Volumes of Revolution

Theory: Volumes of Revolution:

Let $y = f(x)$ be a curve in the (x, y)-plane, from $x = a$ to $x = b$. Rotate this curve around the x-axis to form a solid shape. Consider moving along the x-axis; how does the amount of volume to your left change as x increases?

Show that between x and $x + \delta x$, the change in volume is approximately $\pi f(x)^2 \delta x$.

Hence show that the rate-of-change of volume is given by

$$\frac{\mathrm{d}V}{\mathrm{d}x} = \pi f(x)^2.$$

Hence show that the total volume is given by

$$\pi \int_a^b f(x)^2 \, \mathrm{d}x.$$

Worked Examples:

By rotating the curve $y = \sqrt{r^2 - x^2}$ for $-r \leq x \leq r$ around the x-axis, derive the well-known formula for the volume of a sphere:

$$V = \frac{4}{3}\pi r^3.$$

Exercises:

1. Rotate the region bounded by $y = \sqrt{x}$, $y = 3$, and the y-axis about the y-axis. Find the area of the resulting solid.
2. Rotate the curve $y = \sin(x)$ about the x-axis, for $0 \le x \le \pi$. Find the volume of the resulting shape.
3. Rotate the curve $x = \sqrt{y}e^{y^2}$ about the y-axis. Find the volume between $y = 1$ and $y = 2$.
4. Rotate the curve $y = \tan(x)$ about the x-axis. Find the volume bounded between $x = 0$ and $x = \frac{\pi}{4}$.