4.2 Application to Financial Time Series

In order to detect an anomaly, we must define what it is. Armed with a proposed definition, we can search for this object which we have defined. Whilst most anomaly detection applications have a clear idea of the anomalies which exist in their field, this is not the case in this application. Hence, we proceed in a slightly different fashion. We begin by proposing a form for an anomaly in a financial time series, and test if this proposed definition corresponds to a meaningful market phenomena.

The prediction of stock prices is not something we have interest in nor capability of doing. To quote Bachelier, 'The exchange reacts on itself, and the current fluctuation is a function, not only of the previous fluctuations, but also of the current state. The determination of these fluctuations depends on an infinite number of factors; it is, therefore, impossible to aspire to mathematical predictions of it'. Hence, it is important for our purpose that we do not even implicitly attempt to do so. With this in mind, we note an important clarification of the objective: our goal is that of detecting anomalous states in financial time series, rather than forecasting their existence.

With this in mind, we now turn to the issue of detection. To detect anomalies, it is only possible with some notion about what 'normal' behaviour is for stock prices. This is most easily done within a collection of stocks whose prices show some common structure. This sort of common structure, usually driven by stocks sharing common drivers (such as banking stocks being sensitive to interest rates, economic activity and policy changes) is readily extractable. Based off of this structure, we postulate a simple model for the values of a collection of stocks at some time.

Let us suppose we have n time series, which in this context are the stock prices of a group of assets we suppose to be correlated. New realisations of their prices are defined by $X_t \in \mathbb{R}^n$. We model X_t as a linear combination of m driving principal components $f_t \in \mathbb{R}^m$, plus some slowly varying trend, $\tau_t \in \mathbb{R}^n$, plus some noise $\epsilon_t \in \mathbb{R}^n$ as

$$X_t = \Lambda_t f_t + \tau_t + \epsilon_t, \tag{4.1}$$

where $\Lambda_t \in \mathbb{R}^{n \times m}$ are loadings on the m principal components at time t. We will again be modelling the τ_t as having independent dimensions. In this case, this simplifying assumption is admissable because of the structure we have already modelled via the principal components. In typical trading conditions $\epsilon_t \sim \mathcal{N}(0, I)$ where I is the n dimensional identity matrix.

Postulate 1 (Anomaly manifestation)**.** *We postulate that an anomaly which manifests in a financial time series can be modelled as $\epsilon_t \sim \mathcal{N}(0, Q)$, where Q is no longer diagonal.*

The reasons for this are two fold. Firstly, it offers some resistance to things which affect a single stock. Large movements caused by changes in fundamentals, e.g. the appointment of a new CEO, will not pass this hypothesis test to be classified as an anomaly. Secondly, depending on the dependencies which arise in Q, it makes the interpretation of signals clear: bet on reconvergence after periods of negative correlation.

4.2.1 Removing Structure from Financial Time Series

To test the postulate, we need some collection of correlated stocks on which we test for the presence of anomalies. There are many options available to us. An obvious choice would be a selection of large oil companies, which feel strong drivers including oil price and supply and demand dynamics. Hence, we will consider 5 oil majors. Namely, BP, Shell, Exxon Mobil, Chevron and Total Energies. Oil is a very tightly knit sector, hence when we plot the price vs time for these, as seen in Figure 4.4 for trading day 12/08/2025, the correlation is manifest.

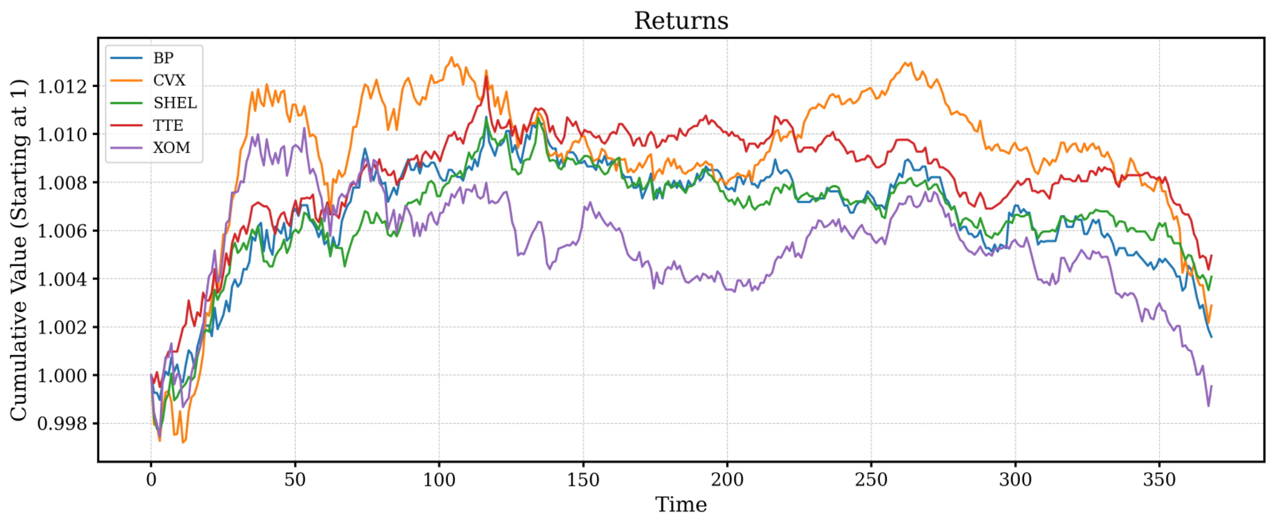

Figure 4.4: Normalised price vs time for 5 oil major companies. x axis displays minutes after market open. Data sourced from Yahoo Finance (2025).

We see the hypothesised common structure, manifesting via our Λ_t and f_t. This can be removed by principal component analysis (Deisenroth, Faisal, and Ong 2020). We fit principal components on the previous six days, and use the first two principal components to remove trend from the unseen, seventh day. We make use of the implementation of PCA from Scikit-Learn in Python (Pedregosa et al. 2011). The resultant series can be seen in Figure 4.5. We have now removed some of the 'explainable' structure from our time series, we are left with the smooth trend and the noise. We now pass the models through a trend removal process, and investigate the residuals. We select the exponential smoother for this process, again with $\alpha = 0.6$, as learned in the previous section.

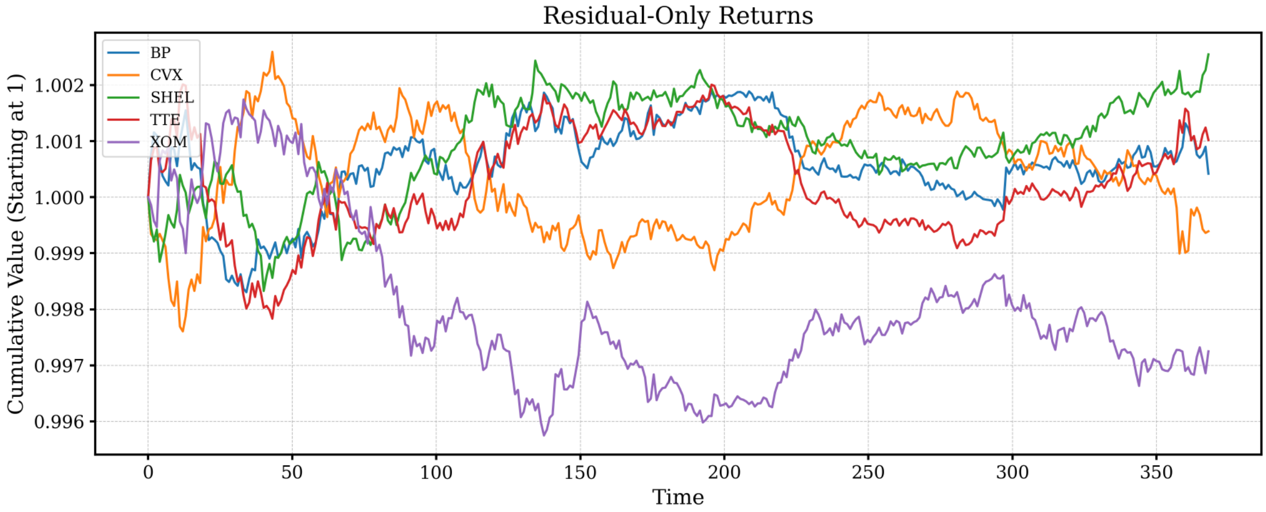

Figure 4.5: Oil prices with first two principal components removed.

Again, we are now testing for spontaneous correlation manifesting what we otherwise assume is white noise. To that end, we apply the algorithm CAPA CC (Tveten, Eckley, and Fearnhead 2022), briefly discussed in Section 2. This algorithm is naturally offline, hence for the first test we apply it to a single trading day of residuals, and allow it to batch process the whole dataset. The reported regions of anomaly are shown in Figure 4.6.

We note a few details. All red regions are paired with green regions. This, at least implicitly, gives an opportunity for factor neutral investing by hedging exposure to already removed principal components.

It is challenging to identify the physicality of these anomalous regions. To test this, a backtest was performed on similar historical data, testing the effectiveness of strategy where we bet on reconverging prices. This is given in Appendix B. We defer this to an Appendix for a few reasons. The most crucial of these is that a single signal, whilst perhaps effective, is only a small part of a monetisable trading algorithm. Performance is therefore very sensitive to a very large number of factors which are unrelated to the quality of the signal, including trade exit and entry logic, stop loss/ take profit orders, and use in parallel with other guard rails. As explained in Appendix B, the algorithm does appear to capture mispricings, but the effectiveness is low as the edge acquired is smaller than can be accessed due to spreads on the assets traded. Hence, we conclude that this model appears to be capturing the behaviour we hope to do so, but it lacks both the accuracy and promptness to be monetisable in its current form.

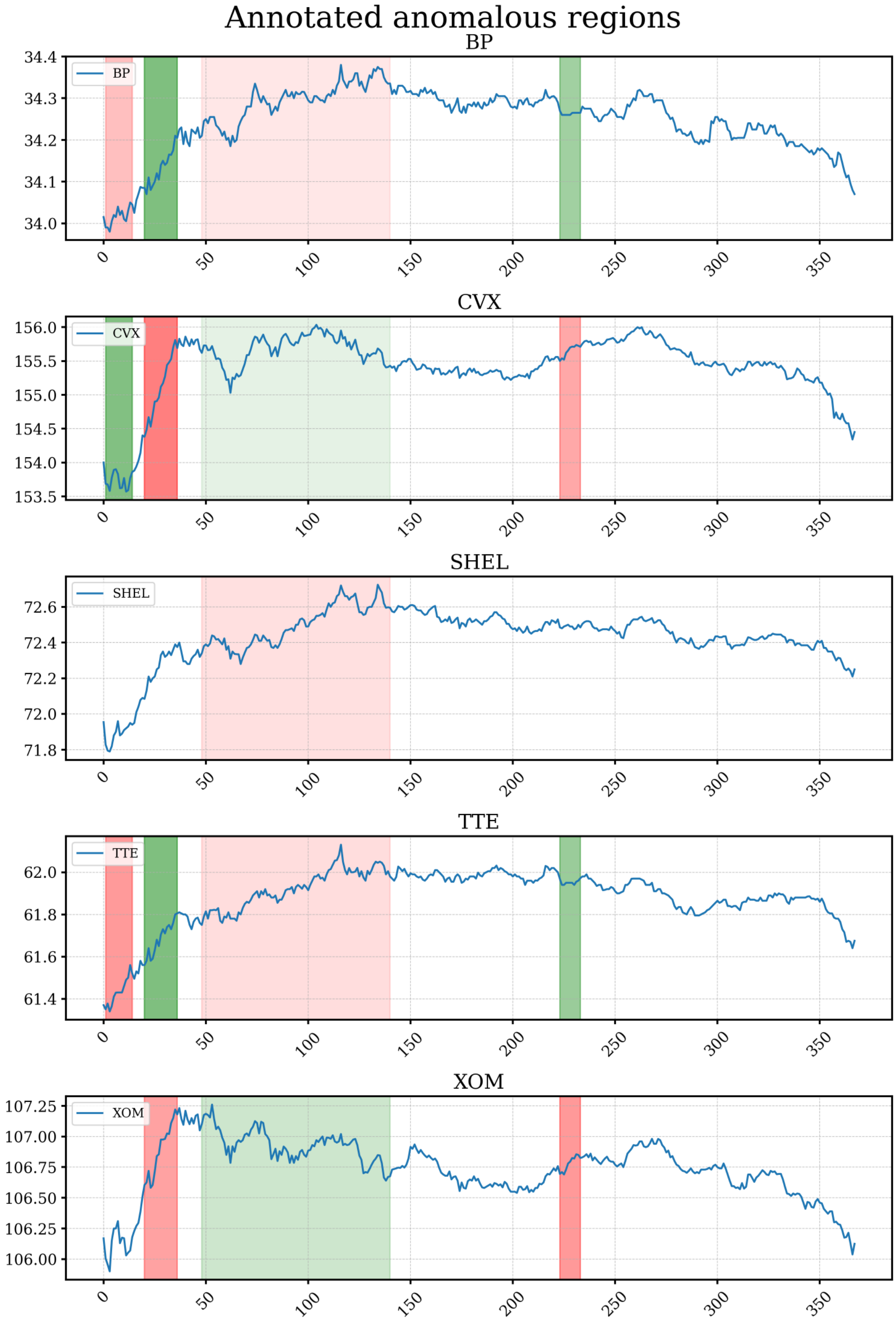

Figure 4.6: Received trading signals from anomalies. A green region indicates anomalously negative residuals (hence a buy signal) and a red region is anomalously positive residuals (hence a sell signal). The opacity of the region is the magnitude of the deviation (i.e. opaque green region is strong suggested buy, nearly transparent red region is weak signal to sell).

4.2.2 Limitations of this Approach

We have learned that our simple model for the manifestation of an anomaly does not seem to be accurate. The choices made in the definition of an anomaly were, in part, inspired by convenience. There already existed an algorithm for detection of changes in covariance structure of some subset of a time series. Choices made with convenience in mind are unlikely to capture the nuance of the problem at hand.

Secondly, the approach of a backtest to confirm or deny the physicality of these market phenomena is coarse. Single signal trading models are uncommon, and hence the efficacy of a signal cannot be purely determined by its ability to perform independently. Approaches for validating both the effectiveness and the physicality of signals, without resorting to backtesting a signal would be of more physical relevance. One possible avenue is to validate signals by measuring their consistency with external data sources, such as order flow. This could, in principle, provide evidence as to whether the signal captures genuine structure rather than noise. This type of validation would hardly be straightforward, but it is a possible avenue to determine if other market participants are reacting to the same discrepancies in value we are.