General Heterogeneous Moran Processes

Vince Knight and Harry Foster

October 10, 2025

1 Introduction

2 Literature Review

3 Model

In this section, we will look at the underlying model of the heterogeneous Moran process. We define the following:

- N **ordered** individuals
- k types $A_1, A_2, \dots A_k$
- A state space S given by the set of ordered N-tuples with entries of type $A_1, A_2, \dots A_k$. Note that $|S| = k^N$
- A fitness function $f : S \to \mathbb{R}^N$

Let $h(V, U)$ denote the Hamming distance between two states V and U. We consider a Markov chain, where for the transition $V = (v_1, v_2, \dots, v_N) \to U = (u_1, u_2, \dots, u_N)$, the transition probability is defined as follows:

$$P(V,U) = \begin{cases} \frac{\sum_{v_i = u_i} f(v_i)}{\sum_{v_i} f(v_i)} & \text{if h(V,U) = 1, differing at position i} \\ 0 & \text{if h(V,U) >1} \\ 1 - \sum_{U \in S \setminus \{\text{V}\}} P(V,U) & \text{if h(V,U) = 0} \end{cases} \tag{1}$$

The final case is given by the following notion. V can transition to itself through the removal of any constituting individual, and the duplication of any individual which shared a type with the removed one. Thus, directly calculating P(V,V) would require a large amount of computational time and effort. However, we can observe that a transition of some sort must occur, and so we simply take the probability of not transitioning to any **different** state as P(V,V).

An important case which proceeds from (1) is that of a transition V $\to U$ where U contains an individual of a *type* not found in V. For example, the transition (0,1) $\to$ (0,2). This would be forbidden by the intuition of a Moran process - i.e the new individual being the duplication of another individual in V. However, we can see that the standard formula yields P(V,U) = 0 because no v_i shares a type with u_i. Following this, we see that (1) correctly gives us P(V,U) = 0 for any **absorbing** V $\neq$ U.