COMP0216 - Systems Engineering for Real-time Systems

Week 2: System Requirements Review (SRR)

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Team 05

Project code repo: https://github.com/Harrishayy/SysEng

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1 Mission Statement & Strategy

1.1 Mission

The mission is to construct an inverted pendulum cart with an incentive to continuously refine plans and build prototypes where the systems can be fine-tuned in order to achieve positive results in all three performance tests (evaluation).

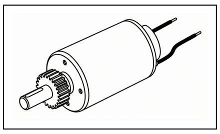
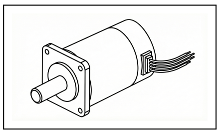
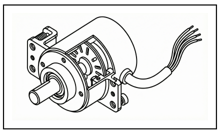
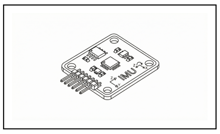
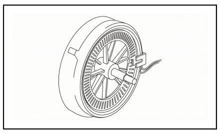
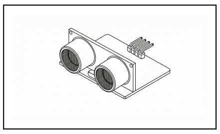
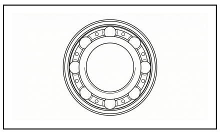
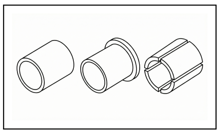
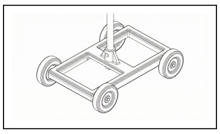
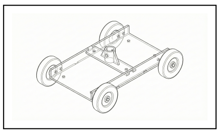
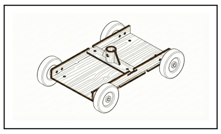
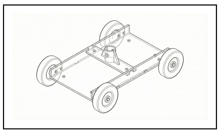
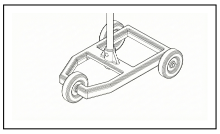
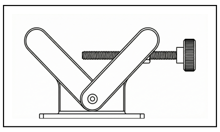
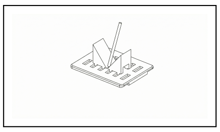
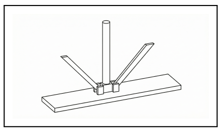
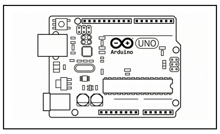
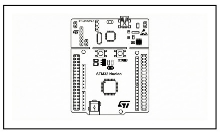
1.2 Strategy

Upon discussing the different project management strategies, the consensus is to partake in a sprint-oriented project management strategy. This is considered to be the rational option as the system is ultimately judged on three performance-driven evaluations that reward rapid iteration and integrated end-to-end behaviour: robust disturbance rejection, high-angle recovery from a non-zero start, and a time-critical sprint-and-stop race with a tight final-position tolerance. A stability-first approach can lead to spending too much time perfecting standing balance, while not putting enough effort into the combined dynamics, switching between modes, and stronger tuning needed to speed up, slow down, and regain balance quickly, which are exactly the skills that end up deciding the final performance.

Sprinting also better matches the project's lifecycle gating: early simulation checks, rapid controller benchmarking, and repeated integration-and-tuning loops. This pace of work pushes the team to make clear choices about motor, sensor, and controller trade-offs, test early whether the design still works when there is noise and disturbance, and keep improving the handover between **launch**, **balance**, and **sprint** behaviours. As a result, major integration problems are less likely to appear at the end, while effort stays focused on improving the results that are actually measured.

2 Morphological Chart (MC)

Table 1: Morphological Chart for the Inverted Pendulum System

Function	Solution 1	Solution 2	Solution 3
Locomotion	 **High Torque DC Motors**	 Stepper Motors	
Angle Sensing	 **Rotary Encoder (Pivot)**	 IMU	
Cart Positioning	 **Wheel Encoders**	 Ultrasonic	
Pivot Support	 **Ball Bearings**	 Bushings	
Chassis Build	 3D Printed PLA	 Laser-cut Acrylic	 **Laser-cut Wood**
Wheels	 **4**	 3	
Launch Fixture	 Screw-Based Fixture	 Tab and Slot	 **Simple Nut-Bolt**
Control Board	 **Arduino/ESP32**	 STM32 Nucleo	

3 Design Component Selection

The hardware configuration prioritizes dynamic response, measurement precision, and structural rigidity to satisfy the sprint and stability requirements.

3.1 Actuation: Dynamic Response

High Torque DC Motors were selected over stepper motors to meet the aggressive acceleration needed for the sprint phase. Steppers suffer from torque drop-off at high speeds and risk skipping steps during rapid load changes. DC motors, paired with the **Arduino** controller, provide the linear torque-speed characteristics and immediate impulse required to recover the pendulum from non-zero angles during launch.

3.2 Sensing: Latency Minimization

Control of an inverted pendulum is highly sensitive to sensor phase lag. A **Rotary Encoder** is used for direct pivot measurement; unlike IMUs, which require filtering algorithms that introduce delay and drift, encoders provide immediate, absolute feedback. Similarly, **Wheel Encoders** were chosen over ultrasonic sensors to avoid acoustic noise and discrete sampling delays, ensuring a smooth velocity signal for the control loop.

3.3 Mechanics: Reducing Non-Linearities

Ball Bearings are used at the pivot to minimize static friction (stiction), allowing the controller to manage micro-movements without complex dead-zone compensation. The chassis is constructed from **Laser-cut Wood** for its high stiffness-to-weight ratio, avoiding the flex common in 3D-printed parts. Finally, a **4-wheel** base passively constrains the roll axis, simplifying the control problem to pitch and translation only.

4 Project Mind Map

To organize the development effort for the Pre-Phase A assessment, the system architecture was decomposed into four interdependent work packages: **Mechanical Design**, **Electronics & Hardware**, **Software & Control**, and **Verification & Testing**.

The mechanical branch dictates the physical constraints, specifying a 60-100cm pendulum with a 50g tip mass to maximize moment of inertia, coupled with a passive V-block mechanism to facilitate the specific launch requirements. This physical layer supports the electronics selection, which prioritizes the integration of DC motors and high-resolution encoders with an Arduino-based controller to ensure low-latency actuation. The software architecture is designed around a discrete state machine transitioning between Idle, Launch, Balance, and Sprint modes supported by simulation in MATLAB/Python and advanced control strategies such as LQR and Kalman filtering. Finally, the verification branch aligns these technical efforts directly with the mission success criteria, establishing testing protocols for Disturbance Rejection (Eval A), Deep Fall Recovery (Eval B), and the Sprint-and-Stop manoeuvrer (Eval C).

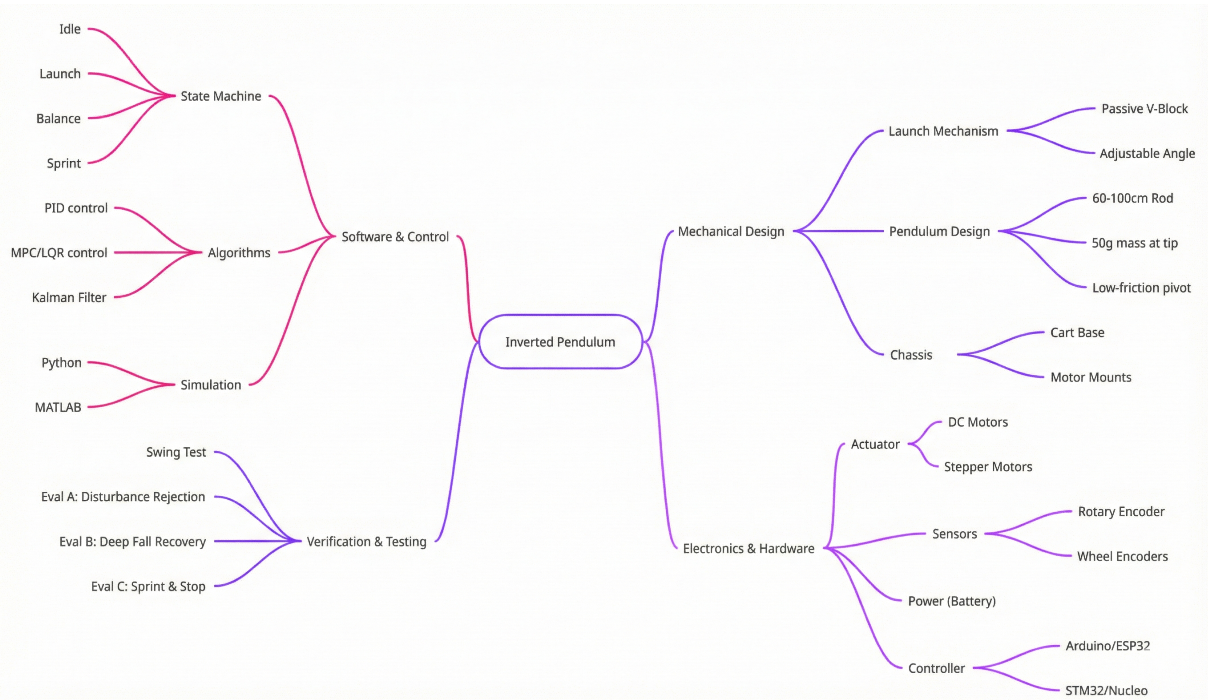
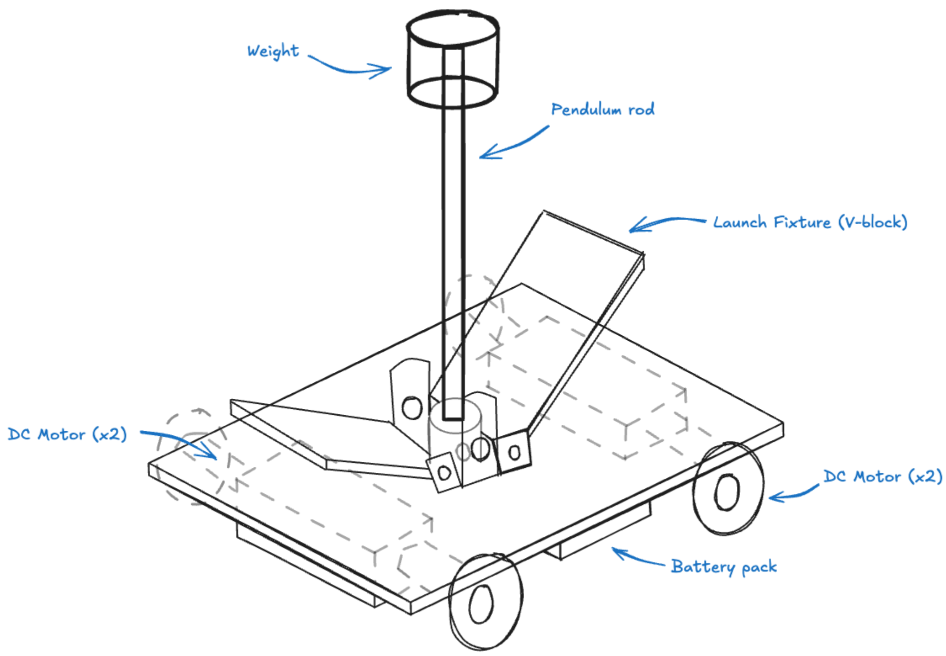
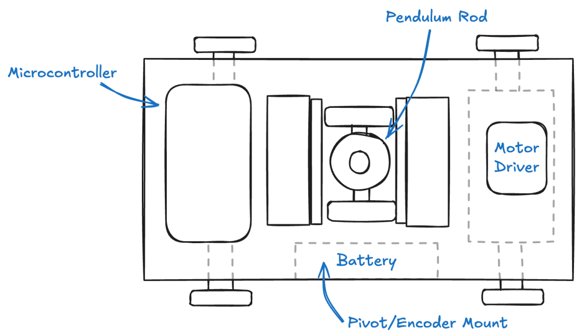


Figure 1: Project Mind Map outlining the Work Breakdown Structure (WBS)

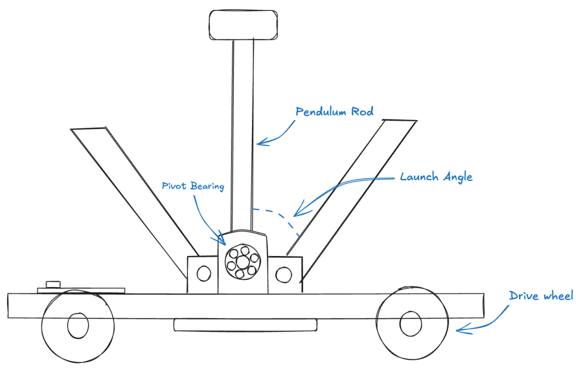
5 Initial Sketches



(a) Main Sketch View



(b) Top View



(c) Side View

Figure 2: Prototype Initial Design

6 System Requirements Specification

The system requirements are derived from the project mission to ensure compliance with the "Sprint" strategy. These are categorized into Mechanical, Electrical, and Software domains.

6.1 Functional Requirements

ID	Requirement Description	Priority
REQ-M-01	The cart must house a pendulum rod of length 60-100cm with a 50g tip mass.	High
REQ-M-02	The pivot mechanism must introduce minimal friction ($\zeta < 0.01$).	Critical
REQ-E-01	The system must be self-contained and battery-powered.	High
REQ-S-01	The controller must stabilize the pendulum from a non-zero start angle ($\theta \geq 15^\circ$).	Critical
REQ-S-02	The system must transition autonomously between Idle, Launch, Balance, and Sprint states.	High

6.2 Performance Requirements

- **Sprint Speed:** The cart must achieve a peak velocity of at least 1.0 m/s to complete the 2m sprint within competitive limits.
- **Stopping Accuracy:** The cart must stop within ±10cm of the finish line while maintaining balance.
- **Recovery Angle:** The system must recover from an initial tilt of 15° without the cart traveling more than 0.5m.

7 Project Management Plan

To manage the "Sprint Strategy" effectively, the project is broken down into specific work packages with defined ownership.

7.1 The "Iron Triangle"

Vertex	**Project Application**
Scope	Stabilise a pendulum rod (60-100cm) and pass Evals A, B, and C
Time	11-week NASA life-cycle with a final demo in Week 10
Cost	Fixed lab components, 4-person team labor, and laser-cut materials.

7.2 A-B-M Time Estimates

Table 4: Activity Duration Estimates with Calculated Expected Time (Hours)

ID	**Work Package / Task**	**A**	**M**	**B**	T_E
1.0 Project Management					
1.1.1	WBS, Network & Gantt Charts	2.0	4.0	8.0	4.33
1.1.2	Risk Matrix & FMEA Analysis	1.5	3.0	6.0	3.33
2.0 Modeling & Simulation					
2.1.1	Derive Equations of Motion	2.0	5.0	12.0	5.67
2.1.2	Python Simulation Environment	4.0	10.0	20.0	10.67
2.1.3	Sensor Noise & Filtering	2.0	4.0	10.0	4.67
2.1.4	PID & LQR Controller Design	3.0	6.0	15.0	7.00
3.0 Mechanical Design					
3.1.1	Cart Chassis CAD (Plywood)	3.0	6.0	12.0	6.50
3.1.2	Pivot & Launch Fixture CAD	2.0	5.0	10.0	5.33

Note: Expected Time calculated as $T_E = \frac{A+4M+B}{6}$.

7.3 Work Breakdown Structure (WBS)

The project is divided into five main phases aligning with the course lifecycle gates.

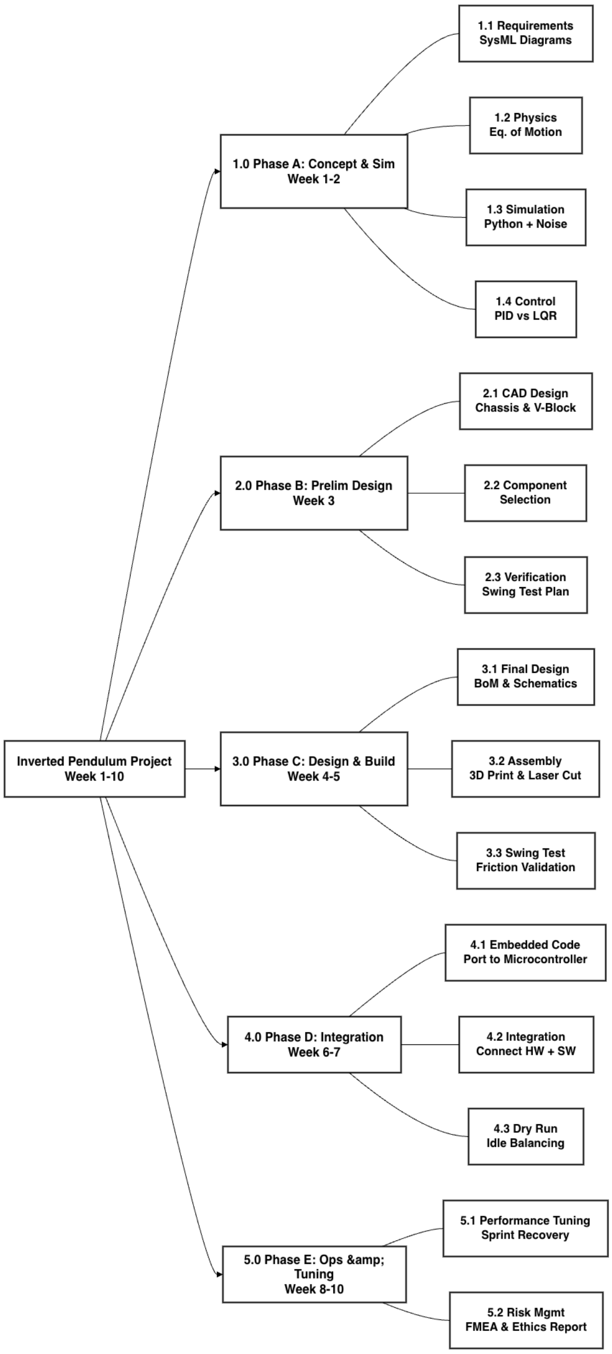


Figure 3: Work Breakdown Structure (WBS) showing hierarchical task decomposition.

7.4 RACI Matrix

The Roles and Responsibilities are assigned to ensure accountability for critical deliverables.

Table 5: Detailed Project RACI Matrix (Weeks 1–11)

Task / Work Package	Jes.	Har.	Ash.	Meh.	Roh.	Inst.	Lab
1.0 Project Management & Planning							
WBS, Network & Gantt Charts	**A**	I	**R**	I	I	I	I
A-B-M Estimates & Critical Path	**R**	C	**A**	C	C	I	I
Risk Matrix & FMEA Analysis	I	I	C	**A**	**R**	I	I
2.0 Modeling & Simulation							
Derive Equations of Motion (EoM)	C	**R**	C	C	**A**	I	I
Python/C++ Sim Environment	C	**R**	**A**	**R**	I	I	I
Sensor Noise & Kalman Filtering	I	**A**	**R**	**R**	I	I	I
PID & LQR Controller Design	**R**	C	**A**	**R**	I	I	I
3.0 Mechanical Design & Prototyping							
Cart Chassis CAD (Plywood)	**R**	**A**	C	C	**R**	C	I
Pivot Hub & Bearing Supports	**R**	I	**A**	C	**R**	C	I
V-Block Launch Fixture Design	**R**	I	**A**	C	**R**	C	I
Supply of Rod and Weights	I	I	I	I	I	I	**A/R**
4.0 Hardware-Software Integration							
Friction Verification (Swing Test)	C	**R**	C	**R**	**A**	C	C
Arduino/STM32 Driver Integration	**A**	C	**R**	**R**	**R**	I	I
Full System Integration (ORR)	C	**R**	**R**	C	**A**	C	C
5.0 Final Evaluation (Week 10)							
Eval A: Disturbance Rejection	**R**	**R**	**R**	**A**	**R**	I	I
Eval B: Deep Fall Recovery	**R**	**R**	**R**	**A**	**R**	I	I
Eval C: Sprint & Stop Race	**R**	**R**	**R**	**A**	**R**	I	I
Final Report & Ethical Analysis	**R**	**A**	**R**	**R**	**R**	I	I

7.5 Network Diagram

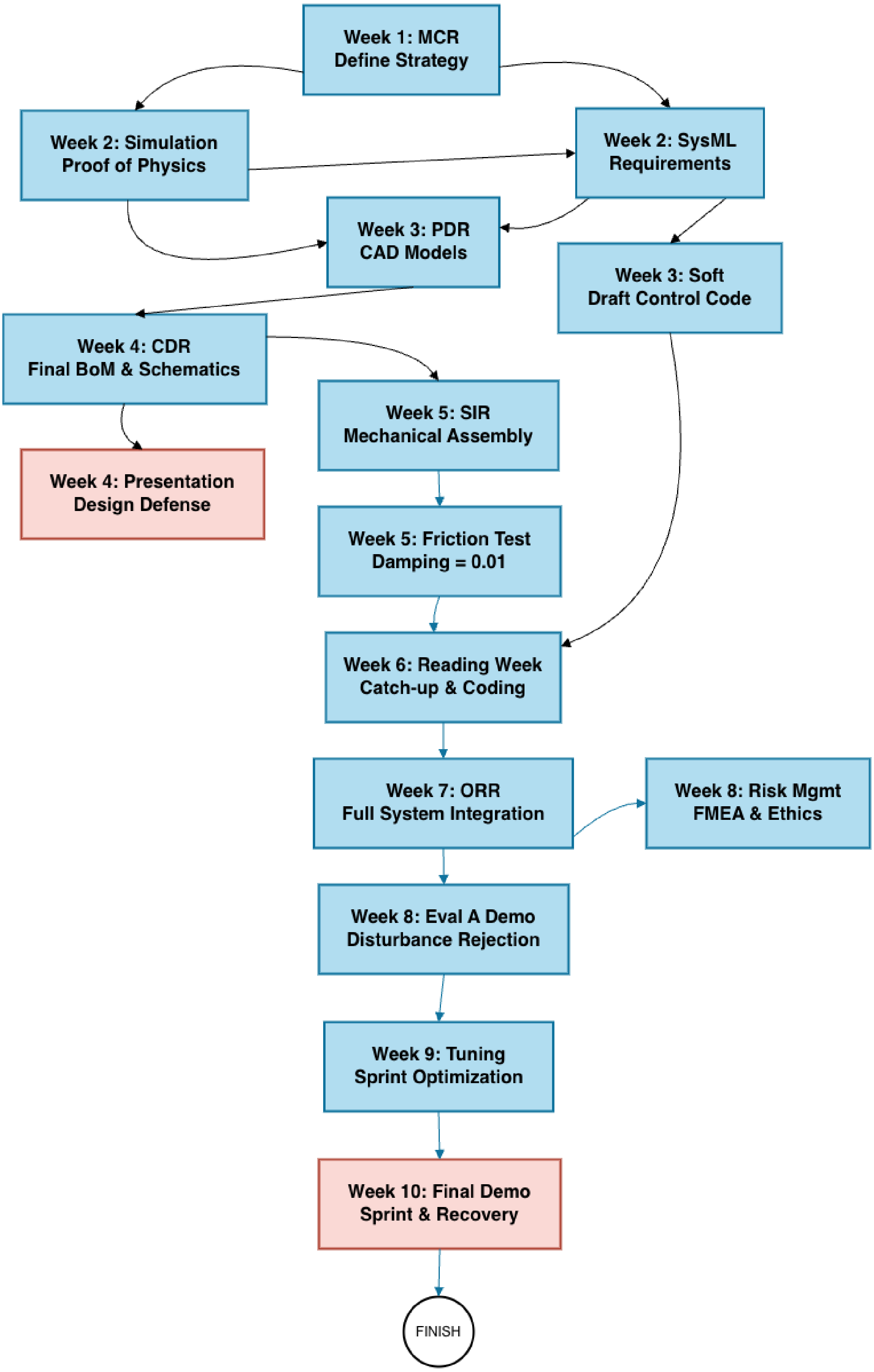


Figure 4: Network Diagram

7.6 Gantt Chart

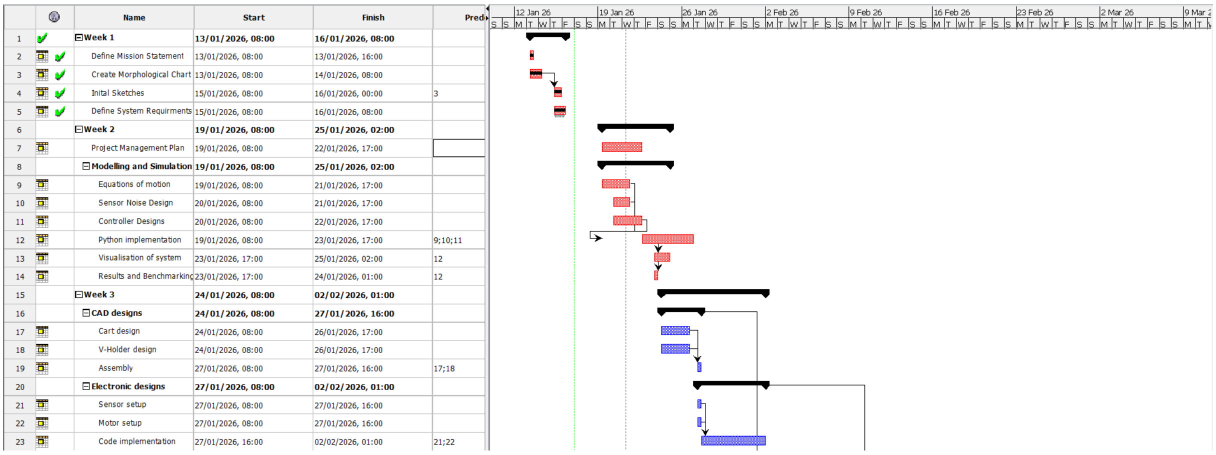


Figure 5: Gantt Chart weeks 1-3

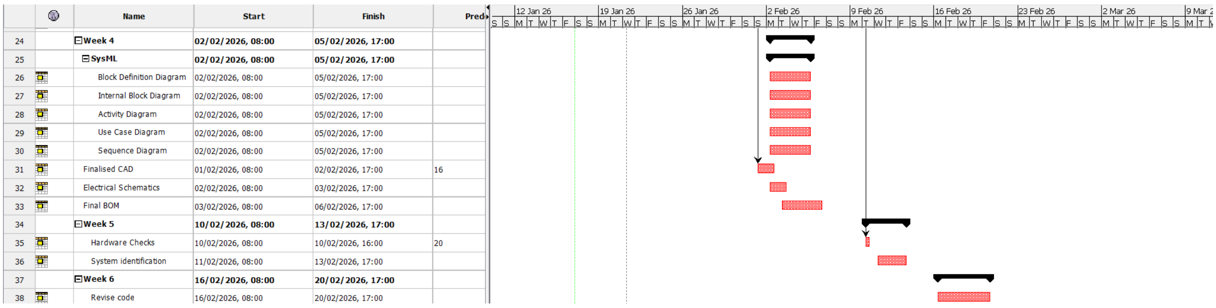


Figure 6: Gantt Chart weeks 4-6

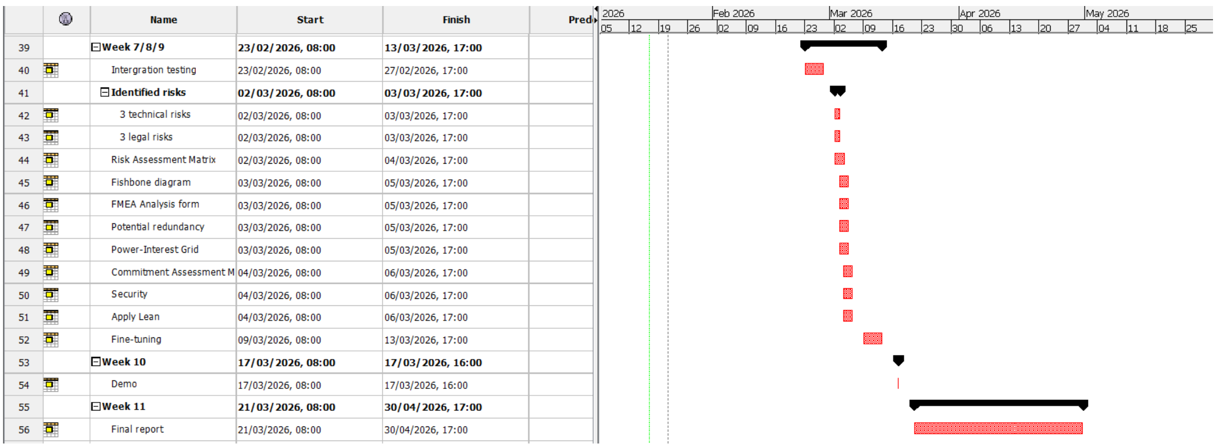


Figure 7: Gantt Chart weeks 7-11

7.7 Critical Path Analysis

Our team identified the Critical Path by calculating the earliest and latest start times for every task. We discovered that our project is 'driven' by the technical simulation and mechanical design.

Because these tasks have zero slack, they are our highest priority. Meanwhile, tasks such as 'Risk Management' have a slack of 3.2 hours, meaning we can reassign team members from those tasks to help troubleshoot the simulation if we run into 'pessimistic' delays without moving our final deadline.

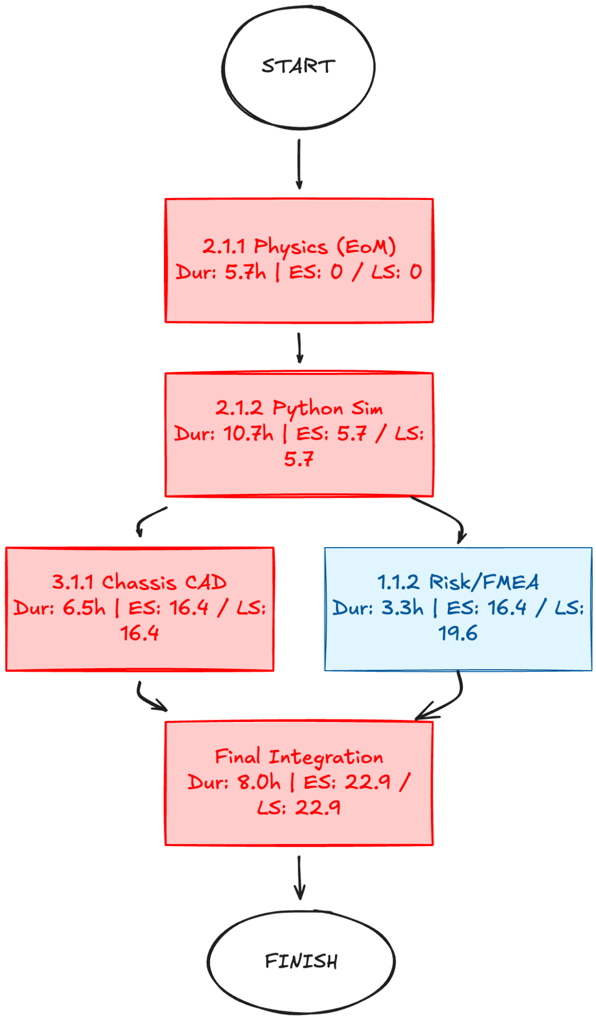


Figure 8: Highlighting the Critical Path

8 Modeling & Simulation

8.1 Equations of Motion

The non-linear dynamics are derived using the Lagrangian method:

$$(M+m)\ddot{x} + mL\ddot{\theta}\cos(\theta) - mL\dot{\theta}^2\sin(\theta) = F - b\dot{x}$$

$$mL^2\ddot{\theta} + mL\ddot{x}\cos(\theta) - mgL\sin(\theta) = -c\dot{\theta}$$

Table 6: System Notation

Symbol	Description	Units
M, m	Cart / pendulum mass	kg
L	Rod length	m
b, c	Cart friction / rotational damping	N·s/m, N·m·s/rad
$x, \dot{x}$	Cart position / velocity	m, m/s
$\theta, \dot{\theta}$	Pendulum angle / angular velocity	rad, rad/s
F	Control force	N

For control design, the system is linearised about the upright equilibrium with state $\mathbf{x} = [x, \dot{x}, \theta, \dot{\theta}]^T$:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \quad \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{b}{M+m} & -\frac{mg}{M+m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{b}{L(M+m)} & \frac{(M+m)g}{L(M+m)} & -\frac{c}{mL^2} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{M+m} \\ 0 \\ -\frac{1}{L(M+m)} \end{bmatrix} \tag{1}$$

Simulation parameters: $M = 1.0$ kg, $m = 0.05$ kg, $L = 0.8$ m, $b = 0.1$ N·s/m, $c = 0.01$ N·m·s/rad.

8.2 Motor Model

Four DC motors drive the cart (3-9 V range, 90 RPM at 4.5 V, wheel radius 0.03 m). The model accounts for back-EMF ($V_{\text{back}} = K_e\omega$), voltage saturation, and current-torque relationship ($\tau = K_t I$). Maximum force at stall: ≈ 6 N.

8.3 Noise Filtering

Gaussian noise ($\sigma_x = 0.005$ m, $\sigma_\theta = 0.01$ rad) simulates sensor conditions. A two-stage filter estimates true state:

Stage 1 – Low-pass filter:

$$y[k] = \alpha y[k-1] + (1-\alpha)x[k], \quad \alpha = \frac{\tau}{\tau + T_s} \tag{2}$$

Stage 2 – Dirty derivative:

$$\dot{y}[k] = \frac{y[k] - y[k-1]}{T_s} \tag{3}$$

Parameters: $\tau_x = 0.1$ s, $\tau_\theta = 0.08$ s, $T_s = 0.02$ s.

9 Control Strategies

All controllers stabilise the pendulum while moving the cart to $x = 2$ m, constrained by motor limits (≈ 6 N max).

9.1 Cascaded PID

Outer loop (position $\rightarrow$ angle setpoint):

$$\theta_{\text{set}} = K_{p,x}(x_{\text{target}} - x) + K_{i,x}\int e_x \, d\tau - K_{d,x}\dot{x}, \quad |\theta_{\text{set}}| \leq 0.12 \text{ rad} \tag{4}$$

Inner loop (angle $\rightarrow$ force):

$$u = K_{p,\theta}(\theta - \theta_{\text{set}}) + K_{i,\theta}\int e_\theta \, d\tau + K_{d,\theta}\dot{\theta} \tag{5}$$

Gains: $K_{p,x} = 1.0$, $K_{i,x} = 0.02$, $K_{d,x} = 2.5$ (position); $K_{p,\theta} = 35$, $K_{i,\theta} = 0.5$, $K_{d,\theta} = 12$.

9.2 LQR

Minimises $J = \int_0^\infty (\mathbf{e}^T \mathbf{Q} \mathbf{e} + u^2 R)\, dt$ where $\mathbf{e} = \mathbf{x} - \mathbf{x}_{\text{set}}$ and $\mathbf{x}_{\text{set}} = [2, 0, 0, 0]^T$.

$$u = -\mathbf{K}(\mathbf{x} - \mathbf{x}_{\text{set}}), \quad \mathbf{Q} = \text{diag}(8, 3, 50, 5), \; R = 0.3 \tag{6}$$

$\mathbf{K}$ obtained via CARE. Higher Q_{33} prioritises angle stabilisation; R limits control effort.

9.3 Pole Placement

$$u = -\mathbf{K}(\mathbf{x} - \mathbf{x}_{\text{set}}), \quad \text{eig}(\mathbf{A} - \mathbf{BK}) = \{-2, -2.5, -3, -3.5\} \tag{7}$$

Real negative poles ensure stable, non-oscillatory response within motor limits.

10 Simulation Results

System simulated for 10 s from $\theta_0 = 10$, target $x = 2$ m.

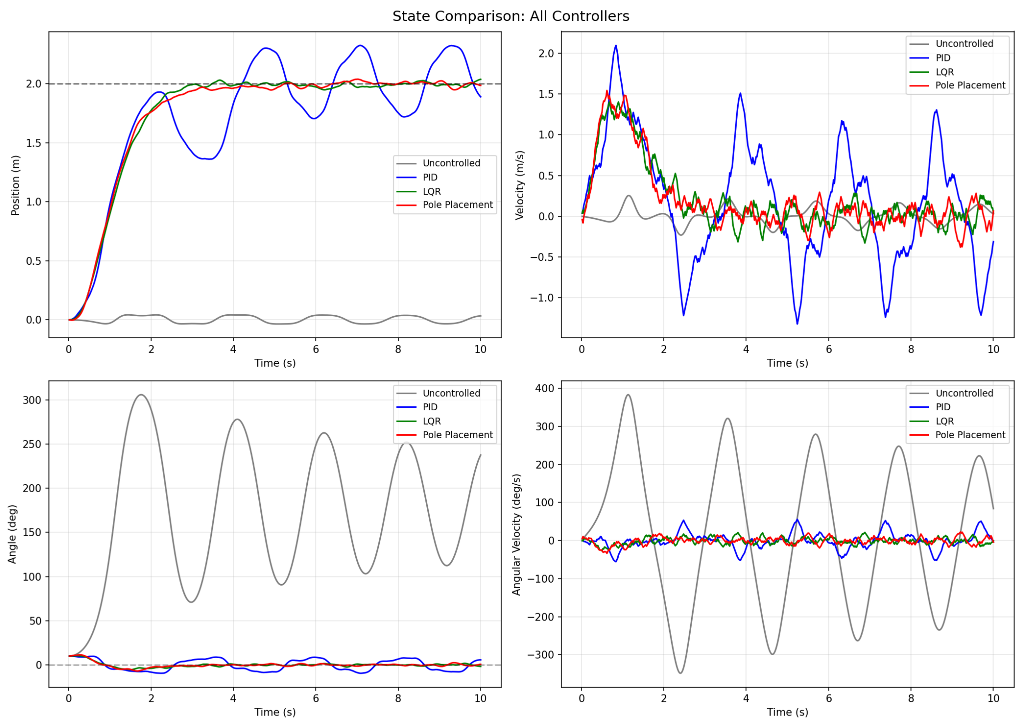


Figure 9: State comparison: position (top-left), velocity (top-right), angle (bottom-left), angular velocity (bottom-right). Dashed line = 2 m target.

11 Controller Comparison

Table 7: Performance Comparison

Metric	**PID**	**LQR**	**Pole Placement**
Position settling (5% band)	Not settled	3.0 s	3.0 s
Position overshoot	0.3 m (15%)	0.05 m (2.5%)	0.05 m (2.5%)
Steady-state oscillation	± 0.5 m	$< \pm 0.1$ m	$< \pm 0.1$ m
Max angle deviation	± 10	± 5	± 5
Peak desired force	13 N	5 N	5 N
Max initial angle recovery	28	23	23
Max disturbance impulse	56 N	57 N	55 N

Key findings:

- **LQR/Pole Placement**: Settle at 2 m in $\sim$3 s with minimal overshoot; tighter angle regulation (± 5); control effort stays within motor limits.
- **PID**: Better large-angle recovery (28 vs 23) but sustained oscillations due to cascaded phase lag interacting with motor saturation.

Table 8: Summary

Criterion	**PID**	**LQR**	**PP**	**Best**
Position settling	Poor	Excellent	Excellent	LQR/PP
Energy efficiency	Poor	Excellent	Very Good	LQR
Large-angle recovery	Excellent	Good	Good	PID
Saturation robustness	Poor	Good	Good	LQR/PP

Chosen Controller: LQR - optimal performance/effort trade-off, systematic tuning via $\mathbf{Q}/R$, superior steady-state behaviour. PID not used due to limit cycle behaviour under actuator saturation.

12 References

References

[1] UCL Computer Science, *COMP0216 Project Manual: Inverted Pendulum*, 2026.

[2] K. Ogata, *Modern Control Engineering*, 5th ed. Prentice Hall, 2010.