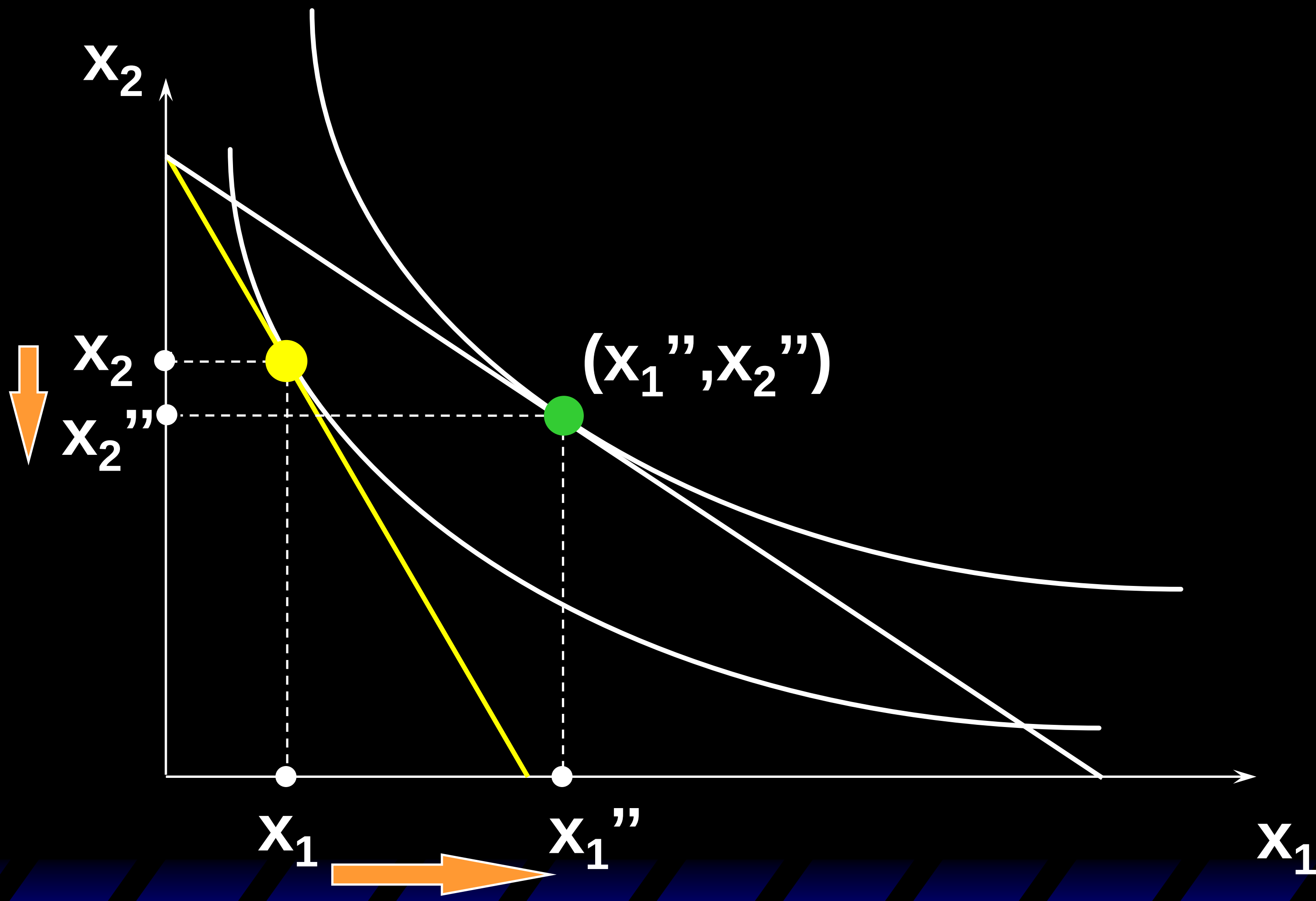
Lecture 6

Slutsky Equation

Effects of a Price Change



Effects of a Price Change

What happens when a commodity's price decreases?

- **Substitution effect: the commodity is relatively cheaper, so consumers substitute it for relatively more expensive other commodities.**

由于相对价格的变化而造成的需求变化被称作替代效应。

Effects of a Price Change

– **Income effect: the consumer's budget of $y can purchase more than before, as if the consumer's income rose, with consequent income effects on quantities demanded.**

由于实际购买力的变化而造成的需求变化被称作收入效应。

Today’s Lecture

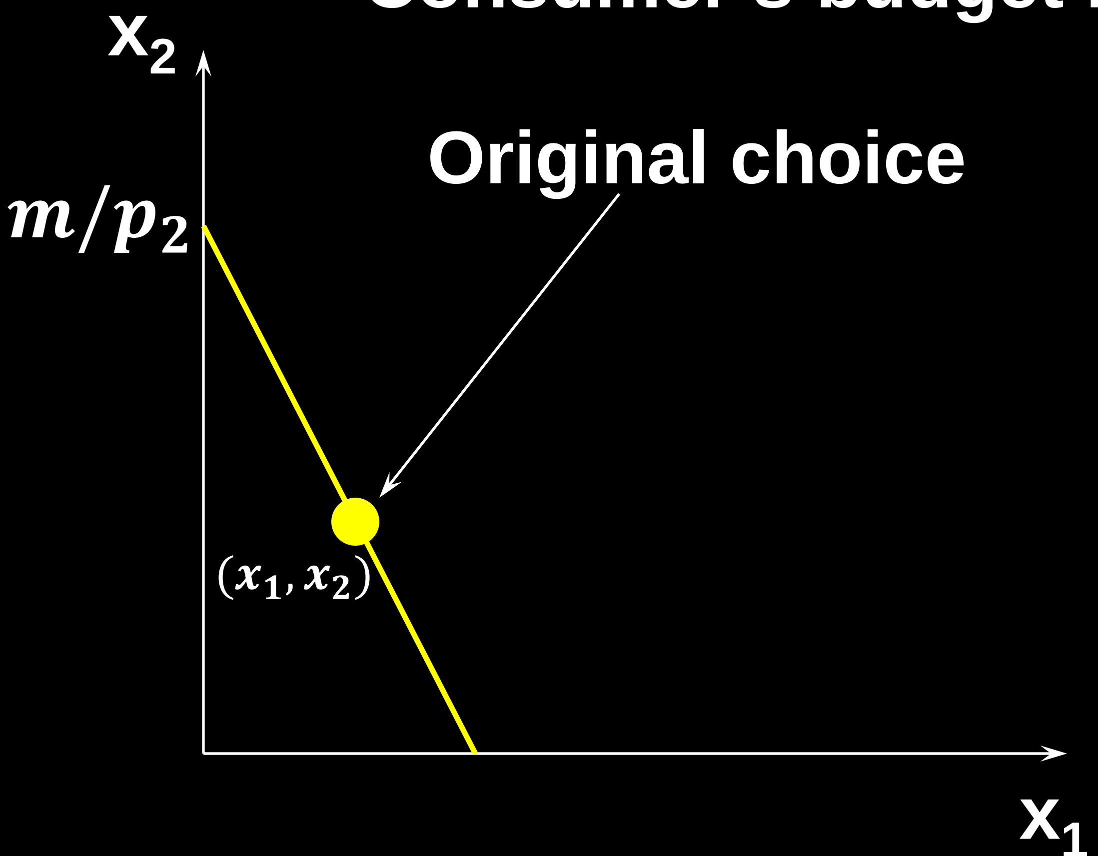
Is to decompose the effect of a price change into:

– **substitution effect**

– **income effect**

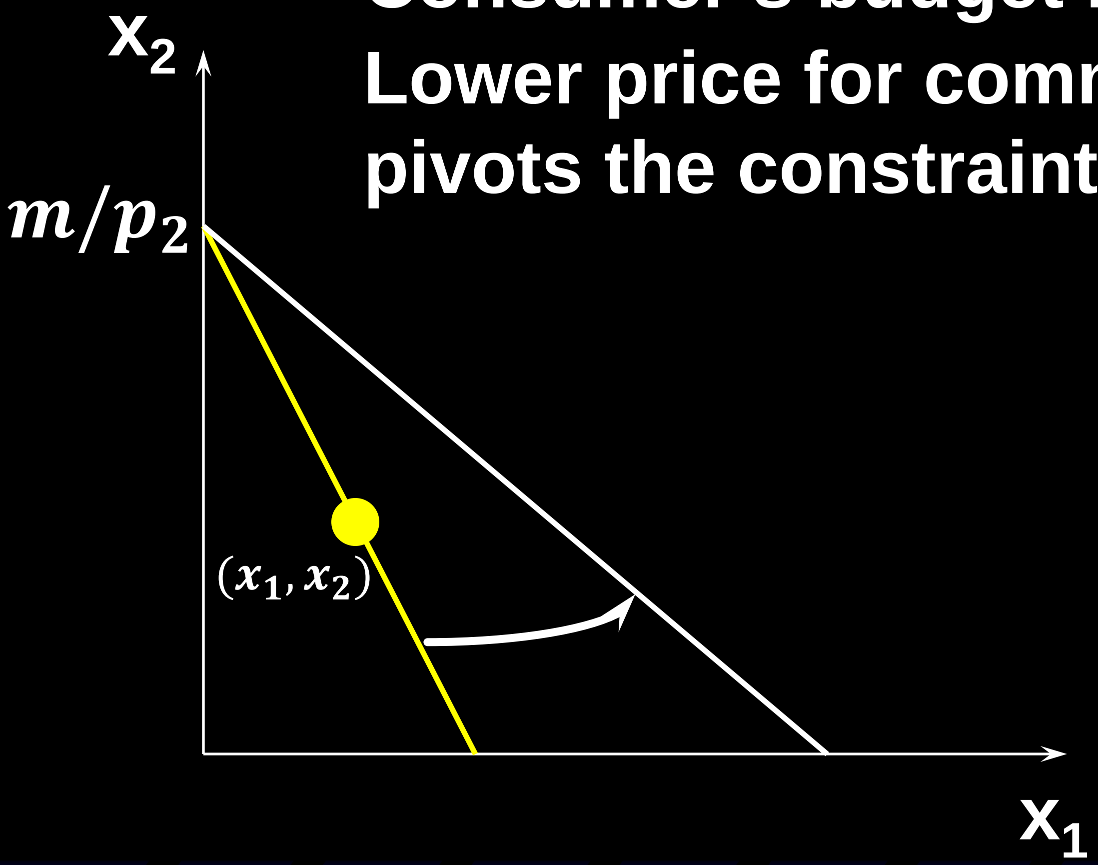
Effects of a Price Change

Consumer's budget is $m.



Effects of a Price Change

Consumer's budget is $m.
Lower price for commodity 1 pivots the constraint outwards.

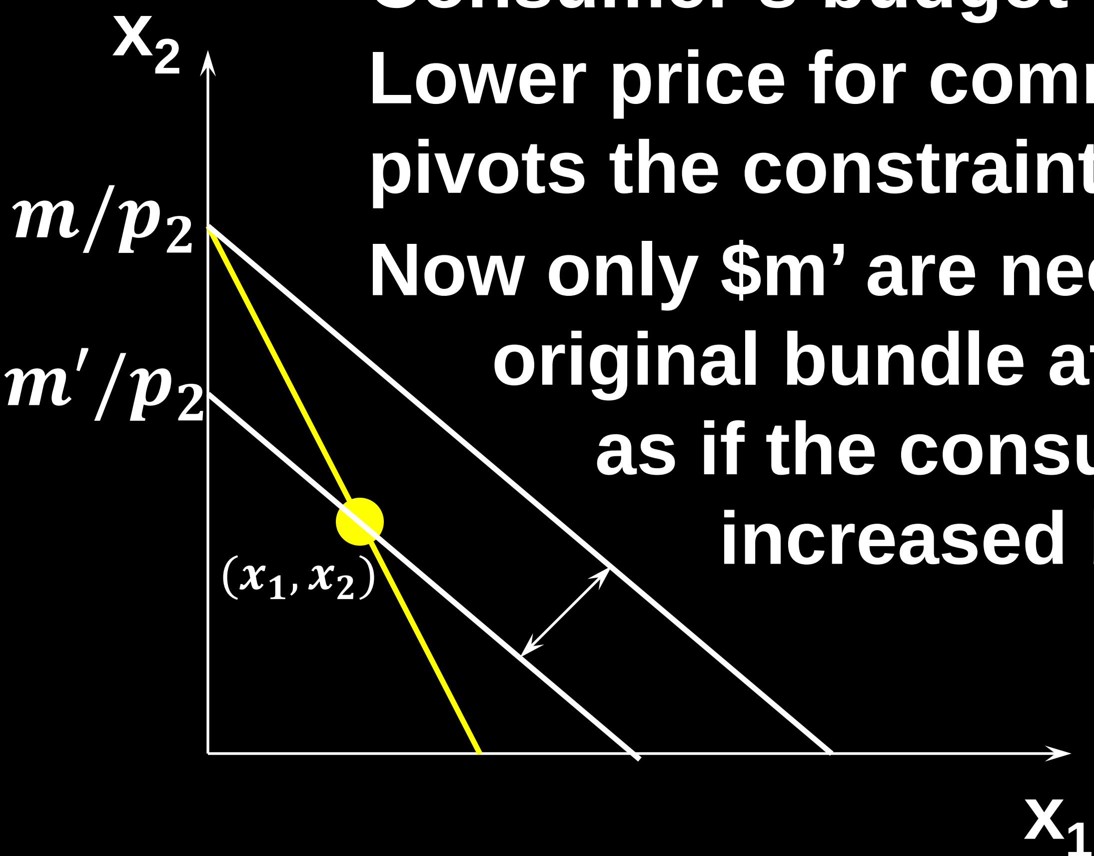


Effects of a Price Change

Consumer's budget is $m.

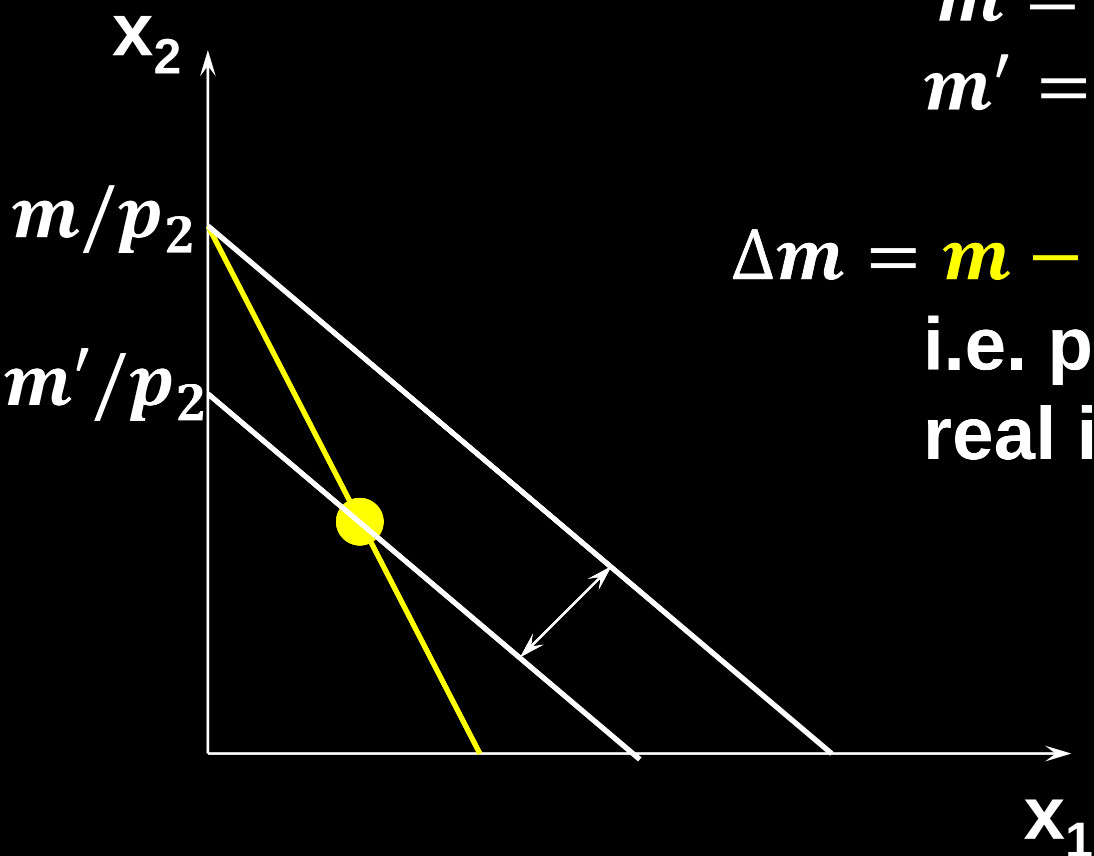
Lower price for commodity 1 pivots the constraint outwards.

Now only $m' are needed to buy the original bundle at the new prices, as if the consumer's income has increased by $m - $m'.



$$m = p_1 x_1 + p_2 x_2$$
$$m' = p_1' x_1 + p_2 x_2$$

Effects of a Price Change



$$m = p_1 x_1 + p_2 x_2$$
$$m' = p_1' x_1 + p_2 x_2$$

$$\Delta m = m - m' = (p_1 - p_1')x_1 > 0$$

i.e. purchasing power / real income increases

Effects of a Price Change

The decrease in p_1 has two effects:

1. $\frac{p_1}{p_2} \downarrow$ **- substitution effect**

2. $\Delta m = m - m' = (p_1 - p'_1)x_1 > 0$

- Changes to quantities demanded due to this 'extra' income are the income effect of the price change.

Effects of a Price Change

Slutsky discovered that changes to demand from a price change are always the sum of a pure substitution effect and an income effect.

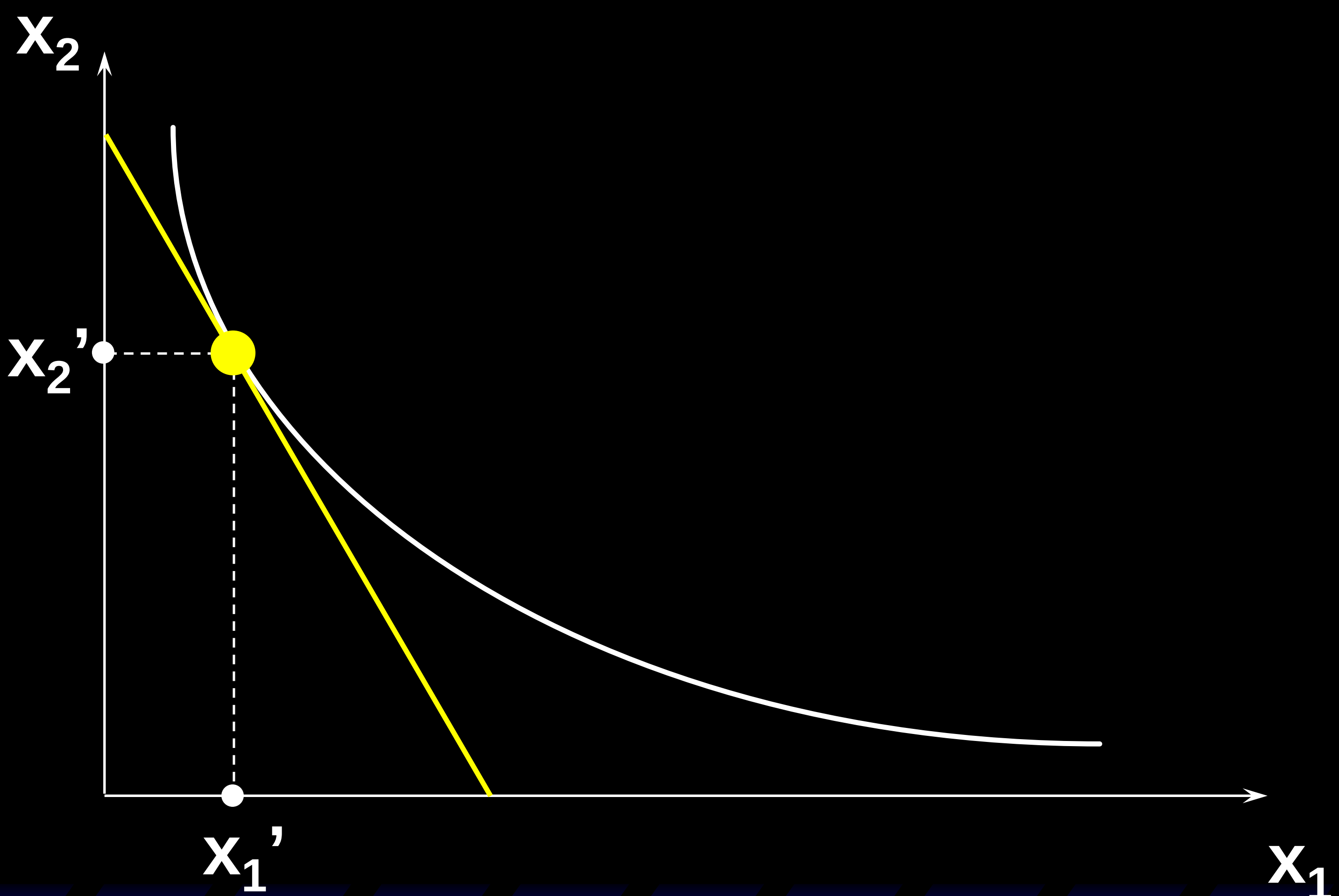
价格变化造成的需求变化等于替代效应和收入效应之和

Pure Substitution Effect

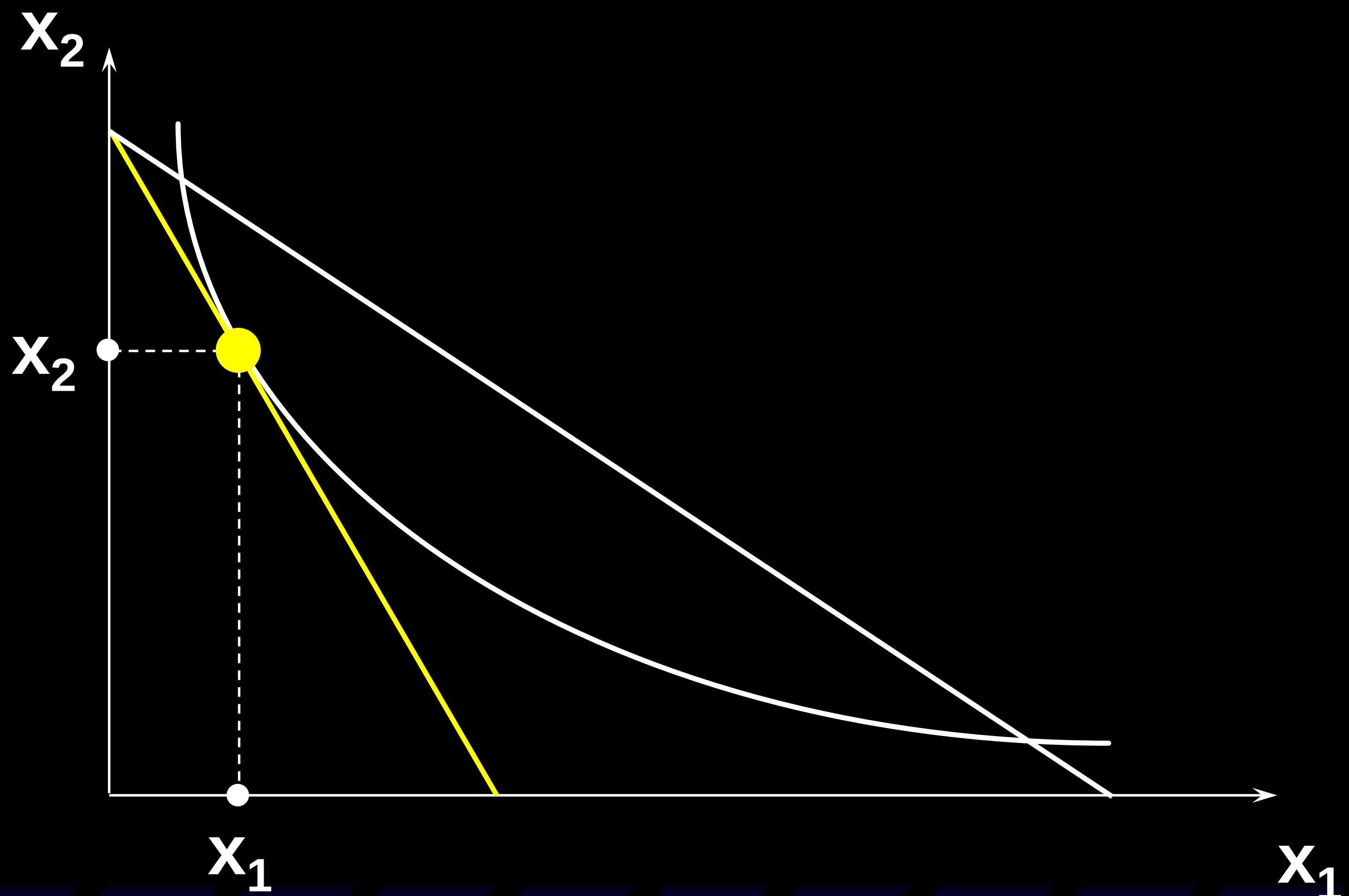
Slutsky isolated the change in demand due only to the change in relative prices by asking "What is the change in demand when the consumer's income is adjusted so that, at the new prices, she can only just buy the original bundle?"

令相对价格变化、同时调整收入使实际购买力保持不变，此时的需求变化完全由替代效应导致。

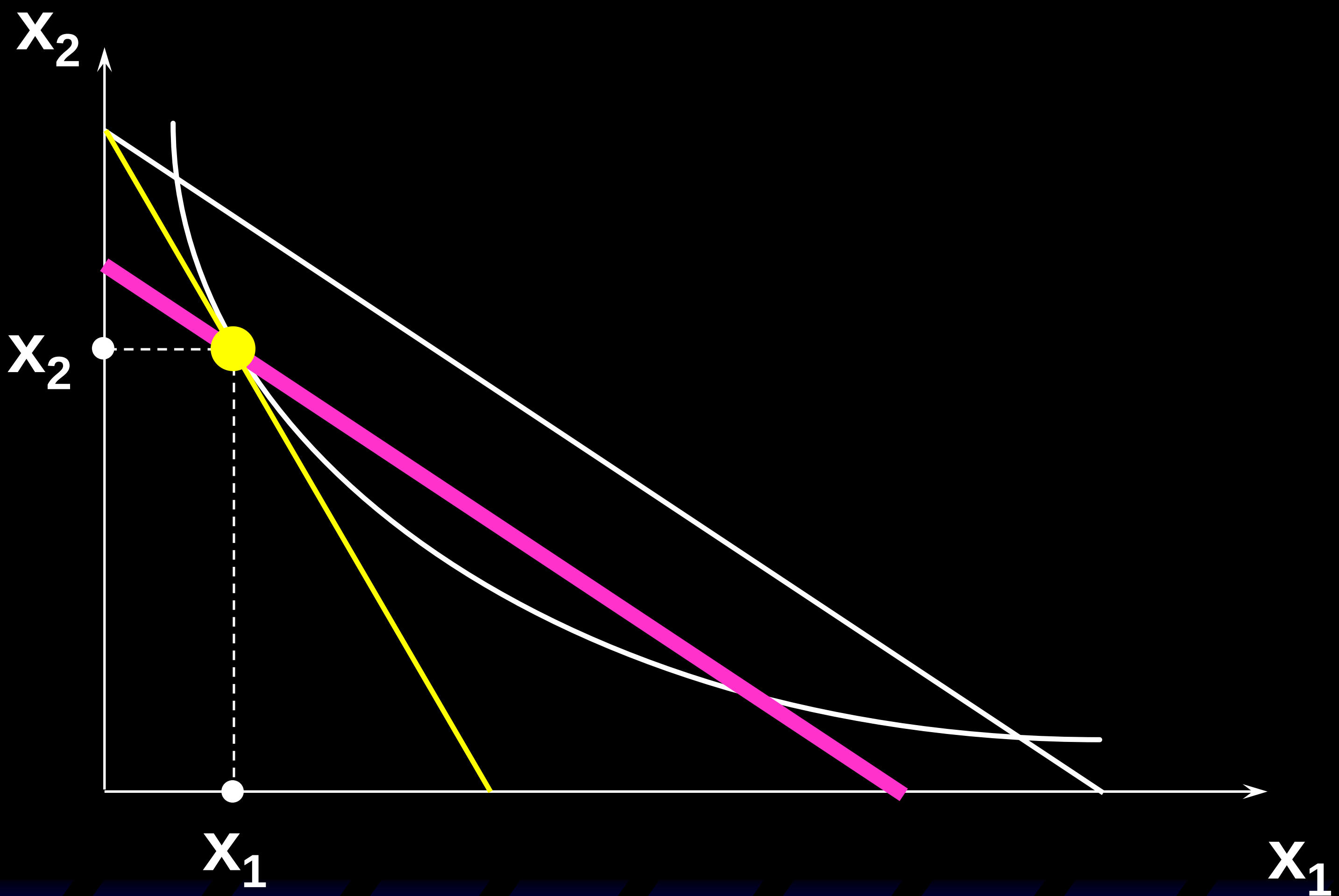
Pure Substitution Effect Only



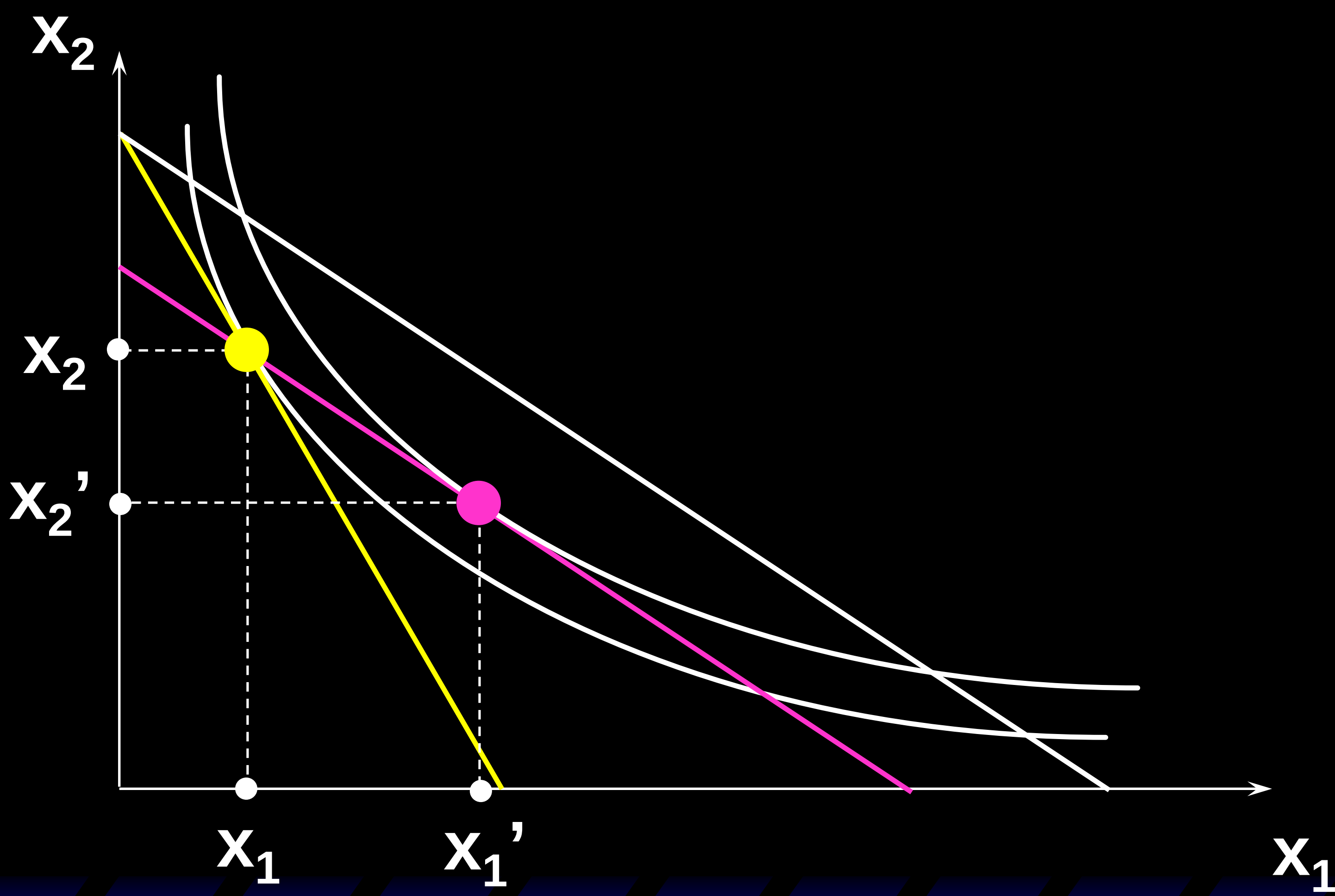
Pure Substitution Effect Only



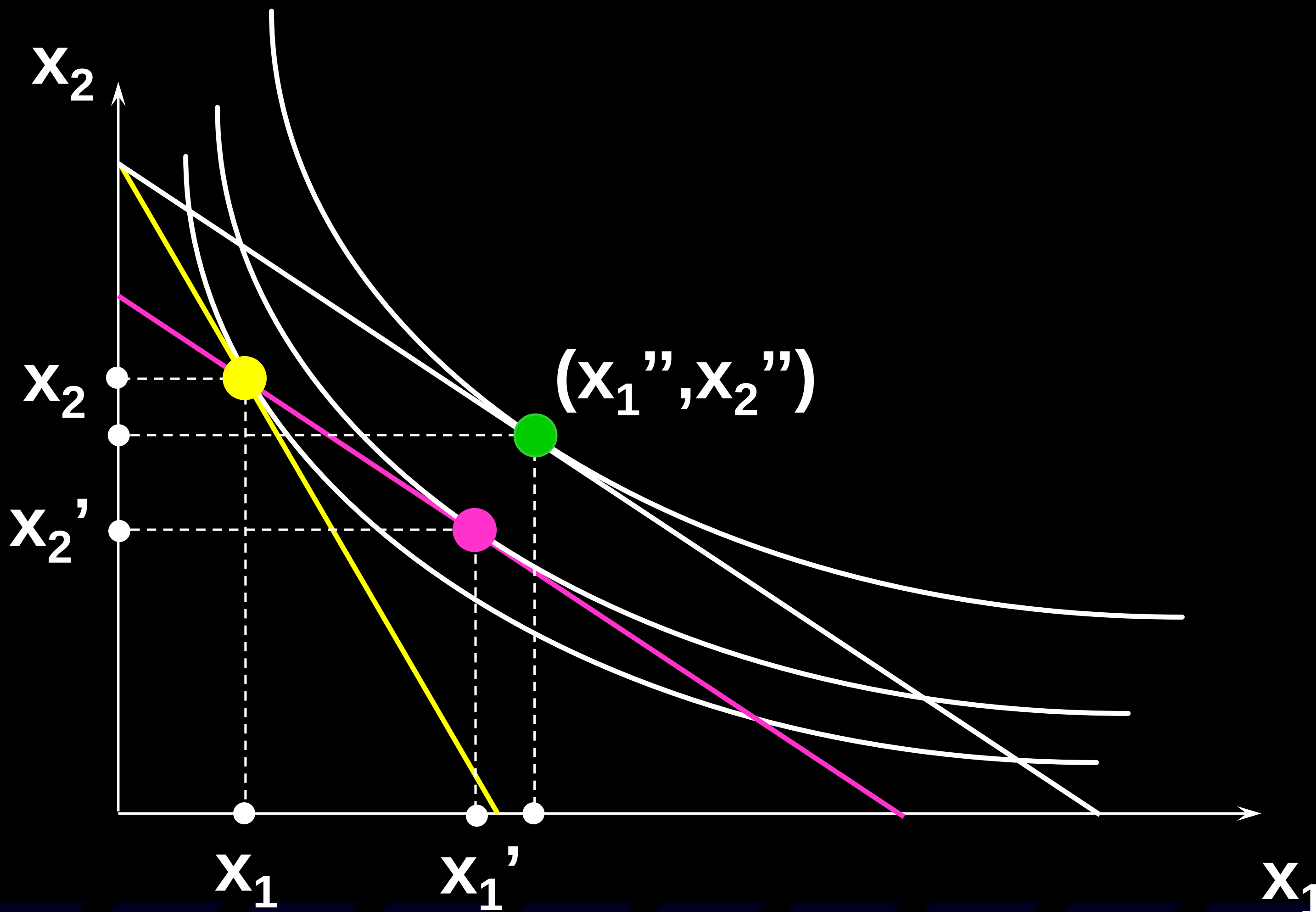
Pure Substitution Effect Only



Pure Substitution Effect Only

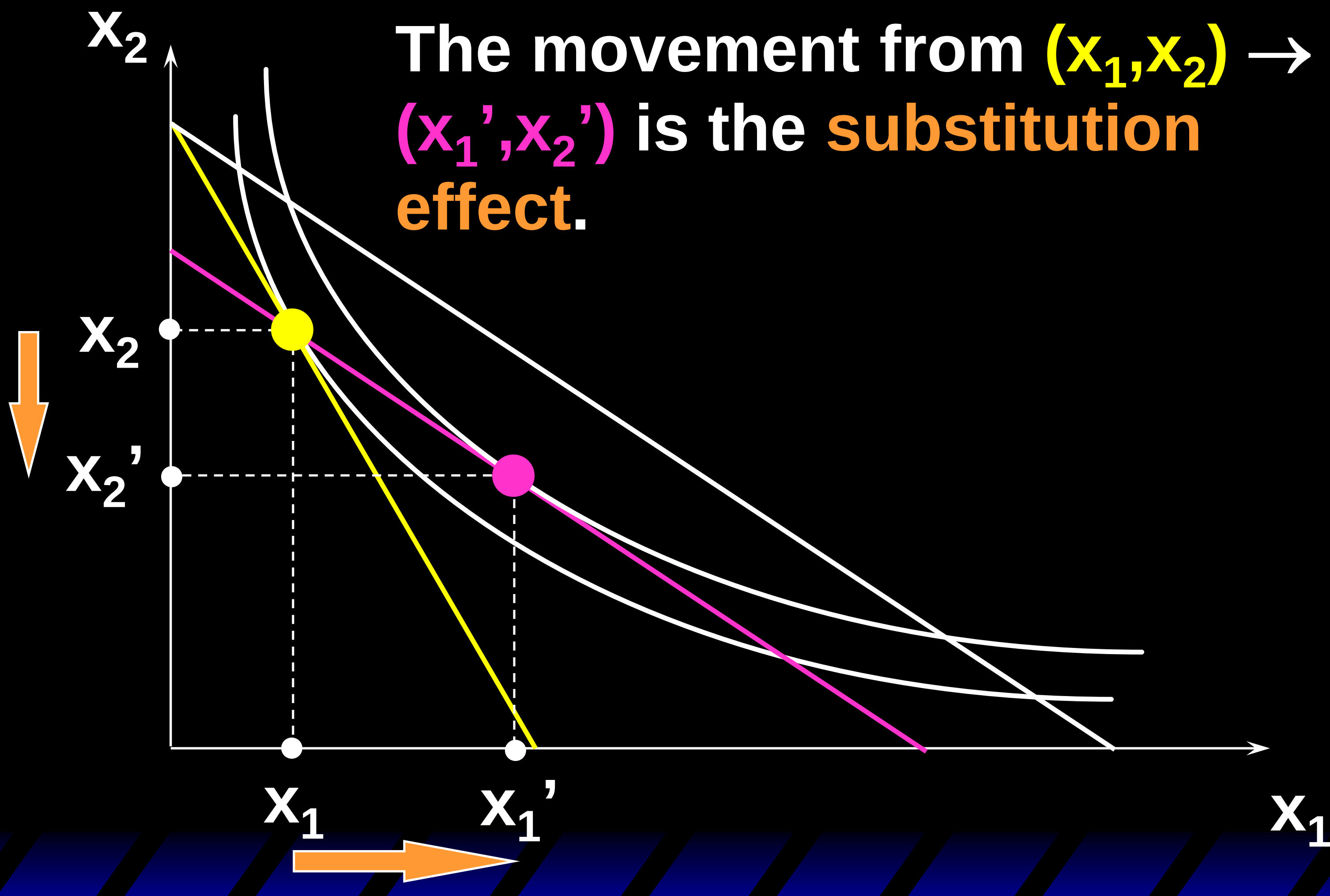


And Now The Income Effect



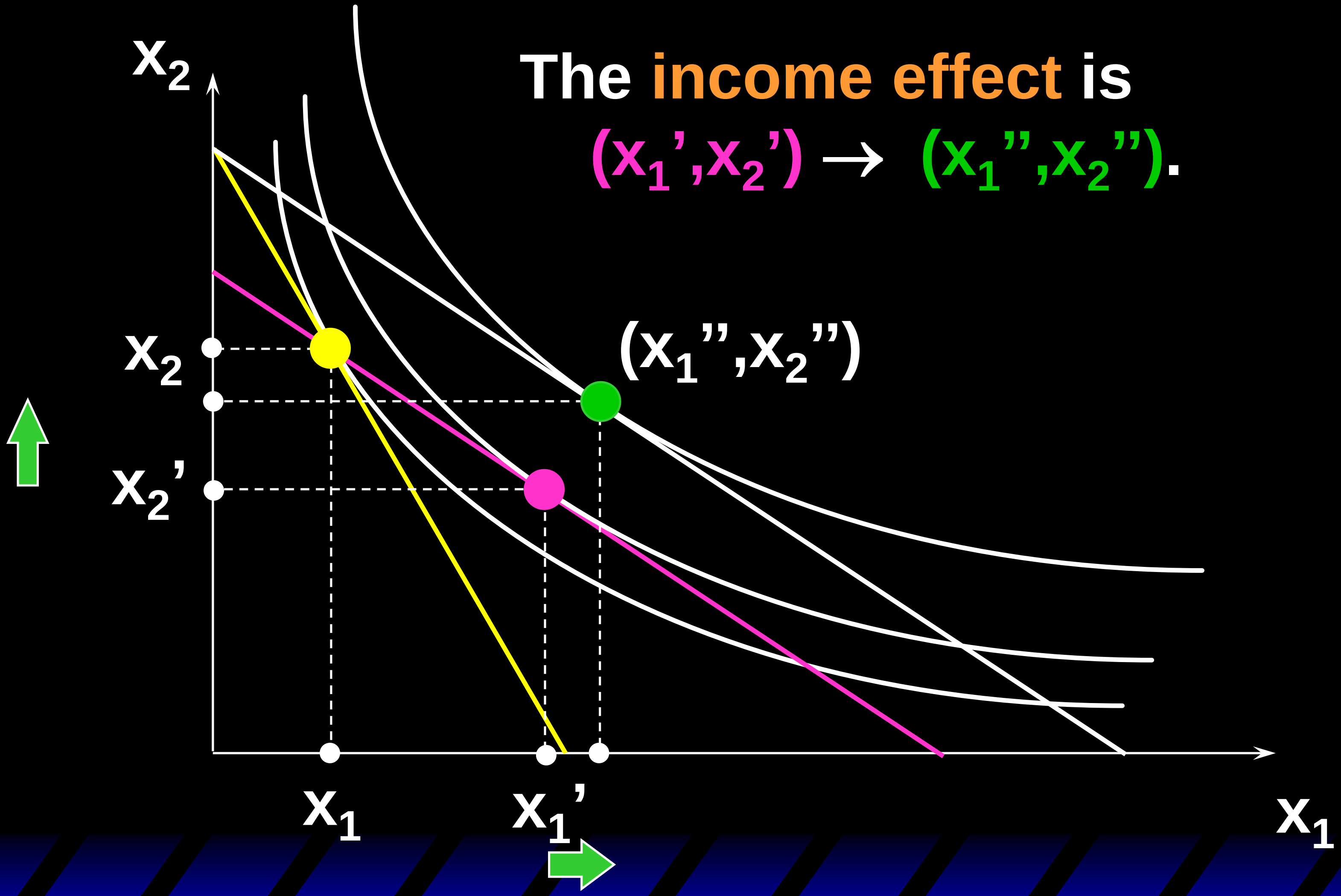
Pure Substitution Effect Only

The movement from $(x_1,x_2) \rightarrow (x_1',x_2')$ is the substitution effect.



And Now The Income Effect

The **income effect** is $(x_1',x_2') \rightarrow (x_1'',x_2'')$.



Total Effect

The change to demand due to lower p_1 is the sum of the income and substitution effects

$$(x_1'', x_2'') - (x_1, x_2) =$$

$$\underbrace{(x_1', x_2') - (x_1, x_2)}_{\text{Substitution Effect}} + \underbrace{(x_1'', x_2'') - (x_1', x_2')}_{\text{Income Effect}}$$

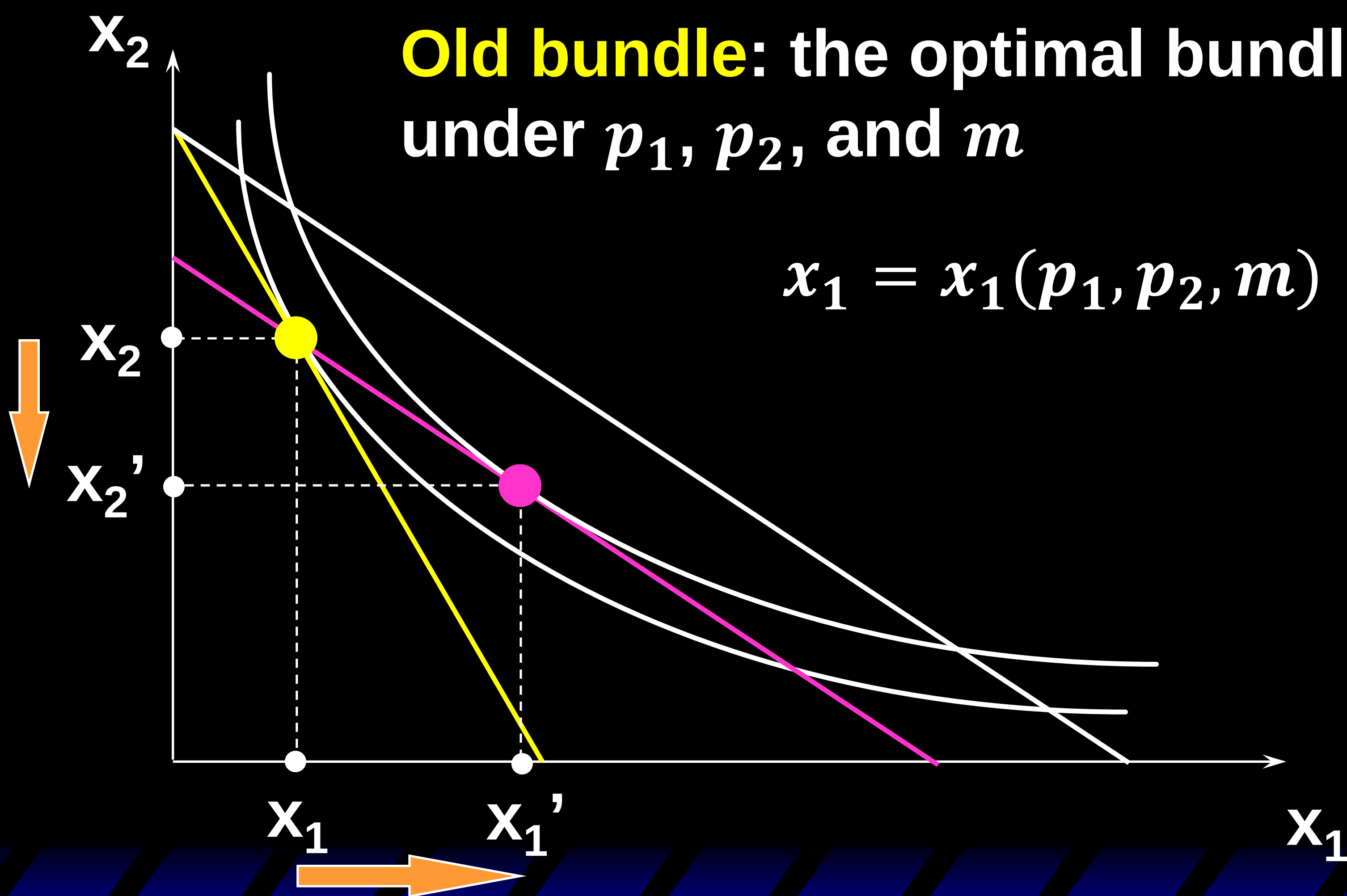
Total Effect on x_1

The change to demand due to lower p_1 is the sum of the income and substitution effects

$$x_1'' - x_1 =$$

$$\underbrace{x_1' - x_1}_{\text{Substitution Effect}} + \underbrace{x_1'' - x_1'}_{\text{Income Effect}}$$

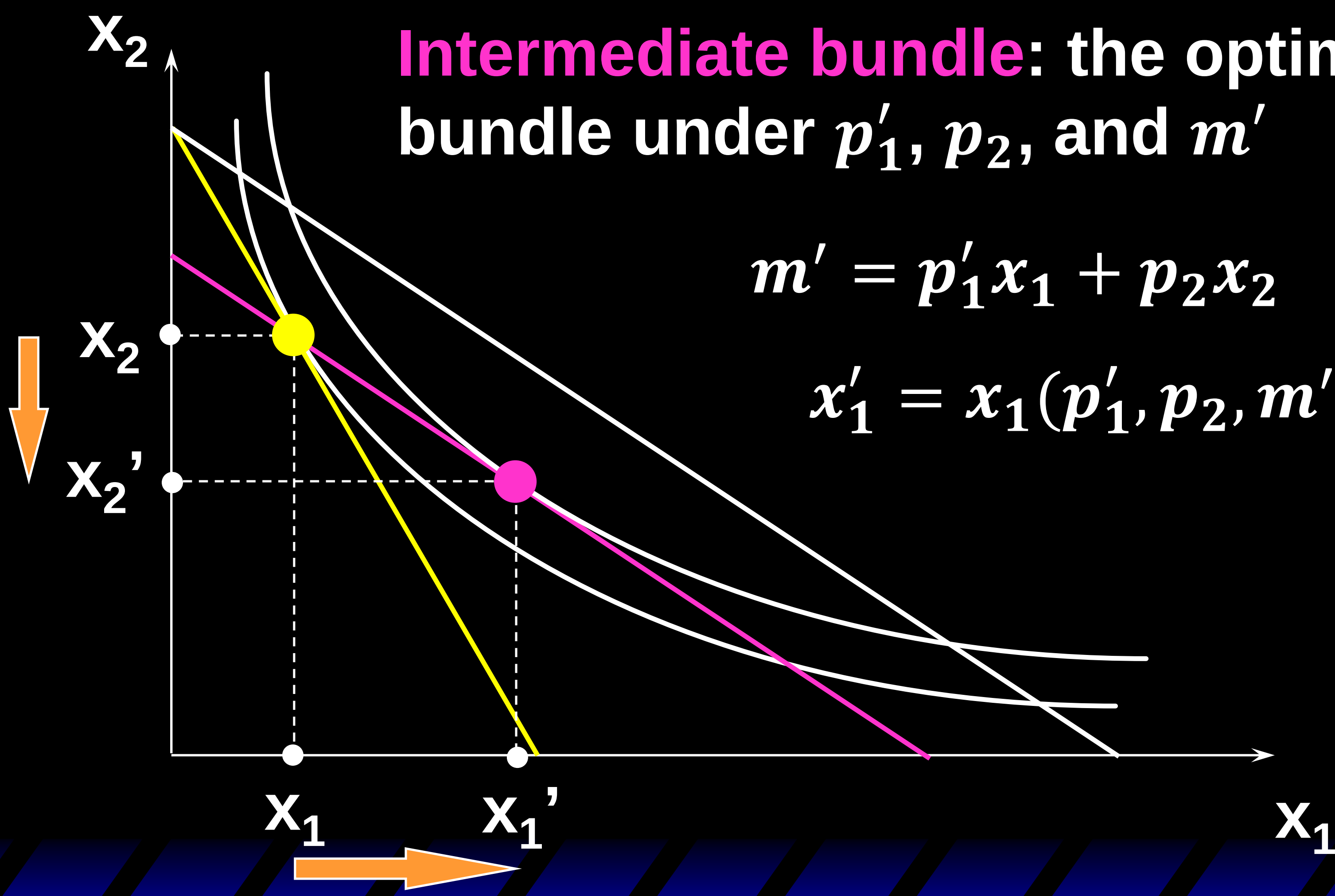
Calculating the Substitution Effect



Old bundle: the optimal bundle under p_1, p_2, and m

$$x_1 = x_1(p_1, p_2, m)$$

Calculating the Substitution Effect



Intermediate bundle: the optimal bundle under p_1', p_2, and m'

$$m' = p_1' x_1 + p_2 x_2$$

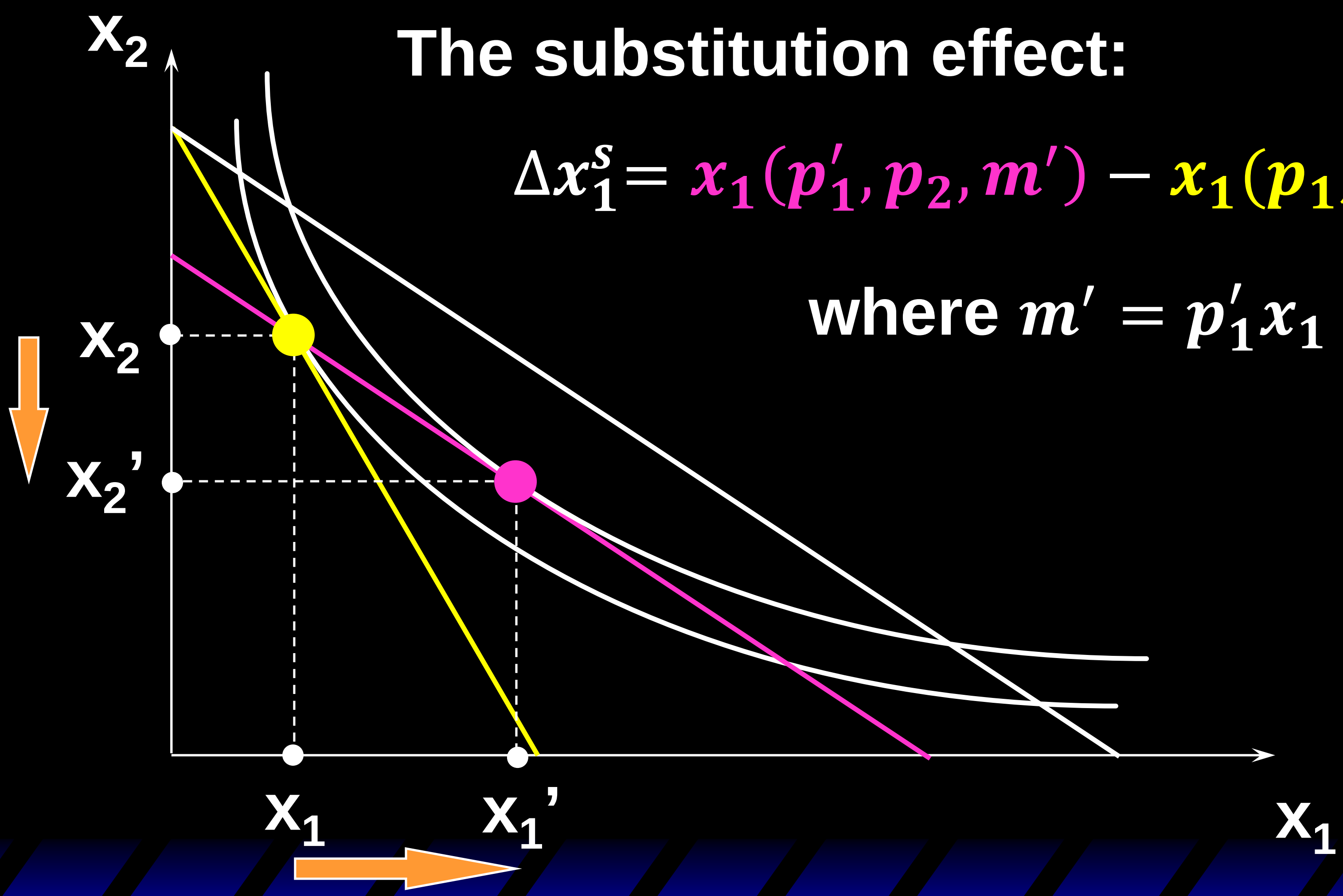
$$x_1' = x_1(p_1', p_2, m')$$

Calculating the Substitution Effect

The substitution effect:

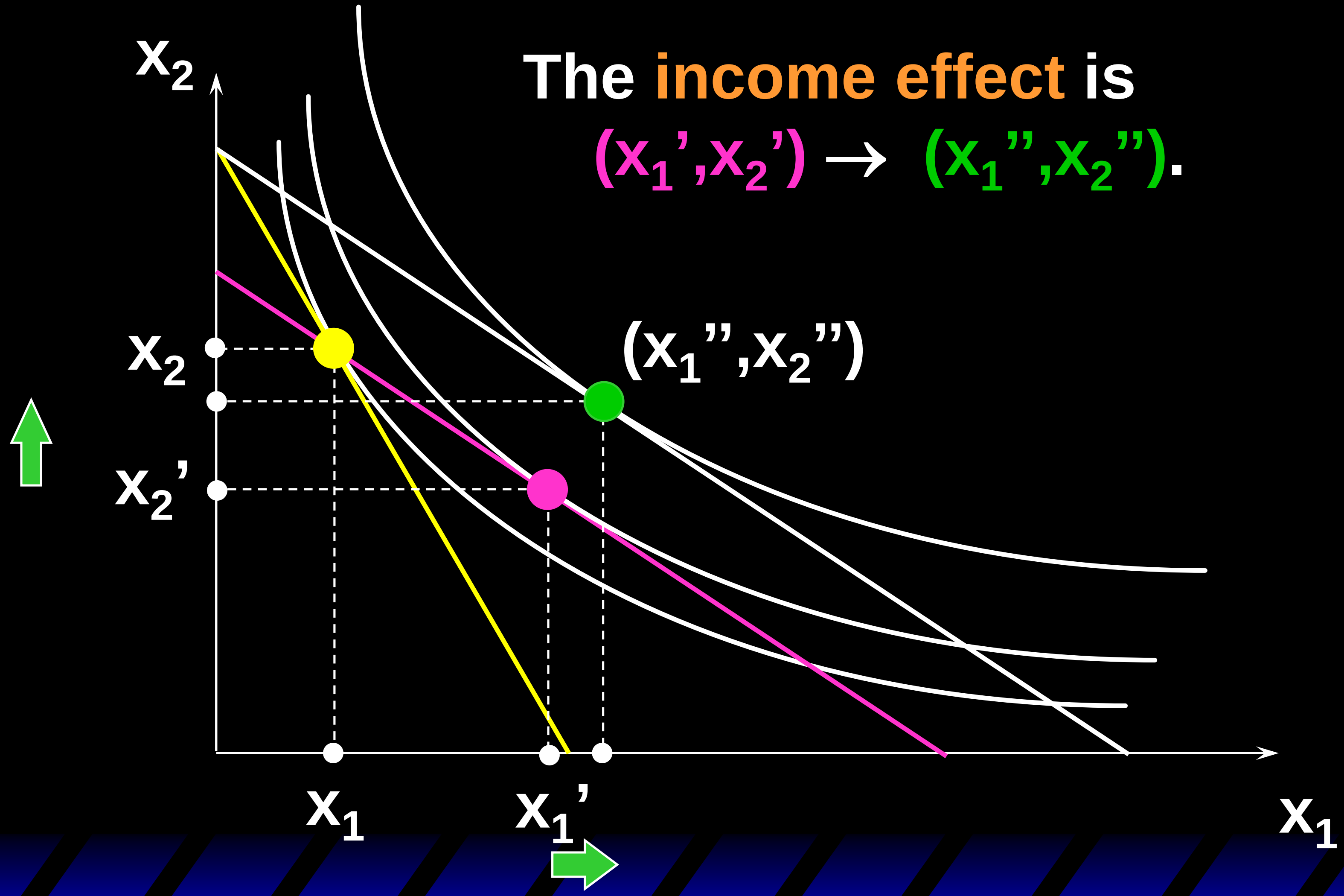
$$\Delta x_1^s = x_1(p_1', p_2, m') - x_1(p_1, p_2, m)$$

where $m' = p_1' x_1 + p_2 x_2$



Calculating the Income Effect

The **income effect** is $(x_1', x_2') \rightarrow (x_1'', x_2'')$.



Calculating the Income Effect

The new bundle: the optimal bundle under p_1', p_2, and m

$$x_1'' = x_1(p_1', p_2, m)$$

The income effect:

$$\Delta x_1^n = x_1(p_1', p_2, m) - x_1(p_1', p_2, m')$$

The Overall Change in Demand

$$\Delta x_1^s = x_1(p_1', p_2, m') - x_1(p_1, p_2, m)$$
$$\Delta x_1^n = x_1(p_1', p_2, m) - x_1(p_1', p_2, m')$$

Slutsky Identity:

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n$$

$$= x_1(p_1', p_2, m) - x_1(p_1, p_2, m)$$

A C-D Example

Cobb-Douglas Utility:

$$U(x_1, x_2) = x_1 x_2$$

Initial prices and income are:

$$p_1 = 2, p_2 = 4, m = 120$$

Then the price of good 1 falls to $p_1' = 1$

Q: What are the substitution and income effects of this price change?

A C-D Example

Step 1: express the quantities demanded as functions of p_1, p_2, m

$$x_1 = \frac{m}{2p_1}$$

$$x_2 = \frac{m}{2p_2}$$

A C-D Example

Step 2: find the old, the intermediate, and the new bundles

Old:

$$\boldsymbol{x_1 = \frac{m}{2p_1} = \frac{120}{2 * 2} = 30}$$

$$\boldsymbol{x_2 = \frac{m}{2p_2} = \frac{120}{2 * 4} = 15}$$

A C-D Example

Step 2: find the old, the intermediate, and the new bundles

Intermediate:

$$m' = p'_1 x_1 + p_2 x_2 = 1 * 30 + 4 * 15 = 90$$

$$x'_1 = \frac{m'}{2p'_1} = \frac{90}{2 * 1} = 45$$

$$x'_2 = \frac{m'}{2p_2} = \frac{90}{8} = 11.25$$

A C-D Example

Step 2: find the old, the intermediate, and the new bundles

New:

$$x_1'' = \frac{m}{2p_1'} = \frac{120}{2 * 1} = 60$$

$$x_2'' = \frac{m}{2p_2} = \frac{120}{2 * 4} = 15$$

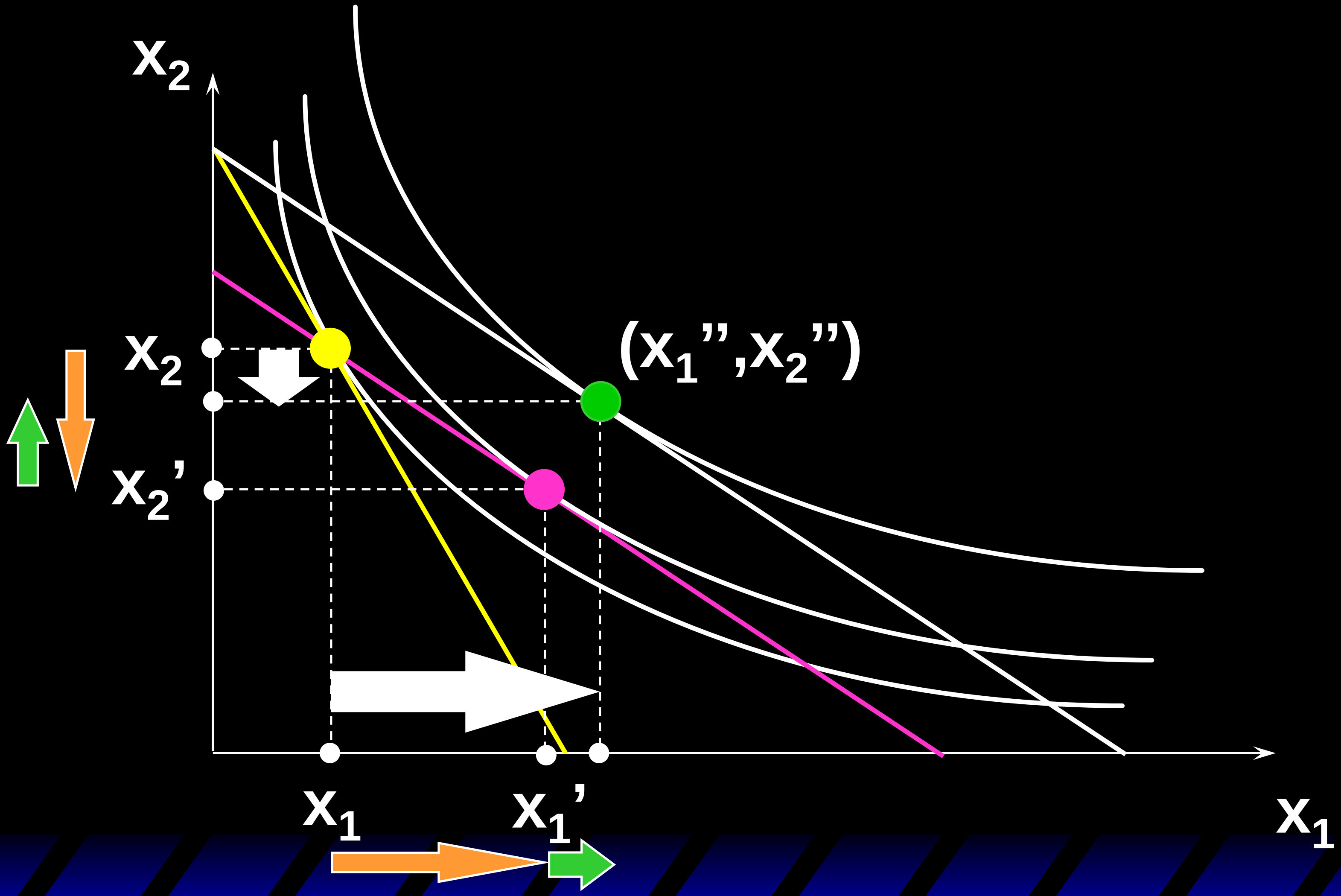
A C-D Example

Step 3: Calculate the substitution and the income effects on x_1

$$\Delta x_1^s = x_1' - x_1 = 45 - 30 = 15$$

$$\Delta x_1^n = x_1'' - x_1' = 60 - 45 = 15$$

A C-D Example

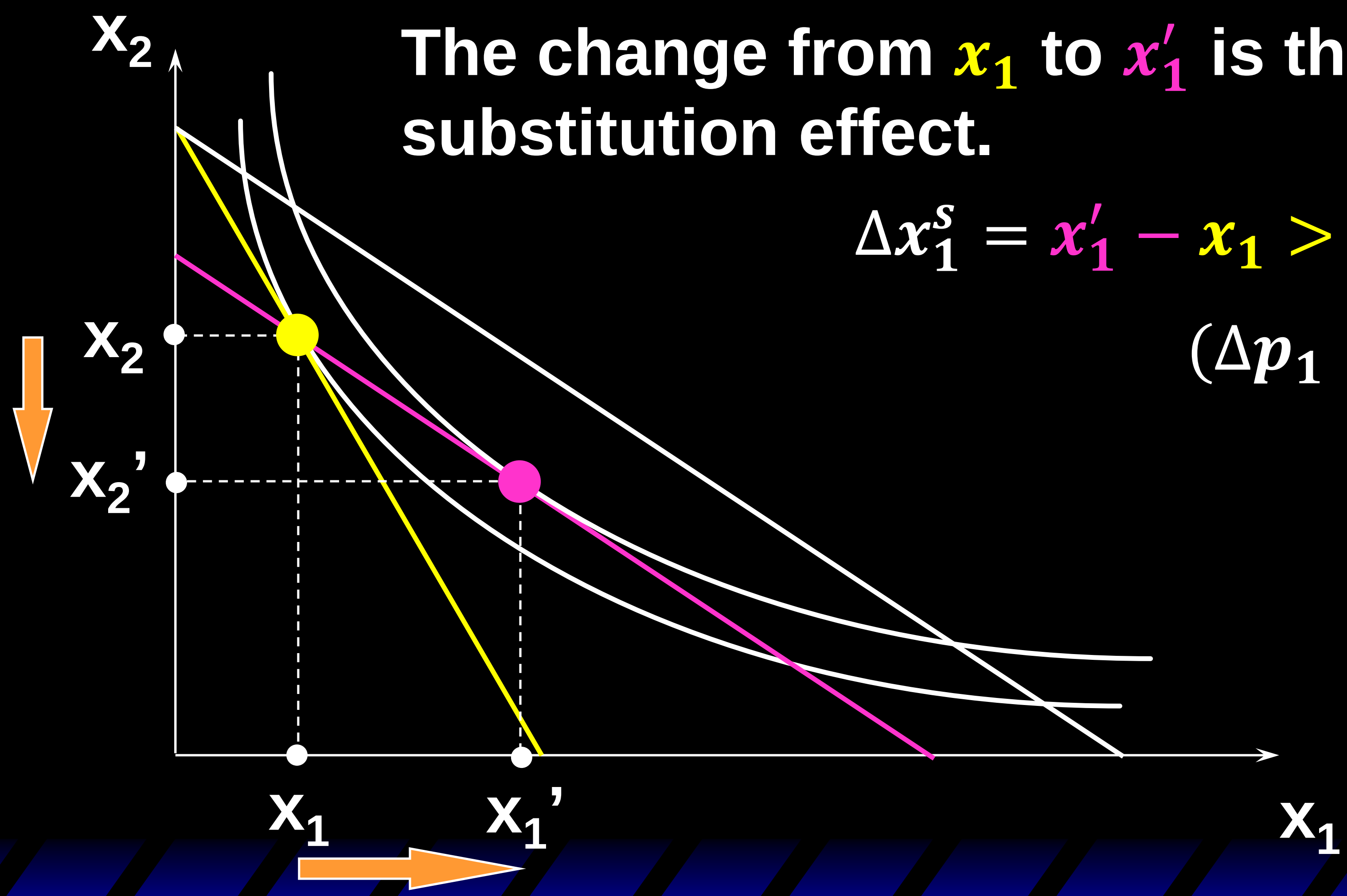


Sign of the Substitution Effect

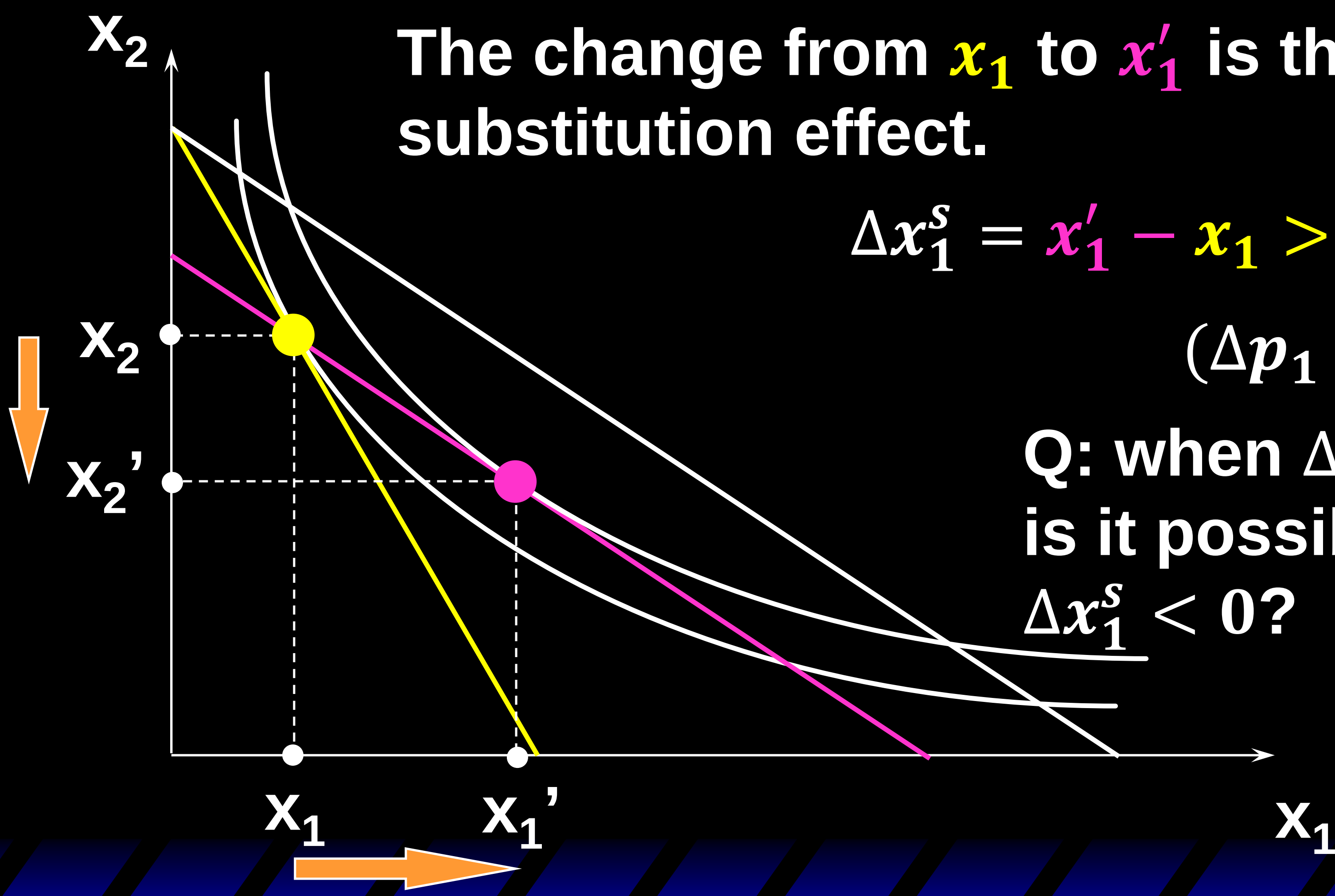
The change from x_1 to x_1' is the substitution effect.

$$\Delta x_1^s = x_1' - x_1 > 0$$

$$(\Delta p_1 < 0)$$



Sign of the Substitution Effect



The change from x_1 to x_1' is the substitution effect.

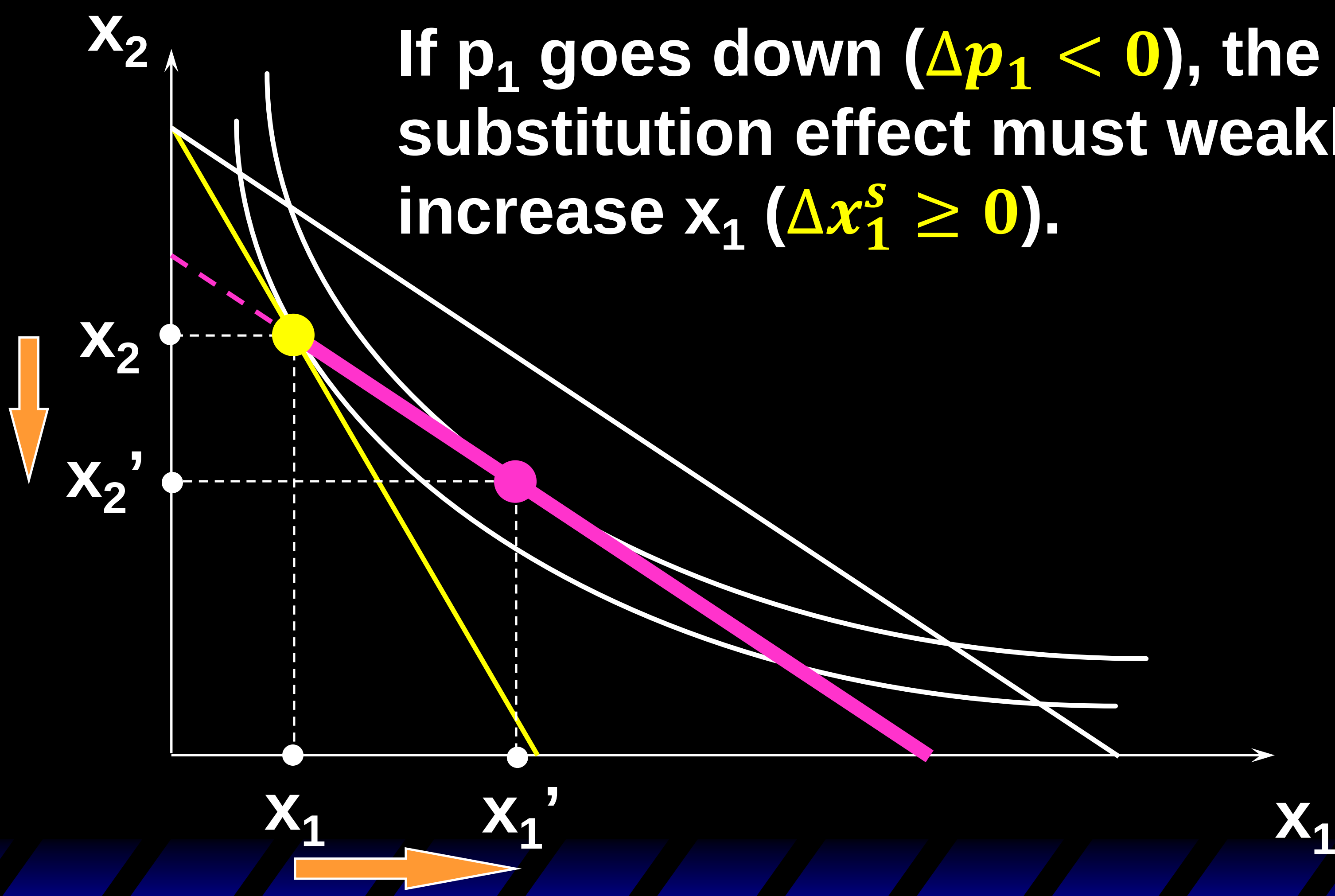
$$\Delta x_1^s = x_1' - x_1 > 0$$

$$(\Delta p_1 < 0)$$

Q: when $\Delta p_1 < 0$, is it possible that $\Delta x_1^s < 0$?

Sign of the Substitution Effect

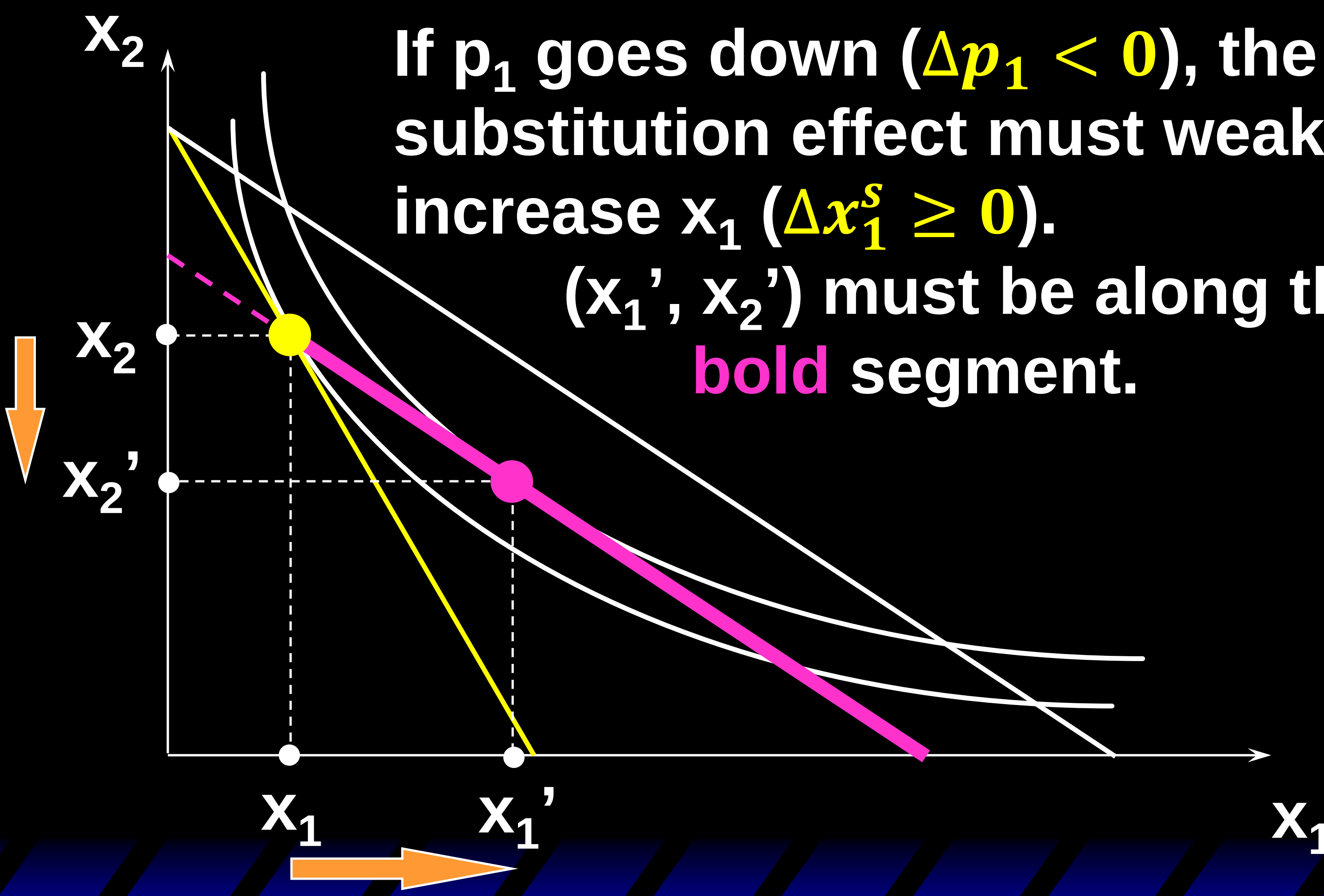
If p_1 goes down ($\Delta p_1 < 0$), the substitution effect must weakly increase x_1 ($\Delta x_1^s \geq 0$).



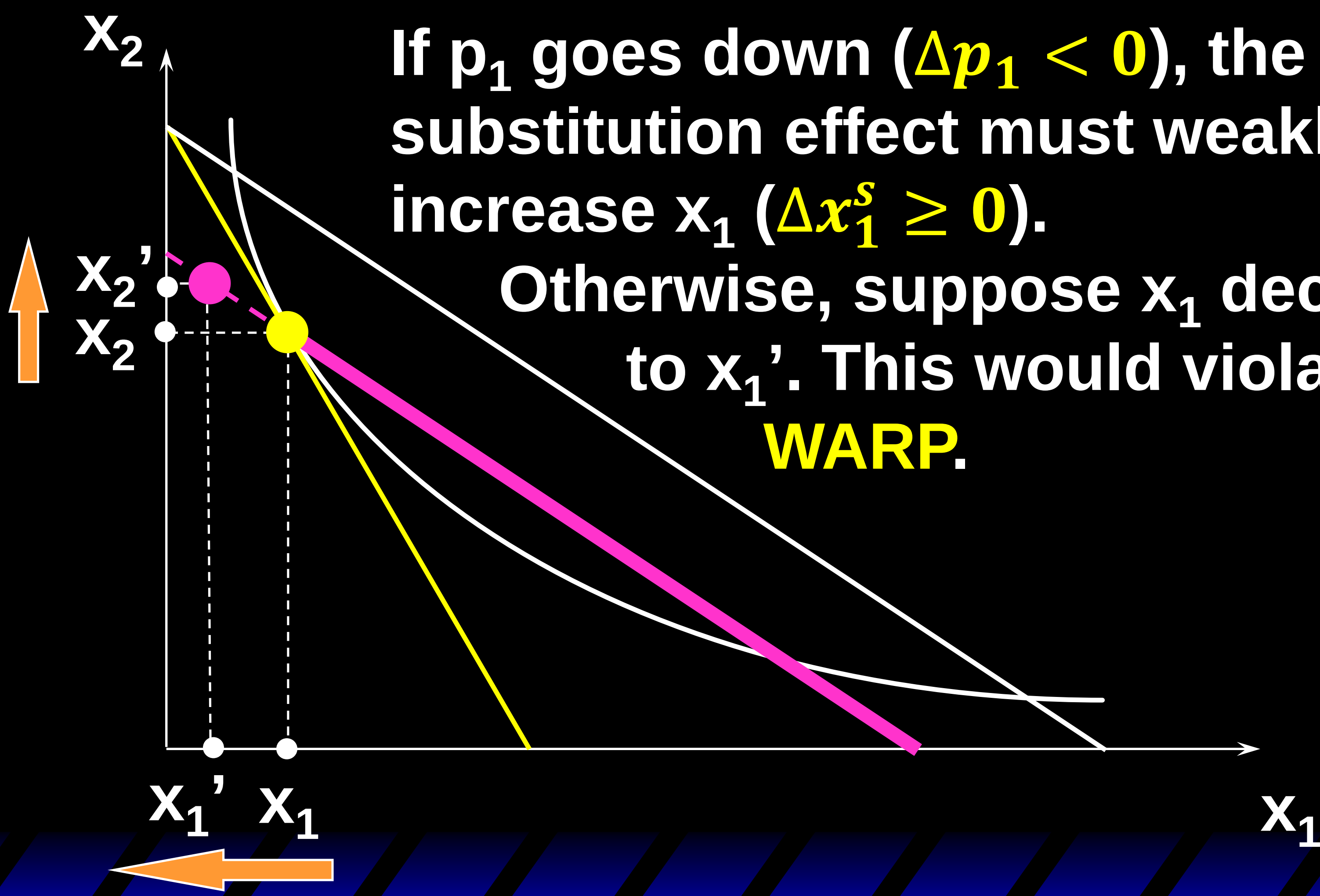
Sign of the Substitution Effect

If p_1 goes down ($\Delta p_1 < 0$), the substitution effect must weakly increase x_1 ($\Delta x_1^s \geq 0$).

(x_1', x_2') must be along the **bold** segment.



Sign of the Substitution Effect



If p_1 goes down ($\Delta p_1 < 0$), the substitution effect must weakly increase x_1 ($\Delta x_1^s \geq 0$).

Otherwise, suppose x_1 decreases to x_1'. This would violate the WARP.

Sign of the Substitution Effect

You can also verify that, if p_1 goes up ($\Delta p_1 > 0$), the substitution effect must weakly decrease x_1 ($\Delta x_1^s \leq 0$).

Sign of the Substitution Effect

You can also verify that,
if p_1 goes up ($\Delta p_1 > 0$), the substitution effect must weakly decrease x_1 ($\Delta x_1^s \leq 0$).

Therefore, the substitution effect and the change in price are in opposite directions.

$$\frac{\Delta x_1^s}{\Delta p_1} \leq 0$$

替代效应和价格变化一定是反向的。否则就违背了弱显示偏好公理。

Income Effects for Normal Goods

- Most goods are normal (i.e. demand increases with income).

Income Effects for Normal Goods

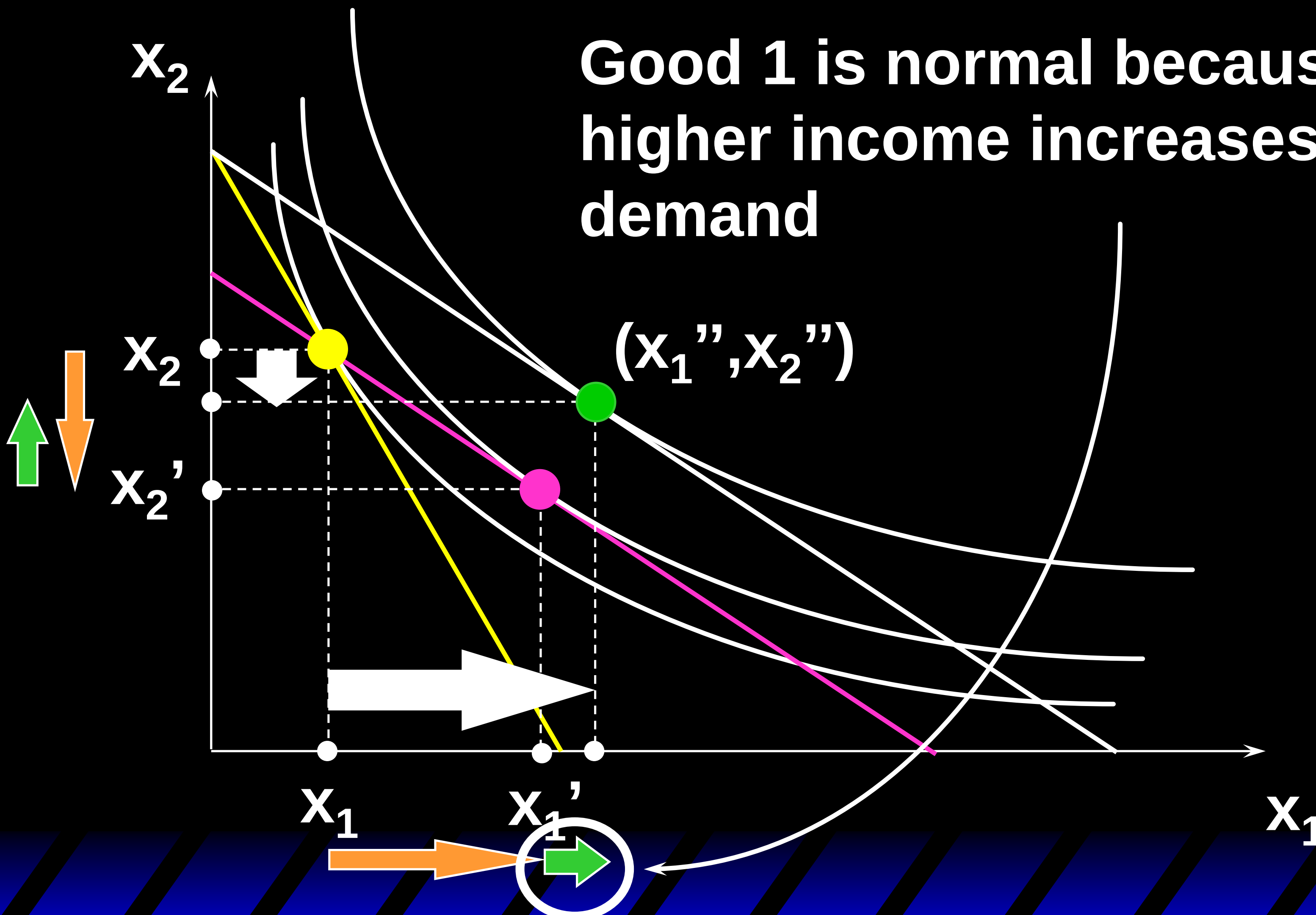
- **If p_1 goes down ($\Delta p_1 < 0$), the real income / purchasing power increases.**

$$m - m' = (p_1 x_1 + p_2 x_2) - (p'_1 x_1 + p_2 x_2)$$
$$= -\Delta p_1 x_1 > 0$$

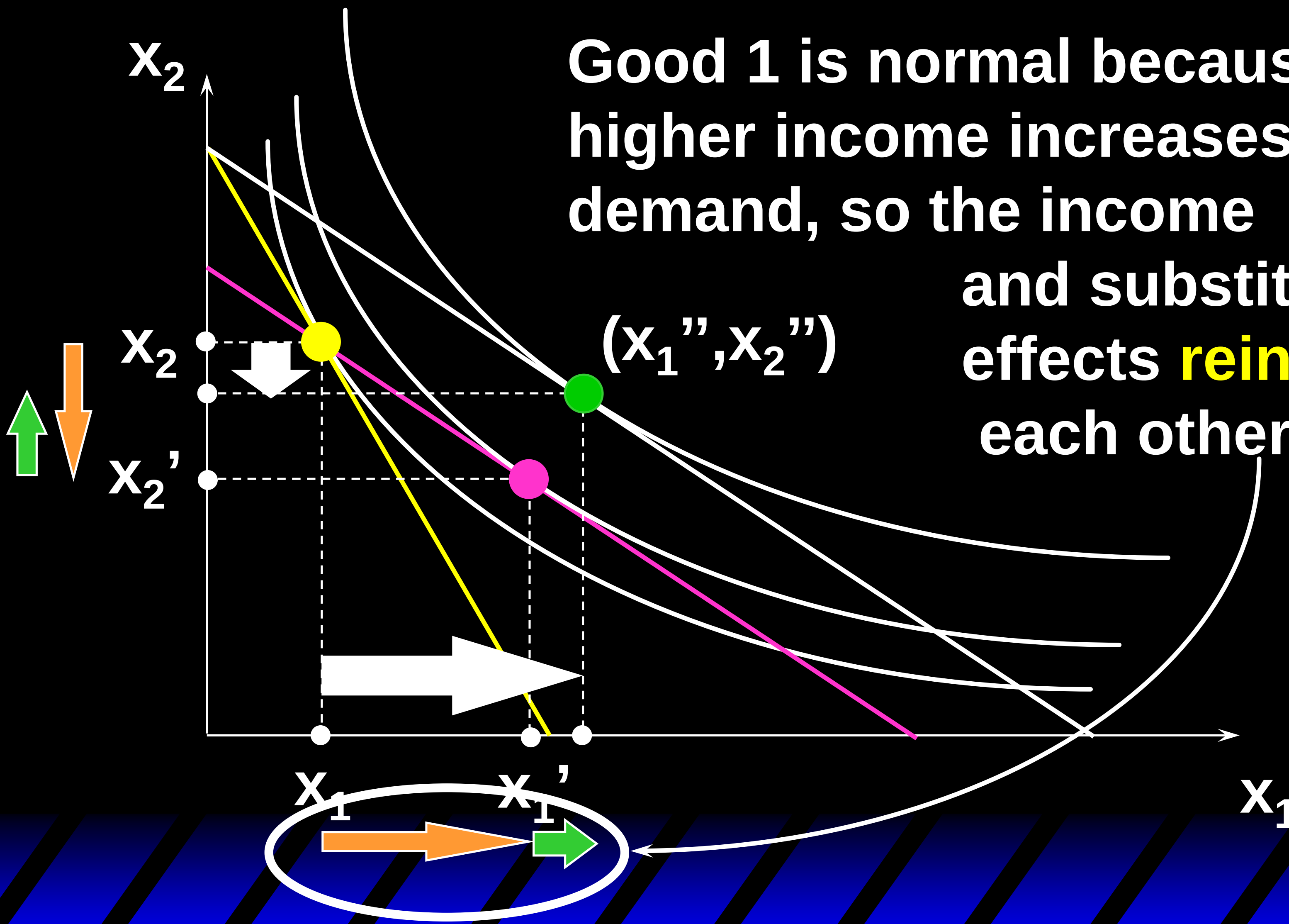
- **The income effect will increase the demand for normal good x_1 ($\Delta x_1^n > 0$).**

Income Effects for Normal Goods

Good 1 is normal because higher income increases demand



Income Effects for Normal Goods



Good 1 is normal because higher income increases demand, so the income and substitution effects reinforce each other.

Total Effects for Normal Goods

Since both the substitution and income effects increase demand when own-price falls, a normal good's ordinary demand curve slopes down.

If $\Delta p_1 < 0$, $\Delta x_1^s > 0$ and $\Delta x_1^n > 0$.

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n > 0$$

$$\frac{\Delta x_1}{\Delta p_1} < 0$$

若一种商品是正常品，则价格变化造成的替代效应和收入效应方向相同。价格的上升（下降）一定会造成净需求的下降（上升）。

The Law of Demand

If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases.

(i.e. normal goods must have downward-sloping demand curves)

需求法则：若需求随收入上升而上升，则需求一定随价格上升而下降。（正常品一定是普通商品）

Income Effects for Inferior Goods

- Some goods are income-inferior (i.e. demand is reduced by higher income).

Income Effects for Inferior Goods

- If p_1 goes down ($\Delta p_1 < 0$), the real income / purchasing power increases.

$$m - m' = (p_1 x_1 + p_2 x_2) - (p_1' x_1 + p_2 x_2)$$
$$= -\Delta p_1 x_1 > 0$$

- The income effect will **decrease** the demand for inferior good x_1 ($\Delta x_1^n < 0$).

若一种商品是低档品，则收入效应与价格变化的方向相同。

Income Effects for Inferior Goods

The substitution and income effects oppose each other when an income-inferior good's own price changes.

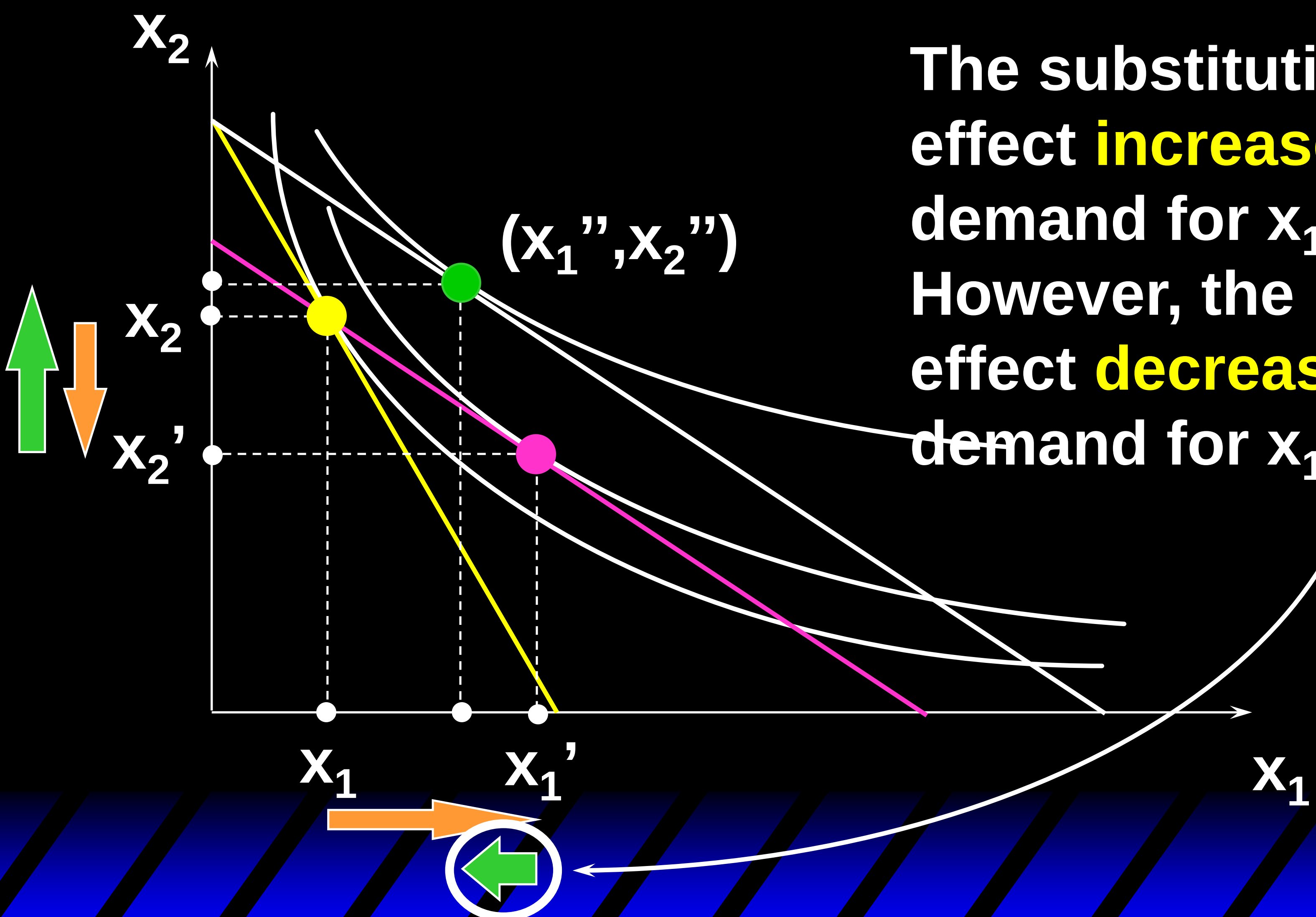
If $\Delta p_1 < 0$, $\Delta x_1^s > 0$ and $\Delta x_1^n < 0$.

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n$$

$$(?) \qquad (+) \qquad (-)$$

$$\frac{\Delta x_1}{\Delta p_1} \; ? \; 0$$

Income Effects for Income-Inferior Goods



The substitution effect increases the demand for x_1. However, the income effect decreases the demand for x_1

Total Effects for Inferior Goods

If $\Delta p_1 < 0$, $\Delta x_1^s > 0$ and $\Delta x_1^n < 0$.

When $|\Delta x_1^n| > |\Delta x_1^s|$,

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n < 0$$

$$\frac{\Delta x_1}{\Delta p_1} > 0$$

i.e. the ordinary demand curve is upward sloping.

Giffen Goods

In rare cases of extreme income-inferiority, the income effect may be larger in size than the substitution effect, causing quantity demanded to fall as own-price falls.

Such goods are Giffen goods.

对低档品而言，若收入效应的大小超过了替代效应，则价格与需求的变化方向相同。这种低档品被称为吉芬商品。

Slutsky equation (rate of change version)

Slutsky Identity:

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n$$

$$\Delta x_1^s = x_1(p_1', p_2, m') - x_1(p_1, p_2, m)$$
$$\Delta x_1^n = x_1(p_1', p_2, m) - x_1(p_1', p_2, m')$$

Slutsky equation (rate of change version)

Slutsky Identity:

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n$$

Divide both sides by Δp_1:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1^n}{\Delta p_1}$$

Slutsky equation (rate of change version)

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1^n}{\Delta p_1}$$

We have already shown that

$$\frac{\Delta x_1^s}{\Delta p_1} < 0$$

Slutsky equation (rate of change version)

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1^n}{\Delta p_1}$$

We want to determine whether

$$\frac{\Delta x_1^n}{\Delta p_1} > 0 \text{ or } \frac{\Delta x_1^n}{\Delta p_1} < 0$$

Slutsky equation (rate of change version)

$$\Delta x_1^n = x_1(p_1', p_2, m) - x_1(p_1', p_2, m')$$

$$= x_1(p_1', p_2, m' + (m - m')) - x_1(p_1', p_2, m')$$

Slutsky equation (rate of change version)

$$\Delta x_1^n = x_1(p_1', p_2, m) - x_1(p_1', p_2, m')$$

$$= x_1(p_1', p_2, m' + (m - m')) - x_1(p_1', p_2, m')$$

Remember that

$$f(x + \Delta x) - f(x) \approx f'(x)\Delta x$$

Slutsky equation (rate of change version)

$$\Delta x_1^n = x_1(p_1', p_2, m) - x_1(p_1', p_2, m')$$

$$= x_1(p_1', p_2, m' + (m - m')) - x_1(p_1', p_2, m')$$

Remember that

$$f(x + \Delta x) - f(x) \rightarrow f'(x)\Delta x$$

$$\Delta x_1^n = \frac{\Delta x_1(p_1', p_2, m)}{\Delta m}(m - m')$$

Slutsky equation (rate of change version)

$$\Delta x_1^n = \frac{\Delta x_1(p_1', p_2, m)}{\Delta m}(m - m')$$

$$= \frac{\Delta x_1(p_1', p_2, m)}{\Delta m}(p_1 - p_1')x_1$$

$$= \frac{\Delta x_1(p_1', p_2, m)}{\Delta m}(-\Delta p_1 x_1)$$

Recall that

$$m - m' = (p_1 x_1 + p_2 x_2) - (p_1' x_1 + p_2 x_2)$$

$$= (p_1 - p_1')x_1$$

Slutsky equation (rate of change version)

$$\frac{\Delta x_1^n}{\Delta p_1} = \frac{\frac{\Delta x_1(p_1, p_2, m)}{\Delta m}(-\Delta p_1 x_1)}{\Delta p_1}$$

$$= -\frac{\Delta x_1(p_1, p_2, m)}{\Delta m} x_1$$

Slutsky equation (rate of change version)

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1^n}{\Delta p_1}$$

$$= \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1(p_1, p_2, m\,)}{\Delta m} x_1$$

Slutsky equation (rate of change version)

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1(p_1, p_2, m\ )}{\Delta m} x_1$$

$(-)$ $(+)$ **if normal**

Therefore,

$\frac{\Delta x_1}{\Delta p_1} < 0$ **for normal goods**

Slutsky equation (rate of change version)

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1(p_1, p_2, m\,)}{\Delta m} x_1$$

$(-)$ $(-)$ **if inferior**

Therefore,

$\frac{\Delta x_1}{\Delta p_1}$ **could be positive or negative.**

Slutsky equation (rate of change version)

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1(p_1, p_2, m)}{\Delta m} x_1$$

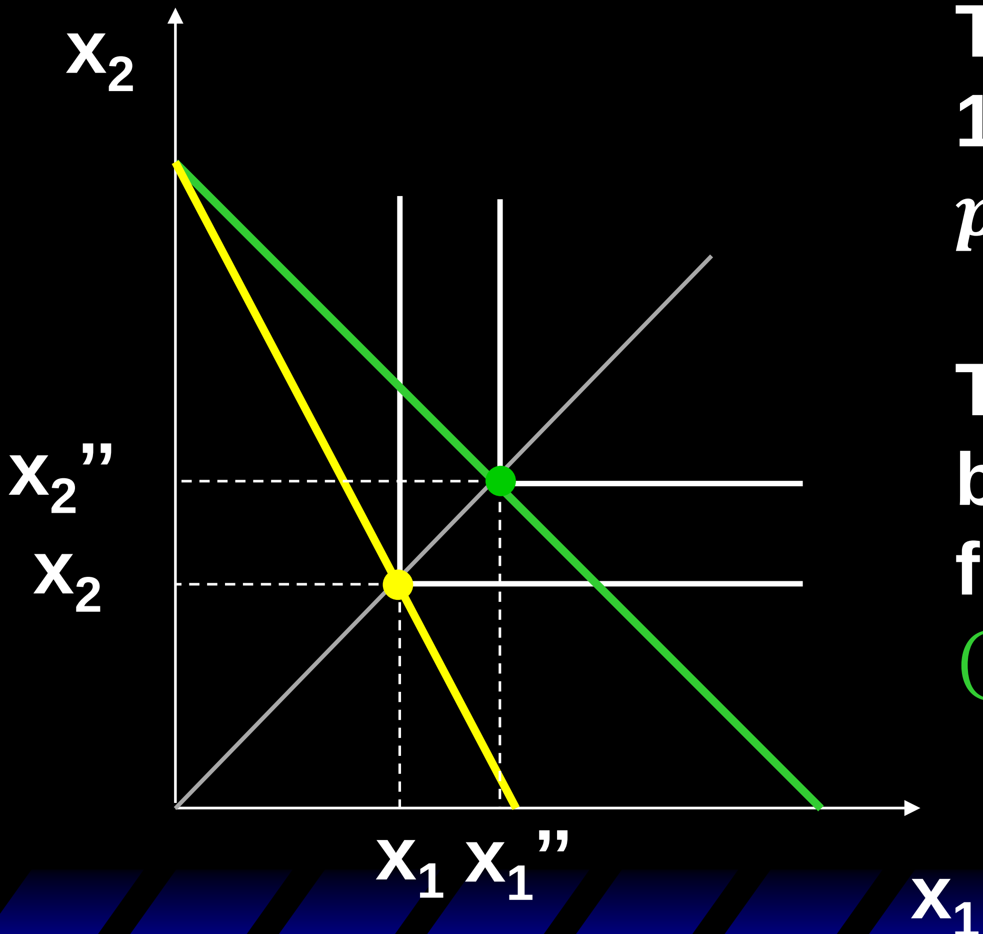
(+) if Giffen (−)

Therefore,

$\frac{\Delta x_1(p_1,p_2,m)}{\Delta m} < 0$ **must hold for Giffen**

i.e. Giffen must be inferior (吉芬商品一定是低档品)

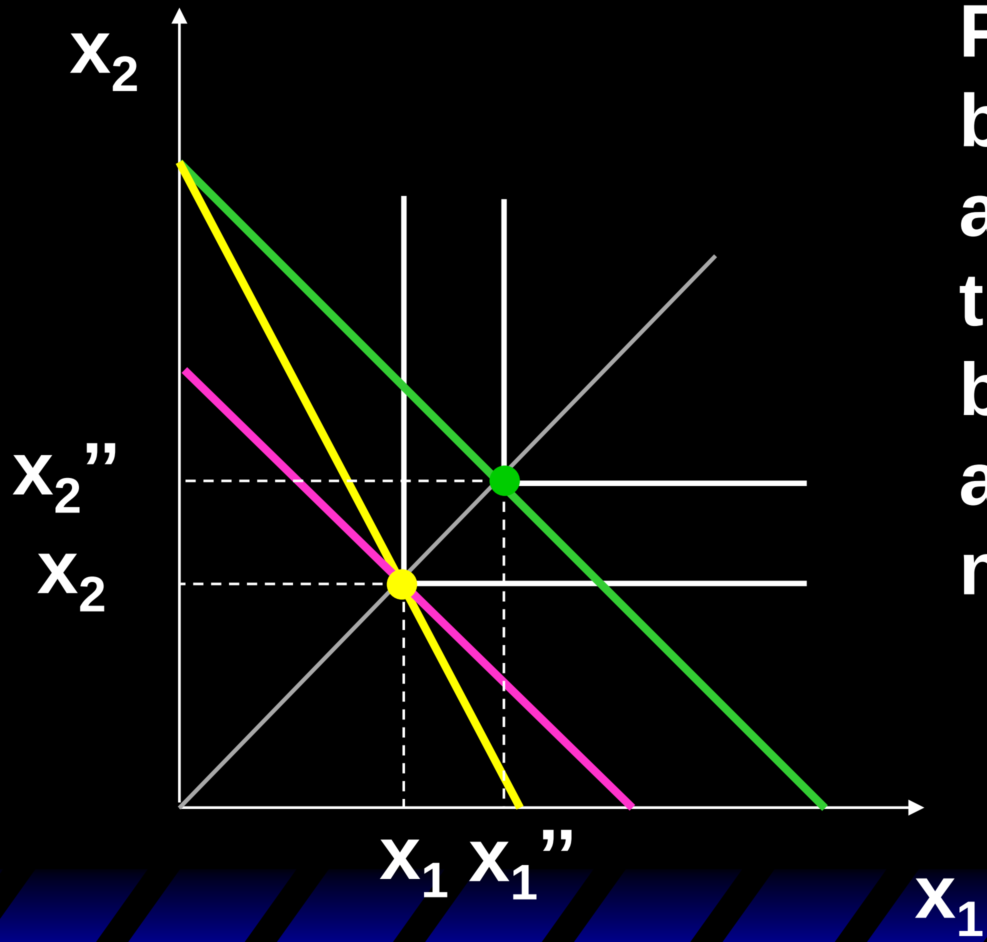
Perfect complements



The price of good 1 decreases from p_1 to p_1'

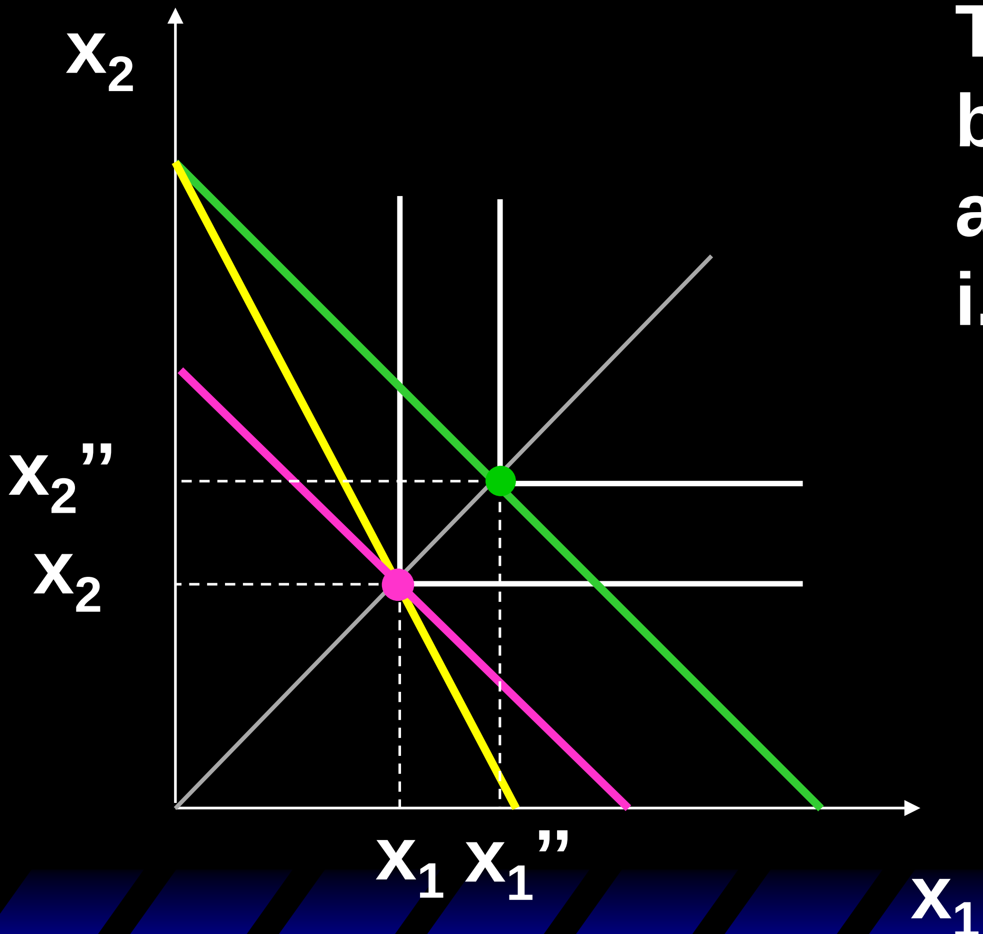
The optimal bundle moves from (x_1, x_2) to (x_1'', x_2'')

Perfect complements



Pivot the old budget line around (x_1, x_2) so that the old bundle is just affordable under new prices

Perfect complements



The "intermediate" bundle is the same as the old bundle. i.e. $x_1 = x'_1, x_2 = x'_2$

Perfect complements

The "intermediate" bundle is the same as the old bundle.
i.e. $x_1 = x_1', x_2 = x_2'$

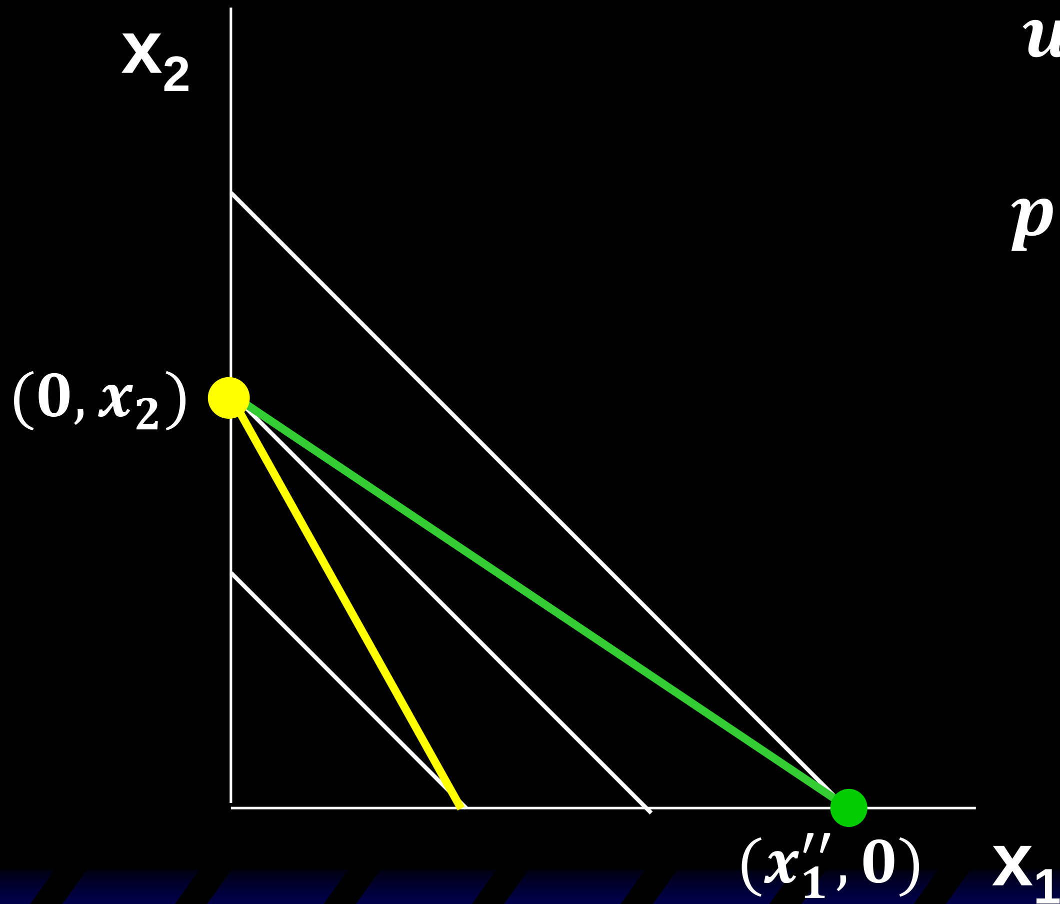
$$x_1^s = x_1' - x_1 = 0$$

- **no substitution effect**

$$x_1^n = x_1'' - x_1' = x_1'' - x_1$$

- Total effect = Income effect

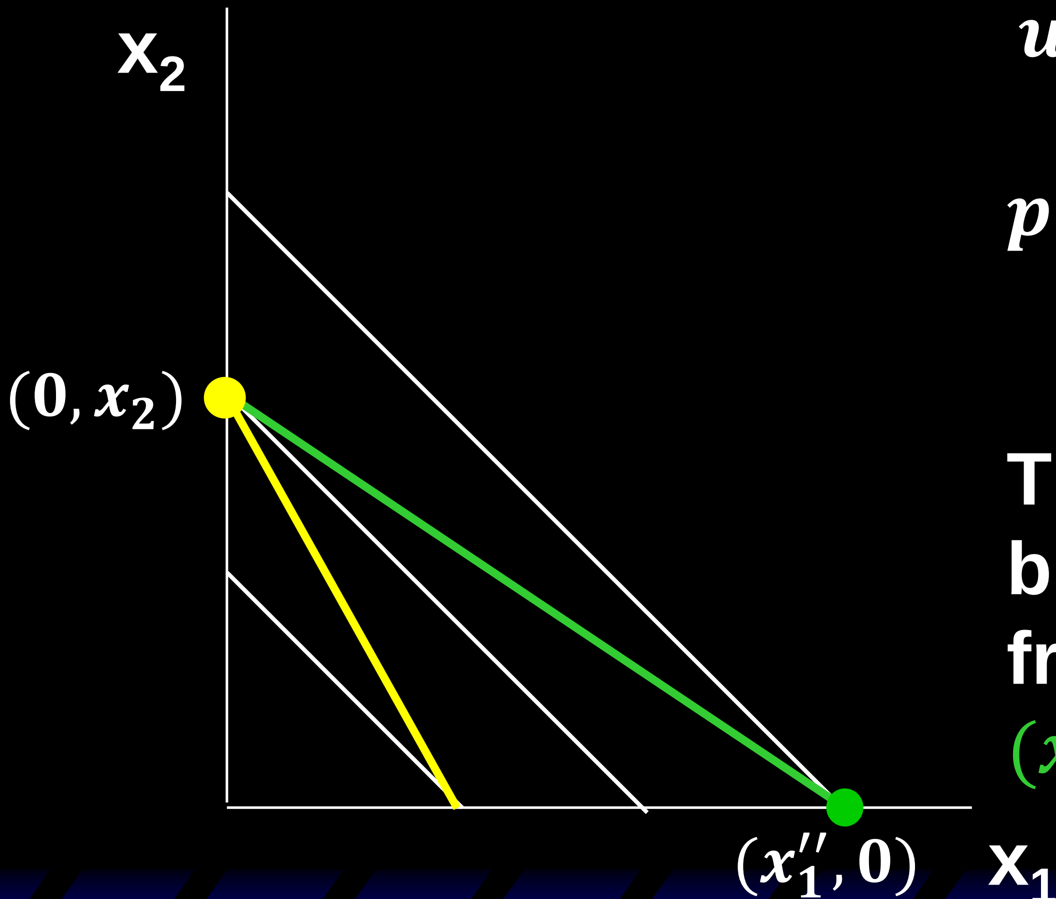
Perfect Substitutes



$$u(x_1, x_2) = x_1 + x_2$$

$$p_1 > 1;\ p_1' < 1;$$
$$p_2 = 1$$

Perfect Substitutes

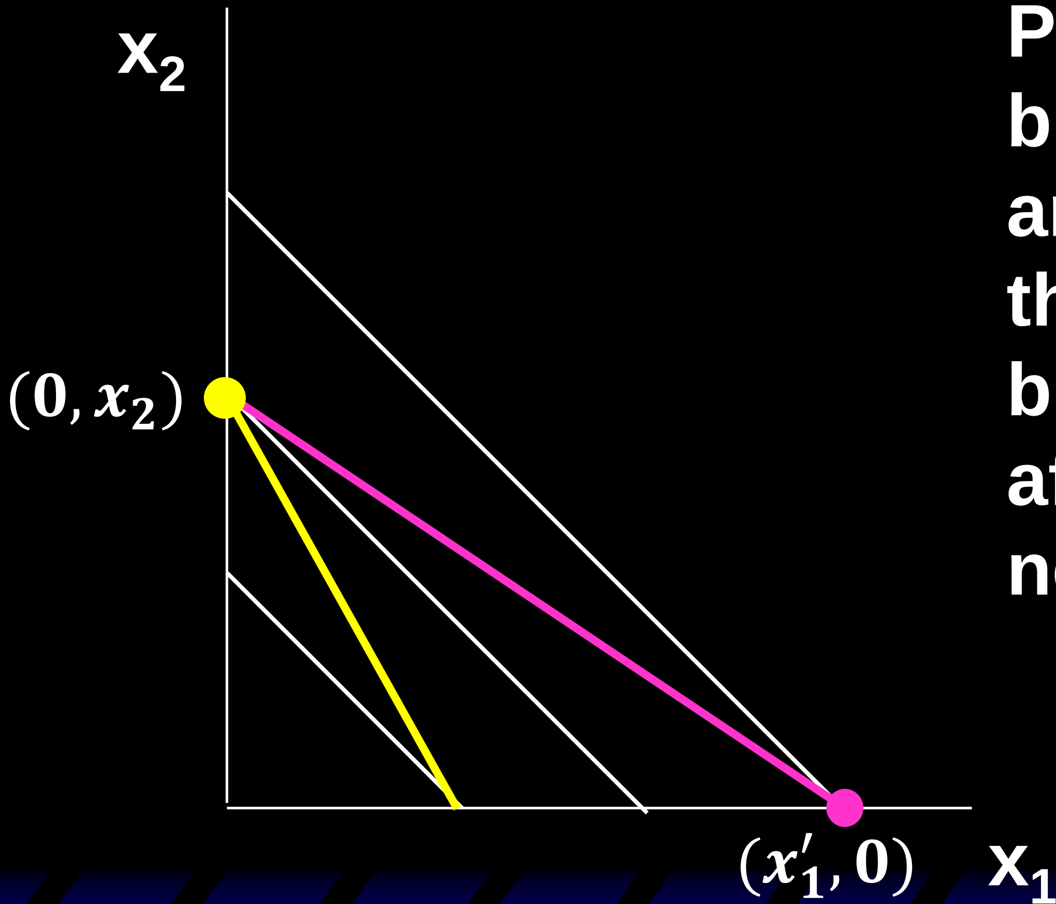


$$u(x_1, x_2) = x_1 + x_2$$

$$p_1 > 1;\ p_1' < 1;$$
$$p_2 = 1$$

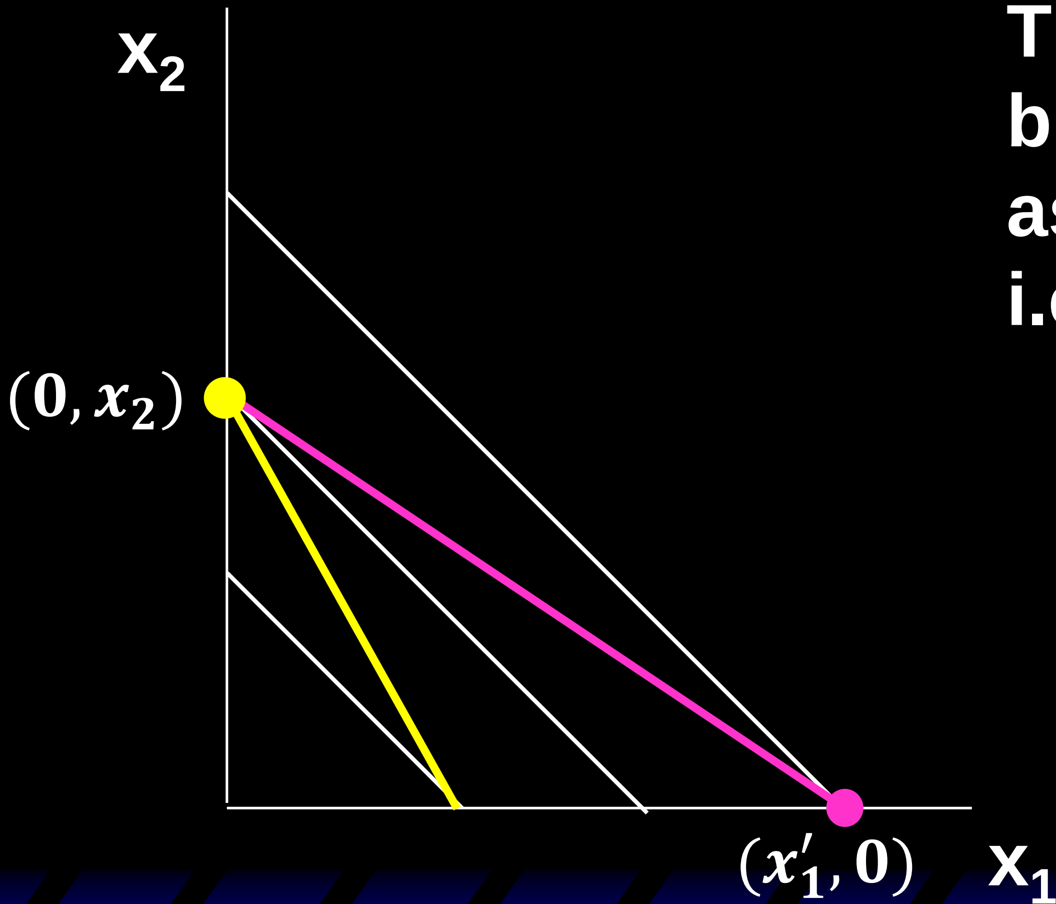
The optimal bundle moves from $(0, x_2)$ to $(x_1'', 0)$

Perfect Substitutes



Pivot the old budget line around $(0, x_2)$ so that the old bundle is just affordable under new prices

Perfect Substitutes



The "intermediate" bundle is the same as the new bundle. i.e. $x_1' = x_1''$, $x_2' = x_2''$

Perfect substitutes

The "intermediate" bundle is the same as the new bundle.
i.e. $x_1' = x_1'', x_2' = x_2''$

$$x_1^s = x_1' - x_1 = x_1'' - x_1$$

- **Total effect = Substitution effect**

$$x_1^n = x_1'' - x_1' = 0$$

- No income effect