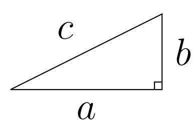
Pythagoras Theorem and Trigonometric Functions

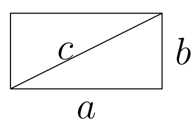
Objective: To be able to use Pythagoras Theorem and SohCahToa.

Warm-up:

Consider the right-angled triangle below, with sides of length a, b, and c.

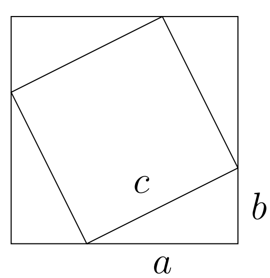


Arrange two identical copies of this triangle as shown:



1. Write an algebraic expression for the area of this rectangle.
2. Hence write an algebraic expression for the area of the triangle.

Now arrange four identical copies of the same triangle as shown:

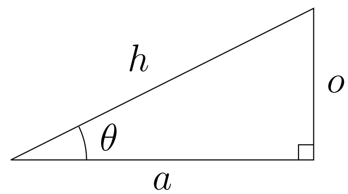


3. Write an algebraic expression for the area of the large square formed from these four triangles.
4. Write an algebraic expression for the area of the smaller, tilted square formed in the middle.
5. Hence write an algebraic equation relating the areas of the two squares and of the triangle.
6. Rearrange this equation to deduce Pythagoras' Theorem: $a^2 + b^2 = c^2$.

7. How many radians are in a circle?
8. How many radians are in $90°$?
9. How many degrees are in $\pi/3$ radians?

Theory - Trigonometry (SohCahToa):

Consider the right-angled triangle below, with an angle θ labelled and sides of lengths o, a, and h (**o**pposite θ, **a**djacent to θ, and the **h**ypotenuse, respectively).



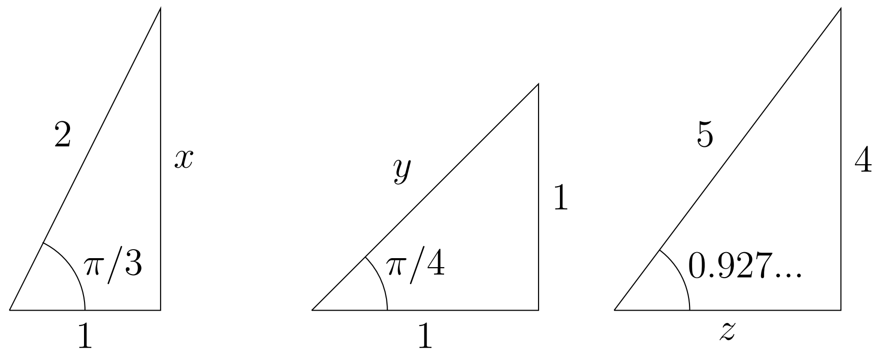
SOH:

CAH:

TOA:

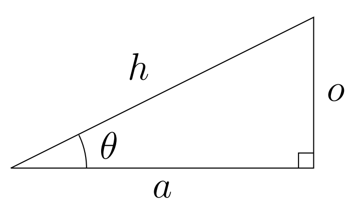
Practice:

Consider the following right-angled triangles (each to scale separately, but not with each other):



1. Calcuate the missing lengths x, y, and z.
2. Find the areas of the 3 triangles.
3. $\sin(\pi/3) =$
4. $\cos(\pi/3) =$
5. $\sin^2(\pi/3) + \cos^2(\pi/3) =$
6. $\tan(\pi/3) =$
7. $\sin(\pi/4) =$
8. $\cos(\pi/4) =$
9. $\tan(\pi/4) =$
10. $\frac{\sin(\pi/4)}{\cos(\pi/4)} =$
11. $\tan(0.927) \approx$

Consider again the general right-angled triangle below.

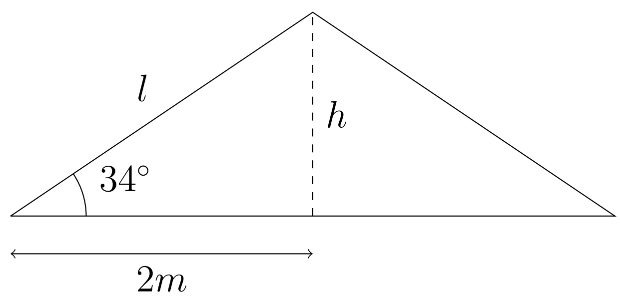


10. Write an expression for $\sin^2(\theta) + \cos^2(\theta)$ in terms of o, a, and h.
11. Simplify this expression, using Pythagoras' Theorem. Compare with question 5.
12. Express $\frac{\sin(\theta)}{\cos(\theta)}$ in terms of o, a, and h. Compare with question 10.

Application - Angles of repose:

When a granular material such as sand is poured onto a level surface, it forms a conical pile. The angle the edge of this pile makes with the ground is called the **angle of repose** of the material. The angle of repose is a property of the material, and tends to be the same in different piles of the same material, even if the piles have very different sizes.

Dry sand typically has an angle of repose of around $34°$ (according to Wikipedia!). If a pile of dry sand has base of radius $2m$, what is its height h? What is the length l of the sloping side? A cross-section of the sand pile is shown below.



Key Points to Remember:

1. In a right-angled triangle with sides of length a, b, and c, where c is the **hypotenuse** (the longest side), **Pythagoras' Theorem** states
$$a^2 + b^2 = c^2$$

2. If θ is an angle in a right-angled triangle (θ should not be the right angle itself), and the sides are o (**opposite** θ), a (**adjacent** to θ), and h (the **hypotenuse**), then the trigonometric functions **sine**, **cosine**, and **tangent**, are defined by **SohCahToa**:
$$\sin(\theta) = \frac{o}{h} \qquad \cos(\theta) = \frac{a}{h} \qquad \tan(\theta) = \frac{o}{a}$$

3. For any angle θ,
$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

4. For any angle θ,
$$\sin^2(\theta) + \cos^2(\theta) = 1$$