Completing the Square "Cheat Sheet"

A step-by-step guide to solving quadratic equations by completing the square, with a running example of $-7x^2 + 7x + 42 = 0$:

1. **Step-by-step instructions:**

2. If the coefficient on the x^2-term is not 1, divide through by this coefficient (including the sign)

3. Take a to be half of the x-coefficient (including the sign)

4. Write out the expansion of $(x+a)^2 = x^2 + 2ax + a^2$ with your particular value of a

5. Subtract a^2 to be left with the first two terms of the original equation

6. Add the constant term from the original equation, so that your new expression is equal to the original one

7. Move the constant terms across to the other side of the equation and simplify

8. Square root both sides, remembering plus or minus

9. Move the remaining constant term to the other side of the equation to leave x isolated on the left

10. Evaluate the two possibilities of the plus or minus to get the roots

1. **Running example:** $-7x^2 + 7x + 42 = 0$

2. $x^2 - x - 6 = 0$

3. Take $a = -\frac{1}{2}$

4. $\left(x - \frac{1}{2}\right)^2 = x^2 - x + \frac{1}{4}$

5. $\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} = x^2 - x$

6. $\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6 = x^2 - x - 6 = 0$

7. $\left(x - \frac{1}{2}\right)^2 = 6 + \frac{1}{4} = \frac{24}{4} + \frac{1}{4} = \frac{25}{4}$

8. $x - \frac{1}{2} = \pm\sqrt{\frac{25}{4}} = \frac{\pm\sqrt{25}}{\sqrt{4}} = \frac{\pm 5}{2}$

9. $x = \frac{1}{2} \pm \frac{5}{2}$

10. $x = \frac{6}{2}$ or $\frac{-4}{2} = 3$ or -2