Kinematics - classification and comparison of motions

Motion → Along a straight line only

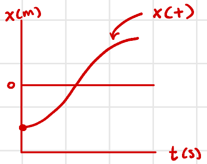
→ Forces (pushes and pulls) cause motion

→ Moving object is either a particle or an object that moves like a particle

Displacement - change from position x_1 to x_2 = $\Delta x = x_2 - x_1$

- Must always include direction (+ or -)
- Vector quantity → needs both direction and magnitude

Position can be plotted as a function of time t : x(t)



Average velocity - $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$

↳ Slope of a straight line that connects two points on x(t)

↳ Also a vector quantity → needs both magnitude and direction

Average speed - $s_{avg} = \frac{\text{total distance}}{\Delta t}$

Instantaneous velocity - $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ v is the rate at which position x is changing with time at a given instant

* Speed is the magnitude of velocity

Acceleration

- Average acceleration → $a_{avg} = \frac{\Delta v}{\Delta t}$
- Instantaneous acceleration → $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

↳ Also a vector quantity → needs both magnitude and direction

→ Large accelerations are sometimes expressed in terms of g units → $1g = 9.8 m/s^2$ (g unit)

Constant acceleration:

$a = a_{avg} = \frac{v - v_0}{t - t_0}$ ⟶ $v = v_0 + at$

$v_{avg} = \frac{x - x_0}{t - 0}$ ⟶ $x = x_0 + v_{avg} t$

$v_{avg} = \frac{1}{2}(v_0 + v)$ ⟶ $v_{avg} = v_0 + \frac{1}{2}at$

⟶ $x - x_0 = v_0 t + \frac{1}{2}at^2$

Equations → Missing Quantity

$v = v_0 + at$ → $x - x_0$

$x - x_0 = v_0 t + \frac{1}{2}at^2$ → v

$v^2 = v_0^2 + 2a(x - x_0)$ → t

$x - x_0 = \frac{1}{2}(v_0 + v)t$ → a

$x - x_0 = vt - \frac{1}{2}at^2$ → v_0

Example problem: car and motorcycle race

Motorcycle: $a_m = 8.40\ m/s^2$, $v_m = 58.8\ m/s$

Car: $a_c = 5.60\ m/s^2$, $v_c = 106\ m/s$

How long does the car take to reach the motorcycle?

1) Write what we're looking for in a mathematical expression

↳ $X_c = X_{m_1} + X_{m_2}$ "At some time t, X_c and $X_{m_1} + X_{m_2}$ will be the same

2) Fill out both sides of the equation

$x - x_0 = v_0 t + \frac{1}{2}at^2$, $x_0 = 0, v_0 = 0$ → $x_c = \frac{1}{2}a_c t^2$

To find x_{m_1} we must find t that it took to reach maximum velocity

$v_m = v_{0m} + at$ → $58.8 = 8.4(t)$ → $t = 7.00\ s$ ← t_{m_1}

$X_{m_1} = v_0 t + \frac{1}{2}at^2$ → $X_{m_1} = \frac{1}{2}at_{m_1}^2$

$X_{m_2} = \Delta X = v_0 t$

3) Put the equations together

$\frac{1}{2}a_c t^2 = \frac{1}{2}a_m (t_{m_1})^2 + v_m(t - t_{m_1})$

$\frac{1}{2}(5.6)t^2 = \frac{1}{2}(8.4)(7)^2 + 58.8(t - 7)$

4) Solve for t

$t = 4.43, 16.56$

↑ Doesn't make sense, $X_{m1} = 7$ ← must be greater than $t = 7$

$\boxed{t = 16.56\ s}$

1-D MOTION

01/13/23

	DIMENSIONS		- UNITS -	EXAMPLE
- Matter (stuff):	Mass	[M]	kg	$\frac{\text{Distance}}{\text{Time}}$ = speed [v] = $\frac{[L]}{[T]}$
- Distance:	length	[L]	m	
	Time	[T]	s	

Force $[F] = \frac{[M][L]}{[T]^2}$

EXAMPLE

$x = a\cos(wt)$ (position: x; time: t)

Find the units of a and w

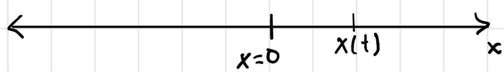
$[L] = a\cos(\underbrace{w\,[T]}_{\text{cannot have dimensions}})$ $\quad w = \frac{1}{[T]} \quad a = [L]$

cannot have dimensions
- inside cos(x), sin(x)
- Argument of the fxn cannot have a dimension

- Every function algebraically goes back to its series

Ex. cos(x)
$\hookrightarrow 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \cdots$

01/18/23



+ sign is important

what could x(t) be:
1) $x(t) = 10\,m \rightarrow v = 0\,m/s$
2) $x(t) = (10 - 5t)\,m \rightarrow v = -5\,m/s$
3) $x(t) = (10 + 2t - 3t^2)\,m \rightarrow v = (2 - 6t)\,m/s \rightarrow a = -6\,m/s^2$

position → velocity

$x(t) \rightarrow v(t) = \frac{dx}{dt} \rightarrow a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

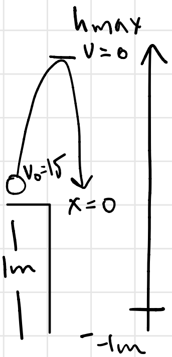
only your velocity tells you what direction you're moving

- when v and a are the same sign, you are speeding up

How to solve problems w/ two objects that meet at the same position and same time: $x_A(t) = x_B(t)$ (setup)

Example problem: Throwing a ball / Free fall

$a = 10\,m/s^2$ downward (+/- depending on direction)



Place where ball is launched
↳ $x=0$ $\quad V_0 = +15\,m/s$

1) What is hmax?
↳ what is x when v=0?
at what time?

IF a is constant
✱ $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$
✱ $v(t) = v_0 + at$
✱ $v^2 = v_0^2 + 2a\Delta x$

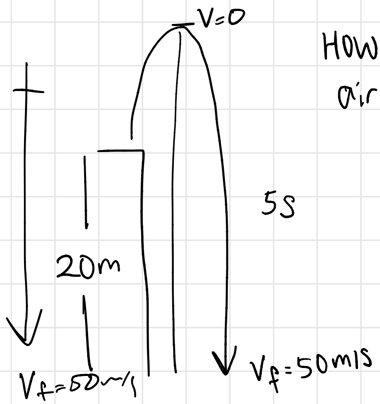
✱ When ball is just moving down, more reasonable to make downward into positive direction.

$\Delta x = v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a\Delta x$
$0 = 225 + 2(-10)\Delta x$
$0 = 225 + 20\Delta x$
$\Delta x = 11.25$

$v(t) = v_0 + at$
$0 = 15 + (-10)t$
$t = 1.5$

$\Delta x = v_0 t + \frac{1}{2}at^2$
$-1 = 15t + \frac{1}{2}(-10)t^2$
$-1 = 15t - 5t^2$



How long was the ball in the air?

$V_f = V_0 + a\Delta t$
$+50 = +10(t)$
$t = 5$

$v^2 = v_0^2 + 2a\Delta x$
$0 = v_0^2 + 2(-10)\Delta x$

$\Delta x = v_0 t + \frac{1}{2}at^2$

$50^2 = 0^2 + 2a(20 + x)$
$2500 = 20(20 + x)$

VECTORS

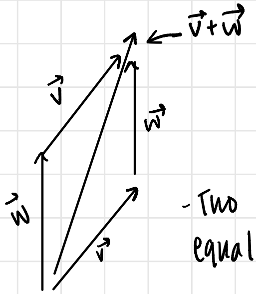
01/19/23

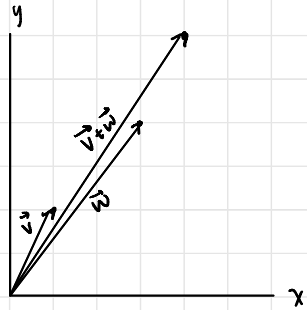
$\vec{v} + \vec{w} = (v_x + w_x, v_y + w_y)$

$\vec{v} = (1,2)$

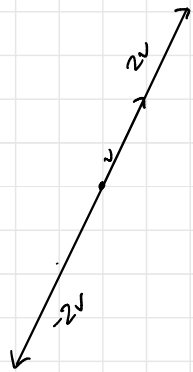
$\vec{w} = (3,4)$

$\vec{v} + \vec{w} = (4,6)$





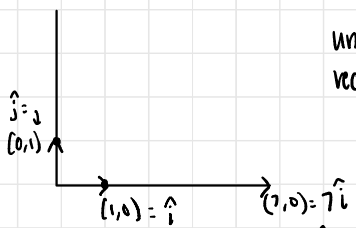
- Two vectors added together is equal to their diagonal



$\vec{v} = (1,2)$

$2\vec{v} = (2,4)$

$-2\vec{v} = (-2,-4)$



multiple tells you the length

unit vector:

vector w/ length 1

$\vec{v} = (-3,4)$

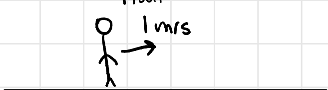
$\vec{v} = (-3,0) + (4,0)$

$\vec{v} = -3\hat{i} + 4\hat{j}$

Relative velocity: What is it moving relative to?

Who is measuring the velocity?

$V_{p,belt} = 1m/s$



$\rightarrow +$

$V_{belt} = 3m/s$

relative to the ground

velocity of the person, relative to the ground

$\vec{V}_{p,ground} = \vec{V}_{belt,ground} + \vec{V}_{p,belt} = 4m/s$

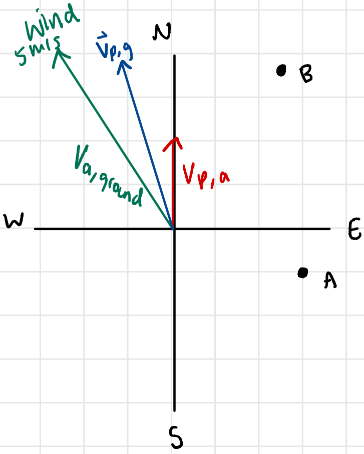
3m/s + 1m/s

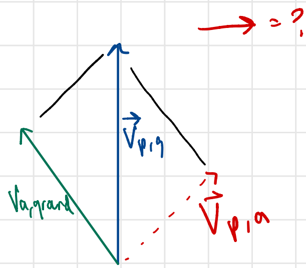
ground = O_2

belt = O_1

O = observer

$$\vec V_{plane,air} + \underbrace{\vec V_{air,ground}}_{\text{wind}} = \underbrace{\vec V_{plane,ground}}_{\text{Speed of plane}}$$



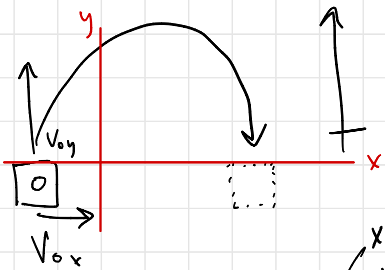


You must cancel the drift from the air

$$\vec V_{p,a} = \vec V_{p,g} - \vec V_{a,g}$$

$$\vec V_{plane,air} + \vec V_{air,ground} = \vec V_{plane,ground}$$

Projectile motion : Ball and cart example



$$\vec a = (a_x, a_y)$$

How far is the cart?

$x(t)$

$a_x = 0$

Added gives us position of the ball

How high is the ball?

$y(t)$

$a_y = -g$

$$x(t) = x_0 + V_{0x}t + \tfrac{1}{2}a_x t^2$$
$$V_x(t) = V_{0x} + a_x t$$
$$x(t) = V_{0x}t$$
$$V_x = V_{0x}$$

$$y(t) = y_0 + V_{0y}t + \tfrac{1}{2}a_y t^2$$
$$V_y(t) = V_{0y} + a_y t$$
$$y(t) = V_{0y}t - \tfrac{1}{2}gt^2$$
$$V_y = V_{0y} - gt$$

Ex.

$V_{oy} = 2m/s$

$V_{ox} = 1m/s$

$g = 10m/s^2$

$a_x = 0$

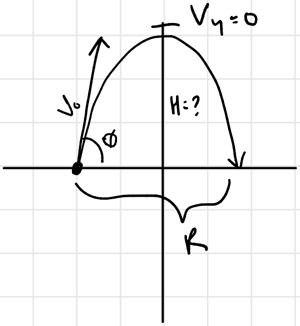
$a_y = -10m/s^2$

$V_x = 1m/s$

$x = t$

$V_y = (2-10t)m/s$

$y = (2t - 5t^2)m$



y component of the object is 0

$H = y_{max} \rightarrow V_y = 0$

$V_y = 2 - 10t$

$0 = 2 - 10t$

$t = \frac{2}{10}$

$t = \frac{V_{oy}}{g}$ time to max height $= \frac{2}{10}$

$y = V_{oy}t - \frac{1}{2}gt^2 = \frac{V_{oy}^2}{2g} = H$

$\cos(90-\emptyset) = \sin\emptyset$

$\sin(90-\emptyset) = \cos\emptyset$

$R = x$ when $y = 0$

when $y=0$ $\quad t = \frac{2V_{oy}}{g}$ $\quad R = \frac{2V_{ox}V_{oy}}{g}$ $\quad \rightarrow R = \frac{2V_0^2\cos\emptyset\sin\emptyset}{g}$

$$\left.\begin{aligned} V_{ox} &= V_0\cos\emptyset \\ V_{oy} &= V_0\sin\emptyset \end{aligned}\right\}$$