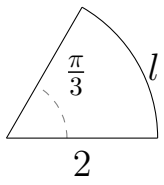
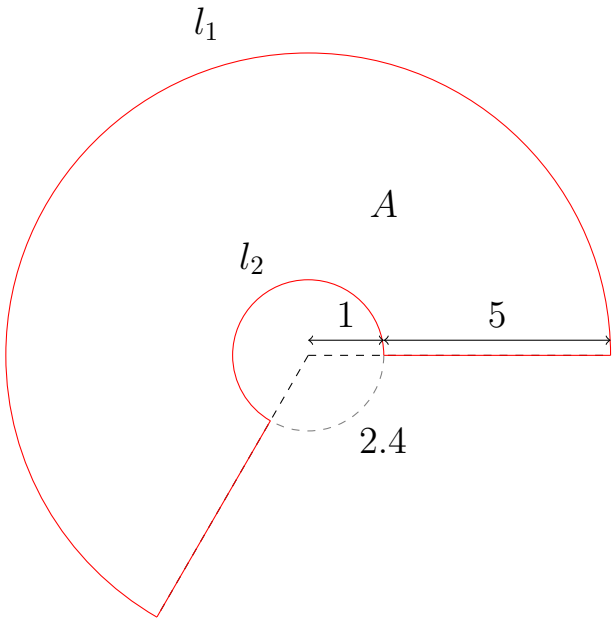
Radians and Trigonometry

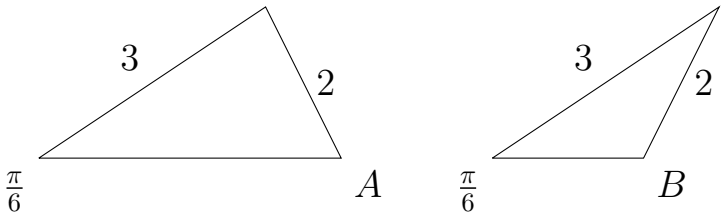
1. Convert the following from radians to degrees:
   (a) 2π
   (b) $\frac{\pi}{3}$
   (c) $\frac{3\pi}{4}$
   (d) $\frac{-7\pi}{6}$
   (e) 2
   (f) -8.32907
2. Convert the following from degrees to radians:
   (a) 270°
   (b) -90°
   (c) 30°
   (d) 44°
3. Calculate the arc length l and area of the sector in the diagram below:



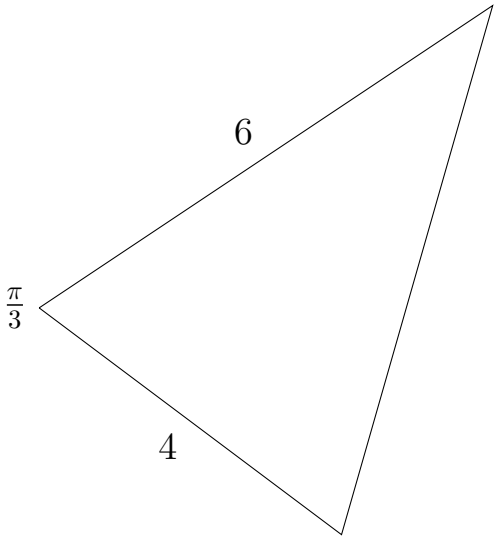
4. Calculate the arc lengths l_1, l_2 and area A between the red curves in the diagram below:



5. Find the angles A and B in the triangles below:



6. Find the missing length in the triangle below, and the area:



7. Solve $\cos^2(t) + \cos(t) = \sin^2(t)$ for $0 \leq t < 2\pi$.

8. Solve $\operatorname{cosec}(x) + a\sin(x) = b$ in terms of a and b, for $0^\circ \leq x < 360^\circ$.

Theory—Sums of Sinusoids of Equal Frequency:

The compound angle formulae split a single sine or cosine into a combination of sines and cosines:

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta),$$

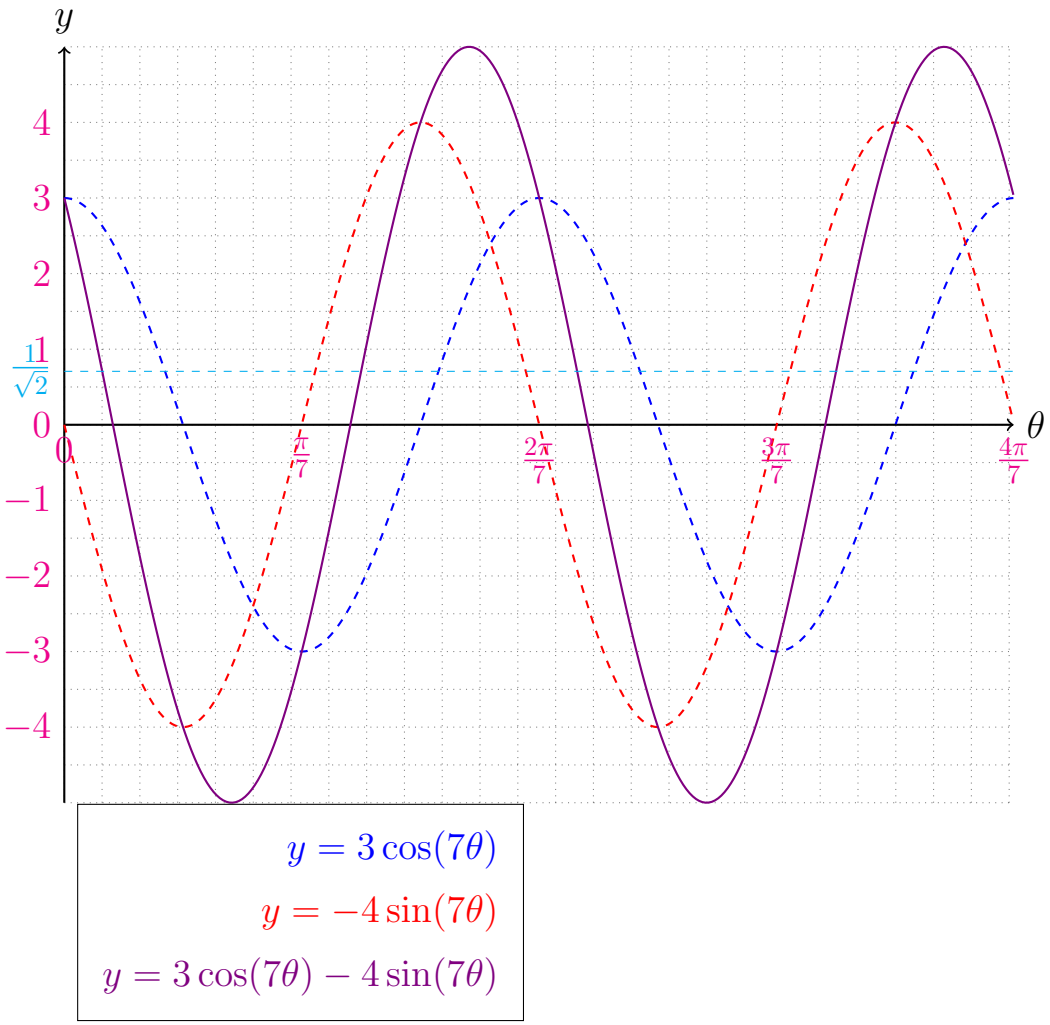
$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta).$$

We can use this in reverse to combine sines and cosines **of the same frequency** into a single sine or cosine.

Write $3\cos(7\theta) - 4\sin(7\theta)$ in the form $R\cos(7\theta+\alpha)$ for some R and α.

Hence solve $3\cos(7\theta) - 4\sin(7\theta) = \frac{1}{\sqrt{2}}$ for $0 \leq \theta < \pi$.

Here we show the graphs of the functions from the last page:



Practice:

1. Express $\sin(3t) - \sqrt{3}\cos(3t)$ in the form $R\sin(3t - \alpha)$.
2. Solve the equation $6\cos(4x) + 8\sin(4x) = -0.5$ for $0 \leq x < \frac{\pi}{2}$.
3. The voltage of mains electricity varies with time by the function $230\sqrt{2}\sin(100\pi t)$. A drive coil on a so-called "3-phase" electric motor receives two alternating voltages, with a time offset between them:

   $$A = 230\sqrt{2}\sin(100\pi t), \qquad B = 230\sqrt{2}\sin\left(100\pi t + \frac{2\pi}{3}\right).$$

   The resulting voltage on the drive coil is $A - B$, the difference between these two voltages. Express the resulting voltage on the drive coil as a single sine function of time; *i.e.*, write $A - B$ in the form $R\sin(100\pi t + \alpha)$. Hence suggest why for high-power applications 3-phase motors are used instead of single phase motors (with just one mains voltage applied).
4. Sound is a pressure wave in the air; for a single pure note at frequency f and amplitude A, the pressure P_1 at a point in the air varies with time according to $P_1 = A\sin(2\pi f t)$. When two sounds are played, with pressure waves P_1 and P_2, the overall effect on air pressure is $P_1 + P_2$.

   Suppose an additional sound is played, with $P_2 = A\sin((2\pi + 1)ft)$. The overall result of these two pressure waves is that the total air pressure is $P_{\text{total}} = P_1 + P_2$. Apply the compound angle formula for sine to P_2 and hence find P_{total}. Describe the sound which results when both these sounds are played together, and hence suggest how noise-cancelling headphones work.