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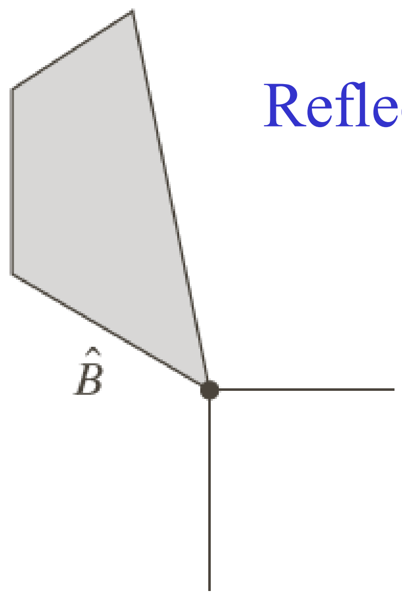
Chapter 9 Morphological Image Processing

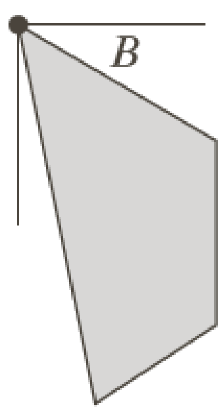
9.1 形態影像處理初步
9.2 膨脹與侵蝕
9.3 開啟與關閉
9.4 HIT-OR-MISS轉換
9.5 基礎形態演算法
9.6 灰階影像之形態影像處理

9.1 Preliminaries

9.1.1 反射與平移

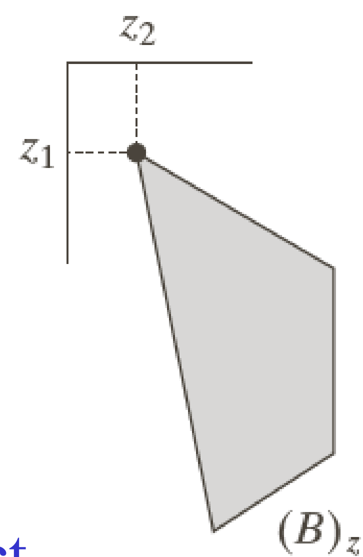


Reflection $\hat{B} = \{w \mid w = -b, \ \text{for } b \in B\}$



Set B

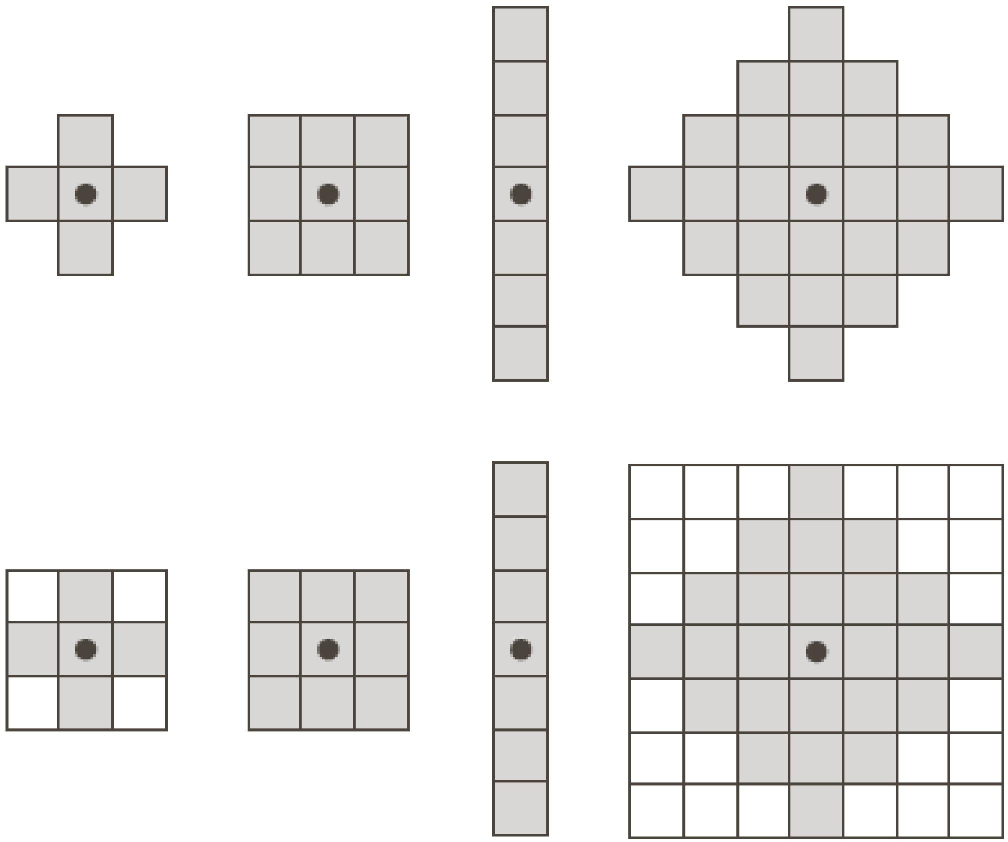
2-D points representing an object



Translation

$$(B)_z = \{c \mid c = b + z, \ \text{for } b \in B\}$$

9.1.2 結構元素 Structure Elements: small sets or subimages used to probe an image under study for properties



SEs

Convert to Rectangular Array

Morphological operation

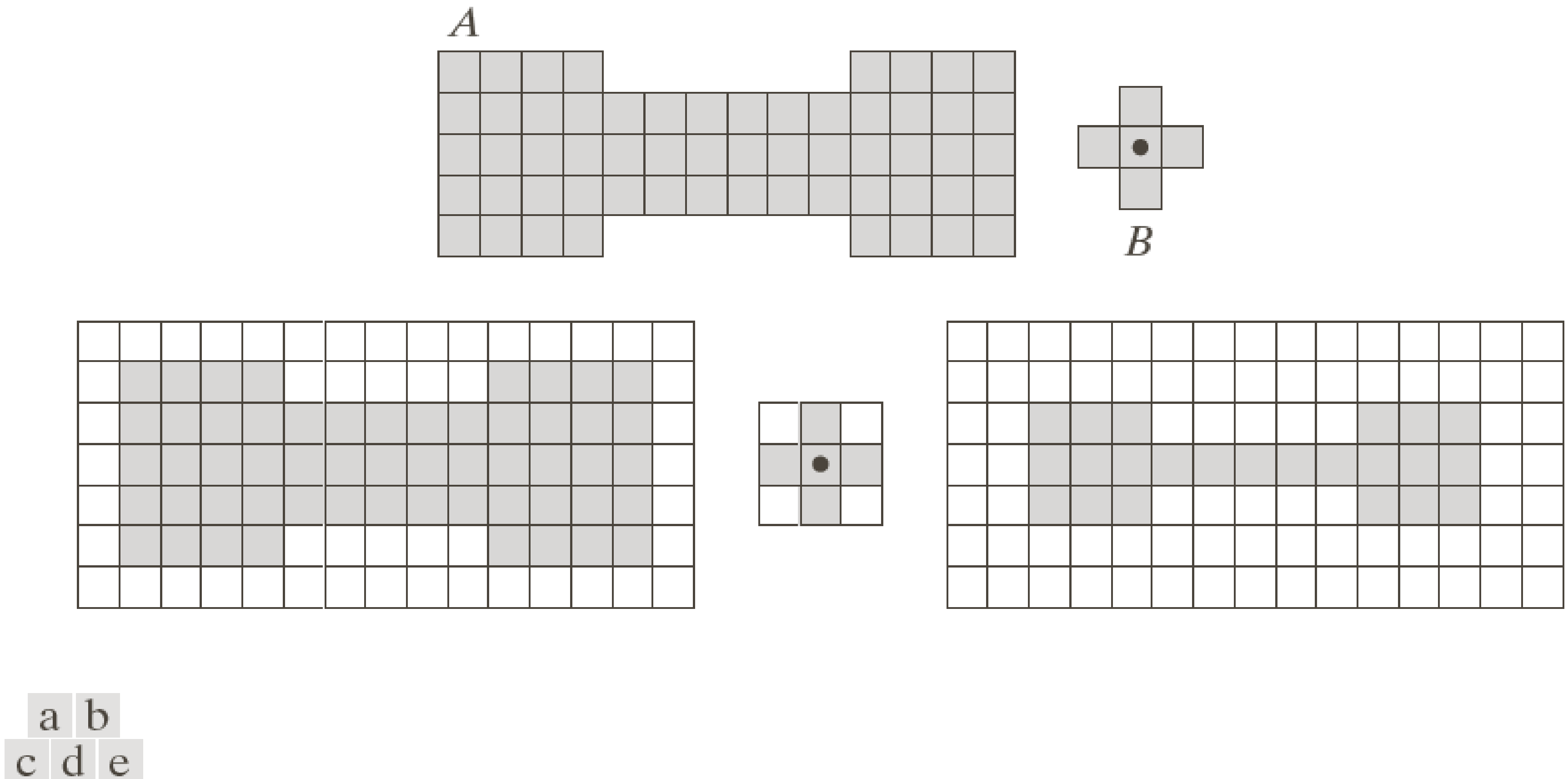


FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

9.2 Erosion and Dilation 侵蝕與膨脹

erosion

$$A \Theta B = \{z \mid (B)_z \subseteq A\}$$

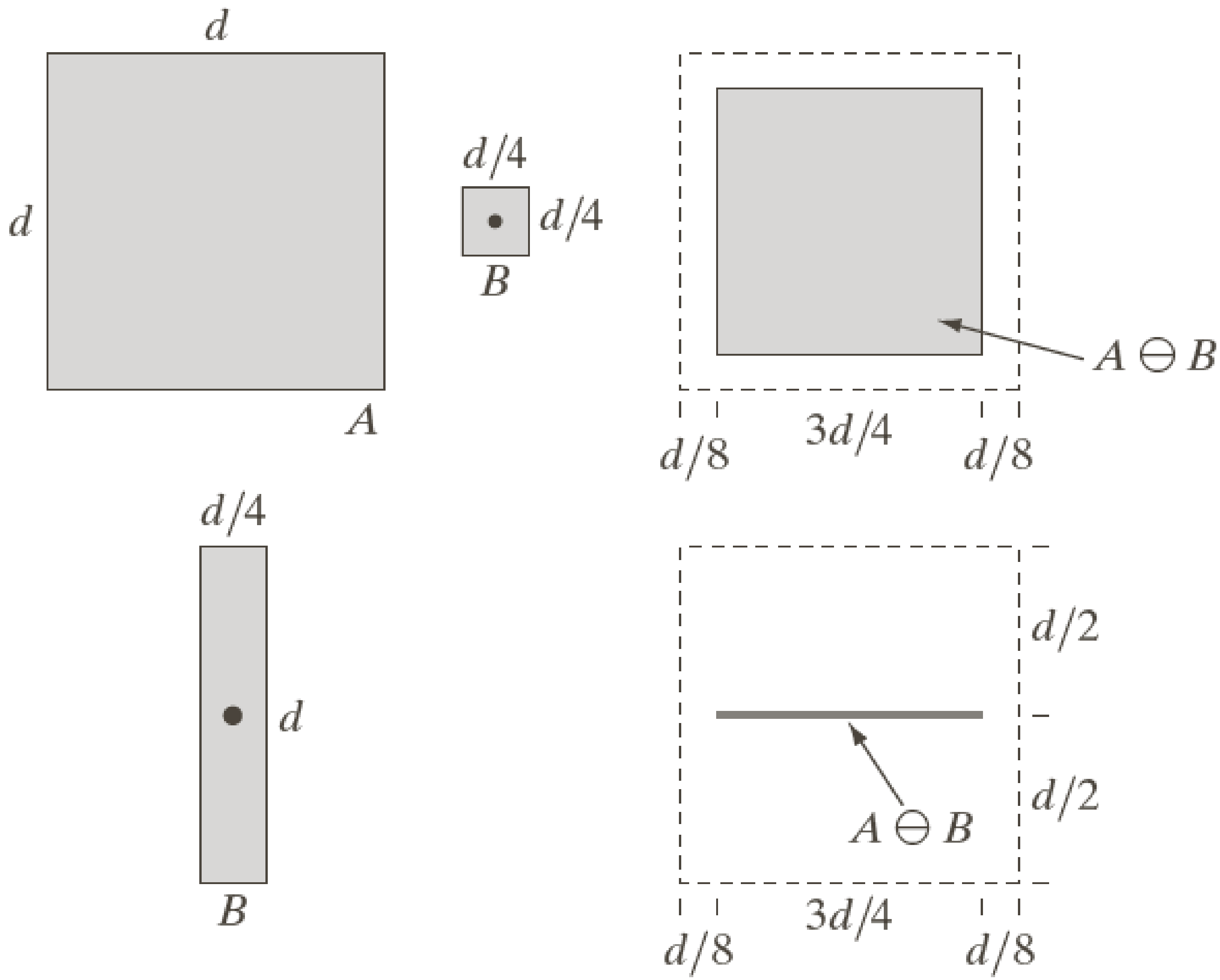
$$A \Theta B = \{z \mid (B)_z \cap A^c = \varnothing\}$$

dilation

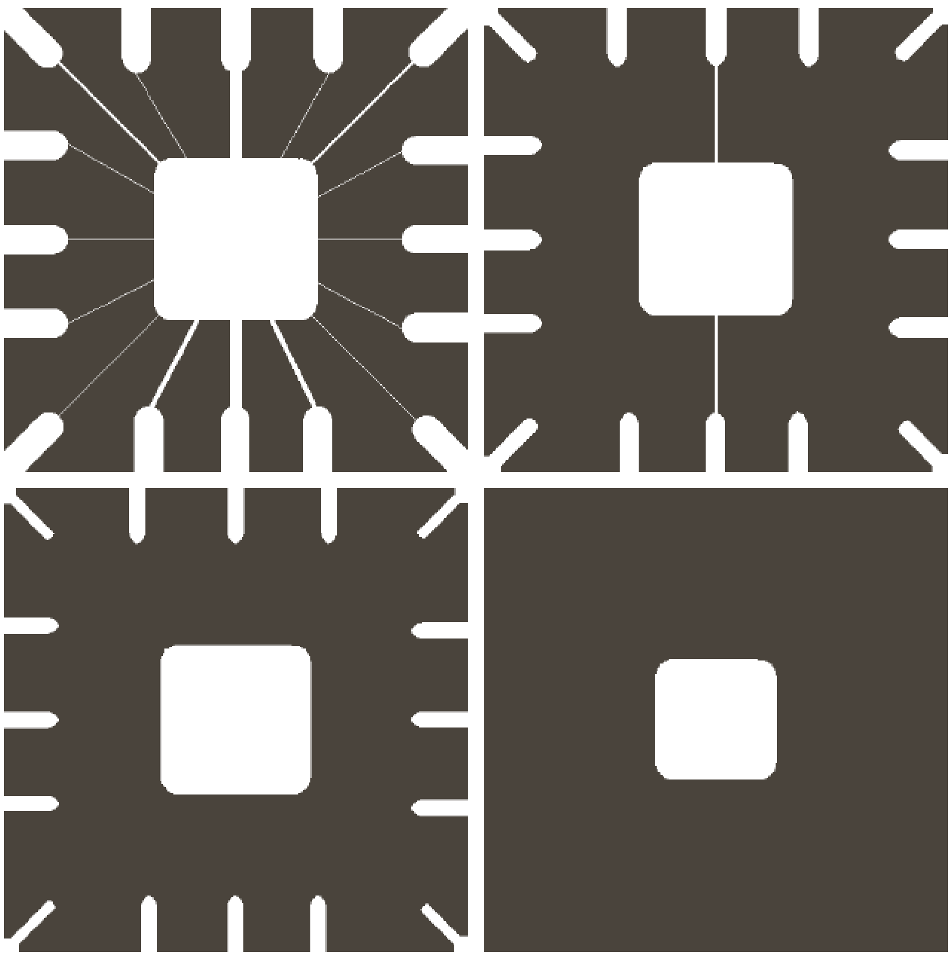
$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \neq \varnothing\}$$

$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

Erosion example



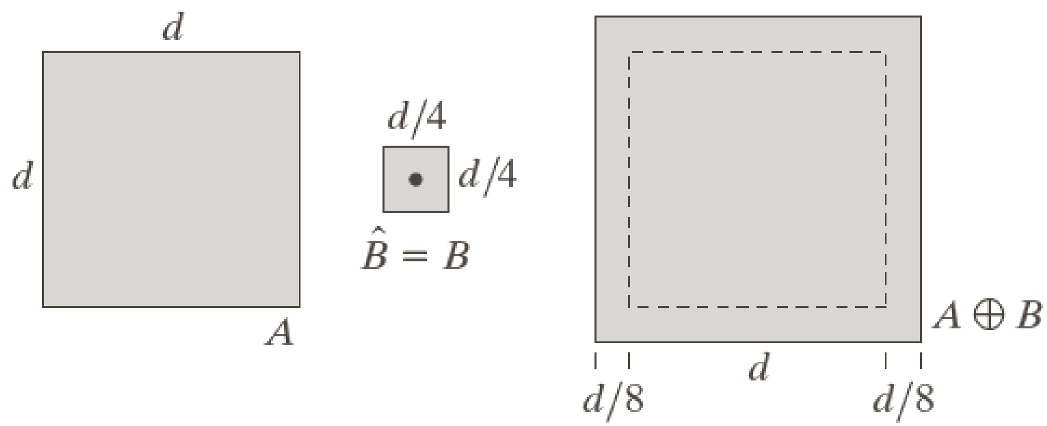
Example 9.1

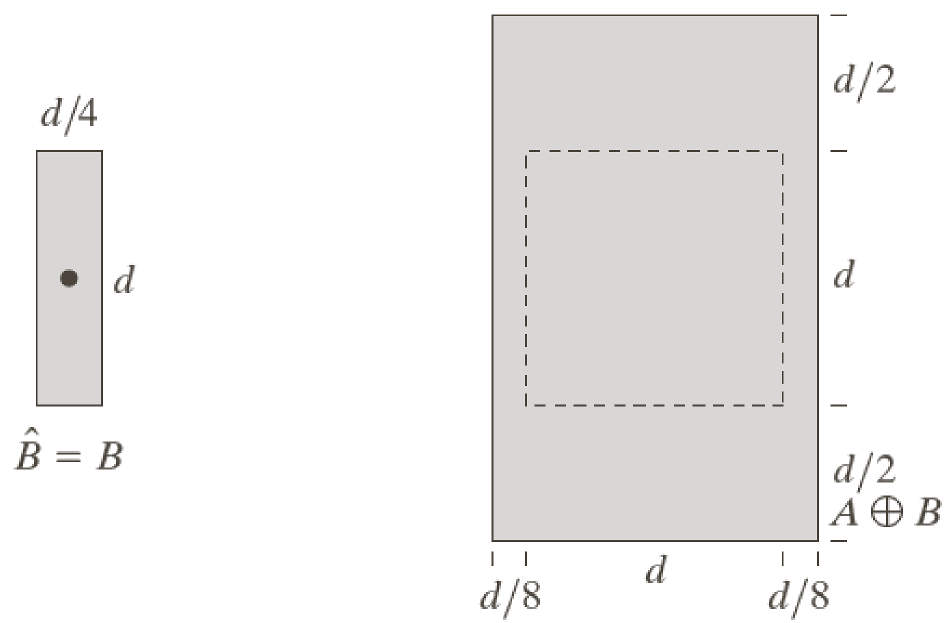


a	b
c	d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45, respectively. The elements of the SEs were all 1s.

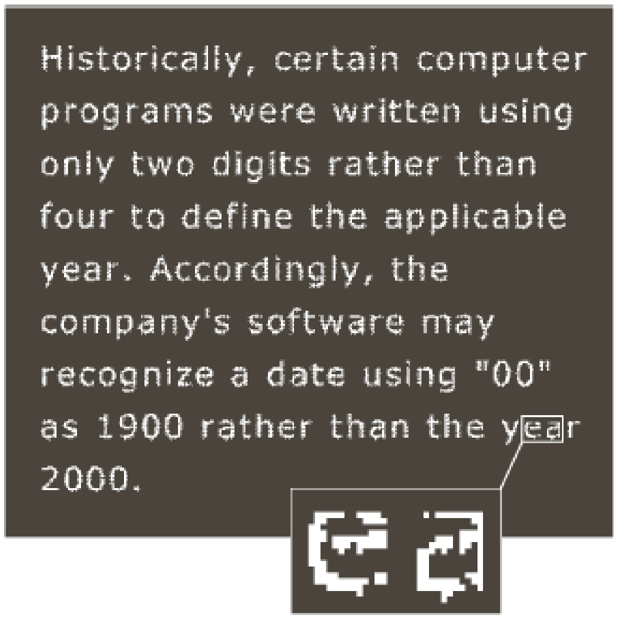
Dilation example

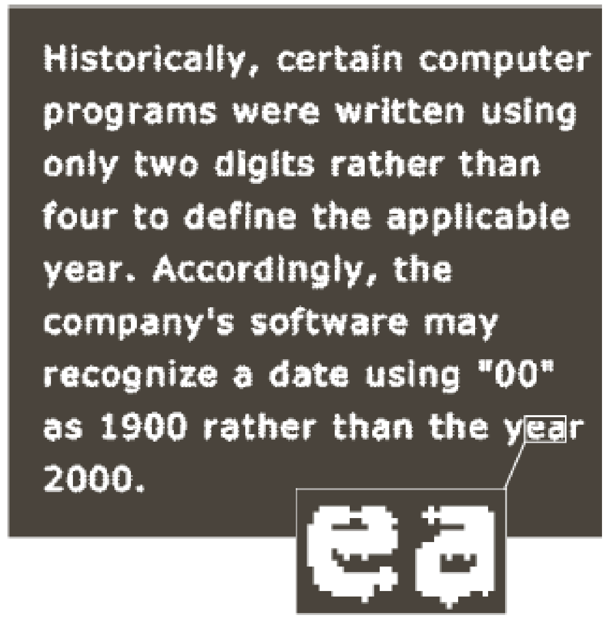




Example 9.2

a c
b





0	1	0
1	1	1
0	1	0

FIGURE 9.7 (a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

侵蝕與膨脹之關係(Duality)

$$(A \Theta B)^c = A^c \oplus \hat{B}$$

$$(A \oplus B)^c = A^c \Theta \hat{B}$$

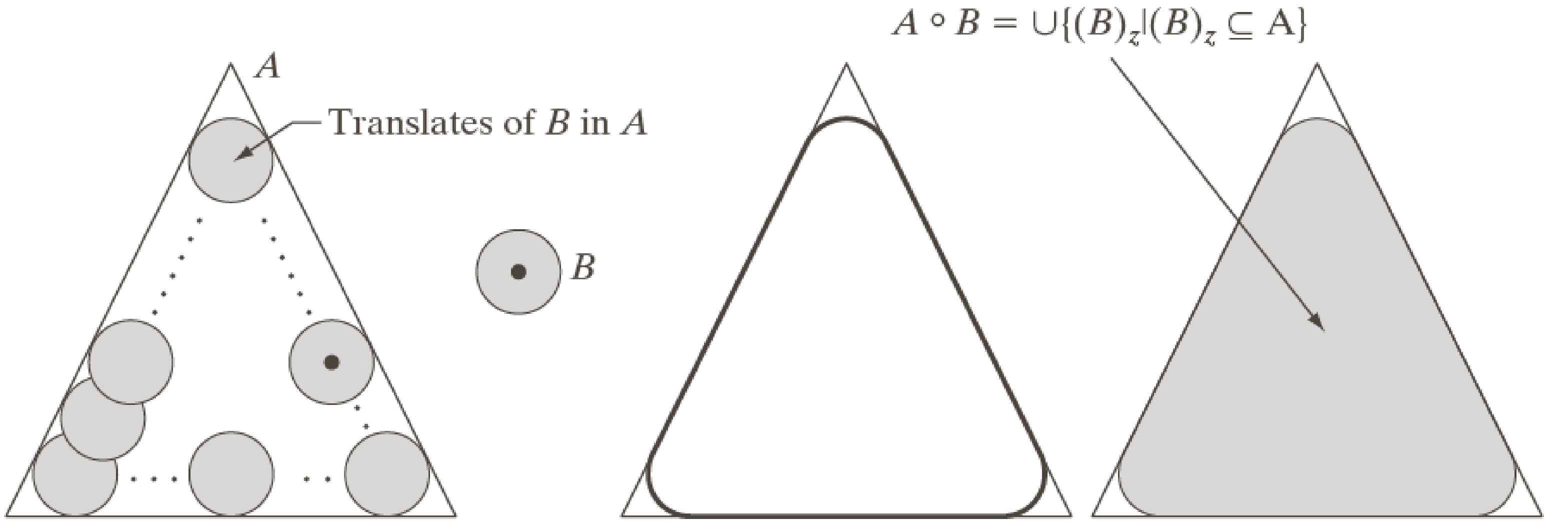
$$\begin{aligned}(A \Theta B)^c &= \{z \mid (B)_z \subseteq A\}^c \\ &= \{z \mid (B)_z \cap A^c = \varnothing\}^c \\ &= \{z \mid (B)_z \cap A^c \neq \varnothing\}^c \\ &= A^c \oplus \hat{B}\end{aligned}$$

9.3 Opening and Closing

Opening

$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = \cup\{(B)_z \mid (B)_z \subseteq A\}$$



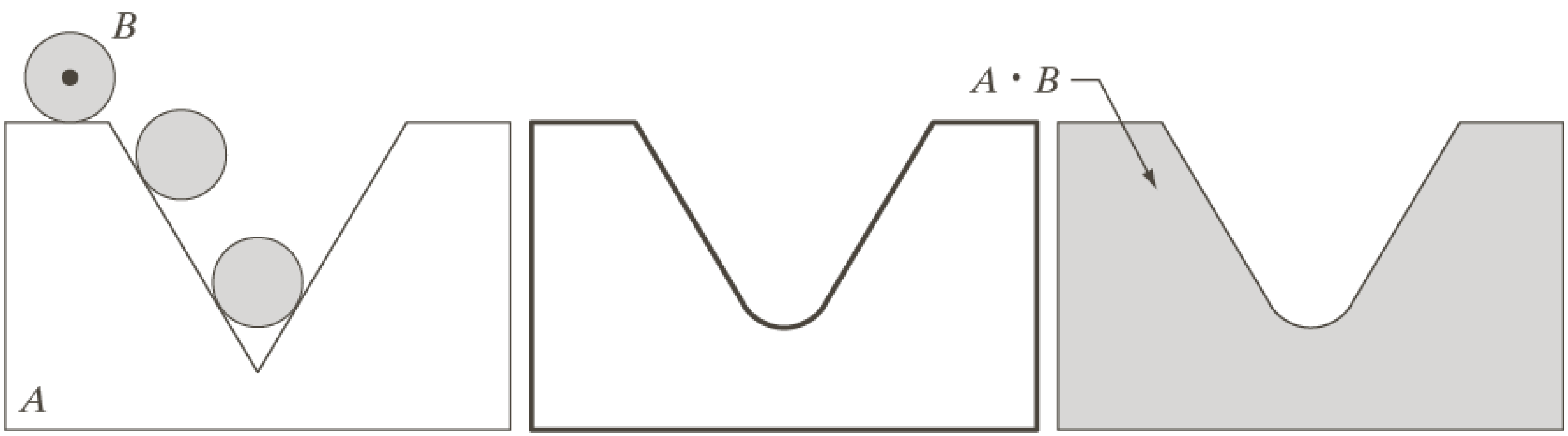
9.3 Opening and Closing

Closing

$$A \bullet B = (A \oplus B) \ominus B$$

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

$$(A \circ B)^c = (A^c \bullet \hat{B})$$



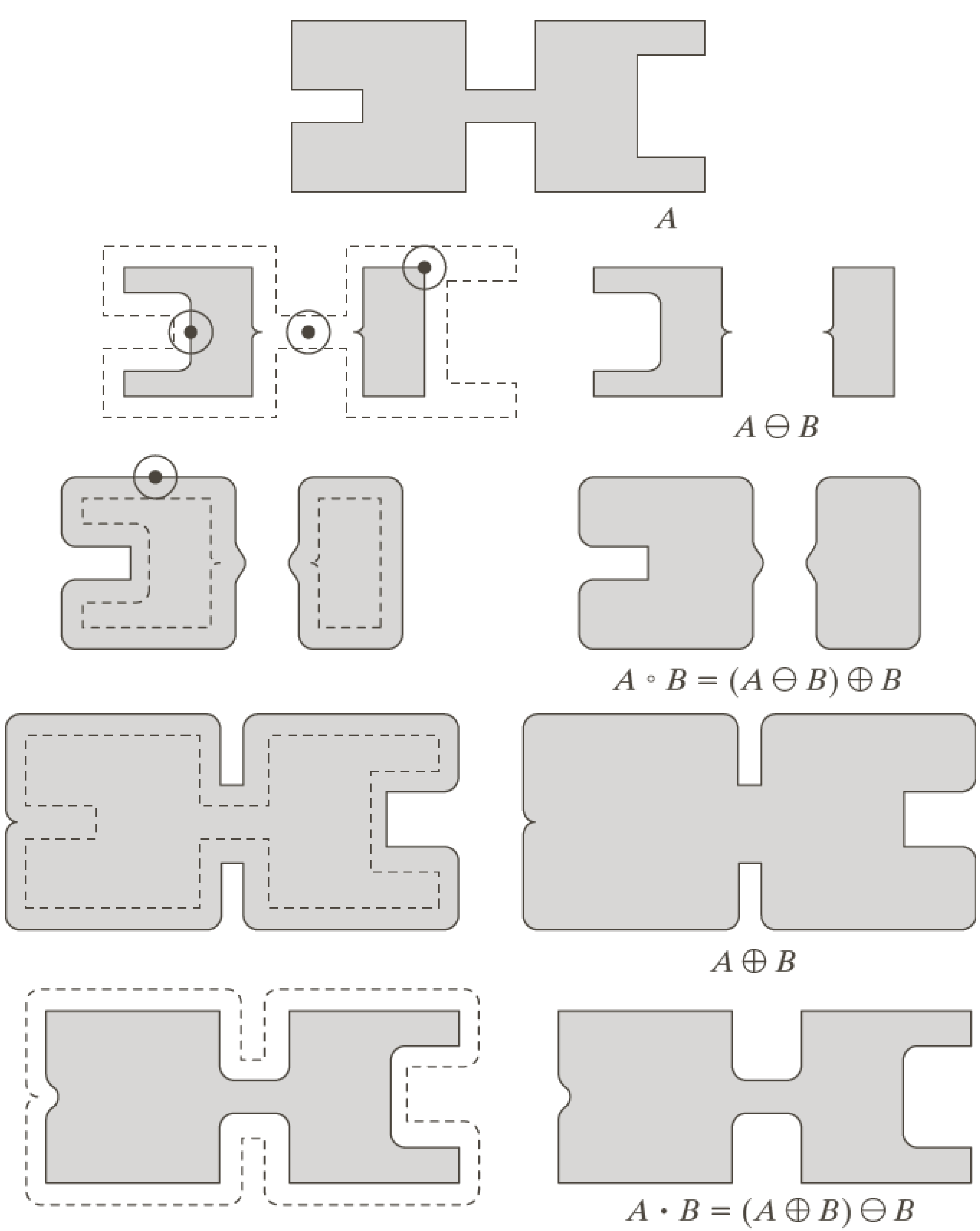


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

開啟與關閉之性質

Opening的性質

1.$A \circ B$為A之子集合

2.若C為D之子集合，則$C \circ B$為$D \circ B$之子集合

3.$(A \circ B) \circ B = A \circ B$

Closing的性質

1.A為$A \bullet B$之子集合

2.若C為D之子集合，則$C \bullet B$為$D \bullet B$之子集合

3.$(A \bullet B) \bullet B = A \bullet B$

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

9.4 HIT-OR-MISS轉換

- **Hit-or Miss為形狀偵測之基礎工具**

令B為D與其背景$(W - D)$之聯集，則HIT-OR-MISS轉換為

$$A \circledast B = (A \ominus D) \cap [A^c \ominus (W - D)]$$

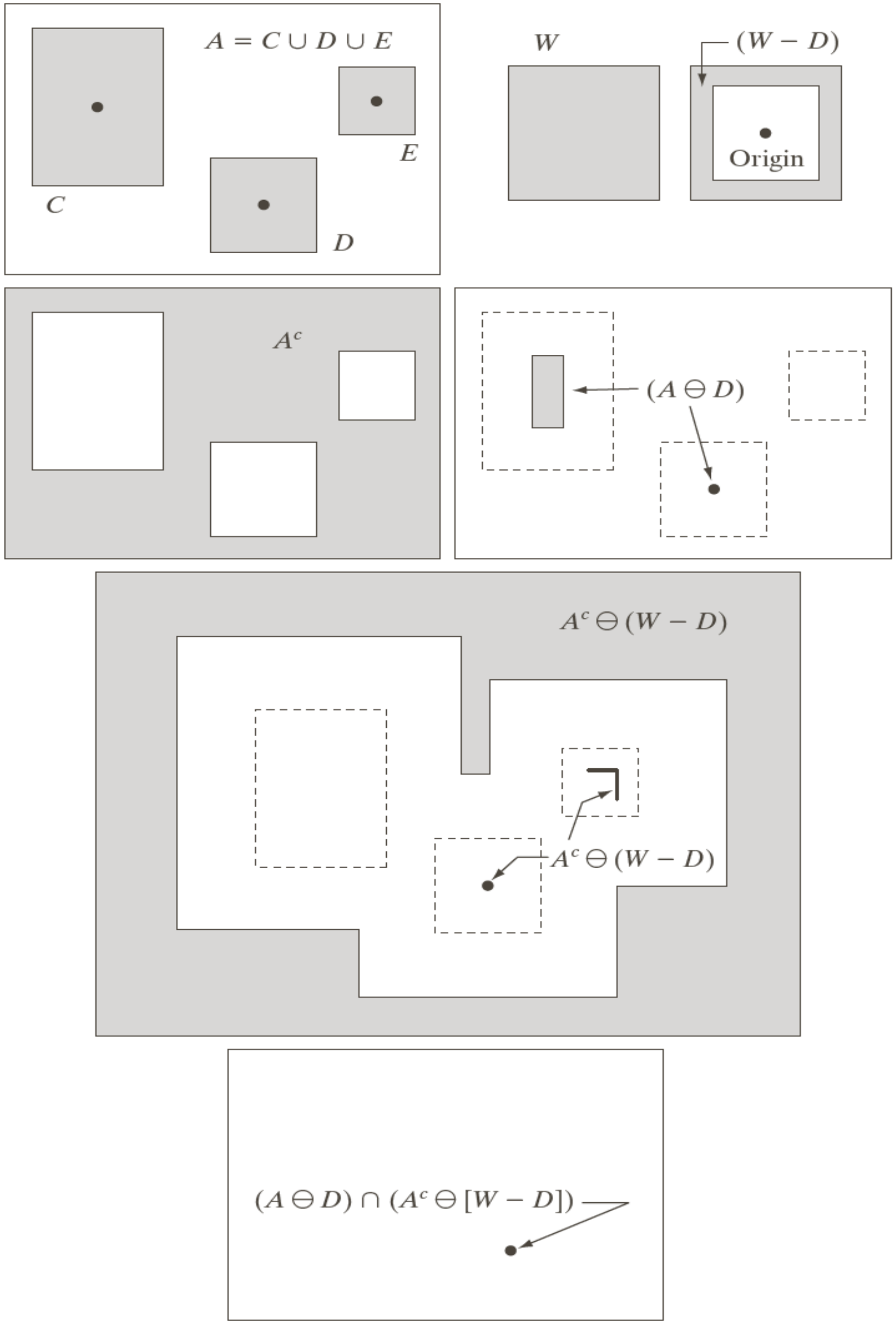
令 B_1=D 且 B_2 =$(W–D)$ 則上式為

$$A \circledast B = (A \ominus B_1) \cap [A^c \ominus B_2]$$

亦可表示為

$$A \circledast B = (A \ominus B_1) - [A \oplus \hat{B}_2]$$

HIT-OR-MISS轉換



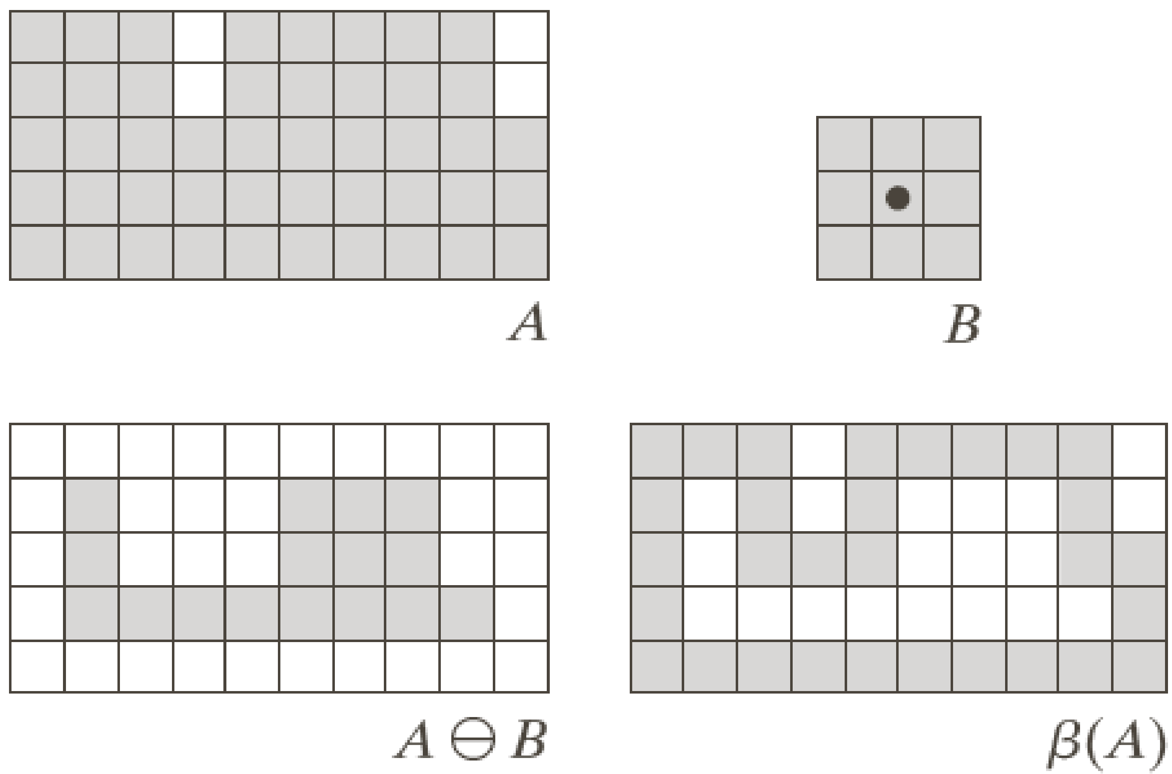
a b
c d
e
f

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

9.5 Some Basic Morphological Algorithms

9.5.1 Boundary Extraction

$$\beta(A) = A - (A \ominus B)$$



a b
c d

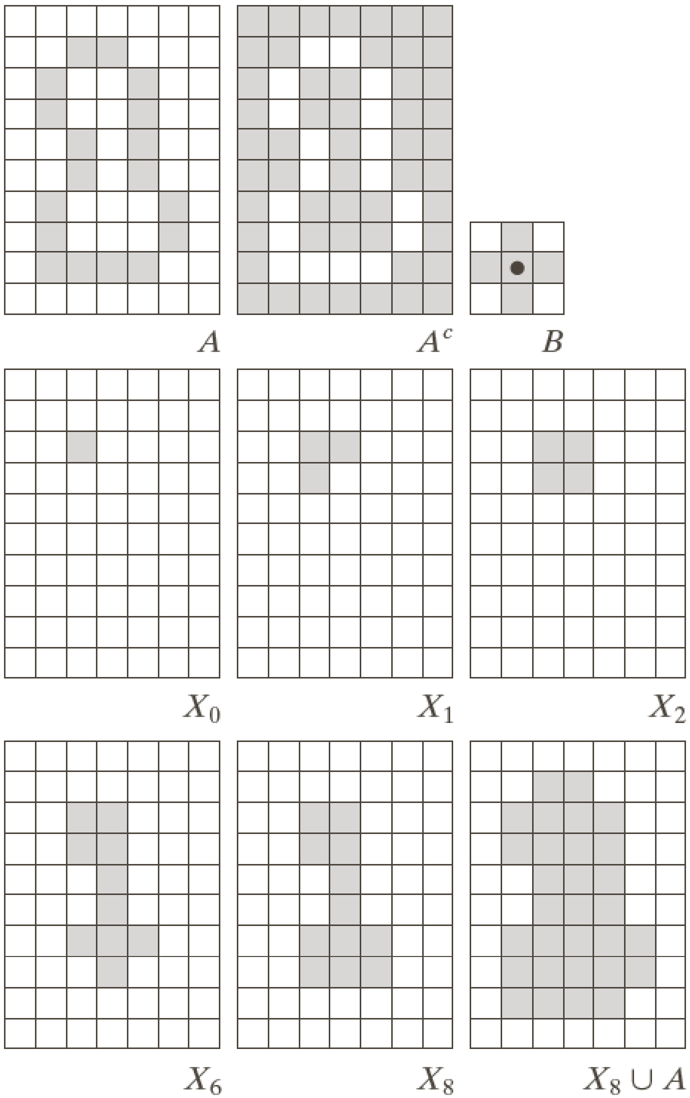
FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

a b

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

9.5.2 區域填充(Region Filling)

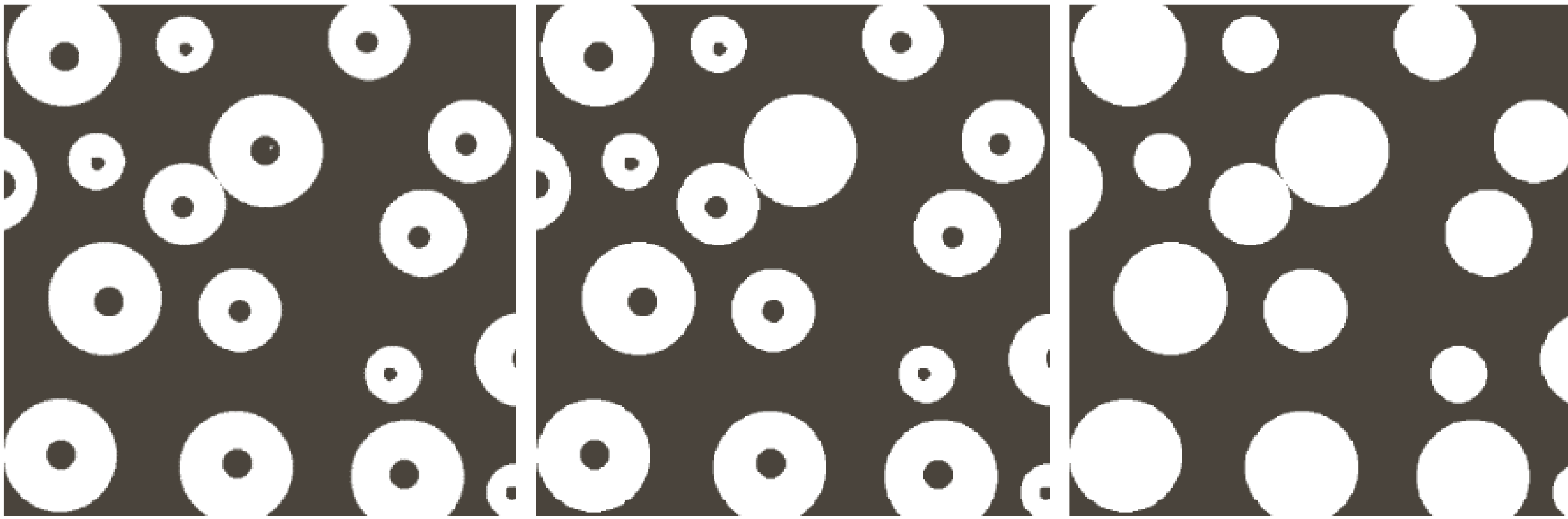
$$X_k = (X_{k-1} \oplus B) \cap A^c$$
$$k = 1,2,3,\ldots$$



a	b	c
d	e	f
g	h	i

FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

9.5.2 區域填充(Region Filling)



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

9.5.3 連接區域之抽取

$$X_k = (X_{k-1} \oplus B) \cap A$$
$$k = 1,2,3,\ldots$$

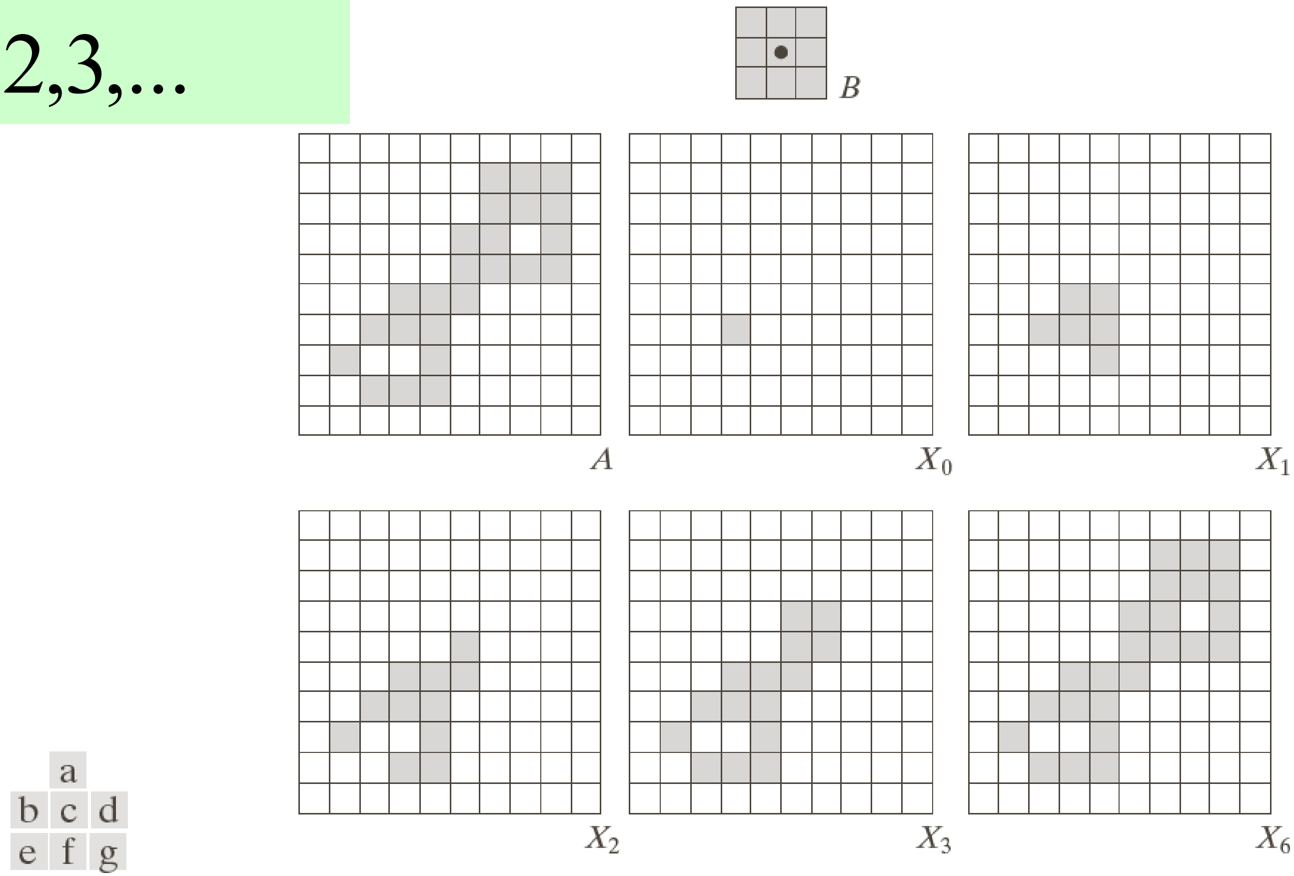
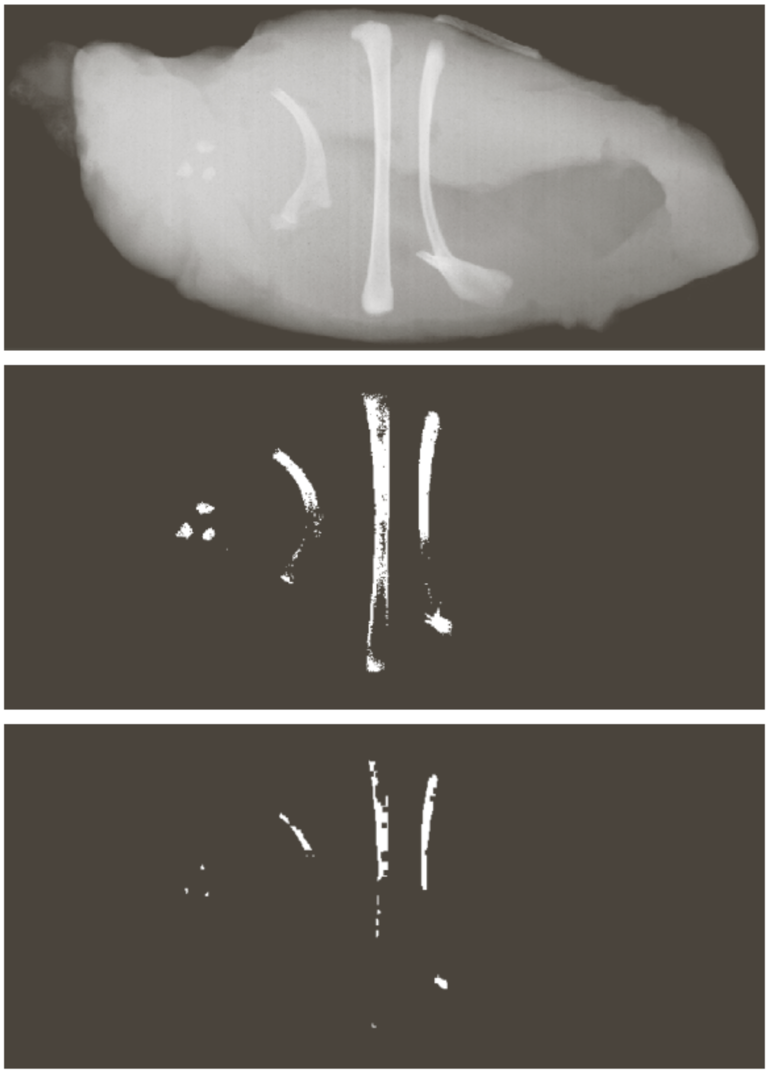


FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).

9.5.3 連接區域之抽取



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

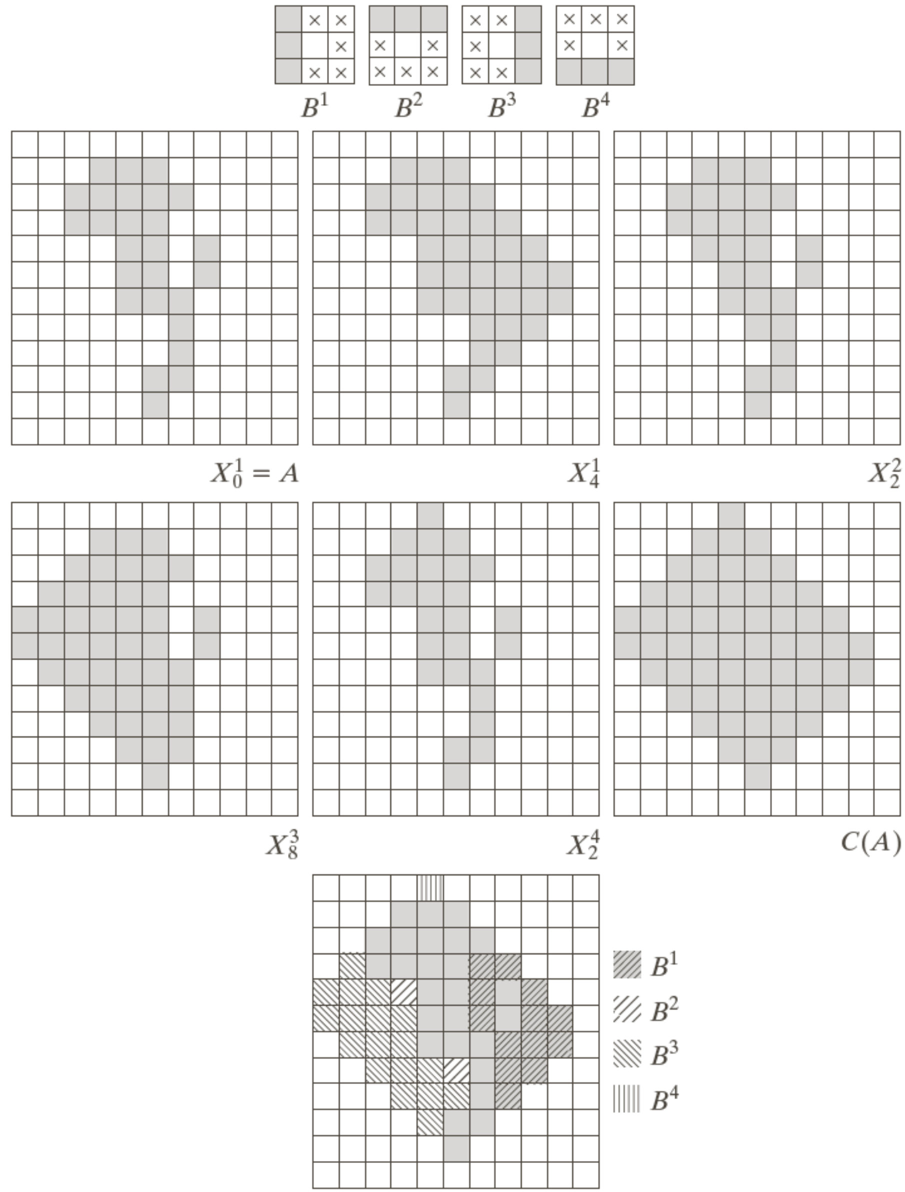
9.5.4 凸形輪廓 (Convex Hull)

$$X_k^i = (X_{k-1} \circledast B^i) \cup X_{k-1}^i$$

$\mathrm{i} = 1,2,3,4$ and

$k = 1,2,3,\ldots$

$$C(A) = \bigcup_{i=1}^{4} D^i$$



9.5.4 凸形輪廓 (Convex Hull)

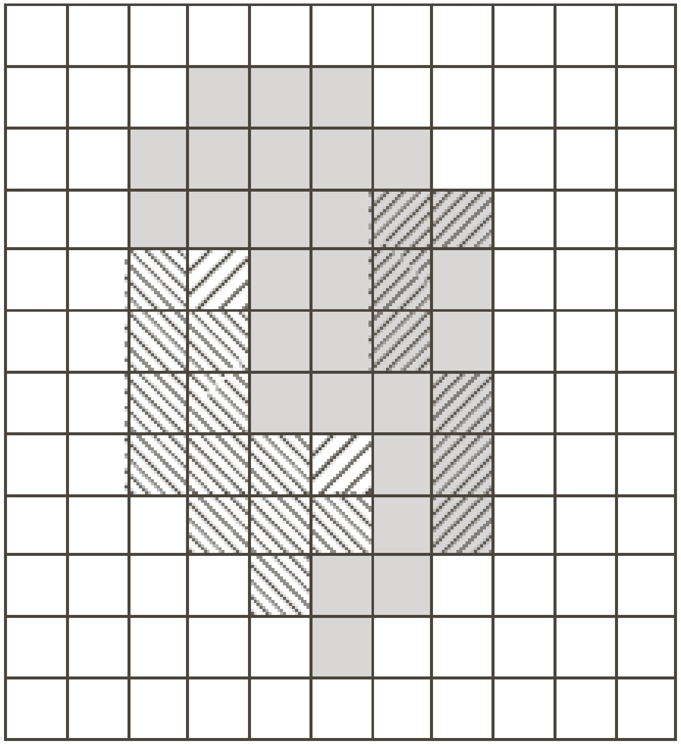


FIGURE 9.20 Result of limiting growth of the convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

9.5.5 細線化 (Thinning)

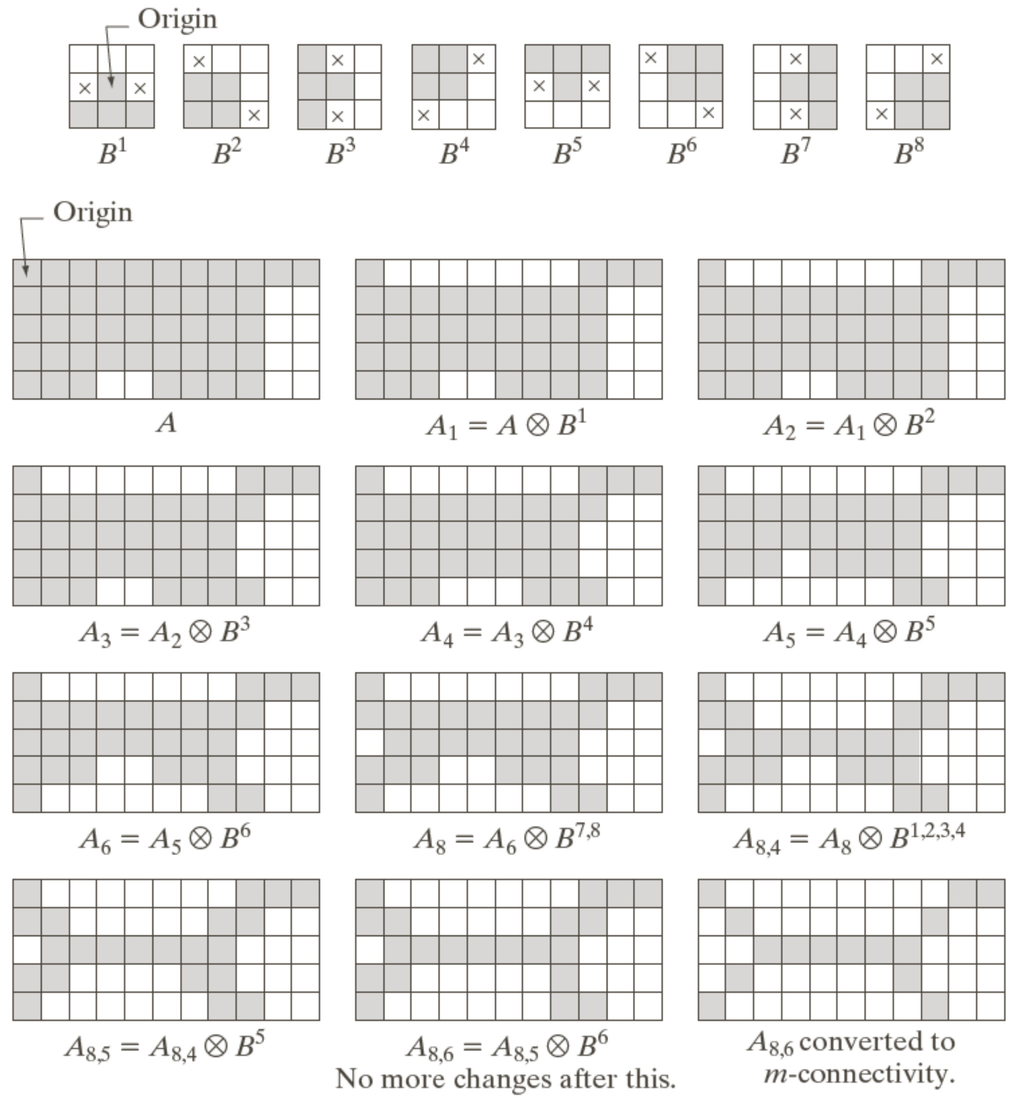
$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c$$

$$\{B\} = \{B^1, B^2, \ldots, B^n\}$$

$$A \otimes \{B\} =$$

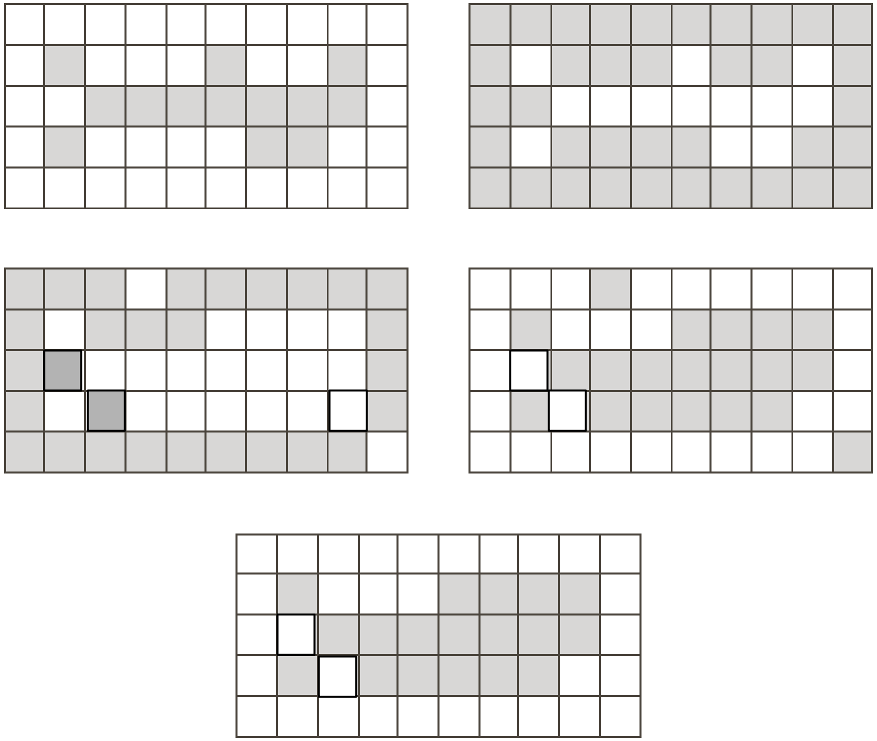
$$((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$



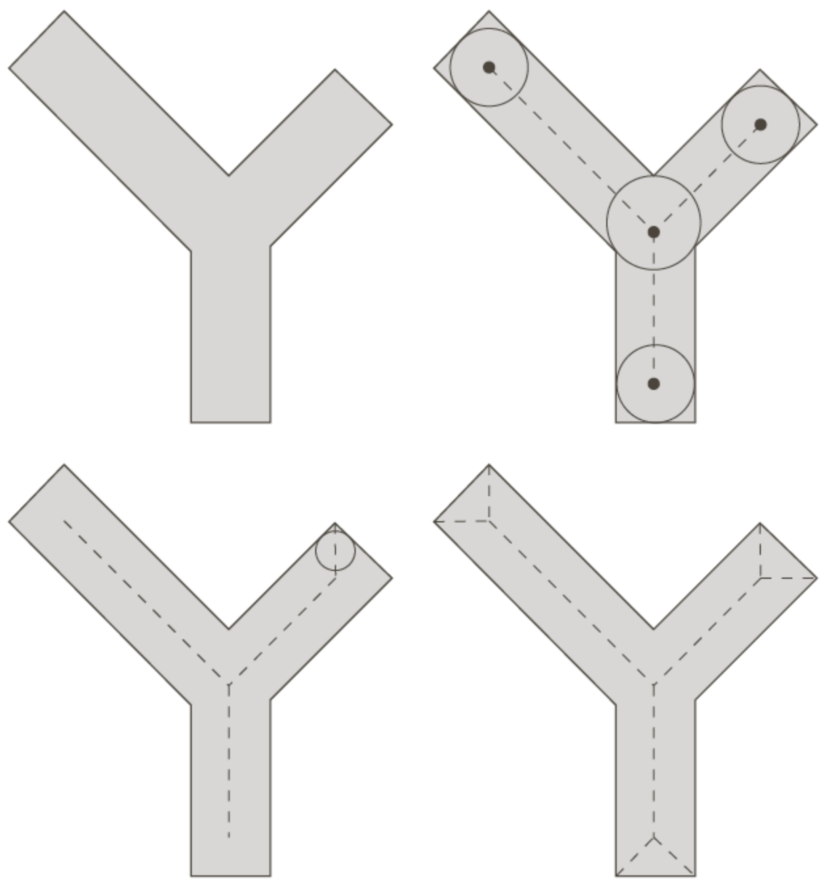
9.5.6 粗線化(Thickening)

$$A \odot B = A \cup (A \circledast B)$$

$$A \odot B = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$



9.5.7 骨架(Skeleton)

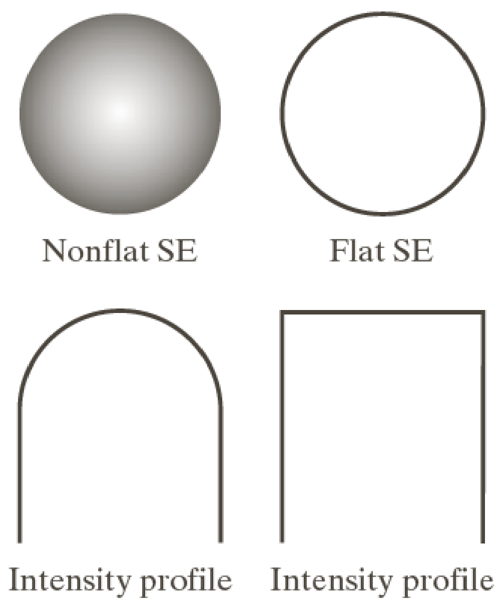


a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.

9.6 Gray-Scale Morphology 灰階影像之形態影像處理

- Structuring elements in gray-scale morphology are used as “probes” to examine a given image for specific properties.
- Structuring elements in gray-scale morphology belong to one of two categories: nonflat and flat.



a b
c d

FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

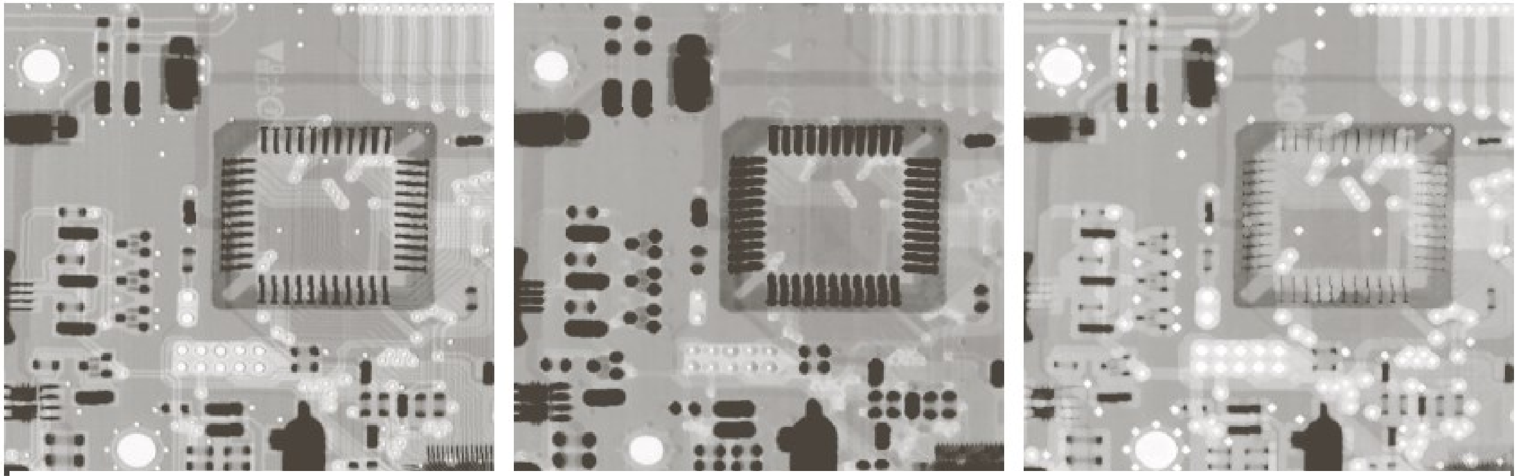
9.6.1 Erosion and Dilation 侵蝕與膨脹

Erosion by a flat structuring element

$$[f \ominus b](x,y) = \min_{(s,t)\in b}\{f(x+s, y+t)\}$$

Dilation by a flat structuring element

$$[f \oplus b](x,y) = \max_{(s,t)\in b}\{f(x-s, y-t)\}$$



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Erosion and Dilation 侵蝕與膨脹

Erosion by a nonflat structuring element

$$[f \ominus b_N](x,y) = \min_{(s,t)\in b_N} \{ f(x+s, y+t) - b_N(s,t) \}$$

Dilation by a nonflat structuring element

$$[f \oplus b_N](x,y) = \max_{(s,t)\in b_N} \{ f(x-s, y-t) + b_N(s,t) \}$$

Erosion and dilation are duals with respect to function complement ation and reflection

$$(f \ominus b)^c = (f^c \oplus \hat{b})$$

$$(f \oplus b)^c = (f^c \ominus \hat{b})$$

9.6.2 Opening and Closing 開啟與關閉

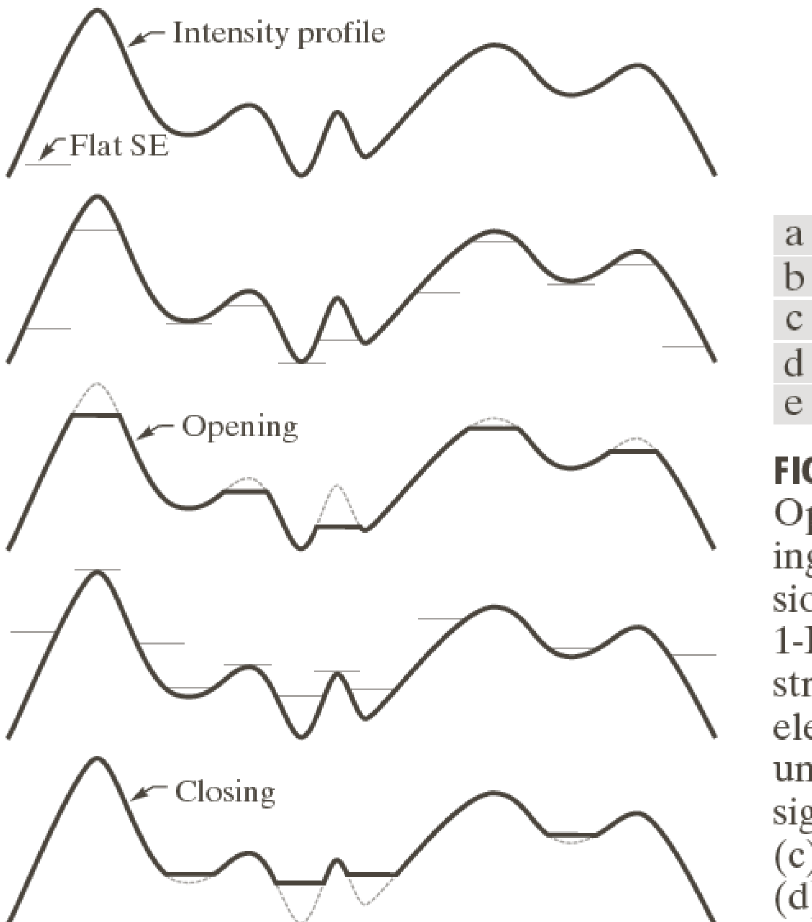


FIGURE 9.36 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

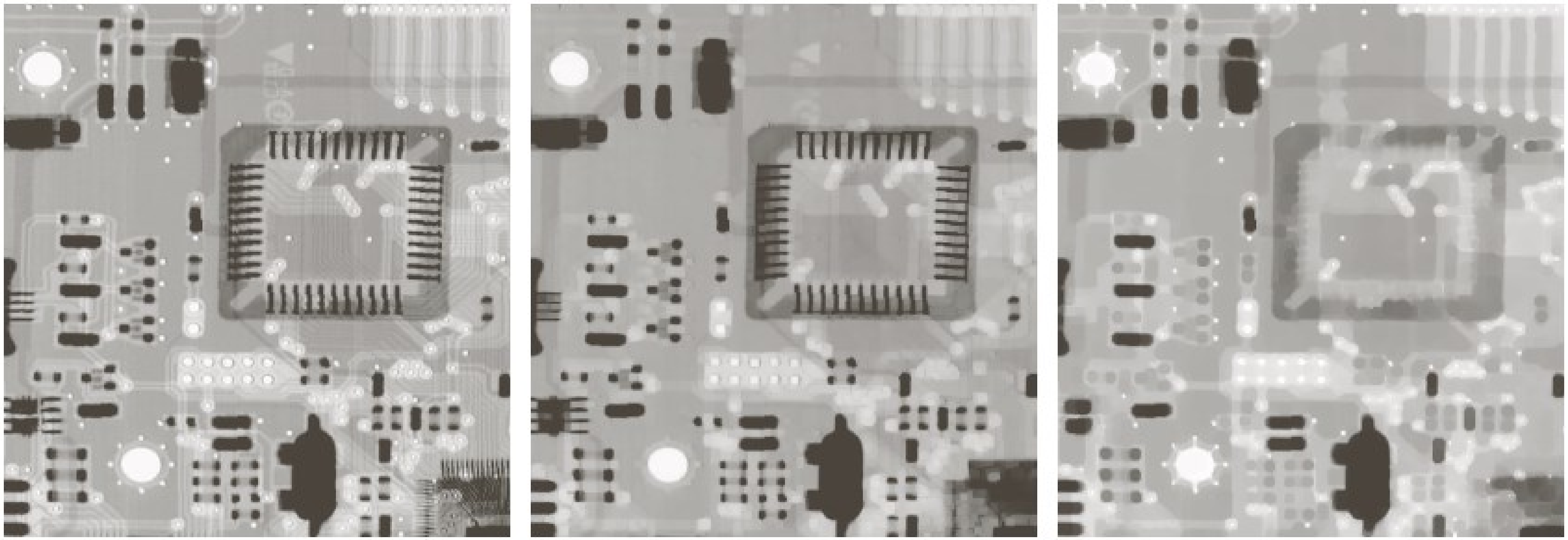
$$f \circ b = (f \ominus b) \oplus b$$

$$f \bullet b = (f \oplus b) \ominus b$$

$$(f \bullet b)^c = f^c \circ \hat{b}$$

$$(f \circ b)^c = f^c \bullet \hat{b}$$

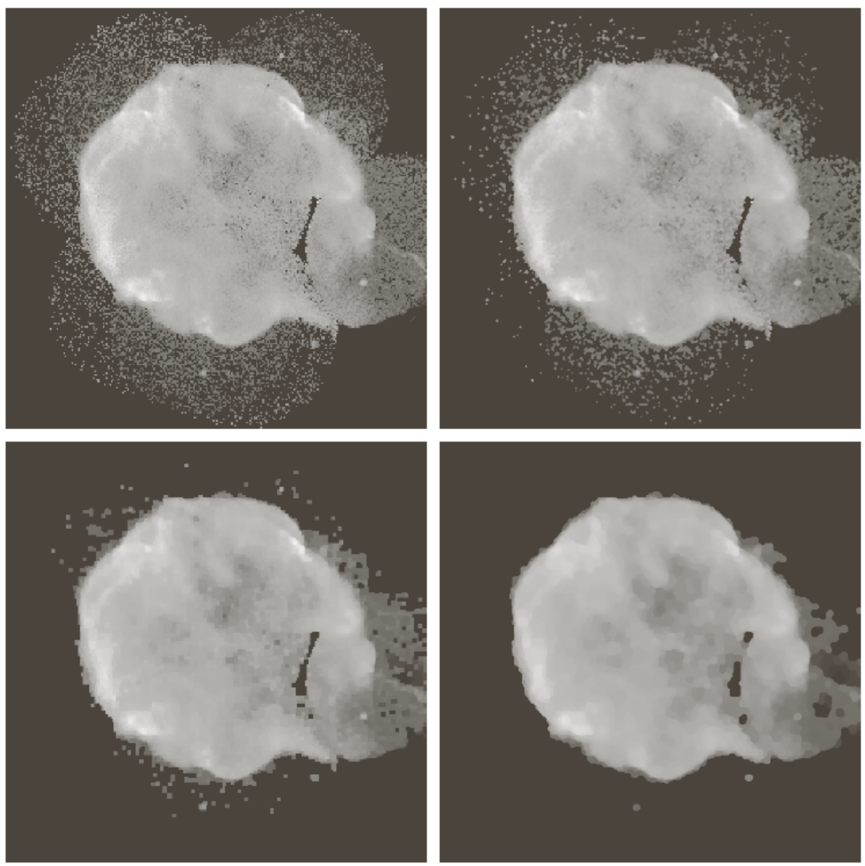
Opening and Closing 開啟與關閉



a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

9.6.3 Morphological Algorithms灰階形態處理之演算法 - Morphological Smoothing 形態平滑化

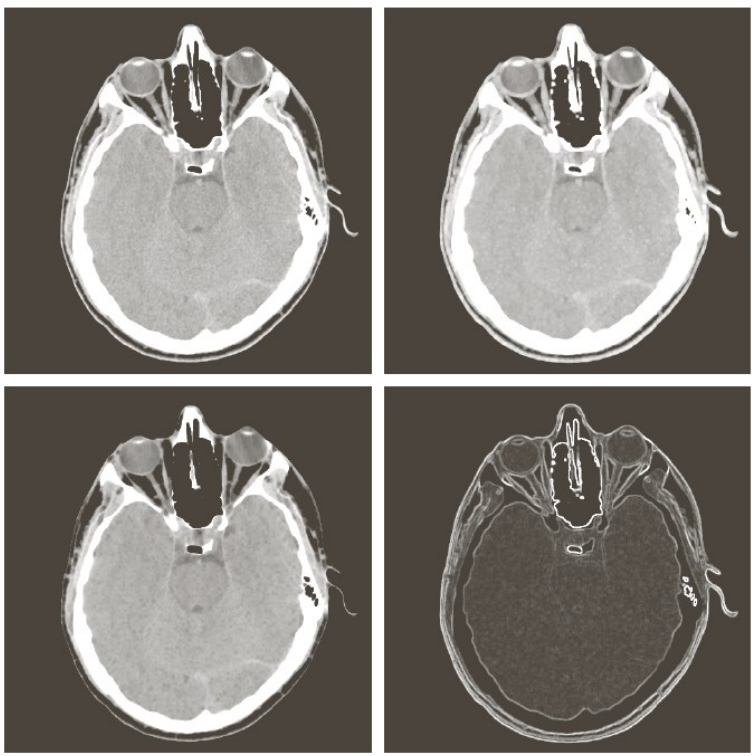


a	b
c	d

FIGURE 9.38
(a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

9.6.3 Morphological Gradient 形態梯度

$$g=(f\oplus b)-(f\ominus b)$$



a	b
c	d

FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)