

Exercise 2.2.4

Consider $F = \{D \rightarrow BK, AB \rightarrow GK, B \rightarrow H, CE \rightarrow AG, H \rightarrow E, K \rightarrow G, EH \rightarrow K, G \rightarrow AH\}$

Prove by Amstrong:

a) $F \models AB \rightarrow GH$

b) $F \models DE \rightarrow AG$

c) $F \models BH \rightarrow EK$

a/ $AB \rightarrow GK \models (f.r\tilde{a}) AB \rightarrow G \text{ (1)}$

$B \rightarrow H \models (t.t) AB \rightarrow AH \models (f.r\tilde{a}) AB \rightarrow H \text{ (2)}$

$(1), (2) \models (h.) AB \rightarrow GH \blacksquare$

b/ $D \rightarrow BK \models (t.t) DE \rightarrow BEK \text{ (1)}$

$K \rightarrow G \models (t.t) BEK \rightarrow GEK \text{ (2)}$

$(1), (2) \models (b.c) DE \rightarrow GEK \models (f.r\tilde{a}) DE \rightarrow G \text{ (3).}$

$D \rightarrow BK \models (f.r\tilde{a}) D \rightarrow K \text{ (4)}$

$K \rightarrow G \text{ (given) (5)}$

$(4), (5) \models (b.c) D \rightarrow G \text{ (6)}$

$G \rightarrow AH \text{ (given) (7)}$

$(6), (7) \models (b.c) D \rightarrow AH \models (f.r\tilde{a}) D \rightarrow A \models (t.t) DE \rightarrow AE \models (f.r\tilde{a}) DE \rightarrow A \text{ (8).}$

$(3), (8) \models (h.) DE \rightarrow AG \blacksquare$

c/ $B \rightarrow H \models (t.t) BH \rightarrow H \text{ (1)}$

$H \rightarrow E \text{ (2) (given)}$

$(1), (2) \models (b.c) BH \rightarrow E \text{ (3).}$

$B \rightarrow H, H \rightarrow E \models (b.c) B \rightarrow E \models (t.t) BH \rightarrow EH \text{ (4).}$

$EH \rightarrow K \text{ (given) (5)}$

$(4), (5) \models (b.c) BH \rightarrow K \text{ (6)}$

$(3), (6) \models (h.) BH \rightarrow EK \blacksquare$