is the set of all strings of 0's and 1's that ... is the set of all strings of ... the DFA of Example 2.4, then $L(A)$ is the set of all strings ... numbers of 0's and 1's are both even. □

2.2.6 Exercises for Section 2.2

Exercise 2.2.1: In Fig. 2.8 is a marble-rolling toy. A marble is dropped at A or B. Levers x_1, x_2, and x_3 cause the marble to fall either to the left or to the right. Whenever a marble encounters a lever, it causes the lever to reverse after the marble passes, so the next marble will take the opposite branch.

* a) Model this toy by a finite automaton. Let the inputs A and B represent the input into which the marble is dropped. Let acceptance correspond to the marble exiting at D; nonacceptance represents a marble exiting at C.

! b) Informally describe the language of the automaton.

c) Suppose that instead the levers switched *before* allowing the marble to pass. How would your answers to parts (a) and (b) change?

*! **Exercise 2.2.2:** We defined $\hat{\delta}$ by breaking the input string into any string followed by a single symbol (in the inductive part, Equation 2.1). However, we informally think of $\hat{\delta}$ as describing what happens along a path with a certain string of labels, and if so, then it should not matter how we break the input string in the definition of $\hat{\delta}$. Show that in fact, $\hat{\delta}(q, xy) = \hat{\delta}\big(\hat{\delta}(q, x), y\big)$ for any state q and strings x and y. *Hint*: Perform an induction on $|y|$.

! **Exercise 2.2.3:** Show that for any state q, string x, and input symbol a, $\hat{\delta}(q, ax) = \hat{\delta}\big(\delta(q, a), x\big)$. *Hint*: Use Exercise 2.2.2.

Exe

$\{0,$

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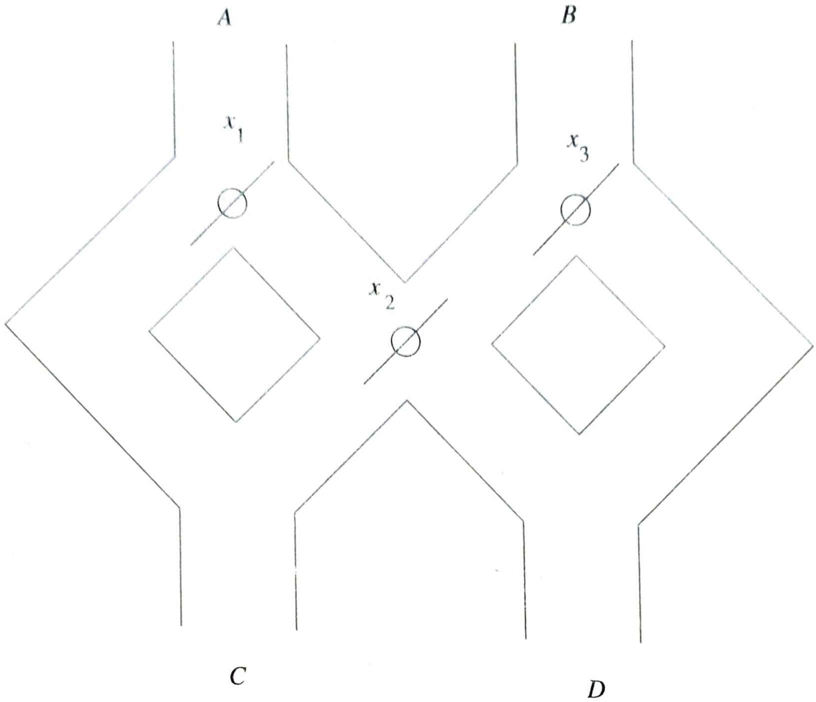

Figure 2.8: A marble-rolling toy

Exercise 2.2.4: Give DFA's accepting the following strings over the alphabet $\{0, 1\}$:

* a) The set of all strings beginning with 101.

b) The set of all strings containing 1101 as a substring.

c) The set of all strings with exactly three consecutive 0's.

! **Exercise 2.2.5:** Give DFA's accepting the following strings over the alphabet $\{0, 1\}$:

a) The set of all strings such that the number of 1's is even and the number of 0's is a multiple of 3.

b) The set of all strings not containing 110.

c) The set of all strings that begin with 01 and end with 11.

d) The set of all strings which when interpreted as a binary integer is a multiple of 3.

!! **Exercise 2.2.6:** Give DFA's accepting the following languages over the alphabet $\{0, 1\}$:

* a) The set of all strings beginning with a 1 that, when interpreted as a binary integer, is a multiple of 5. For example, strings 101, 1010, and 1111 are in the language; 0, 100, and 111 are not.

b) The set of all strings that, when interpreted *in reverse* as a binary integer, is divisible by 5. Examples of strings in the language are 0, 10011, 1001100, and 0101.

Exercise 2.2.7: Let A be a DFA and q a particular state of A, such that $\delta(q, a) = q$ for all input symbols a. Show by induction on the length of the input that for all input strings w, $\hat{\delta}(q, w) = q$.

Exercise 2.2.8: Let A be a DFA and a a particular input symbol of A, such that for all states q of A we have $\delta(q, a) = q$.

a) Show by induction on n that for all $n \geq 0$, $\hat{\delta}(q, a^n) = q$, where a^n is the string consisting of n a's.

b) Show that either $\{a\}^* \subseteq L(A)$ or $\{a\}^* \cap L(A) = \emptyset$.

*! **Exercise 2.2.9:** Let $A = (Q, \Sigma, \delta, q_0, \{q_f\})$ be a DFA, and suppose that for all a in Σ we have $\delta(q_0, a) = \delta(q_f, a)$.

a) Show that for all $w \neq \epsilon$ we have $\hat{\delta}(q_0, w) = \hat{\delta}(q_f, w)$.

b) Show that if x is a nonempty string in $L(A)$, then for all $k > 0$, x^k (i.e., x written k times) is also in $L(A)$.

*! **Exercise 2.2.10:** Consider the DFA with the following transition table:

δ	0	1
$\rightarrow A$	B	A
$*B$	A	B

Informally describe the language accepted by this DFA, and prove by induction on the length of an input string that your description is correct. *Hint*: When setting up the inductive hypothesis, it is wise to make a statement about what inputs get you to each state, not just what inputs get you to the accepting state.

! **Exercise 2.2.11:** Repeat Exercise 2.2.10 for the following transition table:

δ	0	1
$\rightarrow A$	B	A
B	C	A
$*C$	C	C