Notes for Westlake Physics Presentation

Harrison Bachrach

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Abstract

Presentation notes for a lecture given by Harrison Bachrach at Westlake High School.

1 Introduction

1.1 A little about myself

My name is Harrison "Harry" Bachrach and I'm currently a Engineering Physics student at UC Berkeley. I graduated from Westlake High School in 2013, having taken both Physics C: Mechanics and Physics B (the old version of Physics 1 & Physics 2) with Mr. Holloway. I plan on pursuing graduate education in physics, and am specifically interested in Computational Physics.

If you have any questions, feel free to contact me at `hib@berkeley.edu` or on Facebook.

If you're interested in how I got this document to look so nice, you should know that it was typeset with LaTeX, which is the standard typesetting system for academics. Try it out at `http://overleaf.com`.

1.2 Notation

- Quantities denoted by boldface, e.g. $\mathbf{x}(t), \mathbf{p}$, etc., are *vectors*.
- The time average of a function/quantity is denoted with angled brackets, e.g. $\langle \sin \omega t \rangle = 0, \langle \sin^2 \omega t \rangle = \frac{1}{2}$

2 Physics Topics

2.1 Why do waves have momentum?

This is a difficult question to answer. I'm glad that you all are asking questions like this, though. You've been presented a definition of momentum in your classes, that is $\mathbf{p} = m\mathbf{v}$. This does not accurately convey momentum in all its forms which are relevant to the study of physical phenomena. Before we get into this question, we must discuss what momentum *is* in the first place. As I've just stated, the mathematical description we have thus far is not really complete, and breaks down as soon as we leave the world of classical mechanics (and sometimes even within it, depending on your definition of "classical mechanics").

2.1.1 What is momentum?

Before we answer the question for which this section is titled, let's talk about what we're doing when we talk about physics in general. By far, the most important concept in all of physics is that of *symmetry*. It is symmetry that establishes an overwhelming amount of the logic of physics. The directions of electric field, the distribution of speeds in a gas... all of these are explained by arguments justified by the logic of symmetry.

It is for this reason that momentum is of great importance in physics. Momentum, above all else, is a *conserved quantity* (when certain conditions are met). As it turns out, there is a theorem, a theorem which in my opinion is the most important in all of physics, by one Emmy Noether. Noether's theorem states that for every symmetry in a physical system, there is an associated conserved quantity. If one follows the proof of Noether's theorm, one learns that the *conservation of momentum* arises when a system has *translational symmetry*, that is, picking up the system and moving it 10 feet over doesn't change anything!. Similarly, a system's *energy* is conserved when said system has *symmetry in time*, that is, when a system doesn't change just because time passes (assuming we hold everything else still!).

But still, the question remains: what is momentum? Sure, now we know what it's associated with, but what *is* it? Well, this might disappoint you, but what I've told you *is* that very definition. Momentum has many beautiful properties, but when it comes down to it, *momentum is the vector quantity that is conserved when a system is traslationally symmetric or translationally "invariant".*

2.1.2 What's wrong with $m\mathbf{v}$?

As I stated earlier, our classical description of momentum is just not really correct in general. The conservation of $\mathbf{P} = \sum_i m_i \mathbf{p}_i$ can be derived from Newton's Laws of Motion. But what about when Newton's Laws fail?

Let's start by moving over to electrodynamics. Imagine we're in space and we have two electrons with mass m and speeds v. One is traveling from $x = -\infty$ along the x-axis and another, from $z = -\infty$ along the z-axis; both are moving towards the origin (see Figure 1 on page 3 for an illustration). While the charge along the x-axis generates a $\mathbf{B}$-field where the other charge is, the reverse is not true: only one of the charges experiences a Lorentz force $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$. Thus, an "equal and opposite force" is not present! This means that we have some momentum in the $+x$ direction that is unaccounted for. Thus the net "kinetic" momentum of the system (our old-fashioned $\mathbf{P} = \sum_i m_i \mathbf{p}_i$) is *not* conserved! But from our description of momentum, that it depends on translational symmetry, we know that momentum *must* be conserved, as our

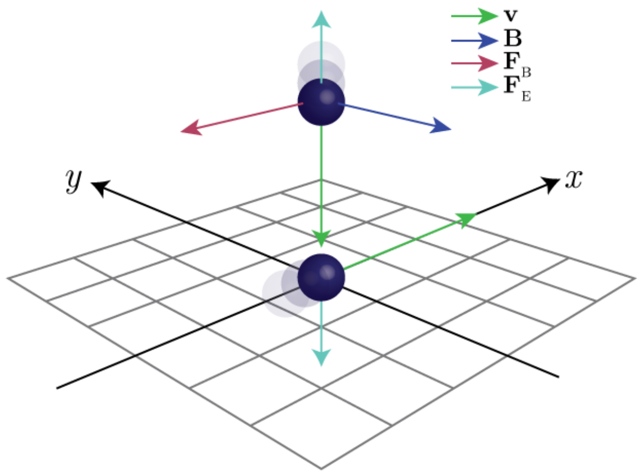


Figure 1: Two charges traveling at (initially) constant velocity towards the origin.

system isn't grounded to any specific location, i.e. the charges could be moving like this anywhere! Thus, there must be some momentum that we aren't keeping track of... but what else is there? As there are no static fields present[1], we turn our attention to waves.

As the charges here accelerate, they generate electromagnetic waves[2], commonly known as light. As the waves are the only remaining objects in this system, they must have the remaining momentum! But wait, how can a wave, which has no mass, have momentum? If we go through a vector calculus analysis of the total energy in a system, we discover this quantity $\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$, known as the *Poynting Vector*. As it turns out, this quantity is incredibly important for studying electromagnetic waves. Strictly speaking, $\mathbf{S}$ represents the power flux density for a wave[3]. Now what the hell is the "power flux density"? It is the essentially a measure of the rate at which energy is flowing through space via an electromagnetic wave. However, as it turns out, this energy density must exert a pressure when it hits another object of magnitude $\langle \mathbf{S} \rangle / c$. Playing with this via thought experiments, we find that this requires that electromagnetic waves must have a momentum density $\mathbf{g} = \langle \mathbf{S} \rangle / c^2$.

[1]Static fields, as it turns out, can also have momentum(!), but special relativistic factors must also be considered. See `http://www.ate.uni-duisburg-essen.de/data/postgraduate_lecture/AJP_2009_Griffiths.pdf`

[2]See `http://commons.wikimedia.org/wiki/File:Electromagneticwave3Dfromside.gif` for an elucidating animation of an electromagnetic or "EM" wave.

[3]Additionally, the direction of $\mathbf{S}$ is the direction of propagation for the wave.

2.1.3 Matter Waves

TO BE COMPLETED

2.2 Why do these muons keep hitting me?

Note: This a common introductory physics example; the bulk of this section is rather sloppily copied from several sources, including Sixty Symbols [4]

Everyone on Earth is being constantly bombarded with tiny (and I mean *tiny*) particles every second known as *muons*. Often symbolized as the Greek letter μ, these elementary particles are very similar to electrons, with charge -1 and spin $\frac{1}{2}$. Unlike the electron, however, the muon is unstable, having a half-life of only $2.19 \cdot 10^{-6}$ seconds.[5].

TO BE COMPLETED

3 College Topics

3.1 Your job at university

When you go to university, you have three responsibilities as a student. I list them here in decreasing order of importance:

1. Take care of your mental and physical health
2. Improve yourself as a person
3. Prepare yourself for the future

These are all important. One should not neglect *any* of the items listed here. But let me make myself clear: they are in this order for a reason. #1 is first because it is what enables you to do #2, which in turn enables you to do #3.

I state these pieces of advice from a position of luck. I have not had to struggle to achieve my access to higher education (or K-12, for that matter). However, it is very possible that you have had similar luck in your lifetime. Regardless, I do believe in the validity of this prioritization of values, as it seems to be sustainable and ultimately more fulfilling than to recklessly focus on one pursuit entirely.

3.2 How to study effectively

At my college and at many others, what I commonly see is a misunderstanding of education. There are two main misconceptions.

[4] A fantastic YouTube channel featuring physicists from the University of Nottingham talking about interesting phenomena in a approachable fashion. I highly recommend watching it, along with Numberphile, its mathematical analogue.

[5] http://cosmic.lbl.gov/SKliewer/Cosmic_Rays/Muons.htm

First are the apathetics. They see their coursework simply as "work"—not as preparation. As a result, people often copy homework solutions, sleep through lecture, skip readings... These students miss the entire point of college. You do not attend university to show how easy the material is for you. If the courses are too easy, you can always find additional challenge, I assure you. To not challenge yourself when you are able to do so is a disservice not only to yourself, but to those who have to fight their way to such an opportunity. I apologize for this unwarranted accusative tone (shifting to the 2nd person was a bit unnecessary) but there are few things in life that upset me more than squandered education.

Second are those who follow the "cannonball approach" to their courses. While these students appreciate the opportunity that a university education affords them, they often see it in the transactional sense. They see courses as walls to power through, describing exams as a test of will. Yes, exams are difficult, but they are not why one attends school. With this approach, the "mental regurgitation" that (let's be honest) *most* high school students perform continues onto college—an incredibly dangerous behavior to continue to practice.

These two categories may seem like groups encapsulating pet peeves of mine, but I assure you, you lose something by falling into one of them.

But let's return to the titular question. Effective studying, when it comes down to it, is a piecemeal process. Your goal is to avoid staying up all night, cramming, and panicking. These activities are very inefficient at long-term informational absorption, and disrupt your other efforts at personal health and personal betterment.

To avoid these unhelpful habits, *consistency* is the name of the game. If you don't understand a concept, it most likely won't suddenly click when studying immediately before finals. Additionally, it's very likely that concept is one that later topics build on top of further along in the course. So this requires being proactive about your understanding. It means asking questions, doing optional homework *when it is assigned*, going to office hours with professors or TAs, etc. It means *not procrastinating*. If you need to create a schedule, do so—it can be very helpful.

This may seem like basic advice, but many don't follow these practices. I mean, I certainly don't *all* the time. We are human! We make mistakes. Don't beat yourself about them—just think about what steps you need to take to not fall into such habits again.

3.3 Living independently

Living on your own for the first time usually yields some unexpected surprises. As exciting as it may be to be independent, there are a lot of small tasks that you may not have really "done" growing up that now fall to you. Doing laundry, ironing your "professional clothes" for career events, picking up cold medicine when you're sick, picking up your room... all of these things take time. If you're living off campus and/or don't have a meal plan, this is often compounded with making meals, getting groceries, cleaning dishes, cleaning your place, paying rent, paying taxes, and a whole bunch of other miscellanea. All these are in addition to your schooling, and if you work, your job.

It can be overwhelming sometimes. The main thing to remember is that most everyone is in the same position as you when you start college, and that many people before you have adjusted, and so will you.

3.4 The difference between high school and college technical courses

Note: Technical courses $\iff$ STEM courses (Science, Technology, Engineering, Mathematics)

High school is not easy. You guys take 6 or 7 classes *every day*, with (often) multiple extracurriculars on top... in your forth year, you get to apply to college and college scholarships additionally... it's not easy.

College is a different sort of challenge. You typically take ~ 4 courses per semester/quarter, which typically only meet for 3-6 hours a week, each. If the course is a lab course, you might have to tack on 3-5 hours for the lab session. Regardless, you have *much* more free time at university. However, this is time that you have to schedule, which is useful, because the things on your plate (especially the simple day-to-day things mentioned above) will take up more time than you expect.

In mathematics and physics, as you get further up, homework will be supplanted with *problem sets*, which instead of 10-20 problems to work through will have 4-6 problems, due on a weekly basis. These are different sorts of problems. These are tests of critical thinking. I've asked professors and graduate students of words of wisdom for high school students, and the response was surprisingly consistent: Math and Physics are not about formulas—they are about ingenuity, creative thinking, and learning how to utilize multiple perspectives to solve problems. There is an immense beauty to this material which I hope you gain at least *some* exposure to at college: I personally study physics because it is, in my opinion, the most beautiful "thing" I have ever seen, worked with, or experienced.

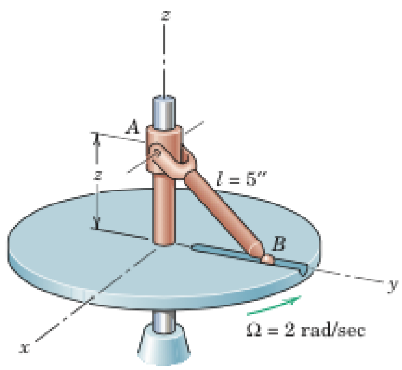
3.5 Physics vs. Engineering

In my experience, there is a fair bit of overlap between engineers and physicists. If you major in engineering, chances are many of your introductory courses will be the same as those of incoming physics students. That being said, there is a distinction which I would like to illuminate here.

In physics courses, we are interested in fairly *general* phenomena. It is quite rare for me to have answers to my problems in terms of pure numbers. In figures 2 and 3 on page 8, I've included two example problems from engineering and physics textbooks examples, respectively. These two problems, while both about similar material, ask for very different quantities and have *very* different degrees of abstraction. While the mechanics problem gives a common system in an engineering environment, with explicitly defined lengths and relatively complex and uniquely defined geometry, the physics problem deals with *any* axially symmetric body, asking for proofs and derivations involving fairly abstract quantities, requiring the switching of frames to arrive at a solution. These two problems are both topics of classical mechanics—an area where (in my opinion) this overlap between engineering and physics is greatest. As one ventures into areas like circuitry (if you're an engineer) or relativity (if you're a physicist) this disparity only grows. Engineering education is motivated by crafting and understanding useful systems or designing them so that they function as desired, while physics is more geared towards describing or deriving phenomena based upon first principles.

In terms of study, while getting a job with a Bachelor's in engineering is relatively common, to get a job *related to physics*, a Ph.D. in physics is generally necessary, though not always. That being said, many obtain employment with only a Bachelor's in physics outside of the field of physics.

7/42 The collar and clevis A are given a constant upward velocity of 8 in./sec for an interval of motion and cause the ball end of the bar to slide in the radial slot in the rotating disk. Determine the angular acceleration of the bar when the bar passes the position for which $z = 3$ in. The disk turns at the constant rate of 2 rad/sec.



Problem 7/42

Figure 2: A problem out of the engineering-focused *Dynamics* by Meriam and Kraige.

10.46 ★★★ We saw in Section 10.8 that in the free precession of an axially symmetric body the three vectors $\mathbf{e}_3$ (the body axis), $\boldsymbol{\omega}$, and $\mathbf{L}$ lie in a plane. As seen in the body frame, $\mathbf{e}_3$ is fixed, and $\boldsymbol{\omega}$ and $\mathbf{L}$ precess around $\mathbf{e}_3$ with angular velocity $\Omega_b = \omega_3(\lambda_1 - \lambda_3)/\lambda_1$. As seen in the space frame $\mathbf{L}$ is fixed and $\boldsymbol{\omega}$ and $\mathbf{e}_3$ precess around $\mathbf{L}$ with angular velocity Ω_s. In this problem you will find three equivalent expressions for Ω_s. **(a)** Argue that $\boldsymbol{\Omega}_s = \boldsymbol{\Omega}_b + \boldsymbol{\omega}$. [Remember that relative angular velocities add like vectors.] **(b)** Bearing in mind that $\boldsymbol{\Omega}_b$ is parallel to $\mathbf{e}_3$ prove that $\Omega_s = \omega \sin\alpha / \sin\theta$ where α is the angle between $\mathbf{e}_3$ and $\boldsymbol{\omega}$ and θ is that between $\mathbf{e}_3$ and $\mathbf{L}$ (see Figure 10.9). **(c)** Thence prove that

$$\Omega_s = \omega \frac{\sin\alpha}{\sin\theta} = \frac{L}{\lambda_1} = \omega \frac{\sqrt{\lambda_3^2 + (\lambda_1^2 - \lambda_3^2)\sin^2\alpha}}{\lambda_1}. \qquad (10.118)$$

Figure 3: A problem out of the physics-focused *Classical Mechanics* by Taylor.