Exercise Sheet 6

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Exercise 1

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Be $Sat(\exists(\Phi W \Psi)) =: T$. First, we show that:

$$T \subseteq Sat(\Psi) \cup \{s \in Sat(\Phi) | Post(s) \cap T \neq \emptyset\}$$

For any $s_0 \in T$ it holds that there is a path $\pi = s_0, s_1, s_2, ...$ s.t. $\pi \models \Phi W \Psi$, i.e. one of the following cases must hold:

- For all $i \geq 0$ it holds that $s_i \models \Phi$. This implies that $s_i \in \{s \in Sat(\Phi)|Post(s) \cap T \neq \emptyset\}$ and consequently $s_0 \in \{s \in Sat(\Phi)|Post(s) \cap T \neq \emptyset\}$.
- There is an $j \geq 0$ s.t. for $0 \leq i < j : s_i \models \Phi$ and $s_j \models \Psi$. Consequently either $s_0 \in \{s \in Sat(\Phi)|Post(s) \cap T \neq \emptyset\}$ or $s_0 \in Sat(\Psi)$.

Second, we show that there is no T' with $|T'| > |T|$ such that T' satisfies the condition stated in the exercise. Let us assume that there is such a set T' with $|T'| > |T|$. This implies that there is a state $s* \in T' \setminus T$. This means on of the following cases must hold:

- $s* \in Sat(\Psi)$. This leads to a contradiction since this implies that $s* \models \Psi$ and thus $s* \in Sat(\exists(\Phi W \Psi))$, leading to a contradiction.
- $s* \in \{s \in Sat(\Phi)|Post(s) \cap T \neq \emptyset\}$. This implies that there are $s_1, s_2, ...$ with $s_1 \in Post(s*)$, $s_{i+1} \in Post(s_i), i \geq 1$ and either there is an $j \geq 1$ s.t. for $1 \leq i < j : s_i \models \Phi$ and $s_j \models \Psi$ or $s_l \models \Phi$ for all $l > 0$. Either way, it follows that $s* \in Sat(\exists(\Phi W \Psi))$, leading to a contradiction.

Both cases lead to a contradiction, thus $Sat(\exists(\Phi W \Psi))$ is the maximal set meeting the condition stated in the exercise.

1 Exercise 2

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a) $\Phi = \neg\exists\Big((\neg\exists\bigcirc\neg\exists(a\exists\Box b))\bigcup\underbrace{(\neg true \wedge \neg(\exists\bigcirc\neg\exists(a\bigcup\exists\Box b))}_{= false}\Big) \wedge \underline{\neg\exists\Box\neg\exists\bigcirc\neg\exists(a\bigcup\exists\Box b)}$

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= true

Exercise 5

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a)

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- for all $2 \leq i \leq 4$: $TS_1 \not\equiv TS_i$ because $\{a\}\{a,b\}^\omega \in Traces(TS_i) \setminus Traces(TS_1)$
- $Traces(TS_2) = Traces(TS_3) \cancel{=} \{a\}(\{a,b\}^+\{a\})^*(\{a\}\{b\}^\omega|\emptyset^\omega)$

= $Traces(TS_4)$ $\{a\}\{a,b\}^\omega \in Traces(TS_i)$ for $2 \leq i \leq 4$ −2

b)

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- Because TS_1 is not trace equivalent we can deduce by contraposition that TS_1 is not bisimulation equivalent to any other TS_i.
- $\underline{TS_2 \equiv_{bisim} TS_3}$ by the relation $\mathfrak{R} := \{(s_0, s_0), (s_0, s_4), (s_1, s_1), (s_2, s_2), (s_2, s_3), (s_3, s_5)\}$ −1
- $TS_2 \not\equiv_{bisim} TS_3$ because $TS_2, s_0 \models \exists next((a \wedge b) \to \forall next(\neg a \wedge \neg b))$ = Φ

For $TS_3 \sim TS_4$, correct.

$TS_2 \models \bar{\Phi}$ $TS_3 \not\models \bar{\Phi}$ −1

- Also $TS_2 \not\sim TS_4$. −2