

$(a \cup b^*)a^nb^n$

1.29 Use the pumping lemma to show that the following languages are not regular.

a. $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

b. $A_2 = \{www \mid w \in \{a, b\}^*\}$

c. $A_3 = \{a^{2^n} \mid n \geq 0\}$ (Here, a^{2^n} means a string of 2^n a's.)

1.51 Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection, and complement.

- a. $\{0^n 1^m 0^n \mid m, n \geq 0\}$
- b. $\{0^m 1^n \mid m \neq n\}$
- c. $\{w \mid w \in \{0,1\}^* \text{ is not a palindrome}\}^8$
- *d. $\{wtw \mid w, t \in \{0,1\}^+\}$

1.52 Let $\Sigma = \{1, \#\}$ and let

$$Y = \{w \mid w = x_1 \# x_2 \# \cdots \# x_k \text{ for } k \geq 0, \text{ each } x_i \in 1^*, \text{ and } x_i \neq x_j \text{ for } i \neq j\}.$$

Prove that Y is not regular.

1.62 Let $\Sigma = \{0, 1, +, =\}$ and

$ADD = \{x=y+z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$

Show that ADD is not regular.

1.68 Let $\Sigma = \{0,1\}$. Show that \overline{C} is not closed under the *union* operation.

- a. Let $A = \{0^k u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$. Show that A is regular.
- b. Let $B = \{0^k 1 u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$. Show that B is not regular.

1.69 Let M_1 and M_2 be DFAs that have k_1 and k_2 states, respectively, and then let $U = L(M_1) \cup L(M_2)$.

- a. Show that if $U \neq \emptyset$, then U contains some string s , where $|s| < \max(k_1, k_2)$.
- b. Show that if $U \neq \Sigma^*$, then U excludes some string s , where $|s| < k_1 k_2$.

1.70 Let $\Sigma = \{0,1,\#\}$. Let $C = \{x \# x^R \# x \mid x \in \{0,1\}^*\}$. Show that \overline{C} is a CFL.

- 1.71
- a. Let $B = \{1^k y \mid y \in \{0,1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for } k \geq 1\}$. Show that B is a regular language.
 - b. Let $C = \{1^k y \mid y \in \{0,1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1\}$. Show that C isn't a regular language.