Application: Differentiator Circuits

Objective: To understand how the properties of the Fourier transform allow us to design a circuit that differentiates the input signal with respect to time.

Recall the Fourier transform (ordinary frequency version):

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(t) e^{-2\pi j \xi t} \, \mathrm{d}t,$$

and its inverse transform:

$$f(t) = \int_{-\infty}^{\infty} \hat{f}(\xi) e^{2\pi j \xi t} \, \mathrm{d}\xi.$$

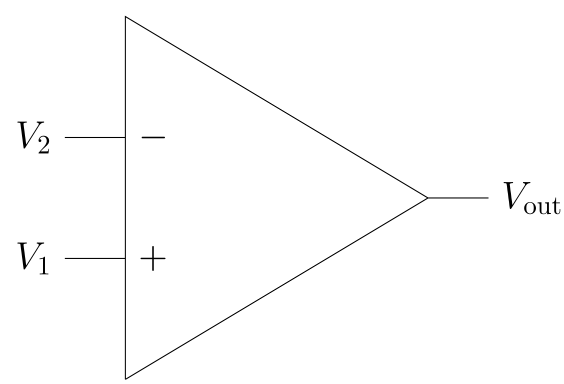
Recall also that

$$\widehat{\frac{\mathrm{d}f}{\mathrm{d}t}}(\xi) = (2\pi j \xi) \hat{f}(\omega).$$

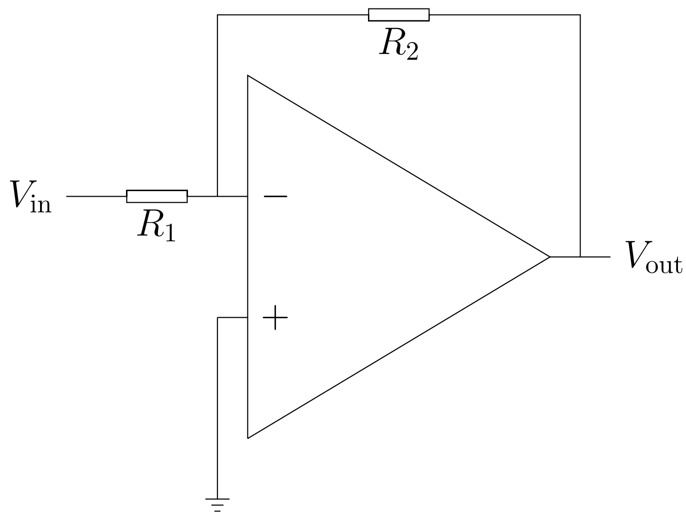
The output voltage V_{out} of an op-amp (shown below) depends on the two input voltages V_1 and V_2 according to

$$V_{\text{out}} = G(V_1 - V_2),$$

where G is some very large number (the gain of the amplifier).



Suppose we connect the output of the op-amp through a voltage divider to the negative input, as shown, and tie the positive input to ground:



We assume that the input impedance of the op-amp is extremely high, so no current flows into or out of the op-amp inputs. Let V_0 denote the voltage at the negative input of the op-amp.

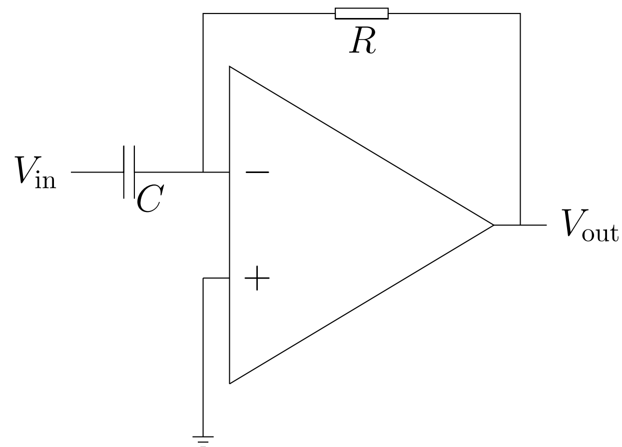
1. By considering current, show that
$$\frac{V_{\text{out}} - V_0}{R_2} = \frac{V_0 - V_{\text{in}}}{R_1}.$$

2. Hence show that
$$V_{\text{out}} = V_0 \left(1 + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1} V_{\text{in}}.$$

3. By considering the effect on V_{out} if $V_0 > 0$ and if $V_0 < 0$ respectively, show that the action of the circuit is to make $V_0 = 0$.

4. Hence conclude that the output of the circuit is given by
$$V_{\text{out}} = -\frac{R_2}{R_1} V_{\text{in}}.$$

5. Now suppose that R_1 is replaced by a capacitor, C. Recall that the impedance of a capacitor when a sinusoidal voltage of frequency ξ is applied is given by
$$\frac{1}{2\pi\xi j C}.$$



Show that if V_{in} is a sinusoid of amplitude A, then the amplitude of V_{out} is given by

$$-2\pi\xi jRCA.$$

6. Suppose that $V_{\text{in}} = f(t)$ is some time-varying function. Show that the Fourier transform of V_{out} is given by

$$-2\pi\xi jRC\hat{f}(\omega).$$

7. Hence conclude that the (time-domain) output of the circuit is

$$V_{\text{out}}(t) = -RC\frac{\mathrm{d}V_{\text{in}}}{\mathrm{d}t}.$$

8. Suppose instead that we leave R_1 as a resistor and replace instead R_2 by the capacitor C, as shown below. Suppose the circuit is switched on at time 0, and before that has 0V input and output. By carrying out a similar frequency-domain analysis, show that the output of the circuit is now given by

$$V_{\text{out}}(t) = -\frac{1}{RC}\int_0^t V_{\text{in}}(\tau)\,\mathrm{d}\tau,$$

where τ is a dummy variable of integration.