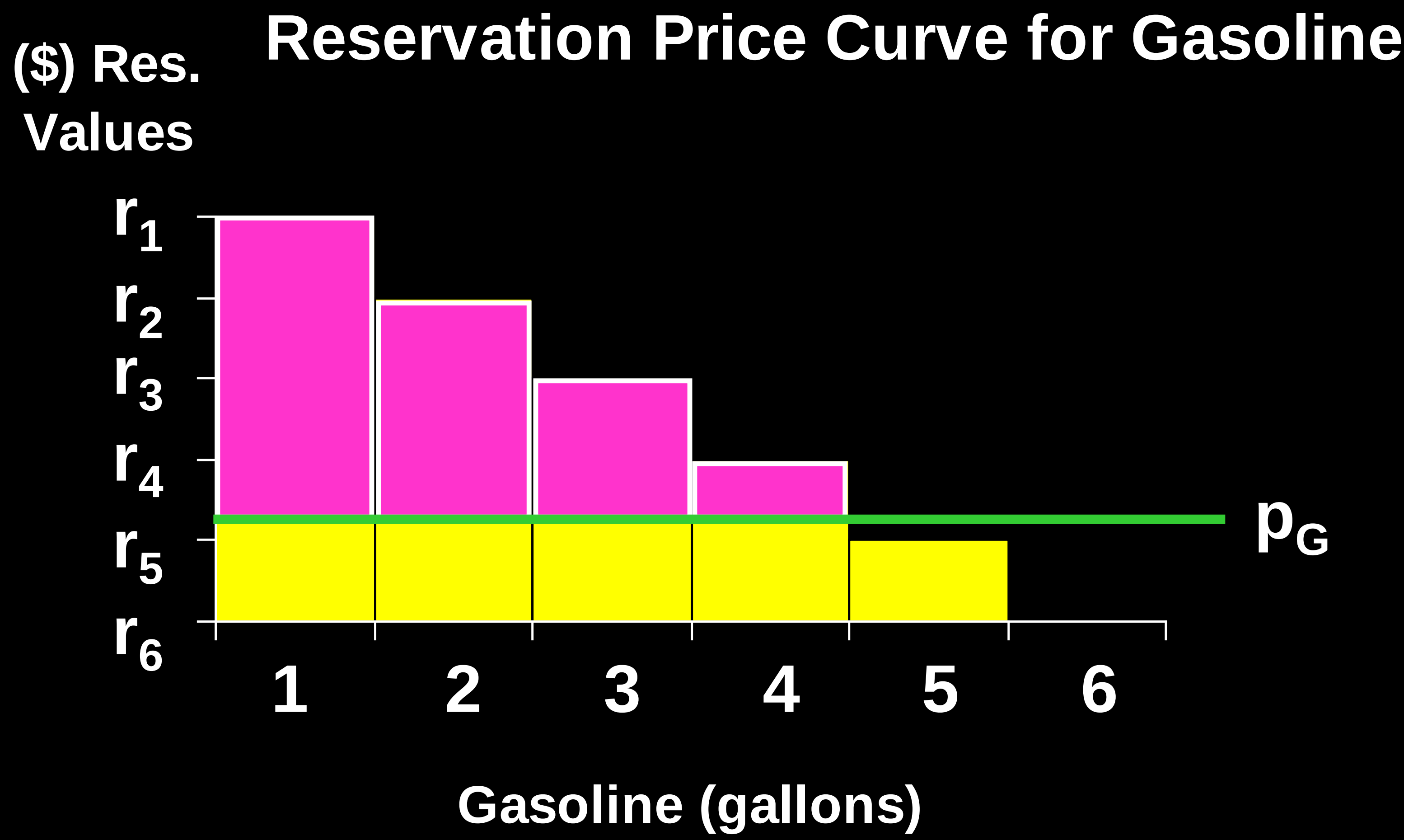
$ Equivalent Utility Gains

- What is the monetary value of our consumer's gain-to-trading in the gasoline market at a price of $\$p_G$?

$ Equivalent Utility Gains



$ Equivalent Utility Gains



$ Equivalent Utility Gains

- **The dollar equivalent net utility gain for the 1st gallon is** $\$(r_1 - p_G)$
- **and is** $\$(r_2 - p_G)$ **for the 2nd gallon,**
- **and so on, so the dollar value of the gain-to-trade is**

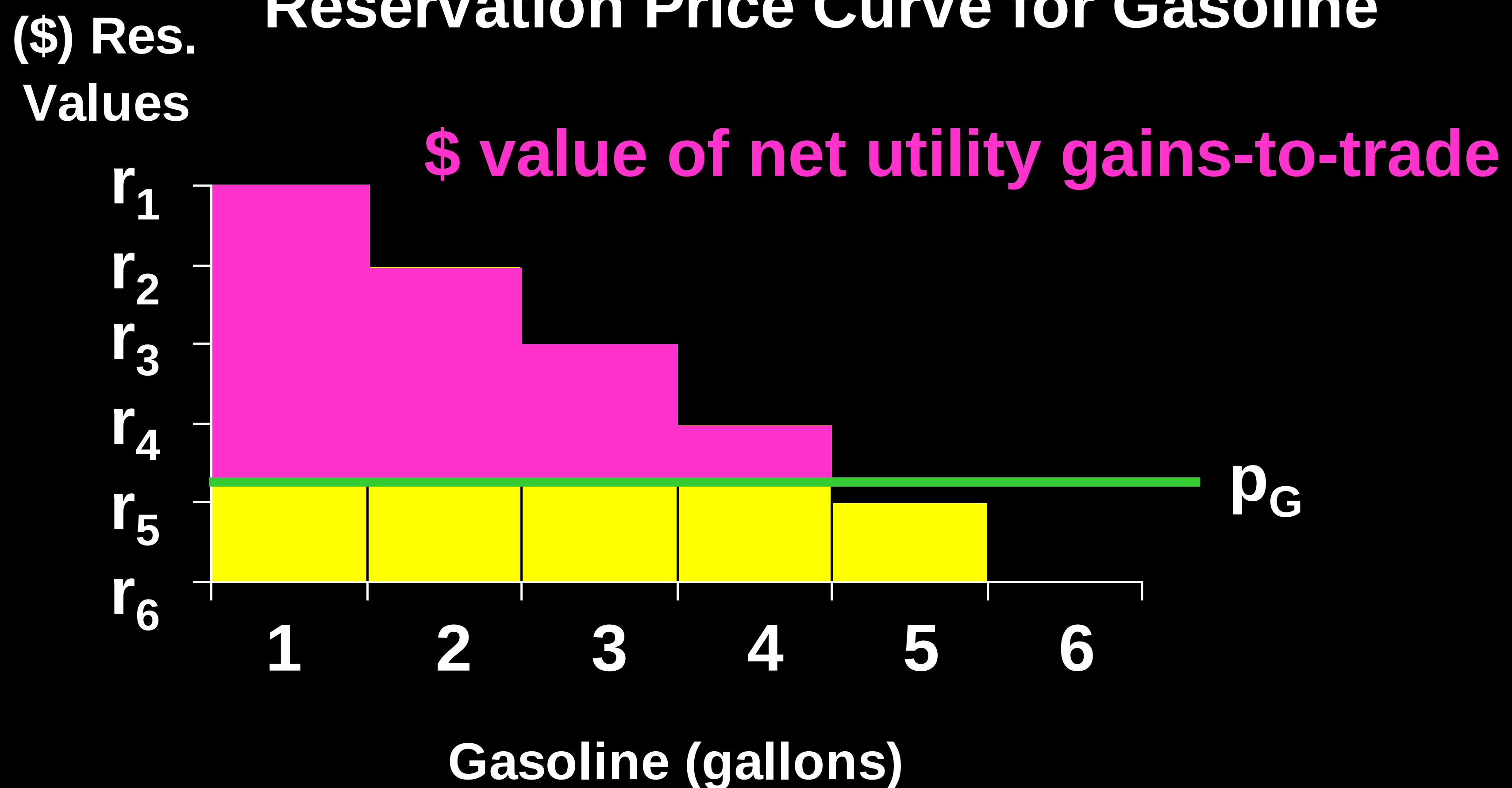
$$(r_1 - p_G) + (r_2 - p_G) + \cdots + (r_n - p_G)$$
$$= r_1 + r_2 + \cdots + r_n - p_G n$$

for as long as $r_n - p_G \geq 0$

.

$ Equivalent Utility Gains

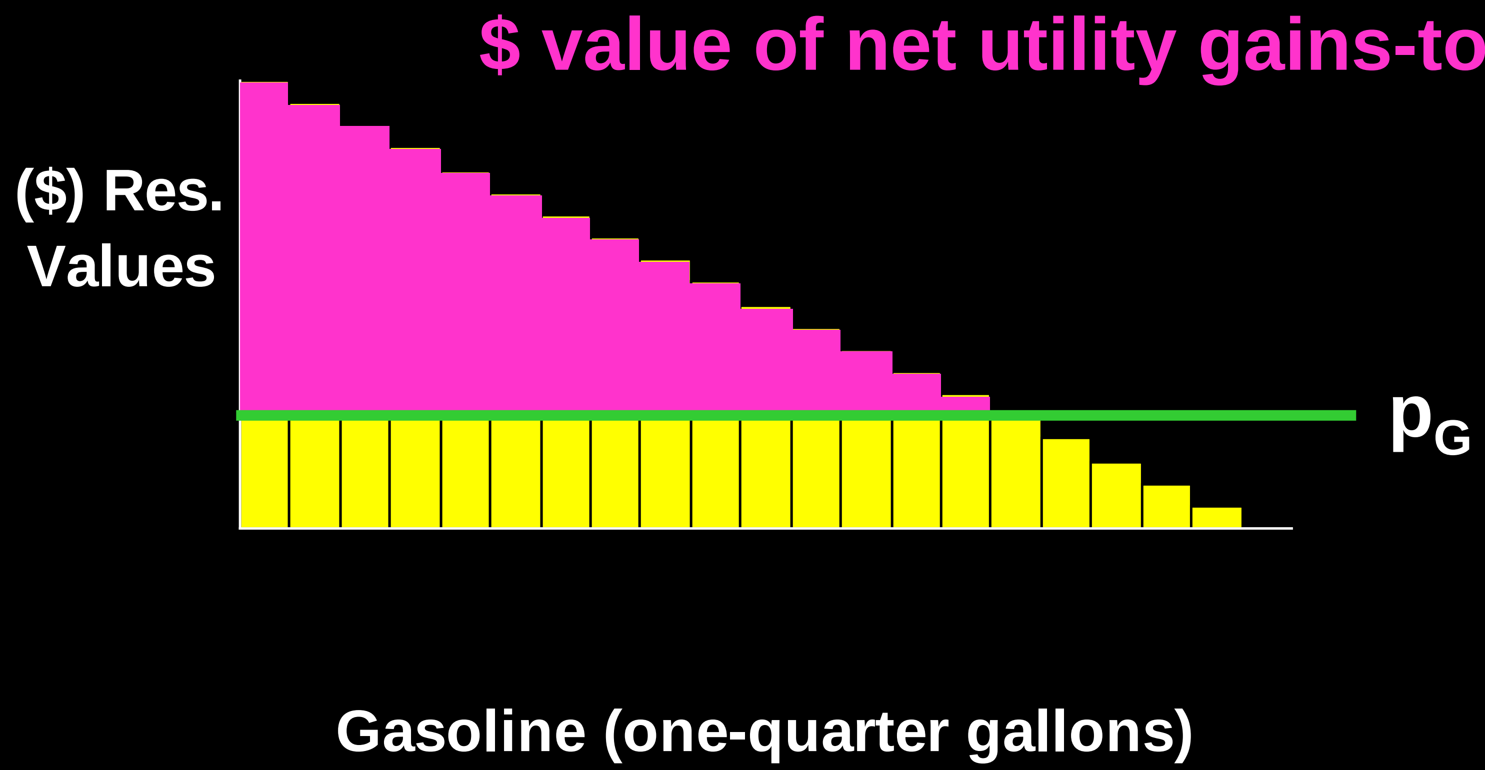
Reservation Price Curve for Gasoline



消费者从交易中所获得的净效益（净效用的等价货币值）被成为消费者剩余。

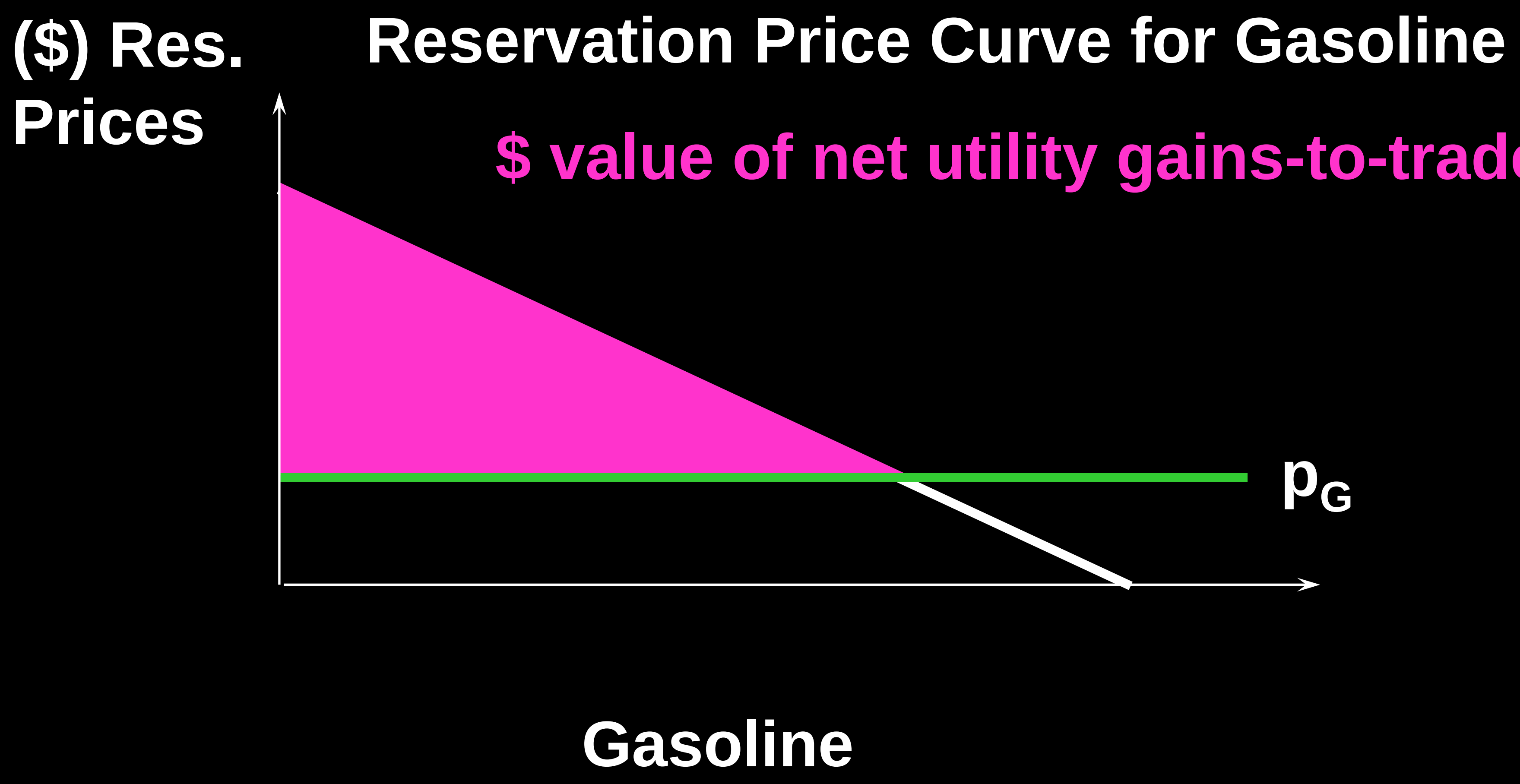
$ Equivalent Utility Gains

Reservation Price Curve for Gasoline

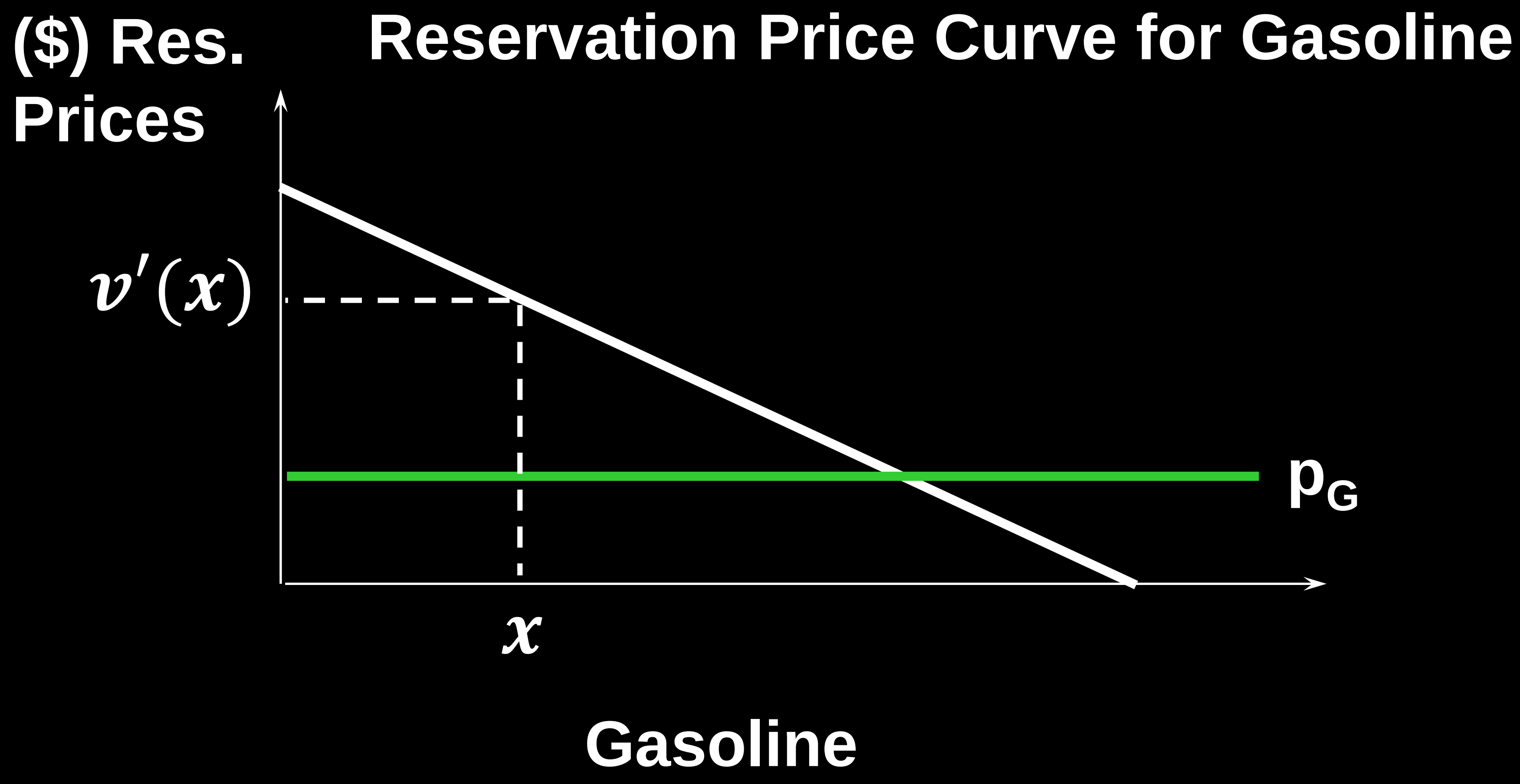


无限细分，近似于连续

$ Equivalent Utility Gains



$ Equivalent Utility Gains



在连续的情况下，第x单位的保留价格等于该单位的边际效用$v'(x)$

$ Equivalent Utility Gains

- **Utility is quasi-linear:**

$$U(x, y) = v(x) + y$$

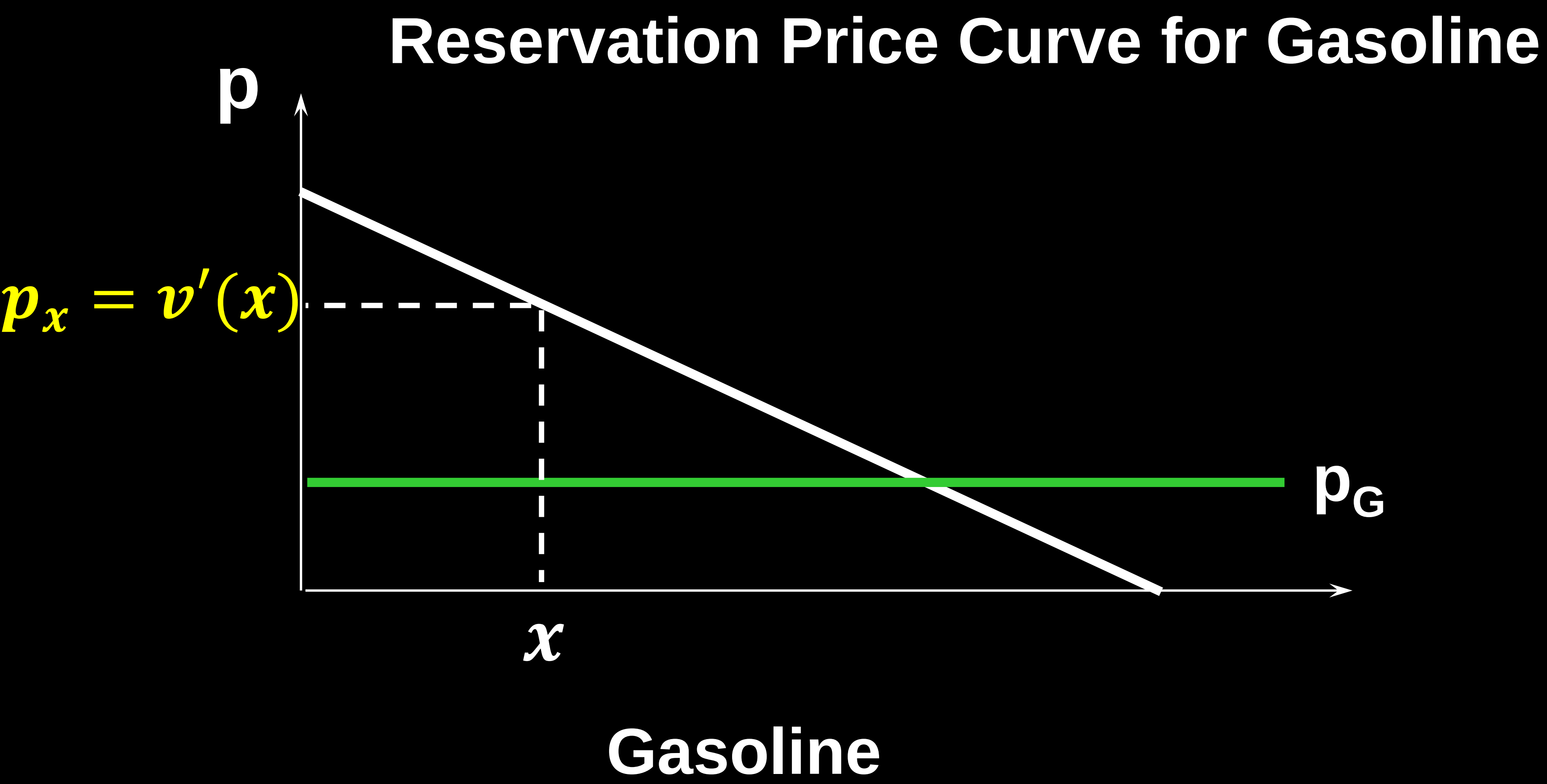
- **Optimality gives**

$$\frac{v'(x)}{1} = \frac{p_x}{1}$$

Demand function for x:

$$v'(x) = p_x$$

$ Equivalent Utility Gains



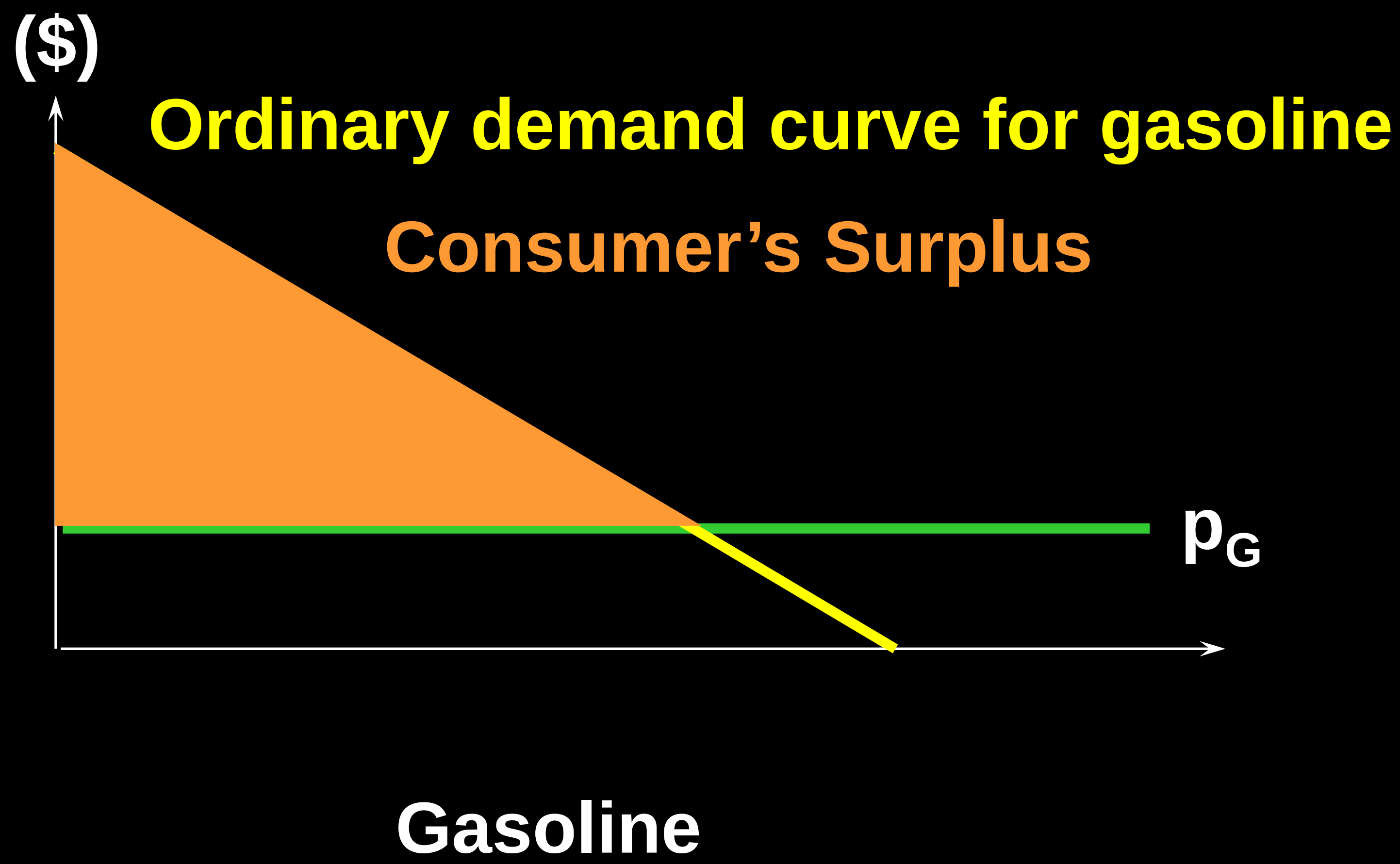
当效用函数为拟线性时，个体需求曲线恰好是保留价格曲线。消费者剩余是需求曲线以下、价格线以上的部分。

$ Equivalent Utility Gains

- Unfortunately, estimating a consumer's reservation-price curve is difficult when utility is **not quasi-linear**,
- so, as an approximation, the reservation-price curve is replaced with the consumer's ordinary demand curve.

当效用函数不是拟线性时，我们用个体需求曲线来作为保留价格曲线的近似替代

Consumer's Surplus

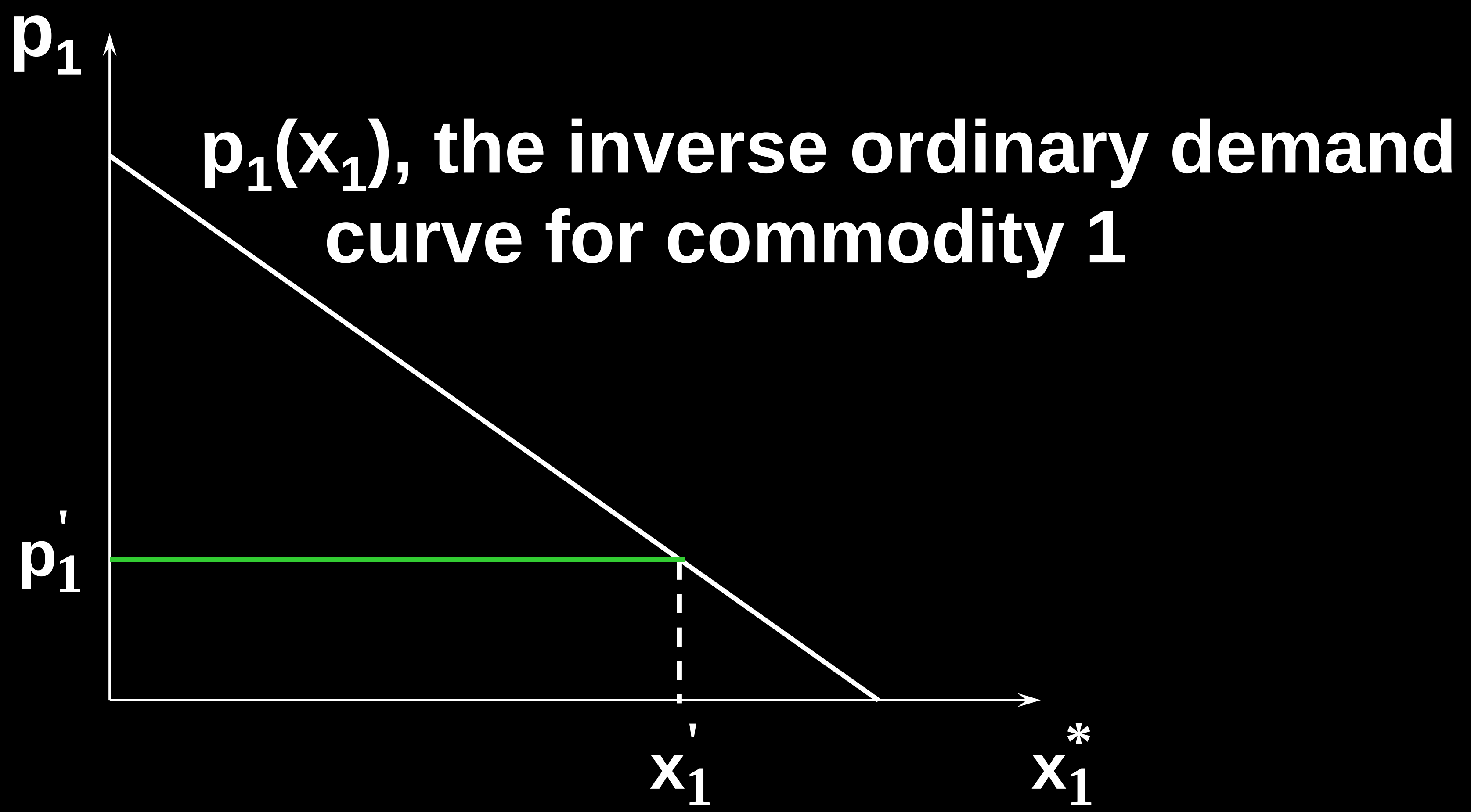


Consumer's Surplus

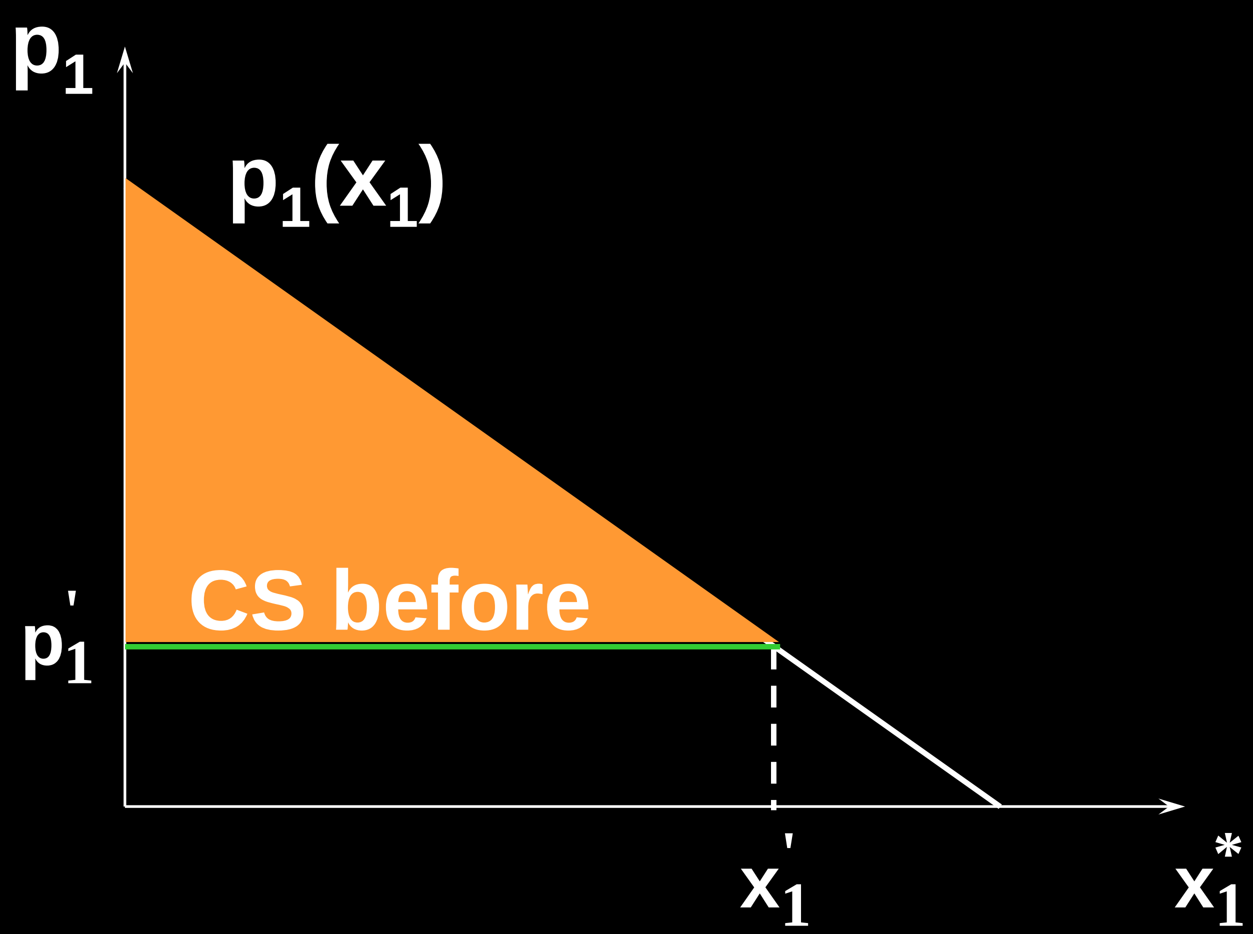
- Approximating the net utility gain area under the reservation-price curve by the corresponding area under the ordinary demand curve gives the **Consumer's Surplus measure of net utility gain**.

我们用个体需求曲线以下、价格线以上的面积来作为消费者剩余的近似替代，衡量消费者从贸易中所得效用的货币价值

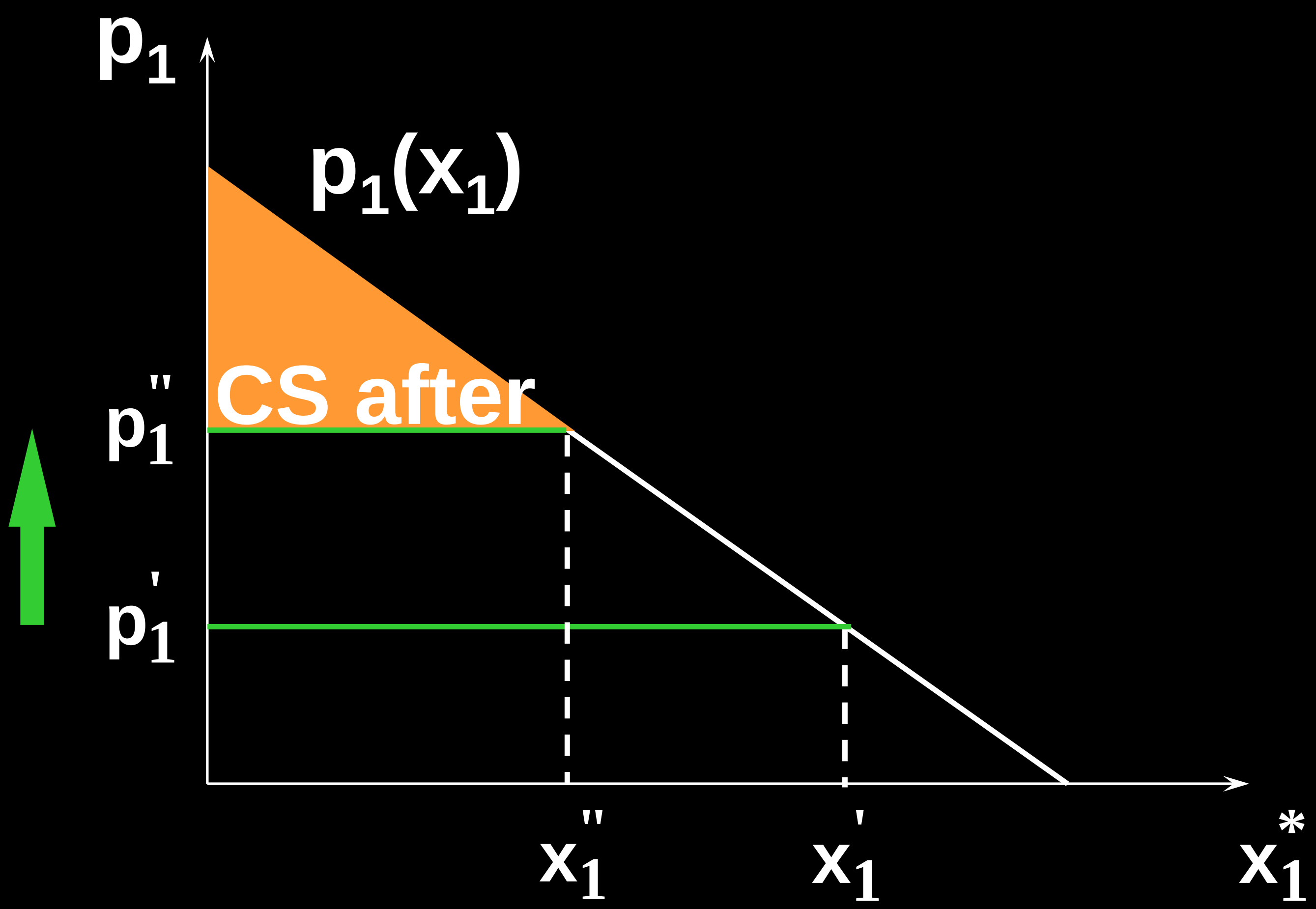
Changes in Consumer's Surplus



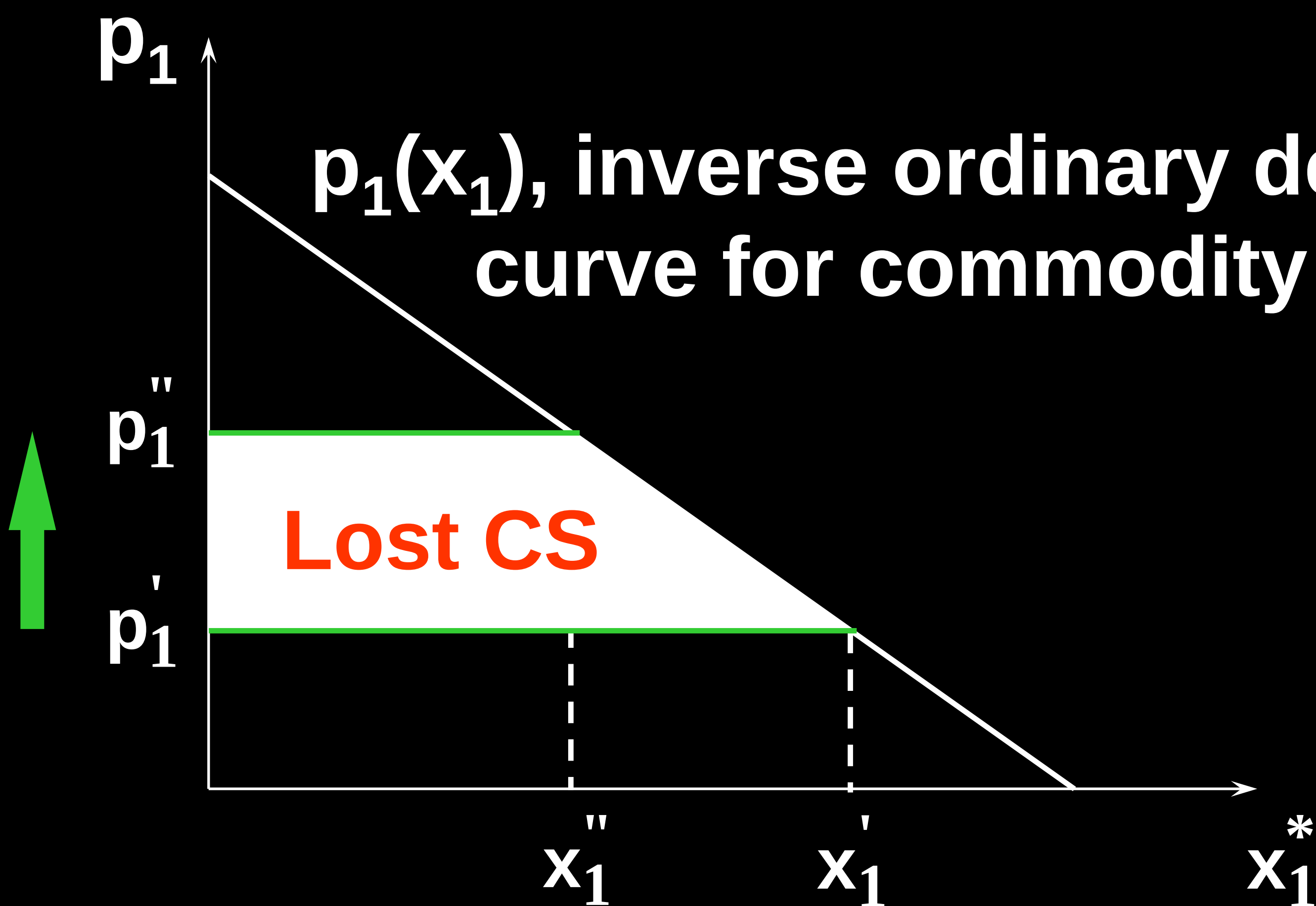
Changes in Consumer's Surplus



Changes in Consumer's Surplus



Changes in Consumer's Surplus



价格上升导致消费者剩余下降

Compensating Variation and Equivalent Variation

- **Two additional dollar measures of the total utility change caused by a price change are Compensating Variation (补偿变化) and Equivalent Variation (等价变化).**

Compensating Variation

- p_1 rises.
- Q: What is the least extra income that, at the **new prices**, just restores the consumer's original utility level?

价格上升时，我们至少需要补偿消费者多少货币才能使他的效用水平保持不变？

Compensating Variation

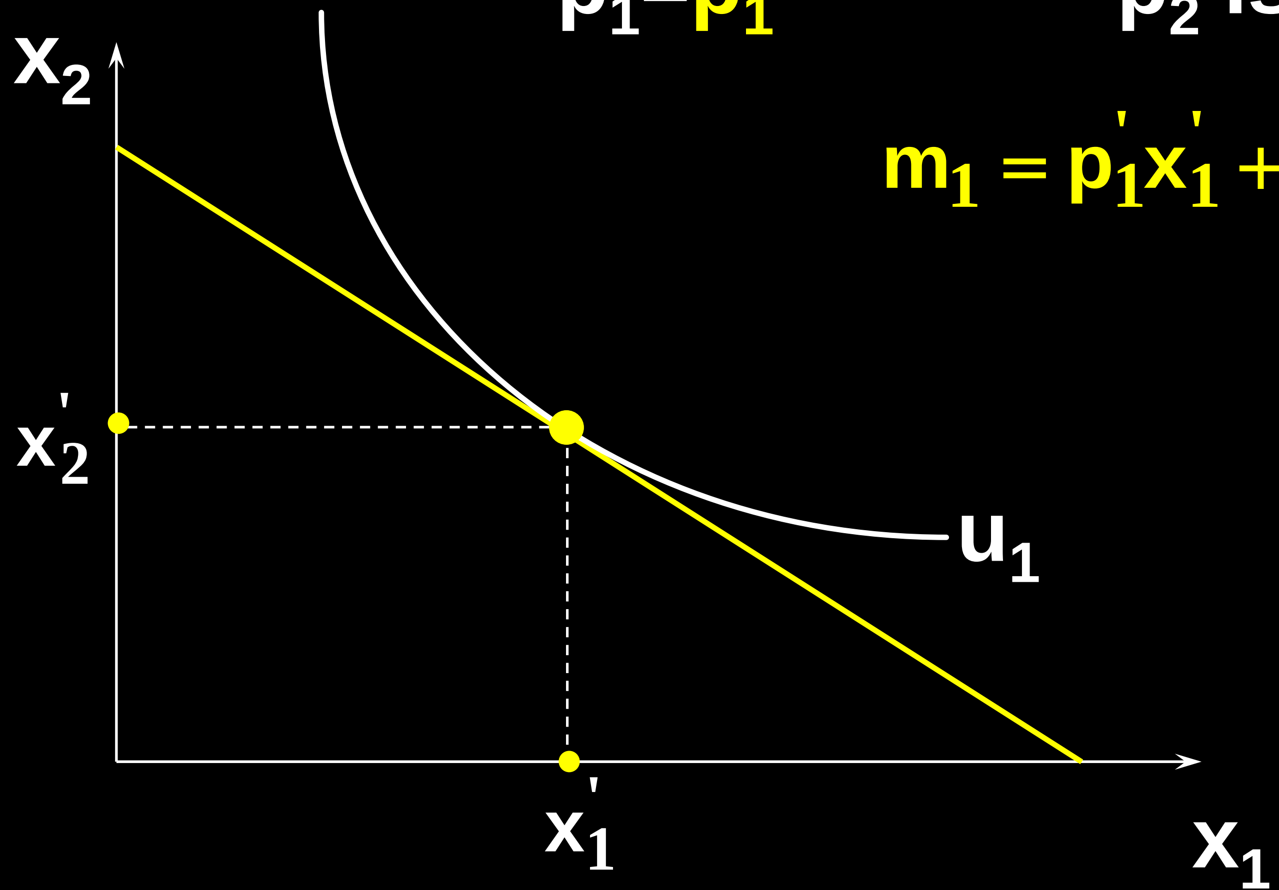
- p_1 rises.
- Q: What is the least extra income that, at the new prices, just restores the consumer's original utility level?
- A: The Compensating Variation.

CV is the amount of money you have to give the consumer after the price rise to compensate them for the price increase

Compensating Variation

$p_1 = p_1'$ $\qquad$ p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$



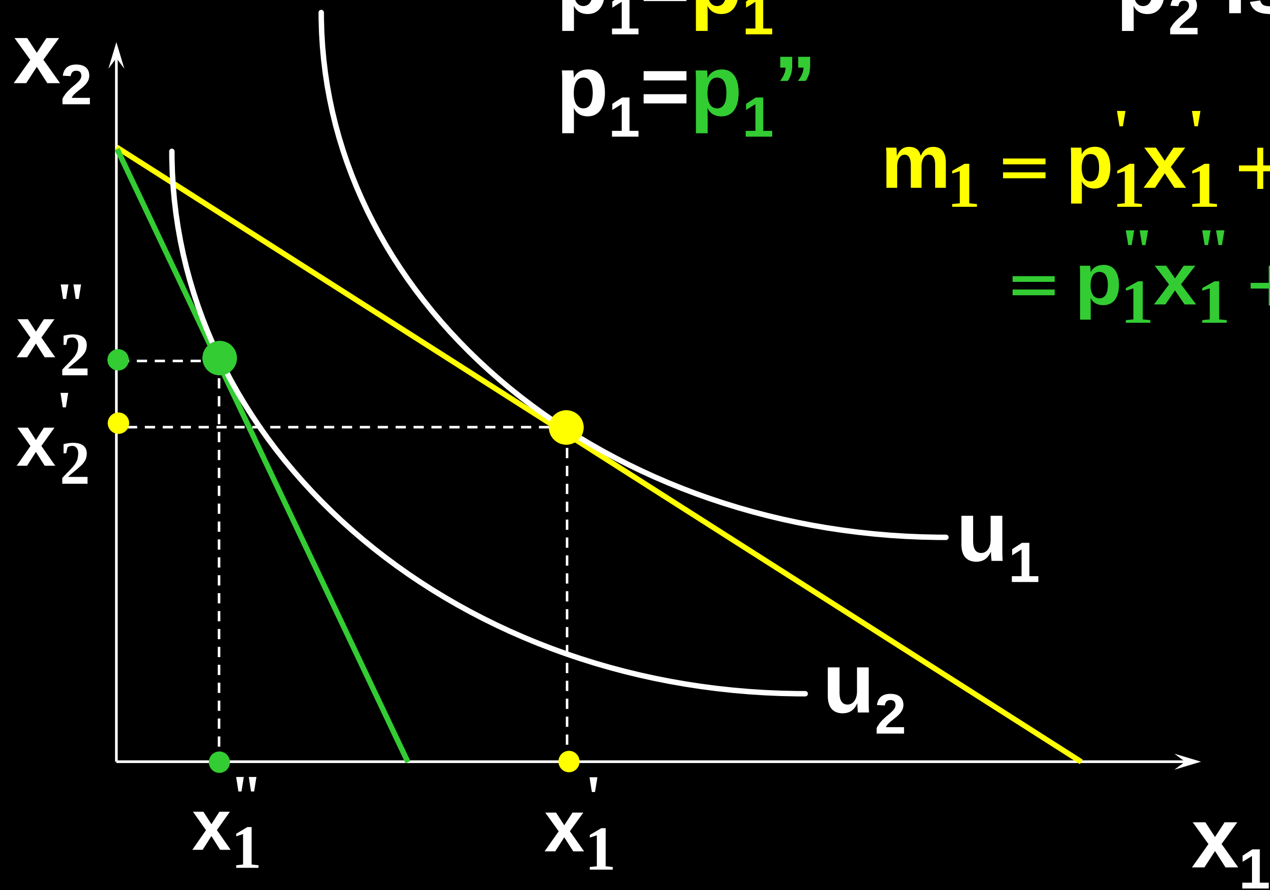
Compensating Variation

$p_1 = p_1'$

$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$



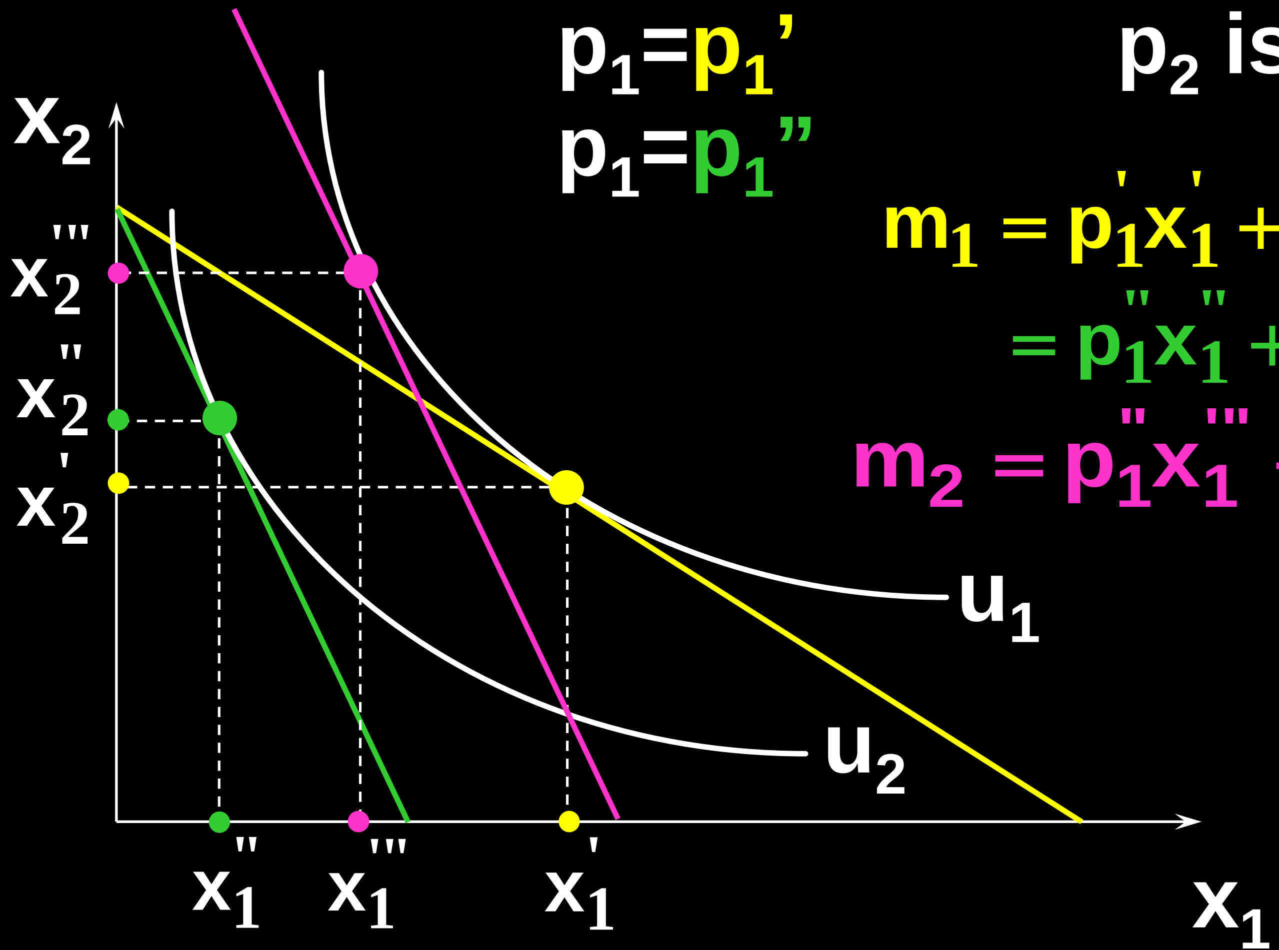
Compensating Variation

$p_1 = p_1'$

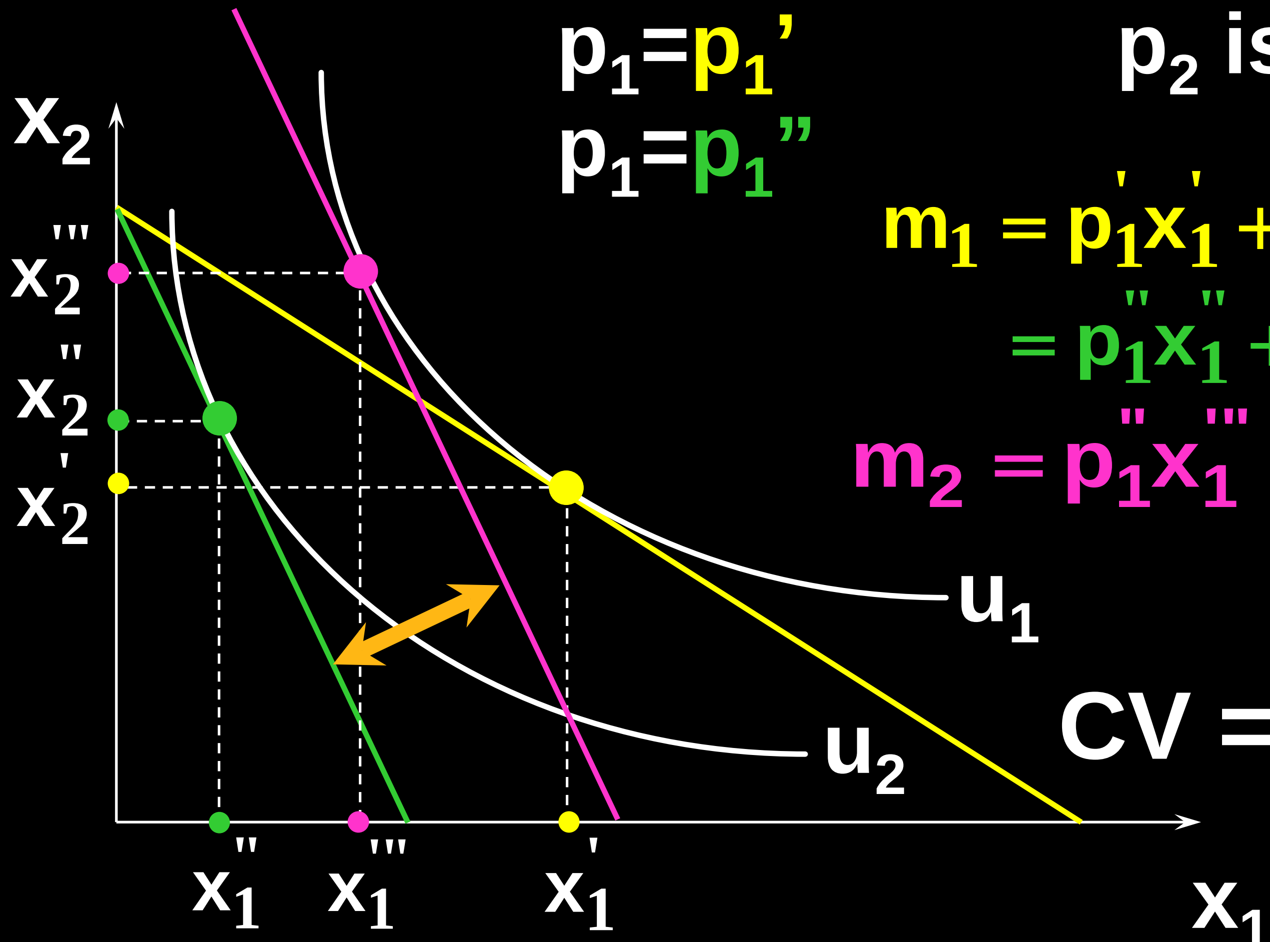
$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$
$$m_2 = p_1'' x_1''' + p_2 x_2'''$$



Compensating Variation



p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$
$$m_2 = p_1'' x_1''' + p_2 x_2'''$$

CV = m_2 - m_1.

Equivalent Variation

- p_1 rises.
- Q: What is the highest amount of income that the consumer is willing to pay to avoid the price change?
- A: The Equivalent Variation.

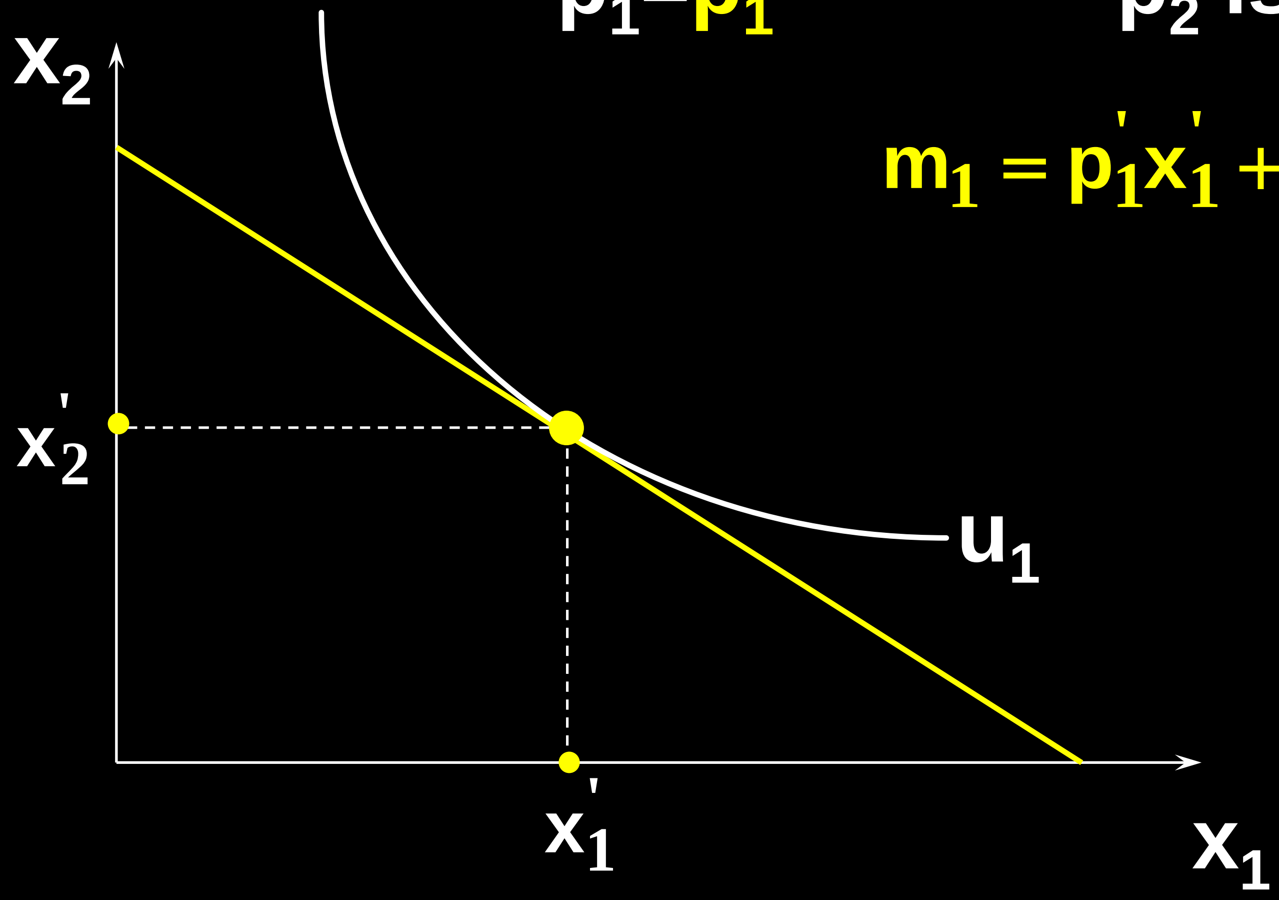
EV is the amount of income a consumer would be just willing to give up to avoid the price increase

消费者为避免价格上升所愿意支付的最高货币值被成为EV

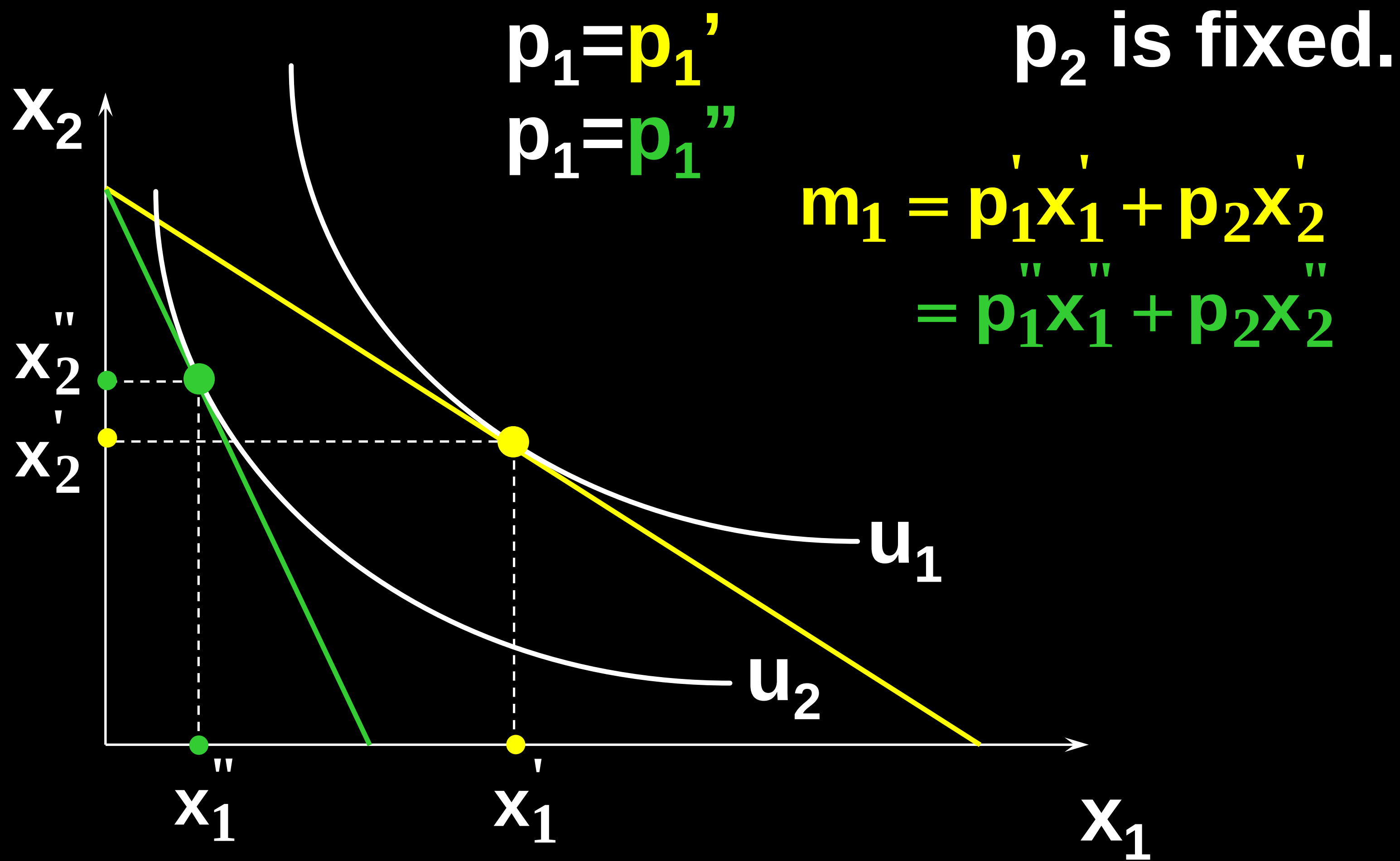
Equivalent Variation

$p_1 = p_1'$ $\quad$ p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$



Equivalent Variation



Equivalent Variation

$p_1 = p_1'$

$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$
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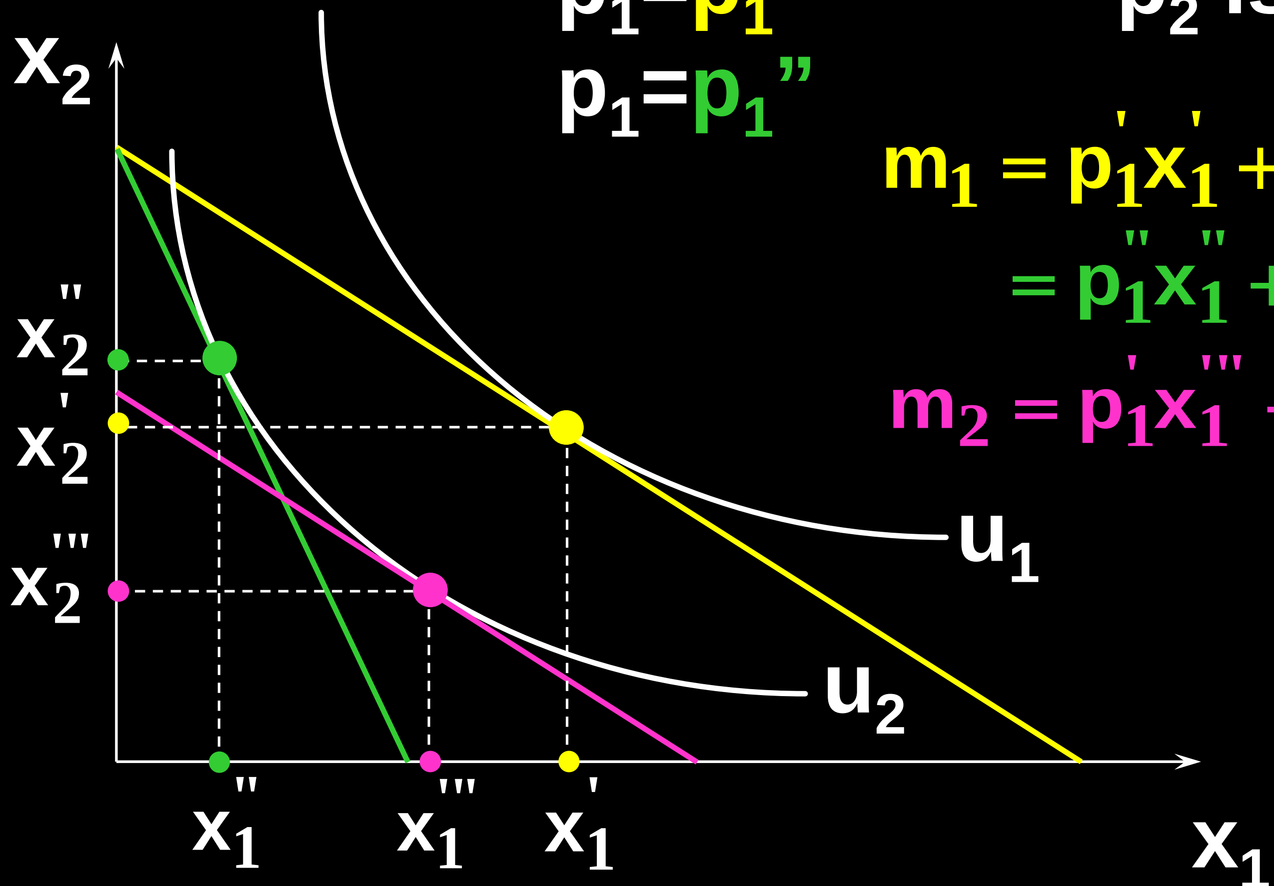

Equivalent Variation

$p_1 = p_1'$

$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
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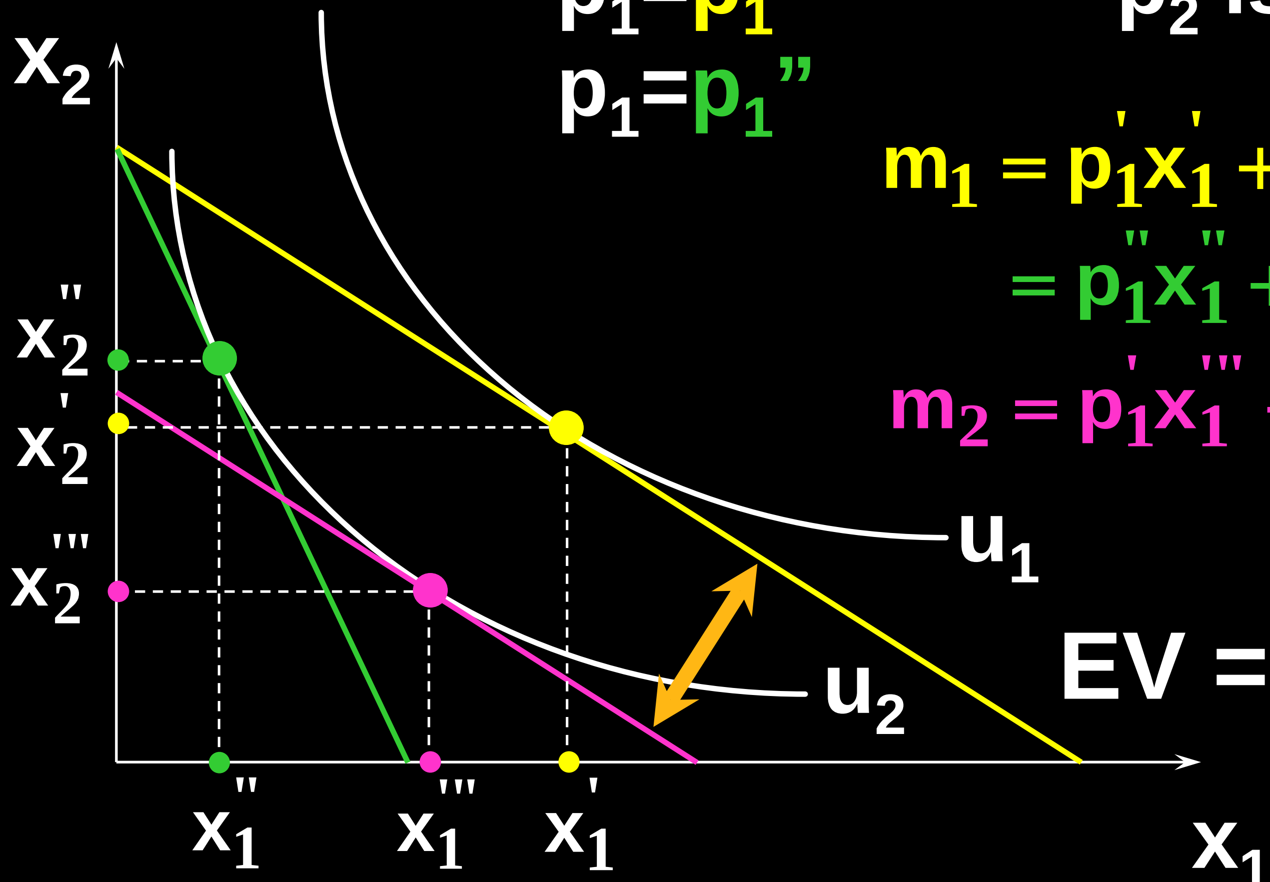


EV = m_1 - m_2.

A C-D Example

$$U = x_1^{1/2} x_2^{1/2}$$
$$p_1 x_1 + p_2 x_2 = m$$

Originally, prices are (1,1) and income is 100.

Then p_1 increases to 2. What are the CV and EV?

A C-D Example

$$U = x_1^{1/2} x_2^{1/2}$$

$$p_1 x_1 + p_2 x_2 = m$$

$$x_1^* = \frac{m}{2p_1}, x_2^* = \frac{m}{2p_2}$$

A C-D Example

$$U = x_1^{1/2} x_2^{1/2}$$
$$p_1 x_1 + p_2 x_2 = m$$

$$x_1^* = \frac{m}{2p_1}, x_2^* = \frac{m}{2p_2}$$

When $p_1 = p_2 = 1$ and m=100,

$$x_1^* = x_2^* = 50$$
$$U^* = 50^{1/2} 50^{1/2} = 50$$

价格上升前的效用值为50

A C-D Example

$$U = x_1^{1/2} x_2^{1/2}$$
$$p_1 x_1 + p_2 x_2 = m$$

$$x_1^* = \frac{m}{2p_1}, x_2^* = \frac{m}{2p_2}$$

When $p_1' = 2, p_2 = 1$ and m=100,

$$x_1^{**} = 25, x_2^{**} = 50$$
$$U^{**} = 25^{1/2} 50^{1/2} = 25\sqrt{2}$$

价格上升后的效用值为$25\sqrt{2}$

A C-D Example

CV is the extra income we need to give the consumer so that she has the **original** utility at the **new** prices.

A C-D Example

CV is the extra income we need to give the consumer so that she has the **original** utility at the **new** prices.

$$U = x_1^{1/2} x_2^{1/2}$$

$$x_1^* = \frac{m}{2p_1}, x_2^* = \frac{m}{2p_2}$$

After getting $CV, the consumer buys the following bundle **at the new prices**.

$$x_1 = \frac{m+CV}{2p_1'} = \frac{100+CV}{4}, x_2 = \frac{m+CV}{2p_2} = \frac{100+CV}{2}$$

A C-D Example

After getting \$CV, the consumer buys the following bundle at the new prices,

$$x_1 = \frac{m+CV}{2p_1'} = \frac{100+CV}{4}, \; x_2 = \frac{m+CV}{2p_2} = \frac{100+CV}{2}$$

and achieves the following utility level:

$$U = \left(\frac{100+CV}{4}\right)^{1/2}\left(\frac{100+CV}{2}\right)^{1/2} = \frac{100+CV}{2\sqrt{2}}$$

A C-D Example

After getting \$CV, the consumer buys the following bundle at the new prices,

$$x_1 = \frac{m+CV}{2p_1'} = \frac{100+CV}{4},\ x_2 = \frac{m+CV}{2p_2} = \frac{100+CV}{2}$$

and achieves the following utility level:

$$U = \left(\frac{100+CV}{4}\right)^{1/2}\left(\frac{100+CV}{2}\right)^{1/2} = \frac{100+CV}{2\sqrt{2}}$$

Set $U = \frac{100+CV}{2\sqrt{2}} = U^* = 50,$

$$CV = 100\sqrt{2} - 100 = \$41$$

A C-D Example

EV is the amount of money the consumer is just willing to pay to avoid the price changes.

i.e. After giving away $EV, the consumer achieves the new utility level at the old prices.

A C-D Example

After giving away \$EV, the consumer achieves the new utility level at the old prices.

$$U = x_1^{1/2} x_2^{1/2}$$

$$x_1^* = \frac{m}{2p_1},\ x_2^* = \frac{m}{2p_2}$$

After giving away \$EV, the consumer buys the following bundle at the old prices.

$$x_1 = \frac{m - EV}{2p_1} = \frac{100 - EV}{2},\ x_2 = \frac{m - EV}{2p_2} = \frac{100 - EV}{2}$$

A C-D Example

After giving away \$EV, the consumer buys the following bundle at the old prices,

$$x_1 = \frac{m - EV}{2p_1} = \frac{100 - EV}{2},\ x_2 = \frac{m - EV}{2p_2} = \frac{100 - EV}{2}$$

and achieves the following utility level:

$$U = \left(\frac{100 - EV}{2}\right)^{1/2}\left(\frac{100 - EV}{2}\right)^{1/2} = \frac{100 - EV}{2}$$

A C-D Example

After giving away \$EV, the consumer buys the following bundle at the old prices.

$$x_1 = \frac{m - EV}{2p_1} = \frac{100 - EV}{2},\ x_2 = \frac{m - EV}{2p_2} = \frac{100 - EV}{2}$$

and achieves the following utility level:

$$U = \left(\frac{100 - EV}{2}\right)^{1/2} \left(\frac{100 - EV}{2}\right)^{1/2} = \frac{100 - EV}{2}$$

Set $U = \frac{100 - EV}{2} = U^{**} = 25\sqrt{2},$

$$EV = 100 - 50\sqrt{2} = \$29.3$$

A C-D Example

$\Delta\mathbf{CS} = ?$

$$U = x_1^{1/2} x_2^{1/2}$$

$$p_1 x_1 + p_2 x_2 = m$$

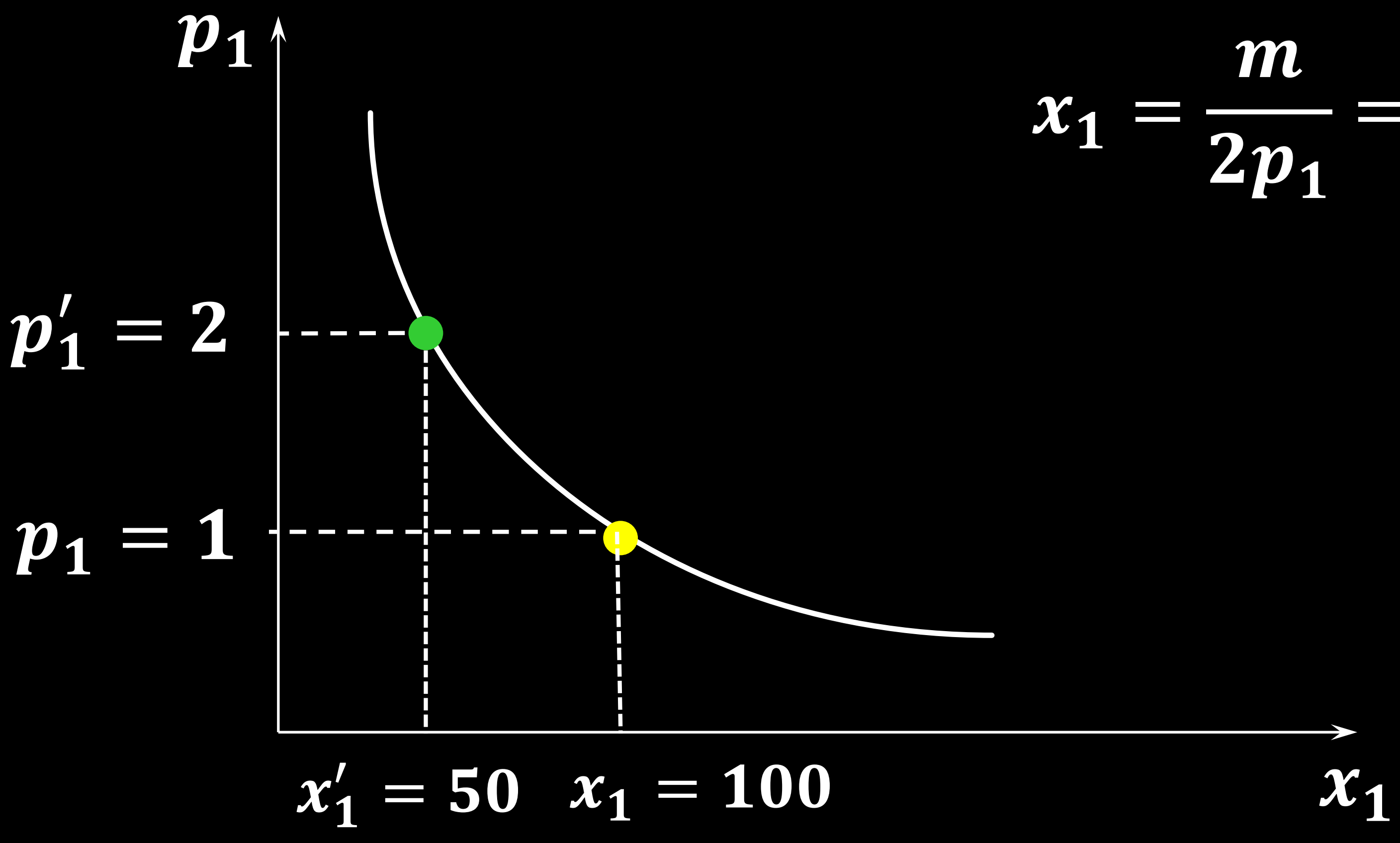
$$x_1^* = \frac{m}{2p_1}, x_2^* = \frac{m}{2p_2}$$

Demand function for x_1:

$$x_1 = \frac{m}{2p_1} = \frac{50}{p_1}$$

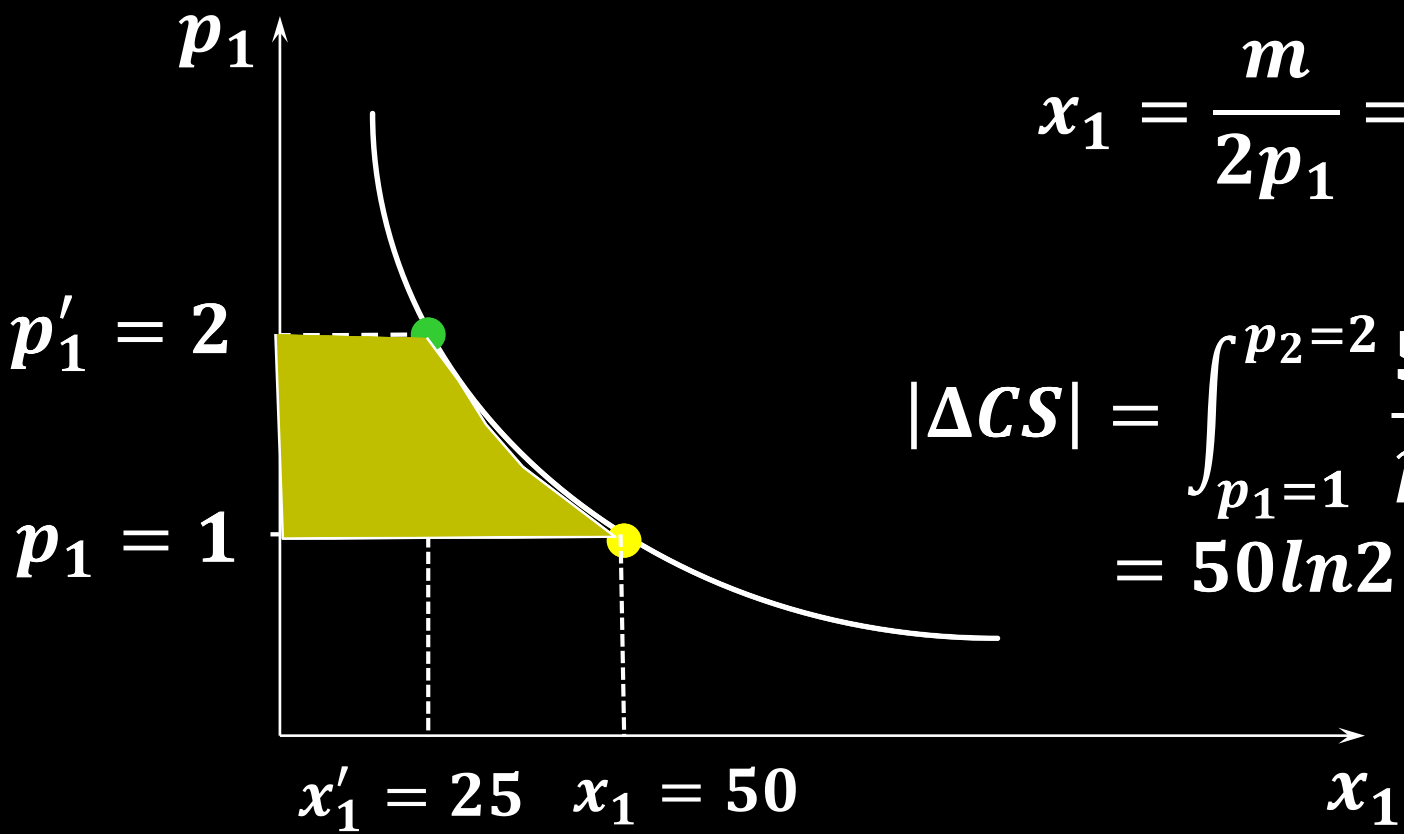
A C-D Example

$\Delta\mathbf{CS} = ?$



$$x_1 = \frac{m}{2p_1} = \frac{50}{p_1}$$

A C-D Example

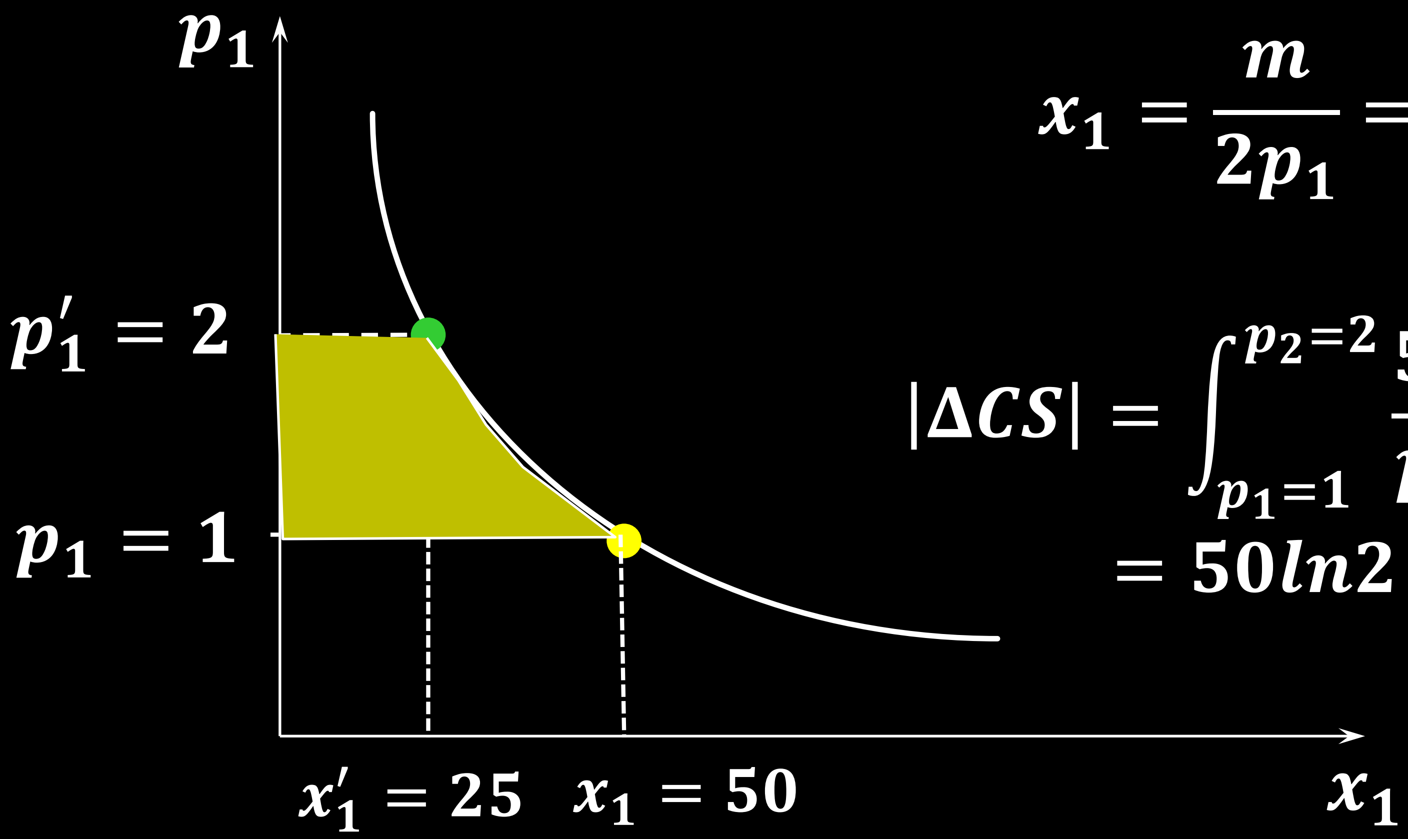


$$x_1 = \frac{m}{2p_1} = \frac{50}{p_1}$$

$$|\Delta CS| = \int_{p_1=1}^{p_2=2} \frac{50}{p_1} dp_1$$

$$= 50ln2 = \$34.7$$

A C-D Example



$$x_1 = \frac{m}{2p_1} = \frac{50}{p_1}$$

$$|\Delta CS| = \int_{p_1=1}^{p_2=2} \frac{50}{p_1} dp_1$$
$$= 50ln2 = \$34.7$$

|EV| < |ΔCS| < |CV|

Consumer's Surplus, Compensating Variation and Equivalent Variation

When the consumer has non-quasilinear utility,

$$|EV| < |\Delta CS| < |CV|$$

When the consumer has quasilinear utility,

$$CV = EV = \Delta CS.$$

Consumer's Surplus, Compensating Variation and Equivalent Variation

Assume $u(x, y) = v(x) + y$. The demand is

$$p_x = v'(x)$$

When $p_x = p'$, consumer demands $(x^*, m - p'x^*)$. When $p_x = p'' > p'$, consumer demands $(x^{**}, m - p''x^{**})$.

Consumer's Surplus, Compensating Variation and Equivalent Variation

Assume $u(x, y) = v(x) + y$. The demand is

$$p_x = v'(x)$$

When $p_x = p'$, consumer demands $(x^*, m - p'x^*)$. When $p_x = p'' > p'$, consumer demands $(x^{**}, m - p''x^{**})$.

CV:

$$v(x^*) + m - p'x^* = v(x^{**}) + m + CV - p''x^{**}$$

$$CV = [v(x^*) - v(x^{**})] - [p'x^* - p''x^{**}]$$

Note: 增加的收入CV不改变对x的选择，全部用来增加对y的消费

Consumer's Surplus, Compensating Variation and Equivalent Variation

Assume $u(x, y) = v(x) + y$. The demand is

$$p_x = v'(x)$$

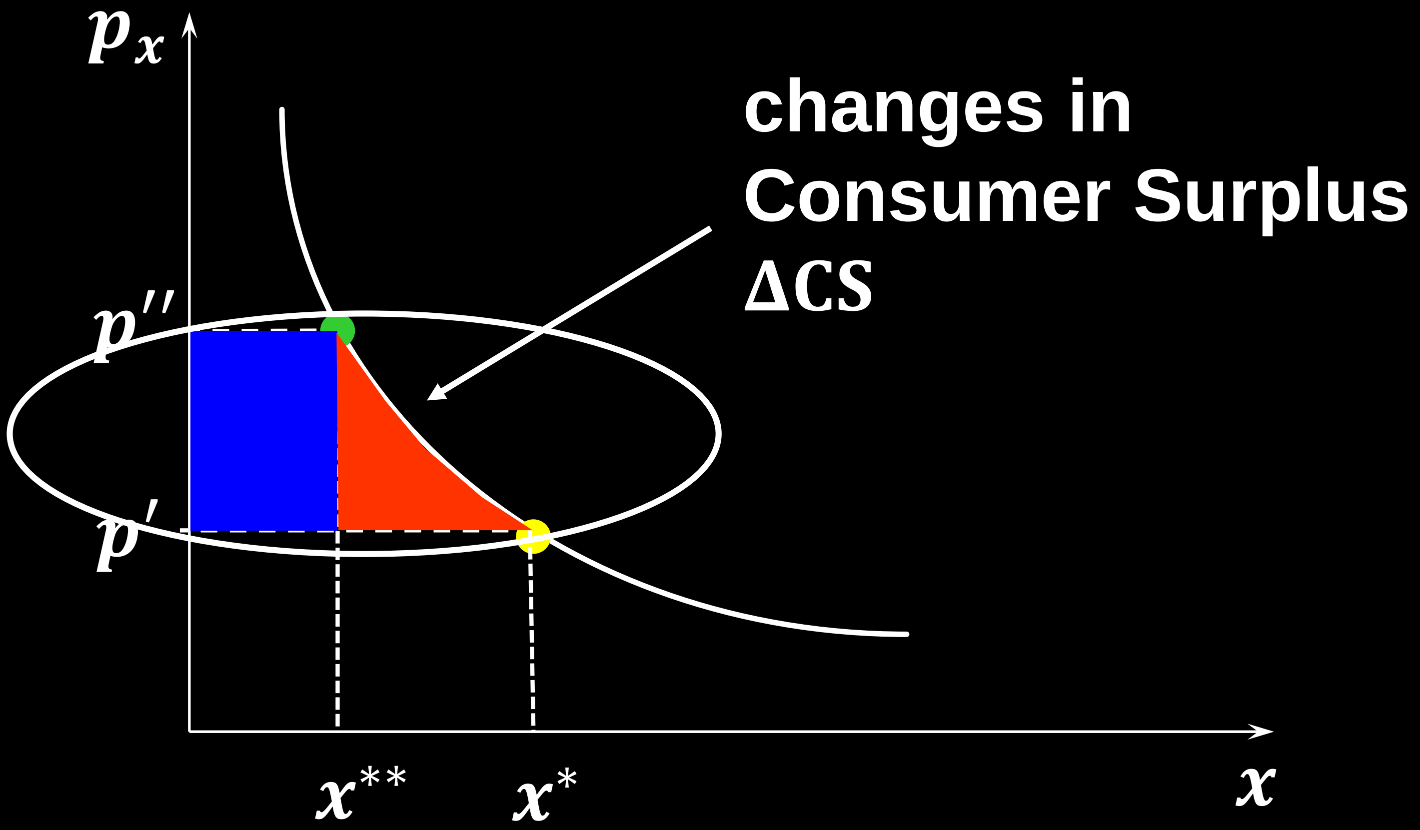
When $p_x = p'$, consumer demands $(x^*, m - p'x^*)$. When $p_x = p'' > p'$, consumer demands $(x^{**}, m - p''x^{**})$.

EV:

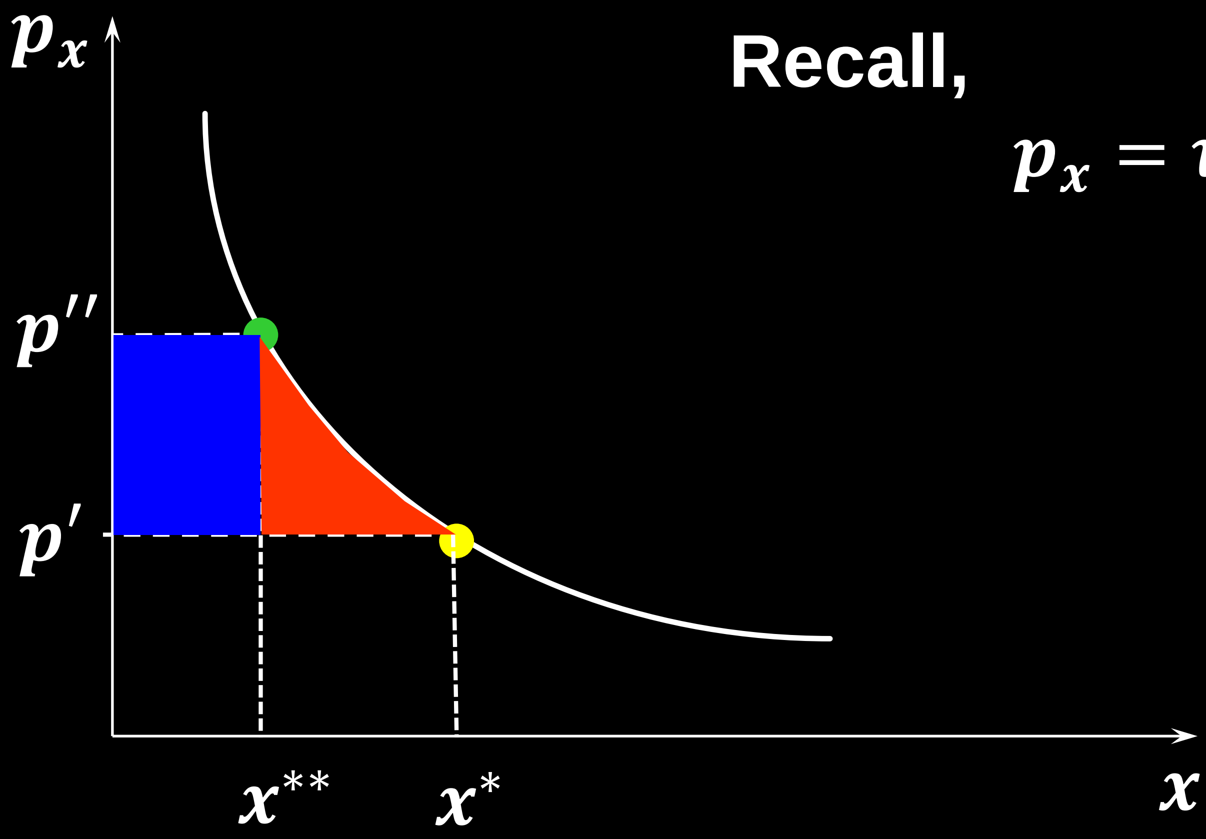
$$v(x^*) + m - p'x^* - EV = v(x^{**}) + m - p''x^{**}$$

$$EV = [v(x^*) - v(x^{**})] - [p'x^* - p''x^{**}] = \mathbf{CV}$$

Consumer's Surplus, Compensating Variation and Equivalent Variation



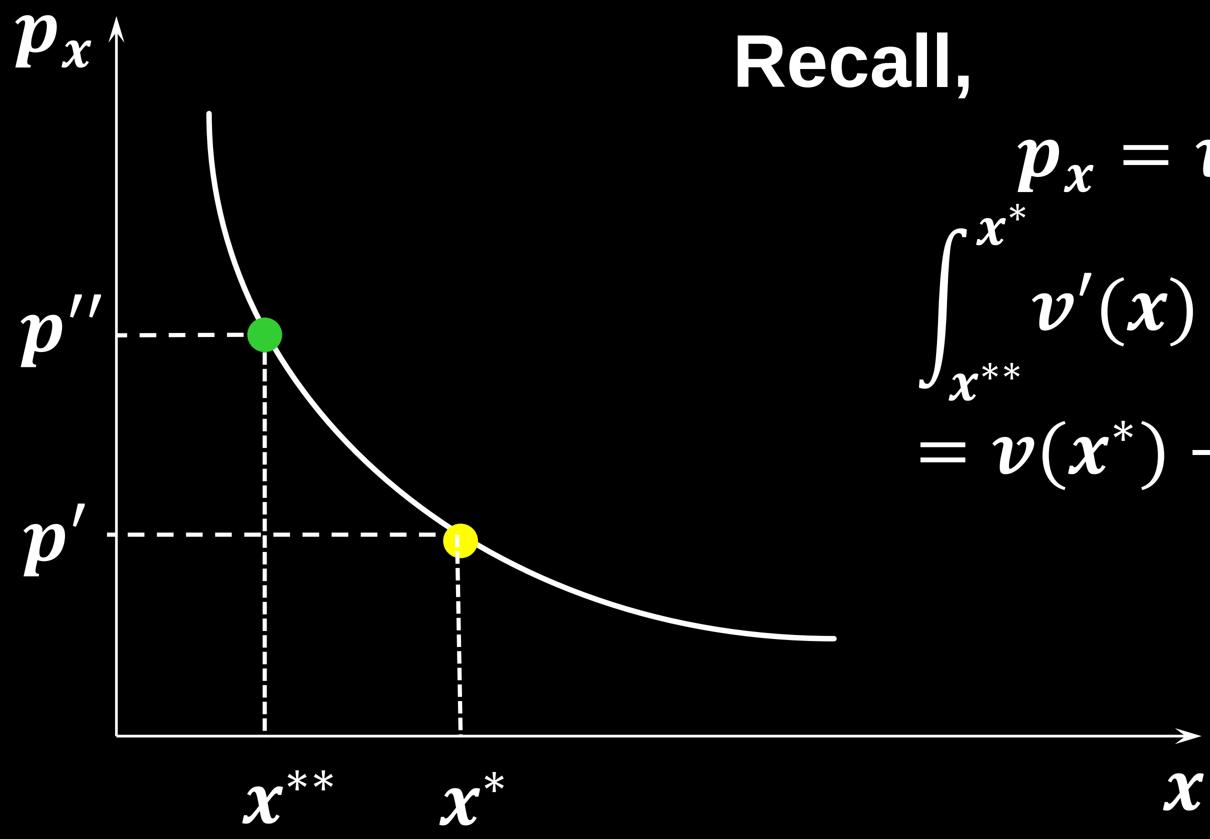
Consumer's Surplus, Compensating Variation and Equivalent Variation



Recall,

$$p_x = v'(x)$$

Consumer's Surplus, Compensating Variation and Equivalent Variation



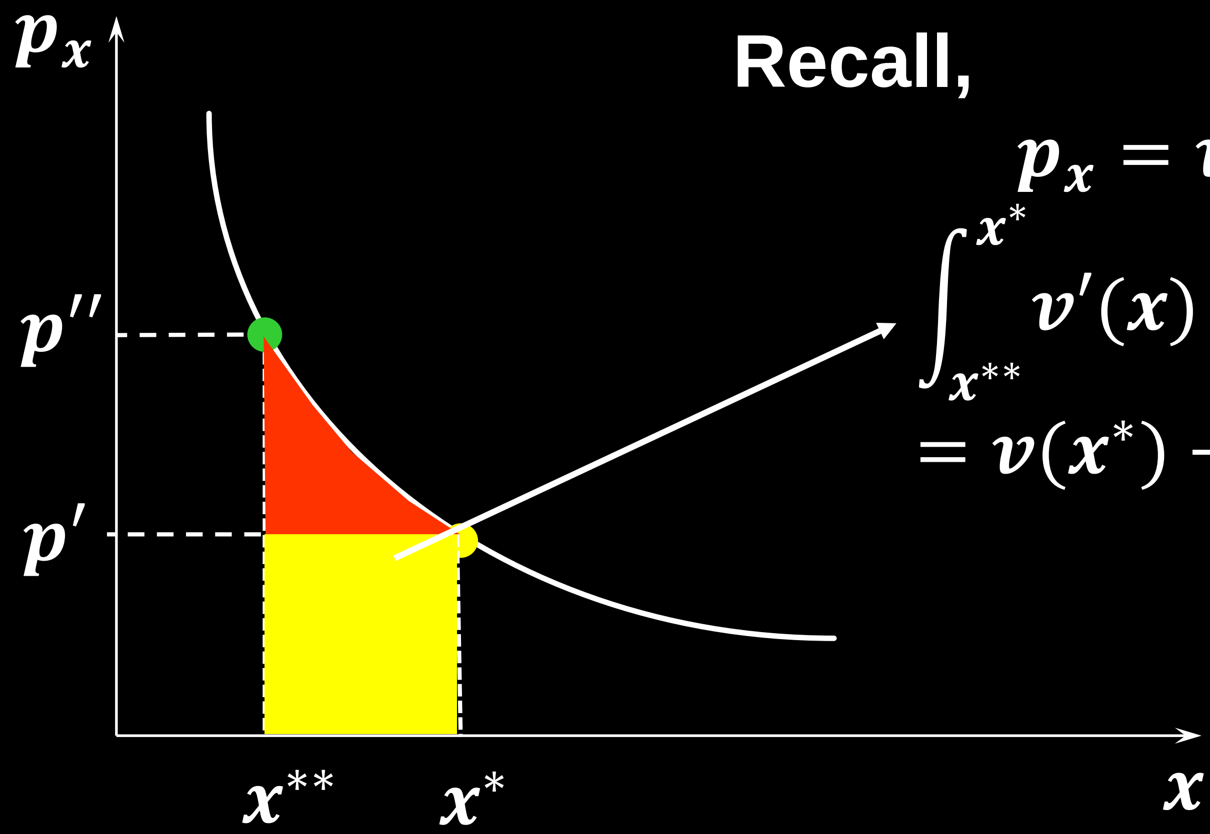
Recall,

$$p_x = v'(x)$$

$$\int_{x^{**}}^{x^*} v'(x)\ dx$$

$$= v(x^*) - v(x^{**})$$

Consumer's Surplus, Compensating Variation and Equivalent Variation



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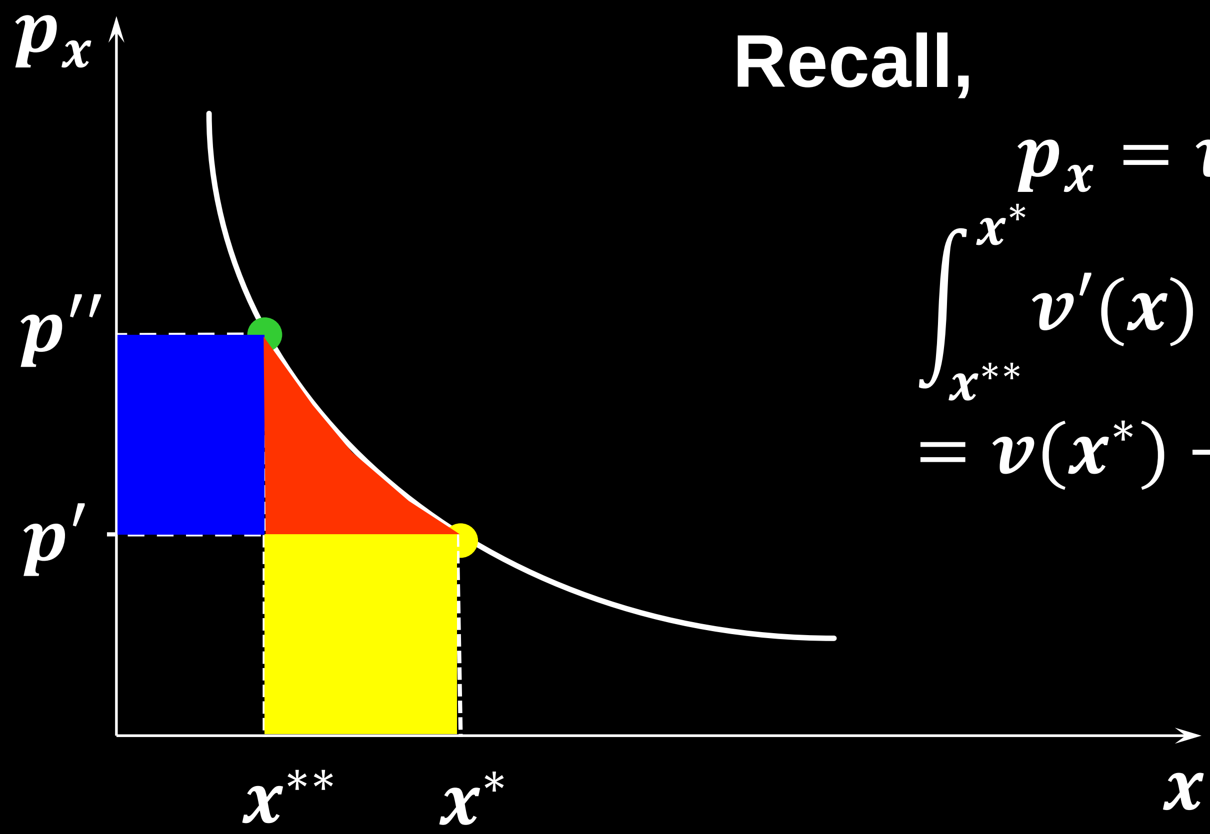


Recall,

$$p_x = v'(x)$$

$$\int_{x^{**}}^{x^*} v'(x)\ dx$$

$$= v(x^*) - v(x^{**})$$

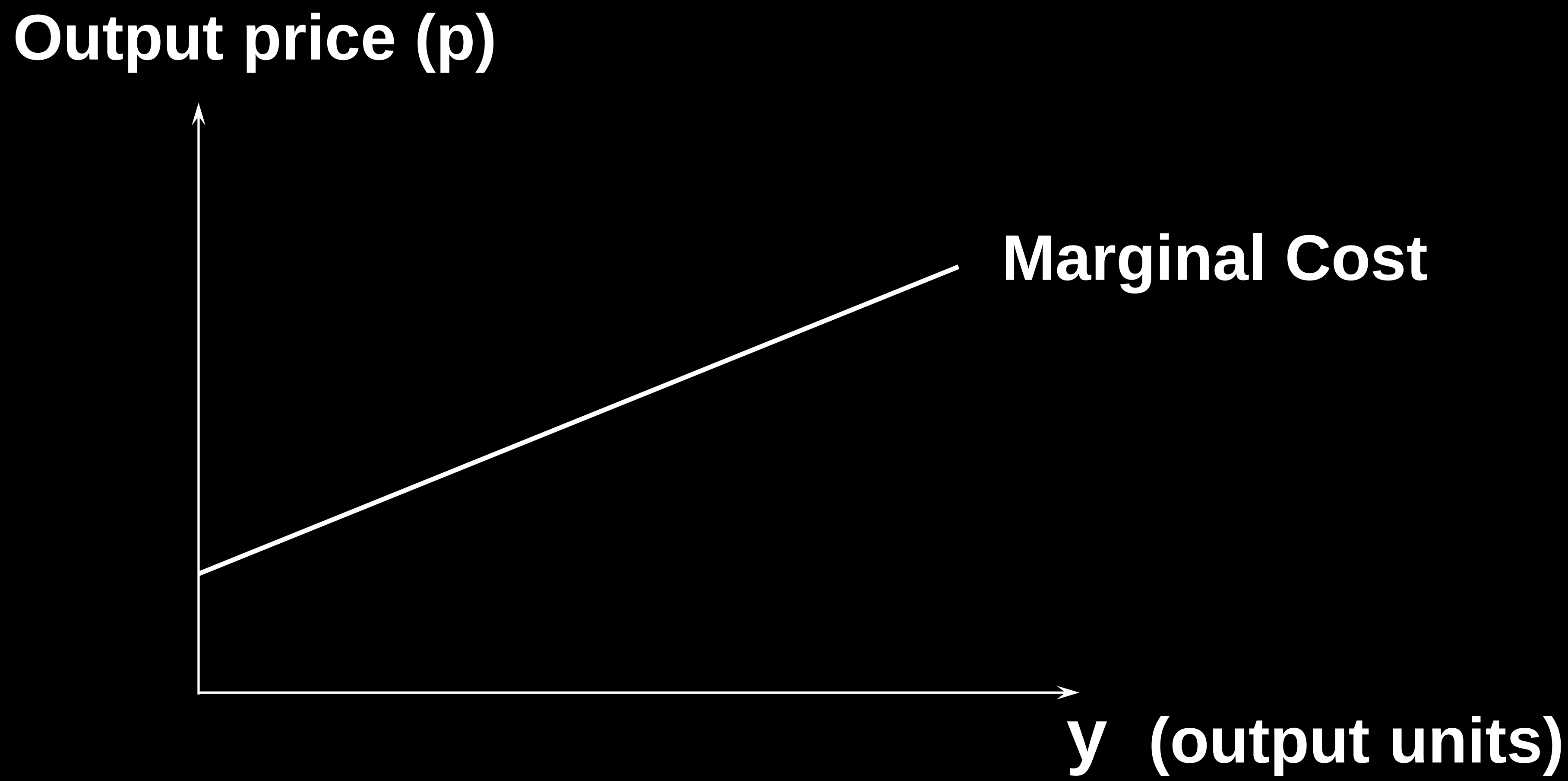
Then,

$$\Delta CS = [v(x^*) - v(x^{**})] - [p'x^* - p''x^{**}] = CV = EV$$

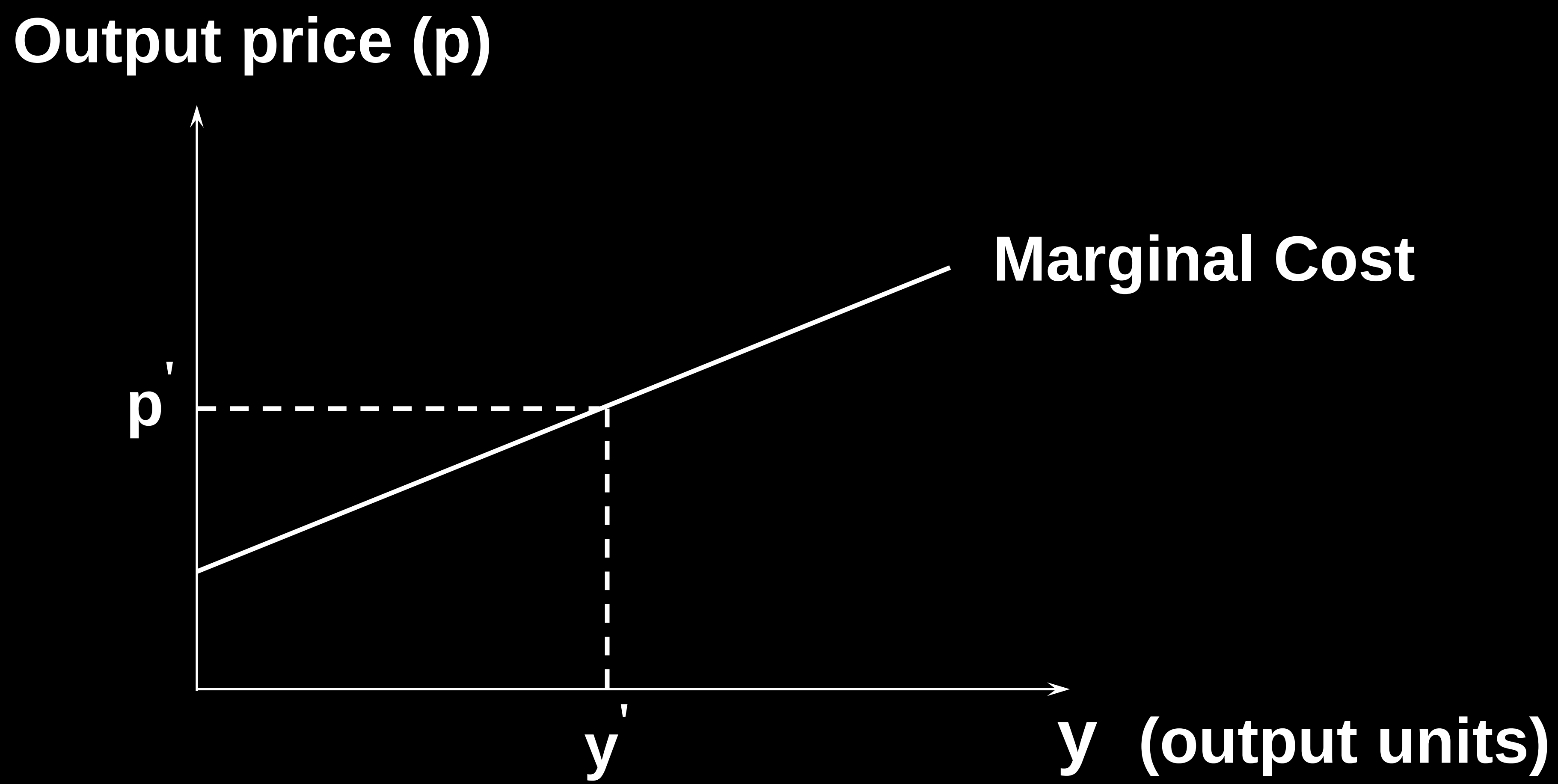
Producer's Surplus

- Changes in a firm's welfare can be measured in dollars much as for a consumer.

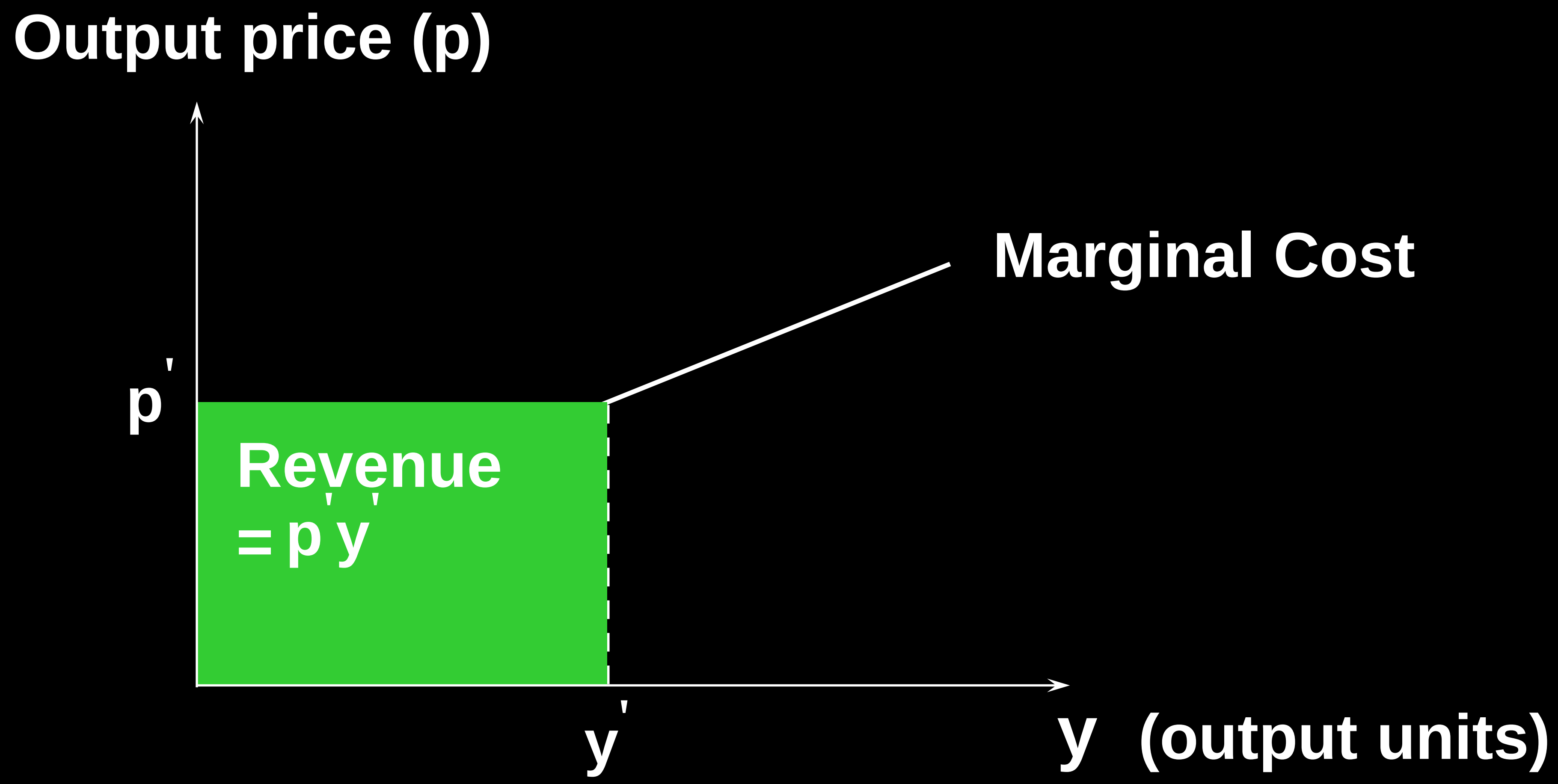
Producer's Surplus



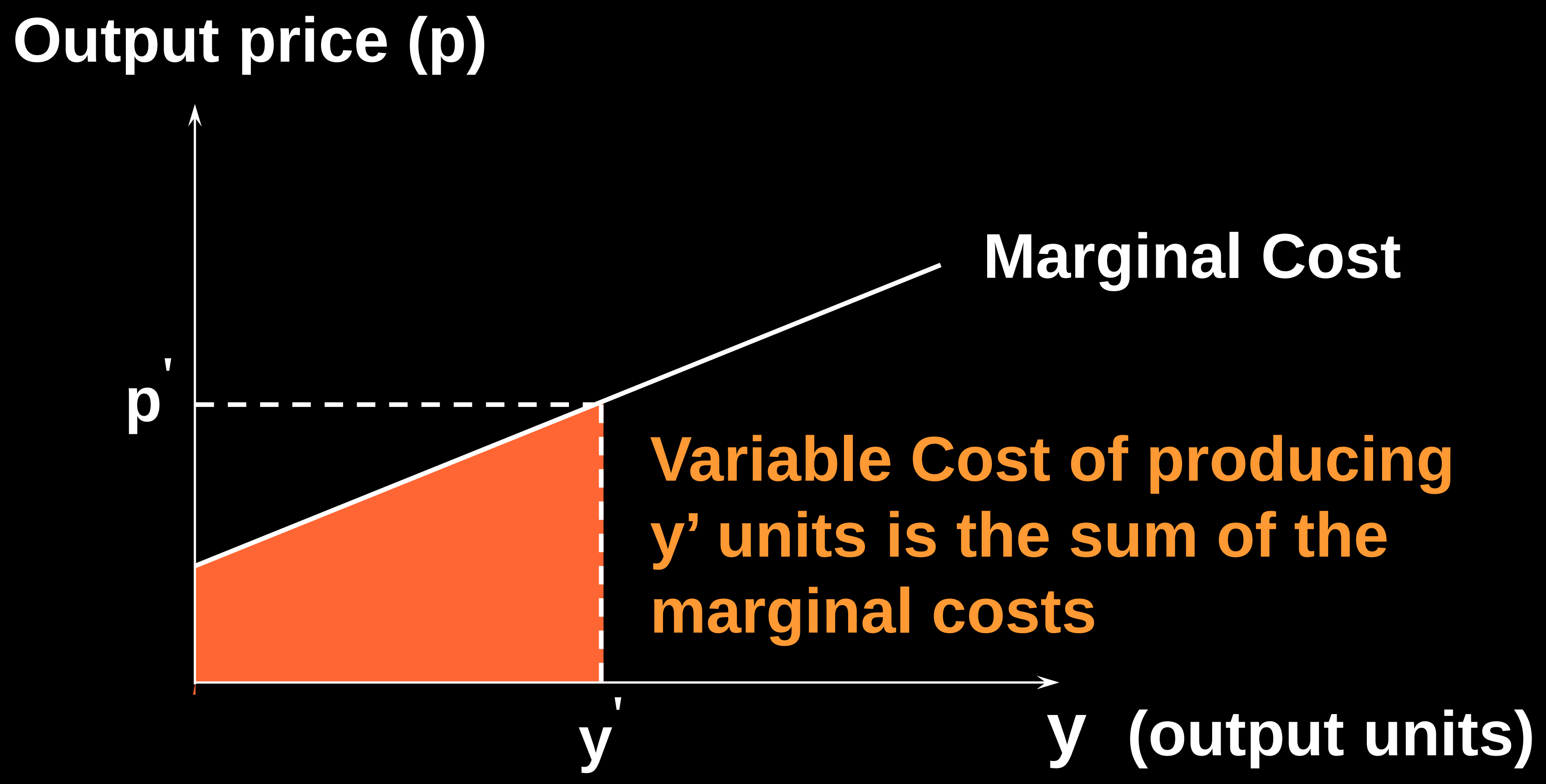
Producer's Surplus



Producer's Surplus



Producer's Surplus



Producer's Surplus