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# Open Clusters' Stellar Ages through Bayesian Hierarchical Modelling and Machine Learning

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**Abstract**

**Context.** Recent space missions like Kepler have allowed a vast amount of asteroseismic data on the individual stars in open clusters to be extracted, with incredibly low observational uncertainties. As the study of stellar ages in open clusters is intrinsically connected to stellar physics and evolution, it is important to find some more accurate and computationally efficient methods to compute said ages given asteroseismic data.

**Aims.** To develop and demonstrate the accuracy and efficiency of using Bayesian Hierarchical Modelling on stellar populations in determining ages.

**Methods.** We will apply Bayesian Hierarchical Modelling with some hyperprior deduced from literature and the open clusters' observed astroseismic data as constraints on our estimation on the stars' stellar properties, like age. We will replace traditional forward grid modelling with a custom trained neural network on these grids in order to greatly lower computational cost.

**Preliminary results.** Using the ages of 31 red giants in open cluster M67 presented in Stello et al. (2016), we computed the population to have mean age $4.76\pm0.42$ Gyr and a spread $1.69\pm0.42$ Gyr.

**Implications.** With a much faster and at least equally accurate method of age estimation developed, the stellar ages in a large number of open cluster can be analyzed. This vast volume of new stellar ages should aid the studies of stellar evolution process and stellar physics.

# 1 Introduction

The study of open clusters has always been an important topic in modern astrophysics, as it relates tightly to theories of stellar evolution and hence finding stellar ages (Hippel, 2005). Open clusters are thought to be clusters of stars that formed from the same molecular cloud, causing them to have a relatively uniform age and chemical composition compared to galactic field stars (Renaud, 2018). This provides a convenient population of stars that we already know are of similar stellar parameters such as age and metallicity. With asteroseismic data becoming readily available thanks to ESA's CoRoT mission (Baglin et al., 2008), NASA's Kepler and K2 missions (Borucki et al., 2008) and TESS mission (Ricker et al., 2014), and PLATO (Rauer et al., 2014) mission in the future (for the coverage of CoRoT, Kepler and K2, see Davies and A. Miglio (2016) page 1), open clusters will continue to be a very effective calibration tool for our stellar evolution models. This is also the reason why open clusters were chosen as the stellar population to be studied in this project, as they already have their age measured extensively in literature, which provides hyperprior values to our estimates of individual star ages.

Stellar evolution models are models that aim to reproduce observed parameters of stars like oscillatory modes' peak frequencies, large/small frequency separations and $T_{eff}$ from fundamental stellar parameters like M, R and age. Asteroseismology is the study of detectable surface oscillation of stars that has some regular periods, with correlation to the internal physics of stars (William J. Chaplin and Andrea Miglio, 2013).

## 1.1 Bayesian hierarchical modelling

Bayesian hierarchical modelling (BHM) is a statistical technique of fitting multiple parameters of a population of objects simultaneously by considering all populations in the same pool at once (Gelman and Pardoe, 2006). Estimations made by BHM usually turn out to be more accurate than some other non-pooled methods due to the nature of the estimation being simultaneously constraint by the hyperpriors inputted and the observed populations. BHM has previously been applied on the problems of Initial-Final Mass Relationship of star clusters (Si, David A van Dyk, et al., 2018) and ages of galactic white dwarfs (Si, David A. van Dyk, et al., 2017), but has yet been applied on open cluster populations to resolve its mean age, spread and individual stellar ages. In this project, we would attempt to employ BHM coupled with machine learning on estimating these age properties of open clusters.

# 2 Methods

## 2.1 Overview

The overarching process of the BHM involved in this project is illustrated in figure 1. The inputs into this machineary will be 1. Some hyperpriors of stellar parameters of the open cluster that is to be estimated by the BHM process, derived from previous work on the stellar parameter, 2. fixed stellar parameters that are not to be estimated, derived from previous work as well and 3.

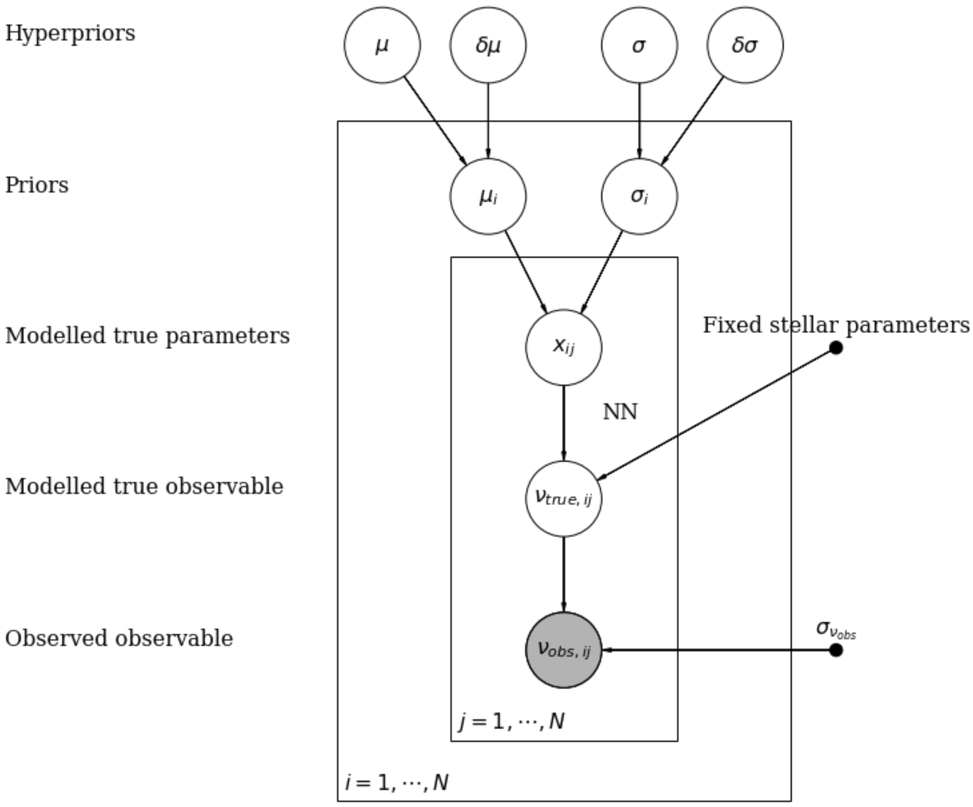


Figure 1: A simplified graphical demonstration of the processes involved in the project. This is not a (fully correct) probabilistic graphical model, but just a demonstration of the flow of the variables.

real asteroseismic observed data of a population of stars from that open cluster and the estimated observational uncertainty. The process goes as followed:

1. For each stellar parameter, $X$ to be estimated, it has a corresponding hyperprior that is a set of mean value $\mu$ and spread $\sigma$ and some uncertainties $\delta\mu$, $\delta\sigma$ correspondingly. If no prior knowledge can be inferred from previous work, the uncertainties would be set many orders of magnitudes higher than $\mu$ and $\sigma$, to prevent bias in the case of the lack of prior knowledge.

2. A list of priors $\{P(\mu_i, \sigma_i)\}$ are then created under normal distribution $N(\mu, \delta\mu)$ and $N(\sigma, \delta\sigma)$.

3. Each of these priors then generate a population of modelled true stellar parameters $\{x_{ij}\}$ of the parameter $X$ with accordance to the normal distribution $N(\mu_i, \sigma_i)$.

4. With the addition of stellar parameters that are not to be estimated provided as fixed parameters, it makes up for the full list of stellar parameters (eg. M, R, age, log g, $T_{eff}$) required for forward modelling and obtain observable asteroseismic data (eg. individual p-mode peak frequencies) about the stellar populations, giving $\{\nu_{true,ij}\}$

5. Artificial noise in accordance to estimated observational noise $\sigma_{\nu_{obs}}$ of the asteroseismic data are added to the modelled true observables to give modelled observed values $\{\nu_{obs,ij}\}$.

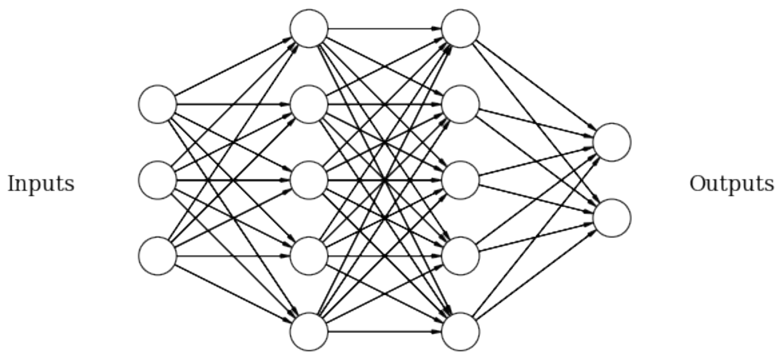


Figure 2: An example of a fully connected neural network. This NN contains 2 hidden layers, with the layer on the far left being the input and the one on the far right the output.

6. This population is then compared with the real population of observed data to give a posterior for that particular population of stars under the prior $P(\mu_i, \sigma_i)$.

After repeating steps 3-6 sufficiently large enough times, a chosen list of priors $\{P(\mu_i, \sigma_i)\}$ can be sampled to plot each estimated stellar parameter's posterior density distribution curve on both their $\mu$ and $\sigma$ values, from which we can draw conclusions about the open cluster's stellar population's fundamental properties like ages.

### 2.2 Machine learning as an alternative to stellar evolution models

Currently the two main approaches to obtain the stellar properties (eg. M, R, age, log g) from observables are either with interpolation on a grid of pre-evolved stellar models (demonstrated in W. J. Chaplin et al. (2013), Davies and A. Miglio (2016), Kallinger et al. (2010), Lagarde et al. (2012) etc.), or evolving sample stars from their initial conditions repeatedly while minimizing the difference between the evolved model's observables and the real observed values with a "merit function" (demonstrated in Appourchaux et al. (2015), W. H. Ball and Gizon (2017), W H Ball, Themeßl, and Hekker (2018) etc.). However, both of the approaches have their drawbacks: The former includes source of error through interpolation and the latter is very computationally expensive.

An answer to the problems above might come in the form of machine learning (ML) through neural networks (NNs). ML is the process where a machine/program's performance in some task, measured with a defined measurement index, improves over time as the machine/program gain "experience" in said task (Mitchell, 1997). ML is often used to approximate and hence make predictions of some trends/behaviours in data without needing to fully understand the origin and structure of the data itself, given a vast enough volume of data the learning algorithm can train on.

NN is a form of ML algorithm which consists of multi-layered net of "nodes" or "neurons", each with individual inputs and outputs. The outputs of each node is determined by the node-specific weights and biases and activation functions, operating on its inputs. Figure 2 displays an example of a 4-layer (2 hidden layers) NN. Cybanko's universal approximation theorem suggests that a sufficiently large, single hidden layer feedforwrad NN with finite number of nodes is able to approximate any continuous function (Cybenko, 1989), which includes the correlations between stellar parameters and observables in concern of this project. Our NN, once built, will be trained on a conversion grid between stellar parameters and observables, validated by a loss function that compares the NN and the grid's values. The perk of using a NN over the previous two methods is that, once properly trained, the NN will be able to return the stellar paramenters of a given set of observables of a star in a matter of seconds with potentially the same level of accuracy, according to previous studies that applied NN to other stellar populations:

Hendriks and Aerts (2019) employed a multi-layered NN in forward modelling manner to bridge stellar parameters and oscillation frequencies of coherent modes of relatively high mass stars (2-20 $M_{\odot}$). Bellinger et al. (2016) used random forests, another technique of ML, to backwards model main sequence solar-like stars. Verma et al. (2016) used a multi-layered NN in backward modelling for solar-like stars as well. All of these studies commonly concluded their ML methods provided accurate and much faster results to a star's stellar parameters given a set of observables. Hence, we would replace the normal grid interpolation method linking between "modelled true parameters" and "modelled true observables" in figure 1 with a supervised NN.

## 3 Results and Implications

The age estimations of 31 red giants in open cluster M67 were found in Stello et al. (2016). With a simple BHM, we were able to estimate the stars to have a mean age $3.90 \pm 0.32$ Gyr and a spread $1.01 \pm 0.42$ Gyr. The mean value very much agrees with previous findings of 3.6-4.6 Gyr in VandenBerg and Stetson (2004) and 3.8-4.3 Gyr in Barnes et al. (2016), it is also not far from the value $3.46 \pm 0.13$ Gyr deduced by Stello et al. (2016), but the spread is a bit larger than estimated in those studies. After training our NN and setting up the BHM framework, we expect to be able to resolve the mean, spread and individual stellar ages of many more open clusters from just astroseismic data, in best case scenario down to a few Myr of uncertainty. This vast number of accurate stellar age estimations, would allow the further study of any age-related problems.

## 4 Timeline

1. Autumn week 1-4: background research and proposal writing
2. Autumn week 5-7: BHM on M67
3. Autumn week 8-11: research for grid/stellar models for NN
4. Term break: building NN and optimizing its architecture
5. Spring week 1-5: training NN with validating algorithm (loss function) and program, fine tuning its structure, getting results from full BHM framework
6. Spring week 6-9: potential extension of the project
7. Spring week 10-11: final write up

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