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Exercise Sheet 5

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Exercise 1 (LTL Satisfaction):

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- $TS \not\models \varphi_1$. Be $\pi_1 := (\pi_1)_i$ with $(\pi_1)_i = s_2$ if $2|i$ and else $(\pi_1)_i = s_4$ for $i \geq 0$. Then $\pi_1 \not\models \varphi_1$.
- $TS \models \varphi_2$. From any state except s_4 all transitions lead to either s_4 or to a state in which c holds. Any transition from s_4 leads to a state in which c holds.
- $TS \models \varphi_3$. $\bigcirc\neg c$ holds only for paths with the prefix s_1, s_4 or s_2, s_4. Any transition from s_4 leads to a state in which c holds, thus $\bigcirc\bigcirc c$ holds for any path with one of said prefixes.
- $TS \not\models \varphi_4$ Be $\pi_4 := (\pi_4)_i$ any path over TS with $(\pi_4)_0 = s_2$. Then $\pi_4 \not\models \varphi_4$, since $\{c\} \not\models a$.
- $TS \models \varphi_5$. In all states but s_1 $\cancel{a \vee b}$ $b \vee c$ holds. For any path s_1 can only ever occur as the initial state. Thus for any path starting in s_2 $\cancel{a \vee b}$ always holds. $s_1 \models a$. As s_2 can only occur as an initial state, we can follow that $TS \models \varphi_5$.
- $TS \not\models \varphi_6$ Be π_6 some path over TS starting with $s_1 s_4 s_2 \ldots$. Then $\pi_6 \not\models \bigcirc\bigcirc b$ and $s_1 \not\models b \vee c$. Thus, $\pi_6 \not\models \varphi_6$.
- $TS \not\models \varphi_7$ Be $\pi_7 := (\pi_7)_i$ any path over TS with $(\pi_7)_0 = s_2$. It holds that the initial state s_2 $\not{m}odelsb$. Since there is no previous state which could have "released"the requirement that b has to hold, this implies that $\pi_7 \not\models \varphi_7$.

Exercise 2 (Fairness and LTL):

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a) The following paths satisfy *fair*:

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- $\pi_1 := (\pi_1)_i$ with $(\pi_1)_i = s_0$, if $2|i$, and $(\pi_1)_i = s_1$ else.
- $\pi_{2,j} := (\pi_{2,j})_i$ with $(\pi_{2,j})_i = (\pi_1)_i$, if $i < j$ and $(\pi_{2,j})_i = s_2$ for $i \geq j$ for some $j \in \mathbb{N}, 2|j$.
- $\pi_{3,j} := (\pi_{3,j})_i$ with $(\pi_{3,j})_i = s_3$, if $i < j$ and $(\pi_{2,j})_i = s_4$ for $i = j$ and $(\pi_{2,j})_i = s_5$ for $i > j$ for some $j \in \mathbb{N}_{>0}$.

b)
- $TS \not\models_{fair} \varphi_1$. Since $s_0 \models b$ and $s_1 \models b$, it holds that $\pi_1 \not\models \varphi_1$ ($\Box\neg b$ is never established). 9/9
- $TS \models_{fair} \varphi_2$.
  For π_1 b holds in every state on the path. Thus $\pi_1 \models \varphi_2$.
  For $\pi_{2,j}, j \in \mathbb{N}, 2|j$ it holds that $(\pi_{2,j})_i \models b, i < j$ and $(\pi_{2,j})_i \models \neg b, i \geq j$. Thus $\pi_{2,j} \models \varphi_2$
  For $\pi_{3,j}, j \in \mathbb{N}_{>0}$ it holds that $(\pi_{2,j})_i \models b, i \leq j$ and $(\pi_{2,j})_i \models \neg b, i > j$. Thus $\pi_{3,j} \models \varphi_2$
  qed.
- $TS \not\models_{fair} \varphi_3$. Since $s_0 \models b$ and $s_1 \models b$, it holds that $\pi_1 \not\models \varphi_3$ ($\Box\neg b$ is never established).

c) From part b) we can directly deduce that both $TS \not\models \varphi_1$ and $TS \not\models \varphi_{\cancel{1}\,3}$ due to the fact that the fairness assumption only restricts the set of considered paths. 7/7
Further $TS \models \varphi_2$ does hold. The only initial, infinite path not satisfying the fairness property is $\pi_4 = s_3, s_3, s_3, s_3 \ldots$. Since $s_3 \models b$, it follows that $\pi_4 \models \Box b$ which qua definition implies that $\pi_4 \models \varphi_2$. From part b) we know that all other initial paths of TS model φ_2 and thus it holds that $TS \models \varphi_2$.

Aufgabe 3 (Model Checking LTL property):

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(a)

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$$
\begin{aligned}
\neg\varphi &= \neg\Box(a \to ((\neg b)U(a \wedge b))) \\
&\equiv \neg\neg\diamond\neg(a \to ((\neg b)U(a \wedge b))) \\
&\equiv \diamond\neg(a \to ((\neg b)U(a \wedge b))) \equiv \diamond\neg(\neg a \vee \ldots) \\
&\equiv \text{true}U(a \wedge \neg((\neg b)U(a \wedge b))) := \psi
\end{aligned}
$$

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$$\begin{aligned} cl(\psi) = \{&true, false, a, \neg a, b \neg b, (a \wedge b), \neg(a \wedge b), \\ &\psi_1 := (\neg b)U(a \wedge b), \neg\psi_1, \\ &\psi_2 := \cancel{\neg} a \wedge \neg\psi_1, \neg\psi_2, \\ &\psi, \neg\psi\} \end{aligned}$$

(b) Using notation from lecture i.e. $\neg a, \neg b \in s_1$, and $a, b \in s_6$ follow-up mistake

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	a	b	$(a \wedge b)$	$\psi_1 = (\neg b)U(a \wedge b)$	$\psi_2 = \neg a \wedge \neg\psi_1$	$\psi = \text{true}U\psi_2$
s_1	0	0	0	0	1	1
s_2	0	0	0	1	0	0
s_3	0	0	0	1	0	1
s_4	0	1	0	0	1	1
s_5	1	1	1	1	0	0
s_6	1	1	1	1	0	1

(c) $G_\psi := (Q, \Sigma, \delta, Q_0, \mathfrak{F})$ where

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- $Q := \{s_1, ..., s_6\}$
- $\Sigma := 2^{AP}$
- $Q_0 := \{s_1, s_3, s_4, s_6\}$
- $\mathfrak{F} := \{\{s_1, s_5, s_6\}, \{s_1, s_2, s_4, s_5\}\}$ s_4

and δ given by:

State	Letter	State
s_1	$\emptyset$	$\{s_2\}$
s_2	$\emptyset$	$\{s_2, s_3, s_5, s_6\}$
s_3	$\emptyset$	$\{s_1, s_2, s_3, s_5, s_6\}$
s_4	$\{b\}$	$\{\}$
s_5	$\{a, b\}$	$\{ \}$
s_6	$\{a, b\}$	$\{ \}$

(d)

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Aufgabe 4 (CTL Semantics):

Aufgabe 5 (Equivalence CTL and LTL):