

1.4

Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.

- a. $\{w \mid w \text{ has at least three } a's \text{ and at least two } b's\}$
- ^Ab. $\{w \mid w \text{ has exactly two } a's \text{ and at least two } b's\}$
- c. $\{w \mid w \text{ has an even number of } a's \text{ and one or two } b's\}$
- ^Ad. $\{w \mid w \text{ has an even number of } a's \text{ and each } a \text{ is followed by at least one } b\}$
- e. $\{w \mid w \text{ starts with an } a \text{ and has at most one } b\}$
- f. $\{w \mid w \text{ has an odd number of } a's \text{ and ends with a } b\}$
- g. $\{w \mid w \text{ has even length and an odd number of } a's\}$

- 1.5 Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.

- ^Aa. $\{w \mid w \text{ does not contain the substring } ab\}$
- ^Ab. $\{w \mid w \text{ does not contain the substring } baba\}$
- ^Cc. $\{w \mid w \text{ contains neither the substrings } ab \text{ nor } ba\}$
- d. $\{w \mid w \text{ is any string not in } a^*b^*\}$
- e. $\{w \mid w \text{ is any string not in } (ab^+)^*\}$
- f. $\{w \mid w \text{ is any string not in } a^* \cup b^*\}$
- g. $\{w \mid w \text{ is any string that doesn't contain exactly two } a's\}$
- h. $\{w \mid w \text{ is any string except } a \text{ and } b\}$

- 1.6 Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{0,1\}$.

- a. $\{w \mid w \text{ begins with a } 1 \text{ and ends with a } 0\}$
- b. $\{w \mid w \text{ contains at least three } 1s\}$
- c. $\{w \mid w \text{ contains the substring } 0101 \text{ (i.e., } w = x0101y \text{ for some } x \text{ and } y)\}$
- d. $\{w \mid w \text{ has length at least } 3 \text{ and its third symbol is a } 0\}$
- e. $\{w \mid w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length}\}$
- f. $\{w \mid w \text{ doesn't contain the substring } 110\}$
- g. $\{w \mid \text{the length of } w \text{ is at most } 5\}$
- h. $\{w \mid w \text{ is any string except } 11 \text{ and } 111\}$
- i. $\{w \mid \text{every odd position of } w \text{ is a } 1\}$
- j. $\{w \mid w \text{ contains at least two } 0s \text{ and at most one } 1\}$
- k. $\{\epsilon, 0\}$
- l. $\{w \mid w \text{ contains an even number of } 0s, \text{ or contains exactly two } 1s\}$
- m. The empty set
- n. All strings except the empty string

- 1.7 Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, the alphabet is $\{0,1\}$.

- ^Aa. The language $\{w \mid w \text{ ends with } 00\}$ with three states
- b. The language of Exercise 1.6c with five states
- c. The language of Exercise 1.6l with six states
- d. The language $\{0\}$ with two states
- e. The language $0^*1^*0^*$ with three states
- ^Af. The language $1^*(001^*)^*$ with three states
- g. The language $\{\epsilon\}$ with one state
- h. The language 0^* with one state

- 1.8 Use the con