Sequences and Series

1. Write down the binomial expansion of $(a + 1)^n$ for positive integer n. Hence show that
$$\sum_{i=0}^{n} \binom{n}{i} = 2^n.$$

2. Find the coefficient of $\sqrt{5}$ in $(3 - \sqrt{5})^4$.

3. The coefficient of x^3 in $(2x - 3p)^9$ is $\frac{21}{2}$. Find the value of p.

4. The coefficient of t^8 in $(t + 2)^n$ is 18. Find the value of n.

5. In a geometric sequence, the first term is 1, the third term is the sum of the first two terms, and the sum to infinity exists. Find the sum to infinity of the sequence.

6. In a geometric sequence, the sum to infinity is k times the first term. Find the common ratio, in terms of k.

7. There is a geometric progression $a_0, a_1, \ldots$ such that, in the standard 'A' series of paper sizes, An paper is a rectangle of length a_n and width a_{n+1}. The area of An paper is twice the area of A$(n+1)$ paper (because two sheets of A$(n+1)$ paper can be placed side-by-side to form one sheet of An paper). The area of an A0 sheet of paper is 1m^2. Find the length and width of an A0 piece of paper.

8. An arithmetic series has n terms, with first term a, common difference d, and sum S. A second arithmetic series also has n terms, with common difference $d - 2$ and sum S. What is the first term of this second series?