InFoMM CDT Scientific Computing

Image Classification Task

October 16, 2017

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For example:

- Images of faces (perhaps in different orientations, expressions, lighting
  $\Longrightarrow$ the classes are the people
- Diagnosis from medical images: the classes might be positive/negative
- Military applications: in radar for example - who exactly does that aircraft belong to?
- Document classification from handwriting style.

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- **Unsupervised classification:** Discover clusters in the data without using any prior information
- Our project will focus on **supervised classification**.

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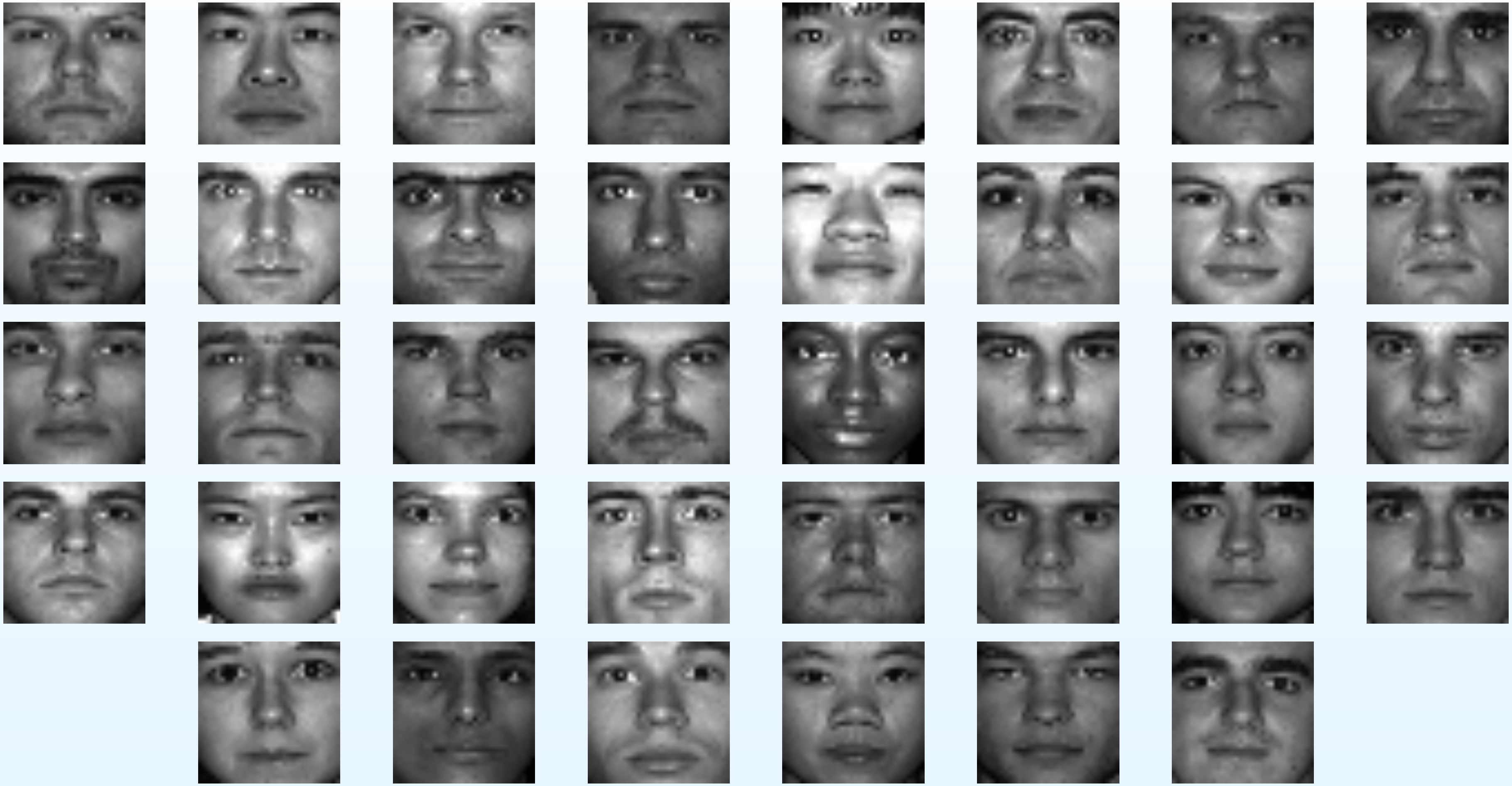
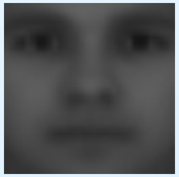
- $\Longrightarrow$ **two sets: training set and test set**

The Yale face database



...and the mean face: 

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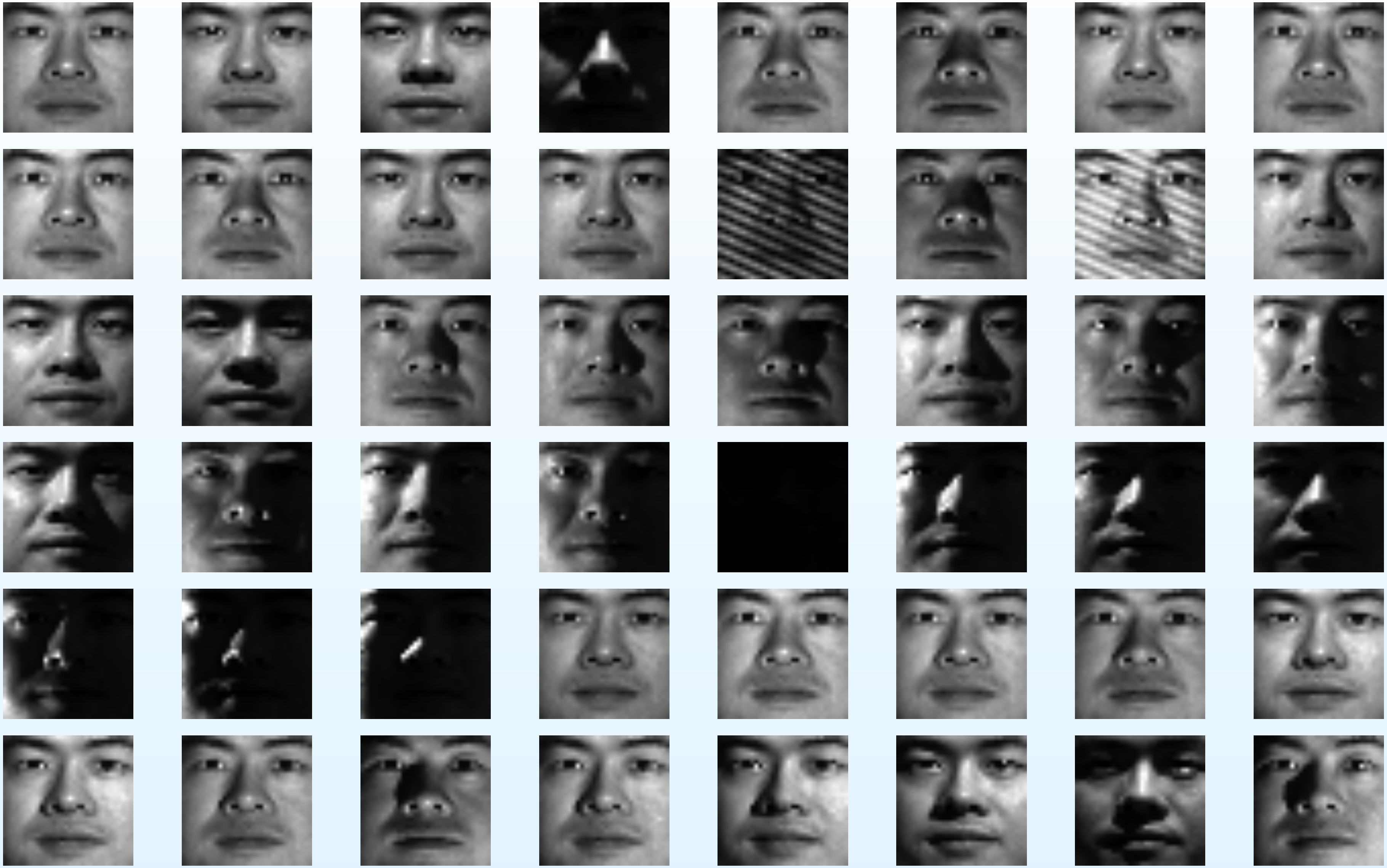

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k-nearest neighbours algorithm

	Training feature vectors	x_i	$i = 1, \dots, m$
Input:	Training labels	g_i	$i = 1, \dots, m$
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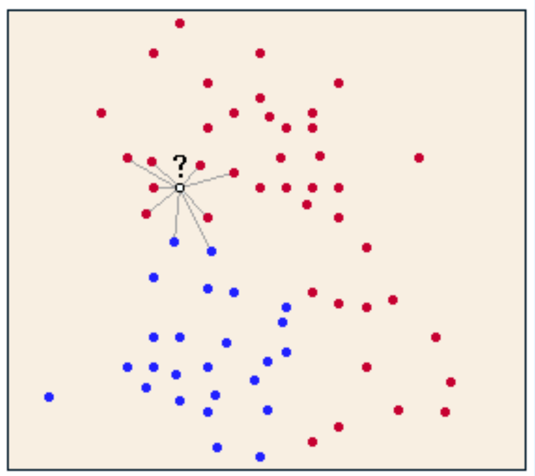
- Find the k feature vectors which are closest to y_j.



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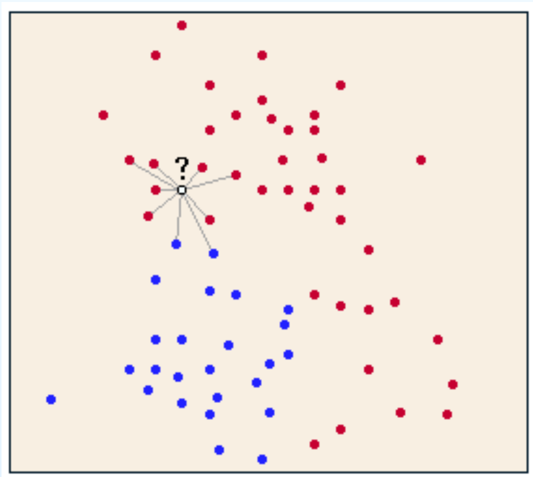

- Assign the most frequent label among the selected x_i.

What is a digital image?

82	93	108	116	131	140	142	139	162	172	171	169	174	155	152	151	159	166	171	182	173	181	179	162	166	156	144	122	113	97	92	71
81	96	106	127	137	153	153	166	160	188	193	194	200	181	198	209	131	200	220	222	220	205	185	175	170	145	141	133	116	110	94	70
72	90	113	119	123	137	145	155	163	163	186	187	194	197	181	190	197	193	208	172	204	196	174	161	139	118	113	103	75	67	93	81
72	97	60	64	73	78	84	85	84	102	105	127	145	163	177	189	190	172	172	138	125	104	84	74	76	69	95	135	127	96	76	46
40	68	103	123	145	141	125	85	70	59	71	85	110	136	154	162	170	158	139	121	91	77	68	84	131	146	137	138	127	123	116	87
93	120	126	126	128	134	134	131	108	93	80	77	92	115	143	170	164	144	121	103	87	87	97	82	82	91	81	85	109	112	111	109
119	109	99	75	68	85	64	56	56	82	91	92	84	90	120	167	156	130	97	91	93	86	84	72	30	48	28	57	59	79	103	109
129	111	76	57	42	109	20	18	51	200	75	138	89	97	92	140	146	111	84	92	158	98	97	189	23	23	33	120	33	54	90	112
135	105	63	59	80	91	95	103	103	105	112	114	115	96	87	128	133	107	82	99	145	123	107	96	93	87	83	92	75	59	65	100
121	123	117	107	112	123	123	127	137	121	112	122	107	74	74	101	108	90	78	91	110	132	114	112	130	120	121	116	111	113	108	111
123	126	124	125	132	142	149	158	151	134	128	119	85	67	82	103	101	87	75	87	98	124	140	143	161	170	163	144	138	127	117	113
124	130	137	132	138	149	160	169	159	153	148	117	75	62	93	108	103	85	68	77	90	128	143	162	165	171	163	159	150	139	137	126
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107	105	118	128	140	147	145	144	161	168	143	99	113	83	135	155	145	99	93	126	101	109	167	176	166	151	139	132	141	132	126	121
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80	72	103	121	108	96	113	114	130	118	109	98	84	83	84	78	81	85	87	83	86	111	115	125	124	101	103	92	140	109	92	89
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74	84	79	85	107	106	109	110	117	92	93	108	127	127	127	133	125	134	137	133	118	110	100	102	132	133	120	128	86	82	89	80
59	65	68	85	114	95	108	119	115	115	118	124	119	115	107	110	116	121	131	133	137	130	123	121	113	123	113	100	89	92	76	67
43	52	60	72	79	98	101	107	112	116	118	125	107	112	109	115	114	124	134	134	137	123	101	112	117	94	97	106	73	67	61	57
36	43	48	52	72	71	86	95	106	105	125	137	144	149	127	124	122	112	126	152	140	132	110	109	100	82	91	68	69	60	55	35
21	23	32	41	46	55	77	81	82	69	113	134	130	116	104	122	124	104	106	139	130	118	115	102	76	76	55	57	50	51	55	33
12	17	26	30	33	58	45	63	66	86	99	125	102	125	131	124	139	129	133	125	89	104	87	69	70	50	59	36	39	37	34	22

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- Vectorize each training image x_i to give a column of X, a $p \times m$ matrix where p is the number of pixels

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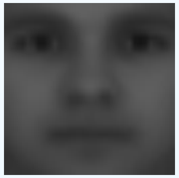

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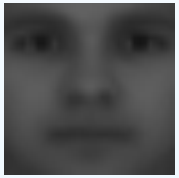

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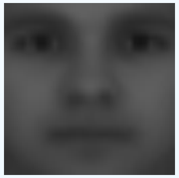

- Subtract the mean from each image: `Xc = X - repmat(Xm,[1 m])`
- Do the same for the test images: `Yc = Y - repmat(Xm,[1 n])`

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- Consider the $p \times p$ **covariance matrix** XX^T
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- If $m < p$, we can first do `[U D] = eig(X'*X)`, and then obtain the eigenvectors of XX^T as `V=X*U`.

Eigenfaces

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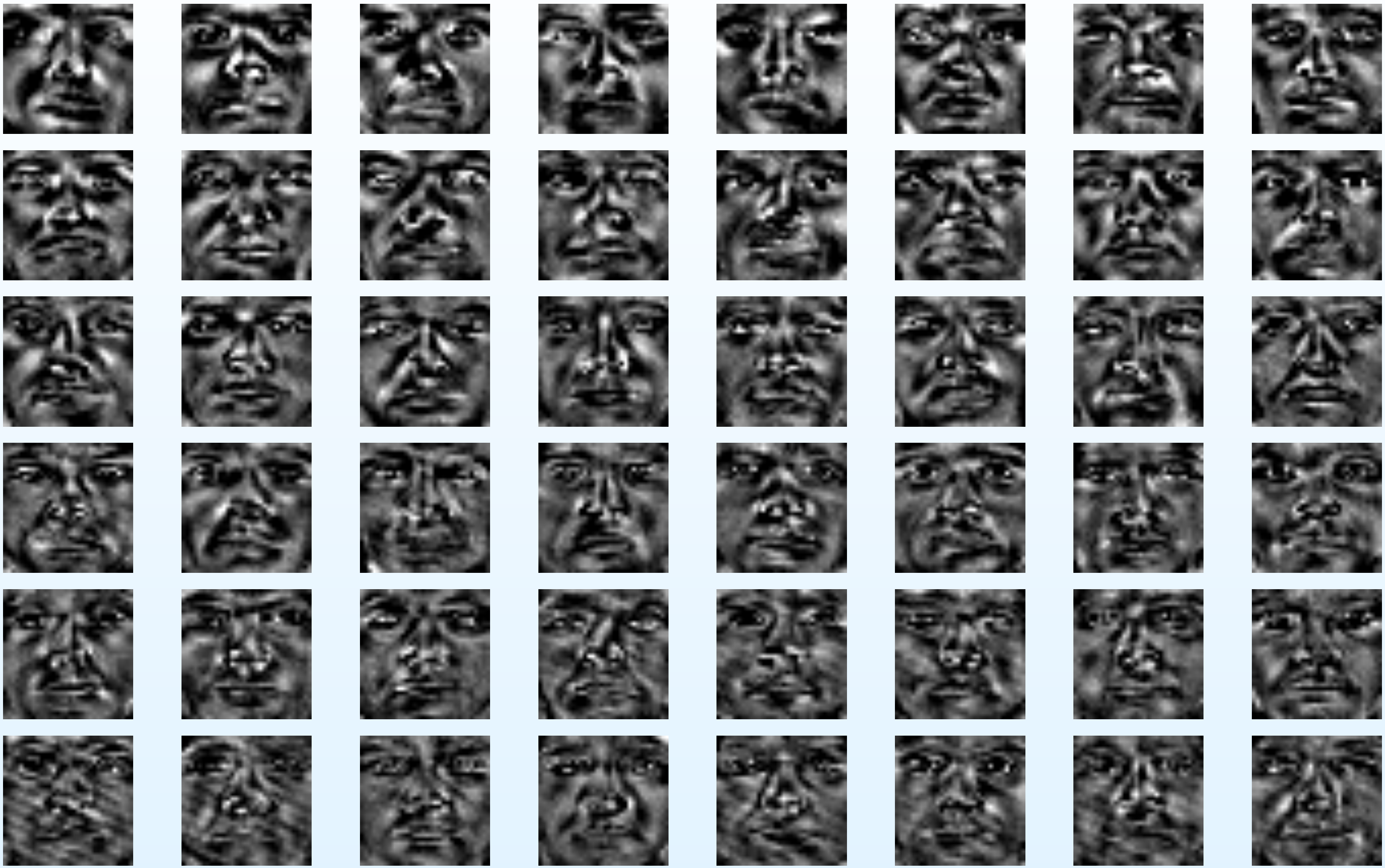

Improvements to eigenfaces

- **Whitening:** Normalize the features:
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- **Removal of first few components:**
  The first few eigenvectors capture lighting effects more than distinguishing people.

Some example matchings



-2.2057	-1.8430	1.2718	-0.3012	-3.9377	1.5653	4.4381	2.5084	0.7554	0.0941	5.2085
-1.5632	-5.5112	4.9246	2.6663	-2.8053	5.2122	7.8306	-0.3192	1.5501	0.0609	5.7439
-0.7094	3.1290	1.5869	5.2848	0.6720	3.1660	3.8727	2.1090	2.9190	0.2547	4.1439
-7.6463	0.3238	-1.4605	5.8808	-6.3703	2.2854	1.4323	4.8371	2.2619	-0.3084	3.6800

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-0.2481	-0.1787	0.9437	0.4573	-0.2593	0.7267	0.0941	-0.0009	0.3398	-0.1720	0.1519
-0.2797	-0.2745	0.9645	0.4286	-0.1054	0.7057	0.0398	-0.1230	0.3175	-0.4393	0.0914
-0.3654	-0.1372	0.8560	0.4541	-0.4664	0.4134	0.0388	-0.0432	0.3673	-0.3042	0.1804
-0.4540	-0.1347	1.2253	0.4417	-0.0277	0.7711	0.0818	-0.0573	0.3836	-0.3561	0.0990

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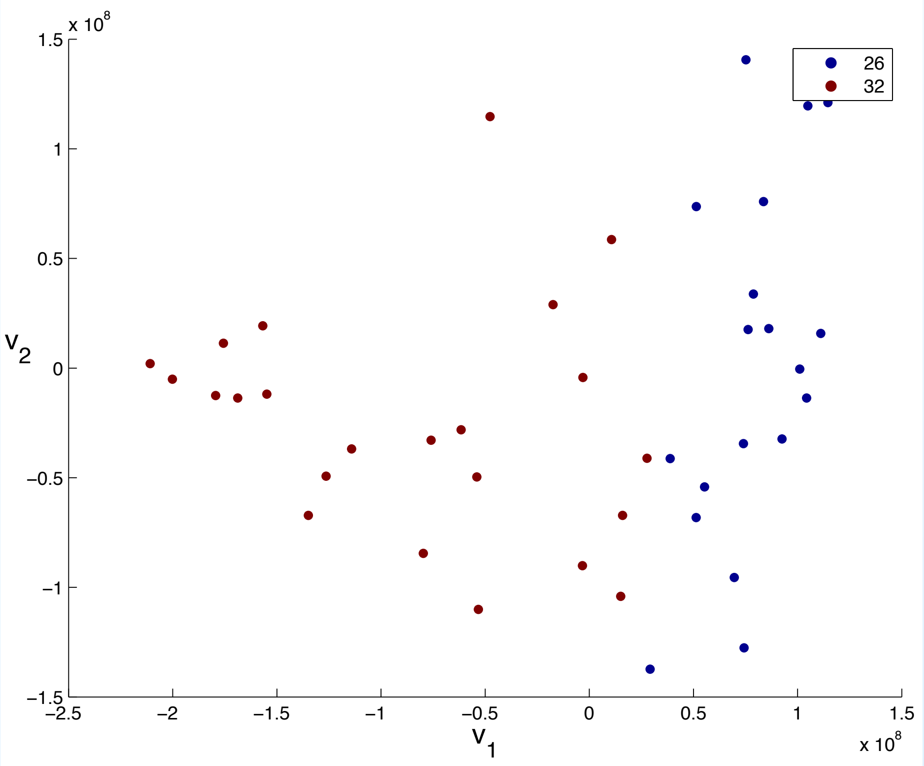

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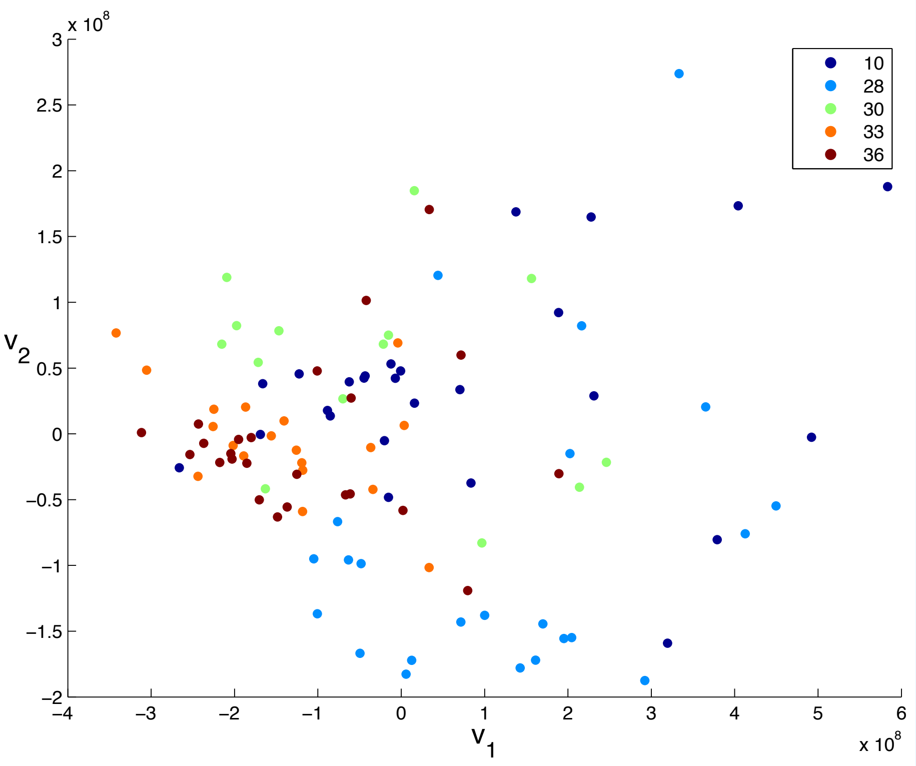

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Provide choices for...

- which digits to test
- the size of training/test sets
- the features to use (e.g. intensity, PCA...)

Possible extensions

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- Extension to other image databases