Solving Quadratic Equations

Objective: To be able to solve quadratic equations.

Recap of previous material:

1. Factorise $x^2 - 6x + 8$.
2. Hence write down the two roots of $x^2 - 6x + 8$.
3. Let $f(x) = x^3 - 7x^2 + 14x - 8$. Evaluate $f(1)$. Hence find a such that $f(x) = (x - a)g(x)$ for some g.
4. With the notation as above, find g.
5. Hence factor f completely into the form $f(x) = (x - a)(x - b)(x - c)$.
6. Hence write down all three roots of f.

Warm-up:

1. Solve $3x + 4 = 11x - 8$.
2. Solve $2x^2 + 4x - 3 = 5x^2 + 4x - 6$.
3. Expand $(x-1)^2$. Hence solve $x^2 - 2x + 1 = 0$.
4. Expand $(x-3)^2$. Hence solve $x^2 - 6x + 9 = 0$.
5. Solve $x^2 - 12x + 36 = 0$.
6. Let a be a constant. Solve $x^2 - 2ax + a^2 = 0$.

Theory - Completing the Square:
Solve $x^2 + 2x + 1 = 0$. Now solve $x^2 + 2x - 3 = 0$.

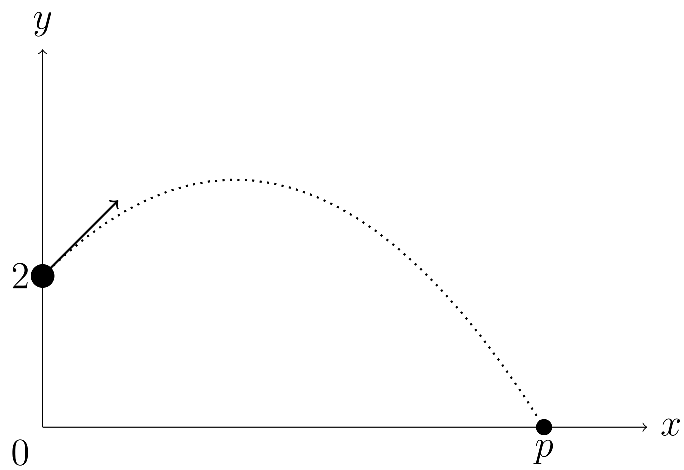
Solve $3x^2 - 18x - 48 = 0$.

Practice:

1. Solve $x^2 + 16x = 36$.
2. Solve $4x^2 - 8x - 7 = 0$.
3. Solve $x^2 + 14x + 50 = 0$.
4. Let b and c be constants. Solve $x^2 + bx + c = 0$ in terms of b and c.
5. Let a, b, and c be constants. Solve $ax^2 + bx + c = 0$ in terms of a, b, and c.
6. Solve $2x^2 + 4x - 3 = 0$ by completing the square. Now solve by the formula you derived in the last question. Compare your answers.

Application - Projectile Motion:

Suppose a particle is thrown from $2m$ above flat ground. Initially, its sideways speed is $5ms^{-1}$, and its vertical velocity is $5ms^{-1}$ upwards. Ignore air resistance, so the only force acting on the particle is gravity, causing it to accelerate downwards at a constant rate of $9.81ms^{-2}$.



Given that the equation of motion is $s = ut + \frac{1}{2}at^2$, where s is the change in position, u is initial velocity, t is time, and a is acceleration, write down an equation for the x-position of the particle at time t and another equation for the y-position of the particle at time t.

Eliminate t from these equations to find an equation linking x and y.

Use this equation to find the position p where the particle lands on the ground.

Key Points to Remember

1. If a is any constant, the unique solution to $x^2 + 2ax + a^2 = 0$ is $x = a$.
2. You can solve a quadratic equation ($e.g.$, $ax^2 + bx + c = 0$ with $a \neq 0$) by completing the square:
   (a) Divide through by a to get
   $$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
   (b) Compare with $\left(x + \frac{b}{a}\right)^2$:
   $$\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
   (c) Rearrange:
   $$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$$
   (d) Take square roots:
   $$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$
   (e) Rearrange to get just x on the left-hand side:
   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
   Note: Completing the square is a <u>process</u> best learned by repeated practice, not by trying to memorise the above instructions!
3. The formula derived above can be used directly instead of the process of completing the square.