Pareto-Weighted-Sum-Tuning: Learning-to-Rank for Pareto Optimization Problems

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Pareto Optimization

- Multiobjective and Multicriteria Optimization
- Criteria often conflict with one another
- Need to consider trade-offs
- Common in decision-making problems

Pareto Optimization

Choice A
- Very High Best Case Performance
- Very Low Worst Case Performance

Choice B
- Lower Best Case Performance
- Higher Worst Case Performance

- Different Objective Functions
- Trade-offs based on objective function used
- How does DM evaluate trade-offs?

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Applications/Domains

- Mechanical/Civil/Structural Engineering
  - Cost vs. Strength
- Medicine/Healthcare
  - Efficacy vs. Side-effects
- Finance/Business
  - Risk vs. Reward

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Mathematical Formulation

- Weighted Sum Approach:

$$\max \sum_{i=1}^{m} \alpha_i c_i$$

$$s.t. \sum_{i=1}^{m} \alpha_i \leq 1$$

α_i = weight on criteria i
c_i = quantity of criteria i

- Common Problem: Figuring out these weights is difficult
- Can rely heavily on heuristics or domain knowledge

(Yang)

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Previous Work

- Analytical Hierarchy Process (AHP)
  - Compare and Rate Criteria Nodes
  - Requires Explicit Quantifying from Decision-Maker

  (Dyer)

- Multinomial-Logit Model
  - Analyze Users' Choices to determine preferences
  - Successful in Marketing, Natural Language Processing, Computer Vision

  (Negahban, Oh, Thekumparampil, and Xu)

Methodology Developed

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Outline of Approach

- Give decision maker specific instances of Objective Value tuples $[t_1, t_2, \ldots, t_x]$

  $t_i = [c_{1i}, c_{2i}]$

- Decision maker ranks tuples → simpler than scoring
- Use Supervised Learning to learn weights based on decision maker's feedback

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Pareto-Weighted-Sum-Tuning (PWST)

Algorithm 1 Pareto-Weighted-Sum-Tuning

Require: n data-points, x data-points at each iteration where $x \leq n$

α-vectors $= \emptyset$

$i = 0$

$[t_1, t_2, ..., t_x] \leftarrow x$ data-points sampled from the data-set and displayed to decision-maker in random order

while $i \leq \lfloor \frac{n}{x} \rfloor$ **do**

(I) $[r_1, r_2, ..., r_x] \leftarrow$ ranking given by decision-maker for the x sampled data-points; this action is equivalent to "correcting" the order predicted by the algorithm

(II) $[\alpha_1, \alpha_2, ..., \alpha_m] \leftarrow$ Ranking-SVM($[t_1, t_2, ..., t_x], [r_1, r_2, ..., r_x]$)

(III) $[t_1, t_2, ..., t_x] \leftarrow x$ data-points sampled from the data-set (Note: in this step, Pareto-Weighted-Sum-Tuning uses the α-values learned to predict r_{true} and displays $[t_1, t_2, ..., t_x]$ according to its predicted order based on $\sum_{i=1}^{m} \alpha_i c_i$, where a higher sum corresponds to a better rank)

end while

$\vec{\alpha} = [\bar{\alpha_1}, \bar{\alpha_2}, ..., \bar{\alpha_m}]$ where $\bar{\alpha_i}$ is the mean value of α_i of vectors in α-vectors

return $\vec{\alpha}$

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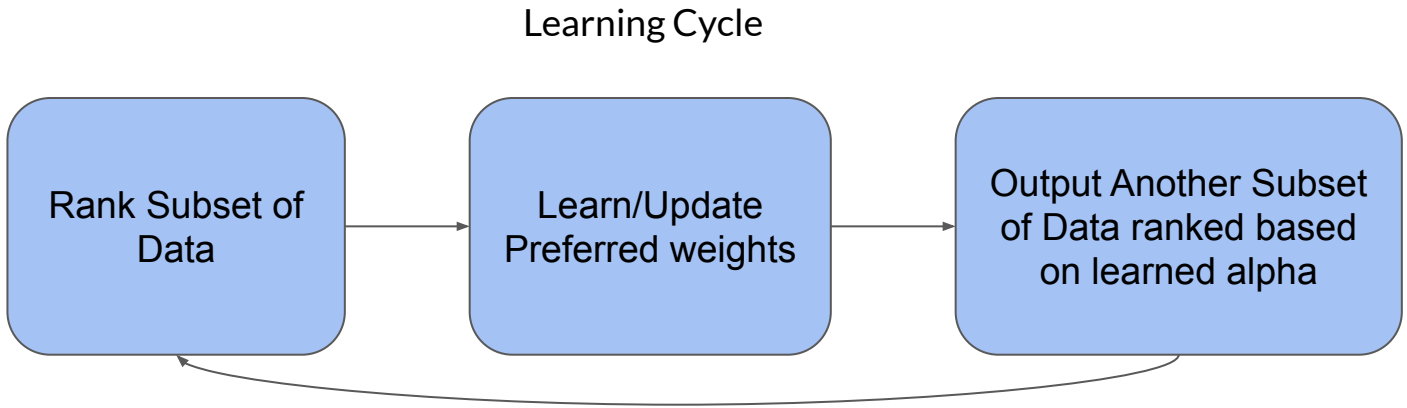
PWST

- Give DM a subset of data
- Example: given n tuples, label from 0-n
- 0 → worst pair; n→ best pair
- Learn latent preferred weights with Ranking-SVM



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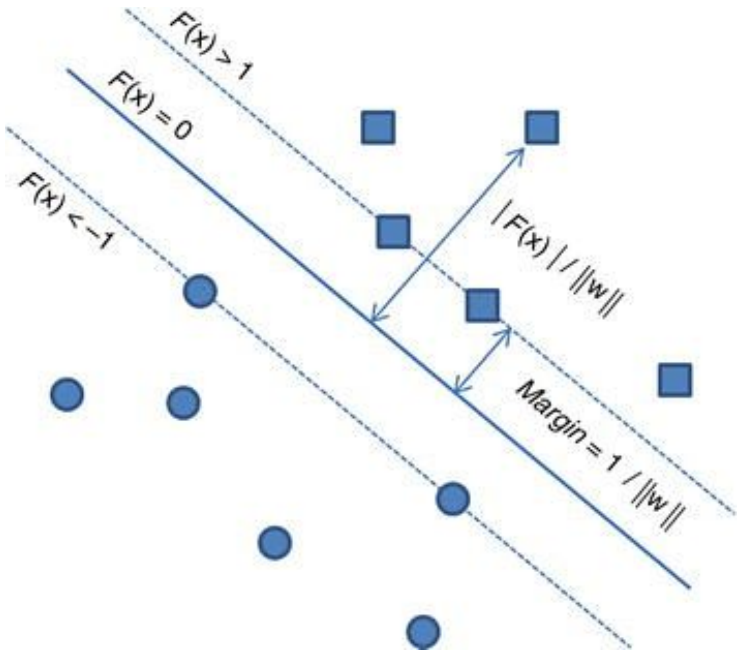
Learn-to-Rank

- Ranking-Support-Vector-Machine
- Learn ranking function f that fits rankings
- Optimize following function:

$$\min -\tau(r_{true}, r_{pred})$$

(Joachims)



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Kendall Tau

- Compare true ranking with predicted ranking
- Kendall Tau:

$$\tau(r_{true}, r_{pred}) = \frac{A - B}{A + B}$$

- A - Number of ordered pairs in both r_{true}, r_{pred}
- B - Number of ordered pairs in both r_{pred}, but not r_{true}

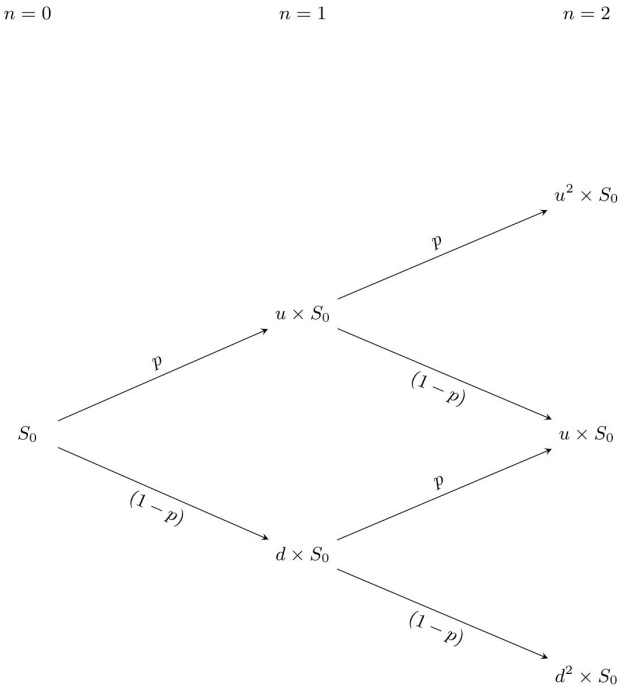
(Joachims)

Example Application

PWST Application Example

- Stock Investment
- Volatile and Hard to Predict
- Binomial Pricing Tree Model

(Muzzioli, S. and Torricelli)



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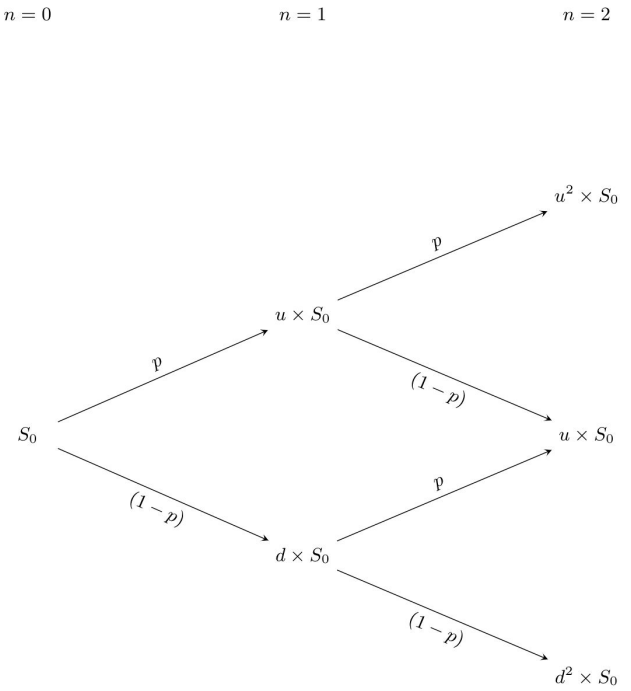
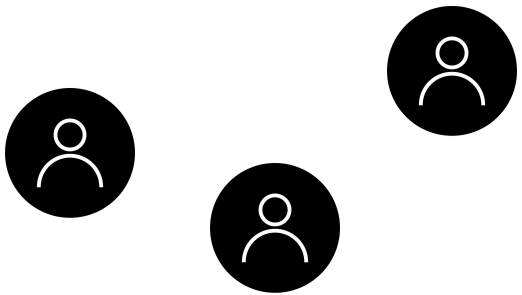
PWST Application Example

- Financial Advisor
- Different Financial Clients
- Different preferences
- Hard to express explicitly





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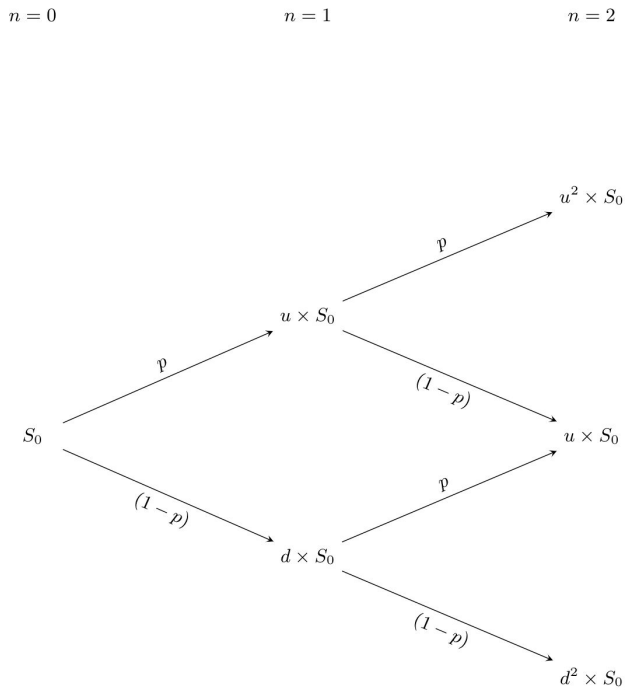
Application Example

- o_i = optimistic expected value
- p_i = pessimistic expected value
- α = client's weight on o
- s_i = price of stock i
- b = budget limit
- x_i = amount of stock i purchased



Application Example

- Expected utility of purchasing stock i:

$$f_i(\alpha) = \alpha \times o_i + (1 - \alpha) \times p_i$$

- Optimization Problem:

$$\max \sum_{i=1}^{s} x_i f_i(\alpha)$$

$$s.t. \sum_{i=1}^{s} s_i x_i \leq b$$

$$x_i \in \{0, 1\}$$

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Application Example

- Must know α to solve problem
- We have data-set of previous stock prices
- Use PWST to find α
- Use integer programming methods to solve once α is known

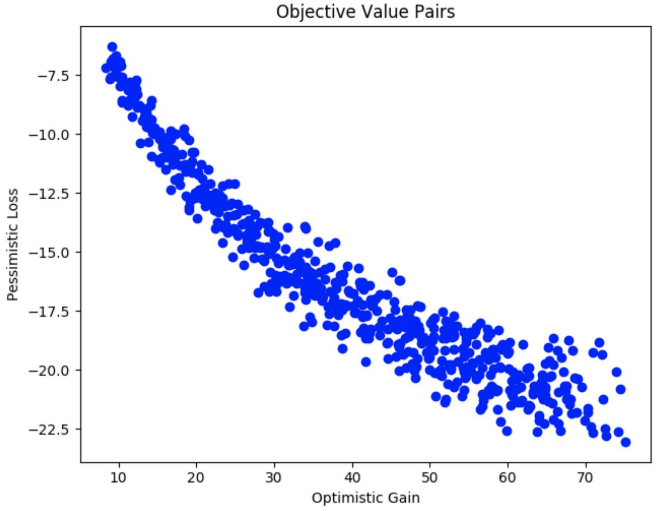
$$\max \sum_{i=1}^{s} x_i f_i(\alpha)$$

$$s.t. \sum_{i=1}^{s} s_i x_i \leq b$$

$$x_i \in \{0, 1\}$$

Data Collection: Simulated Decision-Maker

- Used Binomial Pricing Tree Model to generate data (with noise)
- Python Program to simulate DM
- Initialized with α value
- Given variance level δ
- DM draws from $\text{unif}(\alpha, \delta)$ on each iteration



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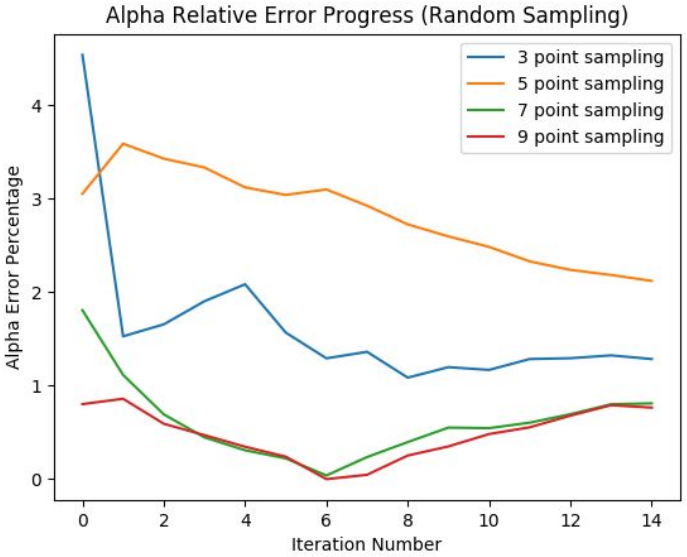
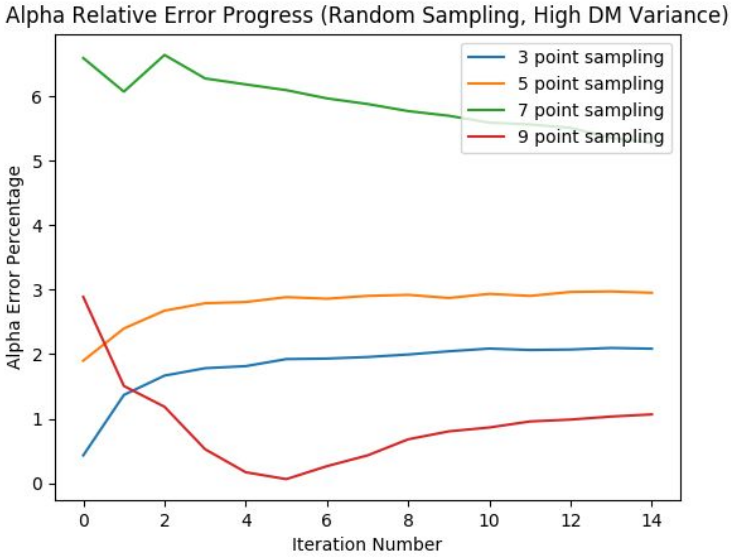
Results with Virtual DM





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Results with Virtual DM

- Some configurations achieve a relative error of below 1 percent after 15 iterations
- Viable approach to further explore
- Can help with weight-tuning problem in Pareto Optimization

Thank you!

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