# Functions

**Composition and Product of Functions:**

Let $f(x) = \sin(x)$, $g(x) = e^{2x}$, and $h(x) = (x+1)^2$. Find:

1. $fg(x)$

2. $gf(x)$

3. $f(x)g(x)$

4. $fh(x)$

5. $hf(x)$

6. $f(x)h(x)$

7. $gh(x)$

8. $hg(x)$

9. $g(x)h(x)$

10. $fgh(x)$

11. $f(x)gh(x)$

12. $fg(x)h(x)$

**Domain and Range; Inverse Functions:**

1. Define the domain and range of a function $f$.

2. Define what it means for a function $g$ to be the inverse of a function $f$.

3. Find the inverses of the following functions, specifying their domains:

   (a) $f(x) = 2x - 3$, $x \in \mathbb{R}$.

   (b) $g(x) = \frac{x+1}{x-1}$, $x > 1$.

   (c) $h(t) = e^{4t+3}$, $t \leq 0$.

   (d) $r(t) = \sin(2\pi t)$, $-\frac{1}{4} \leq t < \frac{1}{4}$.

   (e) $s(x) = x^2 - 6x + 5$, $x \leq 3$.

4. The graph of a function $f(x)$ is shown below. Sketch the graph of $f^{-1}(x)$ on the same axes.
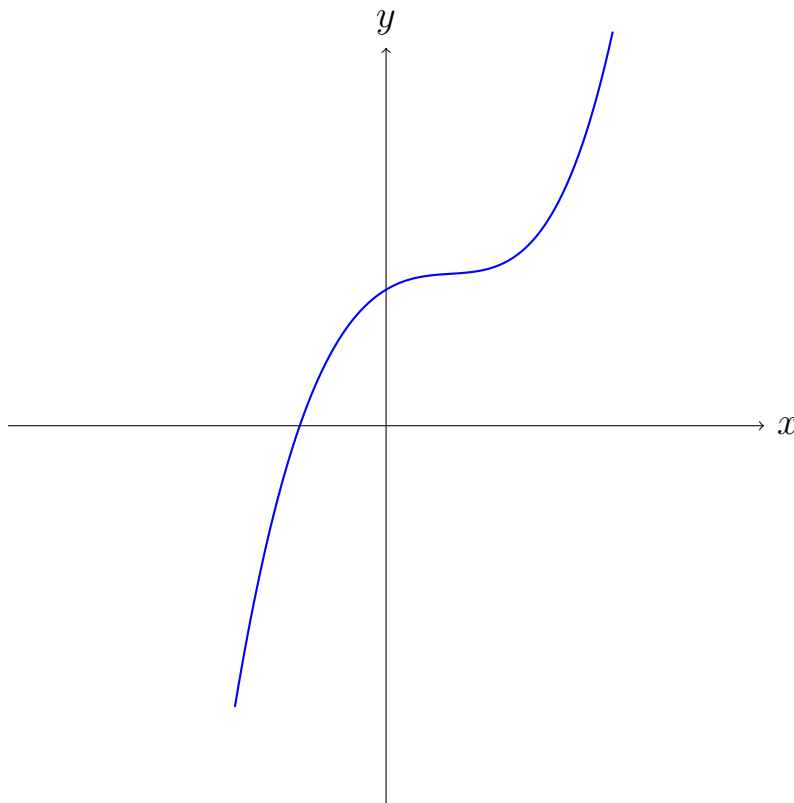
**Transformations of Graphs:**

1. (a) Sketch the graph of $y = \cos(-\theta)$.

   (b) Hence sketch the graph of $y = \cos\left(\frac{\pi}{2} - \theta\right)$.

   (c) Compare this with the graph of $\sin(\theta)$. Explain this result using SOHC-AHTOA.

2. (a) Sketch the graph of $y = x^2$.

   (b) By completing the square, express $x^2 - 4x - 12$ in the form $(x - a)^2 + b$ for some constants $a$ and $b$.

   (c) Hence sketch the graph of $y = x^2 - 4x - 12$.

3. Sketch the graph of $y = \tan(x)$. Hence sketch the graph of $y = \tan\left(x + \frac{\pi}{4}\right) - 1$.

4. Sketch the graphs of $\sin(x)$, $2\sin(x)$, and $\frac{1}{2}\sin(x)$ on the same axes, indicating which is which.

5. Sketch the graphs of $\cos(x)$, $\cos(2x)$, and $\cos\left(\frac{x}{2}\right)$ on the same axes, indicating which is which.