UNIVERSITY OF SURREY

Module PHY3054
Research Techniques in Astronomy

FHEQ Level 6

Coursework 2

Deadline:
Tuesday 5 January 2021 @ 4pm

Instructions

Write a Python code and a report for each of the three problems in the assignment. The suggested length of the report is 4-6 pages, and should not exceed 8 pages. The report should include, for each problem:

1. A description of the problem
2. The methodology used
3. A description and discussion of the results obtained
4. All necessary figures.

The format for the report is pdf, and both a LaTeX and Word template are provided on SurreyLearn. The format for the codes is an executable python file (.py extension). The report and the codes should be submitted to the module's page on SurreyLearn.

IMPORTANT: Any code that generates an error will be given zero marks. You must provide an executable version of your code (a .py file only). Please verify that the codes are written for python 3, and run on the lab computers without errors, especially if you have developed them on a different computer.

Marking scheme

Marks will be assigned in the following way:

- Problem 1: Total = 50 marks. Code = 25 marks. Report = 25 marks.
- Problem 2: Total = 50 marks. Code = 25 marks. Report = 25 marks.
- Problem 3: Total = 50 marks. Code = 25 marks. Report = 25 marks.

Coding marks will be assigned based on both the accuracy of the output (up to 80%, see detailed marks by each question) and the code's efficiency and readability (up to 20%, see rubric on SurreyLearn).

A rubric for the report is available on SurreyLearn for your guidance.

This assignment contributes 50% to the total module mark.

1 Problem 1

Numerical integration and visualisation: the diffraction limit of a telescope [Code: 25 marks, report: 25 marks]
Our ability to resolve detail in astronomical observations is limited by the diffraction of light in our telescopes. Light from stars can be treated effectively as coming from a point source at infinity. When such light, with wavelength λ, passes through the circular aperture of a telescope (which we will assume to have unit radius) and is focused by the telescope in the focal plane, it produces not a single dot, but a circular diffraction pattern consisting of a central spot surrounded by a series of concentric rings.

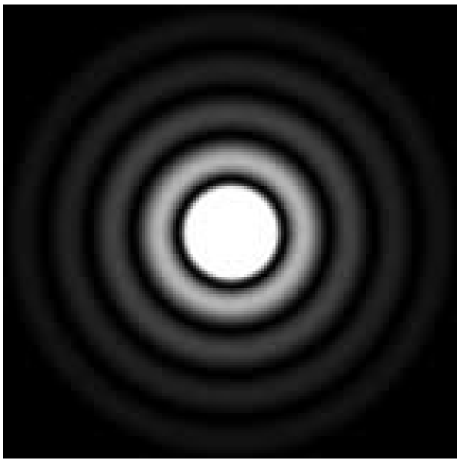

Figure 1: The diffraction pattern produced by a point source of light when viewed through a telescope.

The intensity of the light in this diffraction pattern is given by

$$I(r) = \left(\frac{J_1(kr)}{kr} \right)^2, \tag{1}$$

where r is the distance in the focal plane from the centre of the diffraction pattern, $k = 2\pi/\lambda$, and $J_1(x)$ is the Bessel function

$$J_1(x) = \frac{1}{\pi} \int_0^{\pi} \cos(\theta - x \sin\theta)\, d\theta, \tag{2}$$

where $x \geq 0$.

1. Write a Python function `J1(x)` that calculates the value of $J_1(x)$ using Simpson's rule with $N = 1000$ points and a given x.

[Code: 7 marks]

2. Compare the results for $J_1(x)$ over the range $0 \leq x \leq 20$ with the values returned by `SciPy`'s function `jv(v,z)` for Bessel functions of the first kind of order $v = 1$.

   [Code: 3 marks]

3. Plot, in a single graph, the values for the Bessel function $J_1(x)$ returned by your function `J1(x)` (with blue points) and by `jv(v,z)` (with a black line) over the range $0 \leq x \leq 20$. Add axis labels and a legend.

   [Code: 3 marks]

4. Compute the intensity $I(r)$ and make a density plot of the circular diffraction pattern of a point light source with $\lambda = 500$ nm, in a square region of the focal plane. Consider values of r in the range $0 - 1\mu$m.

   [Code: 12 marks]

2 Problem 2

Generate a discrete sample from a probability density function
[Code: 25 marks, report: 25 marks]
Consider the following probability density function (PDF)

$$f(x) = Ax^{\alpha}. \tag{3}$$

Using the transformation method, perform the following calculations and include them in the report:

1. *Normalise the function.* Find the constant A, expressed as a function of α, such that equation 3 has an area of unity. Assume a range $x_{\text{lo}} < x < x_{\text{up}}$.

2. *Cumulative distribution function (CDF).* Find the CDF $F(x)$ and check that $0 \leq F(x) \leq 1$ in the range $x_{\text{lo}} \leq x \leq x_{\text{up}}$.

3. *Invert $F(x)$.* Find the analytic expression for x in terms of $\alpha, x_{\text{lo}}, x_{\text{up}}$, given a random number R between 0 and 1.

Now write a Python 3 program to perform the following calculations:

1. *Generate random variates.* Generate two samples of $N = 10000$ values for x in the range $1 \leq x \leq 5$: consider $\alpha = -2.35$ for the first sample and $\alpha = -1$ for the second sample.

   [Code: 10 marks]

2. *Histogram.* Plot a histogram for each sample. For each value of α, use two different approaches for binning the data: (1) create histograms with 100 bins that have equal width in x (i.e. $\Delta x =$constant) and (2) create histograms with 100 bins that have equal width in $\log_{10} x$ (i.e. $\Delta \log_{10} x =$constant). Ensure that the distribution is normalised and consider plotting on a log scale where appropriate. Assume that the number of counts in each bin is a Poisson process and plot normalised uncertainties (error bars) for each bin. Discuss the difference in shape of the two binning methods. Is there a reason to prefer any of the two methods? Save all 4 figures to files and include them in the report.

   [Code: 15 marks]

3 Problem 3

Maximise the likelihood [Code: 25 marks, report: 25 marks]
Let us now assume that the sample of x values obtained from the $\alpha = -2.35$ power-law distribution in Problem 2 is a data set. We know that the data are distributed like a power-law, and we are interested in determining the most likely power-law index, and the uncertainty.

1. *Likelihood.* Derive an expression for the total likelihood that the data are drawn from a power-law distribution with index α, and include it in the report. Write a code to compute the likelihood, using the data sample to determine the limits of the range x_{lo} and x_{up}.

   [Code: 5 marks]

2. *Maximum likelihood.* Find the maximum likelihood for a series of 100 values of the slope α in the range $(-4, 0)$. Save a plot of the maximum likelihood to file and include it in the report. Assume that the likelihood curve is approximately Gaussian near the maximum and determine the 1σ uncertainty. How many sigma is the most likely value for α away from the input value $\alpha = -2.35$?

   [Code: 10 marks]

3. *Repeat.* Repeat all previous steps 1000 times. Each time, record the most likely value for α and the corresponding 1σ uncertainty. Plot a histogram of all retrieved slope values and include it in the report. How many times is the most likely value for α less then 1σ away from the input value? Discuss the result.

   [Code: 10 marks]