MECH2610: Engineering Mechanics

Laboratory Report

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Time: 11:00 - 13:00

UNIVERSITY OF LEEDS

Department Name: Mechanical Engineering
University Name: University of Leeds
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Contents

1 Laboratory Assessment

1.1 Tables

Table 1: Strain Gauge Data

Load (N)	Strain Gauge									Bending Moment (Nm)
	1	2	3	4	5	6	7	8	9	
0	0	0	0	0	0	0	0	0	0	0
100	-113	-78	-77	-15	-14	24	24	50	52	17.5
200	220	-150	-150	-27	-27	48	45	99	100	35
300	-332	-225	-225	-40	-39	73	66	151	148	52.5
400	-444	-301	-302	-52	-52	99	88	202	197	70
500	-552	-374	-375	-66	-65	121	111	251	246	87.5
Vertical Height (mm)	0	8	8	23	23	31.7	31.7	38.1	38.21	
Converted Stresses (MPa)	-38.64	-26.18	-26.25	-4.62	-4.55	8.47	7.77	17.57	17.22	

Table 1: presents the data recorded from the experiment. The Vertical Height is the distance of all of the strain gauges from the top surface of the beam under 0 stress. The converted stress uses the recorded strain (under 500N) and the known elastic modulus of the metal, which is 70GPa. Equation 1

$$\sigma = E\varepsilon \tag{1}$$

1.2 Graphs

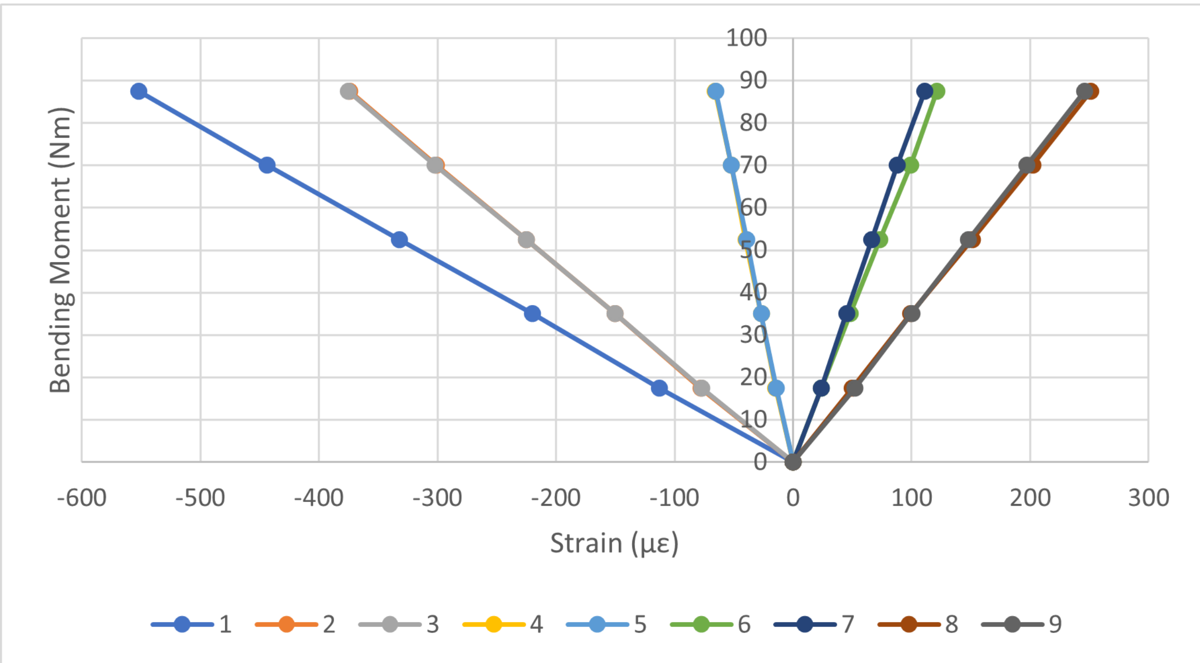


Figure 1: Shows the graph of Bending Moment against Strain for each of the different strain gauges connected to the beam.

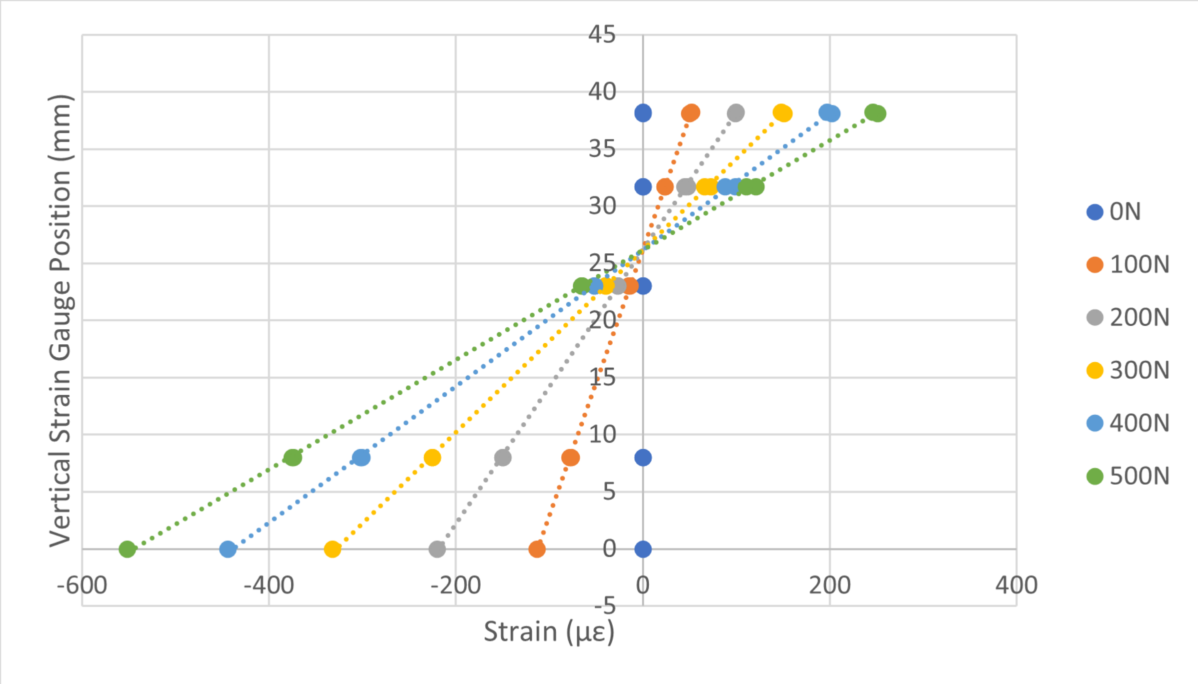


Figure 2: A graph of the vertical strain gauge height on the beam against the strain reading for each of the different loads applied. The Y intercept of this graph represents the point at which there is no strain and so shows the centroid position of the beam as no strain can occur at the pivot point of the beam.

1.3 Theoretical Results

1.3.1 Bending Moment distribution

1.3.2 Maximum Stress

This section shows the calculations used in order to find the maximum theoretical stress on the beam.

Step 1: Find the Centroid position. Firstly it is clear that it is down the centre of the cross-section as it is a symmetrical beam. Next is to find the centroid position in the y axis.

$$\bar{y} = \frac{\sum A\tilde{y}}{\sum A} \tag{2}$$

$A_1 = 6.4 \times 31.7$
$A_2 = 6.4 \times 38.1$
$\tilde{y_1} = 31.7/2$
$\tilde{y_2} = (38.1 - 6.4) + 3.2$

Using equation 2

$$\bar{y} = \frac{A_1\tilde{y_1} + A_2\tilde{y_2}}{A_1 A_2} \tag{3}$$

$$= 26.25mm$$

Step 2: Find the second moment of area of the cross-section. Firstly find the second moment of area of each on the sections of the beam which it has been split into in the diagram above and then add the two values together to find the overall second moment of area.

$$I_z = I_z^1 + I_z^2 \tag{4}$$

$$I_z^1 = I_z'^1 + A_1\hat{y_1}^2 \tag{5}$$

$$I_z'^1 = \frac{bh^3}{12} \tag{6}$$

$I_z^1 = \frac{4\times 31.7^3}{12} + A_1(\tilde{y_1} - \bar{y})^2$

$I_z^1 = 38926mm^4$

Going through the same process for I_z^1.

$I_z^2 = 19084mm^4$

Therefore the overall second moment of area is:

$$I_z = 58009.87mm^4$$

Step 3: Calculate the Theoretical maximum stress on the beam. To do this, Equation 7 is used. In this equation; M is the bending moment, I is the second moment of area, σ is the stress and y is the distance from the centroid. But first all of the terms must be found. I, has just been calculated, σ, is what is being found, y is known and M is what needs calculating next.

$$\frac{M}{I} = \frac{\sigma}{y} \tag{7}$$

In this case Equation 8 is the calculation for bending moment as given in the Laboratory Guidelines. a is given as 350mm and W is the max force, which is 500N.

$$M = \frac{Wa}{2} \tag{8}$$

$M = \frac{500\times 350}{2} = 87500Nmm$

The final value for stress can be found in from Equation **??**

$$\sigma = \frac{My}{I} \tag{9}$$

Substituting in the numbers leaves: $\sigma_{max} = \frac{875000\times 26.25}{58009}$

$\sigma_{max} = 39.59207$

Below is to 2 decimal places the maximum possible stress of the beam by theoretical calculation. $\sigma_{max} = 39.59MPa$

1.4 Experimental Results

1.4.1 Max Stress from experimental y_max

This sections shows one way of calculating the max stress on the beam by using the calculated value for y_{max} obtained through Figure 2. To do this a linear line of best fit is drawn to fit each of the data points for each force recorded. By curve fitting the equation of the lines of best fit can be used to find the y intercept of each of the lines. Then by averaging these values a final experimental value for y_{max} can be found. In this case the value was found as:
$y_{max} = 26.1742$mm The calculation of this value is shown in appendix 1.

Using the same method as in the previous section, including Equations 8 and 9, one experimental stress can be found.
$\sigma_{max} = \frac{875000\times 26.1742}{58009}$

$\sigma_{max} = 39.59456$

Below is to 2 decimal places the maximum possible stress of the beam by theoretical calculation. $\sigma_{max} = 39.59MPa$

1.4.2 Max Stress from Recorded Strain

To find out the maximum stress on the beam, converting the recorded values for strain as shown in Table 1 to values of strain. From these it is possible to see what the maximum stress applied is. The data is shown the Table 1. In the first column the Stress is the highest and as such that is the maximum stress on the beam.
$\sigma_{max} = -38.64MPa$

1.5 Discussion

-what happens with strain gauge values on opposite sides? Should they be identical? if not why not? (2 reasons)
The strain gauges placed at the same heights on the beam both have the same strain reading. As the beam is symmetrical down the centre, this was expected as the force would cause the beam to be displaced by an equal amount on either side of the beam. At each point y is the distance from the central axis and for each strain gauge on either side the size of y is equal for the gauges on the opposite sides. There are however slight deviations in the values and that could be due to a small error in the placement of the strain gauges. i.e. they are not quite level and so there would be a slightly different value. Another reason could be how old the strain gauges are. this could lead to slight differences in the values received from the strain gauges.

-relationship between bending moment and strain at various positions, and why
As the bending moment increases, for each strain gauge, the magnitude of the strain increases. For the strain gauges that are below neutral axis the strain is positive but for the other way the strain is negative. As the strain gauge is further away from the central axis the strain increases. This is because moment is force times by distance and if the distance is increasing the further away from the neutral axis, then the moment will also increase. The graphs are also good a representation of that fact that strain gauges of the same height have the same or a very similar strain.

-how do the different values for strain compare?
The values for strain are all very similar values. Especially the theoretical stress and the stress calculated from the experimental y value. Which were only 0.29% off. The Maximum value of stress was calculated from the strains of the strain gauges. The strain gauge at position 1 was at the maximum distance from the central axis and so would be the point of max stress. The value calculated from this was slightly lower than the theoretical value. This could be due to a number of reasons. Such as the material having a defect at this point hence the strain was altered or there may have been a slight deviation in the Young Modulus value chosen for the steel. Overall however, all of the values are in a similar region with the largest deviation from the theoretical being 2.4%.

2 Buckling Question

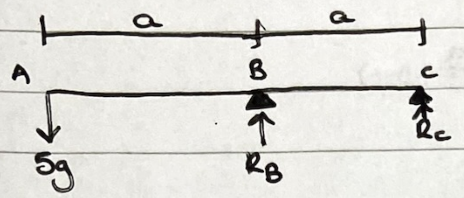
2.1 Types of Bending Failures

This section will discus the difference between failure due to instability, yielding, and plastic collapse. Failure due to instability is where the structural stiffness decreases when the load on the beam is increased. This change in stiffness is because of the large deformation happening throughout the material. When this stiffness reaches 0 the load capacity is reached and so when the stiffness becomes negative the structure will fail. Failure due to yielding occurs when the structure experiences a stress that is higher than its maximum yield stress. It is referred to only when the structure is no longer fit for function. This form of failure does not necessarily mean that the part will have broken into two pieces. Finally, plastic collapse causes immediate failure of the structure because of its inability to arrange stresses to equilibrate the external forces. In other words it causes overall structural instability. It occurs where the moment from the external forces exceed the fully plastic moment of the structural edges.

2.2 Question

Question 2

Split the system into 2 sections. Treat the top beam $\vec{AC}$ as one beam



Not to scale

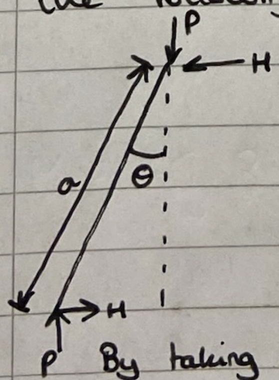
Take moments about C

$$5g \times 2a - R_B \times a = 0$$
$$10ga = R_B a$$
$$\therefore R_B = 10g$$

Balancing forces on the beam

$$5g - R_B - R_c = 0$$
$$5g - 10g = R_c$$
$$-5g = R_c$$

Therefore the force acting on point B by the water bucket is 10g downwards hence this can be equated to P in the following diagram



where H is the support force from the top beam

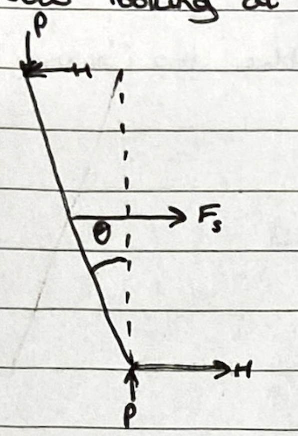
θ can be considered a small angle

$\therefore \cos\theta = 1$

By taking moments

$$Pa\sin\theta = Ha\cos\theta$$
$$Pa\sin\theta = Ha$$
$$\Rightarrow P\sin\theta = H$$

Now looking at the bottom half of the beam



where F_s is the force from the spring

$$F_s = k\frac{a}{2}\sin\theta$$

By taking moments

$$M = Ha\cos\theta + Pa\sin\theta - k\frac{a}{2}\sin\theta\,\frac{a}{2}\cos\theta$$

which simplifies to (small angle theory)

$$M = Ha + Pa\sin\theta - k\frac{a^2}{4}\sin\theta$$

we know $H = P\sin\theta$

$$\therefore \quad M = 2Pa\sin\theta - k\frac{a^2}{4}\sin\theta$$

At the point of collapse $M > 0$

let $M = 0$

$$2Pa\sin\theta = k\frac{a^2}{4}\sin\theta$$

$$2P = k\frac{a}{4}$$

$$P = \frac{ka}{8}$$

From before, $P = 10g$ $a = 355$

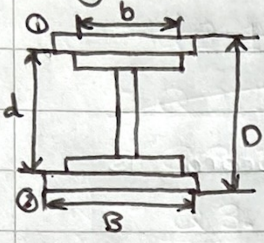
$$\therefore \quad k = \frac{8P}{a} = \frac{80g}{355} = \underline{\underline{2.2\ N/m}}$$

3 Collapse Question

3.1 3) i)

Question 3

System



Not to scale. System symmetrical

Part i) Flanges are at the point of yielding

∴ the bending moment is equal to M_p for one reiforcing plate

$\Rightarrow M_p = \frac{A}{2} h^* \sigma_y$ σ_y is the yield stess

From the diagram A is the area of a reinforcing plate

$\Rightarrow A = B \times \frac{D-d}{2}$

h^* is equal to the distance between the two centroids of the reinforcing plates

$\bar{y}$ is the centroid of the reinforcing plates

$\bar{y}_1 = \frac{D-d}{4}$ from top

$\bar{y}_2 = \frac{D-d}{4}$ from bottom

∴ the difference between the two centroids is h^* which is

$$h^* = D - y_1 - y_2$$

where D is the total length

$$h^* = D - \left(\frac{D-d}{4}\right) - \left(\frac{D-d}{4}\right) = D - \left(\frac{D-d}{2}\right)$$

$$\therefore M_p = \left(\frac{B \times \left(\frac{D-d}{2}\right)}{2}\right) \times \left(D - \left(\frac{D-d}{2}\right)\right) \times \sigma_y$$

For 1 plate

For both plates the area is double

$\therefore$

$$M_p = B\left(\frac{D-d}{2}\right)\left(D - \left(\frac{D-d}{2}\right)\right)\sigma_y$$

The bending moment of the central I beam is not the moment when yielding so

$$M_I = \frac{\sigma_y I}{y_{max}}$$

where: I is the second moment of area of the I beam

y_{max} is the largest distance from the centroid of the I beam

$\therefore$ $I = I$ and

$y_{max} = \frac{d}{2}$

leaving $M_I = \frac{\sigma_y I}{d/2} = \frac{2\sigma_y I}{d}$

$\therefore$ The overall bending moment, M is shown below

$$M = B\left(\frac{D-d}{2}\right)\left(D - \left(\frac{D-d}{2}\right)\right)\sigma_y + \frac{2\sigma_y I}{d}$$

3.2 3) ii)

Question 3

Part 2

At the point wher the two outer surfaces are standing the yield the beam is at the point of first yield.

The stress at the outer most point of the reinforcing plates is equal to the yield stress, σ_y

The stress at the outer surface of the I beam can be found using the ratio of the distance from the centroid.

$\therefore$

$$\sigma_I = \sigma_y \times \frac{d}{D}$$

From this the moment can be found

M_y is the moment at first yield.

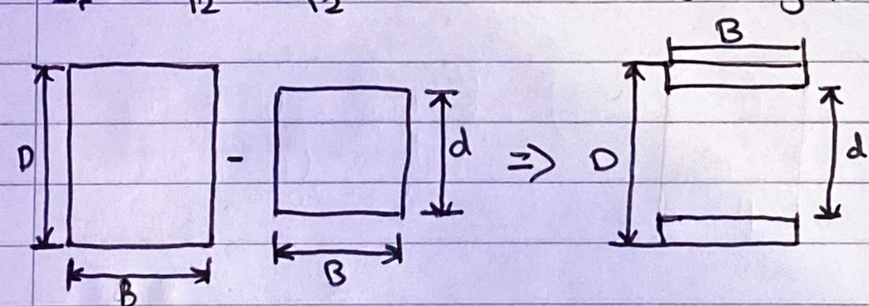
This becomes

$$M_y = \frac{\sigma_y I}{y_{max}}$$

Similar to before the two moments are to be worked out separate and summed

For the reinforcing plates

$$I_r = \frac{BD^3}{12} - \frac{Bd^3}{12}$$ => this is easy to see from this diagram



$I_1 \quad - \quad I_2 \quad = \quad I_r$

y_{max} is different for the two parts

for the I beam $y_{max} = \frac{d}{2}$

for the reinforced plates $y_{max} = D/2$

$$M_{y_I} = \frac{2\sigma I}{d}$$

$$M_{y_r} = \sigma_y \left[\frac{BD^3}{12} - \frac{Bd^3}{12} \right] \frac{2}{D}$$

$$M = \Sigma M_y = M_{y_I} + M_{y_r}$$

$$M = \sigma_y \left[\frac{BD^3}{12} - \frac{Bd^3}{12} \right] \frac{2}{D} + \frac{2\sigma I}{d}$$

From before $\sigma = \sigma_y \frac{d}{D}$

$$M = \sigma_y \left[\frac{BD^3}{12} - \frac{Bd^3}{12} \right] \frac{2}{D} + \frac{2\sigma_y I}{D}$$