Distances in cosmology - what we can learn from them

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Some Online Research Resources

- **http://arxiv.org** - an excellent resource of papers.
- **ADS -http://adsabs.harvard.edu/** – astro paper database
- **SPIRES** – High energy physics database
- **www.sdss.org (http://cas.sdss.org/)** - the SDSS survey and skyserver
- **SIMBAD** – looking for specific objects?
- **http://www.cosmocoffee.com** - cosmology discussion forum, jobs etc…
- **http://scholar.google.com** - general academic resource

Redshift…

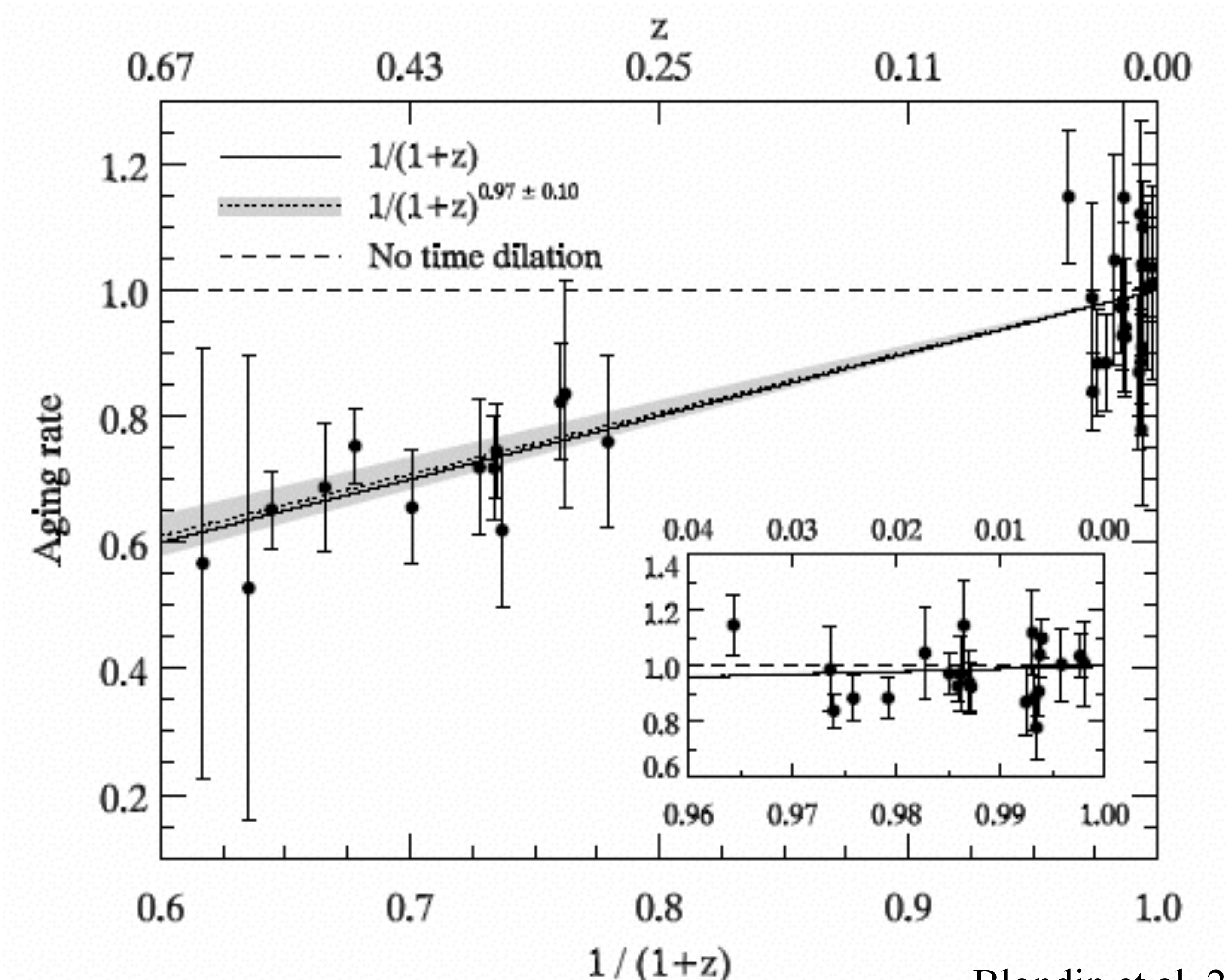
$$z := \frac{\nu_e - \nu_o}{\nu_o},$$

As the universe expands, wavelengths are stretched…it is why the Early universe was radiation dominated (why?)

$$\boxed{1 + z = \frac{a(t_o)}{a(t_e)}.}$$

1+z is the number of times smaller the cosmos was at that time…

Testing cosmic time-dilation with SNIa



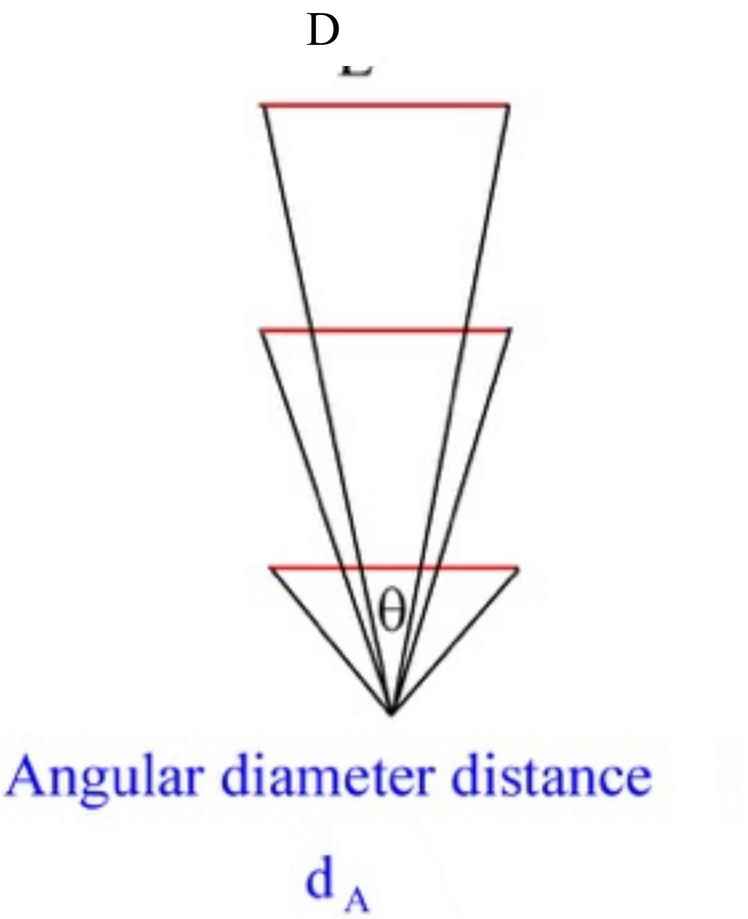
Blondin et al, 2008

Distances…

- Angular-diameter distance (area distance)

$$\boxed{d_A = \frac{D}{\theta}}$$

It is the way we usually measure distances on earth!



Angular diameter distance

- Need "standard rulers" – how can we estimate D?

- E.g. Baryon Acoustic Oscillations

  X-ray clusters

  radio galaxies

  compact radio sources.

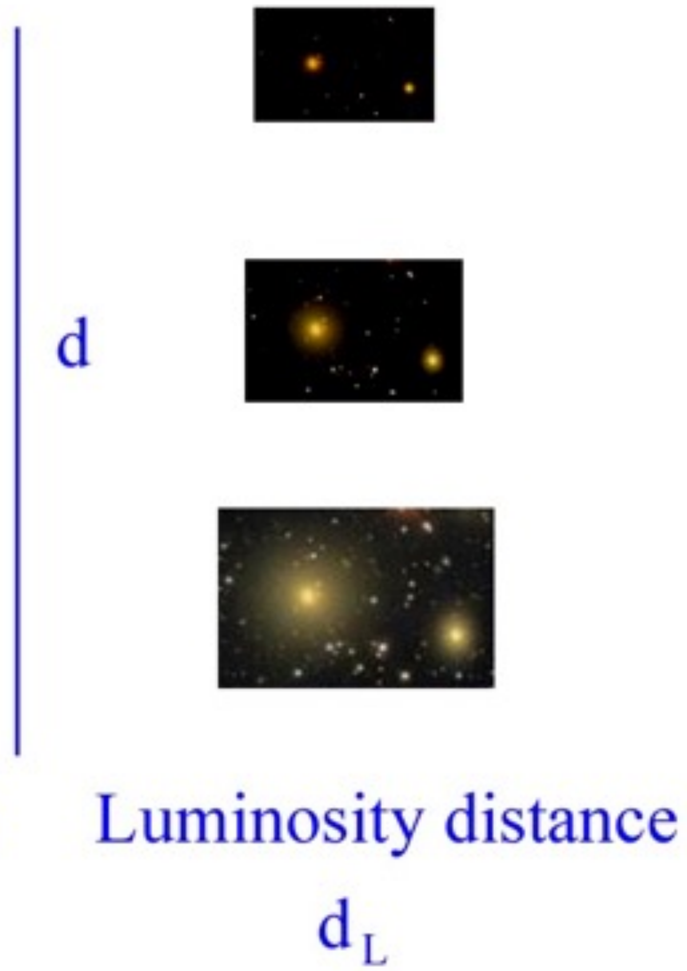
$$d_A = \frac{D}{\theta}$$

All except the first (BAO) rely on complex nonlinear modeling so are not very reliable. We will discuss BAO in detail later in the course.

Luminosity distance

$$d_L = \left(\frac{L}{4\pi \cdot F} \right)^{1/2}$$

- L=intrinsic luminosity
- F = received flux



How to measure distances…

- d_L – "standard candles"

$$d_L = \left(\frac{L}{4\pi \cdot F} \right)^{1/2}$$

- The problem – we cant get the distance unless we know the intrinsic luminosity, *L*.

- SNIa – we can estimate *L* from the decline rate of the lightcurve (brighter SNIa decay faster – See next Lecture).

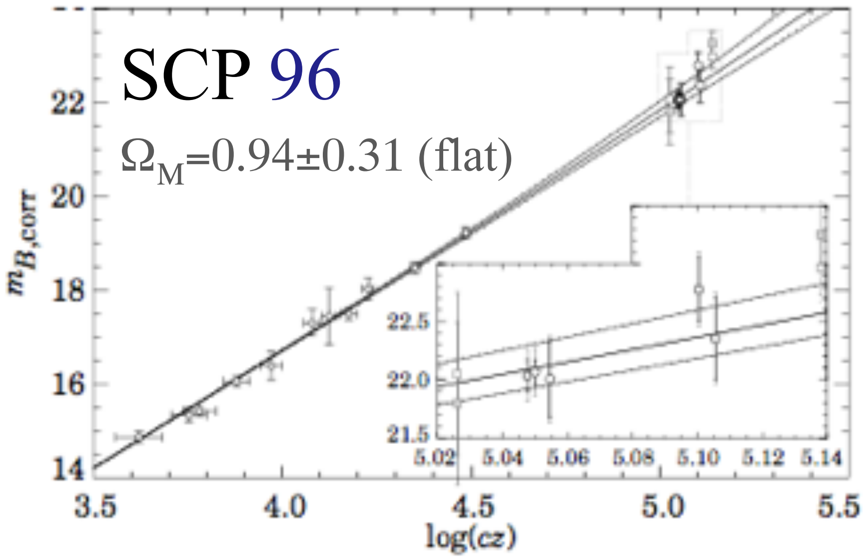
  (Philips, 1993).

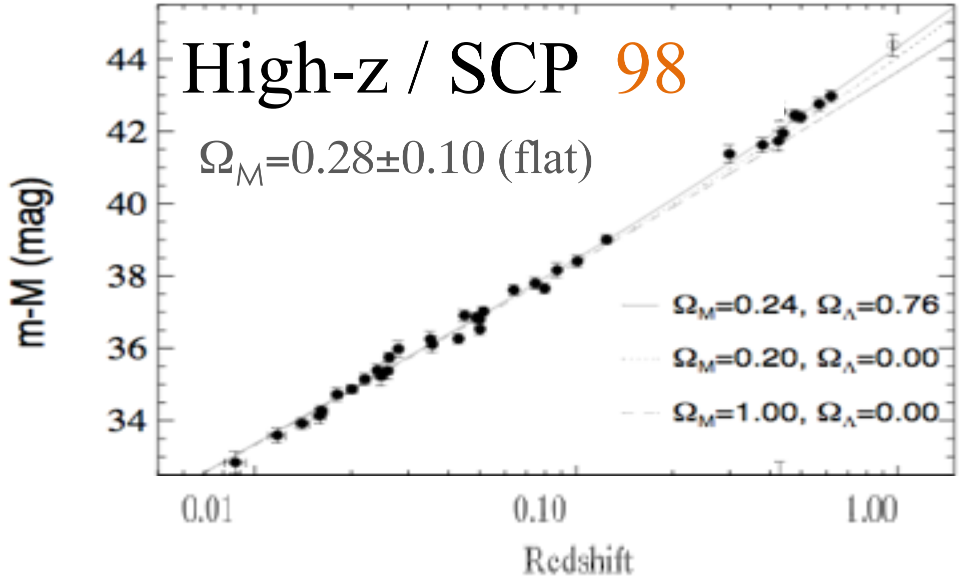
d_L in terms of magnitudes…

- In terms of *absolute (M)* and *apparent magnitudes (m)* we have

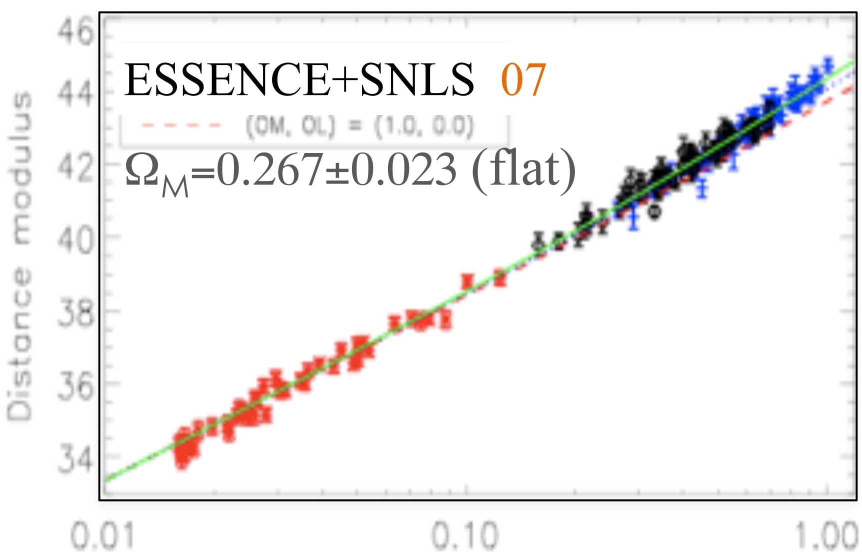
$$m - M = 5\log\left(\frac{d_L}{1\ Mpc}\right) + 25$$

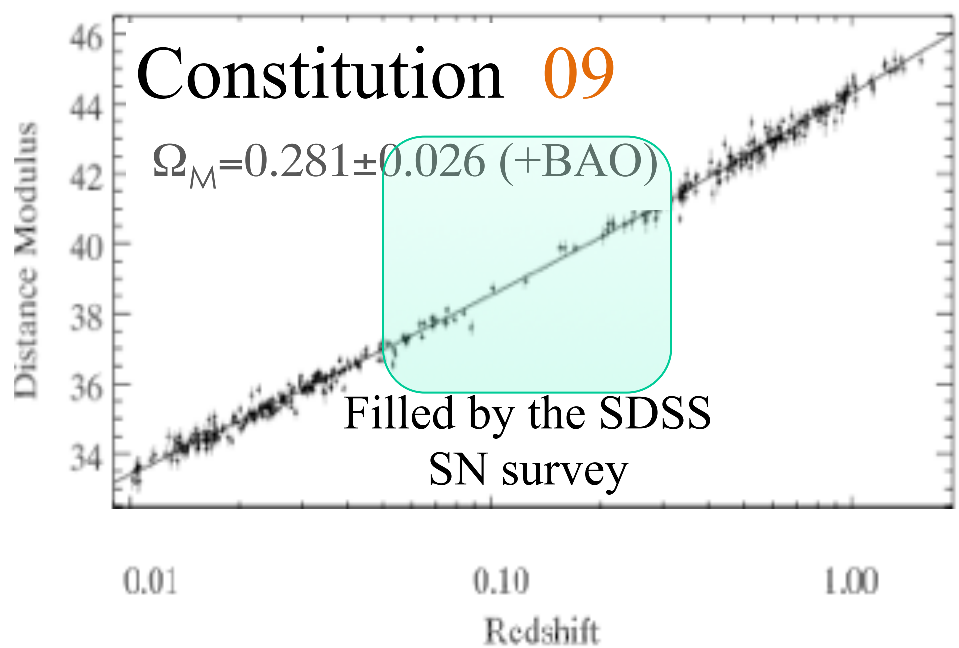
The quantity m-M is the distance modulus, usually denoted μ.



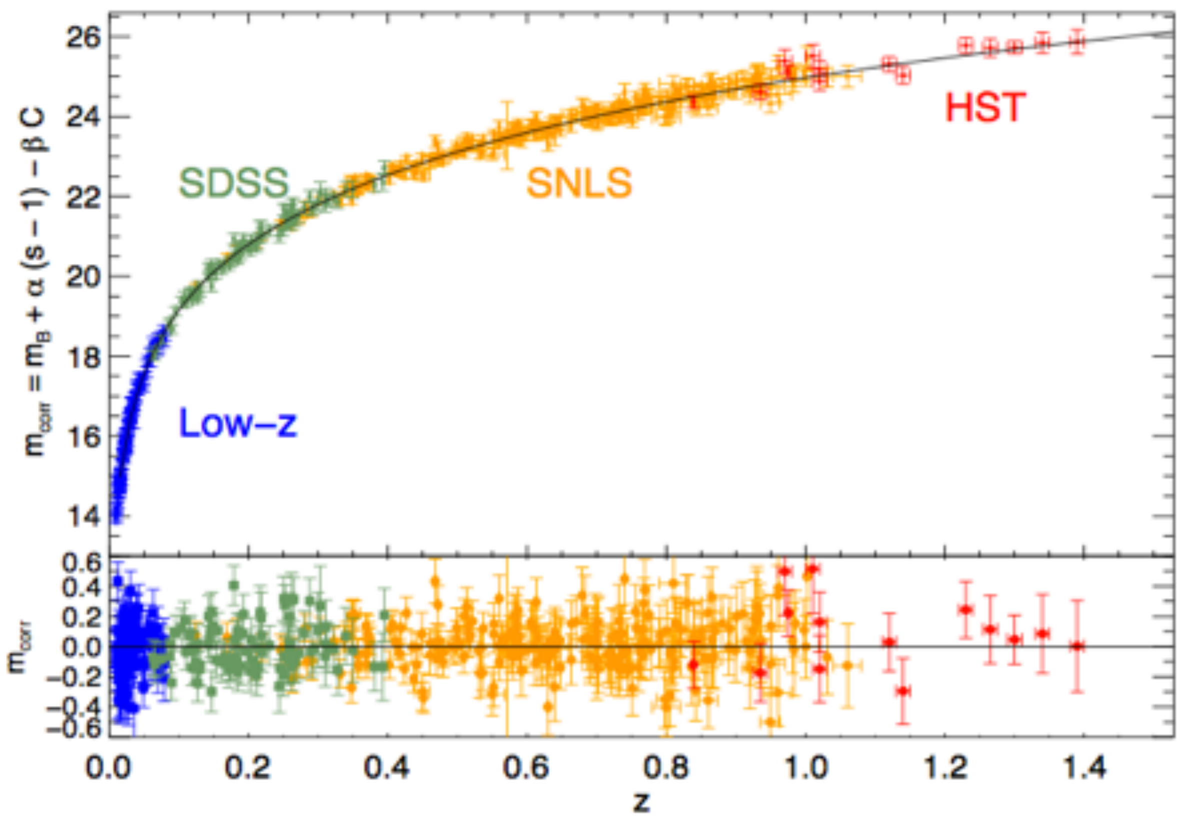


Evolution of the Hubble Diagram 1996-2009





Current state-of-the art



Conley et al, 2011

How are these two distances related?

Surely they must give the same result?

Not quite….but almost:

$$\boxed{d_L = (1+z)^2 d_A}$$

This “distance duality” is
true in any metric theory of gravity (not just General Relativity) as long as photon number is Conserved.

How do we use distances to learn about the cosmos?

The Angular Diameter Distance

$$d_A(z) = \frac{c}{H_0\sqrt{-\Omega_k}\,(1+z)} \sin\left(\sqrt{-\Omega_k} \int_0^z \frac{H_0}{H(z')} dz' \right)$$

c/H_0 is the overall Scale (Hubble scale)

Spatial Curvature (zero = flat)

Expansion rate of the Cosmos (energy densities)

The Luminosity Distance

$$d_L(z) = \frac{c(1+z)}{H_0\sqrt{-\Omega_k}} \sin\left(\sqrt{-\Omega_k} \int_0^z \frac{H_0}{H(z')} dz' \right)$$

What do distances tell us?

If the Universe is flat ($\Omega_k = 0$) then this simplifies to:

$$d_L(z) = (1+z)\frac{c}{H_0}\int \frac{dz'}{E(z')}$$

Distance we measure

What we want

$$E(z) \equiv \frac{H(z)}{H_0}$$

What do distances tell us?

Let's assume the Universe is flat (Ω_k = 0) to start with.

Taylor expanding to second order gives…

$$d_L(z) = \frac{c}{H_0}\left[z + \frac{1}{4}(1 + 3\Omega_\Lambda)z^2 \ldots\right]$$

The Hubble constant, H_0
Sets the overall *scale* for *all*
Distances (Hubble's law
is true for all models)

The *curvature* of the Hubble
Diagram depends on the
Amount of dark energy, Ω_Λ

What do distances tell us?

What happens when we don't assume flatness?

$$d_L(z) = \frac{(1+z)}{\sqrt{-\Omega_K}} \frac{c}{H_0} \sin\left(\sqrt{-\Omega_K} \underbrace{\int \frac{dz'}{E(z')}}_{\chi}\right)$$

Encodes the effect of the curved null geodesics …

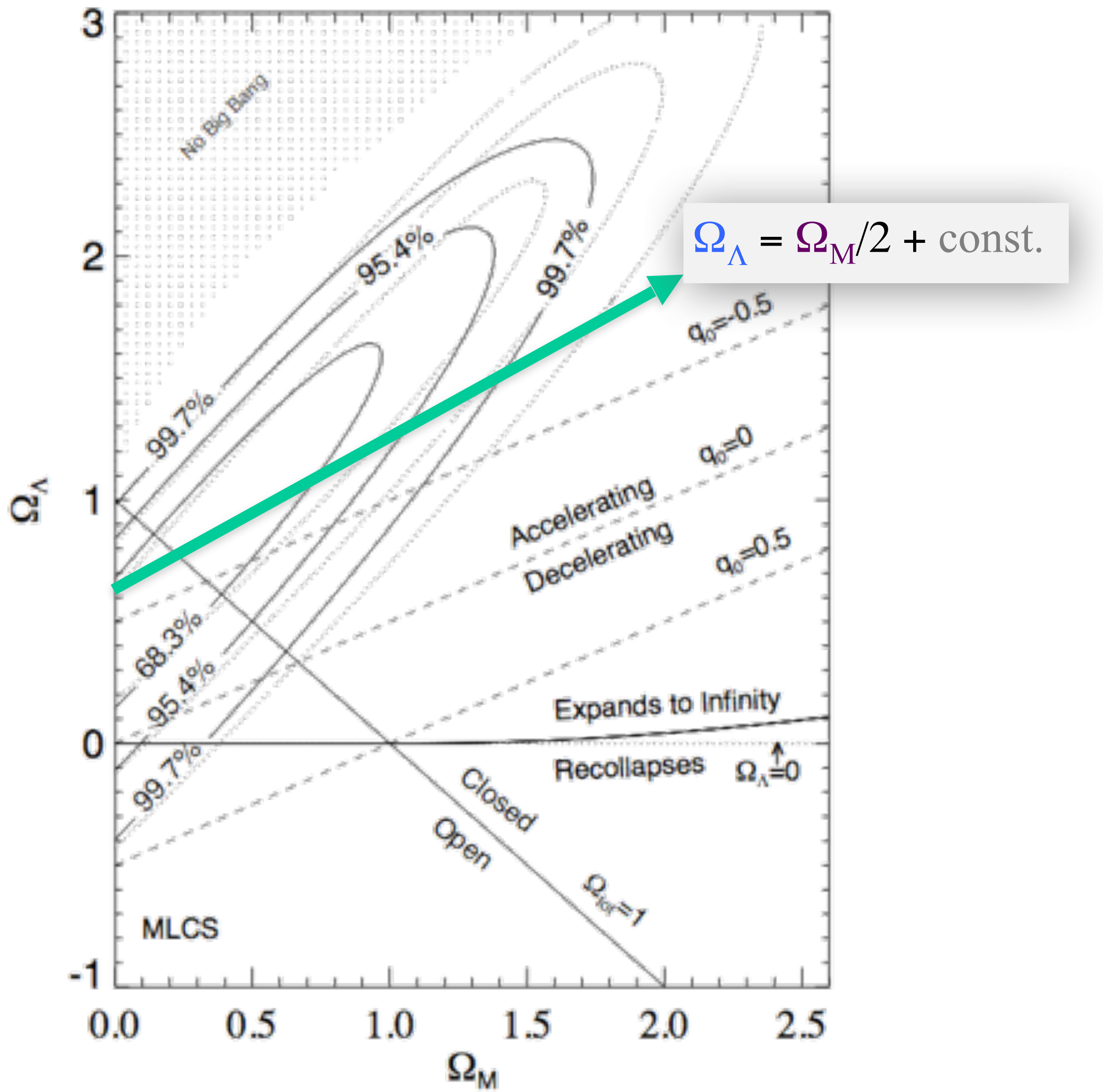
What do distances tell us?

What happens when we don't assume flatness?
Taylor expanding to second order gives…

$$d_L(z) = \frac{c}{H_0}\left[z + \frac{1}{4}(1 + \Omega_K + 3\Omega_\Lambda)z^2 \ldots\right]$$

The *curvature* of the Hubble Diagram now depends on the Amount of dark energy AND the spatial curvature, Ω_K

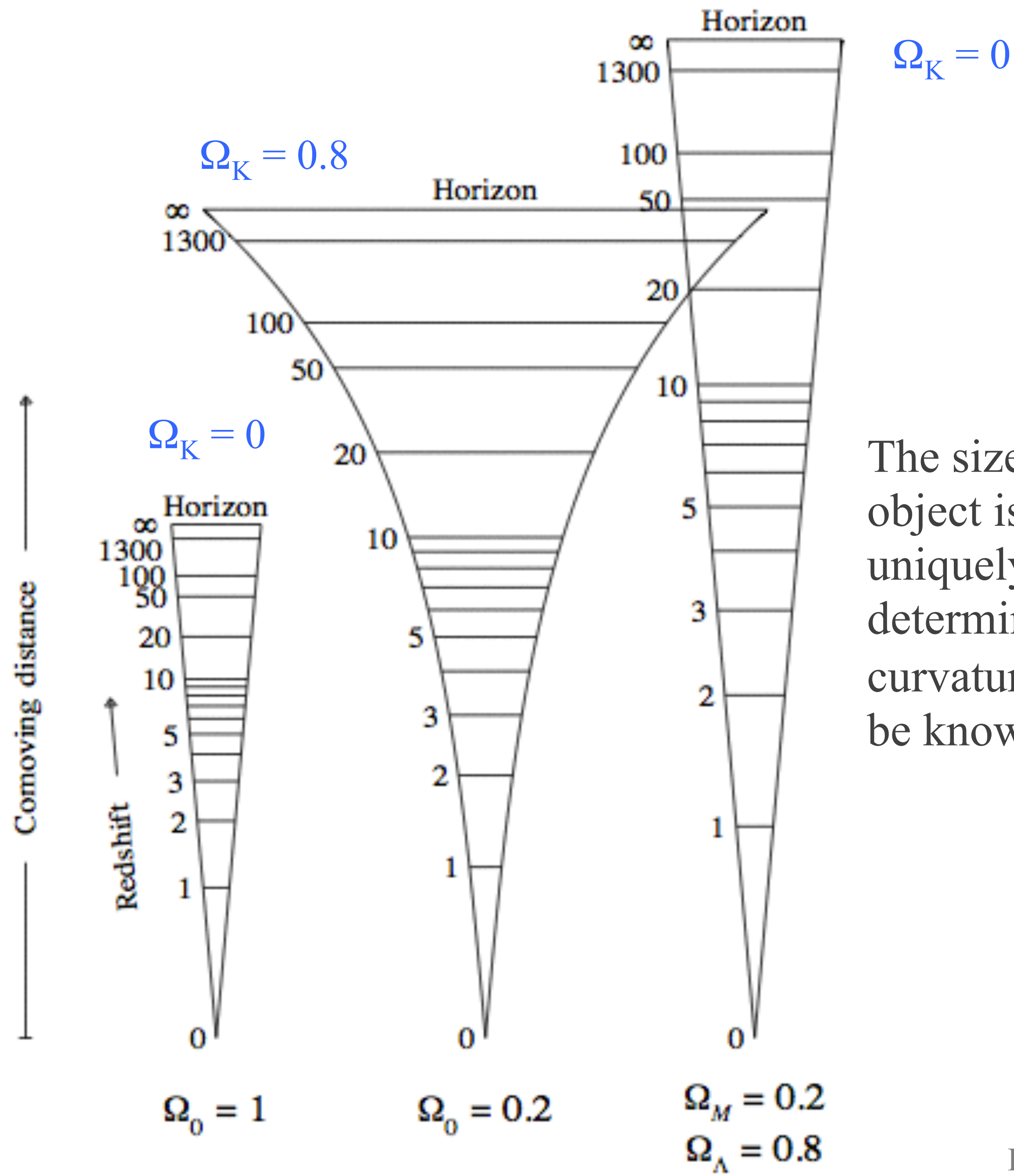
2nd order term is degenerate on the line:
$\Omega_\Lambda = \Omega_M/2 + \text{const.}$



Riess et al, 1998

Dynamics

$$\chi = \int \frac{dz'}{E(z')}$$



The size of an object is not uniquely determined by χ – curvature must also be known

Hamilton 97

Distances and densities..

We showed that in general:

$$d_A(z) = \frac{c}{(1+z)H_0\sqrt{-\Omega_k}} \sin\left(\sqrt{-\Omega_k} \int_0^z \frac{H_0}{H(z')} dz' \right)$$

Where $E(z) \equiv \frac{H(z)}{H_0}$ is the dimensionless

Hubble expansion rate, normalised to unity today.
This is why distances tell us about Dark Energy…

Exercise: derive the flat-space limit of this ($\Omega_k = 0$)

E is for "energy"…

- Where….

$$E^2 = \Omega_m (1+z)^3 + \Omega_{rad} (1+z)^4 + \Omega_k (1+z)^2 + \Omega_{DE} (1+z)^{3(1+w)}$$

In the case where $w = \frac{p}{\rho}$ of dark energy (DE) is assumed to be **constant in time**, and the Ω_i are the density parameters for the various components today.

Dynamical Dark Energy

- In general, *w* can be time-*dependent: w(z)*. Then instead we have

$$H^2(z) = H_0^2(\Omega_m(1+z)^3 + \Omega_{DE} f(z) + \Omega_k(1+z)^2)$$

$$f(z) = \exp\left(3\int_0^z \frac{1+w(z')}{1+z'}\,dz'\right)$$

- **Exercise:** Compute *f(z)* for the case $w(z) = w_0 + w_a\, z/(1+z)$

Geometry – Dynamics Degeneracy

- But note, that even with perfect **distance** measurements there is a perfect degeneracy between **the curvature** (Ω_k) and general **H(z)** (Weinberg, '73)

$$d_L(z) = \frac{(1+z)}{H_0\sqrt{-\Omega_k}} \sin\left(H_0\sqrt{-\Omega_k} \int \frac{dz'}{H(z')} \right)$$

Distance modulus and magnitudes

Apparent
Magnitude

$$\mu = m - M \equiv 5\log_{10}\left[\frac{d_L(z)}{1Mpc}\right] + 25$$

Distance
Modulus

Absolute
Magnitude

d_L converts M → m

Common in Supernova cosmology to plot μ or m_{B} vs redshift

d_L Wish list…

We want objects that are:

- Very Bright…so we can see them across the observable universe
- ~~Known: We know each object's intrinsic luminosity.~~
- Standard: no environmental dependence of the intrinsic luminosity
- Time-invariant: their intrinsic luminosity doesn't change with redshift
- Easy to find…
- Emit most of their light in the optical
  (counter example: binary black holes as GW sirens)

Problem: if we don’t know M
What can we do?

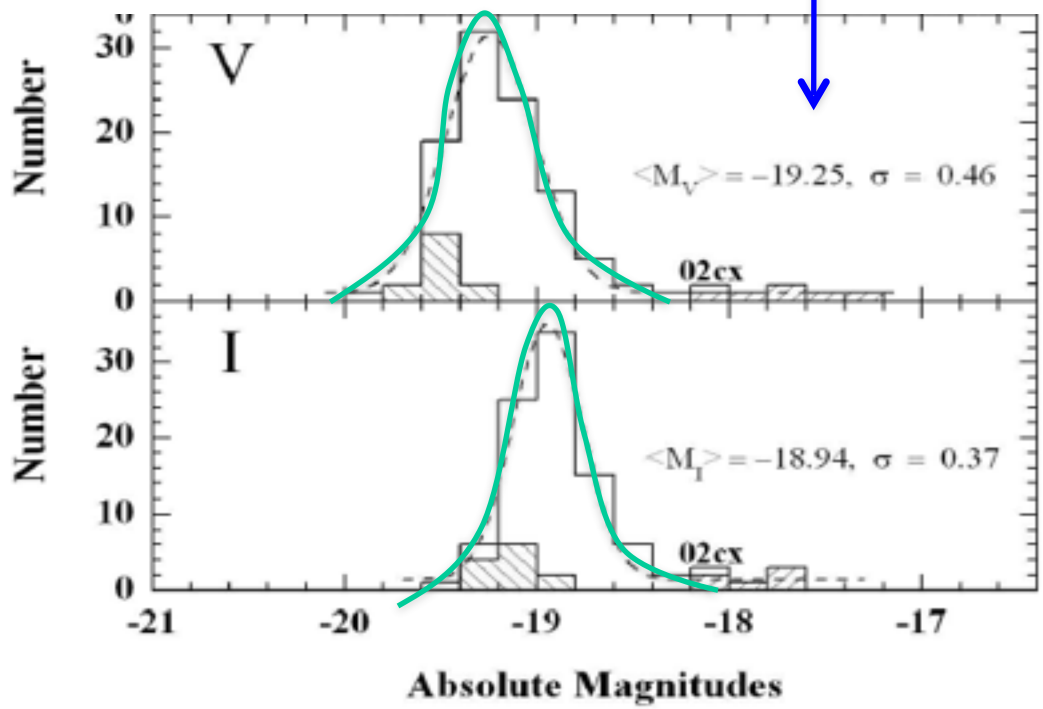
$$m = M + 25 + 5\log_{10} d_L(z)$$

$$= \underbrace{M + 25 + 5\log_{10}\left(\frac{c}{H_0\sqrt{-\Omega_k}}\right)}_{\text{Constants}} \underbrace{+ 5\log_{10}(1+z) + \boxed{5\log_{10}\left(\sin(\sqrt{-\Omega_k}\cdot \overbrace{\chi(z)}^{\int \frac{dz'}{E(z')}})\right)}}_{\text{Redshift dependent}}$$

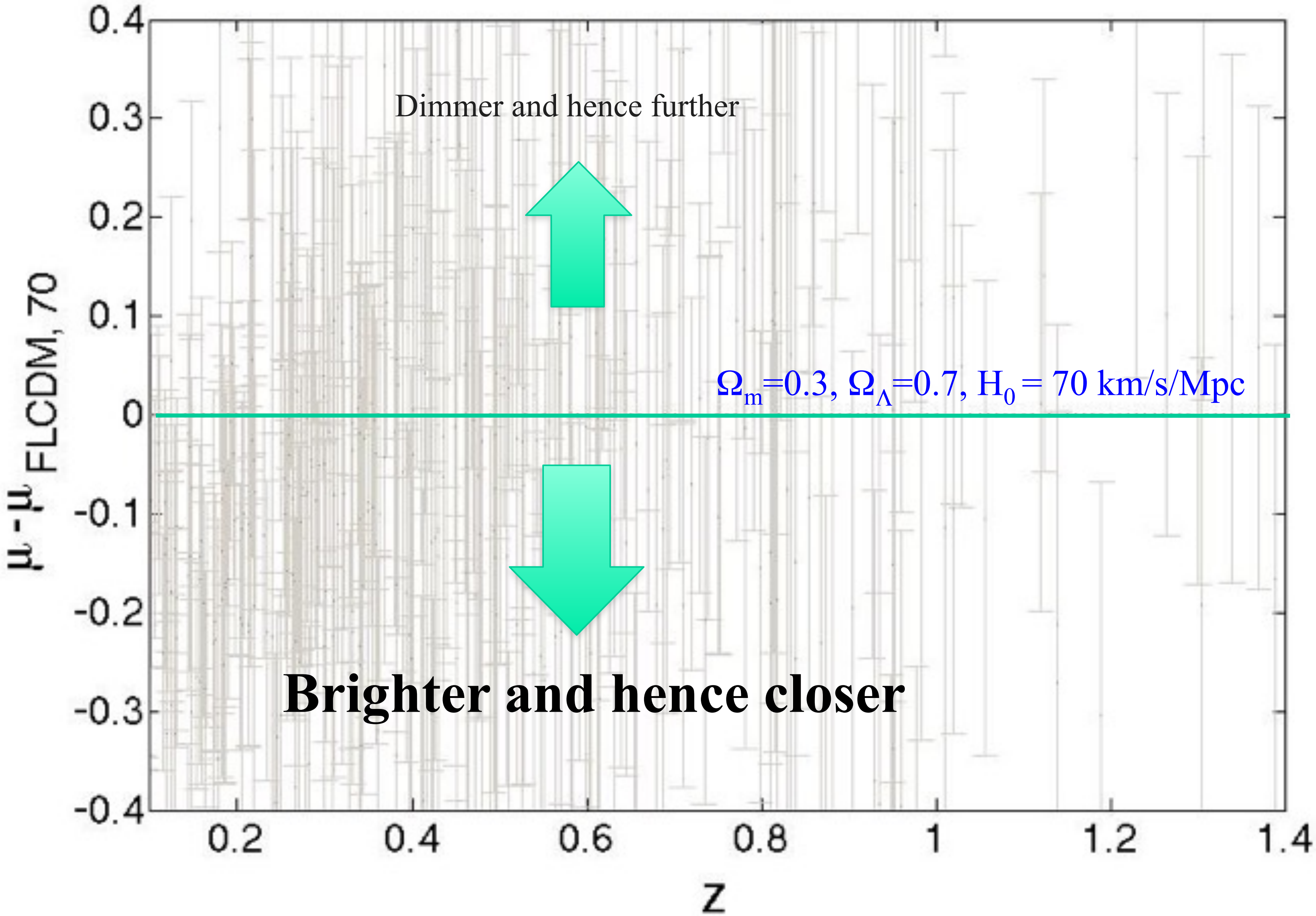
If we subtract the apparent magnitudes of two objects with the same M at different redshifts, all the unknown constants disappear…

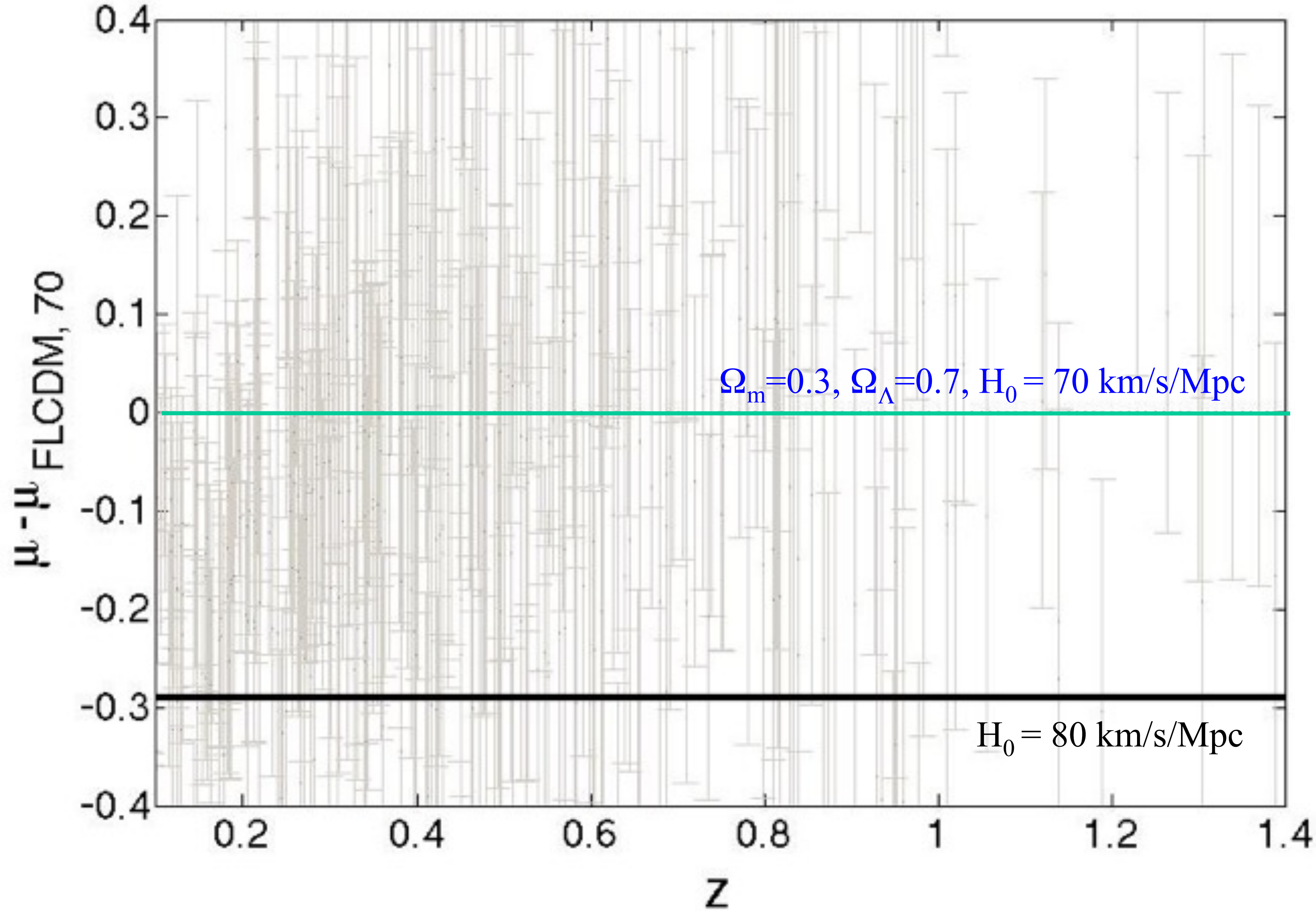
Type Ia Supernovae (SNIa)

- Very Bright…at peak they can outshine their host galaxies
- Stable: Don't seem to evolve with redshift
- Fairly standard:
  $\sigma = 0.4$ mag
- Easy to find…
  ~ 1 per century per galaxy

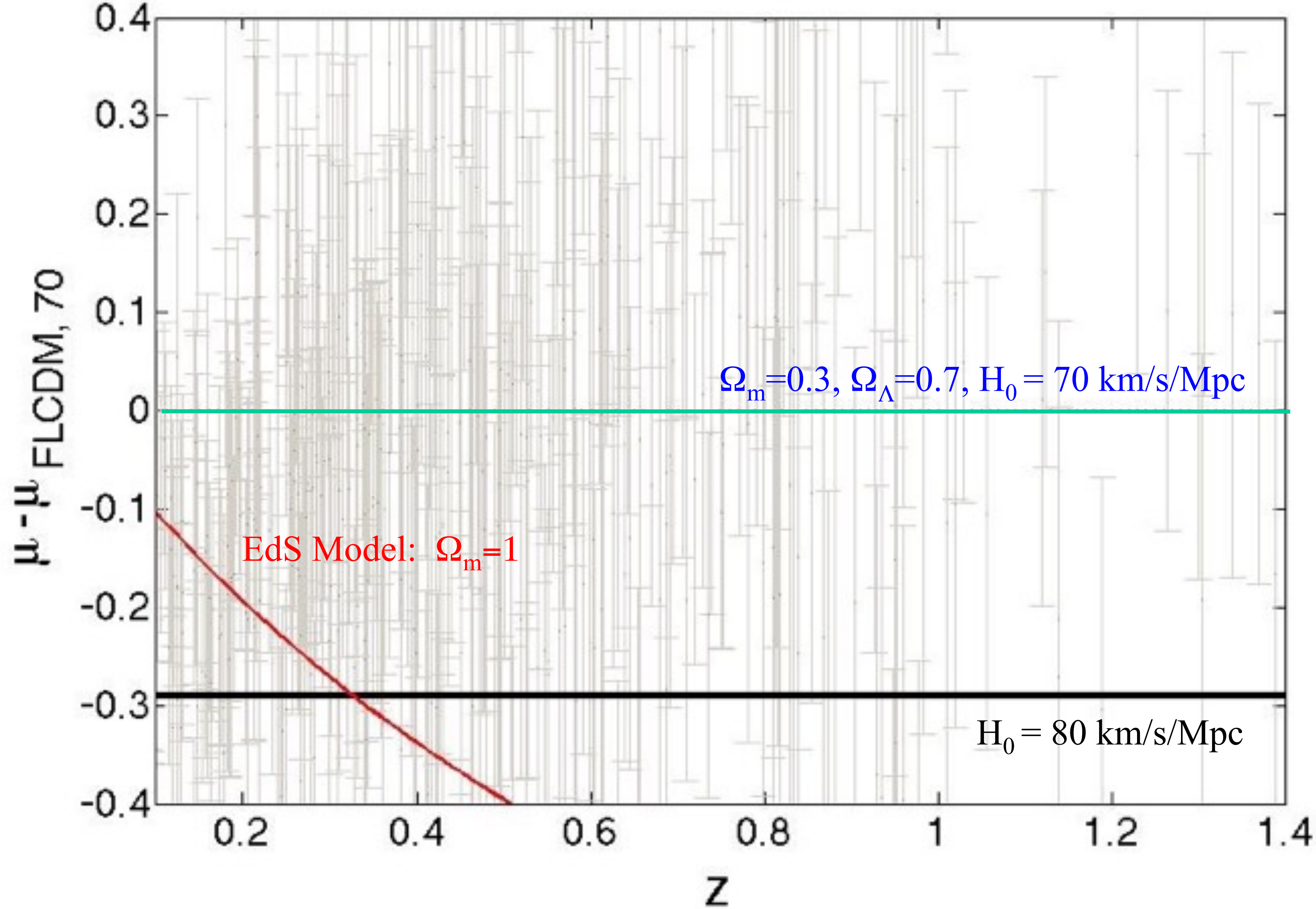


Relative Hubble Diagram

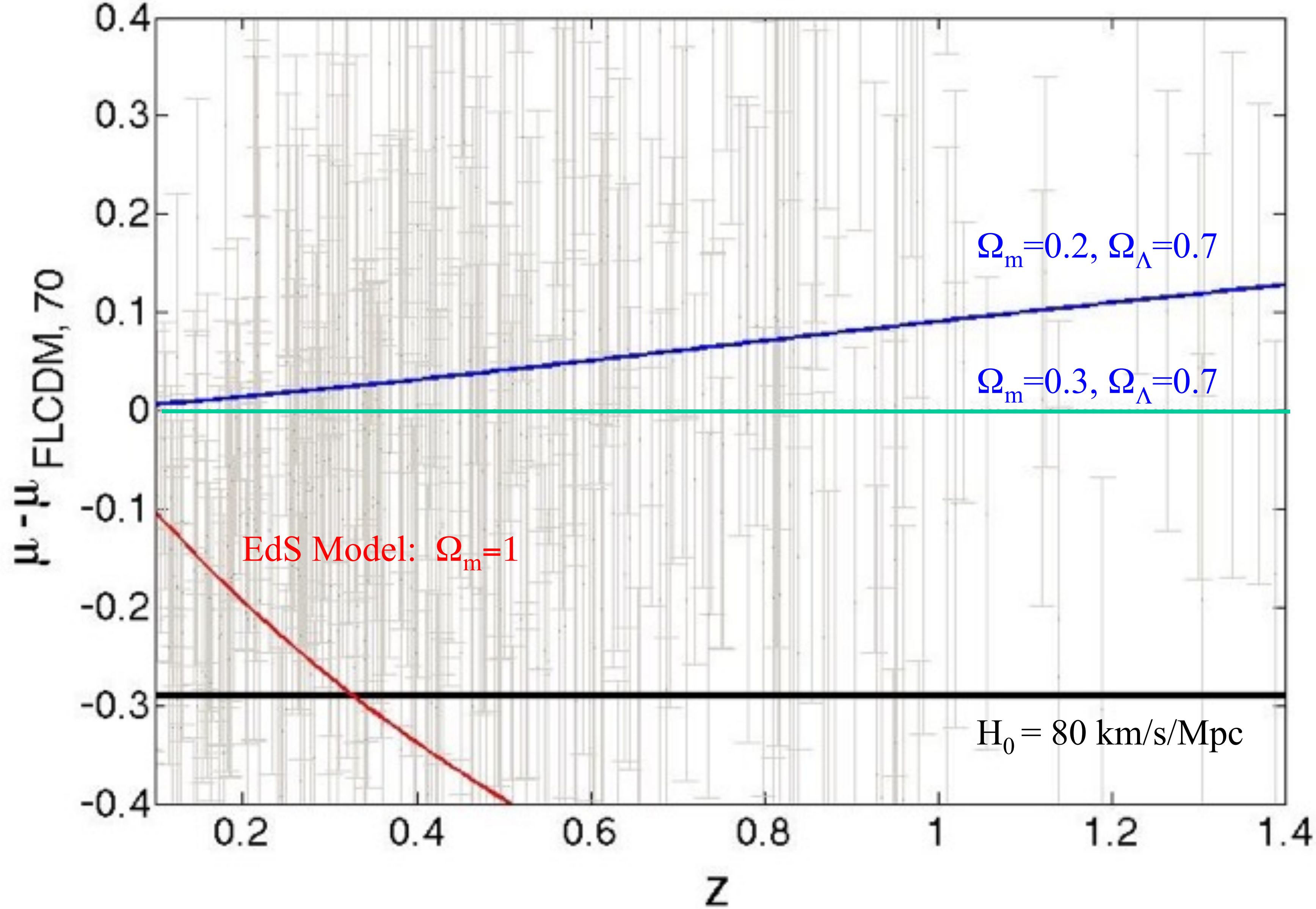




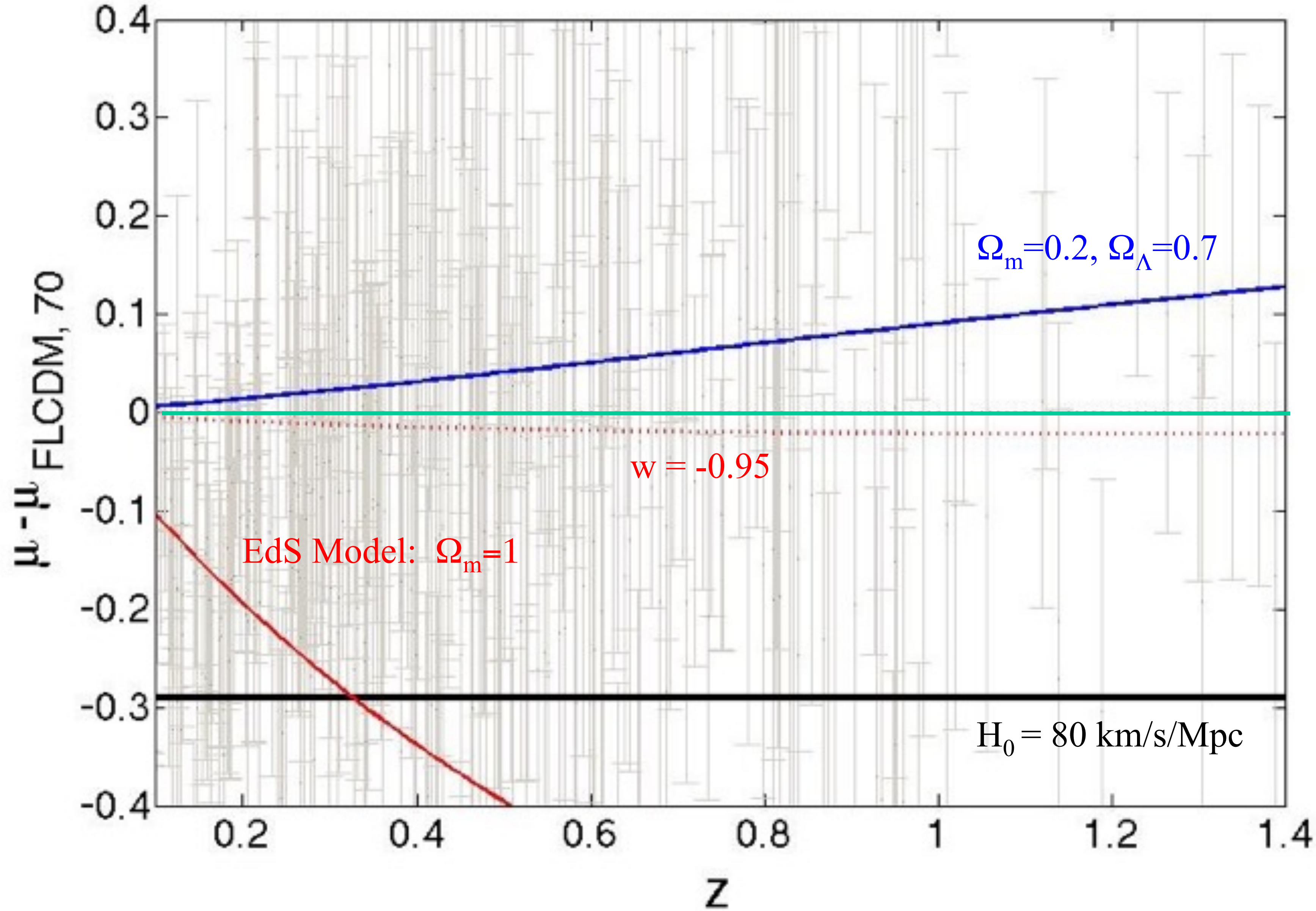
Ω_m=0.3, Ω_Λ=0.7, H_0 = 70 km/s/Mpc
H_0 = 80 km/s/Mpc
$\mu - \mu_{FLCDM,\ 70}$
z
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0.2
0.4
0.6
0.8
1
1.2
1.4



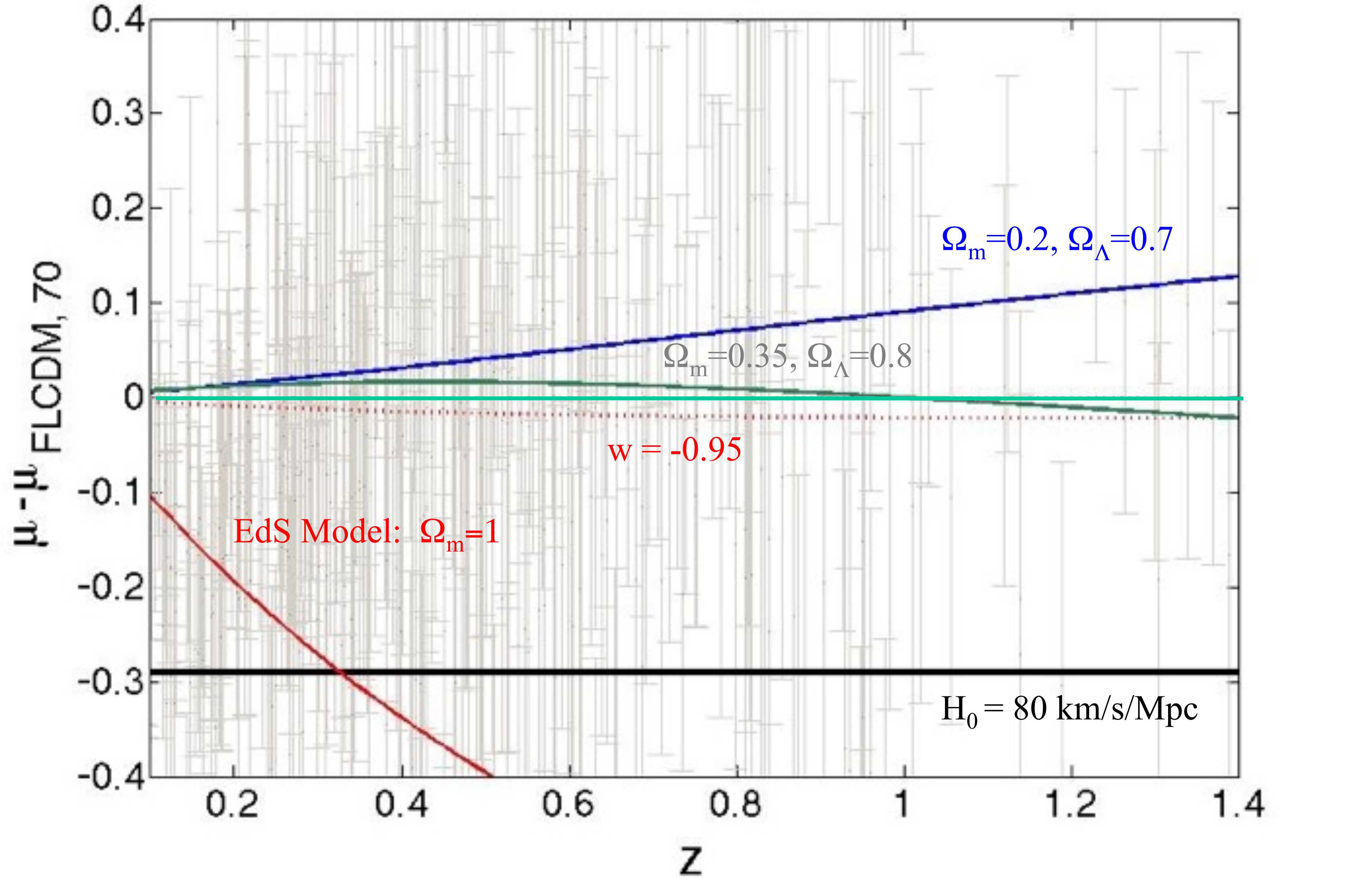
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
μ - μ FLCDM, 70
0.2
0.4
0.6
0.8
1
1.2
1.4
z
Ωm=0.3, ΩΛ=0.7, H0 = 70 km/s/Mpc
EdS Model: Ωm=1
H0 = 80 km/s/Mpc



0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0.2
0.4
0.6
0.8
1
1.2
1.4
z
μ - μ FLCDM, 70
Ωm=0.2, ΩΛ=0.7
Ωm=0.3, ΩΛ=0.7
EdS Model: Ωm=1
H0 = 80 km/s/Mpc



Ω_m=0.2, Ω_Λ=0.7
w = -0.95
EdS Model: Ω_m=1
H_0 = 80 km/s/Mpc
$\mu - \mu_{FLCDM, 70}$
z



Ω_m=0.2, Ω_Λ=0.7
Ω_m=0.35, Ω_Λ=0.8
w = -0.95
EdS Model: Ω_m=1
H_0 = 80 km/s/Mpc
μ - $\mu_{FLCDM, 70}$
z
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0.2
0.4
0.6
0.8
1
1.2
1.4