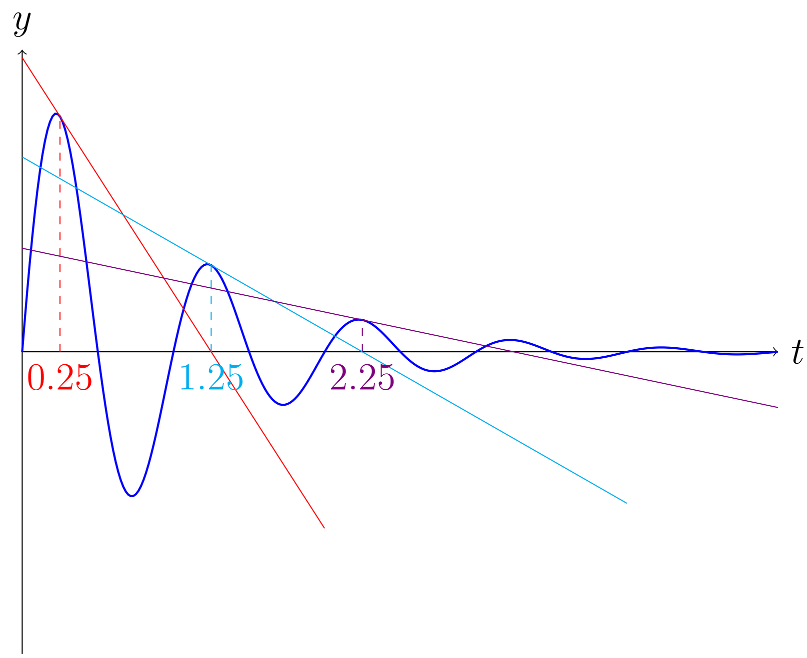
Cautionary example: Let $f(t) = e^{-t}\sin(2\pi t)$. This sort of function occurs in practice as the position of a mass on a spring, with the exponential decay coming from friction or other resistive forces. Clearly $f(t) = 0$ precisely when $\sin(2\pi t) = 0$, so the roots of $f$ are $\frac{n}{2}$ for all integer values of $n$. Starting with $t_0 = 0.25$, perform the Newton-Raphson method. Can you explain what is happening?




$$
\begin{aligned}
t_{n+1} &= t_n - \frac{f(t_n)}{f'(t_n)} \\
&= t_n - \frac{e^{-t_n}\sin(2\pi t_n)}{e^{-t_n}(2\pi\cos(2\pi t_n) - \sin(2\pi t_n))} \\
&= t_n - \frac{\sin(2\pi t_n)}{2\pi\cos(2\pi t_n) - \sin(2\pi t_n)}.
\end{aligned}
$$

If $t_n = n + \frac{1}{4}$, then

$$
\begin{aligned}
t_{n+1} &= n + \frac{1}{4} - \frac{\sin\left(2\pi n + \frac{\pi}{2}\right)}{2\pi\cos\left(2\pi n + \frac{\pi}{2}\right) - \sin\left(2\pi n + \frac{\pi}{2}\right)} \\
&= n + \frac{1}{4} - \frac{\sin\left(\frac{\pi}{2}\right)}{2\pi\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)} \\
&= n + \frac{1}{4} - \frac{1}{0-1} \\
&= (n+1) + \frac{1}{4}.
\end{aligned}
$$

So if we start at $t_0 = 0.25$, then we get $t_n = n + 0.25$ for each $n$, and we never converge to a root! However, if we change our starting point very slightly, the problem disappears. Starting at $t_0 = 0.24$, we converge to the root at 2 in 4 iterations, whereas if we start at $t_0 = 0.26$, we converge to the root at 1 in 3 iterations.