MECH3855 Flight Mechanics - Orbital Assignment

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Contents

1 Part A

1.1 The planets and their orbits

The following data for each of the orbits of the planets around the Sun has been given [1].

Planet	Perihelion, r_p (km)	Aphelion, r_a (km)
Mercury	46001200	69816900
Venus	107477000	108939000
Earth	147098290	152098232
Mars	206669000	249209300
Jupiter	740573600	816520800
Saturn	1.3536e+09	1.5133e+09
Uranus	2.7489e+09	3.0044e+09
Neptune	4.4539e+09	4.5539e+09

Table 1: The data that was provided in the assignment for each of the planetary orbits around the Sun [1].

To calculate the Eccentricity, e, of each planet's orbit, Equation 1 has been used. Where r_a and r_p represent the Aphelion and Perihelion respectively.

$$e = \frac{r_a - r_p}{r_a + r_p} \tag{1}$$

Then to calculate the Specific Angular Momentum, h, the orbital equation shown in Equation 2 has been used. To satisfy the equation the radius has been set equal to the Perihelion and the True Anomaly, θ, is known to be 0 at this point.

$$r = \frac{h^2}{\mu} \frac{1}{1 + e\cos\theta} \tag{2}$$

After setting these values and knowing that $\cos 0 = 1$ Equation 2 can be rearranged to get a new equation to solve for h. This is shown in Equation 3.

$$h = \sqrt{r_p \mu (1 + e)} \tag{3}$$

Before h can be calculated the Gravitational Parameter, μ, must be calculated. As the planets have a heliocentric orbit, an orbit around the Sun, the Gravitational Parameter of the Sun must be used. This can be found using Equation 4.

$$\mu = GM \tag{4}$$

The mass of the Sun has been taken to be 1.989e30 kg [2] and Newton's Gravitational constant, G, is 6.67384e-17 Nkm^2/kg^2. Using this data the Gravitational Parameter of the Sun is calculated to be 1.3274e11 km^3/s^2.

Using Equations 1 and 3 combined with the data in 1 and the Sun's Gravitational Parameter, the Eccentricity and Angular Momentum of each orbit around the Sun can be found. The results are shown in Table 2.

Planet	Eccentricity, e	Angular Momentum, h (km^2/s)
Mercury	0.2056	2.7133e+09
Venus	0.0068	3.7899e+09
Earth	0.0167	4.4556e+09
Mars	0.0933	5.4767e+09
Jupiter	0.0488	1.0154e+10
Saturn	0.0557	1.3773e+10
Uranus	0.0444	1.9522e+10
Neptune	0.0111	2.4450e+10

Table 2: Calculated values for Eccentricity and Angular Momentum for each of the orbits of the planets around the Sun.

1.2 Calculating Orbital Periods

To calculate the Orbital Period for each of the planets Equation 5 can be used. To use this equation the Semi-Major Axis, a, of each planet's orbit needs to be calculated.

$$T = \frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}} \tag{5}$$

The Semi-Major Axis can be calculated using the geometry of the orbit. The Semi-Major Axis is half the distance from the periapsis to the apoapsis of an orbit. This distance is represented by Equation 6.

$$a = \frac{r_p + r_a}{2} \tag{6}$$

Once this has been calculated for each orbit, with the known Gravitational Parameter of the Sun, the Orbital Period for each planet can be calculated using Equation 5. The results of this calculation are shown in Table 3.

Planet	Semi-Major Axis (km)	Orbital Period (s)	Orbital Period (hours)	Orbital Period (Earth days)
Mercury	57909050	7.5997e+06	2.1110e+03	87.9590
Venus	108208000	1.9412e+07	5.3921e+03	224.6725
Earth	149598261	3.1555e+07	8.7652e+03	365.2167
Mars	227939150	5.9348e+07	1.6485e+04	686.8932
Jupiter	778547200	3.7463e+08	1.0406e+05	4.3360e+03
Saturn	1.4334e+09	9.3594e+08	2.5998e+05	1.0833e+04
Uranus	2.8767e+09	2.6608e+09	7.3911e+05	3.0796e+04
Neptune	4.5039e+09	5.2127e+09	1.4480e+06	6.0332e+04

Table 3: Shows all of the necessary values needed to be calculated for each planet to determine the Orbital Period of each planetary orbit as well as the Orbital Period in different sets of units.

1.2.1 Comparing Orbits

This data compares the length of time it takes for each orbit to travel around the Sun. For example, taking one full orbit around the Sun for Neptune would result in multiple orbits for each other planet as Neptune has the longest orbit. It takes Neptune approximately 6033 Earth Days to complete one entire orbit. The number of orbits each of the other planets could complete in this time is shown in Table 4.

Planet	Number of Orbits per 1 Neptune Orbit	Number of complete Orbits per 1 Neptune Orbit
Mercury	685.9142	685
Venus	268.5347	268
Earth	165.1960	165
Mars	87.8337	87
Jupiter	13.9143	13
Saturn	5.5695	5
Uranus	1.9591	1
Neptune	1	1

Table 4: Shows how many orbits each of the planets will complete in the time that Neptune completes one full orbit of the Sun.

1.3 Orbit Radius and Velocity

This section covers the radius of a planet's orbit compared with its speed at different True Anomaly increment values. To create the graphs, the sections between the calculated points have been assumed as straight lines with a linear increase or decrease in radius and velocity. Each graph assumes that the planet is starting at the periapsis of its orbit or a True Anomaly of 0°.

First, increments of 45° have been used for the True Anomaly for the calculations for each planet. With this information, the first thing to calculate is the radius at each True Anomaly increment, starting from 0° to 360°. To calculate the radius the orbital equation, Equation 2 can be used. This can then be plotted on the graph against the True Anomaly.

Having calculated the radius the next thing to calculate is the velocity. At each point along a planet's orbit the velocity, u can be split into two components; the radial component, $\mathrm{u_r}$, and the azimuth or tangential component, $\mathrm{u}_\perp$. Then using Pythagoras' theorem the total velocity can be calculated. This is shown in Equation 7.

$$u^2 = u_r^2 + u_\perp^2 \tag{7}$$

To first calculate the radial and azimuth velocities Equations 8 and 9 respectively can be used.

$$u_r = \frac{\mu}{h} e \sin\theta \tag{8}$$

$$u_\perp = \frac{\mu}{h}(1 + e\cos\theta) \tag{9}$$

From the graphs in Figure 1 it is clear that for each planet the radius increases up to the apoapsis at 180°, this makes sense as the apoapsis is the furthest distance from the focus on an orbit. The velocity decreases up to the same point. This means the velocity is maximum at the periapsis which is expected as at the periapsis the radius is at a minimum and the Angular Momentum of the planet remains constant along the orbit so the velocity must increase to keep a constant Angular Momentum.

Another thing to note from all of the graphs is that as the planet gets further from the Sun, i.e. the orbit radius increases, the velocity of the planet in orbit decreases. This explains why the Orbital Period of the outer planets is much larger than the inner planets.

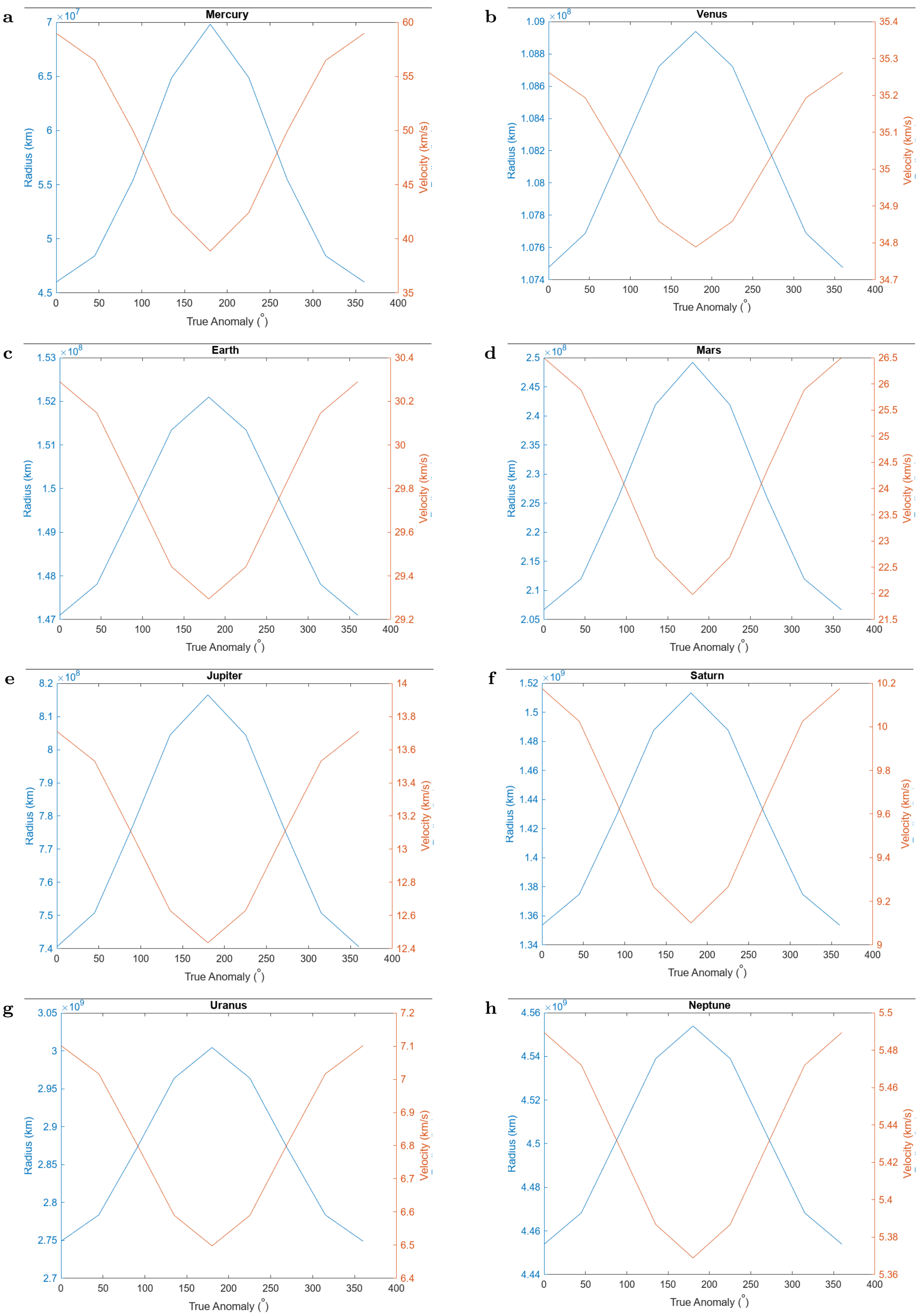

Figure 1: Graph for each of the planets and their orbit radius and velocity with a change in True Anomaly.

1.4 Comparing Orbits Time Periods

This section details a different method to calculate the Orbital Periods for each planet around the Sun. Each orbit has been split into 8 sections of 45°, just as in the previous section. Figure 2 shows two examples of the orbit that is being used (blue) and the orbit that it is trying to represent (orange). A different method must be used to calculate the Orbital Period predicted using this straight-line approximation. This starts with calculating the distance a planet travels in a straight line between two points in the orbit. To do this the radius of each point must be calculated and then the cosine law can be used to determine the displacement between the two points. The radii are calculated using the orbital equation, Equation 2. Once the two radii, r_1 and r_2, have been calculated the distance between the two points, d, can be calculated using Equation 10.

$$d^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\theta \tag{10}$$

Next, the velocity at each point needs to be calculated. This has already been done in the previous section to find the values for the graph so these values can be used again. Therefore, to calculate the velocities, Equations 8 and 9 should be used to find the two components of the velocity. Once this step has been completed the next step is to use Equation 7 to find the total velocity, just as before. For this particular problem, the velocity is assumed to be constant along a straight-line segment and equal to the average velocity between the two points on either side of the straight line. Considering the velocity at the first point, u_1, and the second point, u_2, the average velocity across the region is calculated using Equation 11.

$$u_{avg} = \frac{u_1 + u_2}{2} \tag{11}$$

Then with the distance travelled, d, and the speed of travel, u_{avg}, the time, t, it takes to complete that leg of the journey can be calculated. This is done using Equation 12.

$$t = d/u_{avg} \tag{12}$$

This process can be repeated for each stage of an orbit and then again for each planet. The results of this process are shown in Table 5.

	Orbital Period Theoretical Method			Orbital Period Iterative Method		
Planet	Seconds	Hours	Days	Seconds	Hours	Days
Mercury	7.5997e+06	2.1110e+03	87.9590	7.3453e+06	2.0404e+03	85.0151
Venus	1.9412e+07	5.3921e+03	224.6725	1.8916e+07	5.2546e+03	218.9405
Earth	3.1555e+07	8.7652e+03	365.2167	3.0748e+07	8.5412e+03	355.8838
Mars	5.9348e+07	1.6485e+04	686.8932	5.7741e+07	1.6039e+04	668.2953
Jupiter	3.7463e+08	1.0406e+05	4.3360e+03	3.6492e+08	1.0137e+05	4.2236e+03
Saturn	9.3594e+08	2.5998e+05	1.0833e+04	9.1155e+08	2.5321e+05	1.0550e+04
Uranus	2.6608e+09	7.3911e+05	3.0796e+04	2.5920e+09	7.2000e+05	3.0000e+04
Neptune	5.2127e+09	1.4480e+06	6.0332e+04	5.0797e+09	1.4110e+06	5.8792e+04

Table 5: This table shows the data of the orbits calculated using the theoretical method which uses Equation 5 and the iterative method which uses the iterative calculation method described in Section 1.4

Using the second method assumes that each planet travels straight over each section giving a shorter orbit for each planet compared to a continuous elliptical orbit. Due to the straight line approximation each planet will travel over a shorter distance than if they travelled along their elliptical orbits. As the distance travelled is less a shorter period is expected for each planet. This can be seen in Table 5.

Another assumption that has been made is that the speed of each planet along each straight section is an average between the two sections. Over the shorter distances towards the periapsis the speed changes quite rapidly and the distances are shorter which means the assumption will be more valid. However, towards the apoapsis, the speed fluctuates less but over a longer distance, due to this long distance and the fact that the speed decreases with the inverse square law it will likely be travelling faster over this distance than it would be in reality. This also gives rise to a shorter Orbital Period.

The discrepancy between the two methods is much lower for the planets with smaller orbits. For example, Mercury theoretically has an Orbital Period of 87 Days with only a two-day difference from 85 Days when the iterative method is used, shown in Table 5. Whereas for Neptune the difference between the two values is 1540 Days. The difference can be explained as the distance between distance between two points is much larger for Neptune as the angle is used for both but Neptune is much further from the Sun. This is illustrated in Figure 2, where by taking the scale the distance between the theoretical orbit and the iterative orbit is much bigger. Therefore this assumption is better for smaller orbits.

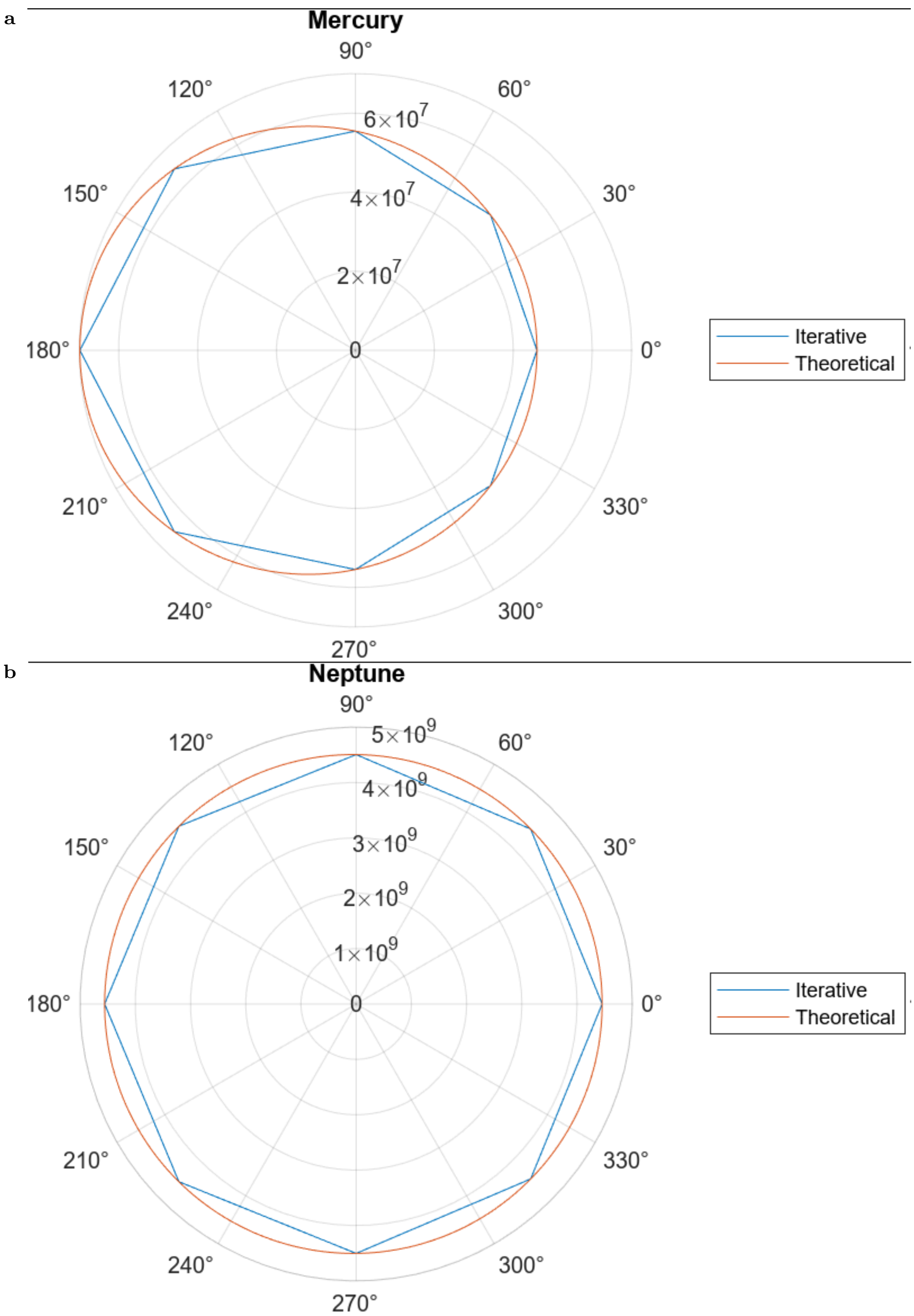

Figure 2: Graph for two planets and their orbits both theoretical and iterative.

1.4.1 Changing the True Anomaly

Using the same method as described before and by splitting each orbit into a different amount of sections, thus changing the True Anomaly difference, different Orbital Period values can be calculated. By comparing the values in Table 6 it is clear that as the True Anomaly gets smaller the Orbital Periods tend to the theoretical calculation performed earlier. This means that the Orbital Period is increasing with smaller True Anomaly values and tending towards the theoretical value, this can be seen from the convergence for each of the Orbital Periods. The convergence is clearly shown in Figure 3 for four of the planets. As the True Anomaly values are smaller the shape of the orbit is becoming more elliptical so the flight path is becoming more accurate. Another diagram shown in Figure 4 shows how by decreasing

	Orbital Period (s) for the different True Anomaly increments					
Planet	$\Delta\theta = 45°$	$\Delta\theta = 20°$	$\Delta\theta = 10°$	$\Delta\theta = 5°$	$\Delta\theta = 1°$	Theoretical Values
Mercury	7.3453e+06	7.5483e+06	7.5868e+06	7.5964e+06	7.5995e+06	7.5997e+06
Venus	1.8916e+07	1.9313e+07	1.9387e+07	1.9406e+07	1.9411e+07	1.9412e+07
Earth	3.0748e+07	3.1394e+07	3.1515e+07	3.1545e+07	3.1554e+07	3.1555e+07
Mars	5.7741e+07	5.9027e+07	5.9267e+07	5.9327e+07	5.9347e+07	5.9348e+07
Jupiter	3.6492e+08	3.7270e+08	3.7415e+08	3.7451e+08	3.7462e+08	3.7463e+08
Saturn	9.1155e+08	9.3109e+08	9.3473e+08	9.3564e+08	9.3593e+08	9.3594e+08
Uranus	2.5920e+09	2.6471e+09	2.6574e+09	2.6599e+09	2.6608e+09	2.6608e+09
Neptune	5.0797e+09	5.1863e+09	5.2061e+09	5.2111e+09	5.2126e+09	5.2127e+09

Table 6: Using the method described in Section 1.4 for different changes in the True Anomaly, the table shows what the Orbital Period is measured at for the different increments compared to the theoretical value calculated previously.

the True Anomaly interval the orbit gets closer to the theoretical orbit. From this figure, it is already becoming difficult to see any differences between the two so with a True Anomaly of 1°the orbits will be almost identical.

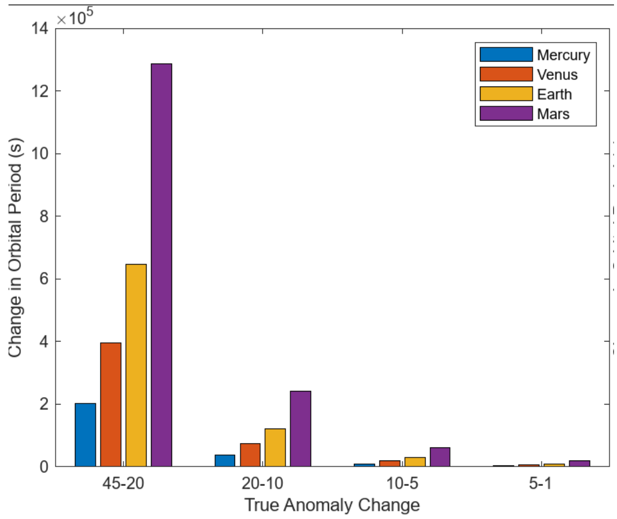

Figure 3: Shows the change in the Orbital Period for the different changes in True Anomaly for the first four planets from the Sun

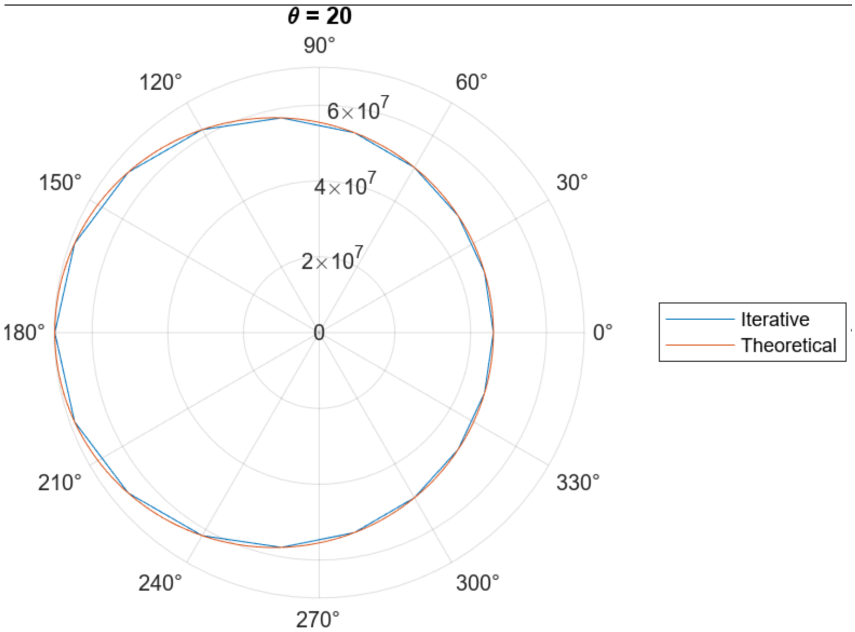

Figure 4: Shows the true orbit of Mercury calculated previously and an iterative orbit that is compared against it to show the differences. This uses a True Anomaly interval of 20°.

1.5 Satisfying Kepler's Third Law

Kepler's Third Law states that the square of the Orbital Period, P,lanet is proportional to the cube of its mean distance from the Sun, represented by the Semi-Major Axis, a. This is summarised nicely by Equation 13.

$$P \propto a^{\frac{3}{2}} \tag{13}$$

A conversion factor between km to Astronomical units of 149597870.7 km/AU [3] and a year has been assumed as the number of days for a single orbit of the Earth around the Sun, 365.22 Days, shown in Table 3 (Calculated in Question 2).

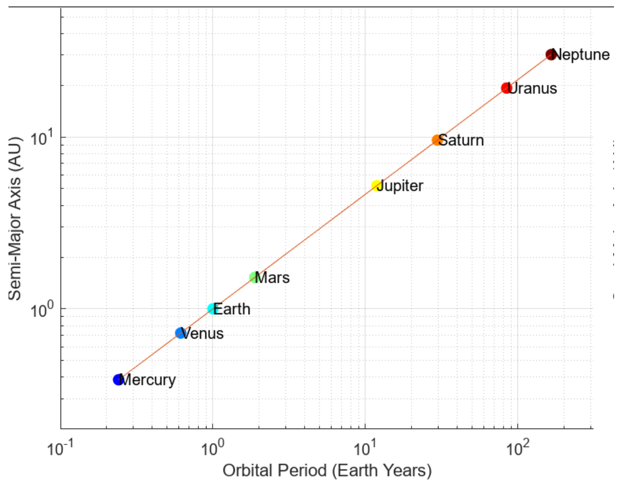

Figure 5: Shows how the Orbital Period for each of the planets is related to the semi-major axis of each orbit.

To show that Figure 5 follows Keplers Law the gradient of the line should be approximately 0.666 or $\frac{2}{3}$. This is because the graph is a log-log graph and the Orbital period is plotted on the x-axis and the Semi-Major axis is plotted on the y-axis. This is because the inverse of this value is 1.5 which is the expected relationship described in Equation 13. As the data is normalised using units of Earth years and Astronomical units (the distance from Earth to the Sun) the expected constant would be equal to 0, as the log of 1 is 0. The derivations where these numbers have been found are shown below. Using the polyfit() function on Matlab with the two parameters as the logged data for the Orbital Period and Semi-Major axis and finding a linear fit the gradient of the line can be found.

$$P = ka^{\frac{3}{2}}$$

$$\log(P) = \log(ka^{\frac{3}{2}})$$

$$\log(P) = \log(k) + \log(a^{\frac{3}{2}})$$

$$\log(P) = \log(k) + \frac{3}{2}\log(a)$$

Then the data is normalised with respect to Earth

$$\log\left(\frac{P}{P_E}\right) = \log\left(\frac{k}{k}\right) + \frac{3}{2}\log\left(\frac{a}{a_E}\right)$$

This can be simplified because k is a constant for all the planets.

$$\log\left(\frac{P}{P_E}\right) = \log(1) + \frac{3}{2}\log\left(\frac{a}{a_E}\right)$$

$$\log\left(\frac{P}{P_E}\right) = \frac{3}{2}\log\left(\frac{a}{a_E}\right)$$

For the graph in Figure 5 the relationship shown in Equation 14 was determined. This shows that the planet's orbit follow Kepler's law.

$$P = a^{\frac{3}{2}} \tag{14}$$

1.5.1 Iterative Time Period

For the Orbital Periods that were calculated using the iterative method the Semi-Major Axis has been considered the same because the method sets to try and mimic the same orbit just with straight lines. To find the Orbital Period in terms of Earth years the time period that was calculated for Earth for each True Anomaly increment has been used (365.2167 in Table 3).

With this data all the graphs in Figure 6 can be constructed. It's expected the graphs will closely follow Kepler's law as the orbits are still elliptical. As the shape is now not a complete ellipse some discrepancies from the correct values are expected, however, not significant amounts. As the True Anomaly tends to 0° it is expected that the orbits will more closely follow Kepler's Law as the shape becomes more and more elliptical as the angle tends to 0.

Figure	Relationship
Figure 6a	$P = a^{\frac{3}{2}}$
Figure 6b	$P = (-0.02836)a^{1.500944}$
Figure 6b	$P = (-0.00561)a^{1.500196}$
Figure 6d	$P = (-0.00141)a^{1.500049}$
Figure 6e	$P = (-0.00035)a^{1.500013}$
Figure 6f	$P = (-0.00002)a^{1.5000005}$

Table 7: This table shows all of the relationships that have been derived for each of the graphs in Figure 6

Table 7 shows that as the interval between the True Anomaly decreases the relationships displayed by the orbits get closer to the relationship expressed by Kepler's Third Law. This is what was expected as detailed above. The differences from Kepler's law are however minimal for each of the different cases. Especially when using a 1°interval the difference is 1 in 50000 for the intercept and the difference for the gradient is 1 in 2000000. Therefore it can be assumed that the orbits represented by this method still follow Kepler's Third Law.

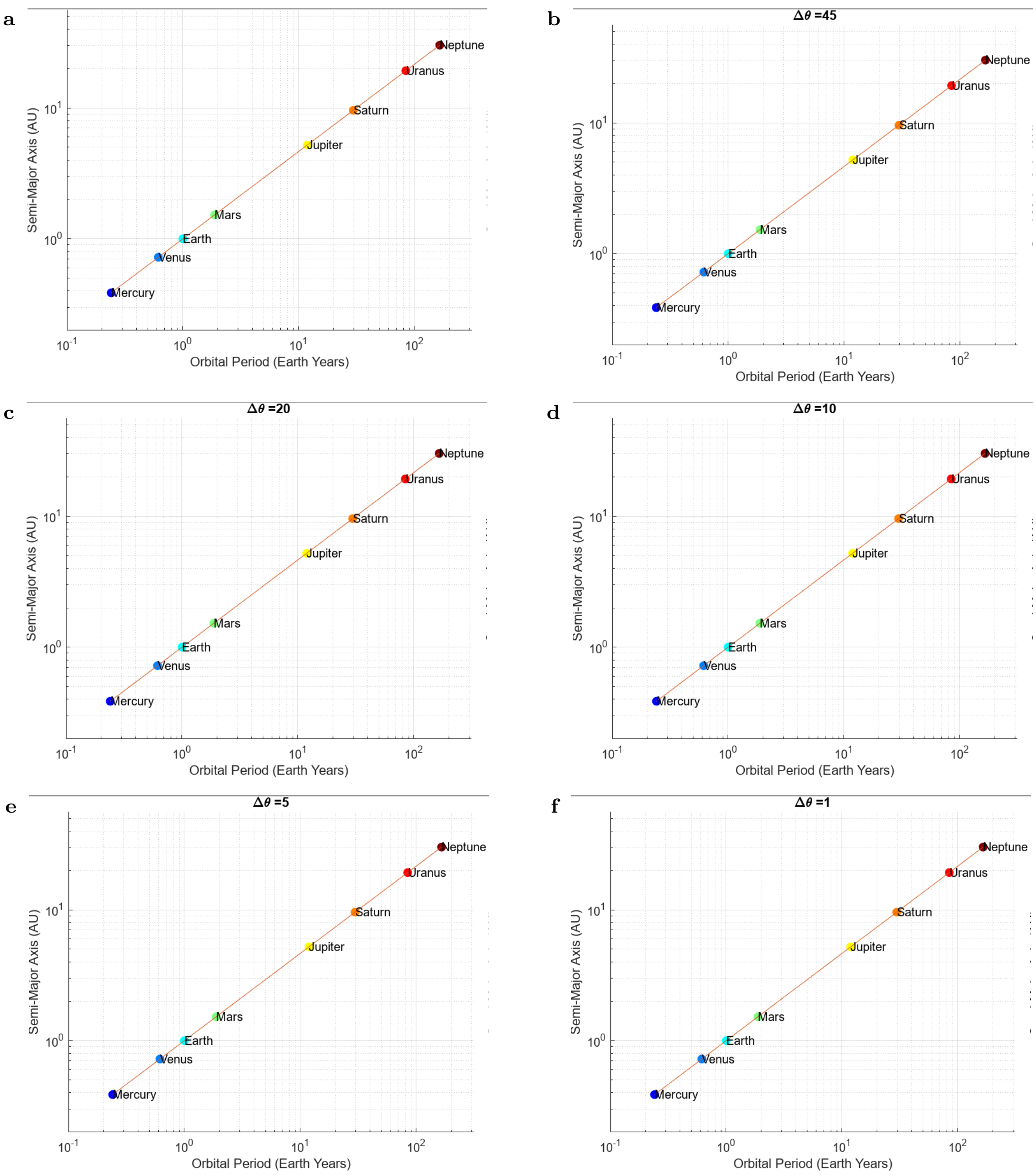

Figure 6: Graph for each of the different True Anomaly values plotting Orbital Period against Semi Major Axis for each of the planets to check if they follow Kepler's Third law. Figure 6a shows the theoretical graph that is also shown in Figure 5

1.6 Orbital Assumptions

For all of the calculations completed in the previous sections of Part A, there have been a few fundamental assumptions for the model to work. These assumptions are highly accurate when representing the orbits of the planets because the amount of days in a year is 365 and for the Orbital Period of the Earth that was calculated 365 days was the result. Lots of the assumptions considered in these calculations won't have a large effect on the results that were obtained. For example, by assuming that all of the planets are at their perigee, shown in Figure 7, the initial set-up of the problem is simplified. However, this will not have much overall effect on each of the different calculations as the actual orbits of the planets aren't being changed. The only difference this will have is that it doesn't reflect the actual planetary positions in the solar system. The actual positions of the planets are shown in Figure 8.

One assumption is to consider 2 body dynamics, not 3 body dynamics. This assumes that there are only interactions between the Sun and the planet; there is no external gravitation force acting between the planets. However, this is not the case and the movement of other planets will affect the movement and position of each of the planets, although this effect appears to be minimal due to the accuracy of the

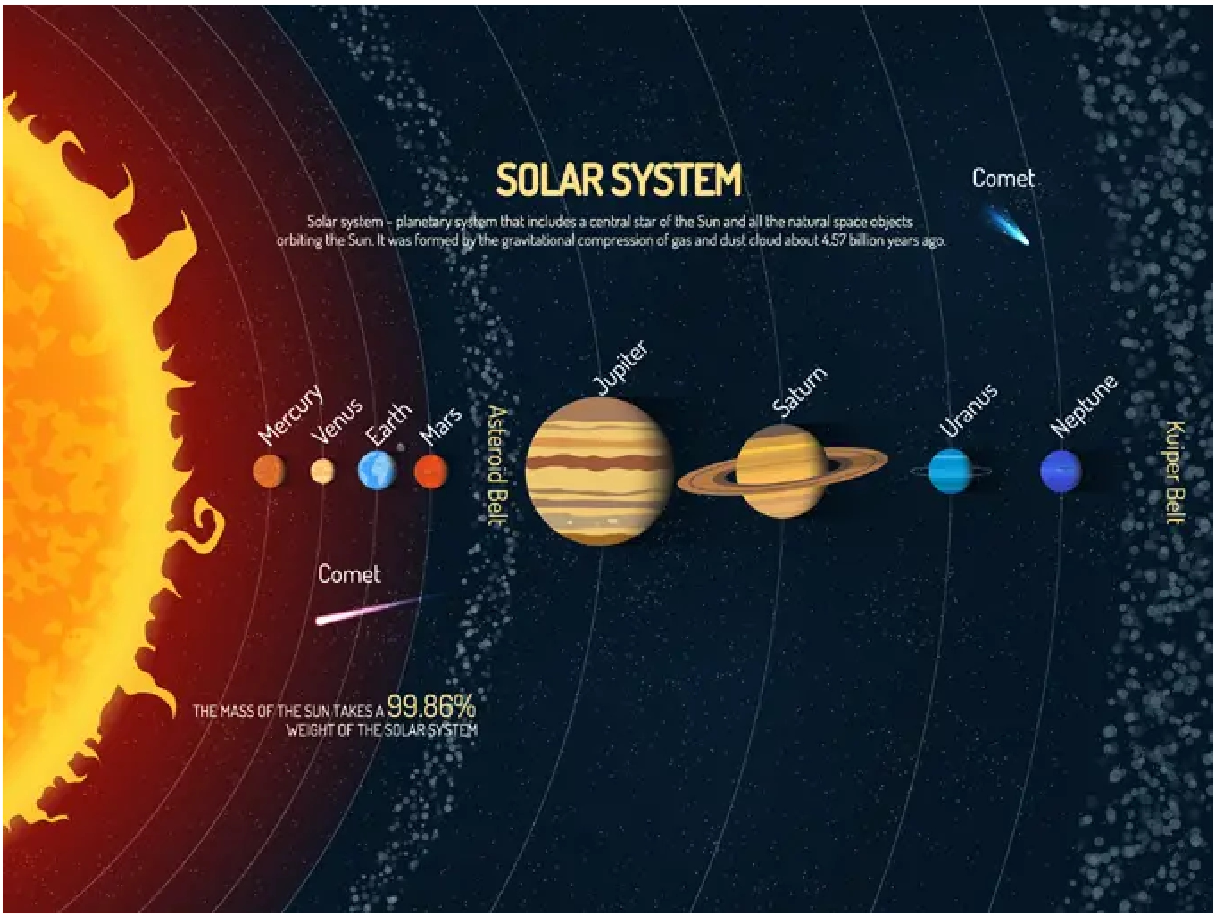

Figure 7: Diagram of the solar system with the planets aligned as is being assumed in Part A [4].

results.

By assuming that all of the planets are co-planar the model of the solar system can be assumed to be 2 dimensional. This means when working out velocity vectors for the orbit that the third dimension is not considered. Although the planets are not co-planar most of the orbits of the planets are planar [6] meaning that a 2 dimensional representation of each of the orbits is accurate. This explains why this assumption has little effect on the results as the planets are still in planar orbits. The different planes of each orbit can be seen in Figure 9.

Another assumption is that the centre of the Sun is at the centre of the inertial coordinate system. However, in reality, the planets also pull the Sun as well as the Sun pulling the planets. This means that the focal point for each planet will be slightly different. This will not have a large effect on the orbits as the mass of the Sun is much larger than the mass of any of the planets, meaning the forces exerted on the Sun will be minimal compared to the force exerted by the Sun. This is why the focal point will only change by a small amount, and will likely still be located within the diameter of the Sun.

The final assumption to consider is the apse line of each of the orbits. It is being assumed that all the planets share a common apse line however this is once again not true. The apse line of the orbits is constantly changing meaning it is unlikely that the planets will have a common apse line, especially all at the same time. This different Apse line is clearly seen in Figure 10.

Each of these would need to be considered in spaceflight. For example, if all of the planets were assumed to be co-planar and a probe was to travel along this plane on a path to Jupiter, it would never intersect with Jupiter. This means the probe could end up far away from the planet aimed for. With regards to timing, travelling through space and over such large distances takes a great deal of time. As the planets are not aligned at their periapsis initially, the spaceflight would need to take place at the correct time so that the final planet is at the interception point at the same time as the spacecraft. Another thing to consider during spaceflight is the interactions with surrounding planets, as the mass of the probe is much smaller than the mass of any planet it will experience different gravitational forces from different planets, especially if it gets too close, causing gravitational perturbations to the flight path of the

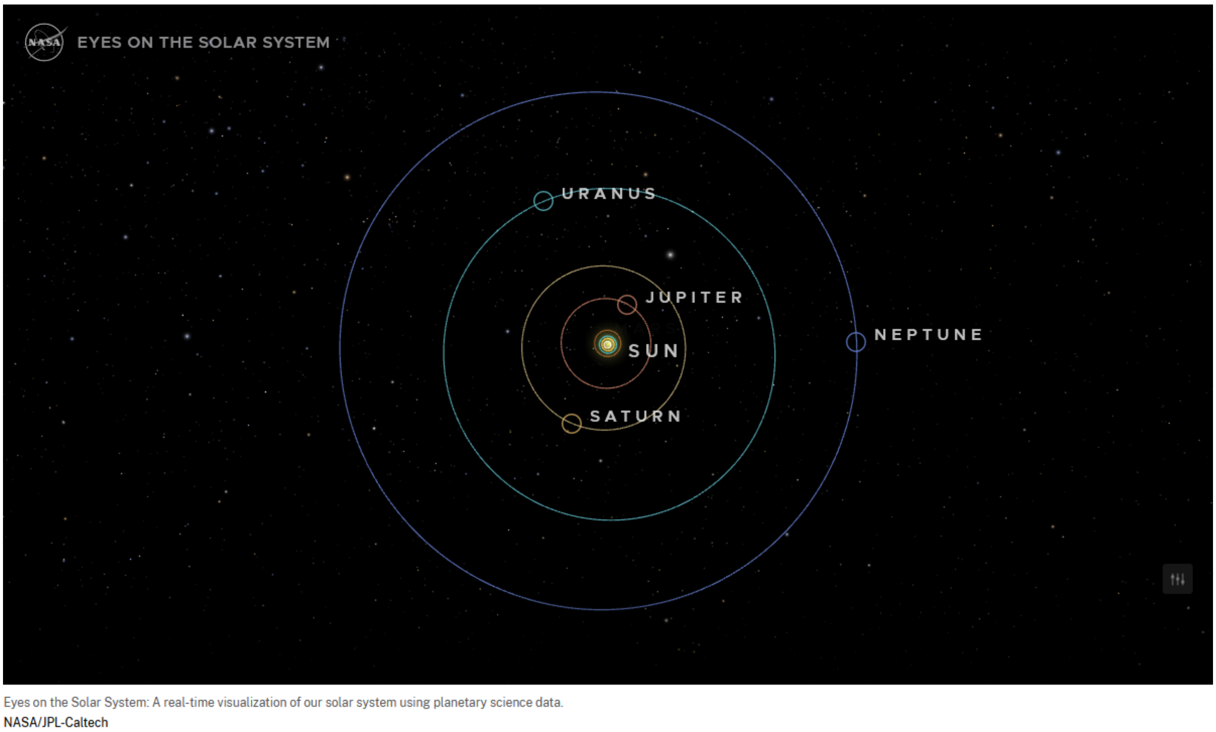

Figure 8: Diagram of the solar system where the planets are in their actual positions, live on 13/04/2024 at 17:00. The inner planets are not visible in this diagram but the idea is shown with the outer planets [5].

spacecraft. So avoiding entering any of the planet's spheres of influence accidentally is very important. Alternatively, these forces can be harnessed to aid the mission and change the speed or direction of the probe. As the planets do not all share an apse line the positioning of the orbits will need to be carefully considered, this is again so that the probe reaches the planet at the correct point in space.

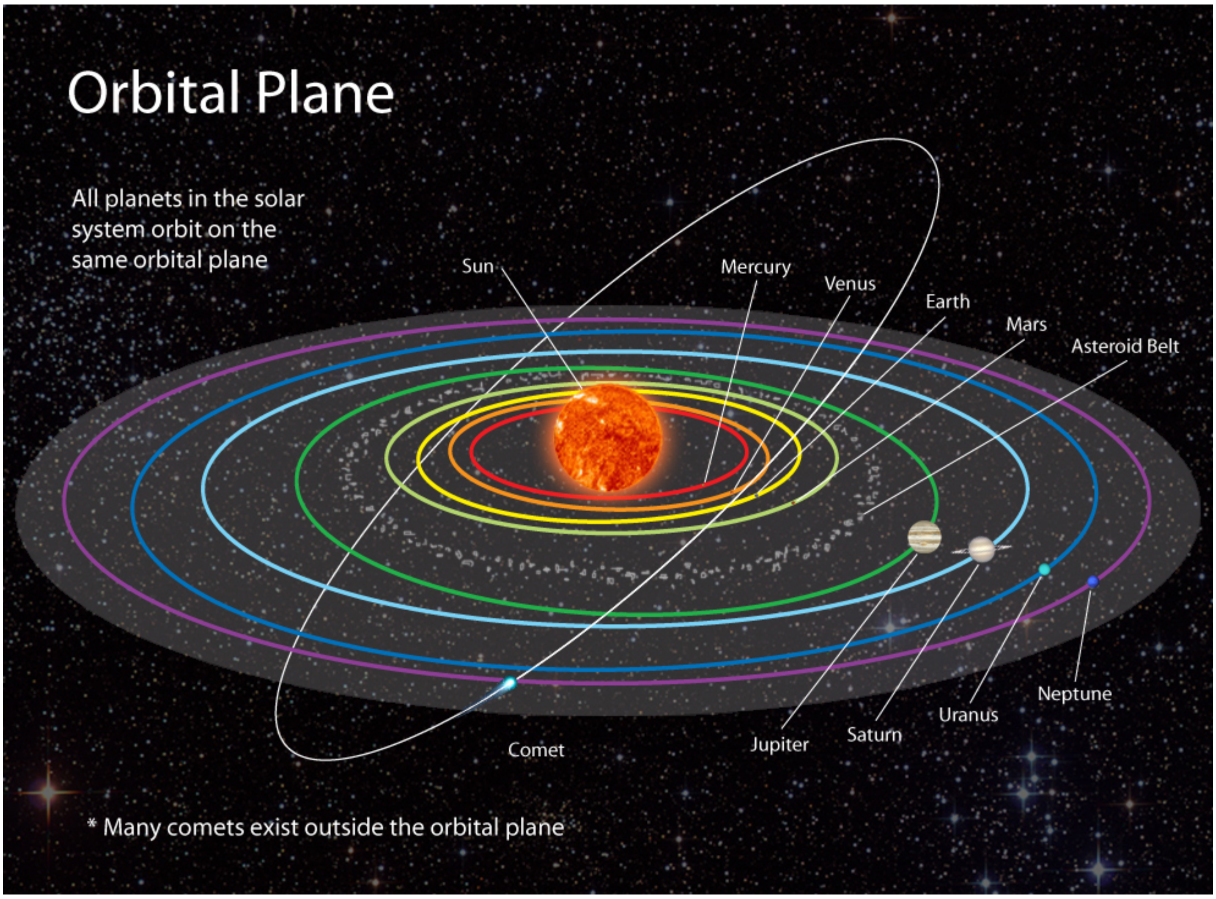

Figure 9: Exaggerated diagram of the solar system and the orbital plane of each planet [6].

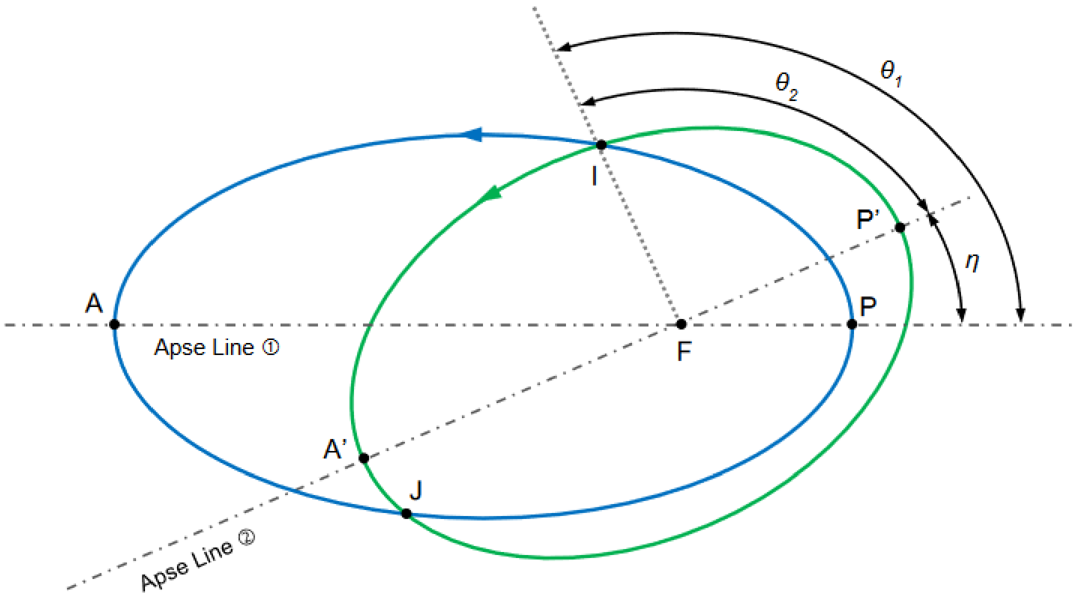

Figure 10: Diagram of two separate orbits with two different apse lines. The apse lines are at an angle of η to each other. This is a representation of the different apse lines, for the planetary orbits it is unlikely that they will cross over live these [2].

2 Part B - Mission to Europa (Moon of Jupiter)

2.1 Launch

The rocket will be launched from the Kennedy Space Center. It is on the east coast the rocket will take advantage of Earth's west-to-east rotation. It is also close to the equator so the rocket can use this Earth's rotation velocity better. The launch will send the spacecraft into a 300km circular parking orbit around the Earth and change the plane of the orbit so it is in-plane with Venus. The launch will have to overcome Earth's gravity by reaching a very high speed in a very short time window without destroying the spacecraft and avoiding space debris.

2.2 Single vs Multi-Stage launch

Multi-stage launches generally allow a higher burnout velocity of the rocket. This can be seen in Figure 11, which shows that a 5-stage rocket has a higher burnout velocity than any fewer stages. This can be subjective depending on the mass ratios between each of the different stages. Another consideration is the complexity of dropping mass from the rocket. Each time the structures are removed from the rocket there is a change of failure, by increasing the number of stages the change of failure increases. The benefit of increasing the number of stages also decreases as shown in Figure 11.

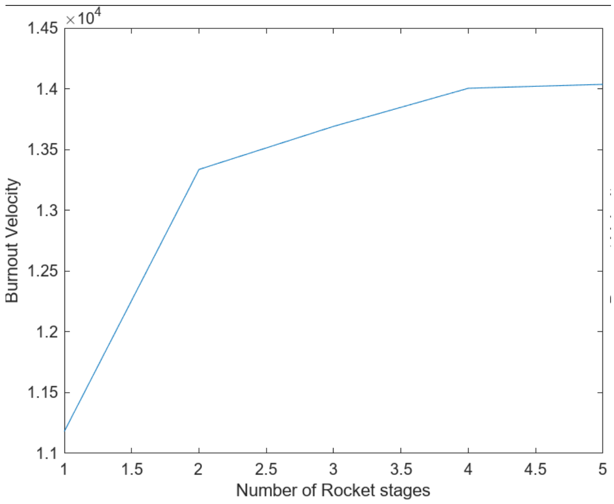

Figure 11: The effect that having more stages of launch for a rocket has on the final burnout velocity of the rocket.

Used for these calculations is the escape velocity (Equation 15) of Earth, which is calculated in Equation 16. This is the required burnout velocity with the lowest weight possible. This does not consider atmospheric resistance which would increase the velocity required.

$$u_{esc} = \sqrt{\frac{2\mu}{r}} \tag{15}$$

$$u_{esc} = \sqrt{\frac{2 \times 398600}{6378}} = 11.18km/s \tag{16}$$

For a 1-stage rocket with a structural mass of 2000kg and a 1500kg payload to reach the escape velocity, a minimum of 32141 kg of propellant would be required. This can be seen in Equation 17. This is with a fuel of Lox-Hydrogen.

$$m_p = (m_s + m_l)\left(1 - e^{\frac{-u_b}{g_0 I_{sp}}}\right) \tag{17}$$

$$m_p = (2000 + 1500)\left(1 - e^{\frac{-11180}{9.81 \times 455}}\right) \tag{18}$$

To compare this with a multistage rocket the same amount of fuel has been used to estimate the burnout velocity.

2.3 Trajectory

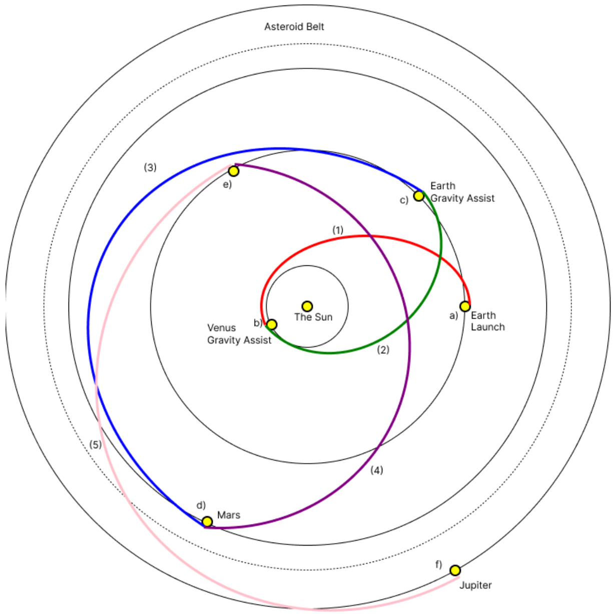

Figure 12: Proposed trajectory to use to get between the planets and reach Jupiter. There will be 6 manoeuvres performed between the planets that are shown in this figure. Starting with a launch from Earth, then a gravity assist (GA) at Venus followed by a GA at Earth, to propel the spacecraft to Mars. Another GA at Mars and a final GA at Earth to send the spacecraft to Jupiter then a final manoeuvre to slow the spacecraft down at Jupiter to enter an orbit around Jupiter. This diagram simplifies all of the planetary orbits to be represented by circular orbits, however, this is not the case in reality. The planetary positions are also not accurate. NOT TO SCALE

Figure 12 shows the mission plan trajectory as follows:

1. Impulse manoeuvre to overcome Earth's gravity, and exit its sphere of influence (SOI) using a hyperbolic escape orbit relative to Earth with a high enough escape velocity and in the correct direction to enter an eccentric orbit around the Sun to transfer to Venus. It requires less energy to Hohmann transfer directly to Venus than Mars therefore travelling to Venus over Mars using an impulse manoeuvre is more fuel-efficient.

2. Perform a gravity assist at Venus to redirect back towards Earth and increase the velocity of the spacecraft.

3. Perform a gravity assist at Earth to propel the spacecraft on an orbit to Mars.

4. Use a gravity assist at Mars to further increase the spacecraft velocity and get back to Earth again.

5. Perform another gravity assist at Earth to again increase velocity and therefore orbit eccentricity so it is redirected on an orbit around the Sun towards Jupiter. This manoeuvre will put the spacecraft on a Hohmann transfer from Earth. This is a very fuel-efficient transfer type but it is a long manoeuvre however, since the spacecraft is not manned the travel time is less important.

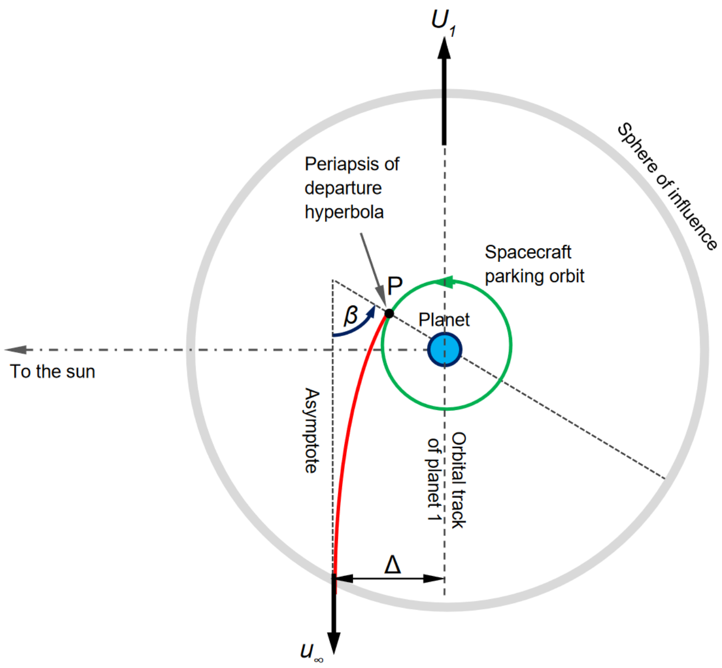

Figure 13: Departure of a spacecraft from an outer planet to an inner planet. The diagram shows the periapsis of the departure hyperbola which is the point Δu must be applied to the spacecraft currently orbiting the departure planet. The excess velocity must be larger than 0 at the edge of the SOI to escape the planet's gravitational pull and enter into an orbit around the Sun. [2]

Figure 13 shows the position that the initial manoeuvre would need to be performed around a circular orbit of Earth to propel the spacecraft on an orbit towards Venus.

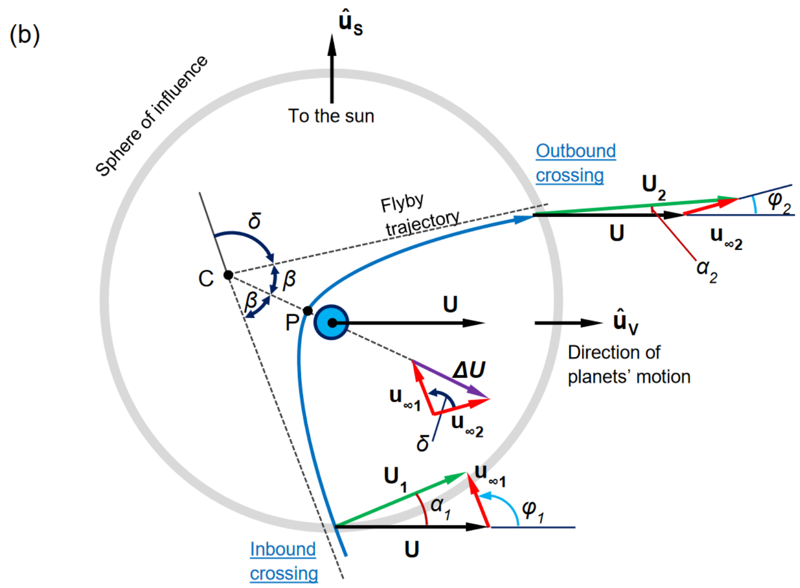

Figure 14: The diagram shows a flyby trajectory around a planet coming into the SOI from behind the planet and using the planet's velocity to increase its speed and change its direction. [2]

The gravity assists are used to increase the speed and change the trajectory of the spacecraft, a diagram of this is shown in Figure 14. The speed of the spacecraft will need to increase by 8.7933 km/s from the speed of Earth at its periapsis, the calculation for this is shown in Appendix A. They will also change the plane of each orbit. For example, Jupiter is not in the ecliptic plane, it is roughly at a

1.3°inclination to the ecliptic plane [7, 8]. If the spacecraft approaches Earth at an angle away from the equator, then a force will act towards the equator to change the plane of the spacecraft's orbit.

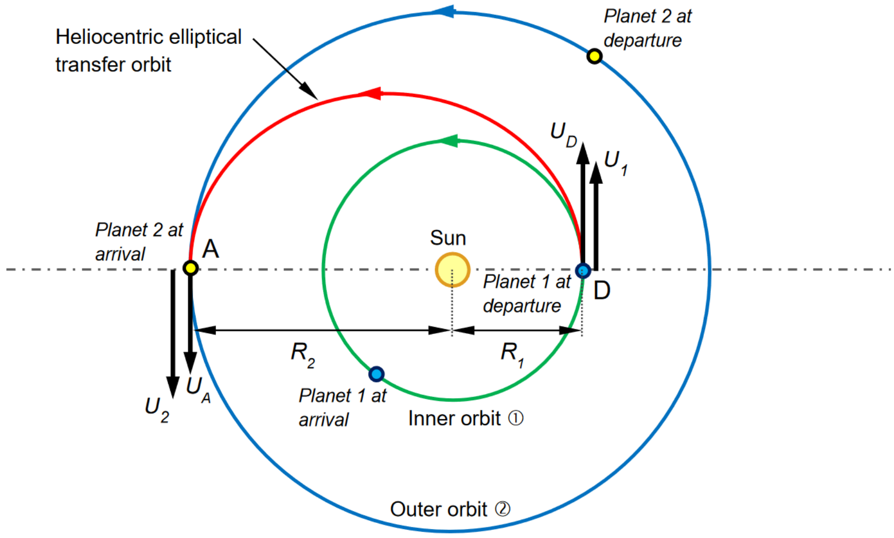

Figure 15: A Hohmann transfer from an inner planet (1) to an outer planet (2). It also shows a diagram of the flight path of the two planets and the spacecraft travelling between the two planets. Planet 1 is Earth and Planet 2 is Jupiter in this instance. This diagram assumes that the orbits of the two planets are co-planar and that both orbits around the Sun are circular. However, this is not the case in reality. [2]

Orbital timing needs to be considered so the spacecraft intercepts all of the planets at the expected point. Figure 15 shows an example of where Planet 2 would need to be at departure from Planet 1 to intersect with the spacecraft at point A. An example calculation has been completed and shown in A. This results in an angle of 97.16 ° from the common apse line which is represented by the dashed line in Figure 15. This is the position Jupiter is required to be at from the final gravity assist at Earth. This is assuming the planets have a common apse line, and periapsis on the same side.

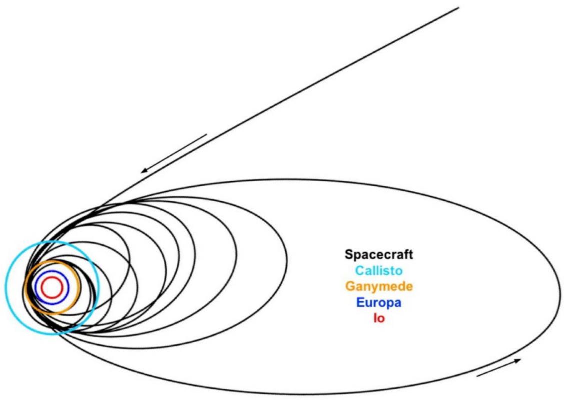

Figure 16: The trajectory of the spacecraft once it has entered the sphere of influence of Jupiter[9]

After reaching Jupiter a similar trajectory to the one shown in Figure 16 will be followed:

1. Enter Jupiter's gravitational influence (SOI) on a hyperbolic approach orbit relative to Jupiter by passing in front of Jupiter to decrease the heliocentric velocity of the spacecraft [8] with a simultaneous impulse manoeuvre to slow the spacecraft down. An example of this manoeuvre is shown in Figure 17 where the spacecraft comes from in front of the planet and uses gravity and an impulse manoeuvre to slow down and enter the green capture orbit.

2. Jupiter has 95 moons [10] so gravity assists at interception with these moons can be used to decrease the size and eccentricity of the orbit so the final orbit will intercept Europa. Figure 16 shows an example using four of these moons.

3. Final manoeuvre at Europa to slow down its velocity and enter an orbit around the moon.

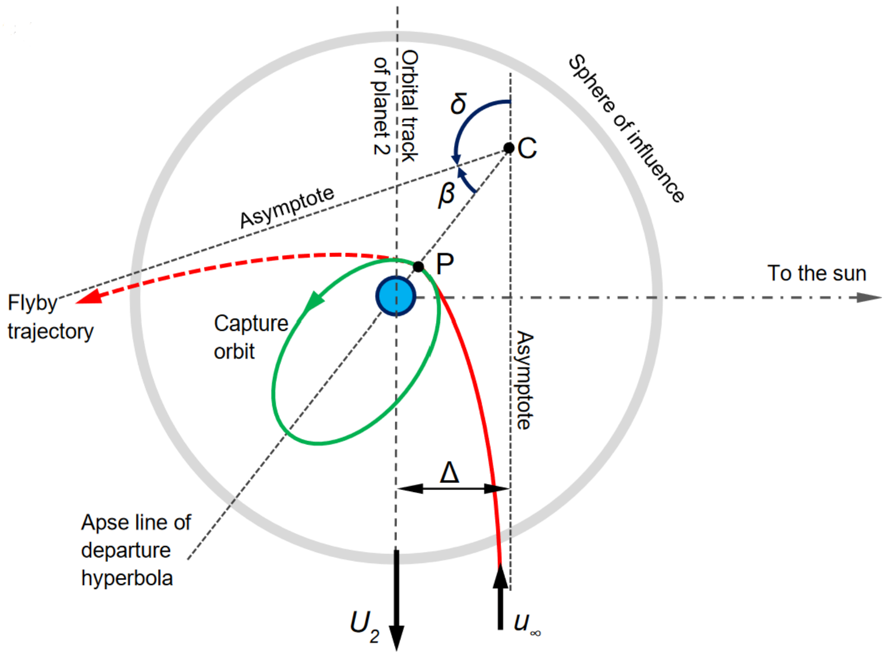

Figure 17: [2]

2.3.1 Challenges

Orbital perturbations should be considered as these can affect the orbit of the spacecraft. For the spacecraft to stay on the correct trajectory corrections may need to be made. A phasing manoeuvre could be implemented to correct this trajectory but they are not practical because of the large Orbital Periods for the planet [7].

Between Earth and Jupiter, there is the asteroid belt, other planets and space debris. Making sure to avoid all of this debris is vital to this mission as one collision could destroy the spacecraft or change the trajectory of the spacecraft too much that it cannot be recovered. The asteroid belt has a significant concentration of asteroids but it is not considered a danger to spacecraft [11]. A higher risk is posed by performing an Earth flyby as there is lots of orbiting space debris. When considering other planets, Mars is between the planets and depending on how close Saturn is to Jupiter it may have more of an influence on the spacecraft's trajectory.

One thing to consider for the two orbits around the Sun is that Jupiter and Earth do not share the same apse line. This is one feature of both planets that is constantly changing due to the other bodies in the solar system having a gravitational effect on the planets.

Another thing to consider about this transfer is the effect of any mistakes, such as having a slightly wrong flyby angle or manoeuvring at the wrong time resulting in the wrong trajectory or velocity. Correcting manoeuvres will need to be performed to counter any mistakes. This will require more fuel and so much to be taken into account before the mission.

One final consideration is space weather. If solar flares are expected, staying out of range is optimal as within range the electromagnetic radiation could affect the systems on board.

2.4 Spacecraft Requirements

Requirement	Description
Probe Housing	• Carry and protect the probe during travel • Release the probe after landing
Thruster	• Change speed of the spacecraft • Change direction of spacecraft
Oxygen Tank	• Provide oxygen for combustion
Solar Panels	• Supply power to the spacecraft
Positioning sensors	• Give accurate spacecraft location and trajectory • Allow spacecraft to perform corrections
Radiation shield	• House scientific equipment • Protect all components from Jupiter's high radiation [9]
Methods to scan the surface	• Collect important data about Europa's Surface • Pick out a suitable landing site
Heating system	• Keep the spacecraft at a working temperature in the cold around Jovian system [12]
Communications system	• Send all information gained back to Earth
Pointing Control system	• Orientate the spacecraft [13]
Alternate Power source	• Provide power to the lander when there is no sunlight to draw power from

Table 8: Requirements for the spacecraft components and what they will be used for.

2.5 Landing

To select an appropriate landing site on Europa the spacecraft carrying the probe will first orbit the planet to locate an ideal place. Once the ideal location has been identified the spacecraft will decent and land. This process is described in Section 2.5.1.

Some of the requirements for the landing site have been detailed below:

- Most likely area where life signs could be found as the radiation is likely to remove lots of signs [14].
- A large flatter area to give the lander a large area and fewer obstacles to worry about.
- A consideration of thrust for the distance from the equator as a plane change manoeuvre would be needed which is fuel expensive meaning the closer to the equator the better.
- Near a plume as this would provide easy access into the water below the ice [15] due to the thinner ice, but plumes do add extra complexity to the landing as they should be avoided.

2.5.1 Landing Technique

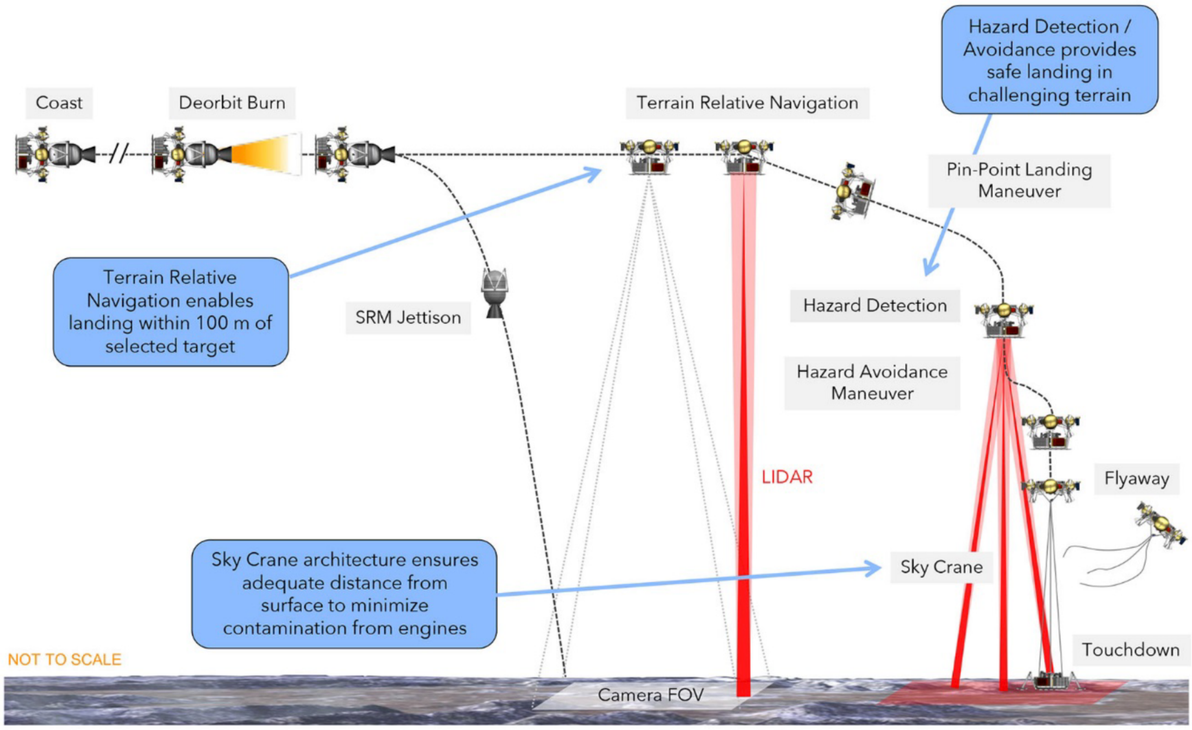

Figure 18: A diagram showing the landing sequence using a sky crane on Europa's surface [9].

An example of the landing sequence to be used can be seen in Figure 18. A sky crane will be used to land the rover. This technique has already been used for two rovers, Curiosity [16] and Perseverance [17]. These rovers both landed on Mars which has an atmosphere less than 1% as dense as Earth's [16]. These are similar conditions to those on Europa and these rovers are a similar weight to the payload being sent [18] in this mission making it a suitable method.

A Appendix 1: Example Calculations

A.1 Assumptions

Orbits are circular - this implies that they share the same apse line Orbits are in the same plane

A.2 Calculations

Orbit	Orbit Periapsis Radius (km)	Orbit Apoapsis Radius (km)	Eccentricity	Specific Angular Momentum (km^2/s)
Earth's Orbit	149600000	149600000	0	4.4555e+09
Jupiter's Orbit	778600000	778600000	0	1.0165e+10
Transfer Orbit	149600000	778600000	0.6777	5.7710e+09

Table 9: All of the information required about each one of the orbits for the calculations being completed in this section.

The Eccentricity and the Specific Angular Momentum displayed in Table 9 have been calculated the same way as described in part A of this report.

A.2.1 Position of Jupiter

To calculate the position of Jupiter first the length of time of the transfer orbit must be found and then compared to the length of time of half an orbit of Jupiter. By doing this the angle that Jupiter will travel through in this time can be calculated.

The time period of an orbit is found using Equation 5. This requires the semi-major axis of the orbits to be known and this can be calculated using Equation 6. As Jupiter is assumed to follow a circular orbit the semi-major axis is the same as the radius of the orbit. This means that the semi-major axis of Jupiter's orbit is 778.6e6 km and the semi-major axis of the transfer orbit is 464.1e6 km.

Using these two equations an orbital period of Jupiter is found to be 3.747e8 s which equates to approximately 4337 days. This means that half of Jupiter's orbit would take around 2169 days. The time of the transfer orbit would take 8.6225e7 s, this is for half of a full orbit along the transfer orbit. By taking the time of the transfer orbit as a percentage of half of Jupiter's orbital period the relative position of Jupiter can be found. This works out to be 0.4602, meaning that at the time of departure from Earth, Jupiter should be 54% along its orbit. This means that at the time of the manoeuvre from Earth, Jupiter should be at a true anomaly, θ from the apse line of around 97.16 °.

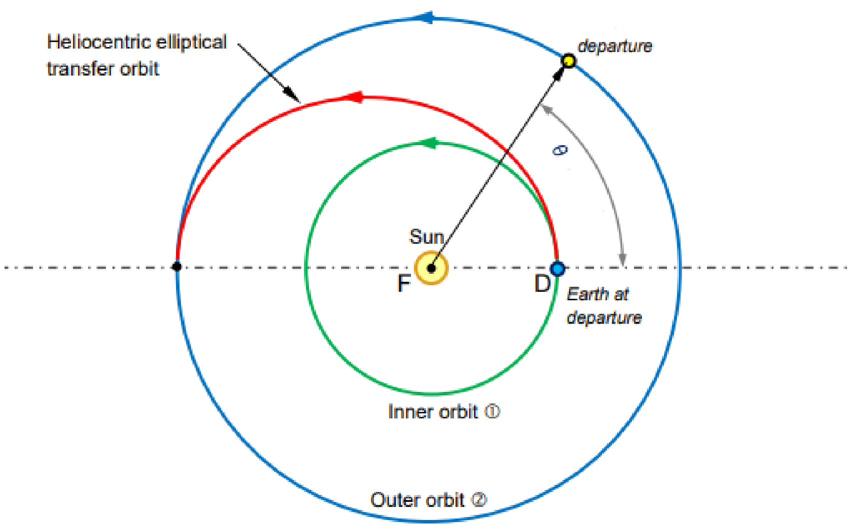

Figure 19: A diagram of the interplanetary Hohmann transfer that is being calculated in this section. The yellow dot shows the position of Jupiter at the time of departure from Earth.[19]

A.2.2 Manoeuvre Velocities

To find out the speed that the spacecraft must be travelling by exiting Earth's sphere of influence the Speed of both of the planets on their orbits need to first be found.

$$U_{planet} = \sqrt{\frac{\mu_{sun}}{R}} \tag{19}$$

Next the speed of departure needs to be calculated. This is the speed that the spacecraft needs to be travelling to be on the heliocentric transfer orbit with a periapsis of Earth's orbit radius and an apoapsis of Jupiter's orbit radius.

$$U_{Departure} = \frac{h_{TransferOrbit}}{R_{Earth}} \tag{20}$$

$$U_{Arrival} = \frac{h_{TransferOrbit}}{R_{Jupiter}} \tag{21}$$

U_{Earth} (km/s)	$U_{Jupiter}$ (km/s)	U_D (km/s)	U_A (km/s)
29.78	13.06	38.58	7.41

Table 10: Each of the velocities at different points in the Hohmann transfer. D represent departure and A represents arrival.

From these values, the excess velocity, u_∞ at the departure point and the impulse velocity at the arrival point can be calculated to determine the total change in velocity required. This value works out to be 8.7933 km/s.

$$u_\infty = U_D - U_{Earth} \tag{22}$$

Using the excess velocity the total impulse velocity from the spacecraft in orbit of Earth can be calculated. This calculation is performed assuming a 300km circular parking orbit of the spacecraft around Earth. For the spacecraft to remain in this orbit it would need to be travelling with a velocity of 7.7258 km/s. This has been calculated using Equation 23.

$$u_{CircularOrbit} = u_c = \sqrt{\frac{\mu}{r}} \tag{23}$$

With the excess velocity known the overall velocity change of the manoeuvre, Δu can be calculated using Equation 24. This results in a total change in velocity of 6.2991 km/s.

$$\Delta u_D = u_c \sqrt{2 + \left(\frac{u_\infty}{u_c}^2\right)} - 1 \tag{24}$$

The point of departure can also be calculated as an angle β shown in Figure 13. This angle was calculated to be 64.17 °.

$$\beta = \cos^{-1}\left(\frac{1}{1 + (r_c u_\infty^2/\mu_E)}\right) \tag{25}$$

The final thing to calculate is the overall ΔU, in other words the total change in velocity that will be required for this Hohmann transfer. This is found to be 14.44 km/s.

$$\Delta U = |U_D - U_{Earth}| + |U_{Jupiter} - U_A| \tag{26}$$

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