北京邮电大学 2022—2023 学年第二学期

《离散数学》期末考试试题（A 卷）

<table>
<tr><td rowspan="4">考试注意事项</td><td colspan="11">一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。</td></tr>
<tr><td colspan="11">二、书本、参考资料、书包等物品一律放到考场指定位置。</td></tr>
<tr><td colspan="11">三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。</td></tr>
<tr><td colspan="11">四、学生必须将答题内容做在试题答卷上，做在草稿纸上一律无效。</td></tr>
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<tr><td>考试课程</td><td colspan="3">离散数学</td><td colspan="2">考试时间</td><td colspan="5">2023 年 6 月 16 日　08:00-10:00</td></tr>
<tr><td>题号</td><td>一</td><td>二</td><td>三</td><td>四</td><td>五</td><td>六</td><td>七</td><td>八</td><td>九</td><td>十</td><td>总分</td></tr>
<tr><td>满分</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>得分</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>阅卷教师</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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1. One choice questions

[20 marks, 2 marks for each]

1) Which of the following compound proposition is a tautology (永真式)? ____

A. $(q \rightarrow p) \rightarrow (p \rightarrow q)$

B. $(\neg p \vee q) \vee (\neg(p \rightarrow q) \wedge q)$

C. $(q \rightarrow p) \vee (p \rightarrow q)$

D. $(p \vee q) \wedge \neg(p \wedge q)$

2) Assume that $\exists x \forall y\, P(x, y)$ is false and that the domain of discourse is nonempty, which of the following statement must also be false? _____

A. $\forall x \forall y\, P(x, y)$

B. $\forall x \exists y\, P(x, y)$

C. $\exists x \exists y\, P(x, y)$

D. $\exists x \forall y\, P(y, x)$

3) Which of the following sets is uncountable? _____

A. N (the set of natural numbers)

B. Z (the set of integers)

C. R (the set of real numbers)

D. Q (the set of rational numbers)

4) Select a theta notation for the number of times the statement $x = x + 1$ is executed in the following algorithm _____.

$$i = n$$

$$\text{while } (i \geq 1) \{$$

$$\text{for } j = 1 \text{ to } n$$

$$x = x + 1$$

$$i = \lfloor i/2 \rfloor$$

$$\}$$

A. $\Theta(\log n)$,　　　　B. $\Theta(n)$,　　　　C. $\Theta(n \log n)$　　　D. $\Theta(n^2)$

5) Suppose that R and S are transitive (传递) relations on a set A, then which of the following relations must be transitive?_____

A. $R \cup S$　　　　B. $R \cap S$　　　　C. $R \circ S$　　　　D. $S \circ R$

6) Which of the given relations is not an equivalence relation (等价关系) on the set $\{1, 2, 3, 4, 5\}$?_____

A. $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 3), (3, 1)\}$

B. $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 5), (5, 1), (3, 5), (5, 3), (1, 3), (3, 1)\}$

C. $\{(x, y) \mid 3 \text{ divides } x + y\}$

D. $\{(x, y) \mid x \text{ and } y \text{ are both even }$　or　$x \text{ and } y \text{ are both odd}\}$

7) Suppose that a candy shop has 5 different kinds of candies, then how many different ways can 6 candies be chosen?_____
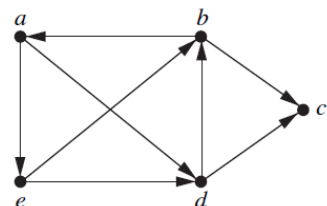
A. 200　　　　B. 210　　　　C.253　　　　D.462

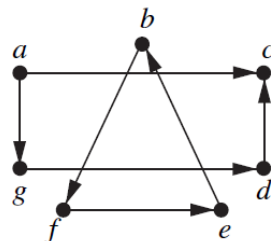8) Which of the following graphs is a strongly connected graph? _____
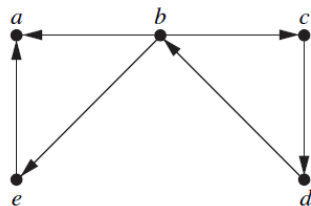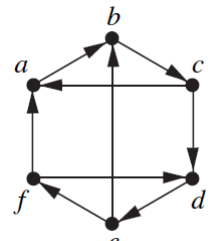
A.　　　　　　　　　　　　　　B.

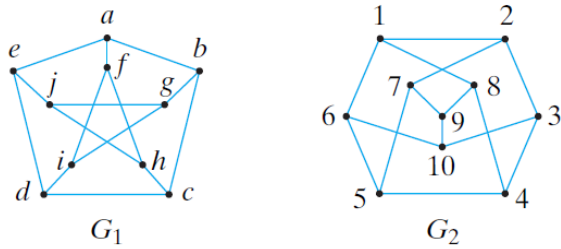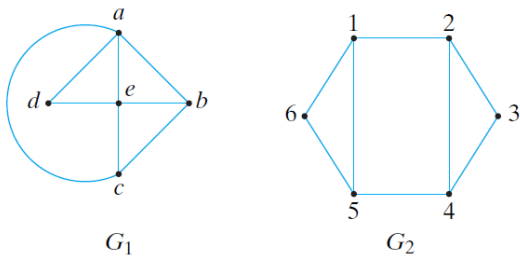C.　　　　　　　　　　　　　　D.

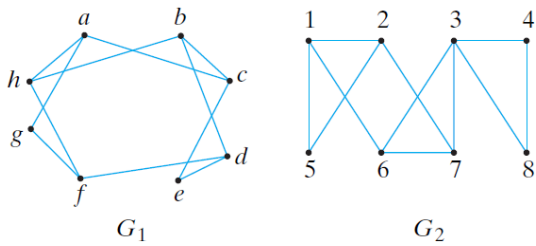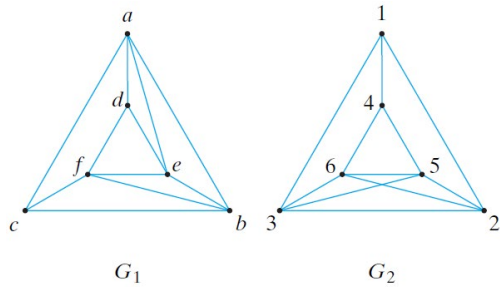9) Which group of the following graphs is isomorphic (同构的)? _____
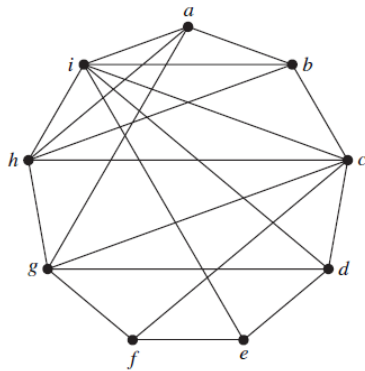
A.

G_1 G_2

B.

G_1 G_2

C.

G_1 G_2

D.

G_1 G_2

10) Consider the graph coloring problem, the chromatic number of the following graph is_____.

A. 3   B. 4   C.5   D.6

2. Fill in the blanks

1) A tree with n vertices has_____edges.

2) Assuming that p and r are false and that q and s are true, then truth value of the following proposition is_____.

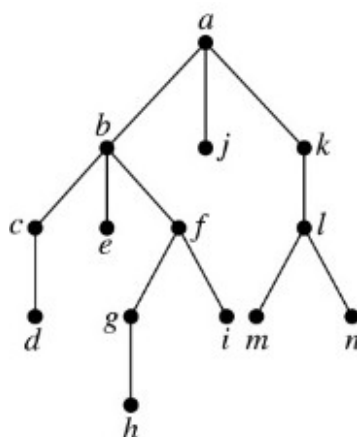$$(s \rightarrow (p \wedge \neg r)) \wedge ((p \rightarrow (r \vee q)) \wedge s)$$

3) Let the domain of discourse (论域) be R, then the truth value of the following statement is_____.

$$\exists x((x > 1) \rightarrow (x^2 > x))$$

4) Let R_1 and R_2 be the relations on $\{1, 2, 3, 4\}$ given by $R_1 = \{(1, 1), (1, 2), (3, 4), (4, 2)\}$ and $R_2 = \{(1, 1), (2, 1), (3, 1), (4, 4), (2, 2)\}$, then $R_1 \circ R_2$ = _____.
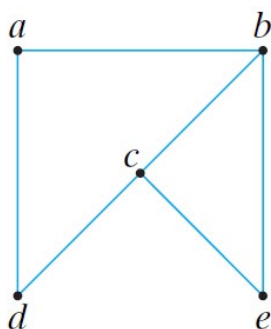
5) A full m-ary tree with n vertices has_____internal vertices (中间节点).

6) List the order of the nodes of the following tree in which the vertices are processed by preorder traversal (前序遍历) _____.
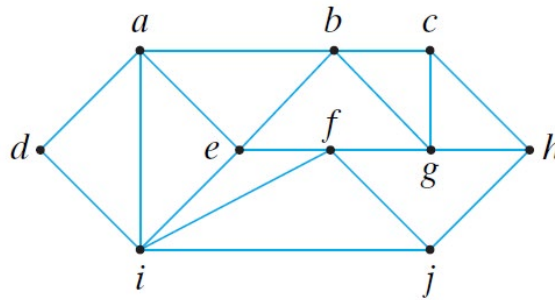

7) Given an expression with postfix (后缀) form $ABC**CDE+/-$, then its prefix (前缀) form is _____.

8) How many paths of length 4 from the vertex d to the vertex e are there in the following graph ?_____.

9) Give a Hamilton circuit (哈密尔顿回路) for the following graph_____.

10) Let *G* be the grammar with the vocabulary $V = \{S, A, a, b\}$, a set of terminals $T = \{a, b\}$, starting symbol *S*, and productions $P = \{S \rightarrow aA, S \rightarrow b, A \rightarrow aa\}$. Then, the language generated by *G* is $L(G) = $ _____.

3. **Show that $(\neg p \wedge (p \vee q)) \rightarrow q$ is a tautology.**

[8 marks]

4. **Suppose that R_1 and R_2 are equivalence relations on a set *A*, prove that $R_1 \cap R_2$ is also an equivalence relation on the set *A*.**

[8 marks]

5. Prove that $\sum_{i=n}^{2n-1}(2i + 1) = 3n^2$, for all positive integers n.

[8 marks]

6. Consider the floor plan of a three-room structure that is shown as follows. Each room is connected to every room that it shares a wall with and to the outside along each wall. Is it possible to begin in a room or outside and take a walk that goes through each door exactly once (经过所有的门仅一次的通路)? If it is possible, give such a path.
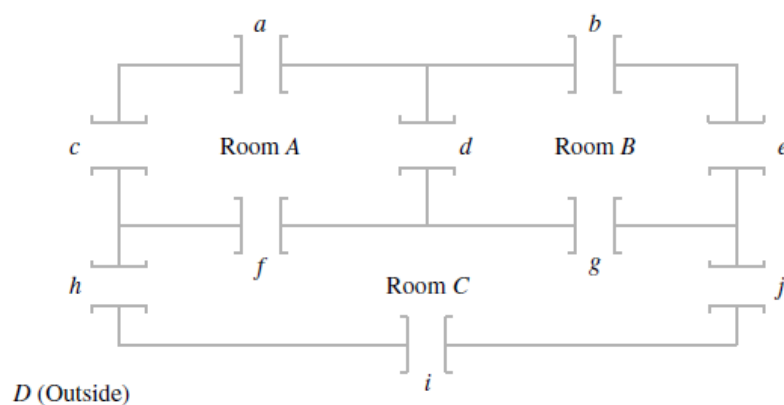
[8 marks]

7. **Firstly, draw the state diagram of the following finite-state machine** $M =$ $(I, O, S, f, g, \sigma_0)$, **where** $I = \{a, b, c\}$, $O = \{0, 1\}$, $S = \{\sigma_0, \sigma_1, \sigma_2\}$, **then write the output string for the given input string** *aabbcc*.

$\quad$ I $\quad$ S	f			g		
	a	b	c	a	b	c
σ_0	σ_0	σ_1	σ_2	0	1	0
σ_1	σ_1	σ_1	σ_0	1	1	1
σ_2	σ_2	σ_1	σ_0	1	0	0

[8 marks]

8. **Solve the recurrence relation** $c_n = 6c_{n-1} - 8c_{n-2}$ **with the initial conditions** $c_0 = 1$ **and** $c_1 = 0$.

[10 marks]

9. **For the Turing machine *T* defined by the five-tuples $(s_0, 0, s_0, 0, R)$, $(s_0, 1,$ $s_1, 0, R)$, $(s_1, 0, s_1, 1, R)$, $(s_1, 1, s_2, 1, L)$, $(s_1, B, s_1, 1, L)$, find the final tape when *T* runs on the following tape, beginning in the initial position (the first nonzero entry from the left).**

	B	B	0	0	0	1	B	0	B	B	
$\cdots$											$\cdots$

[10 marks]