计算物理期末作业

更新电场和磁场

根据下图交替更新电场和磁场，因为python里的稀疏矩阵没有四阶张量，而且最后也没有搞定这四个四阶张量的对角化，所以只做了直接把这$H^{(x)}$和$H^{(y)}$交替作用了上去

$$\frac{\partial}{\partial t}\Psi_{TM}(t) = H\Psi_{TM}(t) = \sum_{i=1}^{n_x-2}{}' \sum_{j=1}^{n_y-2}{}' \left[H^{(x)}(i,j) + H^{(y)}(i,j)\right]\Psi_{TM}(t), \tag{2.77}$$

where

$$H^{(x)}(i,j) = +\frac{\mathbf{e}_{i,j+1}\mathbf{e}^T_{i+1,j+1} - \mathbf{e}_{i+1,j+1}\mathbf{e}^T_{i,j+1}}{\delta\sqrt{\varepsilon_{i+1,j+1}\mu_{i,j+1}}} + \frac{\mathbf{e}_{i+1,j+1}\mathbf{e}^T_{i+2,j+1} - \mathbf{e}_{i+2,j+1}\mathbf{e}^T_{i+1,j+1}}{\delta\sqrt{\varepsilon_{i+1,j+1}\mu_{i+2,j+1}}}, \tag{2.78a}$$

$$H^{(y)}(i,j) = -\frac{\mathbf{e}_{i+1,j}\mathbf{e}^T_{i+1,j+1} - \mathbf{e}_{i+1,j+1}\mathbf{e}^T_{i+1,j}}{\delta\sqrt{\varepsilon_{i+1,j+1}\mu_{i+1,j}}} - \frac{\mathbf{e}_{i+1,j+1}\mathbf{e}^T_{i+1,j+2} - \mathbf{e}_{i+1,j+2}\mathbf{e}^T_{i+1,j+1}}{\delta\sqrt{\varepsilon_{i+1,j+2}\mu_{i+1,j+1}}}, \tag{2.78b}$$

```python
from matplotlib import pyplot as plt
import numpy as np
#更新H
def update_H(Psi, dt):
    i_forward_Psi = np.zeros_like(Psi)
    i_forward_Psi[1:] = Psi[:-1]
    i_backward_Psi = np.zeros_like(Psi)
    i_backward_Psi[:-1] = Psi[1:]
    j_forward_Psi = np.zeros_like(Psi)
    j_forward_Psi[:,1:] = Psi[:,:-1]
    j_backward_Psi = np.zeros_like(Psi)
    j_backward_Psi[:,:-1] = Psi[:,1:]
    Psi[1::2,::2] += -dt * (j_backward_Psi - j_forward_Psi)[1::2,::2] / dx  #H_x
    Psi[::2,1::2] +=  dt * (i_backward_Psi - i_forward_Psi)[::2,1::2] / dy  #H_y
    Psi[1,::2] = Psi[-2,::2] = Psi[::2,1] = Psi[::2,-2] = 0
#更新E
def update_E(Psi, dt):
    i_forward_Psi = np.zeros_like(Psi)
    i_forward_Psi[1:] = Psi[:-1]
    i_backward_Psi = np.zeros_like(Psi)
    i_backward_Psi[:-1] = Psi[1:]
    j_forward_Psi = np.zeros_like(Psi)
    j_forward_Psi[:,1:] = Psi[:,:-1]
    j_backward_Psi = np.zeros_like(Psi)
    j_backward_Psi[:,:-1] = Psi[:,1:]
    Psi[1::2,1::2] += dt * (i_backward_Psi - i_forward_Psi)[1::2,1::2] / dx - dt
* (j_backward_Psi - j_forward_Psi)[1::2,1::2] /dy #E_z
    Psi[1,::2] = Psi[-2,::2] = Psi[::2,1] = Psi[::2,-2] = 0
```

生成波包

初始波包由下面的公式表示：

$$f(x,y,t) = \sin(k(x-x_0-ct))\exp[-((x-x_0-ct)/\sigma_x)^{10} - ((y-y_0)/\sigma_y)^2] \tag{A.28}$$

$$a_{nm} = \frac{4}{\omega L_x L_y} \int_0^{L_x} dx \int_0^{L_y} dy \sin(k_n x) \sin(k_m y) \frac{\partial}{\partial t} f(x, y, t)|_{t=0}, \tag{A.30a}$$

$$b_{nm} = \frac{4}{L_x L_y} \int_0^{L_x} dx \int_0^{L_y} dy \sin(k_n x) \sin(k_m y) f(x, y, t = 0). \tag{A.30b}$$

$$E_z(x, y, t) = \sum_{nm} \sin(k_n x) \sin(k_m y)[a_{nm} \sin(\omega t) + b_{nm} \cos(\omega t)], \tag{A.29a}$$

$$H_x(x, y, t) = \sum_{nm} \frac{ck_m}{\omega} \sin(k_n x) \cos(k_m y)[a_{nm} \cos(\omega t) - b_{nm} \sin(\omega t)], \tag{A.29b}$$

$$H_y(x, y, t) = -\sum_{nm} \frac{ck_n}{\omega} \cos(k_n x) \sin(k_m y)[a_{nm} \cos(\omega t) - b_{nm} \sin(\omega t)], \tag{A.29c}$$

波包参数

```
#初始波包设置,与PDF中一致
Lx = 18
Ly = 12
dx = 0.1
dy = 0.1
x0 = 3.05
y0 = 6.0
c = 1
sigmax = 2.75
sigmay = 2.0
k = 5

#生成格点
x_even = np.arange(dx/2, Lx, dx)
y_even = np.arange(dy/2, Ly, dy)
x_odd = np.arange(0, Lx+dx, dx)
y_odd = np.arange(0, Ly+dy, dy)
X_mn, Y_mn = np.meshgrid(x_odd, y_odd, indexing='ij')
X_Ez, Y_Ez = np.meshgrid(x_even, y_even, indexing='ij')
X_Hx, Y_Hx = np.meshgrid(x_even, y_odd, indexing='ij')
X_Hy, Y_Hy = np.meshgrid(x_odd, y_even, indexing='ij')
n = np.arange(1,51,1)
m = np.arange(1,51,1)
N, M = np.meshgrid(n,m, indexing='ij')

nx = int(Lx/dx)*2 + 1  #x方向格点数
ny = int(Ly/dy)*2 + 1  #y方向格点数
omega = c*np.sqrt((N*np.pi/Lx)**2+(M*np.pi/Ly)**2)
```

计算波包的电磁场

```
def f(x,y,t): #初始波包
    return np.sin(k*(x-x0-t))*np.exp(-((x-x0-t)/sigmax)**10-((y-y0)/sigmay)**2)
def get_anm(n,m): #a_nm
    return np.sum(np.sin(n*np.pi/Lx*X_mn)*np.sin(m*np.pi/Ly*Y_mn)*
((f(X_mn,Y_mn,0.0000001)-f(X_mn,Y_mn,0))/0.0000001)) *
4/(np.sqrt((n*np.pi/Lx)**2+(m*np.pi/Ly)**2)*Lx*Ly)
def get_bnm(n,m): #b_nm
```

```
    return
np.sum(np.sin(n*np.pi/Lx*X_mn)*np.sin(m*np.pi/Ly*Y_mn)*f(X_mn,Y_mn,0)) *
4/(Lx*Ly)
def get_Ez(x,y,t): #电场z分量
    return np.sum(np.sin(N*np.pi/Lx*x)*np.sin(M*np.pi/Ly*y)*
(anm*np.sin(omega*t)+bnm*np.cos(omega*t)))
def get_Hx(x,y,t): #磁场x分量
    return np.sum(c*M*np.pi/Ly/omega*np.sin(N*np.pi/Lx*x)*np.cos(M*np.pi/Ly*y)*
(anm*np.cos(omega*t)-bnm*np.sin(omega*t)))
def get_Hy(x,y,t): #磁场y分量
    return np.sum(-c*N*np.pi/Lx/omega*np.cos(N*np.pi/Lx*x)*np.sin(M*np.pi/Ly*y)*
(anm*np.cos(omega*t)-bnm*np.sin(omega*t)))

#向量化
v_anm = np.vectorize(get_anm)
v_bnm = np.vectorize(get_bnm)
v_Ez = np.vectorize(get_Ez)
v_Hx = np.vectorize(get_Hx)
v_Hy = np.vectorize(get_Hy)
```

```
#计算a_nm, b_nm
anm = v_anm(N, M)
bnm = v_bnm(N, M)
#制备初态电磁场
Ez_init = v_Ez(X_Ez,Y_Ez,t=0)
Hx_init = v_Hx(X_Hx,Y_Hx,t=0)
Hy_init = v_Hy(X_Hy,Y_Hy,t=0)
#制备末态电磁场
Ez_final = v_Ez(X_Ez,Y_Ez,t=10)
Hx_final = v_Hx(X_Hx,Y_Hx,t=10)
Hy_final = v_Hy(X_Hy,Y_Hy,t=10)
```

PDF中波包在$t=0$和$t=10$时刻是:

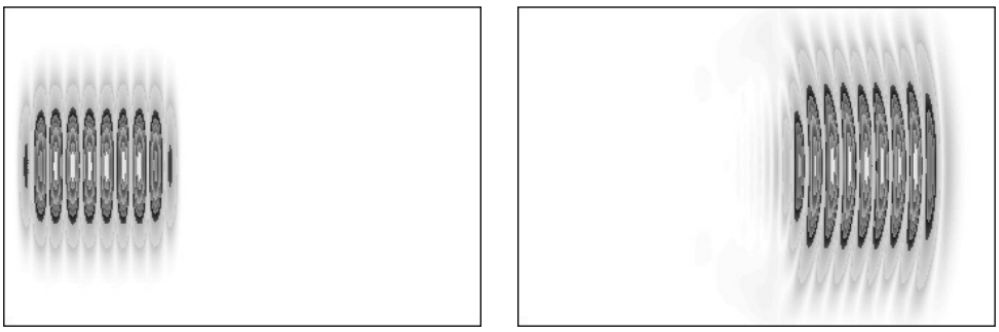
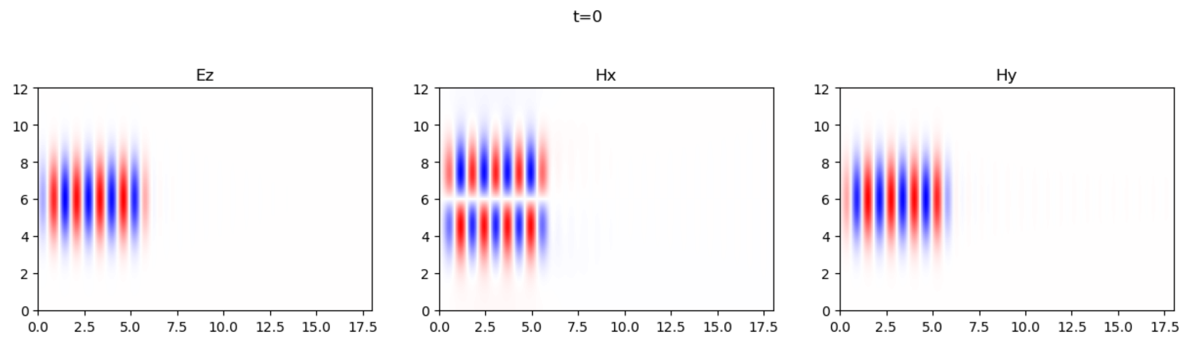


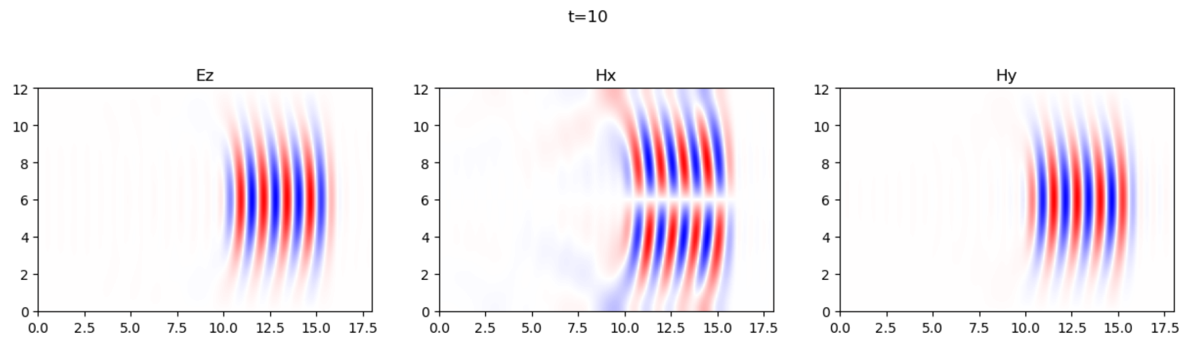
Figure A-2: Example of a wavepacket in two dimensions. The system measures $L = 18\times 12$, with a mesh spacing $\delta = 0.1$. Left: energy distribution density of the initial wavepacket, with widths $\sigma = (2.75, 2.0)^T$, centered at $\mathbf{r}_0 = (3.5, 6.0)^T$ and wave-number $k = 5$. Right: energy distribution at $t = 10$, after integration with the Chebyshev algorithm ($\kappa = 10^{-13}$).

绘制计算出的波包

```python
#画初始波包
fig = plt.figure(figsize=(15,4))
axes = fig.subplots(1, 3)
axes[0].imshow(Ez_init.T, cmap='bwr', origin='lower', extent=[0, 18, 0, 12])
axes[1].imshow(Hx_init.T, cmap='bwr', origin='lower', extent=[0, 18, 0, 12])
axes[2].imshow(Hy_init.T, cmap='bwr', origin='lower', extent=[0, 18, 0, 12])
axes[0].set_title('Ez')
axes[1].set_title('Hx')
axes[2].set_title('Hy')
fig.suptitle('t=0')
```

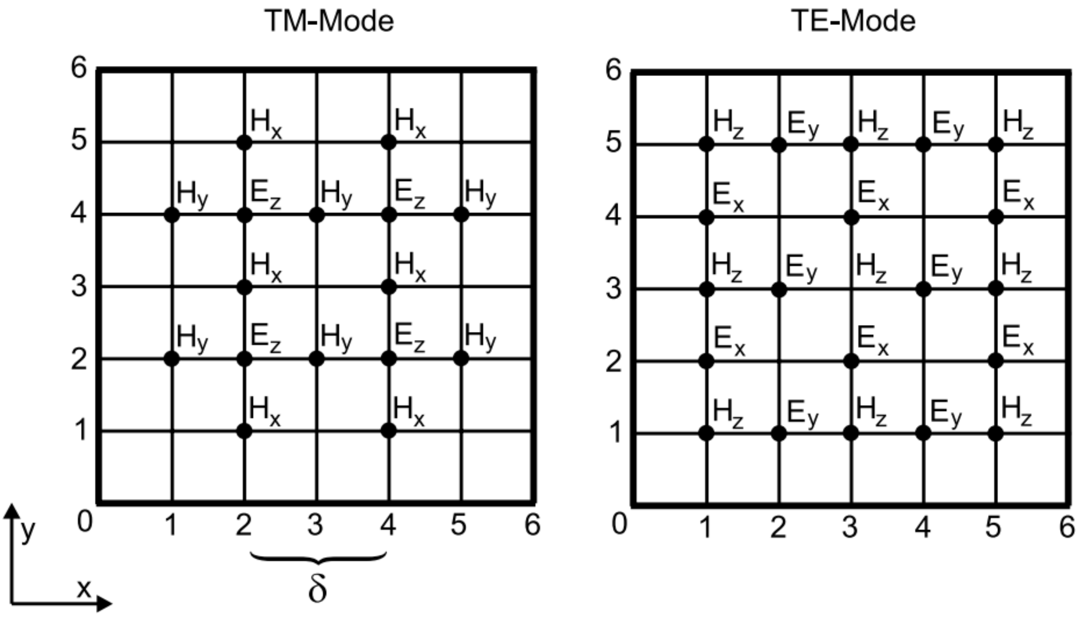



```python
#画t=10时刻的波包
fig = plt.figure(figsize=(15,4))
axes = fig.subplots(1, 3)
axes[0].imshow(Ez_final.T, cmap='bwr', origin='lower', extent=[0, 18, 0, 12])
axes[1].imshow(Hx_final.T, cmap='bwr', origin='lower', extent=[0, 18, 0, 12])
axes[2].imshow(Hy_final.T, cmap='bwr', origin='lower', extent=[0, 18, 0, 12])
axes[0].set_title('Ez')
axes[1].set_title('Hx')
axes[2].set_title('Hy')
fig.suptitle('t=10')
```



波包的演化

按TM-Mode的格点排列方式把E_z、H_x和H_y置入Ψ中




```
#用把E和H填入\Psi
Psi = np.zeros((nx, ny))
Psi[1::2,1::2] = Ez_init  #偶数项是E_z
Psi[1::2,::2] = Hx_init #i偶j奇是H_x
Psi[::2,1::2] = Hy_init #i奇j偶是H_y
```

将update_H和update_E两个函数交替作用在Ψ上，然后绘图

```
dt = 0.01
Psi_odd = np.copy(Psi)
fig = plt.figure(figsize=(15,4))
axes = fig.subplots(1, 3)
for n_frame in range(100):  #生成100张图像
    for i in range(20):     #每张时间间隔dt*20 = 0.2
        update_H(Psi, dt)
        update_E(Psi, dt)

    axes[0].imshow(Psi[1::2, 1::2].T, cmap='bwr', origin='lower', extent=[0, 18,
0, 12], vmin=-120, vmax=120)
    axes[1].imshow(Psi[1::2,::2].T, cmap='bwr', origin='lower', extent=[0, 18,
0, 12], vmin=-10, vmax=10)
    axes[2].imshow(Psi[::2,1::2].T, cmap='bwr', origin='lower', extent=[0, 18,
0, 12], vmin=-120, vmax=120)
    axes[0].set_title('Ez')
    axes[1].set_title('Hx')
    axes[2].set_title('Hy')
    fig.suptitle('t=%.1f'%(0.2*(n_frame+1)))
    fig.savefig(fname='./picture/t=%.1f.png'%(0.2*(n_frame+1)))
```

```
#输出成动图
from PIL import Image
image_files = ['./picture/t=%.1f.png'%(0.2*(n_frame+1)) for n_frame in
range(100)]
images = [Image.open(image) for image in image_files]
images[0].save('output.gif', save_all=True, append_images=images[1:],
optimize=False, duration=100, loop=0)
```

最后结果输出成了动图，以下是其中一些时刻的电磁场