ST116 Tutorial Sheet 2

Submit your solutions to the questions below in the postbox at the Student Support Office by

Friday 28th October 2016, 11:00 a.m.

In your answers you should make sure that

- all variables are clearly defined;
- you explain in words what you are attempting to show and why;
- you use mathematical notation correctly;
- the results are checked for correctness and presented neatly.

If you are unable to answer a question fully, please submit a partial solution and discuss what you did attempt to arrive at a solution and why it did not work.

Please do contribute in the session with your personal tutor. It is a great opportunity to check your understanding, practise your communication skills and get some feedback. Keep in mind that all staff will have been in your shoes at some time in the past!

Please fill out the section below and attach this cover-sheet to your submission. Late submissions or missing cover sheets lead to delays and may mean that your work will not be marked.

Student Name:

Student ID:

Personal Tutor Name:

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1. Combinatorial proof and committee membership

A 'combinatorial proof' is a proof of an identity, in which the number of ways of doing something is calculated in two different ways.

For the purposes of this question, define $\binom{n}{r}$ for integers r and n by

$$\binom{n}{r} = \begin{cases} \dfrac{n!}{r!\,(n-r)!} & \text{if } 0 \le r \le n \text{ (note that } 0! = 1\text{)}, \\ 0 & \text{otherwise.} \end{cases}$$

(a) It is required to select a committee of r members from n candidates. By considering how many of the possible committees contain, or do not contain, a particular candidate, give a combinatorial proof of 'Pascal's identity'

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}.$$

(b) Now it is required to select a committee of r members from m male and n female candidates. Give a combinatorial proof of the identity

$$\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}.$$

Hint: consider how many males are in a committee.

2. The pigeonhole principle

(a) 'The maximum of a finite set of real numbers is at least as big as their average'.

Using whatever notation you think appropriate, prove the above statement.

(b) Make a similar statement about the minimum of the set, and justify your answer.

(c) Prove that if n objects are distributed over m places, and if $n > m$, then some place receives at least two objects.

Note: This is usually referred to as the 'pigeonhole principle', as it means that if m pigeonholes contain a total of $n > m$ pigeons, then at least one pigeonhole contains at least two pigeons. This fact can be surprisingly useful, though statement (a) above is more general. See MA138 Sets and Numbers for more about the pigeonhole principle.

(d) You are given a set of ten distinct two-digit numbers, from which you can choose subsets of numbers. Prove carefully that you will be able to pick two non-empty disjoint subsets whose members have the same sum.

Hint: First calculate the largest and smallest possible such sums, then consider the pigeonhole principle.

End.