Again we have started by assuming the language in question was regular, and we derived a contradiction by showing that some string not in the language was required by the pumping lemma to be in the language. Thus, we conclude that L_{pr} is not a regular language. □

4.1.3 Exercises for Section 4.1

Exercise 4.1.1: Prove that the following are not regular languages.

a) $\{0^n 1^n \mid n \geq 1\}$. This language, consisting of a string of 0's followed by an equal-length string of 1's, is the language L_{01} we considered informally at the beginning of the section. Here, you should apply the pumping lemma in the proof.

b) The set of strings of balanced parentheses. These are the strings of characters "(" and ")" that can appear in a well-formed arithmetic expression.

*c) $\{0^n 10^n \mid n \geq 1\}$.

d) $\{0^n 1^m 2^n \mid n \text{ and } m \text{ are arbitrary integers}\}$.

e) $\{0^n 1^m \mid n \leq m\}$.

f) $\{0^n 1^{2n} \mid n \geq 1\}$.

! **Exercise 4.1.2:** Prove that the following are not regular languages.

*a) $\{0^n \mid n \text{ is a perfect square}\}$.

b) $\{0^n \mid n \text{ is a perfect cube}\}$.

c) $\{0^n \mid n \text{ is a power of 2}\}$.

d) The set of strings of 0's and 1's whose length is a perfect square.

e) The set of strings of 0's and 1's that are of the form ww, that is, some string repeated.

f) The set of strings of 0's and 1's that are of the form uw^R, that is, some string followed by its reverse. (See Section 4.2.2 for a formal definition of the reversal of a string.)

g) The set of strings of 0's and 1's of the form $w\overline{w}$, where $\overline{w}$ is formed from w by replacing all 0's by 1's, and vice-versa; e.g., $\overline{011} = 100$, and 011100 is an example of a string in the language.

h) The set of strings of the form $w1^n$, where w is a string of 0's and 1's of length n.

!! **Exercise 4.1.3**: Prove that the following are not regular languages:

a) $\{0^{n!} | n \geq 1\}$.

b) The set of all strings of 0's and 1's whose length is a prime number.

! **Exercise 4.1.4**: When we try to apply the pumping lemma to a regular language, the proof cannot be completed. Identify what goes wrong when we choose L as one of the following languages:

* a) ε

* b) 0^*1^*

* c) $0^*(10 + 01)^*$

4.2 Closure Properties of Regular Languages

In this section, we shall prove several theorems of the form "if certain languages are regular, and a language L is formed from them by certain operations (e.g., L is the union of two regular languages), then L is also regular." These theorems are often called *closure properties*