MA137   Mathematical Analysis
Examples Sheet 3

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Please deposit your solutions to these questions in your supervisor's slot in the pigeon loft by 3pm on Monday, 24 October, 2016. They will be marked for credit towards the final assignment score for this module. Please do not ask your supervisor to help you with these questions.

1. For what values of x do the following inequalities hold? Whenever possible, write your solution as an interval, or union of intervals.

   (a) $2x + 1 < |x - 2|$.

   (b) $|2x + 1| < |x - 2|$.

   (c) $|(|2x + 1| - 3)| \geqslant 2$.

   (d) $3x - 4x^3 < x^5$.

   (e) $3x^3 - 4x \leqslant x^4$.

   (f) $\dfrac{1}{x} + \dfrac{1}{x + 2} > 0$.

2. Write down an expression which does not involve absolute value signs and which is equivalent to the expression $2x - |(x - |x + 1|)|$.

3. Let $(a_n)_{n \in \mathbb{N}_+}$ be a sequence of *integers* which converges to a limit ℓ. Show that $\exists\, N \in \mathbb{N}_+$ such that $a_n = \ell \; \forall\, n > N$.

4. Let $(a_n)_{n \in \mathbb{N}_+}$ be a sequence for which $a_n \geqslant 0 \; \forall\, n \in \mathbb{N}_+$. Show that if $(a_n)_{n \in \mathbb{N}_+}$ converges to ℓ, then $\ell \geqslant 0$.