Integration by Change of Variables

1 Standard Functions

1. $$\int (x^7 - 3x^2 + x^4)\, \mathrm{d}x =$$

2. $$\int \left(\frac{1}{5}y^3 - 9y^{100}\right)\, \mathrm{d}y =$$

3. $$\int \sqrt{x}\, \mathrm{d}x =$$

4. $$\int \frac{x^{3/2} - x^7}{x^{1/3}}\, \mathrm{d}x =$$

5. $$\int \sin(3t)\, \mathrm{d}t =$$

6. $$\int 7\cos(-2t)\, \mathrm{d}t =$$

7. $$\int e^{7x}\, \mathrm{d}x =$$

8. $$\int \frac{1}{s}\, \mathrm{d}s =$$

2 Trigonometric Integrals

1. $$\int \cos^2(\theta)\, \mathrm{d}\theta =$$

2. $$\int \tan^2(3\theta)\, \mathrm{d}\theta =$$

3. Find the area under the curve $y = \sin(x)\cos(x)$ between $x = 0$ and $x = \frac{\pi}{2}$.

4. Find the area under the curve $x = \sin(2t)\tan(t)$ between $x = \frac{-\pi}{4}$ and $x = \frac{\pi}{4}$.

3 Integration by Substitution

1. $$\int x\sqrt{2x+1}\,\mathrm{d}x =$$

2. $$\int x\cos(x^2)\,\mathrm{d}x =$$

3. $$\int 4te^{\sqrt{t^2-1}}\,\mathrm{d}t =$$

4. $$\int \frac{r^2-3}{r^3-9r+1}\,\mathrm{d}r =$$

5. $$\int \tan(\theta)\,\mathrm{d}\theta =$$

4 Integration by Parts

1. $$\int y\cos(y)\ \mathrm{d}y =$$

2. $$\int (s^2 - s + 1)e^s\ \mathrm{d}s =$$

3. Find the area beneath the graph of $y = xe^x$ and above the graph of $y = x\sin(x)$, for $2 \leq x \leq 5$.

4. Let
$$I = \int e^x \cos(x)\ \mathrm{d}x.$$
Use integration by parts twice to show that $I = e^x(\sin(x) + \cos(x)) + I$. Hence find I.

5. $$\int \ln(x)\ \mathrm{d}x =$$
Hint: use parts with $u = \ln(x)$; what must $\frac{\mathrm{d}v}{\mathrm{d}x}$ be?