3MCT Coursework Assignment 2:
Deadline 4pm. Tuesday 5th January 2021 (week 12)

This assignment involves Fourier Transforms, Neural Networks and Monte Carlo Methods

Q1) Training a neural network with the Metropolis algorithm

In this question, the goal is to write a code which trains a neural network to reproduce the truth table for a three-input exclusive-or (XOR) gate. The truth table is shown in Table 1. It has an output of 0 if there are none or two inputs which are 1. It has an output of 1 if there are one or three inputs which are 1.

A	B	C	OUT
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0

Table 1: 3-input XOR gate

You are free to choose the geometry of the neural network as you wish, though a standard feedforward multilayer perceptron with at least two nodes in the hidden layer can reproduce the table, as can the kind of idea used in the week 6 exercise in which some signals to the output layer bypass the hidden layer.

For a training rule, <u>you must implement this using the Metropolis algorithm</u> and not the backpropagation rule. The Metropolis algorithm minimises some quantity, so you can use it to minimise the error in the neural network. The output of the neural network is in the range [0,1] so for the purposes of displaying the output table, the numbers will need to be rounded to the nearest integer, but they should not be rounded for calculating the error. You will have to decide how to make the random change to the weights in each Metropolis iteration.

You should make written (by hand or on computer) notes of explaining and documenting your choice of neural network layout and your choice of weight alteration in the Metropolis algorithm. You should submit: these notes (as pdf); A code which begins from random weights and trains a neural network to reproduce the 3-input XOR gate using the Metropolis algorithm,

[30 marks]

Q2) Fourier Methods

The following table presents the numerical values of a function sampled at 16 equally-spaced points between 0 and 15π/8. The function is periodic with period 2π.

$f(0)=-0.2$,	$f(\pi/8)= -0.1$	$f(\pi/4)=0.3$,	$f(3\pi/8)=0.2$,	$f(\pi/2)=0.4$,
$f(5\pi/8)=0.5$,	$f(3\pi/4)=0.0$	$f(7\pi/8)=-0.4$,	$f(\pi)=-0.4$,	$f(9\pi/8)=-0.2$,
$f(5\pi/4)=0.1$,	$f(11\pi/8)=0.2$,	$f(3\pi/2)=0.2$,	$f(13\pi/8)=0.1$,	$f(7\pi/4)=0.1$,
$f(15\pi/8)=-0.1$				

Use a Fast Fourier Transform to find Fourier series for this data up to the terms in sin(7x) and cos(7x). To check that you have produced a correct result, plot the continuous function that arises from the truncated Fourier sum, along with the individual points from the table.

You should submit your code, which solves the problem and produces a plot of the results.

[10 marks]

Q3) Monte Carlo Integration

The *unit n-ball* is the region of *n*-dimensional space which is all those points less than a unit distance from the origin. In 1-dimension it is a line between -1 and +1, in 2-dimensions it is a the area within a circle with radius 1, in 3-dimensions, the volume within a sphere with radius 1, and so on.

We already looked an example of Monte Carlo integration to find the area of a unit circle. In this question you should use Monte Carlo integration to find the volume of the unit n-ball for $2 \leq n \leq 15$. Note that the word *volume* is used for the general case, though for $n=2$ it is really an area.

Produce a plot of the results of the volumes against the dimension of the space, n. You can compare your results in your plot with the analytic solution to the integral:

$$V(n) = \frac{\pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}+1\right)},$$

Where Γ is the gamma function (for which you can find an intrinsic routine in Fortran and a library routine in Python).

You need to submit: your code, which produces a plot of your results, comparing with the analytic result.

[10 marks]

MARK SCHEME

This assignment is worth 50% of the entire module. The marks for the questions are allocated as follows. You will be given feedback regarding the marks and a commentary on your code. Note that each mark below will be awarded using the level HE6 grade descriptors as found in the University Code of Practice on Assessment and Feedback, and in the Physics UG Course Handbook. Each marking point below will be awarded a mark in half-integer steps.

Q1) 30 marks
[3 marks] For clear and comprehensive notes on the chosen layout of the neural network and how it maps on to your coded implementation
[3 marks] For clear and comprehensive notes on the chose method of updating the weights an implementing the Metropolis algorithm
[2 marks] For the part of the code which correctly computes the output of the neural network for given input and weights
[4 marks] For the correct implementation of the Metropolis algorithm
[4 marks] for code which is clearly-presented, well-commented and easy to understand
[2 mark] A code which successfully finds a valid solution to the logic gate
[6 marks] Well-presented and useful output (to screen and/or to files as appropriate)
[6 marks] For overall efficient and elegant coding & solution of the problem

Q2) 10 marks
[2 marks] For a code which correctly performs the Fourier transform with the data
[4 marks] For the calculation of the output and presentation of the resulting curve alongside the original data
[4 marks] For an efficient and elegant coding of the problem

Q3) 10 marks
[3 marks] For a correct implementation of the Monte Carlo algorithm for the case at hand with sensible results
[3 marks] For the presentation of the output
[4 marks] For an efficient and elegant coding of the problem