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ch8. #9.

The data ⇒ $\{f_0 = f(-1) = 1,\ f_1 = f'(-1) = 1,\ f_2 = f'(1) = 2,\ f_3 = f(2) = 1\}$

Consider the $H_3 \in \mathbb{P}_3$, We let $H_3(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ to satisfies $H_3^{(k)}(x_i) = f^{(k)}(x_i)$

so the linear system:

$$
\begin{aligned}
H_3(-1) &= -a_3 + a_2 - a_1 + a_0 = f(-1) = 1 \\
H_3'(-1) &= 3a_3 - 2a_2 + a_1 + 0 = f'(-1) = 1 \\
H_3'(1) &= 3a_3 + 2a_2 + a_1 + 0 = f'(1) = 2 \\
H_3(2) &= 8a_3 + 4a_2 + 2a_1 + a_0 = f(2) = 1
\end{aligned}
$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & -1 & 1 \\ 3 & -2 & 1 & 0 \\ 3 & 2 & 1 & 0 \\ 8 & 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

, calculate $\det\left(\left|\begin{bmatrix} -1 & 1 & -1 & 1 \\ 3 & -2 & 1 & 0 \\ 3 & 2 & 1 & 0 \\ 8 & 4 & 2 & 1 \end{bmatrix}\right|\right)$ by cofactor expansion, on 4-th column.

We have: $(-1)\cdot\left|\begin{bmatrix} 3 & -2 & 1 \\ 3 & 2 & 1 \\ 8 & 4 & 2 \end{bmatrix}\right| + 1\cdot\left|\begin{bmatrix} -1 & 1 & -1 \\ 3 & -2 & 1 \\ 3 & 2 & 1 \end{bmatrix}\right|$

$= (-1)\cdot(12 - 16 + 12 - 16 - 12 + 12) + 1\cdot(2 + 3 - 6 - 6 + 2 - 3)$

$= 8 + (-8) = 0$

Hence the determinate is 0, the matrix of linear system is singular ⇒ ① No solution ② Infinite solution

but Hermite-Birkoff interpolating poly is unique and existing, so the system is not sol. for the following data ※

∴ this kind of H_3 does not exist

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ch8 #12

$f(x) = \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$, $r(x) = \frac{a_0 + a_2x^2 + a_4x^4}{1 + b_2x^2}$

Let $r(x) = \frac{a_0 + a_2x^2 + a_4x^4}{1 + b_2x^2} = (a_0 + a_2x^2 + a_4x^4)\left(\sum_{n=0}^{\infty}(-b_2x^2)^n\right) = (a_0 + a_2x^2 + a_4x^4)(1 - b_2x^2 + b_2^2x^4 - b_2^3x^6 + \cdots)$

consider the coefficient for each terms:

constant: a_0

x^2 : $a_2 - a_0b_2$

x^4 : $a_0b_2^2 - a_2b_2 + a_4$

x^6 : $-a_0b_2^3 + a_2b_2^2 - a_4b_2$

$\Rightarrow \therefore r(x) = a_0 + (a_2 - a_0b_2)x^2 + (a_0b_2^2 - a_2b_2 + a_4)x^4 + (-a_0b_2^3 + a_2b_2^2 - a_4b_2)x^6 + \cdots$

For $f(x) = \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + O(x^8)$

$r(x) = a_0 + (a_2 - a_0b_2)x^2 + (a_0b_2^2 - a_2b_2 + a_4)x^4 + (-a_0b_2^3 + a_2b_2^2 - a_4b_2)x^6 + O(x^8)$

compare $f(x) - r(x)$ and solve coefficient ⇒

$$\begin{cases} a_0 = 1 \\ a_2 - a_0b_2 = \frac{-1}{2} \\ a_0b_2^2 - a_2b_2 + a_4 = \frac{1}{24} \\ -a_0b_2^3 + a_2b_2^2 - a_4b_2 = \frac{-1}{720} \end{cases} \Rightarrow \begin{cases} a_0 = 1 \\ a_2 - b_2 = \frac{-1}{2} \\ b_2^2 - a_2b_2 + a_4 = \frac{1}{24} \\ -b_2^3 + a_2b_2^2 - a_4b_2 = \frac{-1}{720} = -b_2(b_2 - a_2b_2 + a_4) \end{cases}$$

$$\Rightarrow \begin{cases} a_0 = 1 \\ \therefore -b_2\cdot\frac{1}{24} = \frac{-1}{720} \\ b_2 = \frac{1}{30} \end{cases}$$

, hence $a_2 - \frac{1}{30} = \frac{-1}{2}$

$\Rightarrow a_2 = -\frac{7}{15}$

and $a_4 = \frac{1}{24} - \frac{1}{900} - \frac{7}{450} = \frac{1}{40}$

The coefficients:

$$\Rightarrow \begin{cases} a_0 = 1 & b_2 = \frac{1}{30} \\ a_2 = -\frac{7}{15} & a_4 = \frac{1}{40} \end{cases}$$

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