ETUDE 10: CUBES

For this etude, we tried some systematic, by-hand approach, to determine the total number of unique cubes. To do this, we would determine the total number of cubes for a given number of "sub-cubes", starting with 0 blue / 8 yellow and working up to 4 blue / 4 yellow incrementally. For each number, we would start with a single blue cube located in the top left corner. We would then add one blue sub-cube at a time (up until our desired number), determining where we could place this whilst ensuring our solutions were unique. For 4 blue sub-cubes for example, we looked at all of the possibilities with four subcubes on the same face, then three sub-cubes on the same face, then two, etc. The catalogue of cubes is given on the final page.

0 and 1 Blue:

There is only one cube that can be made from all yellow cubes. Similarly for 1 blue sub-cube, any sub-cube that we pick to be blue can be rotated in such a way that this sub-cube is located in the top left corner as illustrated in the table.

2 Blue:

For a cube consisting of 2 blue sub-cubes, rotate the cube so that one of these sub-cubes is in the top left corner when front-facing. The second blue subcube can be located in one of three positions:

1. If the second cube is on the same face and directly adjacent, the cube can be rotated to give the first entry.
2. If the second cube is on the same face but diagonally adjacent, the cube can be rotated to give the second entry.
3. If the second cube is not on the same face, it must be located in the opposite "back" corner of the cube, so the cube can be rotated to give the third entry.

3 Blue:

For a cube consisting of 3 blue sub-cubes, there will be two scenarios. Either (at least) two of the blue sub-cubes are directly adjacent to each other, or all blue sub-cubes are diagonally adjacent to each other.

1. With two directly adjacent blue sub-cubes, if the third blue sub-cube is located on the same face as the other two, then it must be a rotation of the first entry. (all three on the same face).
2. With two directly adjacent blue sub-cubes, if the third sub-cube is not on the same face, it must be a rotation of the second entry.
3. If all blue sub-cubes are diagonally adjacent to one another, it must be a rotation of the third entry.

4 Blue:

There are a few more possibilities here:

1. If all blue sub-cubes are located on the same face, it must be a rotation of the first entry.
2. With three blue sub-cubes located on the same face, rotate the cube so that these three are on the front face in an upside "L" shape (top left, top right and bottom left corners) as shown in images 2-5 in the 4 blue column. There are four possibilities for where this last cube must be:
    a. It could be adjacent to one of the three blue sub-cubes. If it is directly adjacent to the top left blue sub-cube, then it must be a rotation of the second cube.
    b. If it is adjacent to the top right or bottom left sub-cube, it must be a rotation of the third cube.
    c. If it is not directly adjacent to one of these cubes (ie, it is on the back face), it must be a rotation of the fourth or fifth cube (the fifth cube has the last blue sub-cube in the back/bottom left out of view).
3. With two blue sub-cubes located on the same face, there is only one arrangement, so must be the sixth cube.
4. With only one blue sub-cube located on each face, all blue sub-cubes must be diagonally adjacent to one another, thus it must be the seventh cube (the last sub-cube is the back/bottom left out of view).

5, 6, 7 and 8 Blue:

We arbitrarily picked 0 blue to start - we could have also chosen 0 yellow and 8 blue. Therefore, the number of cubes from 5 - 8 blue will be the same as 0 - 3 blue with colours reversed. This gives us a total of **23** unique cubes.

0 Blue 8 Yellow	**1 Blue 7 Yellow**	**2 Blue 6 Yellow**	**3 Blue 5 Yellow**	**4 Blue 4 Yellow**