MECH3790 Aerodynamics and Aerospace Propulsion - Jet Propulsion Laboratory Report

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March 18, 2024

Word Count Q3-5: 1491

UNIVERSITY OF LEEDS

Contents

1 Results

1.1 Experimental

RPM	Thrust (N)	Fuel Consumption (g)	Fuel Mass Flow Rate (g/s)	EGT (°C)	Case Pressure (Pa)
55500	0	11	0.3951	592	13790e
74000	1.3182	15	0.5158	540	20684
99000	3.2799	21	0.7274	494	34474e
120000	5.8224	27	0.9285	482	55158e
140000	9.8167	33	1.1313	480	75842e
160000	16.7621	42	1.4483	500	10342
175000	21.9711	49	1.7438	554	13100

Table 1: Experimentally measured results for the Turbojet Engine

RPM	Thrust (N)	Fuel Consumption (g)	Fuel Mass Flow Rate (g/s)	EGT (°C)	Case Pressure (Pa)
56000	0	24	0.8048	512	13000
83000	16.0375	36	1.2329	472	24000
102000	28.1221	44	1.5146	400	37000
120000	44.3702	54	1.8531	392	53000
140000	70.5513	69	2.2280	400	70000
160000	112.9197	83	2.8621	440	103000
180000	155.2882	108	3.7203	508	137000

Table 2: Experimentally measured results for the Turboprop Engine

1.2 Turbojet Cycle Analysis

The data presented in these tables shows the Total Temperatures, T_t, Static Temperatures, T, Isentropic Total Temperatures, T_t' and the Total Pressure, P_t, and Static Pressure P.

Stage	$\mathbf{P_t}$ (Pa)	$\mathbf{T_t}$ (K)	$\mathbf{T_t}$' (K)	**P** (Pa)	**T** (K)
0	101300	297	297	101300	297
1	101300	297	297	N/A	N/A
2	153212.198	346.6906	334.2679	134926.4245	334.3271
3	140955.2222	826.2647	N/A	N/A	N/A
4	107562.1132	783.0926	772.2995	N/A	N/A
5	107562.1132	783.0926	N/A	N/A	N/A
6	107562.1132	783.0926	N/A	101300	771.446

Table 3: Results of the cycle analysis for the Turbojet Engine with an RPM of 99000

Stage	$\mathbf{P_t}$ (Pa)	$\mathbf{T_t}$ (K)	$\mathbf{T_t}$' (K)	**P** (Pa)	**T** (K)
0	101300	297	297	101300	297
1	101300	297	297	N/A	N/A
2	174783.02	363.7836	347.0877	153922.7833	350.8106
3	160800.3784	844.4478	N/A	N/A	N/A
4	112247.2507	786.4284	771.9235	N/A	N/A
5	112247.2507	786.4284	N/A	N/A	N/A
6	112247.2507	786.4284	N/A	101300	766.5243

Table 4: Results of the cycle analysis for the Turbojet Engine with an RPM of 120000

Stage	$\mathbf{P_t}$ (Pa)	$\mathbf{T_t}$ (K)	$\mathbf{T_t}$' (K)	**P** (Pa)	**T** (K)
0	101300	297	297	101300	297
1	101300	297	297	N/A	N/A
2	200310.62	382.1645	360.8734	176403.6813	368.5361
3	184285.7704	882.9613	N/A	N/A	N/A
4	118402.5279	809.055	790.5785	N/A	N/A
5	118402.5279	809.055	N/A	N/A	N/A
6	118402.5279	809.055	N/A	101300	778.1313

Table 5: Results of the cycle analysis for the Turbojet Engine with an RPM of 140000

Stage	$\mathbf{P_t}$ (Pa)	$\mathbf{T_t}$ (K)	$\mathbf{T_t}$' (K)	**P** (Pa)	**T** (K)
0	101300	297	297	101300	297
1	101300	297	297	N/A	N/A
2	230700.62	401.9804	375.7353	203166.6551	387.6452
3	212244.5704	946.6489	N/A	N/A	N/A
4	127230.1575	855.7666	833.046	N/A	N/A
5	127230.1575	855.7666	N/A	N/A	N/A
6	127230.1575	855.7666	N/A	101300	808.4045

Table 6: Results of the cycle analysis for the Turbojet Engine with an RPM of 160000

Stage	$\mathbf{P_t}$ (Pa)	$\mathbf{T_t}$ (K)	$\mathbf{T_t}$' (K)	**P** (Pa)	**T** (K)
0	101300	297	297	101300	297
1	101300	297	297	N/A	N/A
2	256684.07	417.492	387.369	226048.9977	402.6037
3	236149.3444	1018.2061	N/A	N/A	N/A
4	136694.2281	914.2181	888.2211	N/A	N/A
5	136694.2281	914.2181	N/A	N/A	N/A
6	136694.2281	914.2181	N/A	101300	848.2797

Table 7: Results of the cycle analysis for the Turbojet Engine with an RPM of 175000

1.3 Turboprop Cycle Analysis

Stage	$\mathbf{P_t}$ (Pa)	$\mathbf{T_t}$ (K)	$\mathbf{T_t}$' (K)	**P** (Pa)	**T** (K)
0	101300	297	297	101300	297
1	101300	297	297	N/A	N/A
2	155965.532	348.9644	335.9733	137351.1499	336.5199
3	143488.2894	827.4977	N/A	N/A	N/A
4	108145.7406	782.3474	771.0598	N/A	N/A
5	105367.1035	778.2908	N/A	N/A	N/A
6	105367.1035	778.2908	N/A	N/A	N/A
7	105367.1035	778.2908	N/A	101300	770.6749

Table 8: Results of the cycle analysis of the Turboprop Engine with an RPM of 102000

Stage	$\mathbf{P_t}$ (Pa)	$\mathbf{T_t}$ (K)	$\mathbf{T_t}$' (K)	**P** (Pa)	**T** (K)
0	101300	297	297	101300	297
1	101300	297	297	N/A	N/A
2	174783.02	363.7836	347.0877	153922.7833	350.8106
3	160800.3784	844.4478	N/A	N/A	N/A
4	112247.2507	786.4284	771.9235	N/A	N/A
5	107767.0671	780.0591	N/A	N/A	N/A
6	107767.0671	780.0591	N/A	N/A	N/A
7	107767.0671	780.0591	N/A	101300	768.0922

Table 9: Results of the cycle analysis of the Turboprop Engine with an RPM of 120000

Stage	$\mathbf{P_t}$ (Pa)	$\mathbf{T_t}$ (K)	$\mathbf{T_t}$' (K)	**P** (Pa)	**T** (K)
0	101300	297	297	101300	297
1	101300	297	297	N/A	N/A
2	200310.62	382.1645	360.8734	176403.6813	368.5361
3	184285.7704	882.9613	N/A	N/A	N/A
4	118402.5279	809.055	790.5785	N/A	N/A
5	111320.8129	799.1594	N/A	N/A	N/A
6	111320.8129	799.1594	N/A	N/A	N/A
7	111320.8129	799.1594	N/A	101300	780.5476

Table 10: Results of the cycle analysis of the Turboprop Engine with an RPM of 140000

Stage	$\mathbf{P_t}$ (Pa)	$\mathbf{T_t}$ (K)	$\mathbf{T_t}$' (K)	**P** (Pa)	**T** (K)
0	101300	297	297	101300	297
1	101300	297	297	N/A	N/A
2	230700.62	401.9804	375.7353	203166.6551	387.6452
3	212244.5704	946.6489	N/A	N/A	N/A
4	127230.1575	855.7666	833.046	N/A	N/A
5	116324.5623	840.6107	N/A	N/A	N/A
6	116324.5623	840.6107	N/A	N/A	N/A
7	116324.5623	840.6107	N/A	101300	812.0648

Table 11: Results of the cycle analysis of the Turboprop Engine with an RPM of 160000

Stage	$\mathbf{P_t}$ (Pa)	$\mathbf{T_t}$ (K)	$\mathbf{T_t}$' (K)	**P** (Pa)	**T** (K)
0	101300	297	297	101300	297
1	101300	297	297	N/A	N/A
2	265953.02	422.7534	391.3151	234211.7047	407.6775
3	244676.7784	1048.5603	N/A	N/A	N/A
4	140633.5891	940.1824	913.0879	N/A	N/A
5	123734.4041	916.5085	N/A	N/A	N/A
6	123734.4041	916.5085	N/A	N/A	N/A
7	123734.4041	916.5085	N/A	101300	871.8315

Table 12: Results of the cycle analysis of the Turboprop Engine with an RPM of 180000

1.4 Comparison Graphs

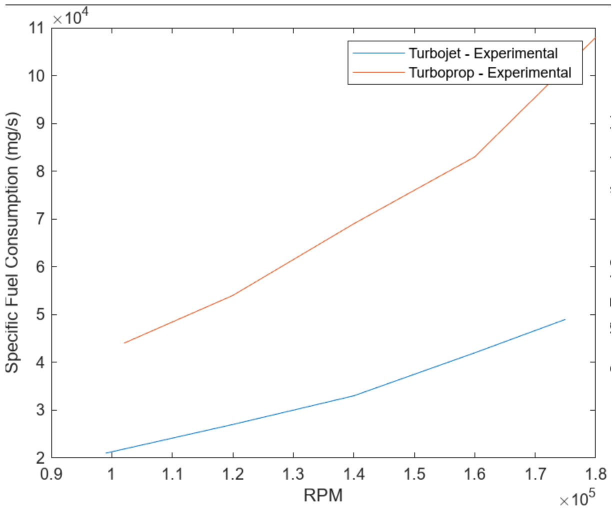

Figure 1: Specific Fuel Consumption is measured for each of the two engines at different operating conditions of the engine (RPM).

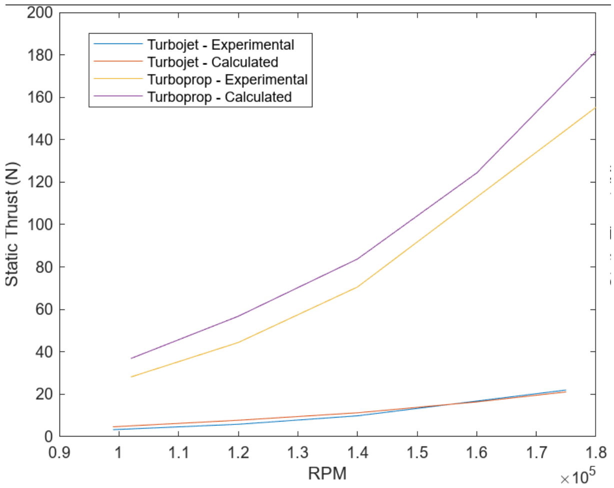

Figure 2: Static Thrust is measured and calculated for each of the engines at different operating points. The Turboprop engine showed a higher thrust value compared to the turbojet engine.

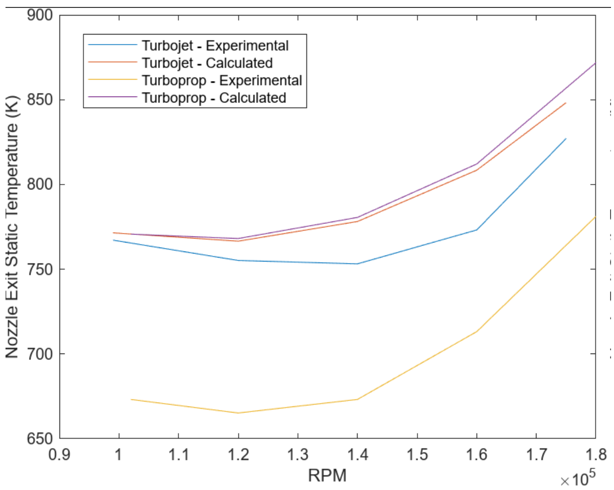

Figure 3: Exhaust Gas Temperature was measured in the lab and calculated from the information about the efficiencies of the two engines and plotted against different RPM values.

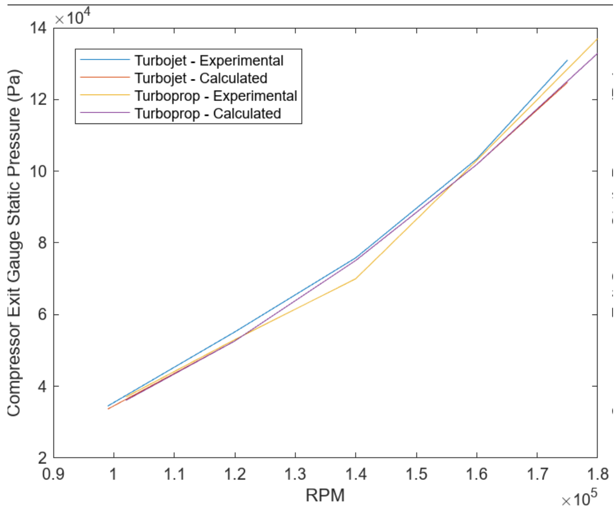

Figure 4: Compressor Exit Static Pressure plotted for different operating points of the engine. The values are very similar between the engines as they contain the same compressor and intake.

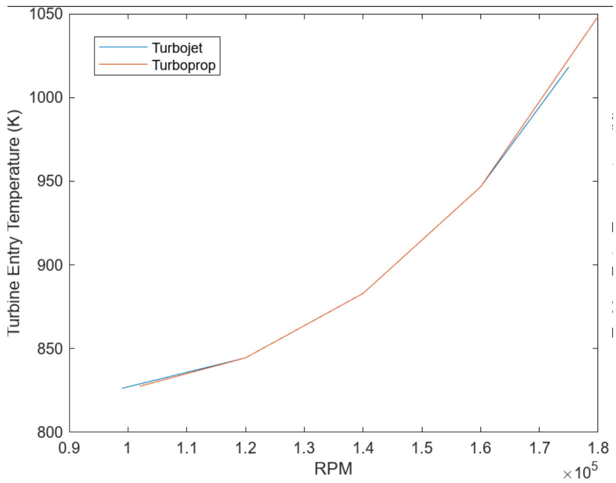

Figure 5: Turbine Entry Temperature at different RPM values. The temperature is calculated leaving the combustion chamber. This is a very important design characteristic for engines due to the material limitations of the turbine blades.

2 Discussion

2.1 Experimental vs Theoretical Differences

First, looking at the static thrust of both engines, Figure 2 shows that the experimental results are closer to the calculated results for the turbojet. However, there are still some differences. These may be due to errors when reading the experimental instrument values. Measuring the RPM was one of the largest challenges as it didn't remain constant at any point. It fluctuated around when measurements were taken with no accurate way to measure this fluctuation and average the data. This could explain the small differences between the thrust as the RPM value taken may be slightly wrong and not match up exactly with the measured value. For the turboprop engine, the values are visibly different. They're almost always the same distance away from the measured thrust for each different RPM. This could be due to how the thrust of the propeller is measured as the thrust calculated is the highest thrust of the propeller, but the measured value may be further downstream allowing the air to lose some of its energy resulting in a lower thrust. The calculation of the thrust also didn't take into account the propeller efficiency only the shaft mechanical efficiency. This could be the factor that could be used to match the calculated to the experimental thrust.

The biggest discrepancies between the calculated and experimental results are with the exhaust gas temperature. Once again, the calculated values for the turbojet are closer to the experimental values. But for both engines, the experimental values are lower than the calculated values. When performing the calculations it is assumed the temperature doesn't change through the exhaust system. This however is unlikely to be the case, most likely there will be a temperature loss through the outer wall to the surrounding air. As the casing is made of metal, an efficient heat conductor, it's likely heat is transferred through the outer casing. As the RPM increases the experimental values become closer to the calculated values. This may be because the engine has been run for a longer time allowing for the different components of the engine to reach a similar temperature resulting in fewer losses. Or it could be due to the heat being generated from the shaft rotating at the higher RPM values. Finally, it could be due to the increasing rate of combustion at higher RPM values in the engine, this will cause a higher overall temperature through the exhaust system and with higher fuel and mass flow rates the gas isn't in the exhaust system as long before it reaches the temperature sensor, so it's closer to the value calculated. This may also have been affected by the assumed ambient conditions, which weren't measured for each test and were assumed to be constant, so further discrepancies could have been introduced here. There are also potential losses from friction between the air and the surface of the exhaust system.

The same calculation is followed for both engines to calculate the compressor exit static pressure. This is because all of the components in this section of the cycle analysis are the same for both engines. The values for the static pressure both experimental and calculated are very similar. These values are similar to the values that are measured by the experimental instruments. There are only slight discrepancies between these values. This again could be due to incorrect RPM values. Alternatively, it could also be due to slight differences in the Mach number for different RPM values, as it has been assumed as constant for each different engine condition. Lastly, it could be because of boundary layer formations in the compressor around the sensor which may cause slight differences in the pressure measured due to the flow turbulence and different Mach numbers, which would result in slightly different ratios of static and dynamic pressure causing these slight differences.

2.2 Engine Performance Differences

Most of the components are the same between the two engines, from the intake to the high-power turbine that drives the compressor, all of the components are the same. The only difference is adding in the low-power turbine that drives the propeller shaft in the turboprop engine. The engine components and processes are similar from the intake to the compressor and this can be seen as the compressor exit static pressure is almost identical for both of the engines. This makes sense as the air is going through the same components up to this point in both engines.

From the measured data shown in Figure 1 it's clear that the turboprop uses more fuel than the turbojet as the mass flow rate of fuel is higher. This is expected as the air-to-fuel ratio for both of these engines is the same and the air mass flow rate through a turboprop engine is much lower than through a turbojet. This is due to the energy of the flow being transferred to the secondary turbine that drives the propeller. This means the kinetic energy of the air will decrease as well as the dynamic pressure so the air will leave the nozzle at a much slower rate. To keep a constant air-to-fuel ratio the mass flow rate of fuel must increase, which explains why the turboprop uses a higher amount of fuel.

The way the engines produce thrust is different which explains why such different values are obtained. The thrust generated is proportional to both the mass flow rate of air and the velocity of the air. The turboprop generates a lot of its thrust from the propeller whereas the turbojet produces most of its thrust from the gas exiting the nozzle and the overall momentum and pressure changes through the engine. As both of the engines being tested in this example are static it makes sense that the turboprop produces a higher thrust as the propeller accelerates a much larger mass of air than the turbojet due to the size of the propeller so at the lower speeds it can produce a much higher thrust. So even though the turbojet maximises the velocity of the air, the mass flow rate isn't high enough to combat the thrust produced by the turboprop.

The exhaust gas temperature exiting the two engines is very different, the turboprop has a much lower exit temperature than the turbojet. This is likely due to a cooling effect caused by the propeller. The propeller pushes air over the engine, and some of the heat from the engine will be transferred to this cooler air. As this air is constantly being replenished there will be a high temperature gradient so the heat transfer will remain higher. This energy transfer will also cause the temperature of the gas in the exhaust to cool. The air is also in the exhaust system for longer due to the lower velocity giving it a larger time frame to dissipate some of the heat energy before reaching the sensor. This loss will take place in the turbojet but not to the same effect as the turboprop.

3 Turbojet Thrust Specific Fuel Consumption (TSFC)

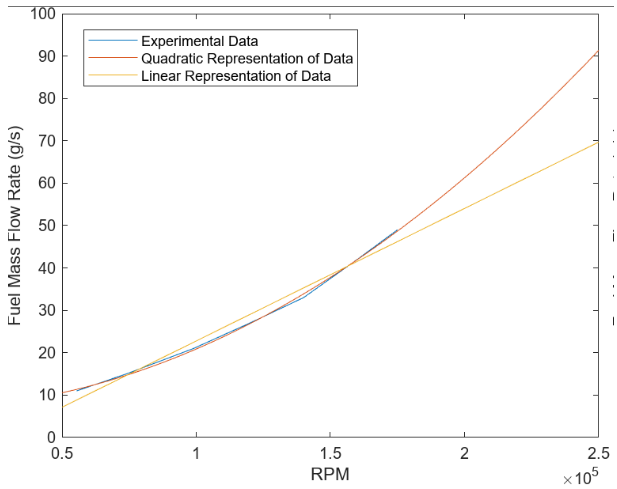

Figure 6: Fuel Mass Flow rate interpolated against a range of RPM values.

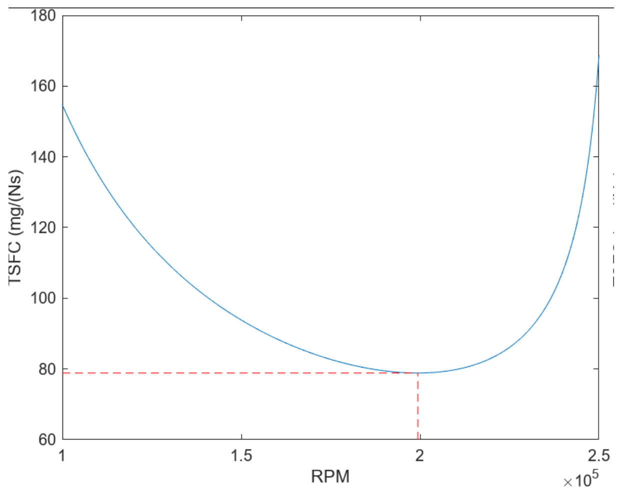

Figure 7: Thrust Specific Fuel Consumption of the Turbojet engine against the RPM it is being run at.

To create this graph the RPM values were extrapolated between 100000 and 250000 and the fuel mass flow rate was assumed to follow a quadratic sequence and extrapolated to larger fuel consumption where no experimental data was taken. A quadratic sequence has been used as it best aligns with the experimental data shown in Figure 6. Using these values the same cycle analysis calculation has been performed to find the TSFC at different engine working conditions. Finally, the calculated TSFC has been plotted against the RPM and presented in Figure 7.

Therefore the optimum engine working condition is at 199370 RPM with a TSFC of 78.8465 mg/(Ns). This means it takes approximately 79mg of fuel to produce 1 newton of fuel each second.

This point represents the optimum operating point of the engine because it uses the minimum of fuel to produce the highest amount of thrust. This can be shown by taking an RPM of 150000, 199370, and 220000 and finding the fuel required to produce 1N of force per second. This can be read from the graph to give the data in Table 13. Each of the values in the table on either side of the optimum condition requires a higher amount of fuel to produce the same amount of thrust. This point occurs at the minimum point on the graph and represents the maximum amount of thrust that the engine can produce to give the highest thrust.

This represents just one optimum condition which may not necessarily be the optimum condition for the engine. If the engine is used on a plane at different altitudes the lowest values of TSFC may not be the optimum aircraft efficiency condition to travel the furthest distance for example. Or if the plane is a military aircraft running at various RPM values due to air combat the highest thrust condition may be

RPM	TSFC (mg/(Ns))
150000	93.8266
199370	78.8465
220000	83.0945

Table 13: Values read from Figure 7

the optimum. This is where other factors will need to be considered, such as the outcomes wanted from the engine.

4 Turboprop cycle analysis

4.1 Stage Diagram

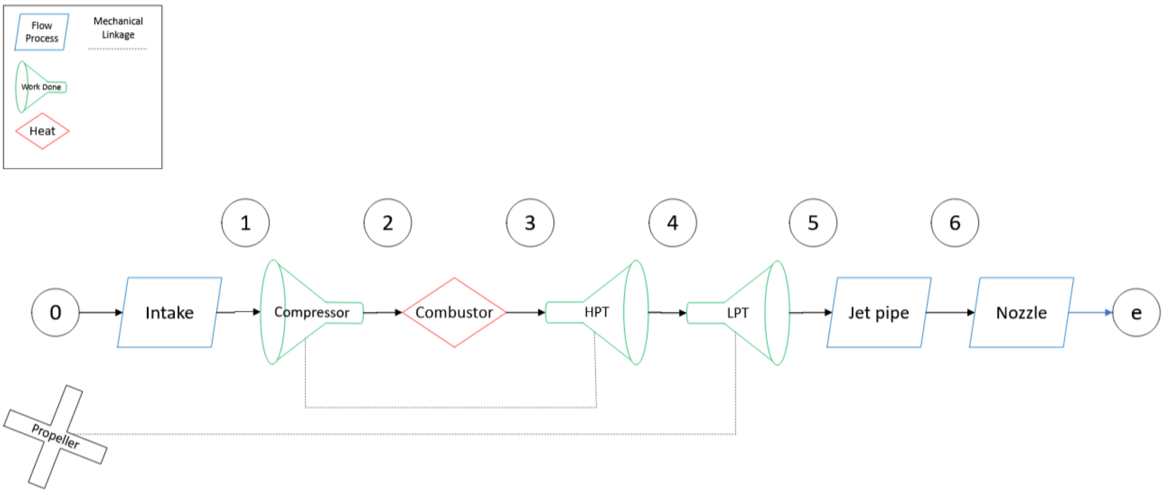

Figure 8: Stage Diagram of the Turboprop engine being analysed in this calculation. HPT and LPT represent the high-powered turbine that drives the compressor and the low-powered turbine that drives the propeller respectively. e represents the exit conditions from the nozzle of the engine.

4.2 Engine Characteristics

Property	Value	Unit
RPM	170000	
Intake Pressure Recovery, PR_i	100%	
Compressor Pressure Ratio, PR_c	2.4454	
Compressor Exit Mach Number, M_c	0.43	
Compressor Isentropic Efficiency, η_c	75%	
Combustor Efficiency, η_{cc}	50%	
Combustor Pressure Recovery, PR_{cc}	92%	
Fuel Lower Heating value, LHV	42.9	MJ/kg
Air-to-Fuel Ratio, AFR	31.2050	
Fuel Mass Flow Rate, $\dot{m}_{fuel}$	3.2912	g/s
Air Mass Flow Rate, $\dot{m}_{air}$	102.7019	g/s
HPT Isentropic Efficiency, η_{HPT}	80%	
HPT Mechanical Efficiency, $\eta_{m,HPT}$	98%	
Power Division, ξ	40%	
LPT Isentropic Efficiency, η_{LPT}	80%	
LPT Mechanical Efficiency, $\eta_{m,LPT}$	90%	
Jet Pipe Pressure Recovery, PR_{jp}	100%	
Nozzle Pressure Recovery, PR_n	100%	
Propeller Diameter	0.65	m

Table 14: All of the Engine Performance Characteristics and their values for this Turboprop analysis.

Some of the values have been calculated using the specific formula provided for this engine. To calculate the Compressor Pressure ratio the following formula has been used based on the provided compressor pressure map.

$$PR_c = 6\text{e}{-11} \times RPM^2 - 3\text{e}{-6} \times RPM + 1.2214 \qquad (1)$$

RPM	Mass Flow Rate of Fuel (g/s)
160000	2.8621
180000	3.7203

Table 15: Snapshot of Table 2 showing the data required to find the mass flow rate of fuel at an RPM of 170000

Similarly, a different plot has been provided for the air-to-fuel ratio. This time the equation used to find this value is shown below.

$$AFR = -18\text{e}{-10} \times RPM^2 + 39\text{e}{-5} \times RPM + 16.925 \quad (2)$$

Using these two equations for an RPM value of 170000 as given in Table 14 the following calculations can be done to work out the values of PR_c and AFR which are shown in Table 14.

$$PR_c = 6\text{e}{-11} \times 170000^2 - 3\text{e}{-6} \times 170000 + 1.2214$$

$$PR_c = 2.4454$$

$$AFR = -18\text{e}{-10} \times 170000^2 + 39\text{e}{-5} \times 170000 + 16.925$$

$$AFR = 31.2050$$

The next value to be calculated before the cycle analysis can start is the mass flow rate of the fuel. This is determined by linearly interpolating the experimental data shown in Table 2 for an RPM value of 170000, a snapshot of this data is shown in Table 15. To do this see the calculation below. As an RPM of 170000 is exactly between the data in Table 15 a simple equation finding the middle value of the two mass flow rates of fuel can be done as it is being assumed that the mass flow rate is increasing linearly between these two points.

$$\dot{m}_{fuel} = \frac{2.8621 + 3.7203}{2} = 3.2912$$

The final value to calculate is the mass flow rate of air. This can be calculated using the mass flow rate of fuel and the air-to-fuel ratio.

$$\dot{m}_{air} = AFR \times \dot{m}_{fuel} = 31.2050 \times 3.2912 = 102.7019 g/s$$

4.3 Ambient Conditions

Property	Value	Units
Altitude	0	km
Static Pressure P_0	101.3	kPa
Static Temperature T_0	297	K
Mach Number M_0	0	

Table 16: Ambient conditions of the air entering the engine intake.

4.4 Gas Properties

Property	Value	Units
γ_{cold}	1.4	
$C_{p,cold}$	1005	J/(kg K)
R_{cold}	287.0	J kg^{-1} K^{-1}
γ_{hot}	1.333	
$C_{p,hot}$	1150	J/(kg K)
R_{hot}	287.3	J kg^{-1} K^{-1}

Table 17: Shows all of the gas properties of the air going through the engine under either hot or cold conditions.

4.5 Stage 0 - 1: Engine Intake

The engine examined is at rest at sea level, meaning that the initial Mach number is 0 and the total temperature and total pressure are equal to the static temperature and pressure. This leaves:

$$P_{t0} = P_0 = 101300Pa$$

$$T_{t0} = T_0 = 297K$$

Next looking at the intake, with a pressure recovery throughout of 100% there will be no losses of temperature or pressure in this intake. Therefore:

$$P_{t1} = P_{t0} = 101300Pa$$

$$T_{t1} = T_{t0} = 297K$$

4.6 Stage 1 - 2: Compressor

With the compressor intake values now known the outputs can be found. First of all, using the Pressure Ratio of the compressor, listed in Table 14, the total pressure exiting the compressor can be found as shown below.

$$P_{t2} = PR_c P_{t1} = 2.4454 \times 101300 = 247719.02Pa$$

Then by first assuming isentropic relationships, the ideal total temperature can first be found and then corrected using the isentropic efficiency of the compressor to find the actual total temperature. The ' notation represents ideal properties.

$$T'_{t2} = T_{t1}\left(\frac{P_{t2}}{P_{t1}}\right)^{\frac{\gamma_{cold}-1}{\gamma_{cold}}} = 297 \times \left(\frac{248630.72}{101300}\right)^{\frac{1.4-1}{1.4}} = 383.454259K$$

$$T_{t2} = \frac{(T'_{t2} - T_{t1})}{\eta_c} + T_{t1} = \frac{(383.454259 - 297)}{0.75} + 297 = 412.2723454K$$

4.7 Stage 2 - 3: Combustor

The total exit pressure can be found in a similar way to the compressor, but rather than a pressure increase, there is a pressure loss. By using the combustor pressure recovery value the exit pressure from the combustor can be calculated.

$$P_{t3} = PR_{cc} P_{t2} = 0.92 \times 247719.02 = 227901.4984Pa$$

Next using the equation for the combustor efficiency the exit temperature from the combustor can be found. In this case, the actual heat added is equal to the change in enthalpy of the gas through the compressor and the ideal heat added is the total chemical energy that could be released from combustion with a particular mass flow rate.

$$\eta_{cc} = \frac{\dot{Q}_{actual}}{\dot{Q}_{ideal}} = \frac{(\dot{m}_{air} + \dot{m}_{fuel})C_{p,hot}(T_{t3} - T_{t2})}{\dot{m}_{fuel}LHV}$$

Where $\dot{Q}$ is the heat release rate. By combining this equation with the equation for the air-to-fuel (AFR) ratio the efficiency can be expressed in terms of the AFR.

$$AFR \equiv \frac{\dot{m}_{air}}{\dot{m}_{fuel}}$$

$$\eta_{cc} = \frac{(AFR+1)C_{p,hot}(T_{t3} - T_{t2})}{LHV}$$

This equation can be rearranged for the combustor exit temperature (T_{t4}) as all of the other values are known.

$$T_{t3} = \frac{LHV\eta_{cc}}{(AFR+1)C_{p,hot}} + T_{t2}$$

$$T_{t3} = \frac{42.9e6 \times 0.5}{(31.2050+1)1150} + 412.2723454 = 991.4424716K$$

4.8 Stage 3 - 4: High Powered Turbine

Another energy balance can be applied here as the power required for the compressor is equal to the power generated by the high-power turbine. The power for both the turbine and the compressor has been directly linked to the change in enthalpy across the compressor on the left of the equation and the change in enthalpy of the turbine on the right-hand side of the equation.

$$\dot{m}_{air}C_{p,cold}(T_{t2} - T_{t1}) = (\dot{m}_{air} + \dot{m}_{fuel})\eta_{m,HPT}C_{p,hot}(T_{t3} - T_{t4})$$

By rearranging and using the AFR relationship again the energy balance equation can be rearranged for the turbine exit temperature.

$$T_{t4} = T_{t3} - \frac{(C_{p,cold}(T_{t2} - T_{t1}))}{\left(1 + \frac{1}{AFR}\right)\eta_{m,HPT}C_{p,hot}}$$

$$T_{t4} = 991.4424716 - \frac{(1005(412.2723454 - 297))}{\left(1 + \frac{1}{31.2050}\right) \times 0.98 \times 1150} = 891.8404487K$$

To calculate the total pressure first the isentropic temperature must be calculated with the isentropic efficiency value of the turbine that has been provided and then this value can be used with an isentropic equation to calculate the total pressure.

$$T'_{t4} = T_{t3} - \frac{T_{t3} - T_{t4}}{\eta_{HPT}} = 991.4424716 - \frac{991.4424716 - 891.8404487}{0.8} = 866.9396375K$$

$$P_{t4} = P_{t3}\left(\frac{T'_{t4}}{T_{t3}}\right)^{\frac{\gamma_{hot}}{\gamma_{hot}-1}} = 227901.4984 \times \left(\frac{866.9396375}{991.4424716}\right)^{\frac{1.333}{1.333-1}} = 133185.6798Pa$$

4.9 Stage 4 - 5: Low Powered Turbine

This turbine is used to drive the propeller and the power of the turbine that goes to the propeller is given by the power division which in this case is 40%. This means that 40% of the power from the turbine goes to driving the propeller. The following formula expresses the power division.

$$\xi = \frac{h_{t4} - h'_{t5}}{h_{t4} - h'_{e}} = 0.4$$

The power generated by the turbine can be found using the equation below.

$$\dot{W}_{LPT} = (\dot{m}_{air} + \dot{m}_{fuel})C_{p,hot}(T_{t4} - T_{t5}) \tag{3}$$

The ideal change in enthalpy across the low-powered turbine by:

$$h_{t4} - h'_{t5} = C_{p,hot}(T_{t4} - T'_{t5})$$

The ideal change in temperature can be calculated using the isentropic efficiency of the turbine and the actual total temperature exiting it.

$$T_{t4} - T'_{t5} = \frac{T_{t4} - T_{t5}}{\eta_{LPT}}$$

By combining these three equations a new equation for the change in enthalpy can be found.

$$h_{t4} - h'_{t5} = \frac{C_{p,hot}(T_{t4} - T_{t5})}{\eta_{LPT}} = \frac{\dot{W}_{LPT}}{(\dot{m}_{air} + \dot{m}_{fuel})\eta_{LPT}}$$

If this pressure turbine was removed and the gases were allowed to expand isentropically through the exhaust system of the engine then the ideal change in enthalpy can be found using the following equation.

$$h_{t4} - h'_{te} = C_{p,hot}(T_{t4} - T'_{e}) = \frac{\dot{W}_{ideal}}{\dot{m}_{air} + \dot{m}_{fuel}}$$

T'_e can be exchanged for an isentropic equation leaving.

$$h_{t4} - h'_{te} = C_{p,hot}\left[1 - \left(\frac{P'_e}{P_{t4}}\right)^{\frac{\gamma_{hot}-1}{\gamma_{hot}}}\right]$$

Then by using the original definition for the power division, and rearranging the equation for the turbine power., the power of the low-power turbine can be found.

$$\dot{W}_{LPT} = \xi(\dot{m}_{air} + \dot{m}_{fuel})\eta_{LPT}C_{p,hot}T_{t4}\left[1 - \left(\frac{P'_e}{P_{t4}}\right)^{\frac{\gamma_{hot}-1}{\gamma_{hot}}}\right]$$

$$\dot{W}_{LPT} = 0.4 \times (102.7019\text{e}{-3} + 3.2912\text{e}{-3}) \times 0.8 \times 1150 \times 891.8404487\left[1 - \left(\frac{101300}{133185.6798}\right)^{\frac{1.333-1}{1.333}}\right]$$

$$\dot{W}_{LPT} = 2.2987kW$$

Now using the equation for the work done by the low-power turbine, Equation 3, the total temperature out of the turbine can be found.

$$T_{t5} = T_{t4} - \frac{\dot{W}_{LPT}}{(\dot{m}_{air} + \dot{m}_{fuel})C_{p,hot}}$$

$$T_{t5} = 891.8404487 - \frac{2.2987e3}{(102.7019\text{e}{-3} + 3.2912\text{e}{-3}) \times 1150} = 872.9822K$$

Using a similar process as with the high-power turbine the ideal total temperature is found and then used in an isentropic equation to work out the actual total pressure at the exit of the low-power compressor.

$$T'_{t5} = T_{t4} - \frac{T_{t4} - T_{t5}}{\eta_{LPT}} = 891.8404487 - \frac{891.8404487 - 872.9822}{0.8} = 868.2677K$$

$$P_{t5} = P_{t4}\left(\frac{T'_{t5}}{T_{t4}}\right)^{\frac{\gamma_{hot}}{\gamma_{hot}-1}} = 133185.6798 \times \left(\frac{868.2677}{891.8404487}\right)^{\frac{1.333}{1.333-1}} = 119643.4824Pa$$

4.10 Stage 5 - 6 - e: Jet Pipe - Nozzle

For both the jet pipe and the exit nozzle it is stated that the exhaust pressure recovery is 100% therefore there is no pressure loss throughout the entire exhaust system.

$$P_{te} = P_{t6} = P_{t5} = 119643.4824Pa$$

There is also no temperature loss in the exhaust system so the final total temperature is the same as the total temperature exiting the low-power turbine.

$$T_{te} = T_{t6} = T_{t5} = 872.9822$$

As this is a Turboprop engine it is assumed that the flow will always fully expand out of the nozzle of the exit static pressure of the air is the same as the ambient air conditions. This is because lots of the power is taken from the air to drive the low-power turbine meaning it loses lots of its momentum allowing it to fully expand out of the nozzle.

$$P_e = P_0 = 101.3kPa$$

It is also possible to check this with a choke test. The critical pressure ratio can be found by the following:

$$P_{crit} = \left(\frac{\gamma_{hot}+1}{2}\right)^{\frac{\gamma_{hot}}{\gamma_{hot}-1}} = \left(\frac{1.333+1}{2}\right)^{\frac{1.333}{1.333-1}} = 1.852$$

Next, the ideal pressure ratio must be determined, this is the ratio that would be necessary for the flow to fully expand:

$$\frac{P_{te}}{P_0} = \frac{119643.4824}{101300} =$$

From the two above calculations, it is clear that $P_{crit} > \frac{P_{te}}{P_0}$ therefore the nozzle is unchoked and our static pressure is equal to the ambient static pressure. The static temperature can be found using the following equation as the nozzle is unchoked.

$$T_e = T_{te}\left(\frac{P_e}{P_{te}}\right)^{\frac{\gamma_{hot}-1}{\gamma_{hot}}} = 872.9822\left(\frac{101300}{11963.4824}\right)^{\frac{1.333-1}{1.333}} = 837.4310K$$

With this, the output velocity of the gas through the nozzle can be found.

$$V_e = \sqrt{2C_{p,hot}(T_{te}-T_e)} = \sqrt{2 \times 1150 \times (872.9822 - 837.4310)} = 285.9505m/s$$

Then the output velocity can be converted to give the Mach number of the flow, using the local speed of sound for a hot gas at sea level, a_e.

$$M_e = \frac{V_e}{a_e} = \frac{V_e}{\sqrt{\gamma_{hot}R_{hot}T_e}}$$

$$M_e = \frac{285.9505}{\sqrt{1.333 \times 287.3 \times 837.4310}} = 0.5049$$

4.11 Engine Outputs

To find the thrust that is produced by this engine, we must sum both the thrust from the propeller and the flow out of the nozzle. To find the thrust from the propeller, F_0, the following equation can be used.

$$F_0 = (\dot{W}_B\sqrt{2\rho A_p})^{\frac{2}{3}}$$

To do this the brake shaft power, $\dot{W}_B$, the air density, ρ, and the propeller disc area, A_p must be calculated. First, the density of air at sea level can be assumed to be 1.2256kg/m^3. Next find the brake shaft power, which is determined by:

$$\dot{W}_B = \eta_{m,LPT}\dot{W}_{LPT} = 0.9 \times 2.2987e3 = 2.0699kW$$

Finally, the propeller disc area can be found from the propeller diameter given in Table 14.

$$A_p = \pi r^2 = \frac{\pi d^2}{4} = \frac{\pi \times 0.65^2}{4} = 0.3318$$

Therefore:

$$F_0 = (2.0699e3 \times \sqrt{2 \times 1.2256 \times 0.3318})^{\frac{2}{3}} = 151.5576N$$

The jet thrust, F_N, from the air out of the nozzle can be found using the change in momentum of the gas. There is no thrust from the pressure in this case as the initial static pressure is the same as the exit static pressure.

$$F_N = (\dot{m}_{air} + \dot{m}_{fuel})(V_e - V_0) = (102.7019\text{e}{-3} + 3.2912\text{e}{-3})(285.9505 - 0) = 30.3088N$$

This gives an overall thrust from the turboprop engine of:

$$F = F_0 + F_N = 151.5576 + 30.3088 = 181.8664N$$

The final thing to calculate for this cycle analysis is the Power Specific Fuel Consumption, $PSFC$, of this turboprop engine.

$$PSFC = \frac{\dot{m}_{fuel}}{\dot{W}_{jet} + \dot{W}_A}$$

To do this the jet power, $\dot{W}_{jet}$, and the available power, $\dot{W}_A$, must be calculated. Without being provided with any information about the propeller efficiency or the gear ratio it has been assumed that these are 1 and so the available power is equal to the brake shaft power. To work out the jet power the following equation is used.

$$\dot{W}_{jet} = \frac{(\dot{m}_{air} + \dot{m}_{fuel})V_e^2}{2} - \frac{\dot{m}_{air}V_0^2}{2} = \frac{(102.7019\text{e}{-3} + 3.2912\text{e}{-3})285.9505^2}{2} = 4.3334kW$$

Therefore:

$$PSFC = \frac{3.2912\text{e}{-3}}{4.3334e3 + 2.0699e3} = 5.1407\text{e}{-7}kg/(Nm)$$

Or

$$PSFC = 0.51407mg/(Nm)$$

A Matlab Code

A.1 Turbojet Function

```
function [TSFC,Static_Thrust,T_6,P_2,Tt_3,turbojet_Data] =
    TurbojetFunction(y_cold,C_p_cold,R_cold,R_hot,y_hot,C_p_hot,P_0,T_0,
    M_0,m_air,m_fuel,PR_i,PR_c,n_c,M_c,PR_cc,TET,LHV,n_cc,n_t,n_m,AFR,
    PR_jp)
Intake (0-1)
    a_0 = sqrt(y_cold*R_cold*T_0);
    V_0 = M_0 * a_0;
    Pt_0 = P_0*(1+((y_cold-1)/2)*M_0^2)^(y_cold/(y_cold-1));
    Tt_0 = T_0*(Pt_0/P_0)^((y_cold-1)/y_cold);
    Tt_0_prime = Tt_0;
    Pt_1 = Pt_0 * PR_i;
    Tt_1 = Tt_0;
    Tt_1_prime = Tt_0_prime;
Compressor (1-2)
    Pt_2 = PR_c * Pt_1;
    Tt_2_prime = Tt_1*(Pt_2/Pt_1)^((y_cold-1)/y_cold);
    Tt_2 = (Tt_2_prime-Tt_1)/n_c + Tt_1;
    if not(isempty(M_c))
        T_2 = Tt_2/(1+(M_c^2*(y_cold-1))/2);
        P_2 = Pt_2/(Tt_2/T_2)^(y_cold/(y_cold-1)) - P_0;
    else
        P_2 = "N/A";
        T_2 = "N/A";
    end
Combustor (2-3)
    Pt_3 = PR_cc * Pt_2;
    if isempty(TET) && isempty(AFR)
        return
    elseif not(isempty(TET)) && isempty(AFR)
        Tt_3 = TET;
        m_fuel = m_air/((LHV*n_cc)/(C_p_hot*(Tt_3-Tt_2))-1);
        AFR = m_air/m_fuel;
    elseif isempty(TET) && not(isempty(AFR))
        m_air = AFR*m_fuel;
        Tt_3 = (m_fuel*LHV*n_cc)/((m_air+m_fuel)*C_p_hot)+Tt_2;
    end
Turbine (3-4)
    Tt_4 = Tt_3-(C_p_cold*(Tt_2-Tt_1))/((1+1/AFR)*n_m*C_p_hot);
    Tt_4_prime = Tt_3-(Tt_3-Tt_4)/n_t;
    Pt_4 = Pt_3*(Tt_4_prime/Tt_3)^(y_hot/(y_hot-1));
Jet Pipe (4-5)
    Pt_5 = PR_jp*Pt_4;
    Tt_5 = Tt_4;
Nozzle (5-6)
    Pt_6 = Pt_5;
    Tt_6 = Tt_5;
Choke test
    PR_crit = ((y_hot+1)/2)^(y_hot/(y_hot-1));
    PR_ideal = Pt_6/P_0;
Choked
    if PR_ideal > PR_crit
        M_6 = 1;
        P_6 = Pt_6/PR_crit;
        TR_crit = (y_hot+1)/2;
```

```
        T_6 = Tt_6/TR_crit;
Unchoked
    else
        P_6 = P_0;
        T_6 = (Tt_6/(Pt_6/P_6)^((y_hot-1)/y_hot));
    end
Exit velocity
    V_6 = sqrt(2*C_p_hot*(Tt_6-T_6));
Nozzle Exit Area
    A_e = ((m_air+m_fuel)*R_hot*T_6)/(P_6*V_6);
Thrust
    Static_Thrust = (m_air+m_fuel)*V_6 - m_air*V_0 + A_e*(P_6-P_0);
Thrust specific fuel consumption
    TSFC = m_fuel/Static_Thrust;
Data Output
    headers = ["Stage", "Pt", "Tt", "Tt_prime","P","T"];
    turbojet_Data = [
        "0", Pt_0,Tt_0,Tt_0_prime,P_0,T_0;
        "1", Pt_1,Tt_1,Tt_1_prime,"N/A","N/A";
        "2", Pt_2,Tt_2,Tt_2_prime,P_2,T_2;
        "3", Pt_3,Tt_3,"N/A","N/A","N/A";
        "4", Pt_4,Tt_4,Tt_4_prime,"N/A","N/A";
        "5", Pt_5,Tt_5,"N/A","N/A","N/A";
        "6", Pt_6,Tt_6,"N/A",P_6,T_6];
    turbojet_Data = array2table(turbojet_Data,"VariableNames",headers);
return
```

A.2 Turboprop Function

```
    function [m_fuel,Static_Thrust,T_7,P_2,Tt_3,turboprop_Data] =
       TurbopropFunction(y_cold,C_p_cold,R_cold,R_hot,y_hot,C_p_hot,P_0,
       T_0,M_0,m_air,m_fuel,PR_i,PR_c,n_c,M_c,PR_cc,TET,LHV,n_cc,n_hpt,
       n_m_hp,pd,n_lpt,n_m_lp,AFR,PR_jp,A_p,density)
Intake (0-1)
    a_0 = sqrt(y_cold*R_cold*T_0);
    V_0 = M_0 * a_0;
    Pt_0 = P_0*(1+((y_cold-1)/2)*M_0^2)^(y_cold/(y_cold-1));
    Tt_0 = T_0*(Pt_0/P_0)^((y_cold-1)/y_cold);
    Tt_0_prime = Tt_0;
    Pt_1 = Pt_0 * PR_i;
    Tt_1 = Tt_0;
    Tt_1_prime = Tt_0_prime;
Compressor (1-2)
    Pt_2 = PR_c * Pt_1;
    Tt_2_prime = Tt_1*(Pt_2/Pt_1)^((y_cold-1)/y_cold);
    Tt_2 = (Tt_2_prime-Tt_1)/n_c + Tt_1;
    if not(isempty(M_c))
        T_2 = Tt_2/(1+(M_c^2*(y_cold-1))/2);
        P_2 = Pt_2/(Tt_2/T_2)^(y_cold/(y_cold-1)) - P_0;
    else
        P_2 = "N/A";
        T_2 = "N/A";
    end
Combustor (2-3)
    Pt_3 = PR_cc * Pt_2;
    if isempty(TET) && isempty(AFR)
        return
```

```
    elseif not(isempty(TET)) && isempty(AFR)
        Tt_3 = TET;
        m_fuel = m_air/((LHV*n_cc)/(C_p_hot*(Tt_3-Tt_2))-1);
        AFR = m_air/m_fuel;
    elseif isempty(TET) && not(isempty(AFR))
        m_air = AFR*m_fuel;
        Tt_3 = (m_fuel*LHV*n_cc)/((m_air+m_fuel)*C_p_hot)+Tt_2;
    end
High Pressure Turbine (3-4)
    Tt_4 = Tt_3-(C_p_cold*(Tt_2-Tt_1))/((1+1/AFR)*n_m_hp*C_p_hot);
    Tt_4_prime = Tt_3-(Tt_3-Tt_4)/n_hpt;
    Pt_4 = Pt_3*(Tt_4_prime/Tt_3)^(y_hot/(y_hot-1));
Low Pressure Turbine (4-5)
    %Nozzle assumed to be unchoked as most of the power goes to the
    %propeller rather than to thrust
    P_7 = P_0;
    %Work done by low pressure turbine to drive propeller
    W_lpt = pd*(m_air+m_fuel)*n_lpt*C_p_hot*Tt_4*(1-(P_7/Pt_4)^((y_hot-1)/y_hot));
    %From this the total temperature exiting the turbine can be found
    Tt_5 = Tt_4-W_lpt/(C_p_hot*(m_air+m_fuel));
    Tt_5_prime = Tt_4-(Tt_4-Tt_5)/n_lpt;
    Pt_5 = Pt_4*(Tt_5_prime/Tt_4)^(y_hot/(y_hot-1));
Jet Pipe (5-6)
    Pt_6 = PR_jp*Pt_5;
    Tt_6 = Tt_5;
Nozzle (5-6)
    Pt_7 = Pt_6;
    Tt_7 = Tt_6;
Choke test
    PR_crit = ((y_hot+1)/2)^(y_hot/(y_hot-1));
    PR_ideal = Pt_7/P_0;
Choked
    if PR_ideal > PR_crit
        M_7 = 1;
        P_7 = Pt_7/PR_crit;
        TR_crit = (y_hot+1)/2;
        T_7 = Tt_7/TR_crit;
Unchoked
    else
        P_7 = P_0;
        T_7 = (Tt_7/(Pt_7/P_7)^((y_hot-1)/y_hot));
    end
Exit velocity
    V_7 = sqrt(2*C_p_hot*(Tt_7-T_7));
Nozzle Exit Area
    A_e = ((m_air+m_fuel)*R_hot*T_7)/(P_7*V_7);
Jet Thrust
    Jet_Thrust = (m_air+m_fuel)*(V_7-V_0) + A_e*(P_7-P_0);
Propeller Thrust
    %Brake shaft power
    W_B = W_lpt*n_m_lp;
    %Assume that propeller efficiency and gear ratio are 1
    W_A = W_B;
    Static_Thrust = (W_B*sqrt(2*density*A_p))^(2/3);
Jet Power
    W_jet = ((m_air+m_fuel)*V_7^2)/2 - (m_air*V_0^2)/2;
Power specific fuel consumption
```

```
    SFC = m_fuel/(W_jet+W_A);
Data Output
    headers = ["Stage", "Pt", "Tt", "Tt_prime","P","T"];
    turboprop_Data = [
        "0", Pt_0,Tt_0,Tt_0_prime,P_0,T_0;
        "1", Pt_1,Tt_1,Tt_1_prime,"N/A","N/A";
        "2", Pt_2,Tt_2,Tt_2_prime,P_2,T_2;
        "3", Pt_3,Tt_3,"N/A","N/A","N/A";
        "4", Pt_4,Tt_4,Tt_4_prime,"N/A","N/A";
        "5", Pt_5,Tt_5,"N/A","N/A","N/A";
        "6", Pt_6,Tt_6,"N/A","N/A","N/A";
        "7", Pt_7,Tt_7,"N/A",P_7,T_7];
    turboprop_Data = array2table(turboprop_Data,"VariableNames",headers)
        ;
return
```

A.3 Experimental Data Analysis and Graph creation

```
    clc
clear
Experimental Analysis
%Turbojet
RPM = transpose([55500,74000,99000,120000,140000,160000,175000])
Start_time = [0.17,0.03,0.22,0.08,0.01,0.02,0.05]
End_time = [28.01, 29.11,29.09,29.16,29.18,29.02,28.15]
Initial_fuel = [3802,3751,3687,3639,3577,3501,3429] %g
Final_fuel = [3791, 3736,3666,3612,3544,3459,3380]
%Consumption = diff(Initial_fuel,Final_fuel)
EGT = transpose([592,540,494,482,480,500,554])+273.15
Case_Pressure = [2,3,5,8,11,15,19] %PSI

T = [0,4.3,9.4,20.2];
R = [57200,112000,138800,169900];

Thrust_calculated = transpose([0,((T(2)-T(1))/(R(2)-R(1)))*(RPM(2)-R(1))
    +T(1),((T(2)-T(1))/(R(2)-R(1)))*(RPM(3)-R(1))+T(1),((T(3)-T(2))/(R(3)
    -R(2)))*(RPM(4)-R(2))+T(2),((T(4)-T(3))/(R(4)-R(3)))*(RPM(5)-R(3))+T
    (3),((T(4)-T(3))/(R(4)-R(3)))*(RPM(6)-R(3))+T(3),((T(4)-T(3))/(R(4)-R
    (3)))*(RPM(7)-R(3))+T(3)]);

Case_Pressure = transpose(Case_Pressure.*6894.76);

Consumption = transpose(Initial_fuel-Final_fuel);

time = transpose(End_time-Start_time);

fuel_mass_flow = Consumption./time;

TurbojetData = table(RPM,Thrust_calculated,Consumption,fuel_mass_flow,
    EGT,Case_Pressure)
Theoretical analysis - Turbojet
RPM = transpose([99000,120000,140000,160000,175000])
m_fuel = TurbojetData.fuel_mass_flow(3:end,:)./1000
for i = 1:length(RPM)

    PR_i = 1;
    PR_c = 6e-11*RPM(i)^2-3e-6*RPM(i)+1.2214;
```

```
    m_air = [];
    n_c = 0.75;
    M_c = 0.43; %Mach number out of compressor
    PR_cc = 0.92;
    n_cc = 0.5;
    LHV = 42.9e6;
    n_t = 0.80;
    TET = [];
    n_m = 0.98;

    AFR = -18e-10*RPM(i)^2+39e-5*RPM(i)+16.925;

    PR_jp = 1;

    P_0 = 101.3e3;
    T_0 = 297;
    M_0 = 0;

    y_cold = 1.4;
    C_p_cold = 1005;
    R_cold = 287;
    R_hot = 287.3;
    y_hot = 1.333;
    C_p_hot = 1150;

    [SPC(i),Thrust(i),T_e(i),P_c(i),T_turbine(i),DataJet] =
       TurbojetFunction(y_cold,C_p_cold,R_cold,R_hot,y_hot,C_p_hot,P_0,
       T_0,M_0,m_air,m_fuel(i),PR_i,PR_c,n_c,M_c,PR_cc,TET,LHV,n_cc,n_t,
       n_m,AFR,PR_jp)
end
TurbojetValues = table(SPC,Thrust,T_e,P_c,T_turbine)
Experimental Analysis - Turboprop
%Turboprop
RPM = transpose([56000,83000,102000,120000,140000,160000,180000])
Start_time = [0.18,0,0.05,0.01,0.05,0.13,0.13]
End_time = [30,29.2,29.1,29.15,31.02,29.13,29.16]
Initial_fuel = [3081,2978,2902,2818,2702,2592,2829]
Final_fuel = [3057,2942,2858,2764,2633,2509,2721]
%Consumption = diff(Initial_fuel,Final_fuel)
EGT = transpose([512,472,400,392,400,440,508])+273.15
Case_Pressure = [0.13,0.24,0.37,0.53,0.7,1.03,1.37] %bar

T = [0.9,35.5,75,139.4];
R = [59200,113600,142100,172500];

Thrust_calculated = transpose([0,((T(2)-T(1))/(R(2)-R(1)))*(RPM(2)-R(1))
    +T(1),((T(2)-T(1))/(R(2)-R(1)))*(RPM(3)-R(1))+T(1),((T(3)-T(2))/(R(3)
    -R(2)))*(RPM(4)-R(2))+T(2),((T(4)-T(3))/(R(4)-R(3)))*(RPM(5)-R(3))+T
    (3),((T(4)-T(3))/(R(4)-R(3)))*(RPM(6)-R(3))+T(3),((T(4)-T(3))/(R(4)-R
    (3)))*(RPM(7)-R(3))+T(3)]);

Case_Pressure = transpose(Case_Pressure.*1e5);

Consumption = transpose(Initial_fuel-Final_fuel);

time = transpose(End_time-Start_time);

```

```
fuel_mass_flow = Consumption./time;

TurbopropData = table(RPM,Thrust_calculated,Consumption,fuel_mass_flow,
    EGT,Case_Pressure)
Theoretical Analysis
RPM = TurbopropData.RPM(3:end,:)
m_fuel = TurbopropData.fuel_mass_flow(3:end,:)./1000
for i = 1:length(RPM)
     PR_i = 1;
     PR_c = 6e-11*RPM(i)^2-3e-6*RPM(i)+1.2214;
     m_air = [];

     AFR = -18e-10*RPM(i)^2+39e-5*RPM(i)+16.925;
     n_c = 0.75;
     M_c = 0.43; %Mach number out of compressor
     PR_cc = 0.92;
     n_cc = 0.5;
     LHV = 42.9e6;
     TET = [];
     n_hpt = 0.80;
     n_m_hp = 0.98;
     pd = 0.40; %40% of the power goes to the propeller
     n_lpt = 0.8;
     n_m_lp = 0.90;
     A_p = (0.65^2)/4*pi;
     density = 1.2256;

     PR_jp = 1;
     PR_n = 1;

     P_0 = 101.3e3;
     T_0 = 297;
     M_0 = 0;

     y_cold = 1.4;
     C_p_cold = 1005;
     R_cold = 287;
     R_hot = 287.3;
     y_hot = 1.333;
     C_p_hot = 1150;

  [SPC(i),Thrust(i),T_e(i),P_c(i),T_turbine(i),DataProp] =
      TurbopropFunction(y_cold,C_p_cold,R_cold,R_hot,y_hot,C_p_hot,P_0,
      T_0,M_0,m_air,m_fuel(i),PR_i,PR_c,n_c,M_c,PR_cc,TET,LHV,n_cc,n_hpt,
      n_m_hp,pd,n_lpt,n_m_lp,AFR,PR_jp,A_p,density)
end
TurbopropValues = table(SPC,Thrust,T_e,P_c,T_turbine)
Plot Graphs
figure
plot(TurbojetData.RPM(3:end,:),TurbojetData.Consumption(3:end,:).*1000,
    TurbopropData.RPM(3:end,:),TurbopropData.Consumption(3:end,:).*1e3)
xlabel('RPM')
ylabel('Specific Fuel Consumption (mg/s)')
legend('Turbojet - Experimental','Turboprop - Experimental')

figure
plot(TurbojetData.RPM(3:end,:),TurbojetData.Thrust_calculated(3:end,:),
    TurbojetData.RPM(3:end,:),TurbojetValues.Thrust,TurbopropData.RPM(3:
```

```
end,:),TurbopropData.Thrust_calculated(3:end,:),TurbopropData.RPM(3:end,:),TurbopropValues.Thrust)
xlabel('RPM')
ylabel('Static Thrust (N)')
legend('Turbojet - Experimental','Turbojet - Calculated','Turboprop - Experimental', 'Turboprop - Calculated')

figure
plot(TurbojetData.RPM(3:end,:),TurbojetData.EGT(3:end,:),TurbojetData.RPM(3:end,:),TurbojetValues.T_e,TurbopropData.RPM(3:end,:),TurbopropData.EGT(3:end,:),TurbopropData.RPM(3:end,:),TurbopropValues.T_e)
xlabel('RPM')
ylabel('Nozzle Exit Static Temperature (K)')
legend('Turbojet - Experimental','Turbojet - Calculated','Turboprop - Experimental', 'Turboprop - Calculated')

figure
plot(TurbojetData.RPM(3:end,:),TurbojetData.Case_Pressure(3:end,:),TurbojetData.RPM(3:end,:),TurbojetValues.P_c,TurbopropData.RPM(3:end,:),TurbopropData.Case_Pressure(3:end,:),TurbopropData.RPM(3:end,:),TurbopropValues.P_c)
xlabel('RPM')
ylabel('Compressor Exit Gauge Static Pressure (Pa)')
legend('Turbojet - Experimental','Turbojet - Calculated','Turboprop - Experimental', 'Turboprop - Calculated')

figure
plot(TurbojetData.RPM(3:end,:),TurbojetValues.T_turbine,TurbopropData.RPM(3:end,:),TurbopropValues.T_turbine)
xlabel('RPM')
ylabel('Turbine Entry Temperature (K)')
legend('Turbojet','Turboprop')
```

A.4 Minimum Thrust Specific Fuel Consumption

```
    %Turbojet
RPM = transpose([55500,74000,99000,120000,140000,160000,175000]);
Start_time = [0.17,0.03,0.22,0.08,0.01,0.02,0.05];
End_time = [28.01, 29.11,29.09,29.16,29.18,29.02,28.15];
Initial_fuel = [3802,3751,3687,3639,3577,3501,3429]; %g
Final_fuel = [3791, 3736,3666,3612,3544,3459,3380];
%Consumption = diff(Initial_fuel,Final_fuel)
EGT = transpose([592,540,494,482,480,500,554])+273.15;
Case_Pressure = [2,3,5,8,11,15,19]; %PSI

T = [0,4.3,9.4,20.2];
R = [57200,112000,138800,169900];

Thrust_calculated = transpose([0,((T(2)-T(1))/(R(2)-R(1)))*(RPM(2)-R(1))+T(1),((T(2)-T(1))/(R(2)-R(1)))*(RPM(3)-R(1))+T(1),((T(3)-T(2))/(R(3)-R(2)))*(RPM(4)-R(2))+T(2),((T(4)-T(3))/(R(4)-R(3)))*(RPM(5)-R(3))+T(3),((T(4)-T(3))/(R(4)-R(3)))*(RPM(6)-R(3))+T(3),((T(4)-T(3))/(R(4)-R(3)))*(RPM(7)-R(3))+T(3)]);

Case_Pressure = transpose(Case_Pressure.*6894.76);

```

```
Consumption = transpose(Initial_fuel-Final_fuel);

time = transpose(End_time-Start_time);

fuel_mass_flow = Consumption./time;

TurbojetData = table(RPM,Thrust_calculated,Consumption,fuel_mass_flow,
    EGT,Case_Pressure)

x = TurbojetData.RPM;
y = TurbojetData.Consumption;
%Fit the curve with a linear relationship
m = polyfit(x,y,2);
%Add a 0 to the RC array so the line of best fit crosses the y axis
xi = linspace(50000,250000,2000);
%Calculate y values for line of best fit
yi = polyval(m,xi);

figure
plot(TurbojetData.RPM,TurbojetData.Consumption)
hold on
plot(xi,yi)
m1 = polyfit(x,y,1);
xi = linspace(50000,250000,2000);
yi = polyval(m1,xi);
plot(xi,yi)
hold off
legend('Experimental Data','Quadratic Representation of Data','Linear
    Representation of Data')
xlabel('RPM')
ylabel('Fuel Mass Flow Rate (g/s)')
RPM = linspace(100000,250000,1500);

for i = 1:length(RPM)
    m_fuel = polyval(m,RPM(i))/1000;
    PR_i = 1;
    PR_c = 6e-11*RPM(i)^2-3e-6*RPM(i)+1.2214;

    m_air = [];
    n_c = 0.75;
    M_c = 0.43; %Mach number out of compressor
    PR_cc = 0.92;
    n_cc = 0.5;
    LHV = 42.9e6;
    n_t = 0.80;
    TET = [];
    n_m = 0.98;

    AFR = -18e-10*RPM(i)^2+39e-5*RPM(i)+16.925;

    PR_jp = 1;

    P_0 = 101.3e3;
    T_0 = 297;
    M_0 = 0;

    y_cold = 1.4;
    C_p_cold = 1005;
```

```
    R_cold = 287;
    R_hot = 287.3;
    y_hot = 1.333;
    C_p_hot = 1150;

    [TSFC(i),Thrust(i),T_e(i),P_c(i),T_turbine(i),DataJet] =
       TurbojetFunction(y_cold,C_p_cold,R_cold,R_hot,y_hot,C_p_hot,P_0,
       T_0,M_0,m_air,m_fuel,PR_i,PR_c,n_c,M_c,PR_cc,TET,LHV,n_cc,n_t,n_m
       ,AFR,PR_jp);
end

TSFC = TSFC.*1000000

figure
plot(RPM,TSFC)
hold on

[M,I] = min(TSFC)
RPM_min = RPM(I)
RPM(501)
TSFC(501)
RPM(1201)
TSFC(1200)

x = [100000 RPM_min];
y = [M M];
plot(x,y,'linestyle','--','Color','r')
x = [RPM_min RPM_min];
y = [60 M];
plot(x,y,'LineStyle','--','Color','r')
hold off
xlabel('RPM')
ylabel('TSFC (mg/(Ns)')
```

A.5 RPM 170000 Calculation

```
    RPM = 170000;

x = [160000,180000]
y = [2.8621,3.7203]
p = polyfit(x,y,1)
m_fuel = polyval(p,170000)/1000
PR_i = 1;
    PR_c = 6e-11*RPM^2-3e-6*RPM+1.2214
    m_air = [];

    AFR = -18e-10*RPM^2+39e-5*RPM+16.925
    n_c = 0.75;
    M_c = 0.43; %Mach number out of compressor
    PR_cc = 0.92;
    n_cc = 0.5;
    LHV = 42.9e6;
    TET = [];
    n_hpt = 0.80;
    n_m_hp = 0.98;
    pd = 0.40; %40% of the power goes to the propeller
    n_lpt = 0.8;
```

```
    n_m_lp = 0.90;
    A_p = (0.65^2)/4*pi;
    density = 1.2256;

    PR_jp = 1;
    PR_n = 1;

    P_0 = 101.3e3;
    T_0 = 297;
    M_0 = 0;

    y_cold = 1.4;
    C_p_cold = 1005;
    R_cold = 287;
    R_hot = 287.3;
    y_hot = 1.333;
    C_p_hot = 1150;
Intake (0-1)
    a_0 = sqrt(y_cold*R_cold*T_0);
    V_0 = M_0 * a_0;
    Pt_0 = P_0*(1+((y_cold-1)/2)*M_0^2)^(y_cold/(y_cold-1));
    Tt_0 = T_0*(Pt_0/P_0)^((y_cold-1)/y_cold);
    Tt_0_prime = Tt_0;
    Pt_1 = Pt_0 * PR_i;
    Tt_1 = Tt_0;
    Tt_1_prime = Tt_0_prime;
Compressor (1-2)
    Pt_2 = PR_c * Pt_1;
    Tt_2_prime = Tt_1*(Pt_2/Pt_1)^((y_cold-1)/y_cold);
    Tt_2 = (Tt_2_prime-Tt_1)/n_c + Tt_1;
    if not(isempty(M_c))
        T_2 = Tt_2/(1+(M_c^2*(y_cold-1))/2);
        P_2 = Pt_2/(Tt_2/T_2)^(y_cold/(y_cold-1));
    else
        P_2 = "N/A";
        T_2 = "N/A";
    end
Combustor (2-3)
    Pt_3 = PR_cc * Pt_2;
    if isempty(TET) && isempty(AFR)
        return
    elseif not(isempty(TET)) && isempty(AFR)
        Tt_3 = TET;
        m_fuel = m_air/((LHV*n_cc)/(C_p_hot*(Tt_3-Tt_2))-1);
        AFR = m_air/m_fuel;
    elseif isempty(TET) && not(isempty(AFR))
        m_air = AFR*m_fuel
        Tt_3 = (m_fuel*LHV*n_cc)/((m_air+m_fuel)*C_p_hot)+Tt_2
    end
High Pressure Turbine (3-4)
    Tt_4 = Tt_3-(C_p_cold*(Tt_2-Tt_1))/((1+1/AFR)*n_m_hp*C_p_hot);
    Tt_4_prime = Tt_3-(Tt_3-Tt_4)/n_hpt;
    Pt_4 = Pt_3*(Tt_4_prime/Tt_3)^(y_hot/(y_hot-1))
Low Pressure Turbine (4-5)
    %Nozzle assumed to be unchoked as most of the power goes to the
    %propeller rather than to thrust
    P_7 = P_0;
    %Work done by low pressure turbine to drive propeller
```

```
    W_lpt = pd*(m_air+m_fuel)*n_lpt*C_p_hot*Tt_4*(1-(P_7/Pt_4)^((y_hot
        -1)/y_hot))
    %From this the total temperature exiting the turbine can be found
    Tt_5 = Tt_4-W_lpt/(C_p_hot*(m_air+m_fuel));
    Tt_5_prime = Tt_4-(Tt_4-Tt_5)/n_lpt;
    Pt_5 = Pt_4*(Tt_5_prime/Tt_4)^(y_hot/(y_hot-1));
Jet Pipe (5-6)
    Pt_6 = PR_jp*Pt_5;
    Tt_6 = Tt_5;
Nozzle (5-6)
    Pt_7 = Pt_6;
    Tt_7 = Tt_6;
Choke test
    PR_crit = ((y_hot+1)/2)^(y_hot/(y_hot-1));
    PR_ideal = Pt_7/P_0;
Choked
    if PR_ideal > PR_crit
        M_7 = 1;
        P_7 = Pt_7/PR_crit;
        TR_crit = (y_hot+1)/2;
        T_7 = Tt_7/TR_crit;
Unchoked
    else
        P_7 = P_0;
        T_7 = (Tt_7/(Pt_7/P_7)^((y_hot-1)/y_hot))
    end
Exit velocity
    V_7 = sqrt(2*C_p_hot*(Tt_7-T_7));
Exit Mach number
    M_e = V_7/sqrt(y_hot*R_hot*T_7)
Nozzle Exit Area
    A_e = ((m_air+m_fuel)*R_hot*T_7)/(P_7*V_7);
Jet Thrust
    Jet_Thrust = (m_air+m_fuel)*(V_7-V_0) + A_e*(P_7-P_0);
Propeller Thrust
    %Brake shaft power
    W_B = W_lpt*n_m_lp;
    %Assume that propeller efficiency and gear ratio are 1
    W_A = W_B;
    Static_Thrust = (W_B*sqrt(2*density*A_p))^(2/3);
Jet Power
    W_jet = ((m_air+m_fuel)*V_7^2)/2 - (m_air*V_0^2)/2;
Power specific fuel consumption
    SFC = m_fuel/(W_jet+W_A);
Data Output
    headers = ["Stage", "Pt", "Tt", "Tt_prime","P","T"];
    turboprop_Data = [
        "0", Pt_0,Tt_0,Tt_0_prime,P_0,T_0;
        "1", Pt_1,Tt_1,Tt_1_prime,"N/A","N/A";
        "2", Pt_2,Tt_2,Tt_2_prime,P_2,T_2;
        "3", Pt_3,Tt_3,"N/A","N/A","N/A";
        "4", Pt_4,Tt_4,Tt_4_prime,"N/A","N/A";
        "5", Pt_5,Tt_5,"N/A","N/A","N/A";
        "6", Pt_6,Tt_6,"N/A","N/A","N/A";
        "7", Pt_7,Tt_7,"N/A",P_7,T_7];
    turboprop_Data = array2table(turboprop_Data,"VariableNames",headers)
```