Model Checking
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Exercise Sheet 8

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Exercise 1 (Quatsimulation/2 Quotient):

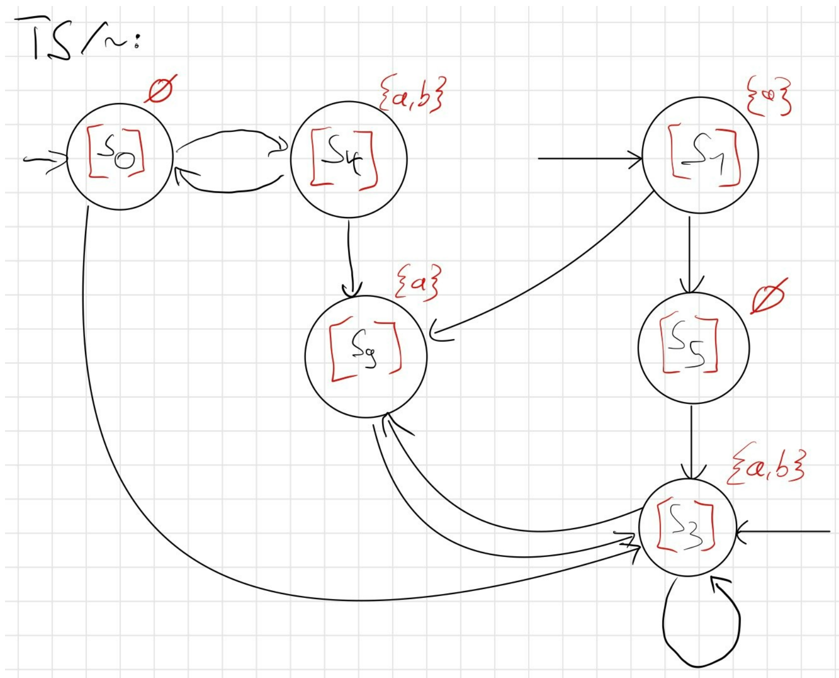
17/20

a)

10/10

The bisimulation equivalence can be given via the following equivalence classes:

$$[s_0]_{\sim_{TS}} = \{s_0\}$$
$$[s_1]_{\sim_{TS}} = \{s_1\}$$
$$[s_4]_{\sim_{TS}} = \{s_4\}$$
$$[s_5]_{\sim_{TS}} = \{s_5\}$$
$$[s_3]_{\sim_{TS}} = \{s_3, s_8, s_2, s_7, s_{10}\}$$
$$[s_9]_{\sim_{TS}} = \{s_9, s_6, s_{11}\}$$



b)

7/10

not a CTL-formula -1,5

$$\Phi_{[s_0]_{\sim_{TS}}} = \neg a \wedge \neg b \wedge \exists(\bigcirc\bigcirc(\neg a \wedge \neg b)$$
$$\Phi_{[s_1]_{\sim_{TS}}} = a \wedge \neg b \wedge \exists(\bigcirc(a \wedge \neg b)$$
$$\Phi_{[s_4]_{\sim_{TS}}} = a \wedge b \wedge \exists(\bigcirc(\neg a \wedge \neg b))$$
$$\Phi_{[s_5]_{\sim_{TS}}} = \neg a \wedge \neg b \wedge \neg\exists(\bigcirc\bigcirc(\neg a \wedge \neg b)$$

-1.5

$$\Phi_{[s_3]_{\sim_{TS}}} = a \wedge b \wedge \exists(\bigcirc(a \wedge b))$$
$$\Phi_{[s_9]_{\sim_{TS}}} = a \wedge \neg b \wedge \forall(\bigcirc(a \wedge b))$$

Exercise 2 (Quotient Algorithm of the night):

25/30

a)

9/10

Initial partition:

$$\Pi_{Old} := \Pi = \{A = \{0, 1, 6, 9\}, B = \{2, 3, 4, 5, 7, 8, 10, 11\}\}$$

First iteration:

$$\begin{aligned}
&Refine(\Pi, A):\\
&\qquad \Pi = \{A = \{0,1,6,9\}, C = \{3,8\}, D = \{2,4,5,7,10,11\}\}\\
&Refine(\Pi, B):\\
&\qquad \Pi = \{A = \{0,1,6,9\}, C = \{3,8\}, E = \{2,4,7,10\}, F = \{5,11\}\}\\
&\quad \Pi_{Old} \neq \Pi
\end{aligned}$$

Second iteration:

$$\begin{aligned}
&\Pi_{Old} := \Pi\\
&Refine(\Pi, A): \text{ no change}\\
&Refine(\Pi, C):\\
&\qquad \Pi = \{G = \{1,6,9\}, H = \{0\}, C = \{3,8\}, E = \{2,4,7,10\}, F = \{5,11\}\}\\
&Refine(\Pi, E):\\
&\qquad \Pi = \{I = \{1,6\}, J = \{9\}, H = \{0\}, C = \{3,8\}, E = \{2,4,7,10\}, F = \{5,11\}\}\\
&Refine(\Pi, F):\\
&\qquad \Pi = \{I = \{1,6\}, J = \{9\}, H = \{0\}, C = \{3,8\}, K = \{2,4,10\}, L = \{7\}, F = \{5,11\}\}\\
&\Pi_{Old} \neq \Pi
\end{aligned}$$

Third iteration:

$$\begin{aligned}
&\Pi_{Old} := \Pi\\
&Refine(\Pi, I): \text{ no change}\\
&Refine(\Pi, J): \text{ no change}\\
&Refine(\Pi, H): \text{ no change}\\
&Refine(\Pi, C): \text{ no change}\\
&Refine(\Pi, K): \Pi = \{M = \{1\}, N = \{6\}, J = \{9\}, H = \{0\},\\
&\qquad\qquad C = \{3,8\}, K = \{2,4,10\}, L = \{7\}, F = \{5,11\}\}\\
&Refine(\Pi, L): \text{ no change}\\
&Refine(\Pi, F): \text{ no change}\\
&\Pi_{Old} \neq \Pi
\end{aligned}$$

Refine(Π, M) splits $\{3,8\} \to \{3\},\{8\}$ -1

In the fourth iteration all blocks remain unchanged. Done.

b)

12/15

Initial partition:

$$\begin{aligned}
&\Pi_{Old} := \{\underline{S}\}\\
&\quad \Pi = \{\underline{A = \{0,1,6,9\}}, B = \{2,3,4,\cancel{5},7,8,10,\cancel{11}\}, \{5,11\}\} \quad -1
\end{aligned}$$

First refinement:

$$\begin{aligned}
\Pi &= Refine(\Pi, A, S \setminus A)\\
&= \{A, \underline{B_1 = \{3,8\}}, B_3 = \{2,4,7,10\}, B_4 = \{5,11\}\}\\
\Pi_{old} &= \{A, \underline{B}\}
\end{aligned}$$

Second refinement:

$$\begin{aligned}
\Pi &= \{A_1 = \{1,6,9\}, A_2 = \{0\}, B_1 = \{3,8\}, B_3 = \{2,4,7,10\}, \underline{B_4 = \{5,11\}}\}\\
\Pi_{old} &= \{A, B_1, \underline{B' = \{2,4,5,7,10,11\}}\}
\end{aligned}$$

Third refinement:

$$\Pi = \{A_{12} = \{9\}, A_{13} = \{1,6\}, A_2 = \{0\},$$
$$B_1 = \{3,8\}, B_{31} = \{2,4,10\}, \underline{B_{33} = \{7\}}, B_4 = \{5,11\}\}$$
$$\Pi_{old} = \{A, B_1, \underline{B_3}, B_4\}$$

Fourth refinement:

$$\Pi = \{\underline{A_{12} = \{9\}}, A_{133} = \{1\}, A_{132} = \{6\}, A_2 = \{0\},$$
$$B_1 = \{3,8\}, B_{31} = \{2,4,10\}, B_{33} = \{7\}, B_4 = \{5,11\}\}$$
$$\Pi_{old} = \{\underline{A}, B_1, B_{31}, B_{33}, B_4\}$$

Fifth refinement:

$$\Pi = \{A_{12} = \{9\}, \underline{A_{133} = \{1\}}, A_{132} = \{6\}, A_2 = \{0\},$$
$$B_1 = \{3,8\}, B_{31} = \{2,4,10\}, B_{33} = \{7\}, B_4 = \{5,11\}\}$$
$$\Pi_{old} = \{A_{12}, \underline{A' = \{1,6,0\}}, B_1, B_{31}, B_{33}, B_4\}$$

Sixth refinement:

$$\Pi = \{A_{12} = \{9\}, A_{133} = \{1\}, \underline{A_{132} = \{6\}}, A_2 = \{0\},$$
$$B_1 = \{3,8\}, B_{31} = \{2,4,10\}, B_{33} = \{7\}, B_4 = \{5,11\}\}$$
$$\Pi_{old} = \{A_{12}, A_{133}, \underline{A'' = \{6,0\}}, B_1, B_{31}, B_{33}, B_4\}$$

Seventh refinement:

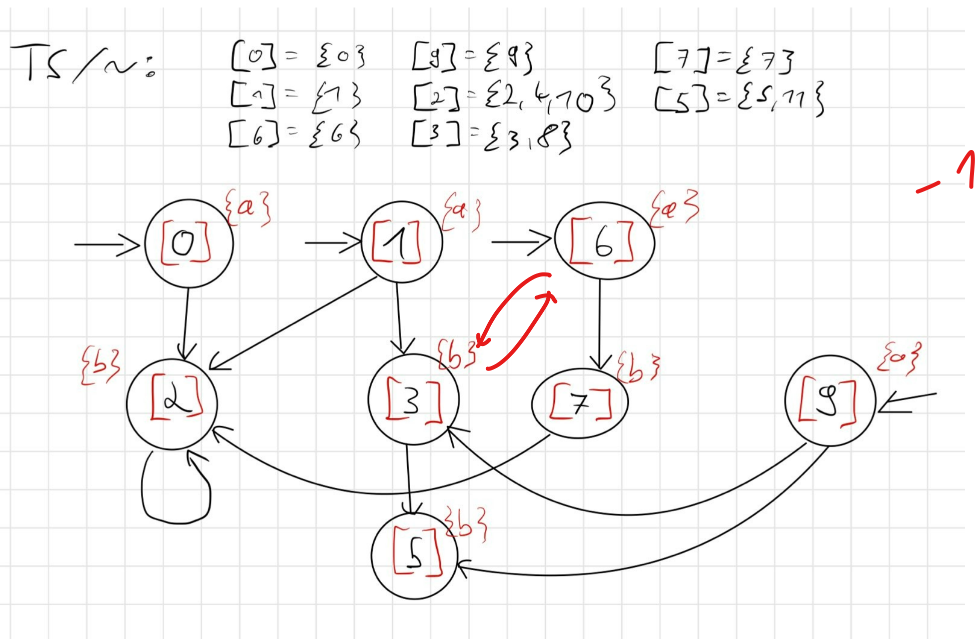
$$\Pi = \{A_{12} = \{9\}, A_{133} = \{1\}, A_{132} = \{6\}, A_2 = \{0\},$$
$$\{3\},\{8\}\; B_1 = \cancel{\{3,8\}}, B_{31} = \{2,4,10\}, B_{33} = \{7\}, B_4 = \{5,11\}\}$$
$$\Pi_{old} = \{A_{12}, A_{133}, A_{132}, A_2, B_1, B_{31}, B_{33}, B_4\}$$

-2

$\Pi_{old} = \Pi$. Done.

c)

4/5



-1

Exercise 3 (Markus Equiva-Lanz):

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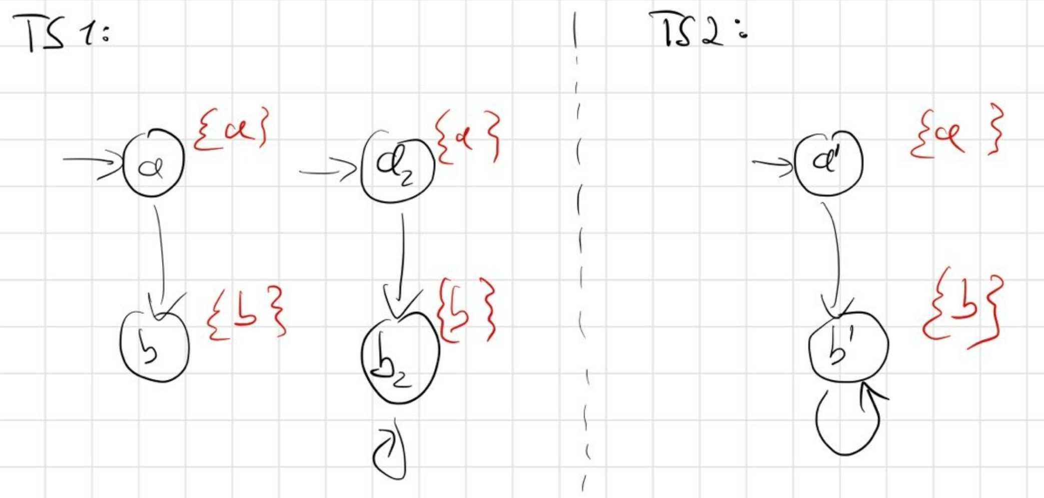
a)

8/8

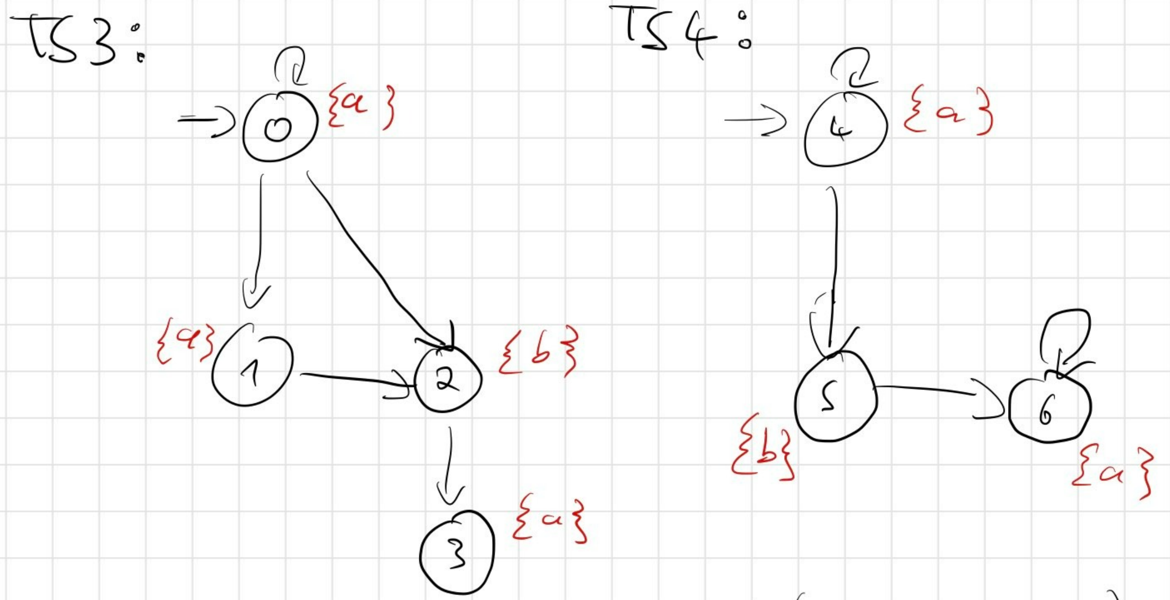


1. Obviously $a' \leq_{TS1 \oplus TS2} a_2$ and $a_2 \leq_{TS1 \oplus TS2} a'$.
Also $a \leq_{TS1 \oplus TS2} a'$ for simulation relation
$R = \{(a, a'), (b, b')\}$.
$\Rightarrow TS1 \preceq TS2$ and $TS2 \preceq TS1 \Rightarrow TS2 \simeq TS1$ ✓

2. Be R a bisimulation for $(TS1, TS2)$
$\Rightarrow (a, a') \in R \Rightarrow (a', a)$ due to simmetry
since $b' \in Post(a') \Rightarrow (b', b) \in R$
since $b' \in Post(b') \Rightarrow (b', x) \in R$ for some $x \in Post(b)$
$Post(b) = \emptyset \Rightarrow$ ↯
Thus TS1 and TS2 are not bisimilar. ✓

b)

8/8



$Traces(TS3) = Traces(TS4) = \mathcal{L}\left(a^\omega + (aa^*b).a^\omega\right)$ ✓

Be R some bisimulation relation for $(TS3, TS4)$

$\Rightarrow (0,4) \in R$. $1 \in Post(0)$ and $L(1) = \{a\}$

$\Rightarrow (1,4) \in R \Rightarrow (4,1) \in R$. $4 \in Post(4) \Rightarrow$

$(4,x) \in R$ for some $x \in Post(1) = \{2\}$

$L(2) = \{b\} \neq \{a\} = L(4)$ ↯ ✓

$\Rightarrow$ TS3 and TS4 are not bisimilar.