Math Mini Quiz

Graded

Student

HARRIS DOAN

Total Points

22 / 25 pts

Question 1

Calculus **4** / 4 pts

1.1 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

1.2 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

1.3 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

1.4 Integration **1** / 1 pt

✔ **+ 1 pt** Correct

Question 2

Probability and Random Variables **4** / 4 pts

2.1 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

2.2 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

2.3 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

2.4 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

Question 3

common distribution **2** / 3 pts

3.1 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

3.2 (no title) **0** / 1 pt

✔ **+ 0 pts** Incorrect

3.3 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

Question 4

Probability Theorem **3** / 3 pts

4.1 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

4.2 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

4.3 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

Question 5

(no title) **4** / 5 pts

5.1 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

5.2 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

5.3 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

5.4 (no title) **1** / 1 pt

✔ **+ 1 pt** Correct

5.5 (no title) **0** / 1 pt

✔ **+ 0 pts** Incorrect

Question 6

(no title) **3** / 3 pts

6.1 **(no title)** **1** / 1 pt

✔ **+ 1 pt** Correct

6.2 **(no title)** **1** / 1 pt

✔ **+ 1 pt** Correct

6.3 **(no title)** **1** / 1 pt

✔ **+ 1 pt** Correct

Question 7

(no title) **1** / 2 pts

7.1 **(no title)** **0** / 1 pt

✔ **+ 0 pts** Incorrect

7.2 **(no title)** **1** / 1 pt

✔ **+ 1 pt** Correct

Question 8

(no title) **1** / 1 pt

✔ **+ 1 pt** Correct

Q1 Calculus

4 Points

Q1.1

1 Point

Let $f(x) = \frac{1}{x+a}, x = e^{5u} + 2u.$ What is $\frac{\partial f}{\partial u}$?

- (●) $\frac{-5e^{5u}-2}{(e^{5u}+2u+a)^2}$
- ( ) $\frac{-e^{5u}-2}{e^{5u}+2u+a^2}$
- ( ) $\frac{-5e^{5u}}{(e^{5u}+2u+a)^2}$
- ( ) $\frac{-5e^{5u}-2}{e^{5u}+2u+a}$

Q1.2

1 Point

For the function $f(x) = x^3 + 6x^2 + 5$ defined over $-4 \leq x \leq 4$, what is the **global maximum** of $f(x)$?

- ( ) 0
- ( ) 5
- (●) 165
- ( ) 37

Q1.3
1 Point

What is the gradient $\nabla f(x, y)$ of the function $f(x, y) = \exp(2x) - xy$? ($\exp(x)$ denotes the function e^x)

- $\begin{pmatrix} \exp(x) - y \\ -y \end{pmatrix}$
- $\begin{pmatrix} 2\exp(2x) \\ -x \end{pmatrix}$
- $\begin{pmatrix} 2\exp(2x) - y \\ -y \end{pmatrix}$
- ◉ $\begin{pmatrix} 2\exp(2x) - y \\ -x \end{pmatrix}$

Q1.4 Integration
1 Point

Calculate $\int \frac{x}{-x^2+1} dx$.

- $-2ln|-x^2+1|+C$
- $ln|-x^2+1|+C$
- ◉ $-\frac{1}{2}ln|-x^2+1|+C$
- $ln|x|+C$

Q2 Probability and Random Variables

4 Points

Q2.1

1 Point

If $E[XY] = E[X]E[Y]$, without any additional assumptions, then random variable X and Y are

- ○ Independent
- ○ Both independent and uncorrelated
- ◉ Uncorrelated
- ○ Neither independent nor uncorrelated

Q2.2

1 Point

Given random variables X_1, X_2, without any additional assumptions, let $Y = X_1 + X_2$. Then $\mathrm{Var}[Y] = \mathrm{Var}[X_1] + \mathrm{Var}[X_2]$.

- ◉ False
- ○ True

Q2.3

1 Point

If a biased coin $P(X = 1) = \frac{3}{4}$ is tossed n times independently (head takes value 1 and tail takes value 0). Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{i=n} X_i$ denote the average number of heads over the n trials, then the expectation of $\bar{X}$ equals to $\frac{3}{4}$

- ○ False
- ◉ True

Q2.4
1 Point

If a biased coin $P(X=1)=\frac{3}{4}$ is tossed n times independently (head takes value 1 and tail takes value 0). Let $\bar{X}=\frac{1}{n}\sum_{i=1}^{i=n}X_i$ denote the average number of heads over the n trials, then the variance of $\bar{X}$ equals to

- (●) $\frac{3}{16n}$
- ( ) $\frac{3}{16}$
- ( ) $\frac{3}{16n^2}$
- ( ) $\frac{3}{16\sqrt{n}}$

Q3 common distribution

3 Points

Q3.1

1 Point

Let X be a random variable distributed according to a Gaussian or normal distribution with probability density function: $P(x) = \frac{1}{s\sqrt{6\pi}} \exp\left(-\frac{x^2}{6s^2}\right)$. The variance of X is $3s^2$

- ○ False
- ◉ True

Q3.2

1 Point

If we model the number of meteors seen as a Poisson distribution, the average number of meteors per hour is roughly constant. If we expect to see a meteor every 10 minutes, use the Poisson distribution to find the probability of seeing at least 3 meteors in one hour.

- ○ $1 - \frac{10^0 e^{-10}}{0!} - \frac{10^1 e^{-10}}{1!} - \frac{10^2 e^{-10}}{2!}$
- ◉ $\frac{10^3 e^{-10}}{3!}$
- ○ $1 - \frac{6^0 e^{-6}}{0!} - \frac{6^1 e^{-6}}{1!} - \frac{6^2 e^{-6}}{2!}$
- ○ $\frac{6^3 e^{-6}}{3!}$

Q3.3
1 Point

With a success rate of 0.6, what is the probability that we hit the target exactly twice out of three attempts?

- ○ $3 \cdot 0.6 \cdot 0.4^2$
- ◉ $3 \cdot 0.6^2 \cdot 0.4$
- ○ $0.6^2 \cdot 0.4$
- ○ $0.6 \cdot 0.4^2$

Q4 Probability Theorem

3 Points

Q4.1
1 Point

Let A, B, C denotes 3 events. $P(A, B, C) = P(B|A, C) \times P(C|A) \times P(A)$

- ○ False
- ◉ True

Q4.2
1 Point

We have 40 treasure chests of type A and 60 treasure chests of type B. Each treasure chest of type A holds 30 gold coins and 70 silver coins, and each treasure chest of type B holds 20 gold and 80 silver coins. Choose a treasure chest uniformly at random, and pick a coin from that chest uniformly at random. If the coin is gold, then what is the probability that you chose a chest of type A?

- ◉ 50%
- ○ 40%
- ○ 70%
- ○ 60%

Q4.3
1 Point

X and Y are two independent random variables with expectation 0: $E(X) = E(Y) = 0$. Let random variable $Z = \frac{X}{3} + Y$. If $Z = 2$, then $E[X|Z = 2] = 6$ must be true.

- ○ True
- ◉ False

Q5
5 Points

Given the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & 3 \end{pmatrix}$$

and vector

$$\mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

Q5.1
1 Point

what is the determinant of A

- ○ 1
- ○ -1
- ○ 2
- ◉ 0

Q5.2
1 Point

what is the trace of A

- ○ -2
- ○ 4
- ◉ 3
- ○ -5

Q5.3
1 Point

What are all the eigenvalues of A

- ○ $\{-2, 0, 1\}$
- ◉ $\{0, 1, 2\}$
- ○ $\{-1, 2\}$
- ○ $\{-1, 0, 2\}$

Q5.4
1 Point

Which of the following is an eigenvector of A associated with eigenvalue 1:

- ○ $v = \sqrt{\frac{1}{3}}(1, 1, 0)$
- ○ $v = \sqrt{\frac{2}{3}}(1, 2, 4)$
- ◉ $v = \frac{1}{3}(1, 1, 1)$
- ○ $v = \sqrt{\frac{1}{3}}(1, 0, 0)$

Q5.5
1 Point

let $f(x) = b^T x + 5$ where x is a vector of length 3. What value does $\frac{\partial f}{\partial x}$ take when $\mathbf{x} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

- (●) $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$
- ( ) $\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$
- ( ) 5
- ( ) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

Q6
3 Points

Consider the vector $\mathbf{y} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

Q6.1
1 Point

Compute $\mathbf{y}^T\mathbf{y}$

- ○ 9
- ○ 16
- ○ 25
- ◉ 34

Q6.2
1 Point

Compute $||\mathbf{y}||_2$

- ○ 3
- ○ 6
- ○ 5
- ◉ $\sqrt{34}$

Q6.3
1 Point

Compute $\mathbf{y}\mathbf{y}^T$

- $\begin{pmatrix} 15 \\ 25 \end{pmatrix}$
- $\begin{pmatrix} 3 & 5 \\ 5 & 3 \end{pmatrix}$
- $\begin{pmatrix} 9 & 15 \\ 15 & 25 \end{pmatrix}$
- $\begin{pmatrix} 9 \\ 15 \end{pmatrix}$

Q7
2 Points

Let A be a 15×30 matrix.

Q7.1
1 Point

Which of the following is a possible rank for A ?

- ○ Both 10 and 20
- ○ 20
- ◉ Neither 10 nor 20
- ○ 10

Q7.2
1 Point

Let $M = AA^T$ and $v \neq \vec{0}$ denote a nonzero vector. What is the range of all possible values for $v^T Mv$?

- ○ ≤ 0 for all possible vector $v \neq \vec{0}$
- ◉ ≥ 0 for all possible vector $v \neq \vec{0}$
- ○ (-∞, ∞)

Q8
1 Point

$f(n) = e^n, g(n) = 2^n$

- ○ Both $f(n) = O(g(n))$ and $g(n) = O(f(n))$
- ○ Neither $f(n) = O(g(n))$ nor $g(n) = O(f(n))$
- ◉ $g(n) = O(f(n))$
- ○ $f(n) = O(g(n))$