# The Counterfactual and Treatment Effects:

## The Conceptual Framework

Yi, Junjian

Peking University

Feb 25, 2022

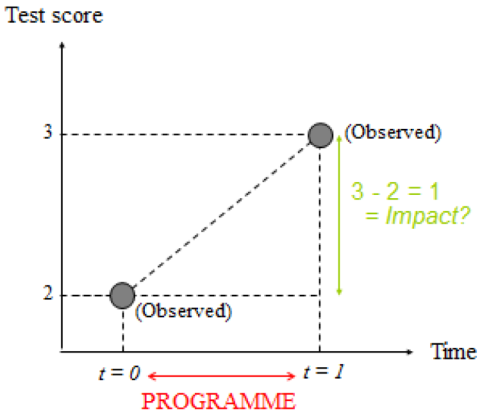
# Policy Evaluation and Casual Inferences

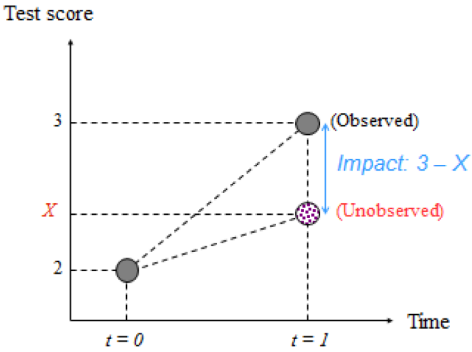
How much did people benefit from the policy?

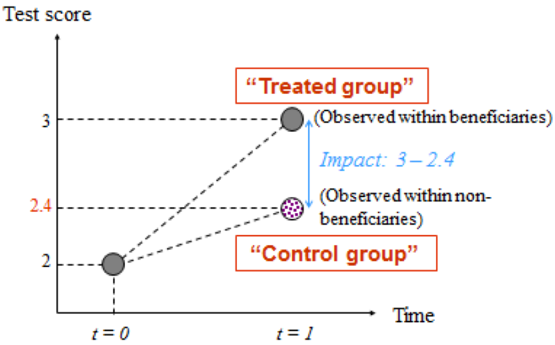
**The Counterfactual:** Need to know what would have happened in the absence of program

- Ideally, need to observe 2 times in the same world: 1 with the policy, and 1 without the policy
- *Ceteris paribus*

# The Counterfactual: Time Series




Test score
3
X
2
(Observed)
Impact: 3 – X
(Unobserved)
Time
t = 0
t = 1


Test score
"Treated group"
3
(Observed within beneficiaries)
Impact: 3 – 2.4
2.4
(Observed within non-beneficiaries)
2
"Control group"
Time
t = 0
t = 1

# The Counterfactual: Cross-section

**Statistical association**

| ID | T | Y |
|---|---|---|
| 1 | 0 | $y_1$ |
| 2 | 0 | $y_2$ |
| 3 | 0 | $y_3$ |
| 4 | 1 | $y_4$ |
| 5 | 1 | $y_5$ |
| 6 | 1 | $y_6$ |

$$\Delta = E(Y_i|T=1) - E(Y_i|T=0) = (\Sigma_{i=4}^{6} y_i)/3 - (\Sigma_{i=1}^{3} y_i)/3$$

# Potential Outcomes

$$Y_i = Y_{0i} + (Y_{1i} - Y_{0i})T_i$$

| ID | T | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|
| 1 | 0 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 0 | $y_{02}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 1 | $y_{15}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

# Average Treatment Effects (ATE)

$$Y_i = Y_{0i} + (Y_{1i} - Y_{0i})T_i$$

| ID | T | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|
| 1 | 0 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 0 | $y_{02}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 1 | $y_{15}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

$$ATE = E(Y_{1i} - Y_{0i}) = E(Y_{1i}) - E(Y_{0i}) = (\Sigma_{i=1}^{6} y_{1i})/6 - (\Sigma_{i=1}^{6} y_{0i})/6$$

# Average Treatment Effects on the Treated (ATT)

| ID | T | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|
| 1 | 0 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 0 | $y_{02}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 1 | $y_{15}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

$$
\begin{aligned}
ATT &= E(Y_{1i} - Y_{0i} | T = 1) \\
&= E(Y_{1i} | T = 1) - E(Y_{0i} | T = 1) \\
&= (\Sigma_{i=4}^{6} y_{1i})/3 - (\Sigma_{i=4}^{6} y_{0i})/3
\end{aligned}
$$

# Average Treatment Effects on the Un-treated (TUT)

| ID | T | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|
| 1 | 0 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 0 | $y_{02}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 1 | $y_{15}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

$$
\begin{aligned}
TUT &= E(Y_{1i} - Y_{0i} | T = 0) \\
&= E(Y_{1i} | T = 0) - E(Y_{0i} | T = 0) \\
&= (\Sigma_{i=1}^{3} y_{1i})/3 - (\Sigma_{i=1}^{3} y_{0i})/3
\end{aligned}
$$

# The Unobservability of Potential Outcomes

| ID | T | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|
| 1 | 0 | $y_{01}$ | $y_{01}$ | . |
| 2 | 0 | $y_{02}$ | $y_{02}$ | . |
| 3 | 0 | $y_{03}$ | $y_{03}$ | . |
| 4 | 1 | $y_{14}$ | . | $y_{14}$ |
| 5 | 1 | $y_{15}$ | . | $y_{15}$ |
| 6 | 1 | $y_{16}$ | . | $y_{16}$ |

# Selection Bias

$$\underbrace{E[Y_i|D_i=1]-E[Y_i|D_i=0]}_{\text{Observed difference}} = \underbrace{E[Y_{1i}|D_i=1]-E[Y_{0i}|D_i=1]}_{\text{Average treatment effect on the treated}}$$
$$+\underbrace{E[Y_{0i}|D_i=1]-E[Y_{0i}|D_i=0]}_{\text{Selection bias}}$$

$$\Delta = ATT + \text{Selection Bias}$$

# Key Concepts

- Heterogeneity: ATT vs. TUT
- Selection: $\Delta$ vs. ATT
- Heterogeneity + Selection: $\Delta$ vs. ATE

# Selection Bias: James Heckman (Nobel laureate, 2000)

# Indirect (Encouraging) Interventions

| ID | Z | T | T(Z=0) | T(Z=1) | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 1 | 1 | 0 | 1 | $y_{12}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | 0 | 0 | 1 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | 1 | 1 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 0 | 1 | 1 | 1 | $y_{15}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | 1 | 1 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

# Intention-to-treat (ITT): $Z \rightarrow Y$

| ID | Z | T | T(Z=0) | T(Z=1) | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 1 | 1 | 0 | 1 | $y_{12}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | 0 | 0 | 1 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | 1 | 1 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 0 | 1 | 1 | 1 | $y_{15}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | 1 | 1 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

$$ITT = E(Y|Z=1) - E(Y|Z=0)$$

# Three Players

| ID | Z | T | T(Z=0) | T(Z=1) | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 1 | 1 | 0 | 1 | $y_{12}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | 0 | 0 | 1 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | 1 | 1 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 0 | 1 | 1 | 1 | $y_{15}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | 1 | 1 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

1. Compliers: $T_0 = 0, T_1 = 1$
2. Always takers: $T_0 = T_1 = 1$
3. Never takers: $T_0 = T_1 = 0$

# Local Average Treatment Effect (LATE): $T \rightarrow Y$

| ID | Z | T | T(Z=0) | T(Z=1) | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 1 | 1 | 0 | 1 | $y_{12}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | 0 | 0 | 1 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | 1 | 1 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 0 | 1 | 1 | 1 | $y_{15}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | 1 | 1 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

$LATE = E(Y_{1i} - Y_{0i} | T_0 = 0, T_1 = 1)$

Wald estimator: $LATE = \frac{ITT}{E(T|Z=1)-E(T|Z=0)} = \frac{E(Y|Z=1)-E(Y|Z=0)}{E(T|Z=1)-E(T|Z=0)}$

# No Never Takers

| ID | Z | T | T(Z=0) | T(Z=1) | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 1 | 1 | 0 | 1 | $y_{12}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | 0 | 0 | 1 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | 1 | 1 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 0 | 1 | 1 | 1 | $y_{15}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | 1 | 1 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

$$LATE = TUT$$

# No Always Takers

| ID | Z | T | T(Z=0) | T(Z=1) | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 1 | 0 | 0 | 0 | $y_{02}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | 0 | 0 | 0 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | 1 | 0 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 0 | 0 | 0 | 1 | $y_{05}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | 1 | 0 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

$$LATE = ATT$$

# Neither Never Takers, Nor Always Takers

| ID | Z | T | T(Z=0) | T(Z=1) | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 1 | 1 | 0 | 1 | $y_{12}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | 0 | 0 | 1 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | 1 | 0 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 0 | 0 | 0 | 1 | $y_{05}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | 1 | 0 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

$$LATE = ATT = TUT = ATE$$

# An Alert: Defiers

| ID | Z | T | T(Z=0) | T(Z=1) | Y | Y(T=0) | Y(T=1) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | $y_{01}$ | $y_{01}$ | $y_{11}$ |
| 2 | 1 | 1 | 0 | 1 | $y_{12}$ | $y_{02}$ | $y_{12}$ |
| 3 | 0 | 0 | 0 | 0 | $y_{03}$ | $y_{03}$ | $y_{13}$ |
| 4 | 1 | 1 | 1 | 1 | $y_{14}$ | $y_{04}$ | $y_{14}$ |
| 5 | 0 | 1 | 1 | 0 | $y_{15}$ | $y_{05}$ | $y_{15}$ |
| 6 | 1 | 1 | 1 | 1 | $y_{16}$ | $y_{06}$ | $y_{16}$ |

No answer to the question, and need to redesign the experiment.

# A Final Remark

Behavioral inferences from the comparisons between $\Delta$, ATE, ATT, TUT, ITT, and LATE are very important!

Need a new framework —-Roy Model

# Construct Proper Counterfactual

- Experimental approach

- Quasi-experimental approaches
  - Conditional independence, regression, and matching
  - Fixed effects, difference-in-differences, and panel data
  - Excludability and instrumental variable method
  - Jump and regression discontinuity design

Construct proper counterfactual, impact evaluation methods, identification strategies

- The manner in which a researcher uses observational data (i.e., data not generated by a randomized trial) to approximate a real experiment (Angrist and Krueger, 1999)

Policy evaluation, causal inferences

# Summary

- Program evaluation
- The counterfactual: The fundamental challenges to program evaluation.
- Treatment effects:
  - ATE
  - ATT
  - TUT
  - ITT
  - LATE